A NOTE ON DUNCAN FOLEY'S CIRCUIT OF CAPITAL

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This note elaborates on Duncan Foley's circuit of capital framework and Deepankar Basu's discrete time version of Foley's model.[1] In the present note, we build a simple version of the model in which its basic properties can be more easily understood. Once the framework has been introduced, we study the consequences of the substitution of a *credit money* for a *commodity money*. A second aspect of the investigation is the discussion of the relationship of Foley's circuit to *classical-Marxian* and *postKeynesian* approaches. The emphasis is on long-term equilibrium (homothetical growth).

Drastic simplifying assumptions are made concerning the pattern of lags. Three capitalists, whose circuits are lagged, are considered so that the recommittal of capital, production, and markets occur at each period. Only one commodity exists. We abstract from the payment of wages. (The consumption of wage-earners is treated as a component of the consumption of inputs in production.)

Section 1 introduces the basic framework in an economy with a money commodity. Section 2 discusses the key issues of realization and availability of money. Two models are presented in Sections 3 and 4, respectively devoted to the classical-Marxian and postKeynesian solutions to the circuit of capital riddle. Section 5 is devoted to a simplified version of the classical-Marxian framework, with a simpler pattern of lags, evocative of our dynamical frameworks. The issue of behaviors remains open, notably concerning the decision to produce and the determination of credits.

1 - The basic circuit of capital model with a commodity money

Section 1.1 introduces our simplifed framework. Section 1.2 generalizes this first approach to the consideration of homethetical growth as in Foley's studies. Section 1.3 makes explicit the relationship to Foley's and Basu's formalisms.

1.1 A simple framework

In this section, money is a commodity such as gold. Discrete time is considered and it is assumed that each step of one atom of value-capital ("capital" for short) in the circuit, from one form to the next, lasts one period.[2] The three traditional forms of capital are considered: money capital /M/, productive capital /P/, and commodity capital /C/. The sequence of events is described in Figure 1:

1. At time $t = 0$, one unit of capital is under the form of money.

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1. D. Foley, "Realization and Accumulation in a Marxian Model of the Circuit of Capital", *Journal of Economic Theory*, 28 (1982), p. 300-319; *Understanding Capital, Marx's Economic Theory*, Cambridge: Harvard University Press (1986); D. Basu, Comparative Growth Dynamics in a Discrete-time Marxian Circuit of Capital Model, Department of Economics, University of Massachussets, Amherst, MA (2011).

2. Obviously, in a given economy the movements of a huge set of such atoms are intertwined, and coexists under the three forms. They may belong to distinct capitalists (or, equivalently, enterprises) or to a same capitalist.

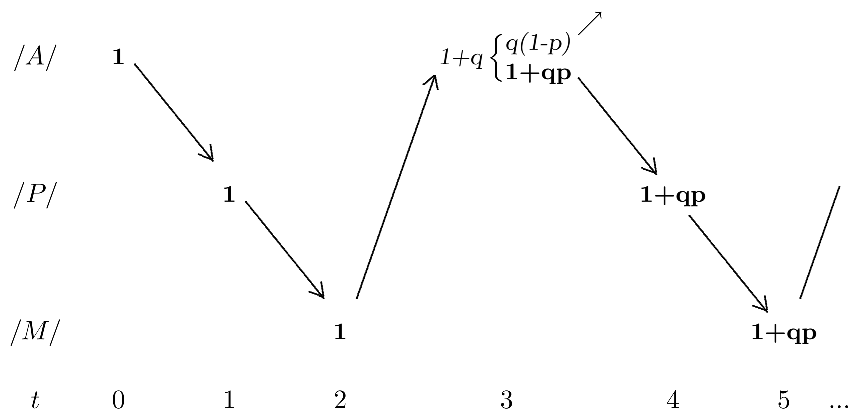
2. One period is required for its transformation into productive capital. (Capitalists purchase inputs on markets.) Thus, at time $t = 1$, one unit of value is under the form /P/ (raw materials, labor force, equipment and structures, and the like).

3. A second period, the production period, is needed for the metamorphosis of these inputs into a commodity. Capital is now under the form /C/.

4. After a new period, the commodity has been sold and the atom of capital has recovered its money form /M/. (Capitalists go the market to sell commodities.) A surplus-value, q, has been extracted during the production process and, at time $t = 3$, the atom of capital is worth $1 + q$. (Account is taken of the surplus-value after the commodity has been sold.) Note that q is here a rate of margin on total cost, neither a rate of margin on variable capital nor a profit rate. One fraction, p, of this surplus-value, a total qp, is kept within the circuit (that is, accumulated), and added to the original atom of 1. The remainder, $(1 - p)q$, is paid out to capitalist households and consumed.

5. A new cycle is initiated with a capital now amounting to $1 + pq$.

Figure 1 The individual circuit of one atom of capital



The times $t = 0, 1...$, in the lower part of the diagram indicate the moments in which the capital stocks are determined. We call "period 1, period 2" and so on, the time spans between the times 0 and 1, 1 and 2, and so on.

In this framework, production is automatic. It only depends on the amount of capital committed to the form /P/. There is no impact of demand (or inventories of unsold commodities). This is equivalent to saying that productive capacities are fully used. Sales are also automatic. In both instances, only time (in our simplified framework, one period) is required.

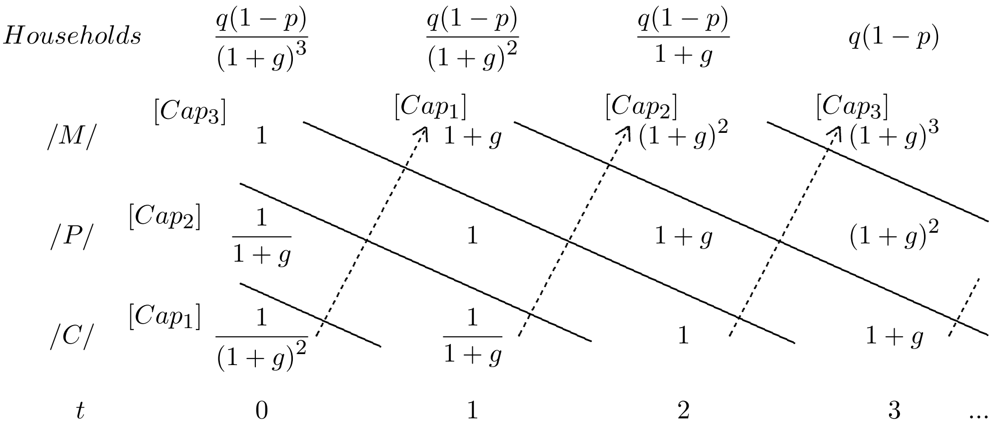
A growth rate is implicit in the sequence of events as described above. Between time $t = 0$ and time $t = 3$, the capital involved in the circuit grew from 1 to $1 + pq$. The grow rate, g, per period (over a sequence of three periods) is determined by the following equation:

$$(1 + g)^3 = 1 + qp \quad \textit{that is} \quad 1 + g = (1 + qp)^{\frac{1}{3}} \tag{1}$$

1.2 Homothetical growth

Three circuits of capital are intertwined, so that one atom of capital exists under each of the three forms in each period. In each period, one of the capitalist acts as buyer, one as seller, and one is not involved in transactions. In this framework, the three atoms of capital necessarily belong to distinct capitalists, since transactions occur. Assuming that the economy grows homothetically, at the rate g, the diagram in Figure 1 can be filled with the corresponding three threads, as in Figure 2. Two atoms of capital Cap_1 and Cap_2, preceding the earlier atom (now Cap_3) of Figure 1 are introduced, with values of, respectively $1/(1+g)$ and $1/(1+g)^2$, according to the assumption of homothetical growth.

Figure 2 Simultaneous circuits of three atoms of capital under the assumption of homethetical growth



One line corresponds to one form of capital, with the exception of the upper line where the amounts of surplus-value paid to capitalist households are shown. As depicted in Figure 1, atoms of capital belonging to distinct capitalists, Cap_1, Cap_2, and Cap_3 follow diagonal downward trajectories, from /M/ to /C/ within the bands suggested by the continuous lines. Their values are not altered. When commodities are sold, the surplus-value is added to the value transmitted, and the total jumps upward in one period as suggested by the dash lines. The division between the surplus-value paid out and the capital recommitted occurs. A new cycle is undertaken for a value mulitiplied by $1+g$.

Consider the atom of capital, $1/(1+g)^2$, held as commodity capital at $t=0$ by the capitalist Cap_1 (at the bottom of the first column). At $t=1$, it has been sold to the capitalist Cap_3 (committing its capital to the form /P/ during period 1 for a value of 1) and to households for a value $\frac{(1-p)q}{(1+g)^3}$. The total sales are, therefore:

$$1+\frac{(1-p)q}{(1+g)^3}\frac{(1+q)}{(1+g)^2}$$

In this total, one can distinguish between $1/(1+g)^2$ of value transmitted and $q/(1+g)^2$ of surplus-value. This surplus-value is divided into the fraction distributed to capitalist households and the fraction accumulated in the circuit, respectively:

$$\frac{(1-p)q}{(1+g)^2} \quad and \quad \frac{pq}{(1+g)^2}$$

One can easily check that the capital recommitted to the circuit is the sum of the value transmitted and accumulated capital:

$$\frac{1}{(1+g)^2} + \frac{pq}{(1+g)^2} = 1+g$$

as shown in Figure 2 (at the top of the second column, line /M/).

Consider the two other atoms of capital that, at time $t = 0$, existed under the forms /P/ and /M/. at time $t = 1$, they migrated to take, respectively, the forms /C/ and /P/, but their values were not altered. Thus, only the surplus-value accumulated in period 1 in the first thread accounts for the variation of the total stock of capital, K_t, involved in the overall economy (summing the three circuits):

$$K_1 - K_0 = \underbrace{(1+g) + 1 + \frac{1}{1+g}}_{\text{Capital at } t=1} - \underbrace{(1 + \frac{1}{1+g} + \frac{1}{(1+g)^2})}_{\text{Capital at } t=0} = 1+g-\frac{1}{(1+g)^2} = \underbrace{\frac{pq}{(1+g)^2}}_{\substack{\text{Accumulated} \\ \text{surplus-value}}}$$

Thus, a profit rate (on flows) can be determined. At $t = 0$, the capital stock is:

$$1 + \frac{1}{1+g} + \frac{1}{(1+g)^2}$$

At $t = 1$, the resulting surplus-value (or profit) is: $q/(1+g)^2$. Thus, the profit rate is:

$$r = \frac{\frac{q}{(1+g)^2}}{1 + \frac{1}{1+g} + \frac{1}{(1+g)^2}}$$

Repeating similar calculations for different periods, one can easily check that it is constant over time. It follows that:

$$r = \frac{g}{p} \quad or \quad g = pr$$

Thus, the well-known Cambridge equation is recovered.

1.3 Foley's and Basu's formalisms

A preliminary remark is that Foley does not use Marx's traditional terminology as above. Instead:

> Productive capital is the stock of value in long-lived plant and equipment, and inventories of raw materials and partly finished goods (valued at costs which include the wages of labor already expended on them), and exists because there is a time lag in the production process. Commercial capital is the inventories of finished commodities awaiting sale, and exists because of the time lag involved in selling itself. Financial capital is the financial

assets of the firm representing value realized in the past in sales but not yet recommitted to production in the form of new costs.[3]

In Foley's original analysis, continuous time is used, but not in Basu's paper, using discrete time as we do. In the framework above all lags (production, sales, recommittal) have been set to 1 for simplicity.

Foley's functions $a(.), b(.)$ et $c(.)$ are Dirac delta functions. With our assumptions concerning lags, one obtains: $a^*(g) = b^*(g) = c^*(g) = \frac{1}{1+g}$. Importing these expressions in Foley's equation (3.11), the equation above for the determination of g is recovered. Imputing these three functions in Foley's equations (3.12), (3.13), and (3.14), one obtains:

$$\begin{array}{ll} \text{Initial stock /P/} & N_0 = \dfrac{1}{1+g} \\ \text{Initial stock /M/} & M_0 = \dfrac{1}{(1+g)^2} \\ \text{Initial stock /A/} & F_0 = 1 \end{array}$$

These values are those shown in diagram 2. The Cambridge equation is equation (3.17) in Foley's article.

2 - Solving the money-realization riddle: Toward models with credit money

Although the sequence of events as in the previous section apparently unfold smoothly, two problems are actually met concerning money and realization (or demand). This is the object of Section 2.1. Section 2.2 is devoted to Foley's treatment of the issue. Section 2.3 introduces an accounting framework in which money and credit are introduced in the balance sheet of enterprises. Section 2.4 briefly addresses a small technical difficulty in the treatment of lags by Foley in the circuit with money. Section 2.5 establishes the relationship with classical-Marxian and postKeynesian frameworks.

2.1 Money and realization

Two related but distinct problems must be emphasized. Considering period 1:

1. The stock of money within the circuit grows from 1 to $1 + g$ as well as the money paid out to households. A first problem is, therefore, posed concerning the origin of the new money, still a commodity.

2. There is no sufficient purchasing power to buy the commodities supposed to be sold in each period. In other words, the supply supported by the atom of capital Cap_1 is

3. D. Foley, "Realization and Accumulation", *op. cit.* note 2, p. 304.

structurally larger than demand. This demand is the sum of the purchase of productive capital by Cap_3 and the consumption of capitalist households:

$$\begin{aligned} \text{Purchasing power (or demand)} &= 1+\frac{q(1-p)}{(1+g)^3} = \frac{1+q}{(1+g)^3} \\ \text{Supply} &= \frac{1+q}{(1+g)^2} \end{aligned}$$

The two issues are tightly related since the demand in period 1 is equal to the total stock of money within the circuit and outside at the time $t=0$. With a money commodity, the supply in period 1 must include the additional money stock, that is, the production of the appropriate amount of gold. Concerning demand, we confront here a basic problem within dynamic frameworks. A sequence of three events is involved, occuring at distinct times—the times at which: (1) money flows back to enterprises by sales; (2) incomes are paid out by enterprises; and (3) the corresponding purchasing power is used to buy. In a growth model, the purchases at time $t=1$ cannot be financed out of the money proceeding from the *sales* in period $t=0$.[4]

The reference to sales in the previous paragraph is intentional. Demand is not only determined by total income, that is, the sum of: (1) the income that has just been paid out, $\frac{q(1-p)}{(1+g)^3}$, which have just been paid out, and (2) the income that has just been accumulated, $\frac{qp}{(1+g)^3}$. All profits previously accumulated, that is, all capital recommitted, are also involved. The flows of value undergoing the metamorphoses in the circuit is not appropriately described by the flows of income but by the flows of sales (in wich value recommitted is included).

2.2 The problem as seen by Foley

Foley formulates the problem as follows:

> The model just outlined implicitly abstracts from the problem of realization, or, in conventional macroeconomic language, of effective demand. It does this by assuming that the time lag between production and sale, b(.), is a given parameter of the system, and that the price at which commodities are sold is not affected by the state of the market. This amounts to assuming that all a firm has to do to sell a commodity is to produce it and wait the appropriate time for it to be taken off the shelf by a buyer.[...].[5]
>
> But here we reach a paradox. [...] Only if g = 0, so that the lag coefficients are equal to 1 would there be enough demand to realize the surplus value in the finished commodities. There is a contradiction in this sense between accumulation at a positive rate and the realization of the commodities produced.[6]

4. A problem is actually met in any framework, not only growth models. In any disequilibrium situation, a lack or excess of money will prevail. Involved here is the notion of the "variation of the net debt" as in G. Duménil, D. Lévy, "Modeling monetary macroeconomics: Kalecki reconsidered", *Metroeconomica*, 63 (2012), p. 170-199. But in the absence of growth, the problem can be solved by the existence of a finite balance of money.
5. D. Foley, "Realization and Accumulation", *op. cit.* note 2, p. 309-310.
6. D. Foley, *ibid.*, p. 309-310.

Foley suggests an alteration of the sequence of events, inversing the realization by sales and the use of the corresponding purchasing power:

> This contradiction can be overcome only if some agents in the system, capitalist firms, the state, or households, spend revenue before it is realized.[7]

In a money-gold economy, agents can draw on their reserves of money commodity:

> [...] draw down stocks of a money-commodity accumulated outside the capitalist production process.[8]

Obviously, a limit will be met in a growth economy. The first way out proposed by Foley is the production of gold, actually the straightforward production of money:

> Marx himself, at the very end of Volume 2 of *Capital* (1893, p. 522-523), proposes one solution, which is taken by Bukharin (1972) in his critic of Rosa Luxemburg. Marx points out that it is not true that all the commodities produced have to be realized being exchanged against money. The money-commodity gold, once produced, is already value in the money form and thus does not need to be sold. If one posits a gold producing sector of exactly the right size, namely with production exactly equal to the difference between money demand and realization expressed in equation (5.64), then the problem of realization is solved. The money demand on the right-hand side of (5.64) is sufficient to realize all the nonmoney commodities, and the rest of commodity production is gold, which does not need to be realized. The gold-producing sector must grow at the same rate as the rest of the system in order to maintain this balance.[9]

We will not enter here into the discussion of the fact that gold must be first purchased by the government and, then, melt into coins, a privilege of the sovereign.

In a credit-money economy, credit moves center stage. The realization problem can be solved as follows:

> [...] the most obvious and common [*way of solving the realization problem*] is for an agent to spend by contracting a debt which it plans to pay out of future revenue. The second is for an agent to draw down stocks of a money-commodity accumulated outside the capitalist production process.[10]

This role of credit is addressed in the following sections. From here, the assumption of a commodity money is gradually abandonned.

2.3 The accountings of credit money

The simplest framework to address the issue of money and credit is to assume that only enterprises (the circuit of capital) borrow. Thus, a stock of money appears in the assets of enterprises, and a stock of loans in their liabilities. After the sales and the realization of profits, but prior to the paying out of a fraction of profits, all money is within the circuit. Under such circumstances, the stocks of money and credits are equal.

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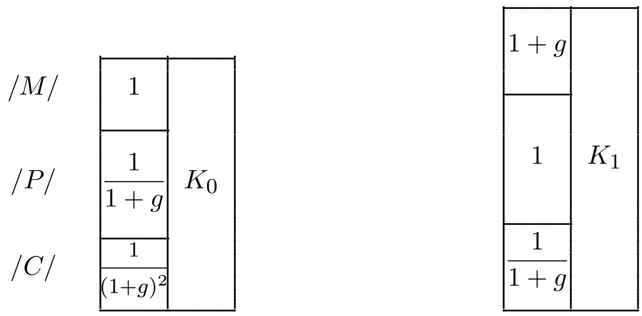
7. D. Foley, *ibid.*, p. 310.
8. D. Foley, *Understanding Capital, op. cit.* note 2, p. 87-88.
9. D. Foley, *ibid.*, p. 88.
10. D. Foley, “Realization and Accumulation”, *op. cit.* note 2, p. 310.

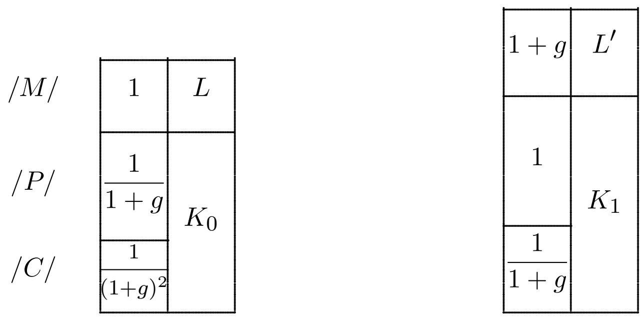
Diagrams 3 and 4 compares the balance sheets of enterprises within commodity-money and credit economies. Diagram 3 accounts for a commodity-money economy. Total capital, K_t, finances the three components of capital. The fraction of surplus-value that is accumulated accounts for the increase, $K_1 - K_0$, that is, the variation of the sum of the three components. In diagram 3, credit money is considered. In this case, accumulation only finances the increase in the sum of capital under the forms /P/ and /C/.

Figure 3 Commodity money: C=/M/+/P/+/C/



Accumulation finances /M/, /P/, and /C/.

Figure 4 Credit money: K=/P/+/C/



Accumulation finances /P/ and /C/.

2.4 Lags in Foley's credit economy

Moving from a commodity money to a credit money, Foley modifies the structure of lags. The lag between $C(t)$, the total flow of value being committed to production, and $S(t)$, the flow of sales is altered.

Within models with commodity money, there is a double lag: (1) from $C(t)$ to the flow of finished output $Q(t)$; and (2) from $Q(t)$ to $S(t)$. Within models with credit money, one fraction of $C(t)$ is sold instantaneously. This is manifest in equations such as 4.3 or 5.2, beginning as follows:

$$S(t) = (1-k)C(t) + ...$$

The reason for this change is not clear. In the framework below, we do not distinguish between the payment of wages and the purchase of other inputs. Both the lags between the committal of capital and the actual purchase of inputs are set to 1.

2.5 The long-term classical-Marxian and postKeynesian ways out of the circuit

The reader will not be surprised to discover that, in our opinion, the resolution of the problems above harks back to traditional frameworks in economics. The first one is the long-term classical-Marxian approach to accumulation and the determination of output. The second such framework is the postKeynesian approach to the determination of effective demand and growth. Concerning Marx, we refer here to the analysis of accumulation in Volumes I and II of *Capital*, to be distinguished from Marx's insights concerning business-cycle fluctuations in the short run, where a link can be found with the short-term Keynesian perspective.

Both production and demand are involved. In the two frameworks, demand is conditioned by the availability of money, within or outside of the circuit. The amount of money is the sum of the money already existing, equal to the value of sales, plus the new credits (more rigorously, what we denote as the variation of the net debt).

Three viewpoints are involved:

1. On a trajectory of homothetical growth, in the classical-Marxian perspective of accumulation, production is decided by the full utilization of existing capital resulting from previous accumulation. The problem is the determination of the amount of credit that will allow for the full sale of output, that is, the equalization of demand to supply. Production is determined by the amount of capital committed. In a model with fixed capital, such a situation corresponds to the prevalence of a normal value of the capacity utilization rate. Credit is determined to adjust demand to supply.

2. In the Keynesian perspective, the levels of demand determine outputs, as enterprises decide on production on the basis of a previous knowledge of demand. Since demand is determined independently of existing productive capacities, these capacities will be used at a capacity utilization rate distinct from normal. This viewpoint was originally devised to address short-term mechanisms, but have been extended to long-term paths, as in the postKeynesian perspective. While, in the Classical-Marxian approach, the purpose of the investigation was the determination of the required amount of credits, at issue in the postKeynesian perspective is the effect on the capacity utilization rate of an exogenous determination of credits.

3. Foley's viewpoint is intermediate. The decision to produce is automatic as in the classical-Marxian tradition (Foley's equation 5.1 in his 1982 paper), allowing for the transition between $C(t)$ and $Q(t)$. Conversely, demand is defined as in postKeynesian models, since the amount of credit is exogenous and given at any level. An unchecked variation of the inventories of unsold commodities follows, as in Foley's equation 5.6.[11] There is no feedback of the existence of inventories on production, since production is determined by the value of committed capital (in Foley's equation 5.1). Along a homothetical growth

11. In this latter equation, the in-going flow is output, $Q(t)$, but the out-going flow is only, $S'(t)$ instead of total demand $S(t)$. The reason is unclear.

path, inventories grow at the same rate as the rest of the economy. While this viewpoint is formally correct, it is economically quite unusual.

In Basu's study, no reference is made to production between page 12 (in Section 3.2 *Baseline Solution: Expanded Reproduction*), where a money commodity is still considered, and page 20 (in Section 4.3 *Maximal Growth Rate of the System*). Thus, the determination of output is not addressed in the two sections: 4.1 *The Realization Problem of the System* and 4.2 *Solution to the Realization Problem with Credit*.

The following sections suggest a full return to either one of the two first perspectives above: either the classical-Marxian growth model, or the postKeynesian adjustment of the capacity utilization rate to demand levels.

3 - The decision to produce: I - The classical-Marxian framework

This first set of models adopts the classical-Marxian viewpoint of accumulation as outlined in the previous section. Two models are presented, either assuming that all profits are accumulated or only one fraction. We recall that the object of the investigation is the determination of the amount of credits to be made available to the economy to ensure that the capacity utilization rate is normal and accumulation results in the homothetical growth path at its maximum growth rate (given the saving rate).

3.1 All profits are accumulated

This first case is illustrated in Figure 5, where the assets of enterprises at times $t = 0$ and $t = 1$ are considered. (Liabilities are represented in the diagram of Figure 4.) As in section 1.2, in period 1, Cap_3 purchases (the two arrows (a)) the inputs necessary to production during period 2 to capitalist Cap_1 holding these inputs under the form /C/.

The growth rate is determined by the equality between supply and demand:

$$\frac{1+q}{(1+g)^2} = 1 \quad \textit{or} \quad 1+g = (1+q)^{\frac{1}{2}}$$

It is easy to check that the profits, here accumulated, $q/(1+g)^2$ are equal the increase of capital under both forms /P/ and /C/:

$$\frac{q}{(1+g)^2} = \left(1 + \frac{1}{1+g}\right) - \left(\frac{1}{1+g} + \frac{1}{(1+g)^2}\right)$$

At $t = 1$, the available money has two origins: (1) the sales of commoditites (as in the doted-line arrow (a); and (2) the credits needed to reach $1 + g$. Since sales amount to 1, these credits must be equal to g, with: $1 + g = (1 + q)^{\frac{1}{2}}$.

Figure 5 The classical-Marxian framework: Accumulating all profits

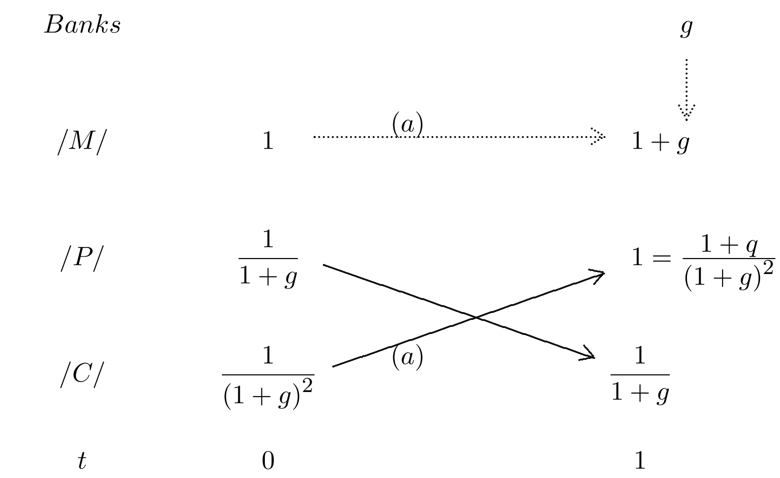
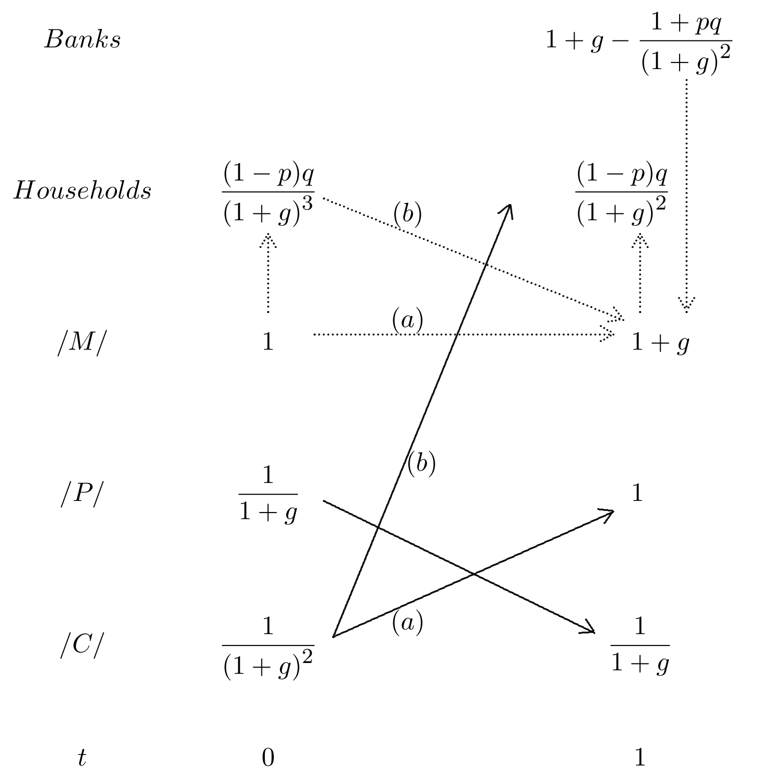


Figure 6 The classical-Marxian framework: Paying out a fraction of profits



The continuous lines denote commodity flows, and the doted lines, money flows. The two arrows (a) describe the two facets of a same transaction, namely the purchases (the use of the available money capital) and sales. The two arrows (b) accounts for the purchases of capitalist households (spending their income) and sales.

3.2 One fraction of profits is paid out

This second case is illustrated in Figure 6. The arrows marked (a) denote the same transaction as in Figure 5. The arrows marked (b) denote the purchase of consumption goods by capitalist households.

As in the previous model, supply is: $\frac{1+q}{(1+g)^2}$. Total demand is the sum of two components (corresponding to arrows (a) and (b)): 1 and $\frac{(1-p)q}{(1+g)^3}$. The equality between supply and demand allows for the determination of the corresponding growth rate, g:

$$\frac{(1-p)q}{(1+g)^3} - \frac{1+q}{(1+g)^2} + 1 = 0$$

This is a third degree equation in $X = \frac{1}{1+g}$. One can check that, with $p = 0$, the growth rate is null. If $p = 1$, the model of the previous section is recovered.

A first theorem follows:

Theorem 1: *There is one and only one solution with a growth rate $g^* > 0$. A larger value of p results in a larger value of g^*.*

Given this value of the growth rate, it is possible to determine the amount of credit required:

1. Enterprises receive the product of sales: $\frac{1+q}{(1+g)^2}$.
2. They pay out a fraction of profits: $\frac{(1-p)q}{(1+g)^2}$.
3. To finance the recommittal of capital, they need $1+g$ in available money.

Thus, the amount of credit necessary, B_1, is given by the following equation:

$$\frac{1+q}{(1+g)^2} - \frac{(1-p)q}{(1+g)^2} + B_1 = 1 + g$$

That is:

$$B_1 = 1 + g - \frac{1+pq}{(1+g)^2}$$

As above, one can check that, for $p = 0$, one has $B_1 = 0$ and, for $p = 1$, one has $B_1 = g$. A second theorem follows:

Theorem 2: B_1 *increases with* p.

To determine the corresponding value of B_0, one only needs to divide B_1 by $1+g$:

$$B_0 = 1 - \frac{1+pq}{(1+g)^3}$$

4 - The decision to produce:
II - The postKeynesian framework

This section is devoted to the postKeynesian way out. As in section 3, two models are presented in sections 4.1 and 4.2.

As mentioned earlier, Foley's framework is the combination of a standard classical-Marxian decision to produce with a postKeynesian treatment of demand. In our opinion, fixed capital must be introduced explicitly and the adjustment rather be realized by the determination of a capacity utilization rate different from normal in the postKeyenesian fashion. For simplicity, in this section, we present a simple model in which only fixed capital is considered (no ciculating capital). A new variable, the capacity utilization rate, is introduced.

4.1 All profits are accumulated

Fixed capital is used at a rate u, determined by demand levels, so that supply equals demand in the Keynesian fashion. We also assume that all profits are accumulated as within classical-Marxian models. The framework is illustrated in the diagram of Figure 7.

Total credit in period 1, B_1, is given. A growth rate immediately follows:

$$g = B_1$$

As in the diagrams in the previous sections, we assume that, at time $t = 0$, there is one unit of money in the assets of enterprises. The equality between supply and demand will prevail if the amount of commodity at time $t = 0$ is $\frac{1}{1+q}$, so that the supply during period 1 be $\frac{1+q}{1+q} = 1$. The components of productive capital have been purchased for this amount. Thus, the stock of productive capital, /P/, at time $t = 1$, is equal to 1. Given the assumption of homothetical growth, the stock of productive capital must be $\frac{1}{1+g}$ at time $t = 0$. At time $t = 1$, the stock of commodity capital /C/ is equal to the stock of /P/ at time $t = 0$, multiplied by the capacity utilization rate, that is, $\frac{u}{1+g}$. Given the growth rate g, commodity capital, /C/, at time $t = 1$, is equal to $\frac{1+g}{1+q}$. A comparison between these two amounts yields:

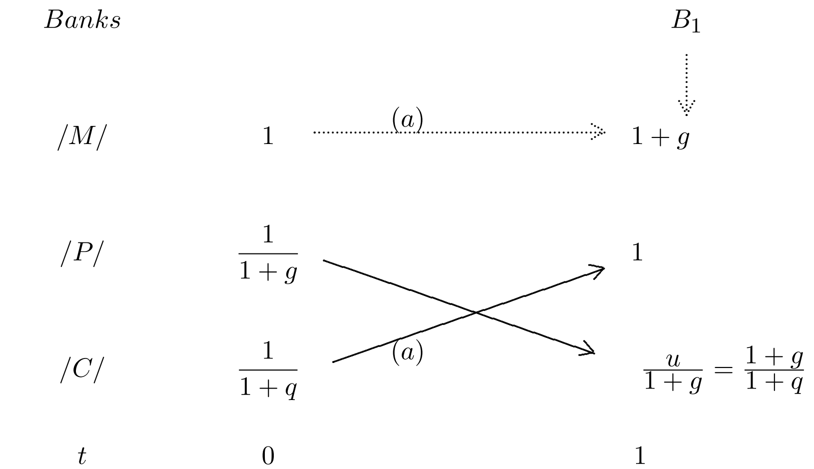
$$u = \frac{(1+g)^2}{1+q}$$

The following results are obtained. Both g and u are increasing functions of the amount of credits B_1. If $B_1 = 0$, one has $g = 0$ and $u = \frac{1}{1+q}$. The maximum value of B_1 corresponds to $u = 1$, the solution of equation:

$$1 = \frac{(1+B_1)^2}{1+q}$$

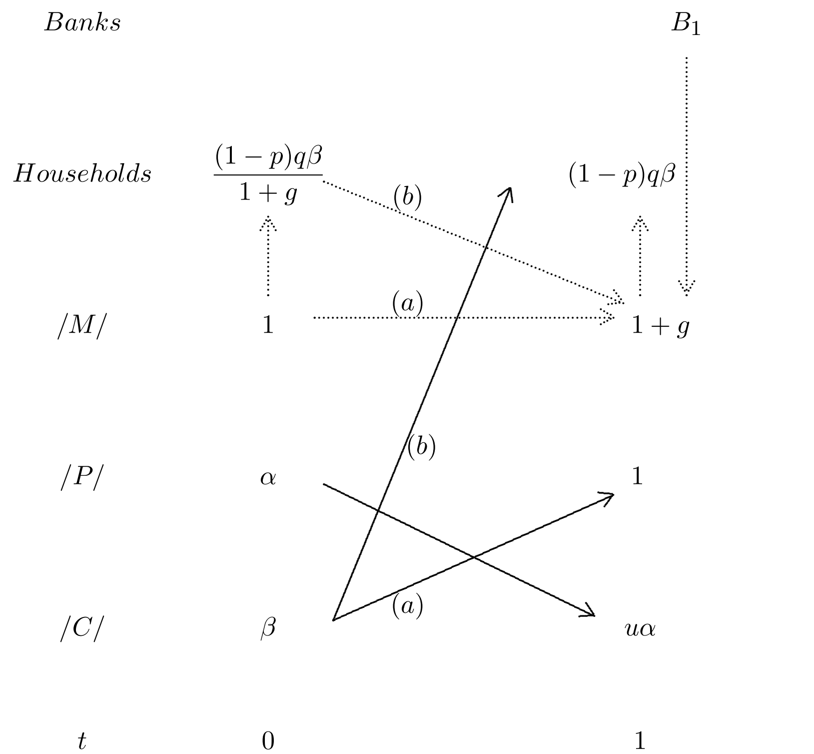
Thus, the amount of credit in Section 3.1 is recovered.

Figure 7 The Keynesian framework: All profits are accumulated and credit is determined exogenously



Arrows are defined as in Figures 5 and 6. The purchase of commodities is limited to investment in fixed capital.

Figure 8 The Keynesian framework: One fraction of profits is paid out



4.2 One fraction of profits is paid out

The same model as in the previous section is considered, but a fraction of profits is paid out. The amount of credits is still exogenous (and limited to enterprises). This framework is illustrated in the diagram of Figure 8.

We denote as α and β the initial values of productive and commodity capital, /P/ and /M/. (We still assume that there is one unit of money capital in the assets of enterprises at time $t = 0$.) The value of commodities to be sold is $(1 + q)\beta$, and profits are $q\beta$. One fraction of these profits is distributed to capitalists for consumption. In the previous period, $(1 - p)q\beta/(1 + g)$ had been distributed.

The four unknowns, α, β, u, and g are solutions of the four following equations:

1. The equality between supply and demand:

$$\beta(1+q) = 1 + \frac{(1-p)q\beta}{1+g}$$

2. The growth at a rate g of the three components of assets:

$$\begin{array}{ll} /A/ & 1+g = \beta(1+q) - (1-p)q\beta + B_1 \\ /P/ & \alpha(1+g) = 1 \\ /M/ & \beta(1+g) = u\alpha \end{array}$$

A third theorem follows:

Theorem: *Both g and u are increasing functions of the amount of credit. Both α et β are decreasing functions.*

The resolution of the above set of equations can be sketched as follows. Three variables, α, u, and g can be eliminated, leading to a single equation in β:

$$B = \beta\left(\frac{q(1-p)}{\beta(1+q)-1} - (1+qp)\right)$$

This function, $B(\beta)$, is a decreasing function of its argument. The same is true of the inverse function $\beta(B)$. Then, g can be expressed as a function of β. One can show that g is a decreasing function of β and, consequently, an increasing function of B.

5 - The simplified classical-Marxian framework

The purpose of the investigation in this section is to simplify the pattern of lags, moving closer to the pattern prevailing in our models.[12] Production takes time, and its duration defines the period. Only one lag (of one period) is considered between the times at which incomes are paid out and the spending of the corresponding purchasing power. Abstraction is made of the possible duration of sales and purchases. This framework is

12. Notably in G. Duménil, D. Lévy, "Modeling monetary macroeconomics", *op. cit.* note 5.

much simpler, as all capitalists act synchronically. Money is credit money. Basic properties are conserved. Below we show that theorems 1 and 2 hold.

The case in which all profits are accumulated is illustrated in Figure 9. The continuous-line vertical arrows represent the transactions among capitalists (the transition from /C/ to /P/). The amount of money in the circuit (equal to 1) remains within the circuit. As in the frameworks in the previous sections, capitalists must borrow. The growth rate is determined by the equality between supply and demand:

$$\frac{1+q}{1+g} = 1 \quad or \quad 1+g = 1+q$$

The profits, $q/(1+g)$, entirely accumulated in this model, are equal to the increase of capital under the form /P/. (There is no need to add the amounts of capital under the forms /P/ and /C/.) One has:

$$\frac{q}{1+g} = 1 - \frac{1}{1+g}$$

At $t = 1$, the available money comes from two sources: (1) the sales of commoditites (as in the doted-line arrow (a); and (2) the credits required to reach $1+g$. Since sales amount to 1, these credits must be equal to g.

The case in which one fraction of profits is paid out is described in Figure 10. The arrows marked (a) denote the same transaction as in Figure 9. The arrows marked (b) describe the purchase of consumption goods by capitalist households. As in the previous model, supply is: $\frac{1+q}{1+g}$. Total demand is the sum of two components (denoted by arrows (a) and (b)): 1 and $\frac{(1-p)q}{(1+g)^2}$. The equality between supply and demand allows for the determination of the corresponding growth rate, g:

$$\frac{(1-p)q}{(1+g)^2} - \frac{1+q}{1+g} + 1 = 0$$

This is a second degree equation in $X = \frac{1}{1+g}$. One can check that, with $p = 0$, the growth rate is null. If $p = 1$, the model of the previous section is recovered. Theorem 1 still holds. For this given growth rate, one can determine the amount of credit necessary:

1. Enterprises receive the product of sales: $\frac{1+q}{1+g}$.
2. They pay out a fraction of profits: $\frac{(1-p)q}{1+g}$.
3. To finance production in the ensuing period, they need $1+g$ in available money.

Thus, the amount of credit necessary, B_1, can be determined:

$$\frac{1+q}{1+g} - \frac{(1-p)q}{1+g} + B_1 = 1+g$$

That is:

$$B_1 = 1+g - \frac{1+pq}{1+g}$$

As above, one can check that, for $p = 0$, one has $B_1 = 0$ and, for $p = 1$, one has $B_1 = g$. Theorem 2 is still valid.

Figure 9 The simplified classical-Marxian framework: Accumulating all profits

$$\begin{tikzcd}
\mathit{Banks} & & g \arrow[d, dotted] \\
/M/ & 1 \arrow[r, dotted, "(a)"] & 1+g \\
/P/ & \frac{1}{1+g} \arrow[dr] & 1 = \frac{1+q}{1+g} \\
/C/ & \frac{1}{(1+g)^2} \arrow[u] & \frac{1}{1+g} \arrow[u, "(a)"'] \\
t & 0 & 1
\end{tikzcd}$$

Figure 10 The simplified classical-Marxian framework: Paying out a fraction of profits

$$\begin{tikzcd}
\mathit{Banks} & & 1+g-\frac{1+pq}{1+g} \arrow[dd, dotted] \\
\mathit{Households} & \frac{(1-p)q}{(1+g)^2} \arrow[dr, dotted, "(b)"] & \frac{(1-p)q}{1+g} \\
/M/ & 1 \arrow[u, dotted] \arrow[r, dotted, "(a)"] & 1+g \arrow[u, dotted] \\
/P/ & \frac{1}{1+g} \arrow[dr] & 1 \\
/C/ & \frac{1}{(1+g)^2} \arrow[u] & \frac{1}{1+g} \arrow[u, "(a)"'] \arrow[uuu, bend right, "(b)"'] \\
t & 0 & 1
\end{tikzcd}$$

The continuous lines denote commodity flows, and the doted lines, money flows. The two arrows (a) describe the two facets of a same transaction, namely the purchases (the use of the available money capital) and sales. The two arrows (b) accounts for the purchases of capitalist households (spending their income) and sales.

Contents