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A STOCHASTIC MODEL OF TECHNICAL CHANGE: AN APPLICATION TO THE US ECONOMY (1869–1989)

Gérard Duménil and Dominique Lévy*
(CNRS-MODEM and CNRS-CEPREMAP)

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ABSTRACT

A stochastic model of technical change is presented that accounts for the profiles of important macroeconomic variables observed in the US economy since the Civil War: labor productivity, the productivity of capital, and the profit rate. No production function exists, and the viewpoint is that of evolutionary economics. Innovation is described as a random, non-biased process, controlled by two parameters. The techniques of production used are selected according to their profitability. Under the assumptions of a rising labor cost and a temporary variation in the profile of innovation, it is possible to reproduce the historical trends of each variable over the three subperiods, 1869–1920, 1920–1960, and 1960–1989. For example, the model explains why the productivity of capital and the profit rate displayed first a downward, then an upward, and again a downward trend. The treatment of a deterministic approximation of the model permits a thoroughly analytical discussion of the various configurations of the variables depending on the values of the parameters.

INTRODUCTION

The purpose of this paper is to develop a stochastic model of technical change and distribution of evolutionary inspiration, and to apply it to the US economy since the Civil War. In this model, firms search for and select new techniques in the vicinity of the prevailing technology. An important aspect of the demonstration is that this simple model, used in simulation, proves to be very capable of accounting for the historical profile of technical change and distribution in the US economy since the Civil War. The basic properties of the model can be studied analytically in a *deterministic approximation* of the stochastic model.

* We thank Mark Glick for his aid in the translation of this text into English.

In the conventional neoclassical theory of growth and technical change, the simultaneous rise of wages and of the capital-labor ratio is only obtained under the assumption that firms optimize along a given production function. This function may vary over time, and it is the purpose of the theory of endogenous growth to "endogenize" this variation. However, these new attempts to streamline the old framework do not modify its foundations. At each point in time, there exists a production function, along which firms choose the optimal factor combination. This representation of technical change is clearly unsatisfactory, and the model in this paper avoids any reference to a production function with factor substitution.

In our model, the overall process of technical change is decomposed into two steps: (1) The *emergence* of new techniques, i.e., the outcome of R&D, called *innovation*, and (2) A *selection* process among the available innovations. Innovation is treated as the very uncertain outcome of R&D, and the emergence of new techniques is described as a stochastic process. No specific pattern is assumed for innovation, except that it is local. In particular, no assumption is made concerning a possible *labor-saving/capital-consuming* bias intrinsic to innovation, which is, instead, assumed to be a completely *neutral* process. The criterion used in the selection of new techniques is that of profitability. The most profitable innovations will dominate as a result of the advantage conferred on the firms that have implemented them.

The analysis of macroeconomic variables purporting to capture the historical evolution of technology and distribution in the US economy reveals a number of characteristic patterns of evolution: for example, a rising labor productivity, a constant or slowly increasing productivity of capital, or a constant wage share. Such regularities are often referred to as *stylized facts* (see, for example, Kendrick J. W., Sato R. 1963, Solow R. M. 1970, and Simon H. A. 1979), and correspond, within the Marxist tradition, to the notion of *historical tendencies* (Marx K. 1894). There is no general agreement, however, concerning the precise nature of these patterns—especially about the profit rate being constant or declining—and, obviously, there are a variety of explanations for this.

Notice that, in describing these historical profiles, there is always a problem of *duration* and *degree*. For example, the assertion that the trend of the profit rate is horizontal is a fairly good description of its profile if the series begins in the aftermath of the Civil War, but this characterization is not accurate if the trend is studied after the 1960s. For this reason, it is important to distinguish various subperiods or stages in these movements. The investigation in this paper covers a period of 121 years (1869–1989) which has been subdivided into three phases. A distinction has been made between the overall *historical trend* over the entire period and trends over periods of shorter duration (though still 30 to 50 years long). The model also proposes a straightforward interpretation of these patterns of evolution.

The paper is composed of three parts. Part 1 presents the stochastic model of technical change: the emergence and selection of new techniques. Part 2 is devoted to the description and reconstruction of the variables that account for technology and distribution. We first introduce the data series, and then show that the model permits the reproduction of the various configurations for the entire period and the three subperiods. This second part takes us one step further in this reconstruction, by linking together the various patterns and by assuming a "smooth" variation of the profile of innovation and endogenizing wages. Part 3 presents the deterministic approximation of the stochastic model, in which the random variables are replaced by their average values. In this dynamical model, it is possible to study analytically the variety of configurations observed, and to interpret the empirical findings in part 2.

1. MODELING TECHNICAL CHANGE

The first purpose of this part is to introduce the model: the emergence of new techniques (section 1.1) and their selection (section 1.2). Section 1.3 will then briefly discuss some elements of comparison with other approaches.

1.1 Innovation

The proposed description of technology is actually very simple. Only one good and a "representative" firm are considered.[1] The good is produced using itself and labor as inputs. At a given point in time a technique of production is represented by two parameters, A and L, which denote the amounts of fixed capital and labor required for the production of one unit of product. (The product here is net of all

[1] This is obviously a departure from the traditional evolutionary perspective (see section 1.3).

commodity inputs and depreciation.) Thus, the productivity of capital is $P_K = 1/A$, and labor productivity is $P_L = 1/L$.

Consider now a new technique (A_+, L_+). For the purpose of its comparison to (A, L), we define the rates of saving on each input, a and l by:

$$A_+ = A/(1 + a) \quad \text{and} \quad L_+ = L/(1 + l).$$

Hence, parameters a and l also denote the growth rates of capital and labor productivities:

$$\rho(P_K) = a \quad \text{and} \quad \rho(P_L) = l. \tag{1}$$

The comparison between the two techniques can be symbolically represented as in panels (a) and (b) of diagram 1, where the black dot (•) corresponds to the old technique. Within region (1) the new technique economizes on each input. Conversely, both inputs are increased within region (4). Regions (2) and (3) describe situations in which economies on one input are made at the cost of an increased utilization of the other.

We now consider the emergence of new techniques. Innovation is the outcome of R&D activities, and there is no given production function or blue print, known *a priori*, before the new technique has been discovered. When the project is initiated, it is not possible to predict the outcome. Thus, a first property of innovation is that it is a *random* process. A second property is that technology is only modified *locally*. Firms search on the basis of the previously existing technology. Third, in the absence of compelling evidence that a different assumption must be made, we assume that innovation is neutral. The probabilities of saving on either input are *a priori* equal.

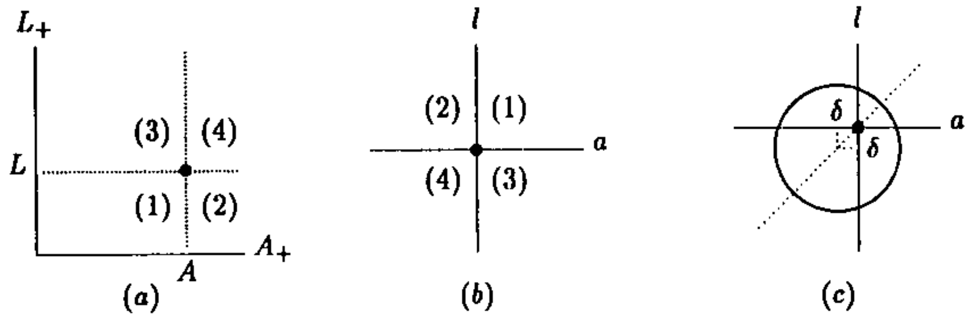


Diagram 1

The precise framework used to account for the emergence of new techniques is represented in panel (c) of diagram 1. The rates of economizing on inputs, (a, l), are modeled as two stochastic variables, which have a uniform distribution within a circle, denoted as the *innovation set*, whose center belongs to the first bisector, in (δ, δ), and whose radius is R (with $|\delta| < R/\sqrt{2}$). This framework clearly incorporates the three above-mentioned assumptions. Innovation is random; search is local, *i.e.*, (a, l) is close to $(0, 0)$ (this amplitude is governed by parameter R); neutrality corresponds to the fact that the innovation set is symmetrical with respect to the first bisector.

1.2 The selection of new techniques

We now turn to the selection of new techniques. As shown in diagram 1, new techniques within region (1) save on each input, and will always be adopted, and all techniques in region (4) will be rejected. This section discusses the decision to adopt a technique corresponding to region (2) or (3), that saves on a single input at the cost of an increased utilization of the other.

We choose the profit rate, r, as the criterion in the selection of new techniques. The *survival* of a new technique is conditioned by its profitability, which generates a cash flow from which growth can be financed. It is also well known that it is easier for a firm with a strong record of profitability to obtain external financing. Larger profit rates are also pertinent to the probability of a firm's survival in competition. The expected profit rate is also an *a priori* criterion in the decision to adopt new innovations, despite the difficulty of its assessment within a "radically" uncertain environment. For simplicity, we will assume that only techniques yielding the best profit rate when the selection occurs are retained.

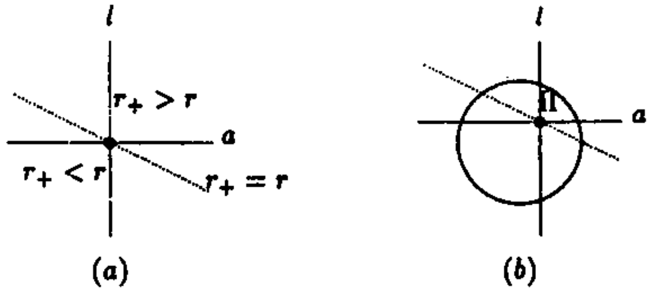


Diagram 2

More specifically, two techniques exist at the beginning of each period, the technique adopted in the preceding period, (A, L), and the new technique (A_+, L_+). If the profit rate, r_+, of the new technique is higher than that of the previous one, the innovation is selected; if it is lower, it is rejected. The borderline corresponds to the condition $r_+ = r$, which is represented by a downward sloping curve crossing the origin in the plane (a, l). We denote this borderline as the *selection frontier*.

With the assumptions made in the previous section, the profit rate can be easily determined. Since only two prices are involved, the price of the unique good and wages, it is possible to consider a single relative price. When labor cost w, *i.e.*, the unit wage deflated by the price of the good, is chosen, one obtains:

$$r = \frac{1 - Lw}{A}. \tag{2}$$

Under the assumption that the innovation set is small, r_+ can be developed linearly in the vicinity of r:

$$r_+ = r\left(1 + \frac{\mu a + l}{\mu}\right) \tag{3}$$

in which:

$$\mu = \frac{Ar}{Lw} = \frac{\text{Profit}}{\text{Wages}} = \frac{1 - \omega}{\omega}$$

denotes the slope of the selection frontier, and $\omega = Lw$ the wage share. The selection frontier is the downward sloping line represented in diagram 2, which rotates from a vertical position for $\omega = 0$, to a horizontal one for $\omega = 1$.

The new technique is retained if it belongs to the upper region, and rejected if it appears within the lower region (and the old technique is maintained). Thus, only the new techniques that fall within the circle and above the line, as shown in panel (b), are selected. This region, Π, will be called the *profitable innovation set*.

Although the innovation set is symmetrical with respect to the first bisector (innovation is neutral), the profitable innovation set, Π, is *not* symmetrical, and imparts a bias to technical change. For example, in panel (b) of diagram 2, the slope of the frontier, μ, is smaller than 1, and savings on labor will tend to be larger than on capital. Accordingly, the capital-labor ratio will exhibit a rising trend.

Within this framework, the *rapidity* of technical change is determined by the characteristics of the emergence and selection of new techniques:

1. *The "distance" of selected innovations from the old technique:* since the average features of selected innovations are represented by the center of gravity of Π, the crucial measure, in this first respect, is the "thickness" of Π (governed by $(R/\sqrt{2}) + \delta$).
2. *The proportion of innovations selected.* The "rate of success" in the profitability test to which innovations are submitted is the ratio of the surface of the profitable innovation set, Π, to that of the entire innovation set (the circle). Abstracting from the variations of the slope of the selection frontier, this ratio is governed by δ/R.

Thus, the rapidity of technical change can be increased either by enlarging the average size of selected innovations, or by raising the number of successes in the selection process. This distinction echoes that of "radical" and "incremental" technical change, or the distinction between *research* leading to actual "discoveries" and the mere *development* of already identified processes, as in R&D.

In the next part, we will use our model to study the major aggregate series accounting for technical change and distribution in the US economy. However, the explanatory power of the model is much broader, and it can also be used at "micro" or "meso" levels of analysis (firms or industries).

1.3 A comparison with other perspectives

In this section, we briefly comment on three characteristics of our model: its differences with other models which assume production functions, the role assigned to endogenous and exogenous mechanisms, and its evolutionary inspiration.

A major point concerning the comparison with the conventional neoclassical model of growth and technical change is that there is no *a priori* description of technical possibilities as in the production function with factor substitution and, consequently, no distinction between the free movement along such a function and a term of exogenous technical progress.[2] In our model, the technology used at a given point in time depends on the technology of the previous period, and not only on the present value of factor prices. In both frameworks, wages affect the outcome, but in different ways. In our model technology depends on the entire trajectory of wages (and prices) in the past. Thus, technical change is *path-dependent*.

Although this rejection of the conventional production function creates an important distinction between our model and the broad variety of "endogenous growth" models, the issue of the *endogenous* or *exogenous* nature of growth and technical change can still be raised in our approach. The selection of new techniques is clearly endogenous in the model, since it is based on the criterion of profitability. The case of innovation, however, is different. The two parameters that control innovation, R and δ, have been considered as given in the presentation of the model. There would, of course, be no objection to an endogenous treatment of these parameters. All candidate mechanisms would be welcome—such as the dependence of innovation on the growth of output (as in the Kaldor–Verdoorn Law) or on the growth of the capital stock per worker (as in Kaldor's technical progress function), the accumulation of "human capital", etc.—provided that they pass empirical muster.

Last, we want to discuss briefly the evolutionary character of the model. Our approach is particularly akin to that of Richard Nelson and Sidney Winter (Nelson R. R., Winter S. G. 1975 and 1982):

1. The basic patterns of the two models are similar: (1) Search and selection are distinguished as two successive steps, (2) The search for new techniques is described in terms of a probability distribution, (3) Search is local, and (4) Profitability is the basic criterion used for selection. There are also some differences. For example, we do not define the entire set of available techniques *ex ante*, and do not resort to "satisficing".
2. One purpose of the present paper is also the investigation of the reasons for a "*negative association between the wage rate and the labor-capital ratio, and a positive association of the wage rate with output per worker*" (Nelson R. R., Winter S. G. 1975, p. 469) and, from a methodological point of view, we fully agree with the view

[2] The model uses a Leontieff-type technology (a "production function" with fixed coefficients and constant returns to scale).

that it is "*possible to develop analytic power with an evolutionary theory*" (Nelson R. R., Winter S. G. 1975, p. 470).

An important exception to an evolutionary perspective is the macroeconomic treatment of the model. However, as shown in Duménil, G., Lévy D. 1993(b), the macroeconomic model in the present paper can be viewed as the aggregate form of a more satisfactory *general disequilibrium model*, in which several goods exist, several enterprises compete in the production of a single good and individual agents *adjust* to disequilibrium in a context of "radical" uncertainty. In such a framework, macroeconomic variables are obtained by aggregating the variables characteristic of individual agents determined at a micro level. The use of "representative agents", in the present paper, is motivated by the desire to analyse the basic properties of the model. Obviously, several issues which are related to the existence of disequilibrium and the multiplicity of agents—that can only be addressed within a general disequilibrium model—are excluded. This is the case, for example, for the impact of the fluctuations in the general level of activity on investment and technical change, and any mechanism related to heterogeneity.

2. HISTORICAL PATTERNS

In this second part, we show that the simple stochastic model in the previous part is sufficient to account for the trends of the variables characterizing technology and distribution in the US economy since the Civil War. Section 2.1 introduces the variables (definitions and sources), and delineates the three periods in which the trends are quite distinct. Section 2.2 shows that the model allows for the reconstruction of all trends in each period, given the growth rates of the labor cost and the various profiles of the innovation set. The three periods are then linked in section 2.3, using continuous profiles of evolution for the innovation set and endogenizing labor cost. Section 2.4 addresses the issue of the confidence interval within which the simulation will most probably lie.

2.1 Technology and distribution since the Civil War

Four variables are used in this paper to account for the historical profile of technology and distribution: wages (labor cost), labor productivity, the productivity of capital, and the profit rate. The unit of analysis is the whole of the US *private economy*.

The definition of the variables is straightforward. *Labor cost* is the total compensation per hour worked by employees, deflated by the Gross National Product (GNP) deflator. The product in each year is defined as GNP minus depreciation and income created by the Government. Employment is the total amount of time worked by employees and self-employed persons. *Labor productivity*, P_L, is the ratio of the product (in constant dollars) and employment. Residential capital is not considered a component of the productive system, and is excluded. Thus, the capital stock is limited to equipment and structures. (Government Owned and Privately Operated capital is also included.) The *productivity of capital*, P_K, is the ratio of the product and the gross capital stock (both aggregates in constant dollars).

Since our primary concern is technical change, our measure of the profit rate employs a broad measure of profit defined as the *product minus labor remuneration* and a narrow definition of capital as the *net stock of fixed capital*. The measure of labor remuneration is determined by multiplying total private employment (*employees + self-employed*) by the average compensation *per* employee. Thus, a wage-equivalent for the self-employed is included within labor income. Profit and capital are in current dollars.

Obviously, these data are more reliable for the more recent years. Prior to 1929 (or 1925), it is necessary to resort to historical studies: Balke N. S., Gordon R. G. (1989) for GNP, Goldsmith R. W. (1952) for the capital stock, Kendrick J. W. (1961) for employment, and Lebergott S. (1964) for compensation. After 1925, information about capital stock and depreciation can be obtained from the Bureau of Economic Analysis. From 1929 onwards, GNP, employment, and total compensation are provided by the National Income and Product Accounts. (A detailed account of the construction of the series is presented in Duménil G., Lévy D. 1991).

The four variables considered, w, P_L, P_K, and r, are displayed in figures 1 to 4, with the trend lines for the entire period and the three subperiods, 1869–1920, 1920–1960, and 1960–1989. The growth rates are presented in table 1.

Considering first the trends over the entire period, the four variables can be clearly separated into two groups. Labor cost and labor productivity display a clear upward historical trend. The profit rate reveals an approximately horizontal trend and, to a lesser extent, the same is also true for the productivity of capital.

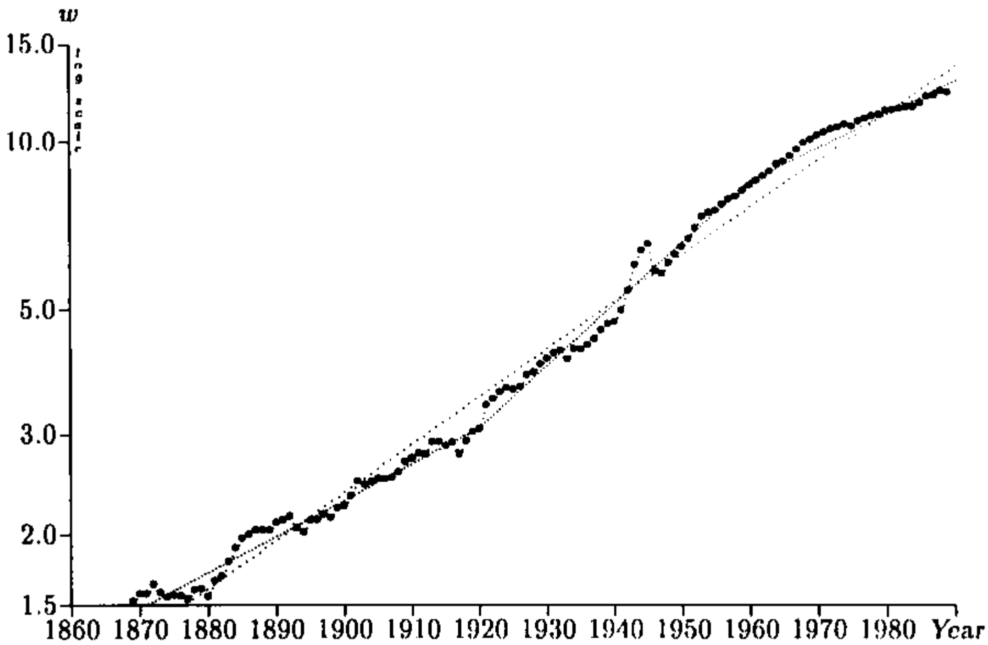


Figure 1: Labor cost: series (●) and trends (·) for various periods

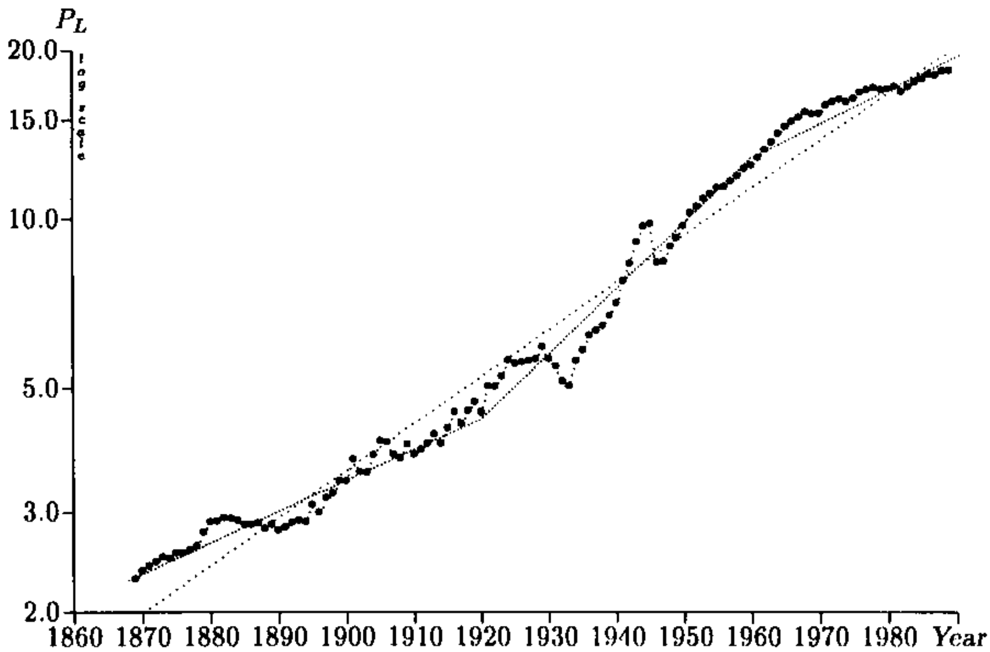


Figure 2: Labor productivity: series (●) and trends (·) for various periods

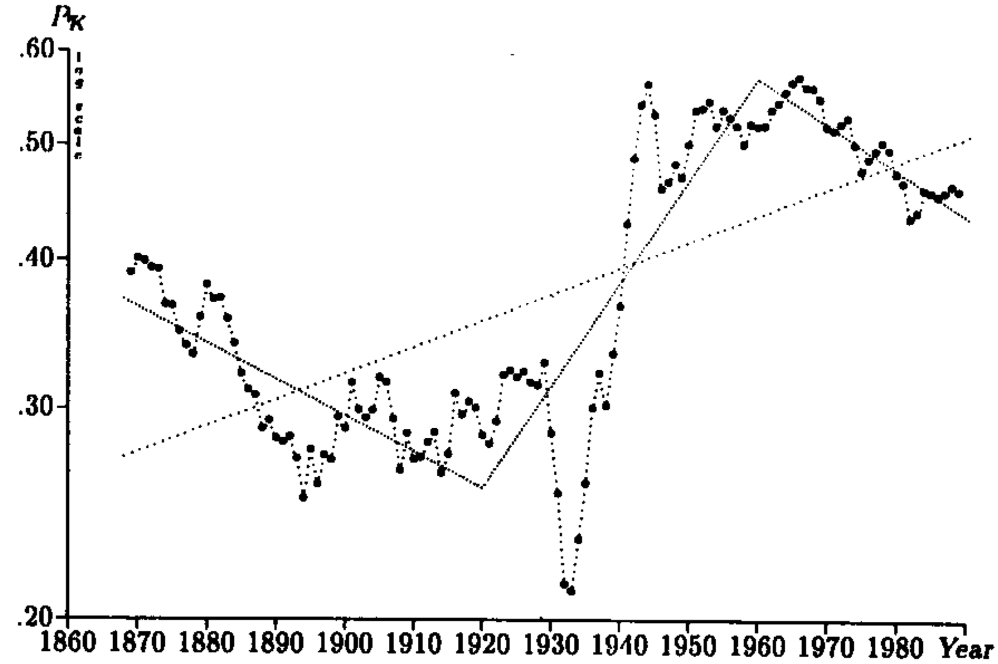


Figure 3: The productivity of capital: series (•) and trends (·) for various periods

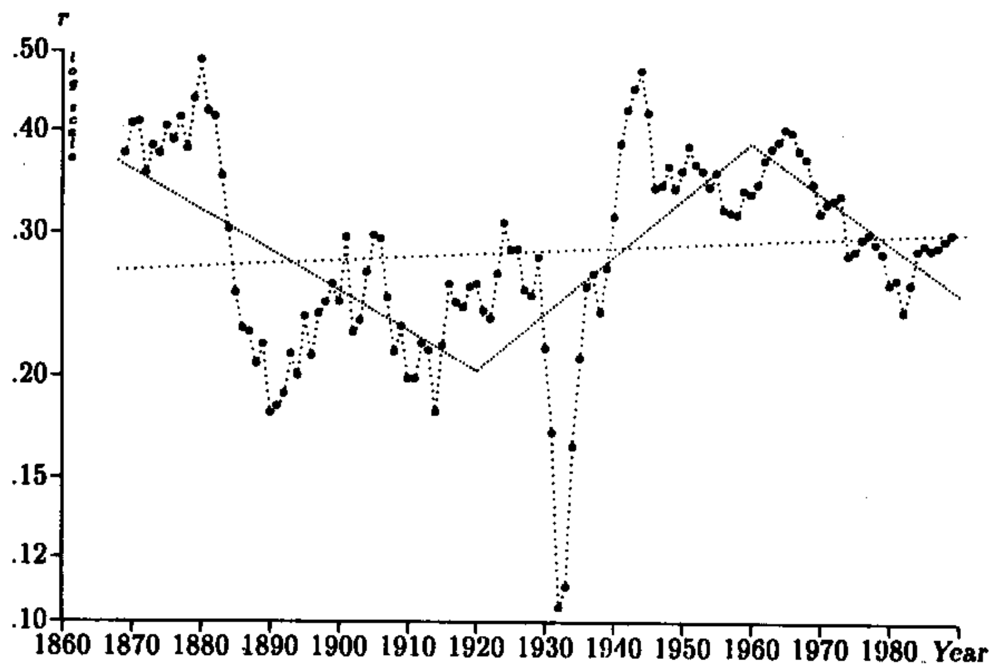


Figure 4: The profit rate: series (•) and trends (·) for various periods

Table 1: Average annual growth rates (% per year)

	1869–1920	*1920–1960*	*1960–1989*	*1869–1989*
$\rho(w)$	1.47	2.53	1.39	1.95
$\rho(P_L)$	1.27	2.68	1.39	1.94
$\rho(P_K)$	−0.70	1.98	−0.88	0.51
$\rho(r)$	−1.14	1.61	−1.42	0.09

Consider now the three subperiods. A striking feature of these series is that the fluctuations around their historical trends reproduce the same pattern in the three stages:

1. From the aftermath of the Civil War to the early 20th century, labor cost and labor productivity grow comparatively slowly, whereas the productivity of capital and the profit rate decline.
2. From the early 20th century to the 1950s, labor cost and labor productivity grow comparatively faster, and the productivity of capital and the profit rate rise.
3. From the 1960s onwards the four variables return progressively to their earlier trends exhibited in the first stage. The first and third stages are very similar.

Two additional features clearly emerge from this description. A first observation concerns the exceptionally favorable profile of the intermediate period, with rapid technical progress and no conflict between the two distributional variables w and r. Second, these trends show that rising labor productivity and declining productivity of capital may be simultaneously observed (as in the first and third periods). This observation stresses the importance of a global approach to technical change, in which *two* categories of inputs (labor and capital) are considered simultaneously (rather than just one when the investigation only focuses on labor productivity, as is often the case). In the intermediate period, the notion of *technical progress* is straightforward, since both productivities rise.

2.2 Reconstructing the trends

In order to reconstruct the variables, we use the model in simulation. More precisely, we proceed as follows:

1. We choose the values of the two parameters, R and δ, which describe the innovation set.
2. We assume that the labor cost grew at its actual average rate of growth during the period (as in the first line of table 1). A third parameter is thus considered, which we denote as ρ_w.
3. At each period a new technique is determined randomly. Its profit rate is computed and compared to that obtained when assuming that the previous technology is maintained. The technology yielding the largest profit rate is adopted. The procedure is initiated by choosing the values of the variables, A and L, which describe the technology in 1868. This procedure is repeated for each period, and a trajectory of technology is generated. The probability that an innovation falls within the profitable innovation set is small at each period. Consequently, the elementary period used in the model is not the year, but the month. (For example, even if one draw out of three falls within the profitable innovation set, on average, the probability that some actual technical change will occur within one year, after twelve draws, is close to 1.)[3]
4. Finally, we calculate the trends of the variables.

The values of parameters R and δ have been determined to allow for a satisfactory reconstruction of the variables. The growth rate of the labor cost, ρ_w, is as in table 1 by assumption.

The results are displayed in table 2, with the growth rates of the variables in the second block and the parameters in the third. Two distinct kinds of simulations have been run: (1) With a single value of parameters R and δ, for the entire period (last column), and (2) With two sets of parameters, one for the first and third periods whose similarity has been stressed in the previous section, and one for the intermediate period. The following comments can be made:

1. Consider first the variables. A comparison with table 1 or the first block of Table 2 shows that there is no difficulty in reproducing their trends, for the entire period or for each subperiod. Although the growth rates for the profit rate are not as good a fit as the others, in

[3] In the reconstruction of the profit rate we do not attempt to reproduce the relative price of GNP *vis-à-vis* capital, and the ratio of gross capital to net capital. These ratios vary in connection to the diminishing service life of fixed capital and the variations of accumulation. Instead of equation 2, we use the formula, $N_1N_2(1 - Lw)/A$, in which N_1 and N_2 correct the two above effects and are determined empirically. (Note that this problem is not met in the reconstruction of the productivity of capital.)

Table 2: Reconstruction of the trends
Annual rates of change of variables (% per year) and parameters

	1869–1920	*1920–1960*	*1960–1989*	*1869–1989*
$\rho(P_L)^*$	1.27	2.68	1.39	1.94
$\rho(P_K)^*$	−0.70	1.98	−0.88	0.51
$\rho(r)^*$	−1.14	1.61	−1.42	0.09
$\rho(P_L)$	1.37	2.65	1.26	2.01
$\rho(P_K)$	−0.53	1.80	−0.80	0.52
$\rho(r)$	−0.77	1.32	−1.61	0.23
ρ_w	1.47	2.53	1.39	1.95
δ	−1.35	0.10	−1.35	−0.19
R	3.15	0.81	3.15	1.31

* Actual values as in table 1

particular during the first period, the model accounts for the various patterns obtained in each period. Both rising, declining, or horizontal productivity of capital and profit rate can be obtained. Various growth rates of labor productivity can be reproduced.

2. As far as parameters R and δ are concerned, one can first notice that the figures for the entire period are consistent with the average of the results for the three periods. The following finding emerges strikingly: *the characteristics of the intermediate period are very different from those of the first and third periods*. (The increased rapidity of technical change in the intermediate period is mainly expressed as an increase in the rate of success in the selection of new techniques every month, rather than a larger size of selected innovations: δ/R rises from −0.43 to 0.12, while $R/\sqrt{2}+\delta$ remains approximately constant (0.88 and 0.67).)

The interpretation of the variations of the specificity of the intermediate period is not straightforward, and will remain beyond the scope of the present analysis. It relates, in our opinion (see Duménil G., Lévy D. 1993(a), Ch. 18), to the emergence of the large modern industrial firm with its new organization, technology, and hierarchical management, as described by Alfred Chandler (Chandler A. D. 1977 and 1990).[4]

[4] This interpretation also echoes the analysis of US leadership in Nelson R. R., Wright G. 1992.

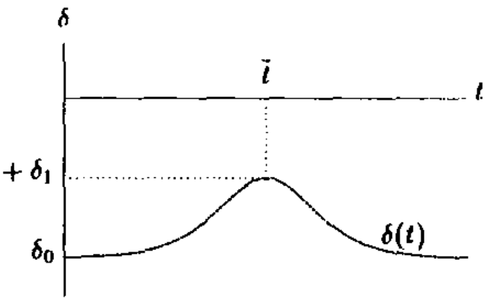
Obviously it also relates to what has often been described in a somewhat reductionistic manner as the Taylor or Ford type of workshop organization.

If the simulation is repeated using different random innovations, with the same values for R and δ, the same overall picture is always observed. The exact drawing of new techniques makes a quantitative difference, but does not alter the basic patterns (see section 2.4 below). It will be the purpose of part 3 to account for this finding, and investigate the *core properties* of the model in its deterministic approximation.

2.3 Linking the three periods

This section adds precision to the above analysis, by linking the three periods into a continuous evolution. The sudden breaks in the growth rate of the labor cost and the profile of innovation assumed in the analysis in the previous section are obviously simple proxies for large but, nevertheless, continuous transitions. We will now introduce into the model more realistic progressive transitions between the periods in these two respects.

Consider first parameters R and δ. We will not attempt to treat their variation endogenously, but rather assume that they followed an exogenous broad historical fluctuation. From a given value in the first period, they were progressively modified during the intermediate period, and then returned to their initial values. For example, the pattern used for δ is represented in diagram 3, together with the exact analytical expression used (the derivative of a logistic). In this expression, $\bar{t}$ denotes the year in which the maximum value of $\delta(t)$ was reached, and Δ provides a measure of the duration of this movement. It is easy to verify that the



$$L(t) = \frac{A \exp\left(-\frac{t-\bar{t}}{\Delta}\right)}{\left(1 + \exp\left(-\frac{t-\bar{t}}{\Delta}\right)\right)^2}$$

$$\delta(t) = \delta_0 + \delta_1 L(t)$$

$$R(t) = R_0 + R_1 L(t)$$

Diagram 3

curve is symmetrical with respect to $\bar{t}$. A similar model is applied to $R(t)$, using the same function $L(t)$ (with the same parameters $\bar{t}$ and Δ).

Consider now the *labor cost*. A straightforward solution would be to use the actual profile of the series, as in figure 1. However, we believe that it is more interesting to treat wages endogenously, since the four variables (P_L, P_K, w, and r) actually form a system of reciprocal interactions, which must be approached globally.

It is not the purpose of this paper to address extensively the issue of wage determination. We will borrow here the equation introduced in Duménil G., Lévy D. (1992 and 1993(a)), which emphasizes a dependence of the growth rate of the labor cost on the level and the growth rate of the profit rate. This dependence expresses the view that large or rising profit rates are favorable for wages, while the opposite holds true for low or declining profit rates:

$$\rho(w) = \alpha + \beta r + \gamma \rho(1 + r). \tag{4}$$

Obviously, the problem of the determination of wages in a capitalist economy cannot be summarized by a simple mechanism or equation. There are obviously many facets to this issue, including class conflicts, institutions, and the labor market situation, and these mechanisms are not exclusive.

It is crucial in this analysis to distinguish between various time frames:

1. In the short term, the growth rate of the real wage (or labor cost) is a function of the fluctuations of the general level of activity (or employment, since the two variables are very strongly correlated, *cf.* Duménil G., Lévy D. 1993(a), 11.8). This relationship corresponds to the widely held view that the labor-market situation affects wages. A threshold difference with the neoclassical model is that we consider the growth rate of wages (as in the Phillips' curve), and not their level. Second, there is no feedback relation, since the level of wages does not affect the supply of employment by firms, and its adjustment does not ensure that the supply of labor be equal to its demand.
2. The model used in equation 4 refers to a quite distinct time frame, that of the *very long term*. It expresses the view that a *feedback* mechanism exists, extending from the profit rate to the determination of wages, which tends to maintain the profit rate under control historically (*cf.* Duménil G., Lévy D. 1993(a), 20.2), despite lengthy fluctuations with declining and rising stages several decades long. Various explanations account for this relationship. In particular, a

> lower profit rate: (1) strengthens the resistance of firms to wage rises, (2) diminishes the pace of accumulation and relaxes the tensions on the labor market and, consequently, on wages, and (3) increases the instability of the macroeconomy, and frequent or severe recessions have a depressing effect on wages. Not only the level of the profit rate is important, but also its trend. For example, a declining trend will be an inducement to resist any rise in wages. An important aspect of this model is that *the supply of labor (or an exogenous growth rate of the available labor force) is not considered*, in sharp contrast with most traditional growth models.

It is, of course, possible to combine these two types of mechanisms, and to include a variable accounting for business fluctuations in equation 4. Such an estimation is provided in Duménil G., Lévy D. (1993(a), 15.A3).

The combination of the *value* and *growth rate* of the profit rate in equation 4 deserves a few additional explanations. As noted in section 2.1 (table 1), there is a clear relationship between the growth rates of the labor cost and the profit rate throughout the three periods, and this first observation suggests a model linking the two growth rates. A more careful examination of this relationship reveals, however, the existence of a *lag*. The movement of the labor cost reproduces that of the profit rate, but with a delay of about 5 years (*cf*. Duménil G., Lévy D. 1993(a), 15.4). For example, the beneficial effects of the high levels of the profit rate in the 1950s and the first half of the 1960s were still felt for a number of years despite the already declining trend of the profit rate. We account for this mechanism by adding the *level* of the profit rate in the equation for the growth rate of the labor cost.

Equation 4 completes the construction of the model. It is possible to fit this model to the data.[5] As shown in figures 5 to 8, and in spite of the rudimentary character of the framework, the quality of the reconstruction is striking.

Figures 5 to 8 illustrate the central thesis of this paper that *a transformation of the characteristics of technical progress in the intermediate period (which stretches from the early 20th century to the 1950s) is sufficient to account for the historical profiles of the main variables that describe technology and distribution in the US since the Civil War*. This result is obtained independently of any *a priori* bias favoring labor or capital, but with a randomly generated neutral innovation process.

[5] One finds $\bar{t} = 39.3$ and $\Delta = 14.5$. Note that the intermediate period, 1920–1960, approximately corresponds to $\bar{t} - 1.4\Delta$ and $\bar{t} + 1.4\Delta$.

Figure 5: Labor cost: series (●) and model (○)

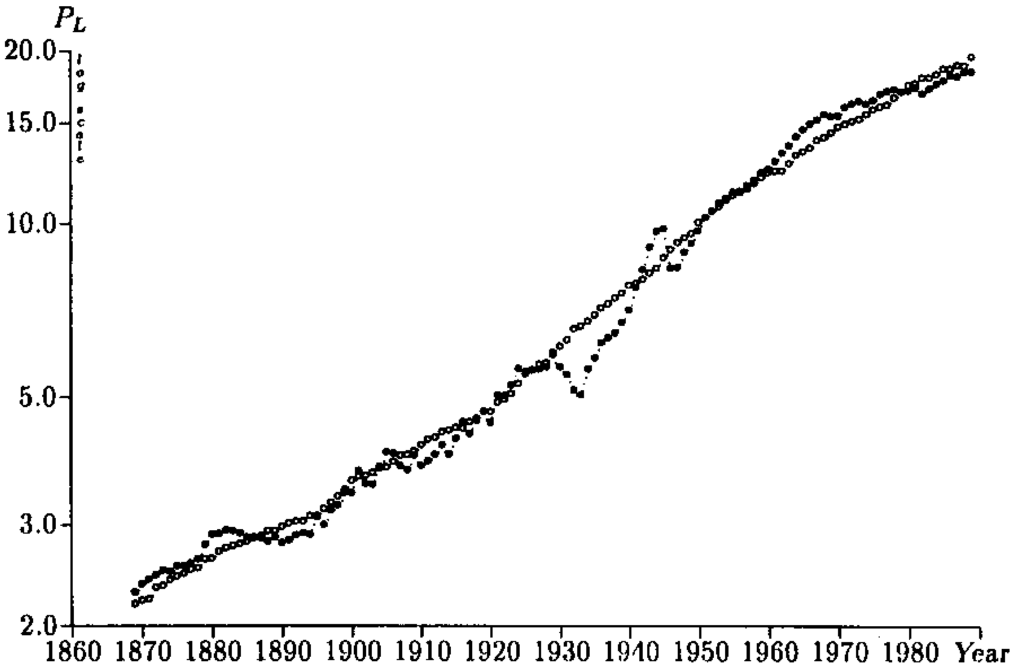


Figure 6: Labor productivity: series (●) and model (○)

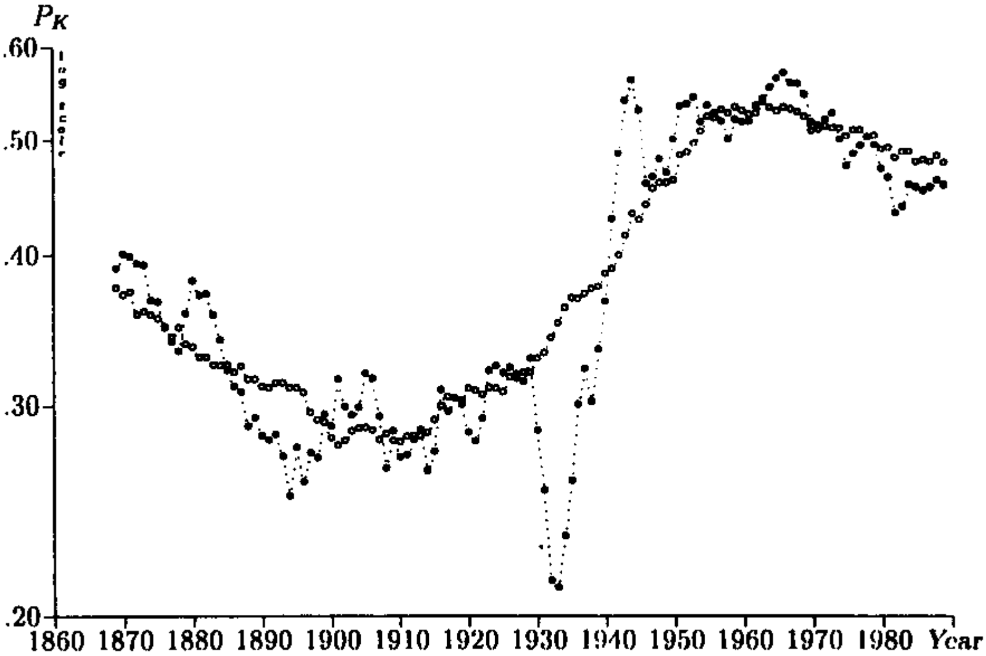


Figure 7: The productivity of capital: series (•) and model (○)

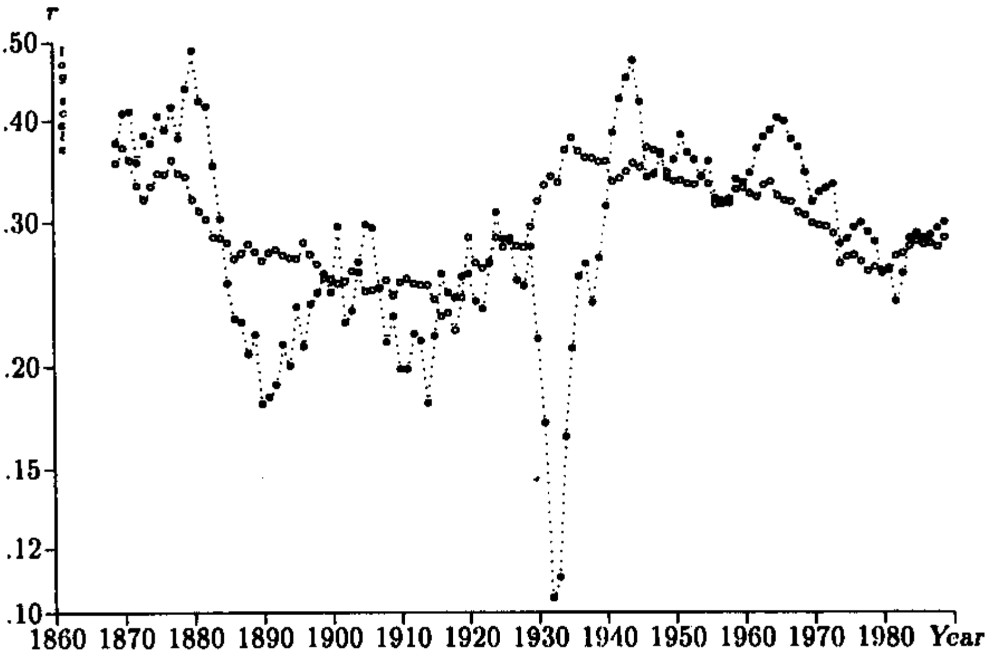


Figure 8: The profit rate: series (•) and model (○)

The additional following remarks can be made about the exact profile of the reconstruction. Obviously, the model does not account for business fluctuations, and one can observe, in particular, the deviation in the middle of the intermediate period corresponding to the Great Depression and World War II. A second observation is that there is a large fluctuation in the movement of the labor cost during the first period (see figures 1 or 5). This fluctuation is not reproduced by equation 4 which purports to represent the long-term variation of wages though it is very evident in the profile of the profit rate (see figures 4 or 8). (In a reconstruction using the actual profile of the labor cost, the fluctuation of the profit rate in the late 19th century is quite adequately reproduced, see Duménil G., Lévy D. 1993(b).) Lastly, one can notice that the (labor) productivity slowdown in recent decades is reproduced by the model, but only partially.

2.4 Confidence interval

The reconstruction in the previous section corresponds to only one possible sample path. One may wonder, however, to what extent the result obtained is contingent. The purpose of the present section is to provide an estimate of the confidence intervals for the estimations.

To this end, we will rerun the simulation in section 2.3, varying only the stochastic variables, keeping all parameters constant (including those characteristic of stochastic distributions). A total of 1000 trajectories are considered.

The results of this investigation are displayed in figure 9 for labor productivity. The two dotted lines represent the upper and lower bounds within which 95 percent of the resulting sample runs lie. As can be expected all paths originate from the same initial value, and progressively deviate with time. The distance between the bounds increases linearly with time.[6] In 1989 an interval of approximately ± 20 percent is obtained. These observations confirm the view that the trajectories depend on the exact drawing, but that this dependency does not question the basic properties of the model.

[6] With d denoting the distance between the two bounds and $\Delta t = t - 1868$, one has:

$$\ln d = \underset{(t\,=\,26.3)}{-3.31} + \underset{(t\,=\,31.2)}{1.004} \ln \Delta t$$

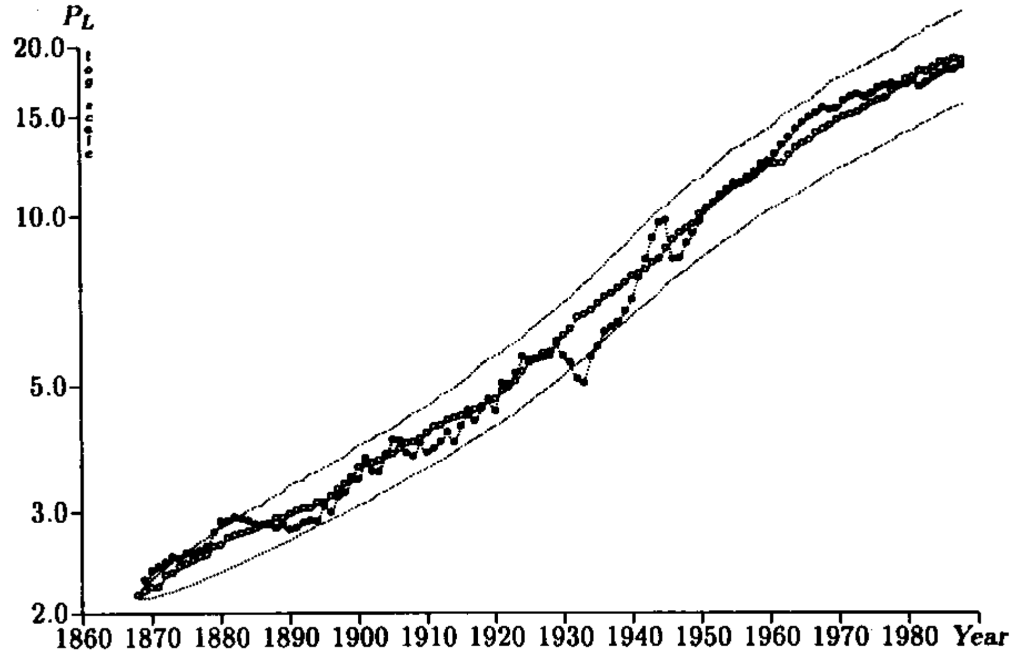


Figure 9: Confidence interval for labor productivity: series (•) and bounds (·)

3. THE DETERMINISTIC APPROXIMATION OF THE MODEL

This last part introduces the deterministic approximation of the stochastic model. The purpose of this investigation is to make explicit some determinants which lead to the various configurations observed in the previous parts. Section 3.1 is devoted to the average values of the rates of economizing on the two inputs a and l. The deterministic dynamical system is then established in section 3.2. The results of this investigation are presented in section 3.3, where the properties of the asymptotic trajectories are discussed with respect to the three parameters, the growth rate of the labor cost, ρ_w, and the two parameters that define the innovation set, R and δ. The proofs of the results given in section 3.3 are provided in the appendix.

3.1 Average values of a and l, and their relation to the wage share

In the deterministic approximation of the model, the values of the stochastic variables at each period are replaced by their average values. We further assume that the innovation set is small (R is small). Under this assumption, the linear development of the profit rate, as in equation 3, can be used.

The average values, $\bar{a}$ and $\bar{l}$, of a and l are defined as follows:

$$\bar{a} = \frac{1}{\pi R^2}\iint_{\Pi} a \, \mathrm{d}a \, \mathrm{d}l \quad \text{and} \quad \bar{l} = \frac{1}{\pi R^2}\iint_{\Pi} l \, \mathrm{d}a \, \mathrm{d}l.$$

Each is a function of the parameters, R and δ, and of the variables, A, L, and w. However, the dependence on A, L, and w is expressed indirectly, only *via* the slope of the selection frontier, $\mu = (1-\omega)/\omega$ and, thus, *via* the wage share ω: $\bar{a} = \bar{a}(\omega)$ and $\bar{l} = \bar{l}(\omega)$.

As shown in diagram 4, one must distinguish between the center of gravity of the profitable innovation set, G, which represents the average of actually adopted innovations, and $G' = (\bar{a}, \bar{l})$, which corresponds to average technical change. The distinction is due to the fact that innovation may fall outside the profitable innovation set, Π, and be rejected (in this case the old technique corresponding to the intersection of the axes, O, is maintained). G′ is the barycenter of G and O, weighted respectively by the surfaces of Π and of the remainder of the innovation set.

A few of the properties of $\bar{a}$ and $\bar{l}$ can be derived geometrically, independently of their explicit expression that will be given in the appendix. The following theorem sets forth some of these properties concerning the characteristics of technical change in relation to the variation of the wage share, *i.e.*, geometrically, by rotating the selection frontier between vertical (for $\omega = 0$) and horizontal (for $\omega = 1$).

Theorem:

(i) *One has:* $\bar{a}(\omega) = \bar{l}(1-\omega)$.

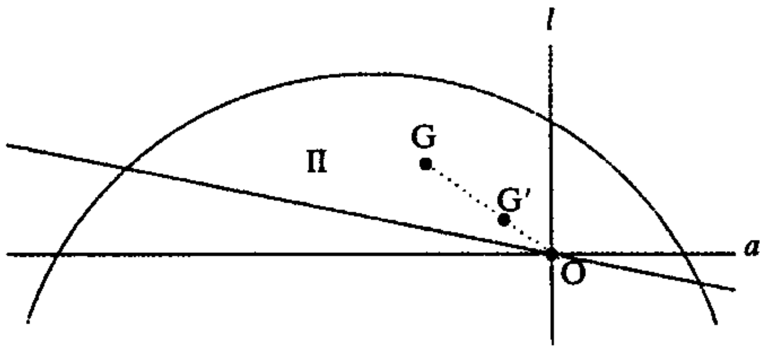


Diagram 4

(ii) *The average rate of economizing on capital, which is equal to the growth rate of the productivity of capital, $\bar{a}(\omega)$, is a decreasing function of ω.*

(iii) *The average rate of economizing on labor, which is equal to the growth rate of labor productivity, $\bar{l}(\omega)$, is an increasing function of ω.*

(iv) *If $\omega = 1/2$, i.e., if the wage share equals the profit share, then $\bar{a} = \bar{l}$, and both are positive.*

(v) *If $\delta < 0$, one has: $\bar{a}(1) = \bar{l}(0) < 0$.*

The proof of this theorem is straightforward (despite the distinction between G and G′). Statement (i) results from: (1) The slope of the frontier, $\mu = (1 - \omega)/\omega$, becomes $1/\mu$, if ω becomes $1 - \omega$, and (2) The innovation set is symmetrical with respect to the first bisector. (ii) and (iii) are evident, as suggested in panel (a) of diagram 5. The equation $\bar{a} = \bar{l}$ in the fourth property is a consequence of (i), for $\omega = 1/2$. The positivity is obvious in panel (b). The equation $\bar{a}(1) = \bar{l}(0)$ follows from (i). The sign of $\bar{a}(1)$ is obvious as shown in panel (c) of the diagram for $\delta < 0$ (the opposite sign prevails for $\delta > 0$).

The economic meaning of the second and third properties in the above theorem must be noted. With a given technology, the rise in the labor cost increases ω. The theorem establishes that, as a result of this rise, the average growth rate of the productivity of capital, $\bar{a}$, is lower, the average growth rate of labor productivity, $\bar{l}$, is higher, and the speed of the substitution of capital for labor is faster. This means that the typical connection between a rising labor cost and the rise of the capital-labor ratio can be established in a model in which the random local search for new techniques is followed by a selection based on profitability, independently of any *a priori* definition of a production function with factor substitution or bias in the innovation process. Since

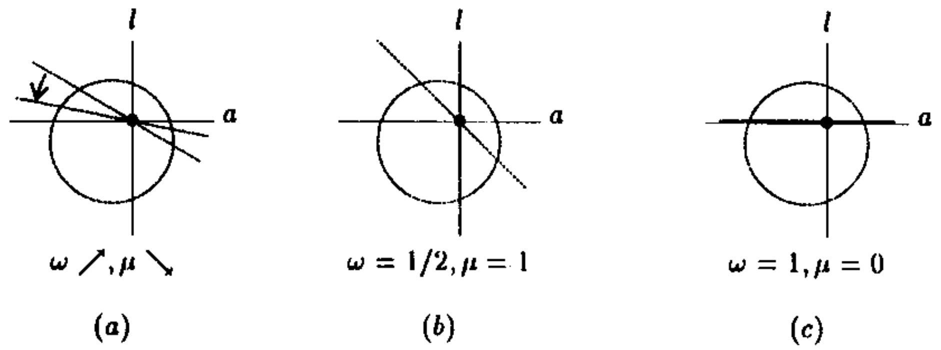


Diagram 5

negative values of $\bar{a}$ can be obtained, the model can account for a declining productivity of capital.

3.2 The dynamical system

After substituting the average values of innovations, $\bar{a}$ and $\bar{l}$, for their stochastic values, a and l, into equation 1, a dynamical system is obtained for the two variables that describe technology, A and L (or P_K and P_L). Under the assumption of a given growth rate of the labor cost, ρ_w, this dynamical system is actually easier to study if the variable L (or P_L) is replaced by the wage share ω ($\omega = w/P_L$):

$$\rho(\omega) = \rho_w - \bar{l}(\omega)$$
$$\rho(P_K) = \bar{a}(\omega). \tag{5}$$

One can notice that the first equation can be studied independently of the second. The fixed point in this equation is a situation in which the wage share is constant (and ω satisfies $\rho(\omega) = 0$) and, thus, the growth rate of labor productivity is also constant and equal to that of wages:

$$\bar{l}(\omega) = \rho_w. \tag{6}$$

As stated in the theorem above, $\bar{l}(\omega)$ is a monotonically increasing function of ω. Therefore, a fixed point exists and is unique, if ρ_w belongs to the interval $[\bar{l}(0), \bar{l}(1)]$. This fixed point corresponds in the second equation to a trajectory in which the productivity of capital, P_K, increases or diminishes asymptotically at a constant rate. Since $r = (1 - \omega)P_K$, the same is true for the profit rate ($\rho(r) = \rho(P_K)$). Since $\rho(K/L) = \rho(P_L) - \rho(P_K)$, the capital-labor ratio also grows at a constant (positive or negative) rate.

In continuous time, the stability of the fixed point is easy to prove. It follows from the third item in the theorem that determines the sign of the derivative of $\bar{l}(\omega)$, and proves that the sign of the (unique) eigenvalue of the Jacobian matrix is negative.

3.3 Results: Asymptotic trajectories

This section is devoted to the discussion of the asymptotic trajectories of the dynamical system with respect to the three parameters, ρ_w, R, and δ

(section 3.3.1). This framework is then applied in section 3.3.2 to the interpretation of the empirical investigation in part 2. The proofs are given in the appendix.

3.3.1 Discussion of the trajectories as functions of ρ_w, δ and R

The following results have been obtained in section 3.2:

1. *For given values of R and δ, and provided that the growth rate of the labor cost is neither too small nor too large, the variables converge toward an asymptotic trajectory in which all variables are either constant or changing at constant growth rates.*
2. *On an asymptotic trajectory, the wage share is constant (or labor productivity grows at the same constant rate as labor cost).*
3. *On an asymptotic trajectory, the productivity of capital and the profit rate grow at the same positive or negative constant rate. The capital-labor ratio also grows at a positive or negative constant rate.*

The present section discusses the value of the wage share and the sign of the growth rates of the other variables on an asymptotic trajectory, as functions of the values of the parameters ρ_w, δ, and R. This discussion is easier when performed *vis-à-vis* the two ratios δ/R and ρ_w/R (or even more conveniently $\pi\rho_w/R$).

As shown in diagram 6, four frontiers, F_0, F_1, F_2 and F_3, delineate three regions, A, B, and C, in the plane (δ/R, $\pi\rho_w/R$):

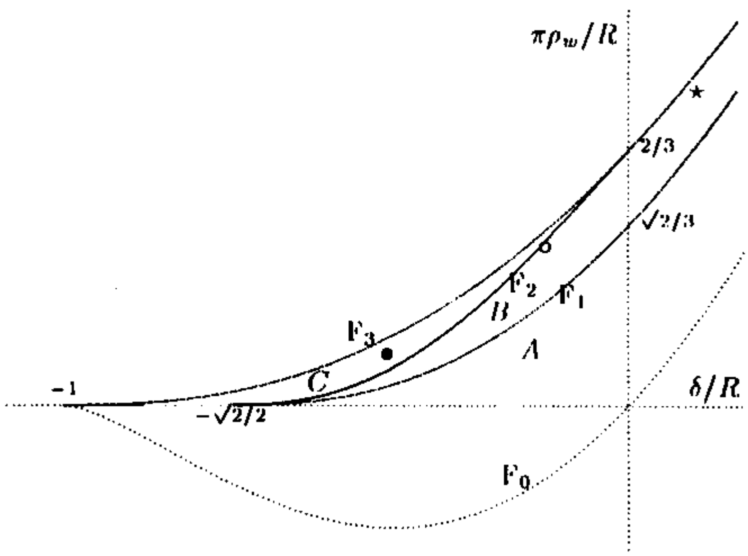


Diagram 6

F_0: This frontier corresponds to the lower boundary of region A. It corresponds to a situation in which the wage share is equal to zero, and the slope of the selection frontier is infinite ($\omega = 0$ or $\mu = \infty$).

F_1: This frontier delineates the boundary between regions A and B. On F_1, the profit and wage shares are equal ($\omega = 1/2$ or $\mu = 1$). In this case, the profitable innovation set is symmetrical with respect to the first bisector, and technical change is not biased. Both labor productivity and the productivity of capital grow at the same rate, which is equal to that of labor cost: $\bar{l} = \bar{a} = \rho_w$ and, consequently, $\rho(K/L) = 0$.

F_2: This frontier separates regions B and C. F_2 is defined by a constant productivity of capital and profit rate: $\rho(P_K) = \rho(r) = \bar{a} = 0$. It entirely belongs to the region $\delta/R < 0$, $\pi\rho_w/R > 0$.

F_3: This frontier defines the upper boundary of region C. The wage share is equal to 1, and the slope of the selection frontier is equal to zero ($\omega = 1$ or $\mu = 0$).

The three frontiers F_0, F_1, and F_3 correspond to three different values of the wage share, 0, 1/2, and 1. Frontier F_2 is associated with the asymptotically constant productivity of capital and profit rate. On F_2, as in all asymptotic trajectories, the wage share is constant, but its value differs, depending on the specific point chosen on this frontier. As is evident from diagram 6, this wage share varies between 1/2 and 1.

Note that, for a given value of δ/R, *i.e.*, on a vertical line in diagram 6, the wage share rises monotonically with $\pi\rho_w/R$, from 0 on F_0 to 1 on F_3 (this follows directly from the theorem in section 3.1 and equation 6). This property allows for the exclusion of both regions below F_0 and above F_3, and the characterization of the others:

Region A: In this first region the productivity of capital and the profit rate grow faster than labor productivity ($\bar{a} > \bar{l}$), the capital-labor ratio declines ($\rho(K/L) < 0$), *i.e.*, *labor is substituted for capital* and the wage share is between 0 and 1/2 ($0 < \omega < 1/2$).

Regions B and C: In these two regions, labor productivity grows faster than the productivity of capital ($\bar{l} > \bar{a}$), the capital labor ratio increases ($\rho(K/L) > 0$), *i.e.*, *capital is substituted for labor*, and the wage share is between 1/2 and 1 ($1/2 < \omega < 1$).

Region B: The productivity of capital and the proft rate rise ($\rho(P_K) = \rho(r) = \bar{a} > 0$).

Region C: The productivity of capital and the profit rate decline ($\rho(P_K) = \rho(r) = \bar{a} < 0$).

3.3.2 Interpretation of the results in part 2

The analysis in the previous section provides a useful framework for the interpretation of the empirical investigation in part 2.

Two dynamics are at work: (1) The process described by system 5, a comparatively fast dynamic, and (2) The variation of the three parameters, ρ_w, R, and δ, which is slower. We assume that the first dynamic has approximately converged.[7] This means that the economy is never very far from the asymptotic trajectory corresponding to the present value of the three parameters.

In this framework, each period can be represented in the plane (δ/R, $\pi\rho_w/R$) by a point corresponding to the parameters that describe its innovation set and growth rate of labor cost (*cf.* table 2), as shown in diagram 6:

1. For all periods, these points (•, ⋆, and ○) are located between F_1 and F_3, where capital is substituted for labor, as is reflected in the rising capital-labor ratio.
2. The point (○) for the entire period 1869–1989 is located close to frontier F_2. Over this period, both the productivity of capital and the profit rate display an approximately horizontal trend.
3. Only one point (•) is used to represent the situation in the first and third periods (for which R and δ are identical, and ρ_w differs only very slightly). The point (⋆) denotes the situation in the intermediate period. These points illustrate the movement from region C to B, and then back to C, corresponding to the declining, increasing, and declining productivity of capital and profit rate.
4. The positive slope of the segment around which the movement occurs echoes the simultaneous increases of δ/R and ρ_w/R during the intermediate period. There was a compensation between these two effects that neutralized to a certain extent the variation of the wage share between the three periods.

CONCLUSION

The analysis of this paper has led to both *theoretical* and *empirical* conclusions, that will be considered now.

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[7] This method is that of temporary equilibrium, but in an unusual time frame.

It is possible to build a framework in which no *a priori* description of available techniques exists (neither a production function, nor a given set of fixed coefficient techniques). Technical progress results from the combination of a random neutral innovation process, followed by the selection of techniques which appear most able to facilitate survival in competition, *i.e.*, the most profitable. Innovation is "local" and the profile of the innovation set is controlled by two parameters. This framework of analysis is thoroughly different from that of the production function and, in particular, there is no distinction between substitution along such a production function, and the shift of this function. The rise of wages affects technology, stimulating the increase of the capital-labor ratio. In addition, the traditional constancy of the wage share is also obtained in our model for given growth rates of the labor cost and profiles of the innovation set. Finally, it appears that the explanatory power of the model is very broad, since it can account for the quite different profiles of the macroeconomic variables.

As far as the empirical investigation of technical change and distribution in the US economy is concerned, the demonstration in this paper can be summarized as follows:

1. There was a continuous tendency for wages to rise, the degree of which was conditioned by the levels and variations of the profit rate.
2. The progress of labor productivity paralleled that of wages.
3. Depending on the profile of the innovation set, different outcomes were possible: the productivity of capital and the profit rate could decline or rise.
4. Since the Civil War, the first configuration (the declining trend of the productivity of capital and the profit rate) was the most frequent. The second was observed in the intermediate period, from the early 20th century to the 1950s.
5. Globally, the historical trends of the variables correspond to a situation close to the boundary between the two cases above, with a nearly horizontal trend of the productivity of capital and profit rate.
6. For a given growth rate of the labor cost and a given innovation set, as was true in each of the three subperiods, the wage share tended to be constant. The growth rate of the labor cost and the innovation set varied between the three periods, but these transformations had offsetting effects on the wage share. Their combined effect was small, and this explains the approximately flat trend of the wage share over the entire period.

APPENDIX: ANALYTICAL DETERMINATION OF THE FRONTIERS

The purpose of this appendix is to study analytically the expressions for the frontiers F_0, F_1, F_2 and F_3. We first provide a few preliminary results, and then derive the properties stated in section 3.3.

Instead of considering directly the average growth rate of labor productivity and of the productivity of capital, $\bar{a}$ and $\bar{l}$, it is easier to begin with the two quantities, $\mu\bar{a} + \bar{l}$ and $\bar{a} - \mu\bar{l}$, which are simpler.[8] One obtains:

$$\mu\bar{a} + \bar{l} = \frac{R}{\pi}\sqrt{1 + \mu^2} f(\beta) \tag{7}$$

$$\bar{a} - \mu\bar{l} = \frac{R}{\pi}\sqrt{1 + \mu^2}\frac{1 - \mu}{1 + \mu}\beta f'(\beta). \tag{8}$$

In these equations, $\mu = (1 - \omega)/\omega$ and:

$$\beta = -\frac{\delta}{R}\frac{1 + \mu}{\sqrt{1 + \mu^2}} \tag{9}$$

$$f(\beta) = \frac{2 + \beta^2}{3}\sqrt{1 - \beta^2} - \beta \arccos \beta \tag{10}$$

and thus:

$$f'(\beta) = \beta\sqrt{1 - \beta^2} - \arccos \beta. \tag{11}$$

Equations 7, 8, 10, and 11, yield:

$$\bar{l} = \frac{R}{\pi}\frac{1}{\sqrt{1 + \mu^2}}\left(f(\beta) - \mu\frac{1 - \mu}{1 + \mu}\beta f'(\beta)\right) \tag{12}$$

$$\bar{a} = \bar{l} - \frac{2}{3}\frac{R}{\pi}\frac{1 - \mu}{\sqrt{1 + \mu^2}}(1 - \beta^2)^{3/2}. \tag{13}$$

The profiles of function f and its derivative, f', are simple: for $0 < \beta < 1$, $f'(\beta)$ is smaller than zero and $f(\beta)$ is a positive and decreasing function of its argument. This result can be proved without

[8] In their derivation, it is convenient to use the new variables (x, y) instead of (a, l), with:

$x = (\mu a + l + \delta(1 + \mu))/\sqrt{1 + \mu^2}$ and $y = (-a + \mu l + \delta(1 - \mu))/\sqrt{1 + \mu^2}$.

difficulty. One determines $f''(\beta) = 2\sqrt{1-\beta^2}$, which is larger than 0. Consequently, $f'(\beta)$ is an increasing function. With $f'(1) = 0$, one has $f'(\beta) < 0$ for $\beta < 1$. Since $f(1) = 0$, the decreasing function $f(\beta)$ is larger than 0 for $\beta < 1$.

Using equations 6 and 12, it is easy to determine the equation for the curves that represent the points in the plane $(\delta/R,\ \pi\rho_w/R)$, for which the wage share (and thus μ) has a given value on the asymptotic trajectory:

$$\pi\frac{\rho_w}{R} = \frac{1}{\sqrt{1+\mu^2}}\left(f(\beta) - \mu\frac{1-\mu}{1+\mu}\beta f'(\beta)\right) \tag{14}$$

with β given by equation 9. This equation permits the determination of the three frontiers F_0, F_1, and F_3, which correspond to particular values of ω:

1. Frontier F_0 is defined by $\omega = 0$ or $\mu = \infty$. For this value of μ, equation 14 yields:

$$\pi\frac{\rho_w}{R} = -\frac{\delta}{R}f'\left(-\frac{\delta}{R}\right). \tag{F$_0$}$$

   This frontier is located in the region $\pi\rho_w/R < 0$ if $\delta/R < 0$ or in the region $\pi\rho_w/R > 0$ if $\delta/R > 0$. Only one minimum exists, and $\pi\rho_w/R$ equals 0 for δ/R equals -1 or 0.
2. Frontier F_1 is defined by $\omega = 1/2$ or $\mu = 1$:

$$\pi\frac{\rho_w}{R} = \frac{1}{\sqrt{2}}f\left(-\sqrt{2}\frac{\delta}{R}\right). \tag{F$_1$}$$

   Since $f' < 0$, the slope of this frontier is positive. It intersects the two axes in $(-\sqrt{2}/2, 0)$ and $(0, \sqrt{2}/3)$.
3. Frontier F_3 is defined by $\omega = 1$ or $\mu = 0$:

$$\pi\frac{\rho_w}{R} = f\left(-\frac{\delta}{R}\right) \tag{F$_3$}$$

   F_3 can be derived from F_1 by a similarity centered in the origin, with a ratio $\sqrt{2}$. The slope of this frontier is also positive. The intersections with the axes are $(-1, 0)$ and $(0, 2/3)$.

The study of F_2 is slightly more complex. This frontier is defined by $\bar{a} = 0$. Its equation cannot be expressed explicitly. Equations 7, 9, and 13 allow for the following implicit system:

$$\pi\frac{\rho_w}{R} = \frac{2}{3}\frac{1-\mu}{\sqrt{1+\mu^2}}(1-\beta^2)^{2/3} \tag{15}$$

$$\pi\frac{\rho_w}{R} = \sqrt{1+\mu^2}f(\beta) \tag{16}$$

$$\beta = -\frac{\delta}{R}\frac{1+\mu}{\sqrt{1+\mu^2}}. \tag{17}$$

We substitute the value of β as in equation 17 into equation 16. After differentiation, one obtains:

$$\mathrm{d}\left(\pi\frac{\rho_w}{R}\right) = \frac{1}{\sqrt{1+\mu^2}}\left(\mu f(\beta) + \frac{1-\mu}{1+\mu}\beta f'(\beta)\right)\mathrm{d}\mu - (1+\mu)f'(\beta)\,\mathrm{d}\left(\frac{\delta}{R}\right).$$

From equations 7, 8, and $\bar{a} = 0$, it follows that:

$$\frac{\beta f'(\beta)}{f(\beta)}\frac{1-\mu}{1+\mu} = -\mu.$$

Consequently, the coefficient of $\mathrm{d}\mu$ in the above equation is equal to zero, and the equation is reduced to:

$$\mathrm{d}\left(\pi\frac{\rho_w}{R}\right) = -(1+\mu)f'(\beta)\,\mathrm{d}\left(\frac{\delta}{R}\right).$$

Since $f'(\beta) < 0$, the slope of F_2 is positive. The intersections with the two axes can be easily determined:

$$\frac{\delta}{R} = 0 \Rightarrow \beta = \mu = 0 \Rightarrow \pi\frac{\rho_w}{R} = \frac{2}{3}$$

and

$$\pi\frac{\rho_w}{R} = 0 \Rightarrow \beta = 1 \Rightarrow \frac{\delta}{R} = -\frac{\sqrt{2}}{2}.$$

One can see that F_2 intersects with either F_1 or F_3 on each axis.

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CEPREMAP
142 rue du Chevaleret
75013 PARIS
France