STABILITY IN CAPITALISM: ARE LONG-TERM POSITIONS THE PROBLEM?

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RÉSUMÉ

LA QUESTION DE LA STABILITÉ DANS LE CAPITALISME: L'ÉQUILIBRE DE LONG TERME EST-IL LE PROBLÈME?

Dans cette étude, nous proposons un cadre pour l'analyse de la stabilité des économies capitalistes, considérée globalement. Le comportement des agents est décrit en terme d'adjustement: ils prennent leurs décisions dans le déséquilibre, réagissant à l'observation du déséquilibre (ce que nous nommons "micro-économie de déséquilibre"). La conception de l'équilibre est celle d'un équilibre de long terme avec des prix de production. On distingue la stabilité du système vis-à-vis de la valeur relative des variables entre les branches (les proportions), de sa stabilité vis-à-vis du niveau général d'activité (la dimension). On construit un modèle de déséquilibre général, puis on déduit deux variantes concernant le court terme (équilibre par les quantités) et un modèle de long terme obtenu à l'issue d'une succession d'équilibres de court terme (succession d'équilibres temporaires). Il apparaît que le capitalisme est très stable vis-à-vis des proportions et instable en dimension, et que cette instabilité prend son origine dans les mécanismes caractéristiques du court terme.

ABSTRACT

THE STABILITY PROBLEM IN CAPITALISM: ARE LONG-PERIOD POSITIONS THE PROBLEM?

In this paper we construct a framework for the analysis of the stability of capitalist economies, considered globally. To this end, the behavior of economic agents is described in term of *adjustment*: agents make decisions within disequilibrium and react to the observation of disequilibrium (what we call disequilibrium microeconomics). The conception of equilibrium is that of a long-term equilibrium with prices of production. We distinguish the stability of the system with respect to the relative values of the variables among industries (proportions) and the stability of the general level of activity (dimension). We build a general disequilibrium model, from which we derive two variants, one concerning short-term equilibrium (by quantities) and the other capturing long-term equilibrium obtained as a result of a succession of short-term equilibria (sequence of temporary equilibria). Capitalism appears very stable with respect to proportions and unstable with respect to dimension, and this instability arises from mechanisms which are characteristic of short-term relations.

MOTS CLEFS : Ajustement, Dynamique, Stabilité, Équilibre de long terme, Équilibre temporaire, Prix de production, Cycle industriel, Crise.

KEYWORDS : Adjustment, Dynamics, Stability, Long-term equilibrium, Temporary Equilibrium, Production prices, Business cycle, Crisis.

J.E.L. Nomenclature: 020,130.

1 - GENERAL GUIDELINES, PURPOSE, AND RESULTS

In this preliminary part, we introduce the overall argument advanced in this study (section 1.1), as well as a summary of the main results (section 1.2).

1.1 GENERAL GUIDELINES AND PURPOSE

In subsection 1.1.1 we discuss the global characteristics of the stability problem and the dichotomy between short and long-term analyses. Subsection 1.1.2 deals with disequilibrium microeconomics as a requirement for the analysis of stability. Subsection 1.1.3 focuses on investment as a source of instability, and this discussion leads to the introduction of monetary and financial mechanisms.

1.1.1 The Overall Stability Problem: Short-Term vs. Long-Term, and Dimension vs. Proportions

Much attention has been devoted in economic theory to the *existence* of *equilibrium*. Best known is the research characteristic of the Walrasian paradigm. However, the same attitude is also evident in the work of the classical economists. (By "classical", we refer here to the work of Smith and Ricardo, and extend the notion to Marx.) For example, the properties of long-term equilibrium prices, or prices of production, have been the object of much investigation, in single and joint production formalisms.[1]

The analysis of *stability* has been the object of far less discussion and concern. In the Walrasian perspective stability problems were approached within the unrealistic framework of *tâtonnement*. Interest in the stability of the classical long-term equilibrium, and the construction of dynamic models, is only recent.

Before 1983, a widespread view concerning the classical analysis of competition was that the process described by the classics (A. Smith, *The Wealth of Nations*, London: Dent and Son (1776), ch. 7, D. Ricardo, *The Principles of Political Economy and Taxation*, London: Dent and Son (1817), ch. 4, K. Marx, *Capital, Volume III*, New York: First Vintage Book Edition (1894), ch. 10), did not work. An unpublished paper by H. Nikaido (H. Nikaido, Refutation of the Dynamic Equalization of Profit Rates in Marx's Scheme of Reproduction, Department of Economics, University of Southern California (1977), now available in H. Nikaido, "Marx on Competition", *Zeitschrift für Nationalökonomie*, 43 (1983), p. 337-362) advanced such a conclusion. A debate in France followed which came to similar

1. The empirical relevance of the concept of long-term equilibrium prices was demonstrated for the U.S. economy (and various European countries) in several studies by H. Ehrbar and M. Glick: Rates of profit tend to gravitate around a uniform rate (*cf.*, for example, H. Ehrbar, M. Glick, "Profit Rate Equalization in the U.S. and Europe: An Econometric Investigation", *European Journal of Political Economy, Europäische Zeitschrift für Politischeökonomie*, Special Issue, 4 (1988), p. 179-201 and "Structural Change in Profit Rate Differentials: The Post World War II U.S. Economy", *British Review of Economic Issues*, 10 (1988), Spring, p. 81-102, and M. Glick, H. Ehrbar, "Long-Run Equilibrium in the Empirical Study of Monopoly and Competition", *Economic Inquiry*, XXVIII (1990), p. 151-162).

negative conclusions.[2] At a conference in Paris in 1983, we presented our first model in which the possibility of the classical convergence process was demonstrated. (Our contribution G. Duménil, D. Lévy, "La concurrence capitaliste: un processus dynamique", *in* J.P. Fitoussi, P.A. Muet (eds.), *Macrodynamique et déséquilibres*, Paris: Économica, 1987, p. 127-154 is available in the proceedings of this Conference, J.P. Fitoussi, P.A. Muet, *Macrodynamique et déséquilibres*, Paris: Économica (1987)). An important step forward was realized during another conference organized in 1984 in Nanterre (France). A number of studies were presented which stirred a new interest in the topic and were prolonged in the following years.[3]

However, the analysis of stability must also address the determination of the general level of activity. Crisis and business cycle theory, in the works of Ricardo and Marx, remained partial and deficient in many respects. Marx's investigation of the business cycle is disseminated in the various parts of his work. Ricardo devoted chapter XIX of his *Principles* (D. Ricardo, *The Principles, op. cit.* note 1) to what he called the "states of distress" of industry. In modern economic theory, the determination of the general level of activity is the domain of macroeconomics.

Several criticisms can be leveled at Keynesian macroeconomics (and, *a fortiori*, at the new (neo)classical macroeconomics), in particular, its adoption of an equilibrium framework of analysis from which dynamics are excluded, to investigate an issue which basically involves a problem of *stability*.[4] A second criticism is that the determination of the general

2. A special issue of *Les cahiers d'économie politique* was devoted to this question, with contributions such as C. Benetti, "La question de la gravitation des prix de marché dans *La Richesse des Nations*", *Cahiers d'économie politique*, 6 (1981), p. 9-31 and J. Cartelier, "Marchandise homothétique, capital financier et loi de Say: de la convergence des prix de marché vers les prix naturels", *Cahiers d'économie politique*, 6 (1981), p. 33-52.

3. R. Arena, C. Froeschle, D. Torre, "Gravitation et reproductibilité: Un point de vue classique", *in* C. Bidard (ed.), *La Gravitation*, Université de Paris X-Nanterre: Cahiers de la R.C.P. "Systèmes de prix de production" (C.N.R.S.) num. 2-3, 1984 available in R. Arena, C. Froeschle, D. Torre, "Formation des prix et équilibre classique, Un examen préliminaire", *Revue économique*, 39 (1988), p. 1097-1117, L. Boggio, "Convergence to Production Prices Under Alternative Disequilibrium Assumptions", *in* C. Bidard (ed.), *La Gravitation*, Université de Paris X-Nanterre: Cahiers de la R.C.P. "Systèmes de prix de production" (C.N.R.S.) num. 2-3, 1984 (see also L. Boggio, "On the Stability of Production Prices", *Metroeconomica*, 37 (1985), p. 241-267 and "Stability of Production Prices in a Model of General Interdependence", *in* W. Semmler (ed.), *Competition, Instability, and Nonlinear Cycles*, Berlin: Springer Verlag, 1986), P. Flaschel, W. Semmler, "Classical and Neoclassical Competitive Adjustment Processes", *The Manchester School*, LV (1987), p. 13-37, R. Franke, "Some Problems Concerning the Notion of Cost-Minimizing Systems in the Framework of Joint Production", *The Manchester School*, LIV (1986), p. 298-307 (see also R. Franke, *Production Prices and Dynamical Processes of the Gravitation of Market Prices*, Frankfurt am Main: Peter Lang (1987)), and our own contribution G. Duménil, D. Lévy, "The Dynamics of Competition: A Restoration of the Classical Analysis", *Cambridge Journal of Economics*, 11 (1987), p. 133-164, The Stability of Long-Term Equilibrium in a General Disequilibrium Model, Cepremap, 8717, Paris (1987) and "The Analytics of the Competitive Process in a Fixed Capital Environment", *The Manchester School*, LVII (1989), p. 34-57).

4. Keynesian economists attempted to provide macroeconomics with dynamic foundations (P.A. Samuelson, "Interactions between the Multiplier Analysis and the Principle of Acceleration", *Review of Economic Statistics*, (May 1939), L.A. Metzler, "The Nature and Stability of Inventory Cycles", *Review of Economic Statistics*, III (1941), p. 113-129, M. Lovell, "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle", *Econometrica*, 29 (1961), p. 293-314 and "Buffer Stocks, Sales Expectations, and Stability: A Multi-Sector Analysis of the Inventory Cycle", *Econometrica*, 30 (1962), p. 267-296, J. Tobin, "Keynesian Models of Recession and Depression", *The American Economic Review*, Papers and Proceedings of the AEA (1975), and E. Malinvaud, *Théorie macro-économique, tome 2*, Paris: Dunod (1982)), but this approach never obtained a real recognition in the profession, probably because of its inability to incorporate the other aspects of economic theory and confront neoclassical microeconomics.

level of activity is treated separately from that of other variables such as relative prices or levels of output among industries.

It is our contention that *stability* is a crucial issue, and that it must be addressed *globally*, in a dynamic model in which disequilibrium is considered. The relevance of the dichotomy between the stability of the classical long-term equilibrium and the stability of the general level of activity cannot be asserted *ex ante*. The problem is to articulate the two types of approaches: stability of long-term equilibrium and stability of the general level of activity. This is what we will attempt *on classical foundations* in a dynamic model. In this project, at least one element of Keynesian macroeconomics will, however, be preserved: the crucial role given to the volatility of investment.

In the discussion of the overall stability problem *the distinction between long term and short term is certainly central*. Keynesian macroeconomics gave to this distinction a conventional and simple meaning, which we will adopt. In the short run, the capital stock is given, and investment is only taken into account as a component of demand. It is in the long run, that the capital stock is be allowed to vary. To this traditional distinction, we add a second, which distinguishes *proportions* and *dimension*. By "proportions", we denote the relative values of the variables among industries: relative prices, outputs, stocks of capital, etc. By "dimension", we mean the general level of these variables and, in particular, the general level of activity. What we call "stability in proportions" and "stability in dimension" are two fundamental aspects of the stability problem. It is clear, for example, that the classics in their analysis of the formation of prices of production are concerned with proportions, and that Keynes abstracted from proportions to concentrate on dimension. (The relationship between proportions and dimension has been the object of much discussion. Disproportion has often been presented as the cause of instability in dimension. Traditional macroeconomics are based on the denial of this relation, since only one commodity is considered.)

In this study we first present a *global model* in which long-term and short-term relations, as well as proportions and dimension, can be discussed. Specifically addressed is the relation between these various aspects of the stability problem. This desire to cover the stability problem globally in a manageable model will require a number of simplifying assumptions. In particular, we will only consider two commodities and assume simple reproduction. We view this model as the "smallest" framework in which the stability problem can be discussed globally. Nevertheless, the model remains rather complex. For example, it is necessary to consider fixed capital to be able to distinguish between the short and long terms and, as will be shown below, it also necessary to provide a treatment of monetary and credit mechanisms.

By "global model", we mean here a model in which the various aspects of the stability problem can be adressed: proportions and dimension, as well as short and long runs. We do not refer to the study of global stability, as opposed to local. In this paper, stability is always meant as "local stability".

1.1.2 Disequilibrium Microeconomics

In order to study the stability of an equilibrium, it is necessary to consider disequilibrium. The stability problem refers to the convergence from disequilibrium to equilibrium.[5]

5. Another approach to the same issues is that of *gravitation*. In gravitation, an economy subject to random shocks remains in a vicinity of equilibrium. From a mathematical point of view, the two problems are equivalent.

Disequilibrium means:

1. The existence of unequal profit rates among industries.

2. Involuntary inventory accumulation. (Disequilibrium on the commodity market means that supply differs from demand and, therefore, that inventories of unsold commodities exist.)

3. Capacity utilization rates which deviate from normal levels. (This variable is rarely considered in theoretical models, but evident in applied economics.)

The economic forces which push the variables toward their equilibrium values are the expression of the individual behavior of agents reacting to disequilibrium. A famous example, in the classical analysis, is when capitalists move their capital from one activity to another, following profitability differentials. Another example is that of enterprises who diminish prices as a result of excess supply. We generalize this approach to decision making — known as "adjustment" — to all types of decisions. Adjustment has always played a role in economic theory, but has never been given central place in the analysis.[6] We call "disequilibrium microeconomics", microeconomics which are extensively based on adjustment to disequilibrium. The general principle of adjustment can be represented as follows:

$$\cdots \rightarrow \begin{pmatrix} \textit{Evidence of} \\ \textit{disequilibrium} \end{pmatrix} \rightarrow \begin{pmatrix} \textit{Modification of} \\ \textit{behavior} \end{pmatrix} \rightarrow \cdots$$

An adjustment procedure can be characterized in three respects: 1) The nature of the disequilibrium considered, 2) The variable which is modified, and 3) The sensitivity to disequilibrium which can be modeled by a "reaction coefficient". Reaction coefficients play an important role in this study. In particular, the conditions for stability are defined with respect to reaction coefficients.

We will not address in this study the issue of *the rationality of adjustment behaviors*, and we will never discuss the determination of reaction coefficients (in particular, their optimal values). All adjustment equations in this paper will be presented as representations of *sensible* behaviors. We devoted two studies (G. Duménil, D. Lévy, The Rationality of Adjustment Behavior in a Model of Capital Allocation: The Classical Investment Function, Cepremap, Larea-Cedra, Paris (1989) and G. Duménil, D. Lévy, "The Rationality of Adjustment Behavior in a Model of Monopolistic Competition", *in* R.E. Quandt, D. Triska (eds.), *Optimal Decisions in Markets and Planned Economies*, Boulder: Westview Press, 1990, p. 224-242) to the derivation of adjustment behaviors as rational behaviors, for the allocation of capital, output, and price decisions (*cf.* equations 6, 7, and 8).

Our models are "general", as opposed to "partial" and can be called "general disequilibrium models". Prices depend on quantities (supply and demand); Capital mobility is guided by profit rates and, thus, depends on prices; Demand is endogenous and is a function of income (consequently of output); the complete loop *Production* → *Demand* and *Demand* → *Production* is modeled.

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6. Adjustment has been used in the Keynesian dynamic models mentioned in a previous footnote and survived in a few heterodox contributions such as J. Kornai, B. Martos, "Autonomous Control of the Economic System", *Econometrica*, 41 (1973), p. 509-528 and *Non-Price Control*, Amsterdam: North Holland (1981), R.H. Day, T. Groves, *Adaptive Economic Models*, New York: Academic Press (1975) (ed.), J. Kornai, *Economics of Shortages*, Amsterdam: North Holland (1980), and A. Simonovits, "Dynamic Adjustment of Supply Under Buyer's Forced Substitution", *Zeitschrift für Nationalökonomie, Journal of Economics*, 45 (1985), p. 357-372.

1.1.3 Investment, Money, and the Capital Constraint

Investment is central in the classical analysis of competition which results in the convergence toward long-term equilibrium. It is interesting to notice that it is also prominent in Keynesian macroeconomics, where investment is depicted as the basic source of instability in capitalism. However, the difference between the Keynesian and classical treatments of investment is striking. In Keynesian models, investment is determined by *demand* prospects, without any reference to the limitation of financial resources.[7] In the classical analysis, investment is subject to a *capital constraint*: the previous accumulation of a limited amount of capital, which can be withdrawn from one industry and accrued to another.

It is our contention that the missing link, between an accumulation model with a capital constraint and a model in which the volatility of investment is primary, is in the existence of money and credit, thus avoiding Say's Law. In order to preserve the classical notion of long-term analysis, it is necessary to assume that enterprises make their investment decisions under the constraint of the availability of capital. But this mechanism must be combined with that of credit. The banking system makes loans to enterprises, who then invest out of this fund. The inducement to borrow responds, as in the Keynesian analysis, to demand expectations (which we measure by the capacity utilization rates). The total amount of credit in the economy is subject to institutional constraints whose forms have greatly evolved historically (from the Gold Standard to modern monetary policy) and to the individual behavior of lenders. The capital constraint is, thus, displaced by credit relations though not suppressed. In our model, these mechanisms are very simply represented by the response of monetary authorities to the variations of the general level of prices.

1.2 RESULTS

In this section, we present the basic results we obtain. In subsection 1.2.1, the overall problem of stability in capitalism is addressed. Some elements of comparison with the classical and Keynesian perspectives are recapitulated in subsection 1.2.2.

1.2.1 Stability in Capitalism

An important result of our investigation is that it is possible to construct a model of global stability and to analyze its properties. But it is also relevant to analytically separate the stability problem into its various aspects, proportions and dimension as introduced earlier, and short and long-term analyses.

The distinction between short and long terms refers to the comparative speed of variation of variables: rapid and slow:

1. In a short-term model derived from the global model, we disconnect long-term mechanisms such as the allocation of capital or the variation of prices. In the short term, adjustments are made by quantities. This analysis is closer to the Keynesian point of view than to the neoclassical perspective (where markets clear *via* prices).

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7. We will not discuss here the well-known fact that Keynes, in the *General Theory* (J.M. Keynes, *The General Theory of Employment, Interest and Money*, London: Macmillan (1936)), actually considers *profitability* prospects (the marginal efficiency of capital), and investment is pursued as long as the profit rate exceeds the interest rate).

2. In the long-term model, also derived from the global model, we assume that short-term equilibrium is obtained at each period. In other words, the route to long-term equilibrium is described as a succession of temporary equilibria.[8]

The main conclusion of this analysis can be summarized as follows: *The crucial problem concerning stability is posed in the short run, and is a problem of dimension. In the short run, as well as in the long run, stability in proportions can be assumed.*

These results imply that the stability of the general level of activity (the problem of business cycle fluctuations) is the weak point of capitalism. Recession is always around the corner. Conversely, the distribution of capital among industries, the availability of commodities on the market, and the setting of relative prices are issues that capitalism can handle rather efficiently.

Even in the simple model presented in this study, the stability problem appears complex. (In)stability in dimension results from the confrontation of various mechanisms: 1) The reaction of production to demand (decision to produce), 2) Some elements of demand which are stabilizing, and 3) Other elements of demand which are destabilizing.

In a model based on adjustment to disequilibrium, one might expect all behaviors to be stabilizing, since they act in a contrary fashion to disequilibrium. This is true for all components of demand with the exception of investment. Indeed, investments are made to correct disequilibria. But this correction will only operate *in the long run*. Investment is accelerated in order to correct for a high level of capacity utilization but, *in the short run*, this decision stimulates demand. A high level of activity provokes a decision which, itself, stimulates activity. The symmetrical situation is easy to imagine: A depressed activity discourages investment and this reaction further cuts into the level of demand.

One can say that investment is "procyclical" in the short run, since it tends to enlarge macroeconomic disequilibria, and "countercyclical" in the long run. We believe, as Keynes himself contended, that the procyclical aspect is dominant. This procyclical effect of investment is counteracted in the model by several countercyclical mechanisms, in particular the reaction of monetary authorities to the variations of the general level of prices.

The fact that (in)stability in dimension represents the main issue in capitalist economies is reflected in the model by the fact that the dominant eigenvalue of the Jacobian matrix is real and close to 1. The associated eigenvector can be easily determined and describes the main component of the dynamics of the system in the vicinity of equilibrium (gravitation, business cycle, and effects of policies). These departures from equilibrium follow quite specific patterns. (For example, a trade-off exists between the average capacity utilization rate and the ratio of inventories to sales, which can be verified empirically).

Money and credit mechanisms play a role in the condition for stability in dimension, but they have no impact on the position of short and long-term equilibria, as well as on

8. This perspective was adopted in H. Sonnenschein, "Price Dynamics Based on the Adjustment of Firms", *The American Economic Review*, 72 (1982), p. 1088-1096, R. Franke, "Some Problems Concerning", *op. cit.* note 3, and R. Arena, C. Froeschle, D. Torre, "Formation des prix", *op. cit.* note 3, but with a succession of temporary equilibria by prices. In the remainder of the literature in which temporary equilibria (Walrasian equilibria or equilibria with fixed prices, or generation models) are considered, the object of the investigation is different: 1) Existence of a temporary equilibrium, 2) Existence of a stationary temporary equilibrium, *i.e.*, repeated without alteration over time, or 3) Convergence toward a Walrasian equilibrium (and not a long-term equilibrium with prices of production).

the conditions for the stability in proportions of these equilibria. (However, they have an influence on the speed of convergence of short-term proportions.)

Although this discussion lies beyond the limits of the present investigation, it is obvious that this analysis provides a basis for a theory of the business cycle. A prominent element in business cycle theory is the acknowledgement that *stability is conditional*, and that capitalism perpetually tests the limits of this stability frontier: We view the business cycle as a succession of periods of stability and instability in a recurrent, but not repetitive, configuration. Since several mechanisms are involved in the condition for stability in dimension, it is not possible to pinpoint the same specific trigger in all crises. Individual management, technology, distribution, credit and monetary mechanisms are all equally likely suspects. Investment is the only procyclical mechanism, but other mechanisms are, depending on the circumstances, more or less countercyclical, and instability can result from a reduction of their stabilizing power.

1.2.2 A Comparison with other Paradigms

These results support Marx's separation between the stability of long-term equilibrium (proportions) and the theory of crisis (dimension), although this latter analysis was never developed into a coherent framework, and the two aspects of stability never articulated. This distinction is less clear in the work of Ricardo, where the "states of distress" of the industry are presented in relation to capital mobility. (This explains why Marx classified Ricardo as a theoretician of "disproportions".)

As was mentioned earlier, Keynes does not adopt the point of view of dynamics, and sticks to the conventional equilibrium approach. In this context, his attitude is radical and biased. Instead of rejecting traditional equilibrium microeconomics, he rejects microeconomics as a whole. Dimension is so dire a problem, that he abstracts from proportions. Capitalism is so unstable that he contends that no stabilizing mechanism exists which is capable of bringing the economy to a normal level of utilization of capacity. "Normal" equilibrium is a theoretical fiction, and the stability problem is confused with a problem of existence. Investment is so volatile, that it must be treated as exogenous, and the whole macroeconomic situation will be determined by "animal spirits", whereas the origin of instability should be sought within the economy itself.

2 - THE MODEL

This part is devoted to the presentation of the model. Section 2.1 introduces the general framework of analysis. In section 2.2, we present a number of simplifying assumptions which render the model manageable. Section 2.3 gives a general view of the basic relations excluding all disequilibrium microeconomics. These behavioral equations are presented in section 2.4. Last, the relation of recursion is defined in section 2.5.

2.1 THE GENERAL FRAMEWORK OF ANALYSIS

The agents are introduced in subsection 2.1.1. The sequence of events (production and market) and the treatment of the commodity market are made explicit in subsection 2.1.2. Monetary and credit phenomena are the object of subsection 2.1.3.

2.1.1 The Agents

Four groups of agents interact in the model: Wage earners, Capitalists, Enterprises, and Banks.

Wage earners sell their labor power to enterprises and then produce. In exchange, they receive a wage in cash. They use their purchasing power to buy consumption goods.

Capitalists control the movement of capital (the cash flow of enterprises, *i.e.*, depreciation allowances and profits). They subtract from these sums a given fraction for their personal consumption. The remainder is re-invested, *i.e.*, is allocated among the different enterprises. With respect to consumption, their behavior is not different from that of wage earners.

Enterprises organize production and make decisions concerning prices and outputs. They pay wages and transfer the cash flow to capitalists. They receive funds from the capitalists and make investment decisions.

Concerning *banks*, no distinction is made between commercial banks and a central bank. The behavior of the banking system in the model is supposed to account globally for its various components.

2.1.2 Production and Market

The model is a *sequential* model. Production and market follow one another:

$$\cdots \to \textit{market } t \to \textit{production } t+1 \to \textit{market } t+1 \to \cdots$$

Traditionally, the commodity market and its relation to production are treated in two basic manners:

1. *Equilibrium by prices (Walrasian paradigm).* Supply *and/ or* demand are functions of prices. The market (actually, the auctioneer) determines prices which insures its clearing.

2. *Equilibrium by quantities (Keynesian paradigm).* Demand is expressed in a preliminary stage of the market and enterprises supply exactly what is demanded. At a macroeconomic level, exogenous demand is expressed first and output is fixed at the precise level which equalizes supply and demand (exogenous and endogenous).

The point of view that we adopt is that of *Disequilibrium.* Production takes time, and is decided before demand is expressed. Supply differs from demand and, before equilibrium is reached, involuntary inventories exist. The *accounting* relation which yields the new value of inventories S_{t+1}, as a function of output Y_t, the initial stock of inventories transmitted from the previous period S_t, and demand D_t, in enterprise i, is the following:

$$S_{t+1}^i = S_t^i + Y_t^i - D_t^i \tag{1}$$

Supply is equal to the sum *Initial stock of inventories* + *Output*: $S_t + Y_t$.

2.1.3 Money, Credit, and Policies

Monetary mechanisms are very complex and treated in the model in a quite primitive framework.

Money stocks could be held by the three groups of agents, wage earners, capitalists, and enterprises. We never consider the stock of money held by enterprises, which is required to finance production and financial relations with capitalists. We assume that enterprises obtain these liquidities from the banks when necessary and deposit their receipts when they are cashed. No balance is conserved from one period to the next. We also assume that enterprises spend for investment the totality of the purchasing power that they receive from capitalists and banks. Therefore, enterprises do not transmit any balance of money at all from one period to the next. The purchasing power of consumers (wage earners and capitalists) results from their income and bank loans. They conserve a fraction of their purchasing power from one period to the next, and a stock of money is, thus, transmitted from period to period.

Three types of credits are considered, for investment, consumption, and operating and financial expenses of enterprises:

1. Banks lend to enterprises which are willing to invest beyond the possibilities available from the allocation of capital by capitalists. The quantity of loans is the result of a negotiation between enterprises and banks. The desire by firms to borrow is measured in the model by their capacity utilization rate. The control of monetary phenomena is achieved by the banking system which reacts to the variations of the general level of prices.

2. Banks also make loans to final consumers and react, in the same manner as for investment, to the variations of the general level of prices, but this type of loan is not essential in the model. They are considered to insure that monetary policy does not favor any industry.

3. Enterprises can always withdraw funds from the banks (short-term borrowings) to finance production or transfer their cash-flow to capitalists. They are never restricted in this respect.

It is implicit in the above presentation of monetary mechanisms that two different channels of funds are considered for the financing of enterprise activity. A first channel corresponds to investment, and a second to short-term transactions. Enterprises can only invest on the basis of funds allocated by capitalists or loans from the banks specifically earmarked for this purpose.[9] Enterprises are always constrained by the availability of funds (under normal circumstances, profitable investment opportunities always exist). They experience what we called in the first part of this study a "*capital constraint*". The second channel corresponds to short-term transactions. Enterprises could be subject to the existence of a *liquidity constraint*, in the sense that they might be obliged, for example, to scale down their activity because of a lack of liquidity. We assume that they never face such situations.

Since final consumers carry over a stock of money from one period to the next and since prices are adjusted, the purchasing power of this balance of money varies over time, and there is a real balance effect in the model. We do not believe, however, that this mechanism is significant. An analysis of this effect requires a more sophisticated treatment

9. Thus, in the enterprise's balance sheet, the sum *Equity* + *Loans for investment* is always equal to the value of fixed capital.

of monetary and financial balances and, in particular, the consideration of debts held by enterprises. In a model in which only internal money is considered, no overall real balance effect exists, only a redistribution of wealth among agents.

In the traditional simple form of the Keynesian model, in particular the IS-LM model, one usually distinguishes between monetary and demand policies. Monetary policy impacts on investment *via* the interest rate, and demand policy is treated as an autonomous increment of consumption. In our model, monetary policy affects investment directly *via* the availability of financing. There is no demand policy since loans to final consumers (in which the state can be included) only respond to the variations of the general level of prices, but it would be easy to add in the model a countercyclical demand policy in relation to the deviations of the capacity utilization rate.

2.2 SIMPLIFYING ASSUMPTIONS

1. Only two industries are considered and in each industry there is only one enterprise. The first industry produces the fixed capital good, and the second the consumption good. We abstract from circulating inputs other than labor. Production combines fixed capital and labor with constant returns. The actual use of one unit of fixed capital can be represented as follows:

$$1\begin{pmatrix}\textit{unit of}\\ \textit{fixed capital}\end{pmatrix} + l^i\begin{pmatrix}\textit{units of}\\ \textit{labor}\end{pmatrix} \rightarrow b^i\begin{pmatrix}\textit{units of}\\ \textit{good } i\end{pmatrix} + (1-\delta^i)\begin{pmatrix}\textit{unit of}\\ \textit{fixed capital}\end{pmatrix}$$

Parameter δ^i denotes the loss of productive power of fixed capital (its depreciation) during the production period. If this unit of fixed capital is used at a capacity utilization rate smaller than 1 ($0 \le u^i \le 1$), the model is the following:

$$1\begin{pmatrix}\textit{unit of}\\ \textit{fixed capital}\end{pmatrix} + l^i u^i\begin{pmatrix}\textit{units of}\\ \textit{labor}\end{pmatrix} \rightarrow b^i u^i\begin{pmatrix}\textit{units of}\\ \textit{good } i\end{pmatrix} + (1-\delta^i)\begin{pmatrix}\textit{unit of}\\ \textit{fixed capital}\end{pmatrix}$$

In a model in which disequilibrium is considered, the capacity utilization rate is a crucial variable (*cf.* figure 1 below). For a stock of fixed capital K^i_t held by enterprise i, its productive capacity is $K^i_t b^i$, its production $Y^i_t = K^i_t b^i u^i_t$, the quantity of labor used $K^i_t l^i u^i_t$, and depreciation allowances are equal to $\delta^i K^i_t p^1_t$.

2. Technology is the same in the two enterprises ($l^1 = l^2$, $b^1 = b^2$, and $\delta^1 = \delta^2$), and their behavior can be described by the same equations.

3. Labor is assumed to be available without limitation, and no other nonreproducible resources exist.

4. The real wage, denoted $\overline{w}$, is assumed to be constant. We use as an auxiliary notation, $w = \overline{w}l$, the real wage for each unit of fixed capital actually used. Wages are paid before production occurs, *i.e.*, on market t, for production occuring in period $t+1$. For enterprise i, one obtains:

$$W^i_{t+1} = K^i_{t+1} w u^i_{t+1} p^2_t$$

5. Profits in enterprise i can be determined as *Sales* − *Wages* − *Depreciation allowances*:

$$\pi^i_t = K^i_t\left(b u^i_t p^i_t - w u^i_t p^2_t - \delta p^1_t\right)$$

6. Capitalists spend all profits for consumption. The mobility of capital only concerns depreciation allowances. Thus, simple reproduction is obtained when equilibrium is reached, and investment is limited to replacement. Strict simple reproduction is not guaranteed in a disequilibrium, because of the existence of loans for investment.

7. Only one capitalist exists, or all capitalists are identical and can be aggregated.

8. Final consumers (wage earners and capitalists) are treated globally, with a propensity to consume a fraction α of their total purchasing power. Thus, only one stock of money will be transferred from one period to the next (*cf.* subsection 2.1.3).

9. Interest rates are set at zero.

1. Notation

Indexes:

i	*Index of a firm (superscript): $i = 1$, fixed production good, and $i = 2$, consumption good*
t	*Index of the period (subscript)*

Variables:

j	*Inflation rate, $j = Y_t p_t / Y p_{t-1} - 1$*
K	*Stock of fixed capital*
$\mathcal{K}$	*Capital reallocated (a purchasing power)*
$\Delta\mathcal{K}$	*Net loans for investment*
M^f, M	*Stocks of money held by final consumers before and after consumption*
ΔM	*Net loans to final consumers (Variation of loans outstanding during the period)*
p	*Price*
π	*Profits*
r	*Rate of profit*
ρ	*Rate of growth of the stock of fixed capital*
$s, \overline{s}$	*Ratio of inventories, Target ratio of inventories*
$u, \overline{u}$	*Capacity utilization rate, Target capacity utilization rate*
W	*Total wages*
x	*Relative prices, $x = p^1/p^2$*
Y	*Output, $Y = Kbu$*
y	*Relative stocks of fixed capital, $y = K^1/K^2$*
z	*Normalized stock of money, $z_{t+1} = M_t^f / K_{t+1}^2 p_t^2$*

Parameters:

α	*Propensity to consume*
b	*Output obtained with one unit of fixed capial for $u = 1$*
δ	*Proportion of the productive capacity of fixed capital lost in one period*
l	*Quantity of labor combined with one unit of fixed capital for $u = 1$*
$\overline{w}$	*Real wage*
w	*Real wage per unit of fixed capital for $u = 1$ ($w = \overline{w}l$)*

Reaction Coefficients:

β	*Price decision (sensitivity to $s - \overline{s}$)*
ε	*Decision to produce (sensitivity to $s - \overline{s}$)*
γ	*Allocation of capital (sensitivity to profit rate differentials)*
ω	*Borrowings for investment (sensitivity to $u - \overline{u}$)*
φ'	*Response of monetary policy to the variation of prices*
φ	*Auxiliary notation: $\varphi = \varphi'\beta$*
σ	*Decision to produce (stickiness of $u - \overline{u}$)*
θ	*Defined in equation 11 (condition for stability in dimension)*

2.3 THE MODEL WITHOUT REACTIONS TO DISEQUILIBRIUM

In this section we present the basic relations of the model abstracting from reactions to disequilibrium. We begin with financial relations between capitalists and enterprises, and between enterprises and banks. The availability of these funds allows enterprises to invest. Demand in the capital-good industry is, thus, determined. Next, we analyze the formation of the purchasing power of consumers, wage earners and capitalists, which is composed of wages, profits, and loans. Consumption goods are purchased from this fund.

Since simple reproduction is assumed, only depreciation allowances are reallocated among the two industries. With K_t denoting total fixed capital in t (in physical terms), the depreciation allowances of enterprise i are $\delta K_t^i p_t^1$, and the total purchasing power $\mathcal{K}$ which is reallocated is:

$$\mathcal{K}_{t+1} = \delta \left(K_t^1 + K_t^2 \right) p_t^1 = \delta K_t p_t^1$$

This total is divided by capitalists into two fractions $\mathcal{K}_{t+1}^1$ and $\mathcal{K}_{t+1}^2$, with $\mathcal{K}_{t+1} = \mathcal{K}_{t+1}^1 + \mathcal{K}_{t+1}^2$. Each industry also receives loans from the bank, which we denote $\Delta\mathcal{K}_{t+1}^i$. This purchasing power (allocated capital plus net loans) is available and used for investment. Gross investment, I_{t+1}^i, of enterprise i, purchased on market t, to be used for production in period $t+1$, is:

$$I_{t+1}^i = \frac{\mathcal{K}_{t+1}^i + \Delta\mathcal{K}_{t+1}^i}{p_t^1} \tag{2}$$

which is added to the depreciated stock of capital in this industry:

$$K_{t+1}^i = (1-\delta)K_t^i + I_{t+1}^i$$

Thus, total demand in the first industry can be determined as:

$$D_t^1 = I_{t+1}^1 + I_{t+1}^2$$

At the end of the previous period, final consumers, capitalists and workers, hold a given stock of money M_t. This purchasing power is increased by wages, W_{t+1}^i, and profits, π_t^i, and net consumer credit ΔM_t. Thus, consumers finally hold:

$$M_t^f = M_t + \pi_t^1 + \pi_t^2 + W_{t+1}^1 + W_{t+1}^2 + \Delta M_t \tag{3}$$

They spend a given fraction, α, of their purchasing power. Thus, αM_t^f is spent and the remainder, $(1-\alpha)M_t^f$, is held until the next market period where it represents the new initial stock of money M_{t+1}. The demand for consumption goods on market t is:

$$D_t^2 = \frac{\alpha M_t^f}{p_t^2} \tag{4}$$

In the assumptions of the model, a number of decisions are automatic: All profits are spent for consumption, all depreciation allowances are reallocated, all funds made available to enterprises are invested, and a given fraction of the purchasing power of consumers is spent for consumption. However, it is necessary to specify: 1) The capacity utilization rates u_{t+1}^i (or outputs), 2) The prices p_{t+1}^i, 3) The capital, $\mathcal{K}_{t+1}^i$, allocated by capitalists to each industry, 4) The loans to enterprises and consumers ($\Delta\mathcal{K}_{t+1}^i$ and ΔM_t).

If one assumes that the value of these variables is maintained from one period to the next, a dynamic model is obtained. It is obvious, however, that its dynamics are divergent. The stock of capital varies and can diverge to zero or to infinity, and supply is not equal to demand. Thus disequilibrium will prevail and will be persistent.

In the following part of this study, we will model the behavior of economic agents responding to disequilibrium, with respect to the four decisions above: 1) and 2) How capacity utilization rates and prices are determined by enterprises, 3) How capitalists proportion their allocation according to profitability differentials, 4) How loans for investment are subject to the needs of enterprises, and how these loans and loans to consumers are both subject to the goals of monetary policy.

2.4 THE MODELING OF BEHAVIORS

We now turn to the modeling of adjustment to disequilibrium. We will first describe the disequilibria to which the agents react and, then, consider successively the agents' various decisions.

2.4.1 Evidences of Disequilibrium

Four disequilibria impact on the behavior of economic agents:

1. *Disequilibrium in profit rates.* Capitalists respond to profitability differentials. Since only two industries are considered these differentials can be characterized by a single variable $\left(r_t^1 - r_t^2\right)$, in which r_t^i denotes the rate of profit of enterprise i.

2. *Disequilibrium between supply and demand.* This disequilibrium is measured by the level of inventories (*cf.* equation 1). This level will be conveniently assessed using the following ratio:

$$s_t^i = S_t^i / K_t^i b \tag{5}$$

in which $K_t^i b$ denotes productive capacity in industry i. Since demand is not invariant, but constantly oscillating, enterprises could not attempt to hold zero inventories without incurring losses. They seek to obtain a certain ratio $\overline{s}$ (the normal or target value of s). Thus, the adequate variable to measure the disequilibrium between supply and demand is $s_t^i - \overline{s}$.

3. *Disequilibrium in the utilization of productive capacities.* The constant fluctuations of demand also explain why enterprises attempt to maintain a certain level of extra capacity. We denote $\overline{u}$ this normal or target value of u. The disequilibrium in the capacity utilization rate is measured by $u_t^i - \overline{u}$.

4. *Disequilibrium concerning the general level of prices.* Any increase or decrease in the general level of prices will be considered as evidence of disequilibrium by the banking system. A tightening or slackening of monetary policy will respond to these variations. The relevant variable is j_t:

$$j_t = \frac{Y_t p_t}{Y_t p_{t-1}} - 1$$

The way these disequilibria will impact on the behavior of economic agents can be summarized as follows:

$r_t^1 \neq r_t^2$	$\longrightarrow$	Capitalists	*Capital Mobility*
$s_t^i \neq \overline{s}$	$\longrightarrow$	Enterprises	*Variation of prices*
$s_t^i \neq \overline{s}$	$\longrightarrow$	Enterprises	*Variation of outputs*
$u_t^i \neq \overline{u}$	$\longrightarrow$	Enterprises	*Variation of outputs*
$u_t^i \neq \overline{u}$	$\longrightarrow$	Enterprises and Banks	*Loans for investments*
$j_t \neq 0$	$\longrightarrow$	Banks	*Loans to consumers and enterprises for investment*

2.4.2 Production

The decision to produce is equivalent to the decision of the new capacity utilization rate, since the stock of capital is given and $Y^i = K^i b u^i$. This decision is made on the basis of the disequilibria on the level of inventories and the capacity utilization rate of the previous period:

$$u_{t+1}^i - u_t^i = -\varepsilon \left(s_t^i - \overline{s} \right) - (1 - \sigma) \left(u_t^i - \overline{u} \right) \tag{6}$$

which can also be written as:

$$u_{t+1}^i - \overline{u} = \sigma \left(u_t^i - \overline{u} \right) - \varepsilon \left(s_t^i - \overline{s} \right)$$

In these equations the reaction coefficients σ and ε have no dimension, since u and s themselves have no dimension. This equation shows, for example, that enterprises will react to inventories larger than normal by diminishing their capacity utilization rate. The degree of this sensitivity to stockpiling is measured by ε.

Reaction coefficient σ models the stickiness of the capacity utilization rate, with $0 \leq \sigma \leq 1$. ($1 - \sigma = 0$ corresponds to maximum stickiness.) The term $(1 - \sigma) \left(u_t^i - \overline{u}^i \right)$ expresses three different types of phenomena: 1) We make the implicit assumption that the demand function which enterprises confront are subject to random shocks and only return slowly to normal levels (autoregressive shocks, *cf.* G. Duménil, D. Lévy, "The Rationality of Adjustment", *op. cit.* note 6), 2) In addition to traditional production costs, enterprises incur disequilibrium costs, such as the cost of stockpiling or the cost of changing production. The existence of this latter cost induces a degree of stickiness in the decision to produce (*cf.* C.C. Holt, F. Modigliani, J.F. Muth, H.A. Simon, *Planning Production, Inventories, and Work Force*, Englewood Cliffs: Prentice-Hall (1960)), 3) Enterprises accelerate or slow their investment rate and simultaneously attempt to restore their capacity utilization rate at normal levels.

2.4.3 Prices

The new ratios of inventories s_{t+1}^i are determined by equations 1 and 5. Price adjustment depends on the disequilibrium between supply and demand:

$$p_{t+1}^i = p_t^i \left(1 - \beta \left(s_{t+1}^i - \overline{s} \right) \right) \tag{7}$$

in which β is a reaction coefficient without dimension. The disequilibrium in capacity utilization rates or the cost of inputs could also be included as arguments in this function.

2.4.4 The Allocation of Capital

In the allocation of capital, capitalists will consider the profit rate, r_t^i, defined as the ratio of profits over the stock of fixed capital[10]:

$$r_t^i = \frac{\pi_t^i}{K_t^i p_t^1} = u_t^i \frac{b p_t^i - w p_t^2}{p_t^1} - \delta$$

The first industry receives $\mathcal{K}_{t+1}^1$ and the second $\mathcal{K}_{t+1}^2$:

$$\begin{aligned} \mathcal{K}_{t+1}^1 &= \left(\delta K_t^1 + \gamma K_t \left(r_t^1 - r_t^2\right)\right) p_t^1 \\ \mathcal{K}_{t+1}^2 &= \left(\delta K_t^2 - \gamma K_t \left(r_t^1 - r_t^2\right)\right) p_t^1 \end{aligned} \tag{8}$$

If the rates of profit r_t^1 and r_t^2 are equal, depreciation allowances, $\delta K_t^i p_t^1$, are reinvested in each industry. A profitability differential provokes a transfer from one industry to the other. The sensitivity of capitalists to profitability differentials is measured by γ. As all other reaction coefficients in this study, γ is normalized and without dimension, so that the model is homogenous with respect to prices and quantities.

2.4.5 Loans and Monetary Policy

We will deal successively with loans for consumption and investment.

We only consider the variation of the net outstanding stock of loans for consumption, ΔM_t, as in equation 3. The banking system reacts to the variation of the general level of prices, j_t, by tightening credit conditions whenever inflation prevails and acting conversely if prices diminish. The size of this variation is proportional to the stock of money, $M_t + \pi_t^1 + \pi_t^2 + W_{t+1}^1 + W_{t+1}^2$, already held by final consumers. The intensity of the response to the variation of the price level is measured by a reaction coefficient φ':

$$\Delta M_t = -(M_t + \pi_t^1 + \pi_t^2 + W_{t+1}^1 + W_{t+1}^2)\varphi' j_t \tag{9}$$

The variation of loans for investments, $\Delta\mathcal{K}_{t+1}^i$, as in section 2.3, responds to two disequilibria concerning the capacity utilization rates, $\left(u_t^i - \overline{u}\right)$, and the variation of the general level of prices, j_t. The first disequilibrium corresponds to the desire of firms to invest, and the second to the willingness of banks to provide the necessary funding. The new loans are proportional to the stock of fixed capital held. The two resulting reaction coefficients are ω and φ':

$$\Delta\mathcal{K}_{t+1}^i = K_t^i p_t^1 (\omega \left(u_t^i - \overline{u}\right) - \varphi' j_t) \tag{10}$$

One can notice that the sensitivity of monetary authority to the variation of prices is the same with respect to the two types of loans. This is equivalent to saying that we assume that monetary policy does not favor any industry in particular.

We will also use two auxiliary notations $\widetilde{s}$ and φ, both related to monetary policy:

10. In the computation of the profit rate in this model, we abstract from inventories, both with respect to the determination of profits in the numerator (appropriated profits instead of realized profits) and determination of capital advanced in the denominator (limited to fixed capital). It would also be possible to compute profits for a normal utilization rate, $\overline{u}$, of fixed capital instead of the actual rate u.

1. We define:

$$\widetilde{s}_t = \frac{Y_t^1 p_{t-1}^1 s_t^1 + Y_t^2 p_{t-1}^2 s_t^2}{Y_t^1 p_{t-1}^1 + Y_t^2 p_{t-1}^2}$$

With this notation, using equation 7, j_t can be replaced by $-\beta(\widetilde{s}_t - \overline{s})$.

2. We will also use $\varphi = \beta\varphi'$, as a synthetic parameter accounting for the overall effect of the disequilibrium between supply and demand:

$$\begin{pmatrix} \textit{Impact on prices of} \\ \textit{Supply} \neq \textit{Demand} \end{pmatrix} \times \begin{pmatrix} \textit{Reaction of monetary authorities} \\ \textit{to price variation} \end{pmatrix}$$

With this notation, $-\varphi' j_t$ can be written $\varphi(\widetilde{s}_t - \overline{s})$.

2.5 THE RELATION OF RECURSION

The above equations define a relation of recursion which allows for the derivation of the value of the variables in period $t+1$ from that in period t.

There are nine variables, u^i, s^i, p^i, K^i for each of the two industries, and the stock of money M^f held by consumers. *A priori*, a double continuum of equilibria exists as a result of the indeterminacy of the general levels of both prices and quantities, and it is possible to reduce the set of variables to seven variables without dimension.

We conserve the two capacity utilization rates u^i and two ratios of inventories s^i which already have no dimension, and consider the relative price, $x = p^1/p^2$, and the relative stock of fixed capital, $y = K^1/K^2$. The stock of money must also be normalized: $z_{t+1} = M_t^f / K_{t+1}^2 p_t^2$. We will also use the two growth rates, $\rho_t^i = (K_{t+1}^i - K_t^i)/K_t^i$, of the stocks of capital. With this notation, the recursion can be written as in table 2.

3 - DIMENSION AND PROPORTIONS

In the study of a dynamic model such as that presented in part 2, one must distinguish between the issues of the existence of an equilibrium and its stability. Section 3.1 is devoted to equilibrium and introduces the study of stability. Section 3.2 considers the two aspects of the stability problem: dimension and proportions.

3.1 EQUILIBRIUM AND STABILITY

In this model, an equilibrium which we call *normal equilibrium* exists, which corresponds to the classical notion of a long-term equilibrium with prices of production:

1. The capacity utilization rates and the ratios of inventories reach their target value: $u^i = \overline{u}$ and $s^i = \overline{s}$.

2. The rates of profits are equal, $r^i = \overline{r} = \overline{u}(b - w) - \delta$ and, prices are equal to production prices. As a result of the assumption of an identical technology within the two industries, prices are equal: $x = 1$ or $p^2 = p^1$.

2. Global Model

The relation of recursion

$$u^1_{t+1} = \overline{u} + \sigma\left(u^1_t - \overline{u}\right) - \varepsilon\left(s^1_t - \overline{s}\right)$$
$$u^2_{t+1} = \overline{u} + \sigma\left(u^2_t - \overline{u}\right) - \varepsilon\left(s^2_t - \overline{s}\right)$$
$$s^1_{t+1} = \frac{1}{1+\rho^1_t}\left(s^1_t + u^1_t - \frac{1+y_t}{y_t}\frac{\delta + \varphi(\widetilde{s}_t - \overline{s})}{b} - \frac{\omega}{b}\left(u^1_t - \overline{u} + \frac{u^2_t - \overline{u}}{y_t}\right)\right)$$
$$s^2_{t+1} = \frac{1}{1+\rho^2_t}\left(s^2_t + u^2_t\right) - \frac{\alpha}{b}z_{t+1}$$
$$x_{t+1} = x_t\left(1 - \beta\left(s^1_{t+1} - \overline{s}\right)\right)\Big/\left(1 - \beta\left(s^2_{t+1} - \overline{s}\right)\right)$$
$$y_{t+1} = y_t(1+\rho^1_t)/(1+\rho^2_t)$$
$$z_{t+1} = (1 + \varphi(\widetilde{s}_t - \overline{s}))\left(\frac{1}{1+\rho^2_t}\left(\frac{(1-\alpha)z_t}{1-\beta\left(s^2_t - \overline{s}\right)} + y_t x_t r^1_t + x_t r^2_t\right) + w\left(y_{t+1}u^1_{t+1} + u^2_{t+1}\right)\right)$$

$$\textit{With}\quad \rho^1_t = \gamma\frac{1+y_t}{y_t}(r^1_t - r^2_t) + \omega(u^1_t - \overline{u}) + \varphi(\widetilde{s}_t - \overline{s})$$
$$\rho^2_t = -\gamma(1+y_t)(r^1_t - r^2_t) + \omega(u^2_t - \overline{u}) + \varphi(\widetilde{s}_t - \overline{s})$$
$$\text{and}\quad r^1_t = u^1_t\left(b - \frac{w}{x_t}\right) - \delta \quad r^2_t = u^2_t\,(b - w)\,\frac{1}{x_t} - \delta \quad \widetilde{s}_t = \frac{x_{t-1}y_t u^1_t s^1_t + u^2_t s^2_t}{x_{t-1}y_t u^1_t + u^2_t}$$

Equilibrium

$$u^i = \overline{u} \quad s^i = \overline{s} \quad (\text{for } i = 1,2) \quad x = 1 \quad y = \overline{y} = \delta/(b\overline{u} - \delta) \quad z = \overline{z} = b\overline{u}/\alpha$$

The Polynomial Characteristic of the Jacobian

$$P(\lambda) = \begin{vmatrix}
\lambda - \sigma & 0 & \varepsilon & 0 & 0 & 0 & 0 \\
0 & \lambda - \sigma & 0 & \varepsilon & 0 & 0 & 0 \\
\begin{matrix}-1 + A(b-w) \\ + \left(\overline{s} + \frac{1}{b}\right)\omega\end{matrix} & -A(b-w) + \frac{\omega}{b\overline{y}} & \begin{matrix}\lambda - 1 \\ + \frac{\varphi\overline{y}}{1+\overline{y}}\left(\overline{s} + \frac{\overline{u}}{\delta}\right)\end{matrix} & \frac{\varphi}{1+\overline{y}}\left(\overline{s} + \frac{\overline{u}}{\delta}\right) & Ab\overline{u} & -\frac{\delta}{b\overline{y}^2} & 0 \\
-B(b-w) & \begin{matrix}-1 + B(b-w) \\ +(\overline{s} + \overline{u})\omega\end{matrix} & \frac{\varphi\overline{y}}{1+\overline{y}}(\overline{s} + \overline{u}) & \begin{matrix}\lambda - 1 \\ + \frac{\varphi}{1+\overline{y}}(\overline{s} + \overline{u})\end{matrix} & -Bb\overline{u} & 0 & \frac{\alpha}{b}\lambda \\
0 & 0 & \beta\lambda & -\beta\lambda & \lambda - 1 & 0 & 0 \\
-C(b-w) - \omega\overline{y} & C(b-w) + \omega\overline{y} & 0 & 0 & -Cb\overline{u} & \lambda - 1 & 0 \\
E\overline{y} - D(b-w) & D(b-w) + E + D'\omega & -w\overline{u}\overline{y}\varphi & -w\overline{u}\varphi - (1-\alpha)\overline{z}\beta & -Db\overline{u} & E\overline{u} + \delta & \lambda - 1 + \alpha
\end{vmatrix}$$

$$\textit{With } A = \overline{s}\frac{b\overline{u}}{\delta}\gamma \quad B = (\overline{u} + \overline{s})(1+\overline{y})\gamma \quad C = (1+\overline{y})^2\gamma$$
$$D' = \overline{u}\left(\frac{b}{\alpha} - w(1+\overline{y})\right) \quad D = D'(1+\overline{y})\gamma \quad E = w(1-\lambda) - b$$

3. The proportions of capital between the two industries are given by: $y = \overline{y} = \delta/(b\overline{u} - \delta)$.
4. The amount of money held by final consumers is given: $z = \overline{z} = b\overline{u}/\alpha$.
5. There is no growth ($\rho^1 = \rho^2 = 0$) and no variation of the general level of prices ($j = 0$).

Since the model is non-linear, the uniqueness of the equilibrium is not guaranteed. Therefore other equilibria can also exist. The consideration of such equilibria is interesting, but it would be impossible to develop such an analysis while preserving the linear forms of the behavioral equations. Likewise, the analysis of the local stability (or the investigation of the various convergence regions, when several equilibria exist) is interesting, but again the results also depend on non-linearities (*cf. e.g., G. Duménil, D. Lévy, "The Macroeconomics of Disequilibrium", Journal of Economic Behavior and Organization, 8 (1987), p. 377-395*).

In order to study the local asymptotic stability of an equilibrium such as that introduced above, it is necessary to determine the Jacobian matrix which expresses the linear approximation of the recursion in the vicinity of equilibrium. Stability is insured if the moduli of all the eigenvalues are smaller than 1. The polynomial characteristic, $P(\lambda)$, of the Jacobian is presented with the recursion. (In the remainder of this study, "stability" will denote "local asymptotic stability".)

$P(\lambda)$ is a polynomial of the seventh degree, whose analysis is difficult. In the next section, we will demonstrate a number of properties of this general formalism. In part 4 a full treatment of stability will be developed for the short and long-term models.

3.2 INSTABILITY IN DIMENSION AND STABILITY IN PROPORTIONS

In this section, we present our general thesis concerning the stability problem in capitalism[11]: *The dominant eigenvalue is always close to 1, and from this property derive both the instability of the system with respect to dimension and its stability with respect to proportions.* In subsection 3.2.1, we study the "stability frontier" (the condition $P(1) = 0$). Subsection 3.2.2 defines the robustness of the conditions which define the stability frontier. In subsection 3.2.3, we determine the eigenvector associated with the eigenvalue 1 which accounts for the dominant component of dynamics in a vicinity of equilibrium. On the basis of this vector, it is possible to contrast the two aspects in the stability problem: dimension and proportions. In subsection 3.2.4 we provide some elements of an empirical verification.

3.2.1 The Dominant Eigenvalue is Close to 1

A necessary and sufficient condition for an eigenvalue to be equal to 1 is $P(1) = 0$, which can also be written $\theta = 1$, with:

$$\theta = \sigma + \varepsilon \frac{F\omega}{\varphi + G\beta} \tag{11}$$

where F and G are two constant parameters independent from the reaction coefficients:

$$F = 1 - \frac{G}{1-\alpha} \quad \text{and} \quad G = \frac{\overline{z}}{1+\overline{y}} \frac{1-\alpha}{1 + b(\overline{u} + \overline{s}) - \delta + w\overline{u}}$$

11. The general condition for stability $||\lambda^i|| < 1 \; \forall i$ can be superseded in three different manners: For one eigenvalue λ^i, $\lambda^i = 1$ or $\lambda^i = -1$, or there exist two complex conjugate eigenvalues of moduli equal to 1.

We call the "stability frontier" the situation in which one eigenvalue is equal to 1 (or θ is equal to 1). In simpler models (as in subsection 4.1.2 below), $P(1) > 0$, *i.e.*, $\theta < 1$, is also a sufficient condition to insure that no real eigenvalue larger than 1 can exist. However, we are not yet able to demonstrate this result in the global form of the model presented here.

Equation 11 and the condition $\theta < 1$ can be interpretated in the following way:

1. In the expression of θ, σ and ε correspond to the effect of demand on supply (*cf.* equation 6). This relation is characteristic of the decision to produce and expresses one aspect of the management of firms. The ratio $\frac{F\omega}{\varphi + G\beta}$ measures the effect of supply on the formation of demand.

2. This last effect can be decomposed into two distinct components, one *procyclical* and the other *countercyclical*:

- Coefficient ω denotes the sensitivity of loans to the capacity utilization rate (*cf.* equation 10). This term represents the procyclical component of the feedback. A high level of activity stimulates lending for investment and, thus, demand to the industry which produces capital goods.
- The two countercyclical mechanisms in the denominator of the ratio correspond to: 1) Monetary policy (coefficient φ as in equations 9 and 10), and 2) The real balance effect (coefficient β as in equation 7). A high level of activity causes inflation and monetary authorities tighten credit conditions. In a similar manner, inflation cuts into the purchasing power of final consumers.[12]

What is determinant in the effect of supply on demand is the relative impact of these pro and countercyclical mechanisms. However, the overall stability of the system also depends on ε and σ (effect of demand on supply).

In the above analysis, we assumed positive values for F and G. This property is only evident for G. The condition $F > 0$ follows from the view defended in the first part of this study, that loans for investment are countercylical in the long run, but procyclical in the short run, and that this latter effect is dominant.

3.2.2 The Robustness of the Condition Defining the Stability Frontier

An important property of the condition defining the stability frontier is that it is very robust with respect to the assumptions made. This is not the case for the conditions which define *global stability* or for the two other conditions corresponding to local stability: one eigenvalue equal to -1 or two complex conjugate eigenvalues with moduli equal to one.

For example, in a model without money ($\alpha = 1$), without credit to consumers, and in which wages are paid *ex post*, equation 4 becomes $D_t^2 = (Y_t p_t - \delta K_t p_t^1)/p_t^2$, and the condition defining the stability frontier is:

$$\theta = \sigma + \varepsilon \frac{\omega}{\varphi}$$

Consider now a growth model (assuming for simplicity that fixed capital does not depreciate, $\delta = 0$), in which money is not treated. Total income, $Y_t p_t$, is divided into two fractions, $\alpha Y_t p_t$ for consumption, and $(1 - \alpha) Y_t p_t$ for accumulation. This fund is allocated proportionally to capacities already installed with a correction responding to three

12. As mentioned in part 1, we do not consider this real balance effect as factually relevant.

disequilibria (profitability differentials, the deviation from normal of capacity utilization rates, and the rate of variation of the general price level) as described in section 2.4. The only modification in equations 8 and 9, is that $\delta K_t^i p_t^1$ must be replaced by $\frac{K_t^i}{K_t}(1-\alpha)Y_t p_t$. One must add $1+b\overline{s}$ to the denominator of equation 2, since the new funds must finance the increased capital stock (1) and inventories ($b\overline{s}$). One then obtains:

$$\theta = \sigma + \varepsilon \frac{\omega + \dfrac{\overline{\rho}}{(1+b\overline{s})\overline{u}}}{\varphi + \dfrac{\overline{\rho} b}{1+b\overline{s}}}$$

We will see below that a similar condition is obtained in the short-term model (*cf.* equation 15).

The basic form of the condition is always:

$$\theta = \sigma + \varepsilon \frac{\textit{Procyclical Mechanisms}}{\textit{Countercyclical Mechanisms}}$$

3.2.3 The Dominant Dynamics in a Vicinity of Equilibrium

The eigenvector, V, associated with the eigenvalue 1 accounts for the primary characteristics of the dynamics in a vicinity of the equilibrium, when the economy is close to the stability frontier ($\theta \simeq 1$). From the value of this eigenvector, it is possible to derive a number of relationships among the variables in a vicinity of equilibrium and for θ close to 1. These relationships account for the features of the economy when it is drawn out of equilibrium. Such departures from equilibrium can correspond to: 1) The usual gravitation process, 2) The business cycle, and 3) The effect of policies attempting, for example, to achieve a level of capacity utilization different from normal (for example, to fight unemployment). The eigenvector is:

$$V = \begin{pmatrix} 1 \\ 1 \\ -\dfrac{1-\sigma}{\varepsilon} \\ -\dfrac{1-\sigma}{\varepsilon} \\ 0 \\ \left(\left(\overline{s} + \dfrac{\overline{u}}{\delta}\right)\left(\omega - \dfrac{1-\sigma}{\varepsilon}\varphi\right) - 1\right)\dfrac{b\overline{y}^2}{\delta} \\ \left(1 - (\overline{s}+\overline{u})\left(\omega - \dfrac{1-\sigma}{\varepsilon}\varphi\right)\right)\dfrac{b}{\alpha} \end{pmatrix} \qquad (12)$$

Although other similar relations also exist in which monetary variables are involved, we will only consider here the relationships between u, s, and x:

1. In a vicinity of equilibrium, the following properties hold:

$$\begin{array}{lll} u^1 - \overline{u} \simeq u^2 - \overline{u} & \textit{or} & u^1 \simeq u^2 \\ s^1 - \overline{s} \simeq s^2 - \overline{s} & \textit{or} & s^1 \simeq s^2 \\ x - 1 \simeq 0 & \textit{or} & p^1 \simeq p^2 \end{array}$$

Thus, the proportions between the two industries are not disrupted when $\theta \simeq 1$.

2. Denoting u the common value of u^1 and u^2, and s the common value of s^1 and s^2, the following relationship holds:

$$(1-\sigma)(u-\overline{u})+\varepsilon(s-\overline{s})=0 \tag{13}$$

In the plane (u,s), a trade-off which goes through $(\overline{u},\overline{s})$ is defined.

If, as we believe is the case in an actual capitalist economy, $\theta \simeq 1$, then the economy will be stable with respect to proportions ($u^1 \simeq u^2$, $s^1 \simeq s^2$, $p^1 \simeq p^2$ and, therefore, $r^1 \simeq r^2$) and unstable with respect to dimension. By unstable with respect to dimension we mean that $u \neq \overline{u}$ and $s \neq \overline{s}$, while u and s are related as in equation 13. Under such circumstances, it is possible to study the overall stability of the system in a model in which only one commodity is considered (as is the case in macroeconomics).

Since the vector V accounts for the response of the economic system to exogenous shocks (random shocks or policies), it is possible to compute the corresponding multipliers. In the linear approximation and for $\theta = 1$, these multipliers diverge. This is equivalent to saying that the economy is very volatile and reacts strongly to any perturbation. It is obvious, however, that the model is non-linear and that these multipliers are not infinite. In our opinion, this property provides some grounding for the understanding of the volatility of the general level of activity which Keynes described in the *General Theory* (J.M. Keynes, *The General Theory, op. cit.* note 7, ch. 22), referring to factors such as the "psychology of financial markets" (p. 320).

3.2.4 Empirical Verification

It is an obvious characteristic of capitalism that it is very unstable in dimension. This is reflected in the constant oscillation of the capacity utilization rate (as shown in figure 1 for manufacturing industries in the U.S. after World War II).

It is possible to verify empirically in a straightforward manner that the dominant eigenvalue is close to 1, by investigating the relationship between the variables when the economy moves out of equilibrium.

Figure 2 presents an empirical verification of equation 13. This figure displays the relationship between u and s for the period 1950-1985, using monthly data. (The variables have been detrended and only inventories of finished goods are considered). It is evident from this figure that a trade-off exists. This observation proves that: 1) The economy is constantly maintained in a vicinity of the stability frontier, 2) Equation 6 is a good model of the decision to produce (since equation 13 is a straightforward consequence of equation 6).

4 - SHORT TERM *VS.* LONG TERM

Section 4.1 deals with the existence and stability of a short-term equilibrium by quantities. In section 4.2, long-term equilibrium is treated as a succession of short-term equilibria. In this framework, we analyze the stability of long-term equilibrium.

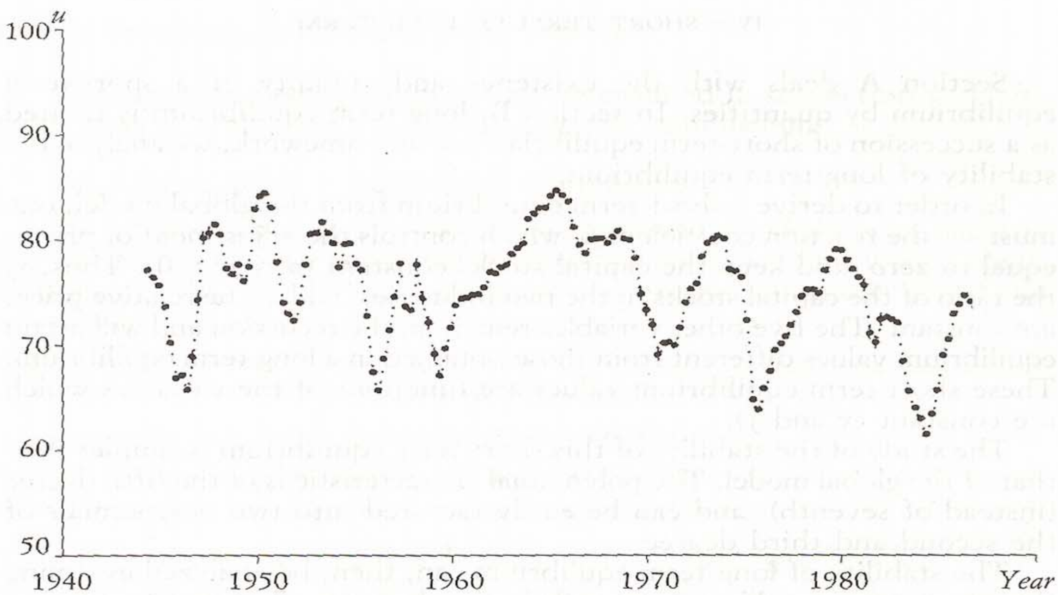

Figure 1 - Capacity Utilization Rate
Manufacturing Industries, Quarterly Data (1948-1985)

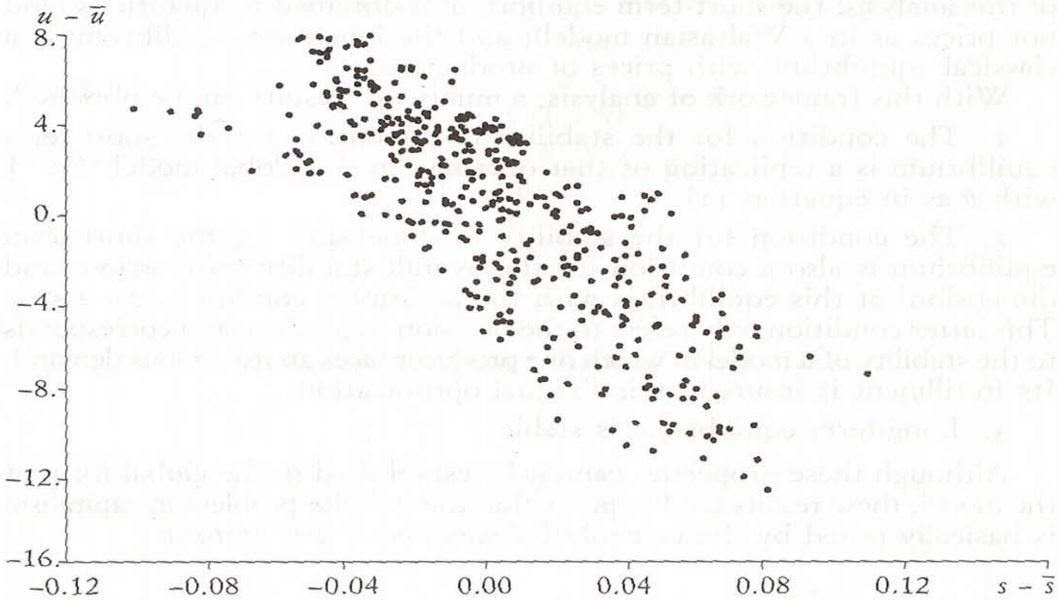

Figure 2 - Plot of $u - \overline{u}$ Against $s - \overline{s}$ (Residual Around the Trends)
Manufacturing Industries, Monthly data, 1950.01-1985.12

In s, the numerator is the inventories of finished goods, and the denominator is shipments slightly smoothened so that the fluctuations of the ratio mirror that of inventories, not shipments.
The trends, $\overline{u}$ and $\overline{s}$, of u and s are determined using the filter of Hodrick-Prescott/Whittaker, with $\lambda = 1000$.
Sources:
Capacity utilisation rate: as in figure 1.
Ratio of inventories:
1958-2004, Manufacturers' Shipments, Inventories, and Orders: U.S. Department of Commerce, Bureau of the Census.
1952-1957, Citibase data.

As already stated in section 1.2.1, in order to derive a short-term equilibrium from the global model, one must disconnect all slow variables:

1. Though investment is considered as a component of demand, the increased fixed capital does not contribute to production. Thus, capital stocks are kept constant ($\rho^1 = \rho^2 = 0$). Consequently, y, the ratio of the capital stocks in the two industries is also constant.

2. All price mechanisms are assumed to be slow and disconnected (β is set to zero and x is constant). Prices have no direct effect on demand *via* a wealth effect (equation 4).

3. Monetary policy is still active, as parameter $\varphi = \beta\varphi'$ is not set to zero in spite of the weakness of β, since φ' is assumed to be large. Parameter φ is applied to the deviation of average inventories from normal (since $\varphi = \varphi' j_t = -\varphi(\widetilde{s}_t - \overline{s})$, section 2.4.5).

The stability of long-term equilibrium can, then, be analyzed assuming that short-term equilibrium prevails in the short run. The problem is that of a *sequence of temporary equilibria*, and only two variables (x and y) are involved in these dynamics. It is important to keep in mind the specificity of this analysis: the short-term equilibrium is obtained by quantities (and not prices as in a Walrasian model), and the long-term equilibrium is a classical equilibrium with prices of production.

With this framework of analysis, a number of results can be obtained:

1. The condition for the stability in dimension for the short-term equilibrium is a replication of that obtained in the global model ($\theta < 1$ with θ as in equation 11).

2. The condition for the stability in dimension for the short-term equilibrium is also a condition for the overall stability (proportions and dimension) of this equilibrium with the additional condition: $\sigma + \varepsilon < 1$. This latter condition only refers to the decision to produce and corresponds to the stability of a model in which one producer faces an exogenous demand. Its fulfillment is insured by individual optimization.

3. Long-term equilibrium is stable.

Although these properties cannot be established in the global form of the model, these results tend to prove that the stability problem in capitalism is basically posed by the control of *dimension in the short-run*.

4.1 SHORT-TERM EQUILIBRIUM BY QUANTITIES

The short-term equilibrium is first determined in subsection 4.1.1. Then, subsection 4.1.2 deals with the stability of this equilibrium.

4.1.1 The Determination of Short-term Equilibrium

We call short-term equilibrium an equilibrium in which prices are constant and the stocks of capital in the two industries are invariant. This is equivalent, in the equations which define the recursion, to the conditions: $\beta = 0$ and $\rho^1 = \rho^2 = 0$. However, as in a Keynesian model, investment is still considered as a component of demand (in the first industry): ω and φ are not modified whenever they appear explicitly. (Notice that γ is only present in the relation of recursion for the global model *via* ρ^1 and ρ^2, since capital mobility has no effect on the overall demand facing the first industry). It is also the case that $x_t = x$ and $y_t = y$, since these two variables are constant. The recursion for the 5 other variables is displayed in table 3.

3. Short-Term Model

The relation of recursion

$$
\begin{aligned}
u^1_{t+1} &= \overline{u} + \sigma\left(u^1_t - \overline{u}\right) - \varepsilon\left(s^1_t - \overline{s}\right)\\
u^2_{t+1} &= \overline{u} + \sigma\left(u^2_t - \overline{u}\right) - \varepsilon\left(s^2_t - \overline{s}\right)\\
s^1_{t+1} &= s^1_t + u^1_t - \frac{1+y}{y}\frac{\delta + \varphi(\widetilde{s}_t - \overline{s})}{b} - \frac{\omega}{b}\left(u^1_t - \overline{u} + \frac{u^2_t - \overline{u}}{y}\right)\\
s^2_{t+1} &= s^2_t + u^2_t - \frac{\alpha}{b} z_{t+1}\\
z_{t+1} &= \left(1 + \varphi(\widetilde{s}_t - \overline{s})\right)\left((1-\alpha)z_t + yxr^1_t + xr^2_t + w(yu^1_{t+1} + u^2_{t+1})\right)
\end{aligned}
$$

Equilibrium

$$
\begin{aligned}
u^{1\star} - \overline{u} &= -\frac{\overline{u}}{\overline{y}(1+\overline{y})}(y - \overline{y})\\
u^{2\star} - \overline{u} &= \frac{\overline{u}}{1+\overline{y}}(y - \overline{y})\\
s^{i\star} - \overline{s} &= -\frac{1-\sigma}{\varepsilon}(u^{i\star} - \overline{u})\\
z^{\star} - \overline{z} &= \frac{b}{\alpha}(u^{2\star} - \overline{u})
\end{aligned}
$$

The Polynomial Characteristic of the Jacobian

$$
P^S(\lambda) = \begin{vmatrix}
\lambda - \sigma & 0 & \varepsilon & 0 & 0\\
0 & \lambda - \sigma & 0 & \varepsilon & 0\\
-1 + \frac{\omega}{b} & \frac{\omega}{b\overline{y}} & \lambda - 1 + \frac{\varphi}{b} & \frac{\varphi}{b\overline{y}} & 0\\
0 & -1 & 0 & \lambda - 1 & \frac{\alpha}{b}\lambda\\
E\overline{y} & E & -\frac{\varphi\overline{y}}{1+\overline{y}}\overline{z} & -\frac{\varphi}{1+\overline{y}}\overline{z} & \lambda - 1 + \alpha
\end{vmatrix}
$$

This dynamic system has a fixed point, short-term equilibrium different from long-term equilibrium. However, the system which allows the determination of the equilibrium values, $u^{1\star}$, $u^{2\star}$, $s^{1\star}$, $s^{2\star}$, and $z^{\star}$, of the variables is nonlinear, and it is difficult to solve explicitly. We assume that the allocation of capital in the short-term equilibrium is not very different from what it would be in a long-term equilibrium. This is equivalent to saying that we only consider small values of $y - \overline{y}$, where $\overline{y}$ denotes the value y in a long-term equilibrium (*cf.* section 3.1), and study a linear development of the recursion in a vicinity of long-term equilibrium. The result is displayed in table 3. One can verify, in this linear approximation of short-term equilibrium, that $\widetilde{s}^{\star} - \overline{s} = 0$, *i.e.*, loans are also stabilized in a short-term equilibrium.

One can notice that short-term equilibrium always belongs to the trade-off defined in equation 13.

4.1.2 The Stability of Short-term Equilibrium

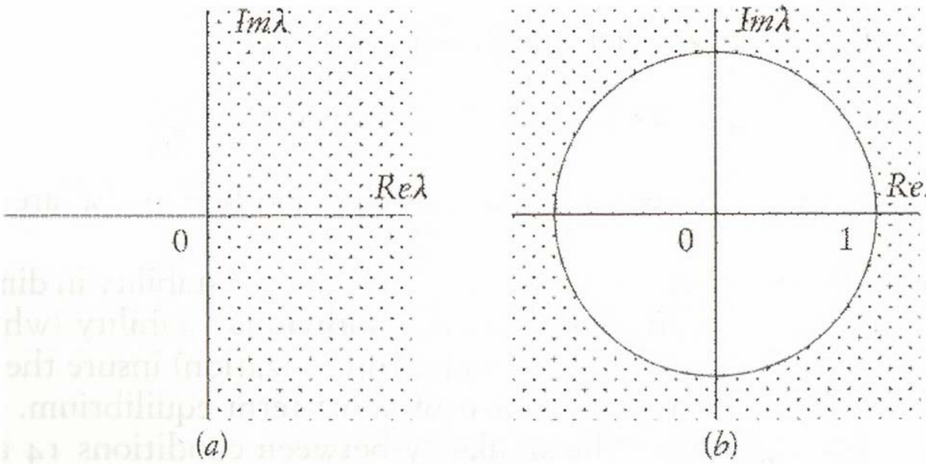

Figure 3 - The Conditions for Stability
with Continuous (a) and Discrete-Time (b) Models

For a model in continuous time (a), stability is insured if Reλ <. In a discrete-time model the condition is $||\lambda|| < 1$. The region corresponding to continuous-time is represented in (b) by the dack line on the circle. We believe this is the only economically meaningful portion of the circle to be investigated.

In order to study the stability of short-term equilibrium, one must consider the Jacobian of the recursion (with five variables: u^1, u^2, s^1, s^2, and z), which is displayed in table 3. It is possible to decompose this polynomial of the fifth degree into two factors of the second and third degrees respectively: $P^S(\lambda) = P_2(\lambda)P_3(\lambda)$, with:

$$P_2(\lambda) = (\lambda - 1)(\lambda - \sigma) + \varepsilon$$

$$P_3(\lambda) = \begin{vmatrix} \varepsilon + (\lambda - 1)(\lambda - \sigma) + \frac{\varphi}{b}(\lambda - \sigma) - \frac{\varepsilon\omega}{b} & \frac{\alpha}{b}\lambda \\ -(w(1-\lambda) - b)\varepsilon - \frac{\varphi}{1+\overline{y}}\overline{z}(\lambda - \sigma) & \lambda - 1 + \alpha \end{vmatrix}$$

Polynomial $P_2(\lambda)$ has two zeros which are either real and between 0 and 1, or complex conjugate. Their moduli are smaller than 1 if:

$$\varepsilon + \sigma < 1 \tag{14}$$

.

Polynomial $P_2(\lambda)$ is, in fact, the Jacobian of the following model of partial equilibrium in which a single enterprise, i, is considered and demand, d^i, is exogenous:

$$u^i_{t+1} = \overline{u} + \sigma(u^i_t - \overline{u}) - \varepsilon(s^i_t - \overline{s})$$
$$s^i_{t+1} = s^i_t + u^i_t - d^i$$

This model corresponds to a specific problem which refers to what could be called short-term "individual stability" or "stability in proportions". We showed in G. Duménil, D. Lévy, "The Rationality of Adjustment", *op. cit.* note 6 that condition 14 is guaranteed by individual optimization. This is equivalent to saying that instability related to condition 14 can only be attributed to deficient management which, we believe, is not at the origin of instability in capitalism.

The study of the three zeros of $P_3(\lambda)$ is more difficult and this complexity is related to the fact that we consider a discrete-time model. In a continuous-time model, instability occurs whenever an eigenvalue has a real part larger than 0 (and the model diverges in $e^{\lambda t}$). If λ is real, a steady growth is obtained. If λ has an imaginary part, then a spiral divergence follows. In a discrete-time model, what corresponds to this condition is the region close to 1 of the circle $||\lambda|| = 1$. What occurs close to $\lambda = -1$ has no counterpart in continuous-time analysis. The variables switch from one period to the next from a large to a small value. We believe that this pattern has no economic relevance and is an artefact of discrete-time models. This statement is equivalent to the view that the reaction coefficients ε, $(1-\sigma)$, ω, and φ, are not too large — a quite natural assumption which is common to most models devoted to the stability of long-term equilibrium. We will restrict the investigation to the vicinity of $\lambda = 1$ which includes two subcases: 1) $\lambda = 1$ proper, which has already been considered in the analysis of the global model (Stability frontier, *cf.* subsection 3.2.1), 2) Two complex-conjugate eigenvalues of moduli equal to 1 and small imaginary parts.

Polynomial $P_3(\lambda)$ has three eigenvalues strictly equal to 1 if $\varepsilon = 1-\sigma = \varphi = \omega = 0$ and they are close to 1 if these parameters remain small. It is easy to derive a development of these three eigenvalues as functions of these parameters. One find one real root, λ^1, and two complex-conjugate roots, λ^2 and λ^3:

$$\lambda^1 = 1 - \frac{\alpha}{b}\left(\frac{1-\sigma}{\varepsilon}\varphi\left(1+\frac{\overline{z}}{1+\overline{y}}\right) - \omega\right)$$

$$\lambda^2 = 1 - \frac{1}{2}\left(\alpha + 1 - \sigma + \frac{\varphi}{b}\right) + i\sqrt{\varepsilon}$$

The condition for $\lambda^1 < 1$ is:

$$\sigma + \varepsilon\frac{\omega}{\varphi\left(1+\frac{\overline{z}}{1+\overline{y}}\right)} < 1 \qquad (15)$$

It is also possible to obtain this equation directly, with the condition $P_3(1) > 0$, *i.e.*, using the same method as in the analysis of the stability frontier (*cf.* subsection 3.2.1). Condition 15 is consistent with condition $\theta < 1$ in the global model. With $\beta = 0$, this condition becomes:

$$\sigma + \varepsilon\frac{F\omega}{\varphi} < 1$$

The difference with equation 15 is due to the fact that ω and φ are set to 0 in the short-term model within the expressions of ρ^1 and ρ^2 and not in the the formation of demand (in conformity with the short-term point of view adopted).

Concerning the two other eigenvalues, one has:

$$||\lambda^2||^2 = ||\lambda^3||^2 = 1 + \varepsilon - \left(\alpha + 1 - \sigma + \frac{\varphi}{b}\right)$$

Condition 14 guarantees that the moduli of both λ^2 and λ^3 are smaller than 1.

It follows from this analysis that the conditions for stability in dimension (which are problematic in capitalism) and individual stability (which can be taken for granted because of individual optimization) insure the overall stability (dimension and proportions) of short-term equilibrium.

It is interesting to note the similarity between conditions 14 and 15. Our thesis, in subsection 3.2.1, that the economy is always close to the stability frontier implies that the coefficient of ε in condition 15 is larger than 1: $\omega > \varphi(1 + w\overline{u})$. This is equivalent to

4. Long-Term Model
(Succession of Temporary Equilibria)

The relation of recursion

$$
\begin{aligned}
x_{t+1} &= x_t - A(y_t - \overline{y}) \\
y_{t+1} &= y_t - B(y_t - \overline{y}) + C(x_t - 1) \\
\textit{with} \quad A &= \beta \frac{1-\sigma}{\varepsilon} \frac{\overline{u}}{\overline{y}} \\
B &= \overline{u} \left(\omega + \gamma(b - w) \frac{(1+\overline{y})^2}{\overline{y}} \right) \\
C &= \gamma(1+\overline{y})^2 b\overline{u}
\end{aligned}
$$

Equilibrium

$$x = 1 \quad y = \overline{y}$$

The Polynomial Characteristic of the Jacobian

$$P^L(\lambda) = \begin{vmatrix} \lambda - 1 & A \\ -C & \lambda - 1 + B \end{vmatrix}$$

saying that *the procyclical effect of investment is stronger than the countercyclical action of monetary authorities.*

4.2 THE ROUTE TO LONG-TERM EQUILIBRIUM AS A SEQUENCE OF SHORT-TERM EQUILIBRIA

Subsection 4.2.1 provides the definition and interpretation of the trajectory toward long-term equilibrium as a succession of short-term equilibria. Subsection 4.2.2 is devoted to the study of the equilibrium and stability of this model.

4.2.1 Definition and Interpretation

In this section, we assume that short-term equilibrium prevails at each period and study the stability of long-term equilibrium under this assumption. In this context, it is common to refer to a sequence of "temporary" equilibria.

Short-term equilibrium (with the approximation that the disequilibrium from long-term equilibrium is small) is defined by the equations displayed in table 3. In the equations for the long-term variables (relative price, x, and ratio of capital stocks in the two industries, y) in the recursion for the global model (*cf.* table 2), we replace the values of the other variables by these short-term-equilibrium values, and obtain the recursion displayed in table 4.

This recursion can be interpreted as follows:

1. The term $-A(y_t - \overline{y})$ corresponds, for example, to the fact that an excess of capital in the first industry ($y_t > \overline{y}$) provokes a relative decline in the price of the commodity produced

by this industry. (Coefficient β in A models the reaction of prices to the disequilibria between supply and demand.)

2. The term $+C(x_t - 1)$ models capital mobility in relation to profitability differentials. If, for example, the price of the first industry is high ($x_t > 1$), *i.e.*, if the profit rate is large in this industry, capital will flow into it. (Coefficient γ in C models the sensitivity of capitalists to profitability differentials).

These two first mechanisms account for the two basic relations in the classical analysis of the formation of prices of production: *Quantities* $\rightarrow$ *Prices* and *Prices* $\rightarrow$ *Quantities*.

3. The term $-B(y - \overline{y})$ represents the direct impact of quantities on quantities. Two mechanisms are combined. One corresponds to the existence of loans and the other to the effect of the capacity utilization rate on the profit rate. If, for example, there is an excess of capital in the first industry ($y_t > \overline{y}$), then $u^{1\star}$ will be small in comparison to $u^{2\star}$. The two effects follow from this disequilibrium:

- More loans will be granted to the second industry and less in the first industry. This corresponds to the presence of coefficient ω in B.
- The disequilibrium in capacity utilization rates is manifested in a profitability differential. The profit rate is lower in the first industry. Capital is displaced from the first industry to the second. This is consistent with the presence of γ in B.

Note that the relation of recursion for the long run is independent from the behavior of final consumers (parameter α) and from monetary policy (coefficient φ). This relation is a function of coefficient ω, with models the sensitivity to the capacity utilization rate in the determination of loans for investment, but only in as much as this behavior impacts on the correction of the capital stock (the stabilizing aspect of investment in the long run).

4.2.2 Equilibrium and Stability

Long-term equilibrium corresponds to $y_t = \overline{y}$ and $x_t = 1$. Its stability can be studied using the polynomial characteristic of the Jacobian displayed in table 4. The three parameters A, B, and C are small since β, ω, and γ are themselves small. Stability is subject to two types of conditions:

1. A and C must be different from 0. If this is not the case, one eigenvalue is equal to 1. This is equivalent to saying that the two basic classical relations between prices and quantities must hold.

2. A second condition is $AC < B$. If this condition is not satisfied, there will be two complex conjugate eigenvalues of moduli larger than one. This condition can also be written:

$$\omega + \gamma \frac{(1+\overline{y})^2}{\overline{y}} \left(b - w - \beta \frac{1-\sigma}{\varepsilon} b\overline{u} \right) > 0$$

It is satisfied if β is small. If β is large, the separate treatment of short-term and long-term equilibria is not possible, and only the global model can be used.

To sum up the discussion concerning the stability of long-term equilibrium in the long-term model, it appears that stability is insured provided that β and γ are different from 0, and β is not too large. When $\beta \neq 0$ and $\gamma \neq 0$, it implies that prices are not fixed and capital is moved as a result of profitability differentials. These two conditions are obvious. A small β is consistent with the distinction between short and long-term models. Note also that $B = 0$, *i.e.*, the absence of direct *Quantity* $\rightarrow$ *Quantity* relation, prohibits stability: At least the direct dependency of the profit rate on the capacity utilization rates is required.

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