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Uncertainty Shocks and Firm Dynamics: Search and Monitoring in the Credit Market

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Uncertainty Shocks and Firm Dynamics: Search and Monitoring in the Credit Market[1]

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Abstract: We develop a business cycle model with gross flows of firm creation and destruction. The credit market is characterized by two frictions. First, entrepreneurs undergo a costly search for intermediate funding to create a firm. Second, upon a match, a costly-state-verification contract is set up. When defaults occurs, banks monitor firms, seize their assets, and a fraction of financial relationships are severed. The model is estimated using Bayesian methods for the U.S. economy. Among other shocks, uncertainty in productivity turns out to be a major contributor to both macro-financial aggregates and firm dynamics.

Keywords: Uncertainty shocks, Financial frictions, Search and Matching, Business Cycles, Firm Dynamics.

Chocs d'incertitude et dynamiques de firmes : frictions d'appariement et coûts d'agence sur le marché du crédit

Abstract : Nous développons un modèle de cycle économique avec flux bruts d'entrées et de sorties de firmes. Le marché du crédit est caractérisé par deux frictions. D'une part, les entrepreneurs doivent effectuer une recherche de financement auprès des banques pour créer une entreprise. D'autre part, la productivité de l'entreprise n'est observée par la banque que par un processus de vérification coûteux, qui se reflète dans le contrat financier. Quand un défaut de paiement se produit, la banque saisit les actifs de l'entreprise et rompt potentiellement sa relation financière avec elle. Le modèle est estimé à partir de méthodes bayésiennes pour l'économie américaine. Entre autres chocs, l'incertitude liée à la productivité apparaît comme une source majeure de fluctuations macro-financières et de dynamique des firmes.

Mots-clefs : Chocs d'incertitude, Frictions financières, Frictions d'appariement et coûts d'agence, Cycles économiques, Dynamique de firmes.

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1 Introduction

A growing macroeconomic literature discusses the role of uncertainty shocks in driving aggregate fluctuations. A common – yet not exclusive – way to define an 'uncertainty shock' is a change in the volatility of firms' productivity, either at the aggregate or individual firm/plant level.[1] Bloom (2009)'s seminal paper provided a milestone to study the effects of such a shock on aggregate investment, labor, and output in particular.[2] Likewise, Christiano et al. (2014) (henceforth, CMR) also consider shocks to the volatility of firms' idiosyncratic productivity – re-labelled as "risk shock" –, but with financial intermediation as a transmission mechanism. Indeed, they develop a business cycle model with a costly-state-verification (henceforth, CSV) contract à la Bernanke et al. (1999) (henceforth, BGG). In this setup, uncertainty affects firms' expected future probability to default on a loan and therefore their current borrowing ability. This model has been particularly successful in replicating important stylized facts of the U.S Great Recession. In particular, a sharp increase in the credit spread together with a deep fall in economic activity. More recent papers, such as Leduc and Liu (2016) assess the effects of productivity volatility on unemployment.

Nevertheless, this literature so far mostly relies on *existing* firms in the economy, and much less on changes in firms' *dynamics* over time. Yet, uncertainty in productivity may matter for the *creation* and *destruction* of firms over the business cycle. Figure 1 depicts these gross flows of firm dynamics during the U.S Great Recession and its aftermath (lower panel), together with the standard output and credit spread variations over the same period (upper panel). A striking feature is the timing asymmetry between firm creation and firm destruction. The latter is characterized by a sharp rise, contemporaneous to the increase in credit spreads, whereas the former by a sluggish drop which lasted long after the recovery in ouptut. Hence, the Great Recession aftermath has not only been characterized by a "jobless recovery" but also a phase of lower business creations altogether.

Motivated by this example, we ask whether uncertainty shocks can explain the observed

[1] Alternatively, Fernández-Villaverde et al. (2015) consider changes in uncertainty about fiscal policy and Basu and Bundick (2017) uncertainty in agents' preferences.

[2] In his model, the existence of non-monotonous capital and labor adjustment costs makes firms occasionally enter into zones of inactivity. As uncertainty grows, investment and hiring are slowed down such that the economy enters a recession.

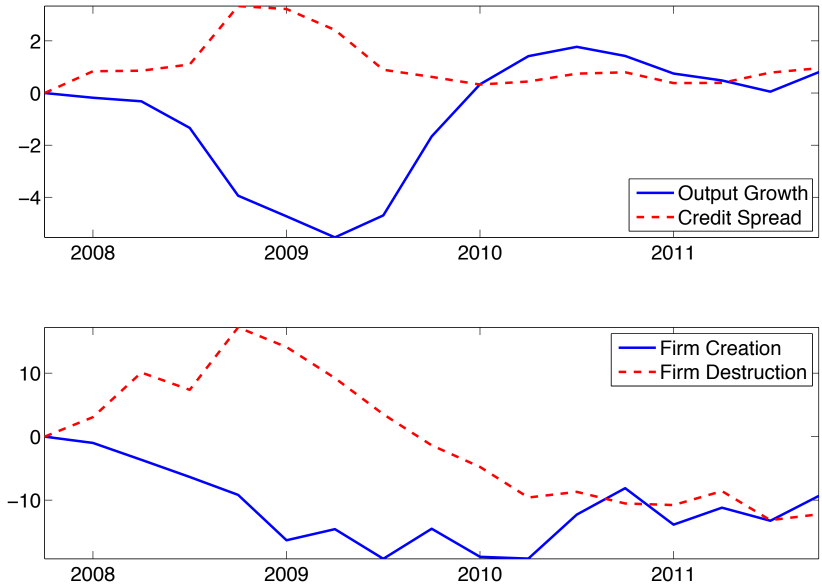

Figure 1: Output Growth, Credit Spread, and Firm Dynamics in the U.S. Great Recession.
Note: Quarterly series, in deviation with respect to 2007Q1. Growth is the year-to-year growth rate of real GDP per capita, in percentage points. Spread is the annualized credit spread (BBA Corporate bond less 10-year Government bond Yields), in percentage points. Firm creation and destruction are levels expressed per capita, in log-deviation. Sources: FRED and BLS, see Appendix A for details.

moments in firm dynamics gross flows, together with the standard macro-finance variables, along business cycles in general. We thus propose a general equilibrium model which aims at reproducing these patterns of firm dynamics during recessions. In particular, we consider a credit market that is characterized by an interplay between two frictions. First, a *search* friction between entrepreneurs and banks.[3] Specifically, new entrepreneurs have to search for banks from whom they could obtain a loan in order to start their business. The role of this search friction is twofold. On the one hand, it allows to disentangle gross flows of firm creation and destruction, as opposite to other standard specifications of firm dynamics with net firm variation only (e.g. Jaimovich and Floetotto (2008); Bilbiie et al. (2012)). This matters if credit conditions affect gross flows in a non-symmetric manner. On the other hand, it also gives value to long-term financial relationships in a model where entrepreneurs

[3]We use the term "bank", for the sake of simplicity, as a financial intermediary in general.

occasionally default on their loan because of negative temporary (uncertainty or other) shocks.

Second, banks *monitor* entrepreneurs in case of a loan default. We here follow the banking literature with optimal debt contracts as a CSV problem. From the early work by Townsend (1979), this approach has been extended to business cycle models by many authors, starting from Carlstrom and Fuerst (1997) and BGG in particular. At the aggregate level, negative shocks tend to make defaults on loan more frequent. This implies a larger monitoring cost that is reflected into a larger credit risk premium. In turn, a costlier access to credit make firms invest less and further reduce the net present value of their assets. As a consequence, a financial accelerator amplifies the size and length of the economic recession. More recently, CMR showed that not only first-moment shocks, but also second-moment shocks – labelled as 'risk shocks' –, can generate such a mechanism. We extend this approach to the question of firm dynamics here, with a distinction between gross firm creation and destruction flows in particular. We then also embed it within a Dynamic Stochastic General Equilibrium (DSGE) model and estimate it with Bayesian techniques for the US economy.

Our contribution to the literature is both theoretical and empirical. At the theoretical level, we examine how search frictions affect the optimal terms of the CSV contract – the size of the loan, the interest rate on the loan, and the default threshold. As compared to BGG-CMR, we generalize their mechanism to the potential presence of search frictions, but nevertheless still nest their (searchless) economy as a particular case. As compared to the search and matching literature applied to credit markets (see the review below), we also contribute in replacing the common Nash bargaining solution with the CSV contract. The latter is closer to both standard banking and macro-finance literatures.

At the empirical level, we apply the Bayesian techniques to estimate our model on US data over the period 1980Q1-2016Q4. This approach has become standard of DSGE models since Smets and Wouters (2003, 2007) in particular. In our case, it has two major advantages. First, it allows to quantify the parameters associated with the search friction on credit markets, and therefore the importance of this mechanism. Search parameters have been extensively studied in the labor market literature but, as far as the credit market is concerned, we are the first to do so, at least in a business cycle context. An entrepreneur's relationship with a bank is estimated to last eleven years on average in our sample. This corresponds to a "search cost" of approximately one third of an entrepreneur's quarterly revenues. At the aggregate level, search costs amount to 3% of entrepreneurial incomes. Second, the Bayesian estimation allows us to estimate the relative contribution of several shocks, including uncertainty shocks, to both firms dynamics and macro-finance variables over time. Uncertainty shocks turn out to be the first contributor to business cycles fluctuations of the credit spread and credit growth, in line with the literature, but also to both flows of firm creation and firm destruction. They also explain an important part of output, investment, and hours worked, between 28% and 43%, often in second position after investment efficiency shocks. Therefore, we confirm the importance of uncertainty shocks for macroeconomic and financial fluctuations, but also show its specific role as a driver of changes in firm dynamics over time.

This paper continues as follows. The rest of this section reviews the literature. Section 2 presents the core of our model, which consists of the optimal loan contracting problem between entrepreneurs and banks in the presence of search frictions. The rest of the general equilibrium environment is standard and relegated to the Appendix. Section 3 describes the steady-state and give more intuition for the mechanisms at play. Section 4 provides simulations and Bayesian estimations. Finally, Section 5 concludes.

Literature review

Our work builds on and contributes to three strands of literature.

First, we develop the search and matching theory in credit markets. Seminal papers on this approach are Den Haan et al. (2003) and Wasmer and Weil (2004). With stylized model setups, they showed how the amplification and propagation properties of search and matching models can be relevant to the analysis of credit markets. Dell'Ariccia and Garibaldi (2005), Herrera et al. (2011), Craig and Haubrich (2013), and Hyun and Minetti (2014) have documented the importance of this approach in the allocation of bank credit to firms. Recently, Petrosky-Nadeau and Wasmer (2013, 2015) have built on credit-market search frictions to explain business cycle facts, in particular puzzling dynamics of the labor market.

We further develop this business cycle approach of credit market frictions to analyze patterns of dynamic variables beyond the labor market. To do so, we nest the search-for-credit within an otherwise standard DSGE model with various real, nominal, and financial frictions. The interplay between these frictions is of particular interest in explaining the dynamics of observed macro-finance and firm dynamics variables. Recently, Reza (2013) and Liberati (2014) also embedded credit-market search frictions into DSGE models. Our distinctive features here are, first, to rely on a BGG-like financial friction, and second, to implement a Bayesian estimation of this framework, in line with the recent literature on DSGE models with financial frictions (e.g see CMR and Del Negro et al. (2015) among many others).

Second, we contribute to the literature on the financial transmission of shocks to the real economy. BGG's 'financial accelerator' essentially stems from the CSV contract à la Townsend (1979). Meanwhile, the credit search literature has also proved the existence of a financial accelerator.[4] Yet, they rely on various other types of contracts, related either to a specific information structure (with a moral hazard problem as in Den Haan et al. (2003)) or an adverse selection problem as in Chamley and Rochon (2011)) or Nash bargaining (as in Wasmer and Weil (2004)). Instead, we here combine the credit search friction with the standard banking (Townsend (1979); BGG) CSV contract. While it nests the searchless economy as a particular case, our model gives an explicit role to long-term financial relationships. Indeed, unlike the BGG-CMR one-period loan contract, the presence of costly search here gives a value to long-term relationships between entrepreneurs and banks. Growing evidence demonstrate the importance of financial relationships in the macroeconomic effect of financial crises, in particular in the US (e.g. Chodorow-Reich (2014); Darmouni (2016)), and in Europe (e.g. Sette and Gobbi (2015)).

Finally, our paper relates to firm dynamics in business cycles. Since the seminal papers by Jaimovich and Floetotto (2008) and Bilbiie et al. (2012), many papers have analyzed causes and consequences of fluctuations in the number of incumbent firms in an economy (e.g Bergin and Corsetti (2008); Lewis (2009); Lewis and Poilly (2012); Lewis and Stevens (2015); Lewis and Winkler (2017)). A subpart of it directly relates firm entry to financial frictions,

[4]This result requires to combine search with another friction. For instance, Den Haan et al. (2003) use an information friction, while Wasmer and Weil (2004) rely on an additional labor market search friction.

starting from Cooley and Quadrini (2001). Poutineau and Vermandel (2015) extended their analysis to business cycles and estimated it. However, with an exogenous firm exit process, this estimation was limited to firm entry data only. In contrast, Rossi (2016) also considers endogenous firm destruction in the presence of financial frictions. Her results from a BVAR estimation are interestingly close to ours, in stressing out the importance of uncertainty shocks in particular. As compared to this literature, our search and matching approach provides an alternative way to formalize firm dynamics, with several advantages. First, while firm entry takes place in monopolistically competive markets in the literature, driven by positive expected profits despite an entry cost, it does so in perfectly competitive markets in our setup. This is useful to analyze the role of financial frictions net of any other markup effects. Second, search and matching allows to analyze both endogenous firm creation and endogenous firm destruction separately. This is particularly important if the two gross firm dynamics margins follow asymmetric patterns over business cycles, as we observed in Figure 1. Third, it gives an intuitive rationale to the existence of a congestion externality in the firm creation process, which has been found particularly important in this literature. Finally, it is worth mentioning that Becsi et al. (2013) also investigated the role of credit search frictions on firm dynamics. However, they do so with a steady-state analysis only, i.e without the business cycle effects and estimations that we are able to provide here.

2 Model

2.1 General equilibrium structure

Our model is a tale of the market for credit where entrepreneurs contract with banks. It is characterized by two frictions, namely a CSV mechanism close to BGG-CMR, and a search process before this optimal contract can be set up. In the main text of this paper, we develop and characterize the equilibrium of this credit market, which is the core part of our model. Then, for the sake of general equilibrium analysis and estimations, we further embed it into a DSGE model. Nevertheless, since these other DSGE parts are standard, we relegate them to the Appendix and just briefly describe it here below.

• Households

There is a unit mass of identical infinitely-lived households. They own firms in different sectors, namely a competitive sector for final goods, a monopolistic sector for intermediate good firms, and a competitive financial intermediation sector. For short, let us call the latter one *banks.* Households derive utility from consumption and leisure. They provide labor to the intermediate good firms, which are themselves the inputs of the final good. Households have two savings vehicles, raw capital and bonds. Their banking sector further uses bonds to supply loans. Finally, they pay taxes to a public authority.

• Entrepreneurs

Their role is to buy raw capital from the households and transform it into 'effective capital', which is then rented out to the intermediate good firms (on a competitive market). This activity is risky, at both the idiosyncratic and aggregate levels. In order to buy raw capital, it is assumed that they use both their own wealth and a bank loan. Since their business is risky, loan defaults will occasionally happen. Entrepreneurs need both to search for banks and to build up a CSV optimal debt contract with them.[5]

• A public authority

It raises taxes, issue bonds, sets the nominal interest rate according to a Taylor-type rule, and consume some final good.

2.2 Entrepreneurs

2.2.1 Population and transitions across states

The total population of entrepreneurs is constant and normalized to unity. Yet, a (time-varying) fraction $(1-\gamma_t)$ of them dies and is born again in every period. Moreover, during their lifetime, entrepreneurs evolve across three distinct states, respectively 'passive', 'unmatched', and 'matched', as follows

[5]Similar entrepreneurs exist in BGG and CMR, without the search friction. In CMR, they are part of the household's family, but not in the original BGG. Here, either case could fit, we choose the latter.

- When new-born, an entrepreneur is always ‘passive’.
- He/she becomes ‘unmatched’ if decides to search for a bank (loan).
- When the bank is found, the entrepreneur is ‘matched’. A one-period debt contract can then be (optimally) set up between both agents, and so holds in each and every period as long this ‘financial relationship’ continues.
- Some entrepreneurs may default on their loan (since their business is risky), among which a fraction separates from their bank. In that case, they become ‘unmatched’ again.[6] They may also decide to become passive again if searching for a new bank is not profitable enough.

Figure 2 summarizes the timeline of ‘unmatched’ and ‘matched’ entrepreneurs.

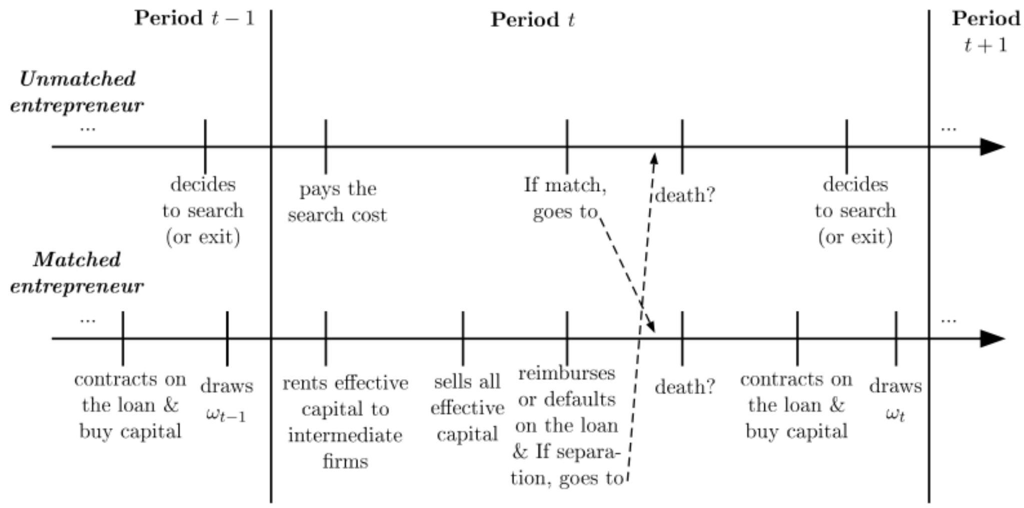

Figure 2: Timeline of matched and unmatched entrepreneurs

[6]The separation rate is exogenous, but conditional on default, which is itself a control variable. Then, firm destruction is endogenous in the model.

2.2.2 Search for a loan

A constant returns-to-scale technology gives the periodic flow of new financial relationships. Specifically, we assume a Cobb-Douglas matching function as[7]

$$credit\,flows_t = z^c \left(u_t^e\right)^{\alpha^c} \left(u_t^b\right)^{1-\alpha^c} \tag{1}$$

where u_t^e denotes the mass of unmatched entrepreneurs at time t, u_t^b the mass of bankers searching for an entrepreneur at time t, z^c the efficiency of the matching process, and $0 < \alpha^c < 1$ a parameter. An (unmatched) entrepreneur's matching probability is thus given by

$$p_t^\theta \equiv \frac{credit\,flow_t}{u_t^e} = z^c \left(u_t^e\right)^{\alpha^c-1} \left(u_t^b\right)^{1-\alpha^c} = z^c \left(\frac{u_t^e}{u_t^b}\right)^{\alpha^c-1} = z^c \theta_t^{\alpha^c-1} \tag{2}$$

where

$$\theta_t \equiv \frac{u_t^e}{u_t^b} \tag{3}$$

is the *credit market tightness*.

Let us assume that an unmatched entrepreneur's decision to search during period t is taken at the end of date $t-1$, with full information on period t search cost, D_t^S, matching probability, p_t^θ, and survival rate, γ_t, all taken as given.[8] Thus, the value at $t-1$ of searching at time t is

$$\mathcal{E}_{t-1}^u = -D_t^S + \gamma_t \beta^e \left[p_t^\theta \mathcal{E}_t^m + \left(1 - p_t^\theta\right) \mathcal{E}_t^u\right] \tag{4}$$

where β^e denotes entrepreneurs' discount factor, and where $\mathcal{E}_t^m$ is the expected present-value of being matched at the end of period t, which depends both on the realization of aggregate shocks during period t and the draw of idiosyncratic productivity at the end of period t.[9]

[7] In the labor market search and matching literature, matching functions between unemployed workers and job vacancies are generally Cobb-Douglas. The early credit market search and matching literature follows this assumption too (e.g Wasmer and Weil (2004)).

[8] As individual entrepreneurs take the financial tightness, and thus the matching probability, as given, the usual search and matching's congestion externality also holds here.

[9] The expectation operator is omitted here as both aggregate innovations and individual draws are i.i.d, such that the decision to search at the end of period $t-1$ is unaffected. The next period Bellman equation for matched entrepreneurs in period t account for those and the law of iterated expectation applies at $t-1$.

2.2.3 Production

Entrepreneurs who are matched in period t decide at the end of period t over their individual capital holding for the next period, denoted K_{t+1} to be purchased at the market price $Q_{K,t}$, taken as given, from both personal wealth N_{t+1} and a one-period debt amount B_{t+1} contracted optimally with the bank at the end of time t. Thus, the capital purchase constraint satisfies

$$Q_{K,t}K_{t+1} = N_{t+1} + B_{t+1} \tag{5}$$

As in BGG-CMR, entrepreneurs may have different levels of wealth (and thus of capital purchases), yet wealth accumulation is not explicit and both types of funding are always required. Hence, entrepreneurs can never become so rich that they would not need intermediation. Similarly, our newly matched entrepreneurs supposedly hold a minimal amount of wealth.[10] The level of wealth, borrowing, and capital purchase are N-type specific, but the leverage resulting from the optimal financial contract will be identical for all entrepreneurs, in BGG-CMR's spirit.

After capital is purchased, an idiosyncratic productivity shock, ω, converts capital, K_{t+1}, into efficiency units, ωK_{t+1}. This random component ω follows a cumulative distribution function $F_t(\omega) \equiv F(\omega, \sigma_{\omega,t})$ with a unit-mean and a standard deviation of $\log \omega$ equal to σ_t. The standard deviation σ_t is itself the result of an exogenous stochastic process because of an uncertainty shock, or "risk shock" using the CMR terminology, as

$$\log \sigma_t = (1 - \rho_\sigma) \log \sigma_\omega + \rho_\sigma \log \sigma_{\omega,t-1} + \varepsilon_{\omega,t} \tag{6}$$

where $\sigma_{\omega,t}$ is determined when the capital purchase is made. Moreover, following CMR, we

[10] One can imagine that newly matched entrepreneurs are given a fixed initial amount of wealth to invest in the project, either by households or a parent company. Alternatively, one can interpret N_{t+1} as the valuation at date t of firms' asset at date $t+1$, which only depends on ex post returns and is thus similar for all entrepreneurs, regardless of the duration of their financial relationship, since individual uncertainty is i.i.d..

consider news on the risk shock that evolves as follows

$$\varepsilon_{\omega,t} = \xi_{0,t} + \xi_{1,t-1} + ... + \xi_{p,t-p} \tag{7}$$

where $\xi_{0,t}$ is the unanticipated component of $\varepsilon_{\omega,t}$ and $\xi_{j,t-j}$ for $j > 0$ is the anticipated (or news) components of $\varepsilon_{\omega,t}$. These shocks are referred to "news shocks", which variances are set to a common value, $\sigma_{\sigma,n}$, for all $j > 1$. The variance of the unanticipated uncertainty shocks, for $j = 0$, is denoted $\sigma_{\sigma,0}$.

In early period $t + 1$, knowing their shock realization, (matched) entrepreneurs rent $u_{t+1}\omega K_{t+1}$ as capital services to intermediate firms, at the real rate r^k_{t+1} taken as given. The rate of capital utilization, u_{t+1}, is chosen optimally by entrepreneurs, according to the aggregate conditions, and thus similar across all N-types. It is equal to 1 in steady-state. The utilization per unit of capital generates a cost $a\left(u_{t+1}\right)$ for the productive entrepreneur, which is increasing and convex. Later in period $t + 1$, the entrepreneur sells the non-depreciated capital, $\left(1-\delta\right)\omega K_{t+1}$ to the households, at the market price $Q_{K,t+1}$. The average/aggregate return per unit of capital invested in period t at date $t + 1$ is denoted R^k_{t+1} and ωR^k_{t+1} at the individual entrepreneur's level.

The debt contract between a matched entrepreneur and the financial intermediary is based on the costly-state verification framework. The contract is stated in the end of period t, before the realization of the idiosyncratic shock, and is settled in the end of period $t+1$. For every state, defined by the realization of ω, with the associated R^k_{t+1}, a matched entrepreneur has to either (i) pay a state-contingent gross interest rate Z_{t+1} on the loan, B_{t+1}, or (ii) default, in which case the bank seizes all entrepreneur's revenue net of a fraction μ spent on monitoring costs. This determines a threshold value $\overline{\omega}$ such that a productive entrepreneur pays back the loan if $\omega > \overline{\omega}_{t+1}$, where

$$R^k_{t+1}\overline{\omega}_{t+1}Q_{K,t}K_{t+1} = B_{t+1}Z_{t+1} \tag{8}$$

and default otherwise ($\omega \leq \overline{\omega}_{t+1}$). Note that the optimal default threshold will be the same for all entrepreneurs, and not N-type specific, since productivity draws are i.i.d across time and across entrepreneurs, as in BGG-CMR.[11]

A fraction s^c_{t+1} of credit relationships among the defaulting entrepreneurs are severed. We assume that the separation rate is exogenous and evolves as

$$\ln s^c_t = (1-\rho_{s^c}) \ln s^c + \rho_{s^c} \ln s^c_{t-1} + \varepsilon_{s^c,t} \tag{9}$$

where $\varepsilon_{s^c,t}$ is an i.i.d $N \sim (0, \sigma_{s^c})$ separation shock. This simplifying assumption has the advantage to express that defaults resulting from low productivity draws do not necessarily lead to separation.[12] Moreover, whenever $s^c = 0$, our model is able to nest CMR without search friction as a particular case. Finally, note that this separation of defaulting entrepreneurs, together with the death of $(1-\gamma_t)m_t$ matched entrepreneurs, determines *firm destruction* in our model.

2.2.4 Long-term value of the credit relationship

At the end of period t, a N-type matched entrepreneur's value reads as

$$\begin{aligned} \mathcal{E}^m_t &= \mathrm{E}_t \left\{ \int_{\overline{\omega}_{t+1}}^{\infty} \left[R^k_{t+1}\omega_t \left(N_{t+1}+B_{t+1}\right) - B_{t+1}Z_{t+1} \right] dF_t + (1-\gamma_{t+1})P_{t+1}C^e_{t+1} \right\} \\ &+\beta^e \mathrm{E}_t \left\{ \gamma_{t+1} \left[\int_{\overline{\omega}_{t+1}}^{\infty} \mathcal{E}^m_{t+1} dF_t + s^c_{t+1} \int_0^{\overline{\omega}_{t+1}} \mathcal{E}^u_{t+1} dF_t + (1-s^c_{t+1}) \int_0^{\overline{\omega}_{t+1}} \mathcal{E}^m_{t+1} dF_t \right] \right\} \end{aligned} \tag{10}$$

where the first term shows the nominal payoff of the entrepreneur net of loan reimbursement conditional on a productivity draw higher than the (endogenous) default threshold. If, on the contrary, the draw is below the threshold, then the payoff for the entrepreneur is zero since the financier seizes whatever has been produced. The second term C^e_{t+1} is the nominal consumption level of the matched entrepreneur if he/she dies in that period, with probability $(1-\gamma_{t+1})$, whose level will be determined later on. This assumption is similar to CMR where all productive entrepreneurs who die consume. Here, it gives entrepreneurs a

[11] Depending on the realization of the idiosyncratic shock, ex post income flows and next period status (matched or unmatched) will vary. Were the default threshold $\overline{\omega}$ fixed, we would have an occasionally binding constraint à la Aiyagari (1994) and need to keep track of the income distribution of entrepreneurs in equilibrium. Instead, $\overline{\omega}$ is endogenous and, in the absence of binding constraint, identically chosen by all (representative) entrepreneurs.

[12] This probability of separation could be made endogenous as an extension of the model.

rationale to undertake costly search activities. Finally, the last three terms in the expression above represent the continuation values if the entrepreneur survives in the next period, with probability γ_{t+1}. The continuation value states that the entrepreneur will remain matched in the next period if his/her current productivity draw is higher than the threshold or the default does not lead to a separation, or will become unmatched again otherwise.

After simplification, we can rewrite it as

$$\begin{aligned} \mathcal{E}_t^m = & \mathrm{E}_t \left\{ \left[1 - \Gamma_t\left(\overline{\omega}_{t+1}\right)\right] R_{t+1}^k \left(N_{t+1} + B_{t+1}\right) + (1-\gamma_{t+1}) P_{t+1} C_{t+1}^e \right\} \\ & + \beta^e \mathrm{E}_t \left\{ \gamma_{t+1} \left[\mathcal{E}_{t+1}^m - F_t(\overline{\omega}_{t+1}, \sigma_t) s_{t+1}^c \left(\mathcal{E}_{t+1}^m - \mathcal{E}_{t+1}^u \right) \right] \right\} \end{aligned} \tag{11}$$

where, following BGG-CMR's notation, $\Gamma_t(\overline{\omega}_{t+1})$ stands for the share of entrepreneurial earnings that goes to the bank and satisfies

$$\Gamma_t\left(\overline{\omega}_{t+1}\right) \equiv \left[1 - F_t\left(\overline{\omega}_{t+1}\right)\right] \overline{\omega}_{t+1} + G_t\left(\overline{\omega}_{t+1}\right), \text{ with } G_t\left(\overline{\omega}_{t+1}\right) \equiv \int_0^{\overline{\omega}_{t+1}} \omega dF_t(\omega) \tag{12}$$

or, net of monitoring cost, $\Gamma_t\left(\overline{\omega}_{t+1}\right) - \mu G_t\left(\overline{\omega}_{t+1}\right) = \left[1 - F_t\left(\overline{\omega}_{t+1}\right)\right] \overline{\omega}_{t+1} + (1-\mu) G_t\left(\overline{\omega}_{t+1}\right)$. Note the last term in equation (11) with a minus sign, which makes it apparent that there is a loss of surplus from severing a financial relationship. Were $s_t^c = 0$ (no separation), this term would vanish and the expected profits boil down to BGG-CMR.

2.3 Banks

Without loss of generality, we assume that each bank is managed by one banker who can monitor only one entrepreneur. Bankers' population is constant over time. As of time t, a mass u_t^b of bankers are "searching" for an entrepreneur, yet at no cost, while others, m_t^b are engaged in a financial relationship. Each banker thus evolves between two states: unmatched and matched.[13] By definition, matched bankers are as numerous as matched entrepreneurs, i.e $m_t^b = m_t^e \equiv m_t$.

[13]Unlike entrepreneurs, bankers are never "passive", because search is assumed to be costless for them. Endogenous entry of bankers would be an interesting extension of this setup.

The value of being unmatched for a banker in the end of period $t-1$ is

$$\mathcal{F}_{t-1}^{u}=\beta_{t,t+1}^{\star}\left[\theta_{t}p_{t}^{\theta}\,\mathcal{F}_{t}^{m}+\left(1-\theta_{t}p_{t}^{\theta}\right)\mathcal{F}_{t}^{u}\right]=\beta_{t,t+1}^{\star}\left[\theta_{t}p_{t}^{\theta}\left(\mathcal{F}_{t}^{m}-\mathcal{F}_{t}^{u}\right)+\mathcal{F}_{t}^{u}\right] \tag{13}$$

where $\beta_{t,t+1}^{\star}$ is the stochastic discount factor from households owning the intermediaries[14], $\theta_t p_t^{\theta}$ is bankers' probability to match with an entrepreneur, and $\mathcal{F}_t^m$ is bankers' value of being matched as of time t.

When matched, a banker lends B_{t+1} to the entrepreneur by borrowing the same amount from the households at the risk-free rate R_t. Even though the financial market is perfectly competitive and intermediaries makes zero profit, an interest rate spread still holds for the monitoring costs associated with borrowers' risky activity. The banker's payoff from the lending activity is either $Z_{t+1}B_{t+1}$ if the entrepreneur draws $\omega > \overline{\omega}_{t+1}$ and does not default, or $(1-\mu)\,R_{t+1}^{k}Q_{K,t}\omega K_{t+1}$ if the entrepreneur draws $\omega \leq \overline{\omega}_{t+1}$ and defaults. If there is no separation with the entrepreneur, and if the entrepreneur survives in $t+1$, the financial relationship continues and a new loan contract is initiated at that time. Otherwise, the bank gets the value of being unmatched (13). Hence, using (12), the value of being matched to a N-type entrepreneur is

$$\begin{aligned}\mathcal{F}_{t}^{m}=\mathrm{E}_{t}\Big\{&R_{t+1}^{k}Q_{K,t}K_{t+1}\left[\Gamma_{t}\left(\overline{\omega}_{t+1}\right)-\mu G_{t}\left(\overline{\omega}_{t+1}\right)\right]-R_{t}B_{t+1}\\&+\beta_{t,t+1}^{\star}\left[\gamma_{t+1}\mathcal{F}_{t+1}^{m}+\left(1-\gamma_{t+1}\right)\mathcal{F}_{t+1}^{u}-\gamma_{t+1}F_{t}\left(\overline{\omega}_{t+1}\right)s_{t+1}^{c}\left(\mathcal{F}_{t+1}^{m}-\mathcal{F}_{t+1}^{u}\right)\right]\Big\}\end{aligned} \tag{14}$$

2.4 Equilibrium

2.4.1 The free entry condition for entrepreneurs

Entrepreneurs enter the credit market until they are indifferent between participating or not. Non-participating entrepreneurs pay no costs and receive no revenues. Therefore the value of staying outside the credit market is zero and, by the free entry condition, the value of searching for a financial intermediary should also be equal to zero: $\mathcal{E}_t^u = 0\ \forall t$, in equilibrium.

[14]Households' problem is similar to CMR and thus relegated to Appendix. In particular, the expression for the stochastic discount factor is given by equation (C.20).

Hence, equation (4) becomes

$$\frac{D_t^S}{p_t^\theta} = \gamma_t \beta^e \mathcal{E}_t^m \tag{15}$$

The free entry condition makes equal the expected costs of successful search (LHS) and the expected value of the financial relationship (RHS), which is given by the equation (11) and becomes with th free entry condition.

$$\begin{aligned} \mathcal{E}_t^m = \mathrm{E}_t \Big\{ & \left[1 - \Gamma_t(\overline{\omega}_{t+1})\right] R_{t+1}^k \left(N_{t+1} + B_{t+1}\right) + \left(1 - \gamma_{t+1}\right) P_{t+1} C_{t+1}^e \\ & + \left[1 - F_t\left(\overline{\omega}_{t+1}\right) s_{t+1}^c\right] D_{t+1}^S / p_{t+1}^\theta \Big\} \end{aligned} \tag{16}$$

Combining (15)-(16), the equilibrium condition for a N-type entrepreneur is

$$\begin{aligned} D_t^S / p_t^\theta = \gamma_t \beta^e \mathrm{E}_t \Big\{ & \left[1 - \Gamma_t\left(\overline{\omega}_{t+1}\right)\right] R_{t+1}^k \left(N_{t+1} + B_{t+1}\right) + \left(1 - \gamma_{t+1}\right) P_{t+1} C_{t+1}^e \\ & + \left[1 - F_t\left(\overline{\omega}_{t+1}\right) s_{t+1}^c\right] D_{t+1}^S / p_{t+1}^\theta \Big\} \end{aligned} \tag{17}$$

where the expected costs of successful search (LHS) equate the expected gains from search (RHS) made up three parts: the share of production revenues received by the entrepreneurs, the entrepreneur consumption if death occurs, and the value of the financial relationship (if not severed).

2.4.2 The optimal financial contract

The optimal contract is characterized by the threshold value of the idiosyncratic shock, $\overline{\omega}_{t+1}$, the gross interest rate on loan, Z_{t+1}, and the level of debt, B_{t+1}, that maximize the expected present-value of a (matched) entrepreneur defined by (16) subject to the financial intermediary's participation constraint. In presence of search frictions, this constraint is restated as

$$\mathcal{F}_t^m - \mathcal{F}_t^u \geq 0$$

which binds in equilibrium, i.e $\mathcal{F}_t^m = \mathcal{F}_t^u = 0$, such that by (14), this implies

$$R_t B_{t+1} = R_{t+1}^k Q_{K,t} K_{t+1} \left[\Gamma_t\left(\overline{\omega}_{t+1}\right) - \mu G_t\left(\overline{\omega}_{t+1}\right)\right] \tag{18}$$

where the left-hand side is known at the end of period t. Note that, in spite of the presence of search frictions, this constraint for the participation of bankers is exactly similar as in BGG, expressing that banks' cost of borrowing on the LHS must be equal in equilibrium to their expected share of the output of the entrepreneur net of monitoring costs. Also note that the funds received by financial intermediaries in each period $(t+1)$ state of nature are assumed to be no less than the funds paid to households in that state of nature, such that the condition (18) holds in each realized $(t+1)$ state of nature and not only in expectation as of time t. This assumption holds in CMR with free entry condition of financial intermediaries on the market for household deposits. Here, it still hold because search frictions are featured on the credit market and not the market for household deposits, where the m matched bankers are in perfect competition.

Hence, the N- type entrepreneur's maximization problem is

$$\begin{aligned}\max_{B_{t+1},\overline{\omega}_{t+1}} \mathrm{E}_t \{ & [1-\Gamma_t(\overline{\omega}_{t+1})] R^k_{t+1}(N_{t+1}+B_{t+1}) + (1-\gamma_{t+1}) P_{t+1} C^e_{t+1} \\ & + \left[1 - F_t(\overline{\omega}_{t+1}) s^c_{t+1}\right] D^S_{t+1}/p^\theta_{t+1} \\ & + \lambda^c_{t+1}\left[-R_t B_{t+1} + R^k_{t+1}(B_{t+1}+N_{t+1})\left[\Gamma_t(\overline{\omega}_{t+1}) - \mu G_t(\overline{\omega}_{t+1})\right]\right]\}\end{aligned} \tag{19}$$

where λ^c_{t+1} denotes the Lagrange multiplier associated with the participation constraint (which depends on the $t+1$ state of nature), and where (5) has been used to introduce the loan variable in the objective function. The first-order condition with respect to B_{t+1} is

$$\mathrm{E}_t\left\{[1-\Gamma_t(\overline{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t} - \lambda^c_{t+1}\left[1 - \frac{R^k_{t+1}}{R_t}\left[\Gamma_t(\overline{\omega}_{t+1}) - \mu G_t(\overline{\omega}_{t+1})\right]\right]\right\} = 0 \tag{20}$$

The first order condition with respect to $\overline{\omega}_{t+1}$ is

$$\begin{aligned}\mathrm{E}_t \{ & R^k_{t+1} Q_{K,t} K_{t+1}\left[\lambda^c_{t+1}\left[\Gamma'_t(\overline{\omega}_{t+1}) - \mu G'_t(\overline{\omega}_{t+1})\right] - \Gamma'_t(\overline{\omega}_{t+1})\right] \\ & - F'_t(\overline{\omega}_{t+1}) s^c_{t+1} D^S_{t+1}/p^\theta_{t+1}\} = 0\end{aligned} \tag{21}$$

with $\Gamma'_t \equiv \frac{\partial\Gamma(\omega_{t+1},\sigma_{\omega,t})}{\partial\omega}|_{\omega=\overline{\omega}_{t+1}}$, $G'_t \equiv \frac{\partial G(\omega_{t+1},\sigma_{\omega,t})}{\partial\omega}|_{\omega=\overline{\omega}_{t+1}}$, and $F'_t \equiv \frac{\partial F(\omega_{t+1},\sigma_{\omega,t})}{\partial\omega}|_{\omega=\overline{\omega}_{t+1}}$. As in BGG-CMR, that the optimal default threshold is chosen the same by all entrepreneurs in equilibrium, regardless of their level of net worth. Further, the level of capital K_{t+1} and the level of borrowing B_{t+1} are N-type specific, but entrepreneurs' leverage is not since, by (5) and (18), we have

$$L_t \equiv \frac{Q_{K,t}K_{t+1}}{N_{t+1}} = \frac{1}{1 - \frac{R^k_{t+1}}{R_t}\left[\Gamma_t\left(\overline{\omega}_{t+1}\right) - \mu G_t\left(\overline{\omega}_{t+1}\right)\right]} \tag{22}$$

which holds the same for all entrepreneurs. The third term of the optimal contract is the loan interest rate, Z_{t+1}, as determined by (8). Note that (18) and (20) are identical to the BGG-CMR models, while the last term in (21) accounts for the effect of search frictions on the optimal contract. However, the BGG-CMR case is still nested as a particular case of our model, should the search cost D^S_t or the separation rate s^c_t be nil. Search frictions make the value of existing financial relationships positive by the existence of a positive probability $F_t\left(\overline{\omega}_{t+1}\right) s^c_{t+1}$ that the entrepreneur must pay the per-period search costs D^S_t again to find a new financial intermediary.

2.5 Aggregation of net worth

Aggregate variables are affected by the mass of entrepreneurs across different states (passive, searching, and producing). In particular, market-clearing for the physical capital requires $\bar{K}_t = m_t K_t$ where $\bar{K}_t$ is the aggregate capital supply from households, m_t the number of matched entrepreneurs where each is demanding K_t units of capital at time t. Similarly for credit, $\bar{B}_t = m_t B_t$ and for the aggregation of net worth, $\bar{N}_t = m_t N_t$.

This implies that the aggregate net worth evolves according to

$$\bar{N}_{t+1} = \frac{m_{t+1}}{m_t}\left\{\gamma_t\left[1 - \Gamma_{t-1}\left(\bar{\omega}_t\right)\right] R^k_t Q_{K,t-1}\bar{K}_t + \bar{W}^e_t\right\} \tag{23}$$

where the first term in the curly brackets is the production revenues from matched entrepreneurs at the end of period $t-1$, who survive with probability γ_t, and the second term $\bar{W}^e_t = m_t W^e_t$ is the aggregate transfer from households to matched entrepreneurs at the end of period t.[15] This curly bracketed part is similar to CMR, except that the aggregate capital

[15] This assumption is similar to BGG-CMR and reflects the idea that a minimum level of wealth is necessary for the feasibility of external finance with asymmetric information.

stock $\bar{K}_t$ and transfer $\bar{W}^e_t$, is proportional to the amount of matched entrepreneur at time t. The additional term $\frac{m_{t+1}}{m_t}$ makes it clear that the accumulation of aggregate net worth also depends on the growth of matched entrepreneurs in this economy. The aggregate leverage is thus identical to the individual leverage since

$$\bar{L}_t \equiv \frac{Q_{K,t}\bar{K}_{t+1}}{\bar{N}_{t+1}} = \frac{Q_{K,t}K_{t+1}m_{t+1}}{N_{t+1}m_{t+1}} = \frac{Q_{K,t}K_{t+1}}{N_{t+1}} = L_t \tag{24}$$

The aggregate resource constraint of this economy, expressed in real terms, is given by

$$Y_t = C_t + G_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\Upsilon^{-t}\overline{K}_t + \bar{D}^M_t + \overline{C}^e_t \tag{25}$$

where the first four terms on the right-hand side stand for households' consumption, public consumption, households' investment in raw capital, and capital utilization costs, which are determined as in CMR[16] and thus relegated to Appendix here.[17] The fifth term stands for the monitoring costs, now proportional to the mass of matched entrepreneurs as

$$\bar{D}^M_t = \mu G(\omega_t)(1 + R^k_t)\frac{Q_{t-1}\bar{K}_t}{P_t} \tag{26}$$

and finally the last term is the aggregate non-survival payoff, in real terms, as

$$\bar{C}^e_t = \frac{1-\gamma_t}{\gamma_t}\Theta\frac{\bar{N}_{t+1}\frac{m_t}{m_{t+1}} - \bar{W}^e_t}{P_t} \tag{27}$$

Indeed, the aggregate level of entrepreneurial assets, hold by matched entrepreneurs, in nominal terms, at the end of period t, is $\left[1 - \Gamma_{t-1}\left(\overline{\omega}_t\right)\right] R^k_t Q_{K,t-1}\bar{K}_t$. By (23), this is equal to $\left(\bar{N}_{t+1}\frac{m_t}{m_{t+1}} - \bar{W}^e_t\right)/\gamma_t$. A fraction $(1-\gamma_t)$ of it is hold by those who die, and a fraction Θ itself consumed. Equivalently, at the individual (matched entrepreneur) level, $C^e_t = \overline{C}^e_t/m_t$ is thus the non-survival payoff entering the Bellman equations, from (10) onward.[18]

[16]The households, monopolistic producers, and public authority are kept unchanged from CMR here.

[17]Note that the credit search costs are assumed non-pecuniary and thus do not appear in (25).

[18]CMR's main text page 39 (and footnote 17) does not have this dying entrepreneur consumption term in the resource constraint but it is indeed present in CMR's Appendix equation (B23) (while (B13) is the model without entrepreneurs). If entrepreneurs were households' members, it could alternatively be assumed that the non-survival consumption $\bar{C}^e_t$ is part of household's consumption C_t.

2.6 Firm dynamics

The number of productive entrepreneurs evolves as

$$m_{t+1} = \gamma_t \left[\int_{\overline{\omega}_t}^{\infty} m_t dF_{t-1}(\overline{\omega}_t) + \int_0^{\overline{\omega}_t} (1 - s_t^c) m_t dF_{t-1}(\overline{\omega}_t) + creditflow_t \right] \tag{28}$$

where the total number of matched entrepreneurs at the beginning of period $t+1$, denoted m_{t+1}, is equal to the number of matched entrepreneurs that did not separate in period t plus the number of new matches. Old and newly matched entrepreneurs face the risk of dying with probability $(1 - \gamma_t)$. Using (2), this can be rewritten as

$$m_{t+1} = \gamma_t \left([1 - F_{t-1}(\overline{\omega}_t) s_t^c] m_t + z^c (u_t^e)^{\alpha^c} \left(u_t^b\right)^{1-\alpha^c} \right) \tag{29}$$

The net creation of firms is measured as

$$net_t = m_{t+1} - m_t \tag{30}$$

and can be expressed as the difference between gross firm creation and destruction flows, i.e

$$net_t = \underbrace{\gamma_t (creditflow)_t}_{\text{Creation Flow}} - \underbrace{[(1 - \gamma_t) + \gamma_t F_{t-1}(\overline{\omega}_t) s_t^c] m_t}_{\text{Destruction Flow}} \tag{31}$$

where firm creation as of time t is the flow of newly matched firms during period t which survive in fraction t, and where firm destruction as of time t is the sum of dying firms, with a probability $(1 - \gamma_t)$, and defaulting firms that separate with the bank with probability $F_{t-1}(\overline{\omega}_t) s_t^c$.

3 Steady-state Analysis

We provide here more intuition on the role of the search friction for credit market outcomes in our economy. First, as compared to the standard CSV case, we examine how the presence of search affects the optimal loan contract. Second, we analyze the determinants of the aggregate mass of unmatched entrepreneurs in the steady-state of our economy, in the spirit of the search and matching literature for unemployed workers.

To simplify the analysis, prices, especially the interest rate, R, and the return on capital, R^k, are taken as given here. Note that they also were in the dynamic derivation of the optimal contract from the viewpoint of individual maximizing agents in Section 2, but become an endogenous variable in the general equilibrium and estimation parts of our model.

3.1 Financial contract optimality conditions

At steady-state, the first-order condition with respect to the amount of debt reads as

$$(1-\Gamma(\overline{\omega}))\frac{R^k}{R} = \lambda^c\left[1-\frac{R^k}{R}(\Gamma(\overline{\omega})-\mu G(\overline{\omega}))\right] \tag{32}$$

The left-hand side of this expression is the entrepreneur's marginal gain of holding debt. Additional capital purchases increase profits, with a share $(1-\Gamma(\overline{\omega}))$ left for the entrepreneur. The right-hand side is the entrepreneur's marginal cost of holding debt, which is equal to the inverse of leverage, $L \equiv \frac{Q_K K}{N} = \frac{B+N}{N} = \left[1-\frac{R^k}{R}(\Gamma(\overline{\omega})-\mu G(\overline{\omega}))\right]^{-1}$ from (22), times the shadow value of wealth, λ^c.

The first-order condition with respect to the default threshold $\overline{\omega}$,

$$R^k QK(\overline{\omega})\left\{\lambda^c\left[\Gamma'(\overline{\omega})-\mu G'(\overline{\omega})\right]-\Gamma'(\overline{\omega})\right\} = F'(\overline{\omega})\, s^c \frac{D^S}{p^\theta} \tag{33}$$

holds for each individual (heterogeneous) entrepreneur's level of capital, and thus requires to use the free-entry condition

$$\frac{D^S}{p^\theta} = \frac{(1-\Gamma(\overline{\omega}))\, R^k QK}{(\gamma\beta^e)^{-1}-1+F(\overline{\omega})\, s^c} \tag{34}$$

simplified with $C^e = 0$ without loss of generality here. The solution for the shadow value of wealth, λ^c, is given by

$$\lambda^c = \frac{\Gamma'(\overline{\omega})}{\Gamma'(\overline{\omega})-\mu G'(\overline{\omega})} + \frac{(1-\Gamma(\overline{\omega}))}{\Gamma'(\overline{\omega})-\mu G'(\overline{\omega})}\frac{F'(\overline{\omega})\, s^c}{(\gamma\beta^e)^{-1}-1+F(\overline{\omega})\, s^c} \tag{35}$$

The first term of this expression is identical to BGG. It stands for the fact that an entrepreneur chooses the optimal value of the default threshold $\overline{\omega}$ depending on its impact on her/his own share of profits. Indeed, an increase of $\overline{\omega}$ implies a fall in the entrepreneur's share $(1-\Gamma(\overline{\omega}))$ since $\Gamma'(\overline{\omega})>0$, and an increase in the bank's share $(\Gamma(\overline{\omega})-\mu G(\overline{\omega}))$, since we restrict the model to the cases where $(\Gamma'(\overline{\omega})-\mu G'(\overline{\omega}))$ is positive as in BGG. The second term is specific to the search friction and can be interpreted as the expected search cost of increasing the risk of default. An increase of $\overline{\omega}$ leads to a higher risk of default F, the magnitude of this increase being given by F', which leads to a separation of the financial relationship with a probability s^c. Upon separation, an entrepreneur loses the value of the financial relationship, which is equivalent to the expected total search costs (the per-period search costs D^s divided by the matching probability p^θ). It shows that the cost of defaulting is higher in an economy with search frictions than without since, in addition to a reduction in the share of returns, measured by Γ', the return of past investment in search activities are lost in case of a separation. This last effect increases the shadow value of wealth as compared to the case without search frictions.

There exists no explicit analytical solution for the optimal $\overline{\omega}$ here as in BGG. Hence, we follow them in introducing the cutoff function $\rho(\overline{\omega})$ solving for

$$\frac{R^k}{R}=\rho(\overline{\omega}^*) \tag{36}$$

which is exactly the equilibrium condition (A.1) of BGG. Hence, combining (32) and (35) to substitute out λ^c and dropping out the argument $\overline{\omega}$, we get

$$\rho(\overline{\omega}^*)=\frac{\left(\frac{\Gamma'}{\Gamma'-\mu G'}\right)[\,1+\overbrace{\frac{1-\Gamma}{\Gamma'}\frac{F's^c}{(\gamma\beta^e)^{-1}-1+Fs^c}}^{\text{search frictions}}\,]}{(1-\Gamma)+\left(\frac{\Gamma'}{\Gamma'-\mu G'}\right)(\Gamma-\mu G)\,[1+\underbrace{\frac{1-\Gamma}{\Gamma'}\frac{F's^c}{(\gamma\beta^e)^{-1}-1+Fs^c}}_{\text{search frictions}}\,]} \tag{37}$$

which is depicted in the top panel of Figure 3, using parameter values resulting from our estimation (see Section 4). This steady-state function has several appealing properties. First, it depends on a single endogenous variable, the cutoff $\overline{\omega}$, for which it therefore gives

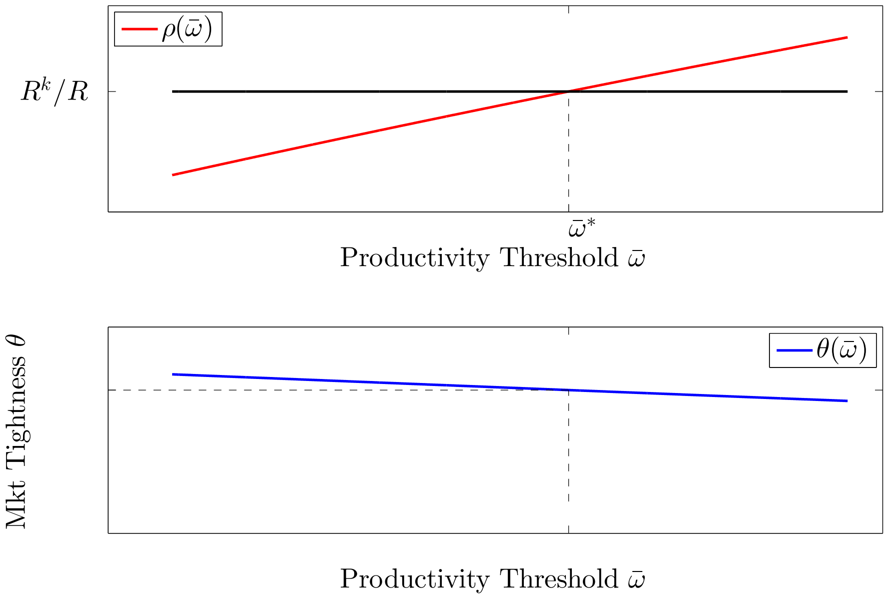

Figure 3: Steady-state Equilibrium: Financial contract

an implicit solution. This does not depend on the (search-specific) credit market tightness θ in particular. Second, it nests BGG as a particular case, whenever $s^c = 0$, i.e whenever defaulting does not induce separations in financial relationships.[19] Third, the effect of search on the equilibrium financial contract is unambiguous. To make this clear, let us compare the solutions with and without separation among defaulting entrepreneurs. Let us assume that a solution exists when there is no separation, $s^c = 0$, and denote it $\overline{\omega}^*_{s^c=0}$, which satisfies $\rho\left(\overline{\omega}^*_{s^c=0}\right) = R^k/R$. Let us also assume that a solution exists for a positive separation rate, $s^c > 0$, and denote it $\overline{\omega}^*_{s^c>0}$, which satisfies $\rho\left(\overline{\omega}^*_{s^c>0}\right) = R^k/R$. Then $\overline{\omega}^*_{s^c>0} < \overline{\omega}^*_{s^c=0}$ since $\rho'\left(\overline{\omega}^*_{s^c=0}\right) > 0$, as in BGG. The economy with separations has a lower equilibrium default cutoff because they imply a potential loss of the matching costs. Implications for the leverage are also straightforward: $L\left(\overline{\omega}^*_{s^c>0}\right) < L\left(\overline{\omega}^*_{s^c=0}\right)$ since $L'\left(\overline{\omega}^*\right) > 0$.

[19]Without the search friction terms, $\rho\left(\overline{\omega}\right)$ becomes identical to BGG's Appendix A.

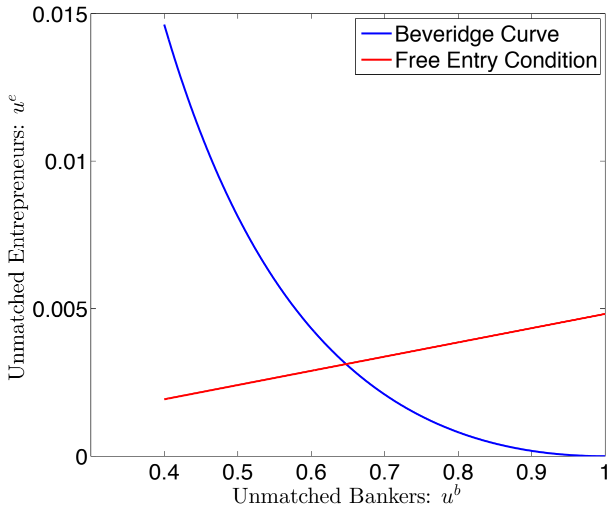

Figure 4: Steady-state Equilibrium: Flow Equilibrium

3.2 Long-run firm creation flow

We follow here the search and matching literature in determining the steady-state value of searching entrepreneurs as the intersection of two equilibrium conditions. Those are depicted in Figure 4, with parameter values taken from our estimation results (see Section 4).

First, (28) reads, in steady-state,

$$p^{\theta} u^{e} \gamma = m \left(1 - \gamma + \gamma F\left(\overline{\omega}\right) s^{c}\right) \tag{38}$$

where the left-hand side stands for the firm creation flow in this long-run economy,[20] while the right-hand side stands for the firm destruction flow, stemming from two sources, namely the exogenous death with probability $(1-\gamma)$ among matched entrepreneurs, m, and the separation with probability s^c among surviving matched entrepreneurs who default on the

[20]Recall that the flow of new matches, $p^{\theta} u^{e}$, is not equal to the firm creation flow in this economy since only a share γ of these new matches $\left(p^{\theta} u^{e}\right)$ survive in each period.

loan with probability $F(\overline{\omega})$. Using the matching function in (1), it can be rewritten as

$$u^e_{BC}\left(u^b,\overline{\omega}\right) = \left[\frac{pop^b - u^b}{z^c\left(u^b\right)^{1-\alpha^c}}\left(\frac{1}{\gamma} - 1 + F(\overline{\omega})\, s^c\right)\right]^{1/\alpha^c} \tag{39}$$

where z^c is the efficiency parameter of the matching function, α^c the elasticity of the matching function with respect to searching entrepreneurs. (39) is the counterpart of the so-called 'Beveridge curve' for the labor market, i.e a decreasing and convex relationship between the two stocks of searching agents on the market, here u^e unmatched entrepreneurs and u^b unmatched banks. A higher default cutoff $\overline{\omega}$ implies a higher firm destruction rate $F(\overline{\omega})$ and thus a higher firm entry by this relationship, everything else equal.

The second equilibrium condition substitutes the definition of the tightness (3) out of the free-entry condition (17), which, taken at steady-state, gives

$$u^e_{FE}\left(u^b,\overline{\omega}\right) = u^b\left[\frac{z^c}{D^S}\frac{R^k N}{(\gamma\beta^e)^{-1} - 1 + F(\overline{\omega})\, s^c}\frac{1-\Gamma(\overline{\omega})}{1-\left(\Gamma(\overline{\omega}) - \mu G(\overline{\omega})\right) R^k/R}\right]^{1/(1-\alpha^c)} \tag{40}$$

The mass of searching entrepreneurs is here a linear function of the mass of unmatched banks. The slope of this line depends on structural parameters and on the equilibrium cutoff $\overline{\omega}$. The impact of $\overline{\omega}$ is however ambiguous because of three effects. First, an increase in $\overline{\omega}$ implies a reduction in the share of profits for the entrepreneur, $1-\Gamma(\overline{\omega})$, which decreases the mass of searching entrepreneurs on the credit market. Second, an increase in $\overline{\omega}$ increases the loan default rate, $F(\overline{\omega})$, which also diminishes the return of search activity and, then, the mass of searching entrepreneurs. The third effect acts in the opposite direction, alleviating the participation constraint of banks by an increase in their profit share, $(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))$. The bottom panel of Figure 3 shows how the credit market tightness θ^* can be deduced given the equilibrium value $\overline{\omega}^*$ solution of (36).

4 Estimation

The model is estimated with Bayesian methods. See An and Schorfeide (2007) and Fernández-Villaverde et al. (2016) for an overview of the methodology. We use the Dynare software package developed by Adjemian et al. (2011) to simulate and estimate the model. We follow the empirical strategy of CMR, which is a reference for the estimation of business cycle models with financial frictions and uncertainty shocks. We extend this strategy to (*i*) the estimation of credit search frictions and (*ii*) the fluctuations of firm creation and destruction.

4.1 Data

We use quarterly observations on thirteen variables covering the period 1980Q1-2016Q4. Appendix A provides details about the different series together with links to the original sources of data. We include eight macroeconomic aggregates, quite standard in Bayesian estimation of business cycle models, e.g. Smets and Wouters (2003, 2007): the growth rates of real GDP per capita, real consumption per capita, real investment per capita, price deflator (inflation), wages, the price of investment, and the level of hours worked together with the short-term risk-free interest rate. Because our estimation period encompasses the zero lower bound period, we use the shadow interest provided by Wu and Xia (2016) for the short-term risk-free interest rate instead of the effective federal funds when it is stuck to zero. We then add three financial variables taken from CMR, i.e the growth rate of credit, the stock market capitalization (as a proxy of entrepreneurial net worth), and the credit spread between the yields of the Baa corporate bonds and the 10 Years Government bonds.[21]

We complete this set of macroeconomic and financial data with the series of establishment births and deaths provided by the Bureau of Labor Statistics. We first construct a series of *firm creation* from 1980 to today by combining the series of new business incorporations provided by Survey of Current Business, which ends in 1995, and the recent series of establishment births provided by the BLS which starts in 1992. Then, we can investigate the dynamics of firm creation for a long period, which includes the last recession.[22] Then, we measure *firm destruction* by the series of establishment deaths provided by the BLS. Unlike most of the firm dynamics literature which considers net entry only, we are able to distinguish gross flows of firm creation and destruction in the data, consistently with our

[21]Unlike CMR, we drop out the slope of the term structure here.

[22]By contrast, many papers have a data sample either ending in the mid-1990s, e.g Bergin et al. (2016), Lewis and Poilly (2012), and Lewis and Stevens (2015), or starting in the mid-1990s, e.g Poutineau and Vermandel (2015).

credit market search model. Unfortunately, the series of firm destruction starts in 1992 against 1980 for all other series. Nevertheless, Bayesian estimations have the advantage to allow for series with different observation periods. Both firm creation and firm destruction flows are taken, per capita terms, as observable variables in the estimation.

Finally, let us define $\mathtt{DATA}_t$ set as

$$\mathtt{DATA}_t = \begin{bmatrix} \Delta \log \mathtt{GDP}_t \\ \Delta \log \mathtt{Consumption}_t \\ \Delta \log \mathtt{Investment}_t \\ \Delta \log \mathtt{Credit}_t \\ \Delta \log \mathtt{Inflation}_t \\ \Delta \log \mathtt{NetWorth}_t \\ \Delta \log \mathtt{InvestmentPrice}_t \\ \Delta \log \mathtt{Wage}_t \\ \log \mathtt{Hours}_t \\ \log \mathtt{Creation}_t \\ \log \mathtt{Destruction}_t \\ \mathtt{CreditSpread}_t \\ \mathtt{R}_t \end{bmatrix}$$

and the set of observable variables as the deviation of $\mathtt{DATA}_t$ with respect to its empirical mean

$$\mathtt{OBS}_t = \mathtt{DATA}_t - \overline{\mathtt{DATA}}$$

4.2 Priors and Posteriors

Some of our model parameters are calibrated and reported in Table 1. The growth rate of the economy μ_z is set to 1.65% and the inflation rate[23] π to 2.4% (the average of the data in annual percent rate). The discount factor of households β is set to 0.9987 as in CMR. The value of households' discount factor β is then deduced from steady-state constraints. The

[23]As in CMR, a shock on the inflation target is calibrated to account for the inflation trend.

Table 1: Calibrated parameters

Symbol	Name	Value
β	Discount rate	0.998
δ	Deprecation rate of the economy	0.025
α	Power on capital in production function	0.40
σ_L	Curvature on disutility of labor	1
Υ	Growth rate of investment specific technological change (Annual percentage)	1.70
μ_z	Growth rate of the economy (Annual percentage)	1.60
λ_w	Steady state markup, suppliers of labor	1.05
λ_f	Steady state markup, intermediate good firms	1.20
γ	Survival rate of entrepreneurs	98.50
w^e	Transfer received by new entrepreneurs	0.005
Θ	Share of assets consumed by entrepreneurs	0.005
π^{target}	Steady state inflation rate (Annual percentage)	2.40
G/Y	Steady-state government spending to GDP ratio	0.20
τ^c	Tax rate on consumption	0.05
τ^k	Tax rate on capital income	0.32
τ^l	Tax rate on labor income	0.24

relative disutility of labor and the total factor productivity are both normalized to unity. The depreciation rate of physical capital δ is set to 2.5 percent. The elasticity of production with respect to the labor input α is 0.40. Entrepreneurs' transfer from households and non-survival consumption, respectively w^e and Θ, are both set to an arbitrary small value of 0.005. Tax and markup rates are taken from CMR who themselves refer to Christiano et al. (2005) and Christiano et al. (2010).

As for our estimated parameters, Tables 2, 3, and 4 report the mean and standard deviation for both prior and posterior distributions – see Appendix B for the mode of the posterior distribution. They also include the 90% confidence interval for the posterior distribution. Our general equilibrium includes three types of frictions, real, nominal, and financial. We estimate parameters in each category. Real friction parameters include the degree of habit formation and the curvatures of the investment adjustment and utilization cost technologies. Nominal friction parameters are related to price and wage stickiness. They include Calvo probabilities and degrees of price indexation in particular. They also include the respective weights of output and inflation gaps in the monetary policy Taylor rule. Priors are aligned with the literature on Bayesian estimation of business cycle models, eg. Smets and Wouters (2003, 2007).

Then, financial friction parameters consist of the monitoring and search activities. As for the monitoring cost and the rate of loan default, we use the same priors as in CMR. In contrast, credit search parameters have not yet been estimated with Bayesian techniques. We propose to estimate four parameters with the following priors. First, the ratio of the entrepreneurs' discount factor to the households discount factor is assumed to have a mean of 0.95, which corresponds to the calibration by Carlstrom and Fuerst (1997). Second, taking the matching rate and the matching probability as given, we deduct two associated structural parameters, namely the search cost D^s and the matching efficiency z^c.[24] Unfortunately, there is no empirical evidence to document these rates to the best of our knowledge as of to date.[25] Finally, our prior distributions assume a mean of 80% of entrepreneurs being financed and producing (versus searching or passibe), a 25% matching probability for searching entrepreneurs, and a mean separation rate s^c of 15%.

Looking at the posteriors, real and nominal parameters are in line with the literature and therefore not discussed here. Financial parameters are however worth more attention. First, regarding the monitoring friction, posterior means are quite different for prior means. Indeed, 5.1% against 0.7% for the loan default rate F, and 7% against 25% for the monitoring costs μ, respectively. On this point, the distance between prior and posterior distributions is larger in our estimation than in CMR. Yet, they are consistent with external observations. The 1985-2016 average delinquency rate on all loans is 3.6%, as reported by the U.S Federal Reserve.[26] Direct monitoring costs as low as 4% are considered in Carlstrom and Fuerst (1997)'s sensitivity analysis, even though values are commonly higher than our estimate in this literature. This can be explained by a sort of reallocation between the monitoring costs and the credit search costs in our model. Indeed, the standard CSV literature generally considers a single financial friction while we have two. Finally, it is worth mentioning the low values of the posteriors standard deviation for these two parameters, when compared with the priors, together with the tightness of the confidence intervals.

Second, let us comment on the estimation results for the credit search parameters. As far as the matching probability p^θ is concerned, both the mean and the standard deviation

[24] Note that the matching rate and the matching probability are endogenous variables in the model. CMR proceed similarly when taking the loan default rate parameter as exogenous, despite being endogenous in the model, to deduct the volatility of idiosyncratic shocks.

[25] Levenson and Willard (2000) show that the duration of the application process for credit is a key feature of credit rationing, yet do not provide an average duration of credit search.

[26] See https://fred.stlouisfed.org/series/DRALACBS and CMR's Figure 8 for a comparison between the theoretical and empirical default rates.

posteriors are very close to the priors. Hence, the data is not particularly informative here. However, the matching rate has a posterior mean of 34%, very much lower than the prior mean of 80%, with tight confidence intervals for the posteriors. Here again, it suggests that when both monitoring and search financial frictions are at play, the respective importance of each goes smaller. The posterior mean of the discount factor ratio between entrepreneurs and households is found to be 27.9%. This number is quite low when compared with the literature, e.g. Carlstrom and Fuerst (1997), but can be explained by an important feature of our model. Indeed, search frictions make the CSV contract dynamic. On the contrary, it is static in the standard BGG model, which can be nested as a particular case of ours when entrepreneurs' discount factor is zero. Therefore, this positive estimate gives support to our approach as a dynamic feature of the CSV contract. Households' discount factor is still much higher than entrepreneurs' and consistent with the log-run values of economic growth, inflation, and the risk-free interest rate.

Finally, the size of search frictions can be interpreted with the ratio $\left(D^s/p^\theta\right) / \left[(1-\Gamma)R^k K\right]$, i.e expressing the matching cost – itself equal to the value of the financial relationship – in terms of per-period flow of entrepreneurial revenue (see equation (17)). The mean of the posterior distribution for this ratio is equal to 0.31. Matching costs thus represent almost a third of the quarterly entrepreneur revenues. If we aggregate for all entrepreneurs, multiplying this ratio by u^e/m, search costs represent 3% of entrepreneur revenues.

Table 2: Results from Metropolis-Hastings: Structural parameters

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
`Calvo wage stickiness:` ξ_w	beta	0.750	0.1000	0.641	0.0204	0.6064	0.6735
`Calvo price stickiness:` ξ_p	beta	0.500	0.1000	0.831	0.0323	0.7837	0.8865
`Price indexing weight on inflation target:` ι	beta	0.500	0.1500	0.789	0.0828	0.6575	0.9207
`Wage indexing weight (inflation):` ι_w	beta	0.500	0.1500	0.397	0.1249	0.1931	0.5995
`Wage indexing weight (growth):` ι_μ	beta	0.500	0.1500	0.953	0.0191	0.9234	0.9837
`Consumption habits:` b	beta	0.500	0.1000	0.814	0.0288	0.7673	0.8619
`Curvature, utilization cost:` σ_a	norm	1.000	1.0000	4.259	0.6473	3.1901	5.3038
`Curvature, investment adjust cost:` $S^{'}$	norm	5.000	3.0000	15.548	2.3670	11.6236	19.4135
`Monetary policy weight on output growth:` $\alpha_{\Delta y}$	norm	0.250	0.1000	0.308	0.0988	0.1440	0.4688
`Monetary policy weight on inflation:` α_π	norm	1.500	0.2500	2.431	0.2203	2.0740	2.8018
`Steady state probability of default:` $F(\bar{\omega})$	beta	0.007	0.0025	0.051	0.0047	0.0432	0.0586
`Monitoring cost:` μ	beta	0.250	0.1000	0.070	0.0056	0.0608	0.0792
`Discount factor ratio:` β^e/β	beta	0.950	0.0200	0.279	0.0177	0.2505	0.3089
`Steady state separation rate:` s^c	beta	0.150	0.0200	0.126	0.0111	0.1081	0.1448
`Steady state matching probability:` p^θ	beta	0.250	0.1000	0.256	0.1013	0.0924	0.4156
`Steady state matched entrepreneurs:` m	beta	0.800	0.1000	0.344	0.0659	0.2348	0.4530

Table 3: Results from Metropolis-Hastings: Standard deviation of structural shocks

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
`Equity:` σ_γ	invg2	0.002	0.0033	0.002	0.0001	0.0014	0.0019
`Price markup:` $\sigma_{\lambda f}$	invg2	0.002	0.0033	0.021	0.0076	0.0113	0.0347
`Investment price:` $\sigma_{\mu\Upsilon}$	invg2	0.002	0.0033	0.004	0.0002	0.0033	0.0040
`Government consumption:` σ_g	invg2	0.002	0.0033	0.019	0.0012	0.0173	0.0213
`Persistent technology:`$\sigma_{\mu z}$	invg2	0.002	0.0033	0.011	0.0007	0.0096	0.0120
`Temporary technology:` σ_ε	invg2	0.002	0.0033	0.006	0.0004	0.0056	0.0069
`Monetary policy:` σ_{xp}	invg2	0.583	0.8250	0.532	0.0361	0.4723	0.5890
`Consumption preferences:` $\sigma_{\zeta c}$	invg2	0.002	0.0033	0.033	0.0036	0.0271	0.0381

(Continued on next page)

Table 3: (continued)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
`Investment efficiency:` $\sigma_{\zeta i}$	invg2	0.002	0.0033	0.037	0.0093	0.0253	0.0498
`Separation rate:` σ_{sc}	invg2	0.233	0.0033	0.246	0.0034	0.2399	0.2511
`Uncertainty unanticipated:` $\sigma_{\sigma,0}$	invg2	0.002	0.0033	0.043	0.0059	0.0330	0.0519
`Uncertainty anticipated (news):` $\sigma_{\sigma,n}$	invg2	0.001	0.0012	0.023	0.0019	0.0204	0.0265
`Measure error shock: Net Worth`	weibl	0.010	5.0000	0.069	0.0042	0.0616	0.0752

Table 4: Results from Metropolis-Hastings: Autocorrelation of structural shocks

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
`Price markup:` $\rho_{\lambda f}$	beta	0.500	0.2000	0.885	0.0392	0.8189	0.9443
`Investment price:` $\rho_{\mu\Upsilon}$	beta	0.500	0.2000	0.983	0.0080	0.9702	0.9956
`Government consumption:` ρ_g	beta	0.500	0.2000	0.979	0.0102	0.9646	0.9953
`Persistent technology:`$\rho_{\mu z}$	beta	0.500	0.2000	0.097	0.0467	0.0201	0.1671
`Temporary technology:` ρ_{ε}	beta	0.500	0.2000	0.988	0.0090	0.9749	0.9994
`Monetary policy:` ρ_{xp}	beta	0.750	0.1000	0.926	0.0072	0.9146	0.9379
`Consumption preferences:` $\rho_{\zeta c}$	beta	0.500	0.2000	0.870	0.0418	0.7999	0.9321
`Investment efficiency:` $\rho_{\zeta i}$	beta	0.500	0.2000	0.726	0.1022	0.5677	0.8871
`Separation rate:` ρ_{sc}	beta	0.500	0.2000	0.753	0.0280	0.7059	0.7979
`Uncertainty unanticipated:` ρ_{σ}	beta	0.500	0.2000	0.948	0.0180	0.9189	0.9763
`Uncertainty anticipated (news):` $\rho_{\sigma,n}$	norm	0.000	0.5000	0.387	0.0861	0.2465	0.5291

4.3 The Role of Uncertainty Shocks

Figure 5 depicts the response of selected variables to an uncertainty shock, using our model with Bayesian estimates. As in CMR, an increase in the cross-sectional dispersion of productivity makes the CSV problem more severe. As the entrepreneurial risk increases, the

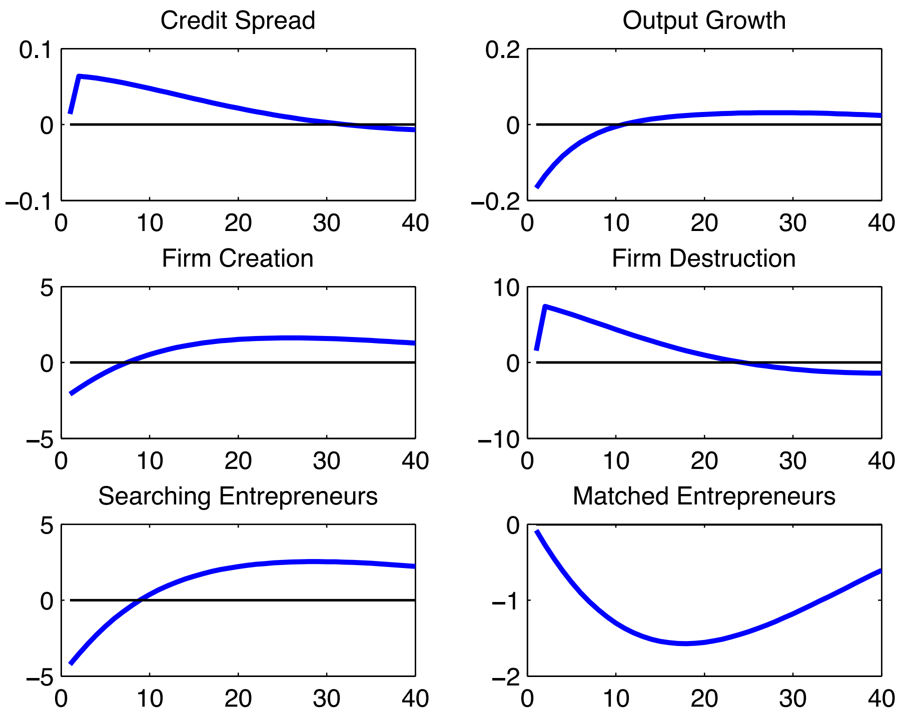

Figure 5: Impulse response functions (IRFs) to a positive uncertainty shock. Percentages of the log-deviation on the vertical axis

risk premium goes up to ensure the participation of bankers to the debt contract. As a consequence, the credit spread increases and the demand for credit falls, leading to a macroeconomic downturn, characterized by a fall in investment and production. Moreover, let us observe the response of firm dynamics to the same uncertainty shock. Because entrepreneurs have to compensate bankers more when the risk goes up, incentives to search are lower, and therefore we observe a reduction in the flow of firm creation. Meanwhile, uncertainty makes defaults on loan more frequent, and thereby firm destruction increases. The combination of lower firm creation on the one hand, and higher firm destruction on the other hand contributes to the slower growth of productive firms in the economy.

To sum up, uncertainty shocks help to understand not only the countercyclical of credit spread, as originally demonstrated by CMR, but also the cyclical behavior of firm creation (procyclical) and destruction (countercyclical), when financial frictions are at play.

Table 5 presents the contribution of shocks to the variance of variables at the business cycle frequencies. In line with the recent literature – e.g. Bloom (2009), CMR, Fernández-Villaverde et al. (2015), Leduc and Liu (2016), and Basu and Bundick (2017) – uncertainty shocks are an important contributor to business cycles. First, they explain most of the variance of financial series, especially the credit spread (96%), but also credit growth and the short-term risk free interest rate. Second, they are an important source of real variable fluctuations, such as the growth rates of real GDP and investment, the level of hours worked, and the inflation rate. For real GDP and investment, they however come in second position, after the investment efficiency shock. Fluctuations in the growth rates of real consumption and wage do not seem driven by uncertainty shocks. Third, uncertainty shocks turn out to be a key driver of firm dynamics. Indeed, they account for 79% of the variance of firm creation and 82% of the variance of firm destruction. In second position, we find the investment efficiency shock for firm creation and the separation shock for firm destruction.

The U.S Great Recession is a particularly interesting episode in our sample. To resonate with the introduction of this paper, we show the contribution of selected shocks to the historical credit spread, growth rate of real GDP, and flows of firm creation and destruction in Figures 1, 6, and 7. Consistently with numerous narratives, uncertainty shocks are found to play a key role during this episode. The rise in uncertainty accounts for a sharp fall in production and investment in 2008 and 2009 in particular. Afterwards, expansionary investment efficiency shocks drive the dynamics of output and investment. When it comes to firm dynamics, uncertainty shocks generate higher variations in firm creation and destruction than the actual variations in these variables. Actually, they are partially compensated by a reduction in the separation rate in the economy. Persistent productivity shocks are also an important driver of slow firm creation recovery in the aftermath of the Great Recession.

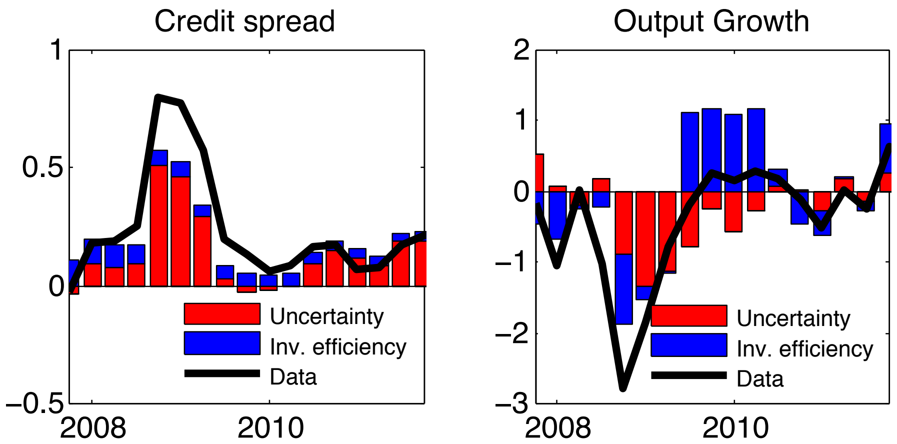

Figure 6: Historical contribution of uncertainty and investment efficiency shocks to credit spread (in deviation in points of percentage), and output growth (log-deviation in percentage) during the Great Recession (2007Q4-2011Q4).

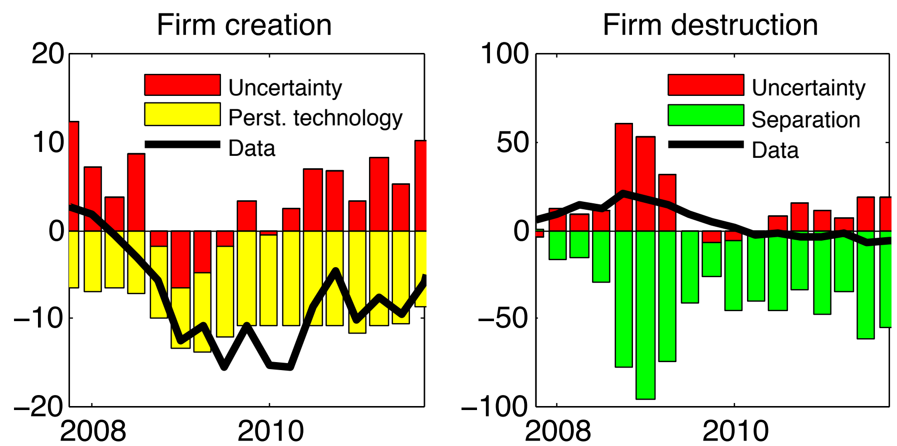

Figure 7: Historical contribution of uncertainty and persistent technological shocks to firm creation and of uncertainty and separation shocks to firm destruction during the Great Recession (2007Q4-2011Q4), log-deviation in percentage.

Table 5: Variance Decomposition (in percent) (Bandpass filter, (8 32))

	σ_{xi}	$\sigma_{\lambda f}$	$\sigma_{\mu\Upsilon}$	σ_g	$\sigma_{\mu z}$	σ_{ε}	σ_{xp}	$\sigma_{\zeta c}$	$\sigma_{\zeta i}$	σ_{sc}	$\sigma_{\sigma,0}$	$\sigma_{\sigma,n}$	σ_{σ}
	`Equity`	`Markup`	`InvP`	`GovC`	`TechP`	`TechT`	`MonPol`	`ConPref`	`InvEff`	`SepRate`	`UncerU`	`UncerA`	`Uncer`
`Gdp`	0.27	6.03	0.15	3.57	4.28	4.52	2.83	9.26	39.93	0.60	4.99	23.54	28.54
`Consumption`	0.02	9.85	0.03	1.84	3.84	19.91	4.13	57.80	0.61	0.02	0.21	1.74	1.95
`Investment`	0.42	2.06	0.03	0.03	0.10	0.22	1.10	0.04	52.12	0.83	7.19	35.85	43.05
`Credit`	14.84	1.91	0.01	0.37	0.05	2.20	0.78	2.89	14.76	14.17	8.43	39.59	48.02
`Inflation`	0.32	29.38	0.00	0.60	0.42	7.16	5.35	12.37	8.71	0.59	5.09	29.90	34.99
`Networth`	2.38	2.39	0.31	0.12	0.18	0.09	6.26	0.60	9.20	3.77	18.46	56.21	74.67
`InvestmentPrice`	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
`Wage`	0.02	16.36	0.00	0.09	72.78	6.48	0.35	0.73	1.31	0.04	0.30	1.54	1.84
`Hours`	0.24	7.70	0.08	2.49	1.85	6.54	2.92	8.62	39.60	0.55	4.40	25.00	29.41
`Creation`	1.16	1.70	0.25	0.10	2.42	0.04	3.42	0.37	8.98	1.65	14.28	65.62	79.89
`Destruction`	2.88	0.04	0.01	0.01	0.01	0.03	0.25	0.06	0.61	13.38	17.37	65.34	82.71
`Creditspread`	0.84	0.05	0.01	0.02	0.02	0.04	0.35	0.07	0.93	0.99	20.41	76.28	96.70
`R`	0.37	13.80	0.01	0.48	0.40	3.64	19.66	11.96	8.72	0.67	5.91	34.35	40.25

Note: Structural shocks are `Equity:` σ_{γ}, `Price markup` $\sigma_{\lambda f}$, `Investment price` $\sigma_{\mu\Upsilon}$, `Government consumption` σ_g, `Persistent technology` $\sigma_{\mu z}$, `Temporary technology` σ_{ε}, `Monetary policy` σ_{xp}, `Consumption preferences` $\sigma_{\zeta c}$, `Investment efficiency` $\sigma_{\zeta i}$, `Separation rate` σ_{sc}, `Uncertainty unanticipated` $\sigma_{\sigma,0}$, `Uncertainty anticipated (news)` $\sigma_{\sigma,n}$, and `Uncertainty` σ_{σ} the sum of $\sigma_{\sigma,0}$ and $\sigma_{\sigma,n}$.

5 Conclusion

This paper builds a general equilibrium model where the credit market is characterized by an interplay between search frictions *à la* Den Haan et al. (2003) and Wasmer and Weil (2004) and a CSV friction (monitoring costs) *à la* BGG-CMR. Search frictions modify the optimal loan contract as compared to the standard CSV case, as they increase the borrower's cost of default by impairing its long-run financial relationship. They also create endogenous firm creation and firm destruction processes in our economy. This environment is particularly suitable to investigate the joint dynamics of macro-financial aggregates and firm dynamics.

We then provide a Bayesian estimation of our model on U.S data over the period 1980-2016. We are particularly interested in assessing the respective roles of our two financial frictions, namely *search* and *monitoring*. Our estimate of the mean monitoring cost is found lower than the CSV literature but completed by the financial search cost, which we are the first to estimate with a business cycle approach. As for the sources of business cycles, we find that uncertainty shocks are a key contributor to business cycles, in line with the literature (e.g. Bloom (2009), CMR, Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017)), but also show their major role in the fluctuations of firm creation and destruction flows.

Further research could improve our analysis on several dimensions. First, firm heterogeneity is quite simplistic here. Transitory idiosyncratic shocks make all firms identical *ex-ante*. A richer environment, with persistent idiosyncratic shocks, would be an interesting extension of our model. In particular, it could make the separation of financial relationships endogenous whereas it is only an exogenous fraction of defaulting entrepreneurs in our model. Second, banks could be given a more active role in the search process. Finally, our distinction between *search* and *monitoring* financial frictions could be particularly relevant when it comes to policy issues. For instance, they allow to disentangle intensive and extensive margins of the credit channel of monetary policy.

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Appendix

A Data

A.1 Macroeconomic and Financial Series

We follow Christiano et al. (2014) for the definition of macroeconomic and financial series.

- `GDP`: Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series), divided by population.
- `Consumption`: US : Real Personal Consumption Expenditures: Nondurable Goods + Real Personal Consumption Expenditures: Services, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series1 + series2 and before 1999, BEA NIPA Table 2.3.3), divided by population.
- `Investment`: US : Real Personal Consumption Expenditures: Durable Goods + Real Gross Private Domestic Investment, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate (Fred series1 + series2 and before 1999, BEA NIPA Table 2.3.3), divided by population.
- `Inflation`: US : GDP Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), logarithmic first difference.
- `InvestmentPrice`: US : Gross Private Domestic Investment Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), divided by GDP Deflator.
- `Hours`: US : Nonfarm Business Sector: Hours of All Persons, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series).
- `Wage`: US : Nonfarm Business Sector: Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted (Fred series), divided by GDP Deflator.
- `R` for the short-term risk-free rates: US : Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted (Fred series).
- `Credit`: US : Nonfinancial Noncorporate Business; Credit Market Instruments; Liability + Nonfinancial Corporate Business; Credit Market Instruments; Liability, Level,

Billions of Dollars, Quarterly, Not Seasonally Adjusted (Fred series1 + series2), divided by GDP Deflator, divided by population.

- `CreditSpread`: US : Moody's Seasoned Baa Corporate Bond Yield, Percent, Quarterly, Not Seasonally Adjusted (Fred series), less 10-year Government Bond Yield.
- `NetWorth` for entrepreneurial net worth: US : Wilshire 5000 Total Market Index, Quarterly, Not Seasonally Adjusted (Fred series), divided by GDP Deflator.
- `Population`: US : Working Age Population: Aged 15-64: All Persons for the United States, Persons, Quarterly, Seasonally Adjusted (Fred series).

A.2 Firm Creation and Destruction Series

To build the firm creation series, we combine two datasets.

- `Creation`: New Business Incorporations (historical series), from the monthly New Business Incorporations series, from 1948M1 to 1994M12, we construct a quarterly sample. The source of the series is the Survey of Current Business, January/February 1996 (Table 13), available on https://fraser.stlouisfed.org/, divided by population.
- `Creation`: Number of establishments births (recent series), From the Bureau of Labor Statistics, we download the total private sector establishments births series (quarterly and seasonaly adjusted), available https://www.bls.gov/, divided by population.

Figure A.1 shows the two series which are then chained and divide by the population series to get a measure of firm creation per capita consistent with our model. To build the firm creation series, we combine two datasets. To build the firm destruction series, we use the same source as for (recent) firm creation series.

- `Destruction`: Number of establishments deaths, From the Bureau of Labor Statistics, we download the total private sector establishments deaths series (quarterly and seasonaly adjusted), available https://www.bls.gov/, divided by population.

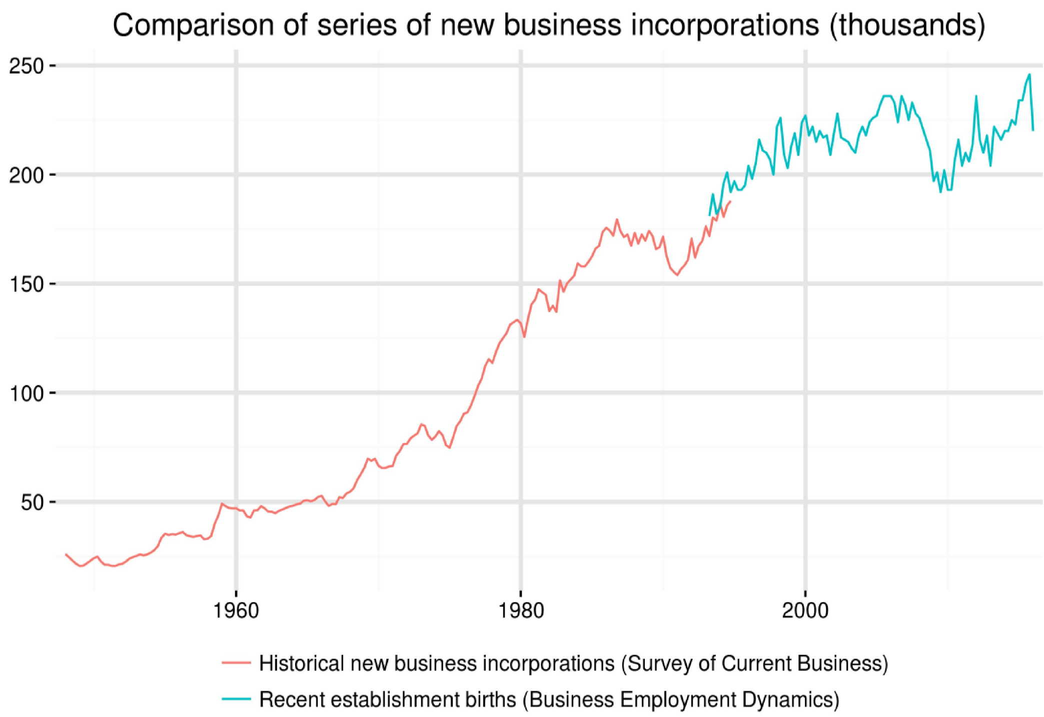

Figure A.1: Firm creation series

B Posterior Distribution: Mode

Table B.1: Results from posterior maximization: Structural parameters (mode)

	Prior			Posterior	
	Dist.	Mean	Stdev	Mode	Stdev
`Calvo wage stickiness:` ξ_w	beta	0.750	0.1000	0.6452	0.0194
`Calvo price stickiness:` ξ_p	beta	0.500	0.1000	0.7961	0.0311
`Price indexing weight on inflation target:` ι	beta	0.500	0.1500	0.7409	0.1016
`Wage indexing weight (inflation):` ι_w	beta	0.500	0.1500	0.3545	0.1187
`Wage indexing weight (growth):` ι_μ	beta	0.500	0.1500	0.9574	0.0199
`Consumption habits:` b	beta	0.500	0.1000	0.7875	0.0220
`Curvature, utilization cost:` σ_a	norm	1.000	1.0000	4.3184	0.6622
`Curvature, investment adjust cost:` $S^{'}$	norm	5.000	3.0000	12.4466	2.4760
`Monetary policy weight on output growth:` $\alpha_{\Delta y}$	norm	0.250	0.1000	0.3154	0.0998

(Continued on next page)

Table B.1: (continued)

	Prior			Posterior	
	Dist.	Mean	Stdev	Mode	Stdev
Monetary policy weight on inflation: α_π	norm	1.500	0.2500	2.6193	0.2028
Steady state probability of default: $F(\bar{\omega})$	beta	0.007	0.0025	0.0517	0.0049
Monitoring cost: μ	beta	0.250	0.1000	0.0683	0.0056
Discount factor ratio: β^e/β	beta	0.950	0.0200	0.2790	0.0182
Steady state separation rate: s^c	beta	0.150	0.0200	0.1241	0.0112
Steady state matching probability: p^θ	beta	0.250	0.1000	0.2168	0.1036
Steady state matched entrepreneurs: m	beta	0.800	0.1000	0.3636	0.0729

Table B.2: Results from posterior maximization: Standard deviation of structural shocks (Mode)

	Prior			Posterior	
	Dist.	Mean	Stdev	Mode	Stdev
Equity: σ_γ	invg2	0.002	0.0033	0.0017	0.0001
Price markup: $\sigma_{\lambda f}$	invg2	0.002	0.0033	0.0142	0.0024
Investment price: $\sigma_{\mu\Upsilon}$	invg2	0.002	0.0033	0.0036	0.0002
Government consumption: σ_g	invg2	0.002	0.0033	0.0191	0.0012
Persistent technology:$\sigma_{\mu z}$	invg2	0.002	0.0033	0.0107	0.0007
Temporary technology: σ_ε	invg2	0.002	0.0033	0.0061	0.0004
Monetary policy: σ_{xp}	invg2	0.583	0.8250	0.5249	0.0362
Consumption preferences: $\sigma_{\zeta c}$	invg2	0.002	0.0033	0.0343	0.0043
Investment efficiency: $\sigma_{\zeta i}$	invg2	0.002	0.0033	0.0603	0.0208
Separation rate: σ_{sc}	invg2	0.233	0.0033	0.2452	0.0035
Uncertainty unanticipated: $\sigma_{\sigma,0}$	invg2	0.002	0.0033	0.0012	0.0007
Uncertainty anticipated (news): $\sigma_{\sigma,n}$	invg2	0.001	0.0012	0.0301	0.0020
Measure error shock: Net Worth	weibl	0.010	5.0000	0.0678	0.0042

Table B.3: Results from posterior maximization: Autocorrelation of structural shocks (Mode)

	Prior			Posterior	
	Dist.	Mean	Stdev	Mode	Stdev
`Price markup:` $\rho_{\lambda f}$	beta	0.500	0.2000	0.9249	0.0212
`Investment price:` $\rho_{\mu \Upsilon}$	beta	0.500	0.2000	0.9834	0.0082
`Government consumption:` ρ_g	beta	0.500	0.2000	0.9812	0.0086
`Persistent technology:`$\rho_{\mu z}$	beta	0.500	0.2000	0.0697	0.0457
`Temporary technology:` ρ_{ε}	beta	0.500	0.2000	0.9796	0.0077
`Monetary policy:` ρ_{xp}	beta	0.750	0.1000	0.9228	0.0077
`Consumption preferences:` $\rho_{\zeta c}$	beta	0.500	0.2000	0.9177	0.0189
`Investment efficiency:` $\rho_{\zeta i}$	beta	0.500	0.2000	0.8947	0.0430
`Separation rate:` ρ_{sc}	beta	0.500	0.2000	0.7630	0.0263
`Uncertainty unanticipated:` ρ_{σ}	beta	0.500	0.2000	0.9500	0.0172
`Uncertainty anticipated (news):` $\rho_{\sigma,n}$	norm	0.000	0.5000	0.4306	0.0681

C General equilibrium

Here, we describe the stationarization procedure and apply it to the full equilibrium set, a part of it being from Section 2 and the other reproduced from the CMR framework.

C.1 Stationarization

We have to account for both the trend of technical progress in the final good sector, z_t^*, and, whenever necessary, the trend of technical progress in the sector of physical capital accumulation, Υ^t. Nominal variables are also divided by the price level, P_t, to be expressed in real terms. Let us denote $\mu_{z,t} \equiv z_t^*/z_{t-1}^*$ the rate of productivity growth in the final good sector and $\pi_t \equiv P_t/P_{t-1}$ inflation. Unless otherwise stated, we use lower case letter for denoting detrended real variables, with an upper bar standing for the aggregate level.

First, output, consumption, investment, and public expenditures thus become $y_t \equiv$

Y_t/z_t^*, $c_t \equiv C_t/z_t^*$, $i_{z,t} \equiv I_t/(z_t^*\Upsilon^t)$, and $g_t \equiv G_t/z_t^*$, respectively. The marginal utility of consumption is further denoted $\lambda_{z,t} \equiv \lambda_t z_t^* P_t$ Aggregate capital, bonds, and entrepreneurial net worth are then $\bar{k}_{t+1} \equiv \bar{K}_{t+1}/\left(z_t^+\Upsilon^t\right)$, $\bar{b}_{t+1} \equiv \bar{B}_{t+1}/\left(z_t^+ P_t\right)$, and $\bar{n}_{t+1} \equiv \bar{N}_{t+1}/\left(z_t^+ P_t\right)$, respectively. Moreover, the price of capital is $q_t \equiv Q_{K,t}\Upsilon^t/P_t$, such that stationarized capital purchases are $q_t\bar{k}_{t+1} \equiv Q_{K,t}\bar{K}_{t+1}/\left(P_t z_t^+\right)$. Other prices, i.e wages and the rental rate of capital, become $w_t \equiv W_t/\left(z_t^* P_t\right)$ and $r_t^k \equiv \tilde{r}_t^k\Upsilon^t$. Net transfers from households to matched entrepreneurs are $\bar{w}_t^e \equiv \bar{W}_t^e/\left(z_t^* P_t\right)$, the non-survival real consumption payoff is $\bar{c}_t^e \equiv \bar{C}_t^e/z_t^*$, and real monitoring costs are $\bar{d}_{t+1}^M \equiv \bar{D}_{t+1}^M/\left(z_{t+1}^*\right)$. Finally, individual non-pecuniary search costs are stationarized as $d_{t+1}^S \equiv D_{t+1}^S/(P_{t+1}z_{t+1}^*)$ to be substituted out in the individual free entry condition.

C.2 Equilibrium equations from the search part of the model

First, we have 6 search-specific variables as $\{\theta,\ p^\theta,\ m,\ u^e,\ u^b\}$ which are pinned down by 6 search-specific equations, stationarized as follows:

- the credit market tightness

$$\theta_t = \frac{u_t^e}{u_t^b}, \tag{C.1}$$

- entrepreneurs' matching probability

$$p_t^\theta = z_t^c\left(\theta_t\right)^{\alpha^c-1}, \tag{C.2}$$

- the evolution of the mass of matched entrepreneurs

$$m_{t+1} = \gamma_t\left\{\left[1-F_{t-1}\left(\overline{\omega}_t\right)s_t^c\right]m_t + z_t^c\left(u_t^e\right)^{\alpha^c}\left(u_t^b\right)^{1-\alpha^c}\right\}, \tag{C.3}$$

- the free entry

$$\frac{d_t^S}{p_t^\theta} = \gamma_t\beta^e \mathrm{E}_t\left\{\left[1-\Gamma_t\left(\overline{\omega}_{t+1}\right)\right]R_{t+1}^k q_t\frac{\bar{k}_{t+1}}{m_{t+1}} + \pi_{t+1}\mu_{z,t+1}^*\left[\frac{\bar{c}_{t+1}^e}{m_{t+1}} + \left[1-F_t\left(\overline{\omega}_{t+1}\right)s_{t+1}^c\right]\frac{d_{t+1}^S}{p_{t+1}^\theta}\right]\right\}, \tag{C.4}$$

- the mass of searching banks

$$u_t^b = pop^b - m_t, \tag{C.5}$$

- the mass of passive entrepreneurs (neither searching for credit nor producing)

$$passive_t = 1 - m_t - u_t^e. \tag{C.6}$$

Then, optimality conditions from the financial contract are stationarized as follows. First, the participation constraint of financial intermediaries is identical to CMR and can be rewritten as

$$\frac{q_t \overline{k}_{t+1}}{\overline{n}_{t+1}} = \left[1 - \frac{R_{t+1}^k}{R_t}\left[\Gamma_t\left(\overline{\omega}_{t+1}\right) - \mu G_t\left(\overline{\omega}_{t+1}\right)\right]\right]^{-1} \tag{C.7}$$

The first-order condition with respect to the amount of debt is also similar to CMR before the substitution of the constraint multiplier λ_{t+1}^c, i.e

$$\mathrm{E}_t\left\{\left[1 - \Gamma_t\left(\overline{\omega}_{t+1}\right)\right]\frac{R_{t+1}^k}{R_t} - \lambda_{t+1}^c\left[1 - \frac{R_{t+1}^k}{R_t}\left[\Gamma_t\left(\overline{\omega}_{t+1}\right) - \mu G_t\left(\overline{\omega}_{t+1}\right)\right]\right]\right\} = 0 \tag{C.8}$$

However, the first order condition with respect to the default threshold is directly modified from search as

$$\mathrm{E}_t\left\{R_t^k q_t k_{t+1}\left\{\lambda_{t+1}^c\left[\Gamma_t'\left(\overline{\omega}_{t+1}\right) - \mu G_t'\left(\overline{\omega}_{t+1}\right)\right] - \Gamma_t'\left(\overline{\omega}_{t+1}\right)\right\}\right\} = \mathrm{E}_t\left\{\frac{G_t'\left(\overline{\omega}_{t+1}\right)}{\overline{\omega}_{t+1}} s_{t+1}^c \frac{d_{t+1}^S}{p_{t+1}^\theta}\right\}. \tag{C.9}$$

with $\frac{G_t'(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} = F_t'\left(\overline{\omega}_{t+1}\right)$.

The time-varying number of matched entrepreneurs also implies to modify some aggregate expressions from CMR. In particular, the aggregate resource constraint is

$$y_{z,t} = g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} + a(u_t)\frac{\overline{k}_t}{\Upsilon \mu_{z,t}^*} + \bar{d}_t^M + \bar{c}_t^e \tag{C.10}$$

where the aggregate detrended monitoring cost is

$$\bar{d}_t^M = \mu G(\omega_t)(1 + R_t^k)\frac{q_{t-1}\overline{k}_t}{\pi_t \mu_{z,t}^*} \tag{C.11}$$

and the aggregate detrended non-survival payoff is

$$\bar{c}_t^e = \Theta\frac{1-\gamma_t}{\gamma_t}\left(\overline{n}_{t+1}\frac{m_t}{m_{t+1}} - \overline{w}_t^e\right) \tag{C.12}$$

Finally, the law of motion for aggregate entrepreneurial net worth is

$$\overline{n}_{t+1} = \frac{m_{t+1}}{m_t} \left\{ \gamma_t \left[1 - \Gamma_{t-1} \left(\overline{\omega}_t \right) \right] \frac{R_t^k q_{t-1} \overline{k}_t}{\mu_{z,t}^* \pi_t} + \overline{w}_t^e \right\}$$

$$= \frac{m_{t+1}}{m_t} \left\{ \frac{\gamma_t}{\mu_{z,t}^* \pi_t} \left\{ q_{t-1} \overline{k}_t \left[R_t^k - R_{t-1} - \mu G_{t-1} \left(\bar{\omega}_t \right) R_t^k \right] + \overline{n}_t R_{t-1} \right\} + \overline{w}_t^e \right\} \qquad \text{(C.13)}$$

where the last equality uses the participation constraint (C.7).

C.3 Other equilibrium equations

This part reproduces the equilibrium conditions of CMR which are not modified from the presence of search frictions.[27]

C.3.1 Households

The representative household maximizes the expected discounted sum of utilities given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \left\{ \log \left(C_t - bC_{t-1} \right) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di \right\}$$

with b the degree of habit formation and $\zeta_{c,t}$ a consumption preference shock, subject to the law of capital accumulation

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left(1 - S\left(\zeta_{I,t} \frac{I_t}{I_{t-1}} \right) \right) I_t$$

and the budget constraint

$$R_t B_t + (1-\tau^l) \int_0^1 W_{i,t} h_{i,t} di + Q_{K,t} \bar{K}_{t+1} + \Pi_t = B_{t+1} + (1+\tau^c) P_t C_t + Q_{K,t} (1-\delta) \bar{K}_t + \frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}} I_t$$

[27] CMR incorporates an additional working capital channel in some expressions but activate it only when financial frictions are mute, as a particular case of their model. Hence, we omit these terms Ψ_L and Ψ_K here. Similarly, the tax rates are time-varying in some versions of their model but are treated as parameters in the estimated version and so they are here.

where τ^c and τ^l are consumption and labor tax rates respectively. After stationarization, the first-order conditions read as

$$\frac{\mu_{z,t}^*\zeta_{c,t}}{c_t\mu_{z,t}^* - bc_{t-1}} - (1+\tau^c)\,\zeta_{c,t}\lambda_{z,t} = b\beta \mathrm{E}_t\left(\frac{\zeta_{c,t+1}}{c_{t+1}\mu_{z,t+1}^* - bc_t}\right) \tag{C.14}$$

for consumption, with $\lambda_{z,t} \equiv \lambda_t z_t^* P_t$ where λ_t is the Lagrange multiplier on the constraint;

$$\zeta_{c,t}\lambda_{z,t} = \beta \mathrm{E}_t\left(\frac{\zeta_{c,t+1}\lambda_{z,t+1}}{\pi_{t+1}\mu_{z,t+1}^*}R_{t+1}\right) \tag{C.15}$$

for short-term bonds, with R_t the nominal interest rate on those bonds; and

$$\begin{aligned} &\frac{1}{\mu_{\Upsilon,t}} - q_t\left[1 - S\left(\frac{\zeta_{I,t}\mu_{z,t}^*\Upsilon i_t}{i_{t-1}}\right) - S'\left(\frac{\zeta_{I,t}\mu_{z,t}^*\Upsilon i_t}{i_{t-1}}\right)\frac{\zeta_{I,t}\mu_{z,t}^*\Upsilon}{i_{t-1}}\right] \\ &= \beta \mathrm{E}_t\left[\frac{\zeta_{c,t+1}\lambda_{z,t+1}}{\zeta_{c,t}\lambda_{z,t}}\frac{q_{t+1}}{\mu_{z,t+1}^*\Upsilon}S'\left(\frac{\zeta_{I,t+1}\mu_{z,t+1}^*\Upsilon i_{t+1}}{i_t}\right)\zeta_{I,t+1}\left(\frac{\mu_{z,t+1}^*\Upsilon i_{t+1}}{i_t}\right)^2\right] \end{aligned} \tag{C.16}$$

for investment, after replacing $Q_{K,t}\bar{K}_{t+1} - Q_{K,t}(1-\delta)\bar{K}_t$ by $\left(1 - S\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right)\right)I_t$ from the capital accumulation into the budget constraint.

After stationarization, the law of capital accumulation reads as

$$\bar{k}_{t+1} = (1-\delta)\,\frac{1}{\mu_{z,t}^*\Upsilon}\bar{k}_t + \left[1 - S\left(\frac{\zeta_{i,t}i_t\mu_{z,t}^*\Upsilon}{i_{t-1}}\right)\right]i_t \tag{C.17}$$

where the adjustment cost function is of the form

$$S(x_t) = e^{\sqrt{(S''/2)}(x_t - x)} + e^{-\sqrt{(S''/2)}(x_t - x)} - 2$$

where S'' is a parameter that determines the curvature of the cost function, and where $x_t \equiv \frac{I_t}{I_{t-1}}\zeta_{I,t}$ is the growth rate of investment multiplied by a shock on the marginal efficiency of investment in producing capital. In stationarized terms, this is

$$S(x_t) = e^{\sqrt{(S''/2)}\left(\zeta_{i,t}\mu_{z,t}^*\Upsilon\frac{i_t}{i_{t-1}} - \mu_z^*\Upsilon\right)} + e^{-\sqrt{(S''/2)}\left(\zeta_{i,t}\mu_{z,t}^*\Upsilon\frac{i_t}{i_{t-1}} - \mu_z^*\Upsilon\right)} - 2 \tag{C.18}$$

with first-order derivative

$$S'\left(x_t\right) = \sqrt{(S''/2)}\left(e^{\sqrt{(S''/2)}\left(\zeta_{i,t}\mu^*_{z,t}\Upsilon\frac{i_t}{i_{t-1}}-\mu^*_z\Upsilon\right)} + e^{-\sqrt{(S''/2)}\left(\zeta_{i,t}\mu^*_{z,t}\Upsilon\frac{i_t}{i_{t-1}}-\mu^*_z\Upsilon\right)} - 2\right) \tag{C.19}$$

C.3.2 Asset pricing

From (C.15), the (real) stochastic discount factor which satisfies $\mathrm{E}_t\left(\beta^\star_{t,t+1}\right) = \frac{\pi_{t+1}}{R_t}$ is

$$\beta^\star_{t,t+1} = \beta\frac{1}{\mu^*_{z,t+1}}\frac{\zeta_{c,t+1}\lambda_{z,t+1}}{\zeta_{c,t}\lambda_{z,t}} \tag{C.20}$$

Therefore, the return on *raw* capital (to the households) can be defined as

$$\mathrm{E}_t\left(\beta^\star_{t,t+1}R^{raw,real}_{t+1}\right) = 1$$

which, by the first-order condition on investment (C.16), is

$$R^{raw,real}_{t+1} = \frac{\frac{q_{t+1}}{\Upsilon}S'\left(\frac{\zeta_{I,t+1}\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)\zeta_{I,t+1}\left(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)^2}{\frac{1}{\mu_{\Upsilon,t}} - q_t\left[1 - S\left(\frac{\zeta_{I,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right) - S'\left(\frac{\zeta_{I,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right)\frac{\zeta_{I,t}\mu^*_{z,t}\Upsilon}{i_{t-1}}\right]}$$

or, calculating an alternative first-order condition on capital as

$$E_t\left(\beta\frac{1}{\mu^*_{z,t+1}}\frac{\zeta_{c,t+1}\lambda_{z,t+1}}{\zeta_{c,t}\lambda_{z,t}}\frac{q_{t+1}}{q_t}\frac{1-\delta}{\Upsilon}\right) = 1,$$

can be also written as

$$R^{raw,real}_{t+1} = \frac{q_{t+1}}{q_t}\frac{1-\delta}{\Upsilon}$$

C.3.3 Entrepreneurs' rent of capital

Entrepreneurs rent the effective capital to the monopolistic competition firms. This defines a (nominal) rate of return on effective capital, in detrended terms, as

$$R^k_t \equiv \frac{(1-\tau^k)(u_t r^k_t - a(u_t)) + (1-\delta)q_t}{\Upsilon q_{t-1}}\pi_t + \tau^k\delta \tag{C.21}$$

where u is the variable utilization rate of effective capital, r^k is the rental rate of effective capital, and $a(u_t)$ an utilization rate cost function as

$$a_t = \frac{r^k}{\sigma_a}(e^{\sigma_a(u_t-1)} - 1) \tag{C.22}$$

with σ_a a parameter. In steady-state, note that $a = 0$ regardless the value of σ_a.[28]

Entrepreneurs chose the optimal utilization rate of capital so as to maximize (C.21), i.e so as to satisfy the first-order condition

$$r_t^k = r_{ss}^k \exp\left(\sigma_a\left(u_t - 1\right)\right) \tag{C.23}$$

with r_{ss}^k the steady-sate value of the rental rate of capital.

C.3.4 Monopolistic producers

- The cost minimization problem

Monopolistic producers, indexed by j, demand effective capital and labor so as to maximize their cost of production, i.e

$$\min P_t \tilde{r}_t^k K_{j,t} + W_t l_{j,t}$$

$$\text{s.t} \quad Y_{j,t} = \varepsilon_t (K_{j,t})^\alpha (z_t l_{j,t})^{1-\alpha} - \varphi z_t^*$$

with φz_t^* a fixed cost of production and ε_t a stationary production technology shock. The (stationarized) first-order conditions are

$$r_t^k = \alpha \varepsilon_t \left(\frac{\Upsilon \mu_{z_t^*} h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t k_t} \right)^{1-\alpha} s_t \tag{C.24}$$

[28] Note that the return on effective capital (to entrepreneurs) can easily be related to the return on raw capital (to households) from (C.3.2), expressed in nominal terms, as $R_{t+1}^k = R_{t+1}^{raw}\left[\frac{(1-\tau^k)(u_{t+1} r_{t+1}^k - a(u_{t+1}))}{(1-\delta) q_{t+1}} + 1\right] + \tau^k \delta$.

with respect to $K_{j,t}$, and

$$w_t = (1-\alpha)\varepsilon_t \left(\frac{h_t (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} \mu_{z,t}^* \Upsilon}{k_t u_t} \right)^{-\alpha} s_t$$

with respect to $l_{j,t}$, substituting $l_t = h_t (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}}$, where s_t are the real marginal costs as

$$s_t = \frac{1}{\varepsilon_t} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \tag{C.25}$$

The stationarized production function finally reads as

$$y_{z,t} = (p_t^*)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}} \varepsilon_t \left[\left(\frac{u_t k_t}{\mu_{z,t} \Upsilon} \right)^{\alpha} \left(h_t \, (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} \right)^{1-\alpha} - \varphi \right] \tag{C.26}$$

- Price maximization and the aggregate price index

The Dixit-Stiglitz demand for the good of producer j is

$$Y_{i,t} = Y_t \left(\frac{p_{j,t}}{p_t} \right)^{-\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}$$

where $\lambda_{f,t}$ is the elasticity of substitution among intermediate goods and is stochastic to allow for a price markup shock.

The price index p_t^* evolves as

$$p_t^* = \left[(1-\xi_p) \left(\frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} + \xi_p \left(\frac{\widetilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} \right]^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}} \tag{C.27}$$

where a fraction ξ_p of monopolistic intermediate producers cannot re-optimize their price in period t and adjust it according to a rule of thumb

$$\tilde{\pi}_t = \left(\pi_t^{target} \right)^{\iota} (\pi_{t-1})^{1-\iota} \tag{C.28}$$

where π_t^{target} is the target inflation rate of the monetary authority, while a fraction $(1-\xi_p)$

re-set their price in period t to the optimal level

$$\widetilde{p}_t = \frac{K_{p,t}}{F_{p,t}} = \left(\frac{1 - \xi_p \left(\frac{\widetilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_{f,t}}}}{1 - \xi_p} \right)^{1-\lambda_{f,t}} \tag{C.29}$$

with $F_{p,t}$ and $K_{p,t}$ auxiliary recursive variables solving

$$F_{p,t} = \zeta_{c,t}\lambda_{z,t}y_{z,t} + \beta\xi_p \mathrm{E}_t \left[\left(\frac{\widetilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_{f,t}}} F_{p,t+1} \right] \tag{C.30}$$

and

$$K_{p,t} = \zeta_{c,t}\lambda_{z,t}\lambda_{f,t}y_{z,t}s_t + \beta\xi_p \mathrm{E}_t \left[\left(\frac{\widetilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} K_{p,t+1} \right] \tag{C.31}$$

C.3.5 Wage maximization and the aggregate wage index

A fraction $(1 - \xi_w)$ of monopoly unions can reoptimize their wage at time t while others cannot and get a wage inflation indexation $\widetilde{\pi}_{w,t}$. Therefore, the aggregate wage index w_t^* evolves as

$$w_t^* = \left[(1 - \xi_w) \left(\frac{1 - \xi_w \left(\frac{\widetilde{\pi}_{w,t}}{\pi_{w,t}} (\mu_{z^*})^{1-\iota_\mu} \left(\mu_{z_t^*}\right)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\widetilde{\pi}_{w,t} \left(\mu_{z^*,t}\right)^{\iota_\mu} (\mu_{z^*})^{1-\iota_\mu}}{\pi_{w,t}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \tag{C.32}$$

where $1/(1 - \lambda_w)$ is the elasticity of substitution among labor inputs and h_t is the aggregate labor input used by intermediate good producers. In the indexing process of wages, the trend of wages is the weighted average of the steady-state value of technological growth rate, μ_{z^*}, with a weight $(1 - \iota_\mu)$, and of the growth rate as of time t, μ_{z^*} with a weight ι_μ. As for the price index, the variables $F_{w,t}$ and $K_{w,t}$ are introduced to characterize the dynamics of nominal wages. They satisfy the following laws of motion

$$\begin{aligned} F_{w,t} = \zeta_{c,t}\lambda_{z,t}\frac{(1 - \tau^l)}{\lambda_w} h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} + \beta\xi_w \mathrm{E}_t \Bigg[& (\mu_{z^*})^{\frac{1-\iota_\mu}{1-\lambda_w}} \left(\mu_{z_{t+1}^*}\right)^{\frac{\iota_\mu}{1-\lambda_w}-1} \\ & \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\left(\widetilde{\pi}_{w,t+1}\right)^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} \Bigg] \end{aligned} \tag{C.33}$$

and

$$K_{w,t} = \zeta_{c,t}\zeta_t \left[(w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_t \right]^{1+\sigma_L} + \beta\xi_w \mathrm{E}_t \left[\left(\frac{\widetilde{\pi}_{w,t+1} \left(\mu_{z_{t+1}^*}\right)^{\iota_\mu} \left(\mu_{z^*}\right)^{1-\iota_\mu}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \right] \tag{C.34}$$

C.3.6 Monetary policy rule

The central bank sets the nominal interest rate according to the following rule

$$\begin{aligned} R_t - R &= \rho_p \left(R_{t-1} - R \right) \\ &\quad + (1-\rho_p) \left[\alpha_\pi \left(\pi_{t+1} - \pi_t^* \right) + \alpha_{\Delta y} \frac{1}{4} \left(g_{y,t} - \mu_z^* \right) \right] + \frac{1}{400} \varepsilon_t^{wp} \end{aligned} \tag{C.35}$$

C.4 Structure of shocks

The price markup, investment price, government consumption, persistent technology, temporary technology, consumption preference, investment efficiency, and separation shocks are driven by the following autoregressive processes

$$\log \left(x_t / x \right) = \rho_x \log \left(x_{t-1} / x \right) + \epsilon_{x,t} \tag{C.36}$$

where ρ_x is the autocorrelation and σ_x^2 the variance of the shock x for $x = \{\lambda_f, \mu\Upsilon, g, \mu_z, \varepsilon, \zeta_c, \zeta_i, s^c\}$. The variances of the equity γ and monetary policy xp shocks are σ_γ and σ_{xp}, respectively. Uncertainty shocks are defined by equations (6) and (7).