Is the IROR a Plausible Approximation of the Profit Rate on Regulating Capital?

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In the analysis of profitability levels and trends, it can be useful to refer to the profit rates obtained by enterprises investing under on-going technical and organizational conditions at a given point in time. In such a perspective, abstraction is made of investments realized during earlier periods and the corresponding profits, as opposed to the determination of profit rates on all vintages in use, as in the average profit rate.

To our knowledge, within the Classical-Marxian literature, attempts have been made to calculate such profit rates within two distinct contexts. A first approach was developed by the authors of this note in the analysis of the long-term historical tendency of the profit rate in the private U.S. economy.[1] In the analysis of the gravitation of profit rates in competition, Anwar Shaikh defended the view that only the profit rates on new investments tend to be equalized, the rates of profit on "regulating capital".[2] In this type of investigation, the main problem is obviously the estimate of the profit rates, on the last vintages of capital, ρ, given the limitation of available data. Which assumptions must be made? Are the results reliable?

The present note is devoted to the methodology put forward by Shaikh, the use of the *Incremental Rate of Return* (IROR), ρ', as approximation of ρ:

$$\rho \approx \rho' \tag{1}$$

The overall idea is that the variation of total profits between two successive periods can be used as estimates of the profits realized on the new investments. Thus, the IROR, the ratio of this variation of total profits to the new investment can be used as estimates of the rate of profit on the new investments.[3]

The contention here is that the assumptions implied in equation 1 are so strong that they basically question the method. The problems follow from the fact that, beside the capability of the newly invested capital to yield additional profits, the variation of total profits also depends on the values the capacity utilization rate, on the variations the capacity utilization rate and of wages for the old vintages, and on the depreciation of these old vintages of fixed capital previously invested. (These four factors correspond to the four terms in the expression of the IROR below in equation 4.)

Justifying the validity of equation 1 in a rather general framework would be difficult. In what follows, we introduce a simple formalism, and we show that, even in this framework, the relevance of equation 1 appears questionable.

We first recall the standard formalism of the profit rate in a simple model, abstracting from the superscript i for distinct industries. Technology is defined by two parameters, labor productivity (Y/L) and the output / capital ratio (Y/K) or, equivalently, $a = Y/K$ and $b = L/K$. The real wage is w. The profit rate, r, can be determined, assuming a normal use of productive capacity:

$$r = \frac{Y - wL}{K} = a - wb$$

1. G. Duménil, D. Lévy, "Post Depression Trends in the Economic Rate of Return for U.S. Manufacturing", *The Review of Economics and Statistics*, LXXII (1990), p. 406-413; and G. Duménil, D. Lévy, "Stylized Facts about Technical Progress since the Civil War: A Vintage Model", *Structural Change and Economic Dynamics*, 5 (1994), p. 1-23.

2. We devoted various studies to the gravitation of prices around prices of production (with average profit rates gravitating around a common value) and checked the theory empirically in G. Duménil, D. Lévy, "The Field of Capital Mobility and the Gravitation of Profit Rates (USA 1948 -2000)", *Review of Radical Political Economy*, 34 (2002), p. 417-436.

3. Our own methodology is described in the papers given in reference above.

The actual capacity utilization rate is u, susceptible of deviating from its "normal" value, $\overline{u} = 1$. The profit rate becomes (assuming that employment is flexible and proportional to output):

$$r = u(a - wb) \tag{2}$$

In the computation of ρ, a vintage model is required. New technology in period t is a_t and b_t, and it is incorporated in the new investment I_t. The rate at which the capital stock is depreciated (or discarded) is δ. One period later, the value of a capital $K_t^0 = I_t$, invested in period t, is reduced to $K_t^1 = I_t(1-\delta)$ as an effect of its depreciation; a new period later, to $K_t^2 = I_t(1-\delta)^2$; and, n periods later, to $K_t^n = I_t(1-\delta)^n$.

When the investment is realized, it is possible to assume that its capacity utilization rate will gravitate around a normal value $\overline{u} = 1$. (If another assumption were made, the value of the investment would be altered.) Making an assumption on the variation of wages is more difficult. The simplest assumption is to use the on-going wage. Thus, the actual last-vintage profit rates, ρ, susceptible of guiding investors (Shaikh's "regulating profit rate") is:

$$\rho_t = a_t - w_t b_t \tag{3}$$

It is unfortunately impossible to compute using only the national account data.

The substitute, the IROR, can be expressed according to its definition:

$$\rho'_t = \frac{P_t - P_{t-1}}{I_t}$$

We first calculate total profits during period t, that is, the sum of profits garnered on all vintages of capital. The conditions, w_t and u_t, prevailing in the period t are used. An additional simplifying assumption must be made concerning the capacity utilization rates on the various vintages of capital in each industry. We assume that capitals are used independently of their age.[4] We use the following notation for the four variables accounting for all vintages (with the on-going values of u_t and w_t): Y_t for output, L_t for employment, W_t for wages, and P_t for profits:

$$P_t = Y_t - W_t = Y_t - w_t L_t$$

To determine Y_t and L_t, sums must be made on all vintages:

$$Y_t = (1-\delta)^t A_t \quad \textit{and} \quad L_t = (1-\delta)^t B_t$$

with:

$$A_t = \sum_{n=0}^{\infty} \frac{K_{t-n}}{(1-\delta)^{t-n}} a_{t-n} \quad \textit{and} \quad B_t = \sum_{n=0}^{\infty} \frac{K_{t-n}}{(1-\delta)^{t-n}} b_{t-n}$$

Profits are:

$$P_t = u_t(1-\delta)^t (A_t - w_t B_t)$$

The variation of profits can, thus, be determined using:

$$A_t = A_{t-1} + \frac{I_t}{(1-\delta)^t} a_t, \quad \textit{and} \quad B_t = B_{t-1} + \frac{I_t}{(1-\delta)^t} b_t$$

4. A declining schedule could be used, but this additional complexity would not add anything to the demonstration.

The IROR, ρ' can now be expressed as a function of ρ. In this calculation, we assume that: (1) between two successive periods, the variations of the two variables, $u_t - u_{t-1}$ and $w_t - w_{t-1}$ are small, and (2) parameter δ is also small. Thus, for industry i (with r denoting the average rate of profit):

$$\rho_t'^i \approx u_t^i \rho_t^i + \frac{K_t^i}{I_t^i}\left(\frac{u_t^i - u_{t-1}^i}{u_t^i} r_t^i - \frac{w_t^i - w_{t-1}^i}{w_t^i}\frac{W_t^i}{K_t^i} - \frac{\delta}{1-\delta} r_t^i\right) \tag{4}$$

The first term shows that, as could be expected, the IROR is linked to the profit rate on the last-vintage capital, after correcting for the capacity utilization rate. The three remaining terms show the respective impacts of the variations of u^i and w^i, and the depreciation of all old capital. Their effect is stronger if the investment is small in comparison to the capital stock (as in K_t^i/I_t^i).

The last step can now be accomplished, inverting the above relationship to determine the expression of ρ_t^i as a function of $\rho_t'^i$ and all other variables:

$$\rho_t^i \approx \left(\rho_t'^i - \frac{K_t^i}{I_t^i}\left(\frac{u_t^i - u_{t-1}^i}{u_t^i} r_t^i - \frac{w_t^i - w_{t-1}^i}{w_t^i}\frac{W_t^i}{K_t^i} - \frac{\delta}{1-\delta} r_t^i\right)\right) \Big/ u_t^i \tag{5}$$

It is the substitute of equation 1 in which the main assumptions have been made explicit. If these simplifying assumptions had not been made, this expression would even be much more complicated.

Two important observations question the relevance of the methodology:

1. *The approximation $\rho_t^i = \rho_t'^i$ is questionable.* An order of magnitude of correcting terms can be obtained considering the case of simple reproduction. One has $K^i = K^i(1-\delta) + I^i$, that is, $K^i/I^i = 1/\delta$. This shows that the last term in equations 4 or 5, $\frac{K^i}{I^i}\frac{\delta}{1-\delta} r_t^i = \frac{r_t^i}{1-\delta}$, is of the same order of magnitude as the variable, ρ^i, under investigation. (In expanded reproduction, K^i/I^i would be slightly smaller than $1/\delta$.) Concerning the two other terms in the parenthesis (in which the variations of u^i or w^i are involved), their value depends on the amplitude of these variations, but due to the coefficient K^i/I^i, it is doubtful that they might be negligible.

2. *A correlation between the $\rho_t'^i$s does not imply a similar correlation between the ρ_t^is.* A priori, this correlation can be determined by any other variable in equations 4 or 5. Suppose that no convergence exists among profit rates and that they remain constant: $\rho_t^i = \rho^i$ (for any value of ρ^i). If business-cycle fluctuations affect similarly all industries (as is empirically the case and as contended by Marx in his reference to the "general" character of crises), all capacity utilization rates are equal: $u_t^i = u_t$ (or, at least, the correlation is very strong between the rates).
Thus, considering only the first term in equation 4, one obtains:

$$\rho_t'^i = u_t \rho^i$$

All ρ'^is are proportional (fully correlated), while the ρ^is are not correlated. The same test can be repeated with the second term in equation 4. The correlation results from the fact that, for all i, this term is proportional to $u_t - u_{t-1}$, independently of any assumption concerning gravitation on the ρ^is. The same is true of the following term, substituting w_t for u_t.