

# Optimal Unemployment Insurance for Older Workers

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## Résumé

Nous discutons des implications de l'effet-horizon sur le profil optimal par âge de l'allocation-chômage. La première réforme envisageable serait de mettre en place une dégressivité des allocations-chômage en fonction de la durée du chômage, mais surtout une dégressivité différenciée en fonction de la distance à l'âge de la retraite. Conformément à l'intuition, la caisse d'allocation-chômage doit imposer plus de dégressivité aux chômeurs plus proches de la retraite, au moins jusqu'à une certaine distance de la retraite. Cette réforme donne en effet surtout des gains à une distance relativement éloignée de la retraite, gain habituel de la dégressivité dans un contexte d'aléa moral. En revanche, pour les individus qui entrent au chômage à proximité de leur retraite, le gain est nul. Cette politique s'avère donc peu efficace pour reculer l'âge à partir duquel le chômeur ne cherche pas activement un emploi. L'efficacité de la politique de dégressivité est donc limitée pour les seniors par la faiblesse de l'élasticité de leur effort de recherche en raison de la proximité de l'âge de la retraite.

Les incitations intrinsèquement faibles à retrouver un emploi poussent alors à prévoir des dispositifs spécifiques aux seniors en matière d'assurance-chômage. Nous plaignons ainsi pour rendre conditionnel le niveau des pensions à la durée du chômage pour rallonger l'horizon des demandeurs d'emploi en fin de carrière: des pensions majorées pourraient venir récompenser leur retour rapide dans l'emploi. Nous présentons des arguments en faveur de l'optimalité de la dispense de recherche d'emploi pour les seniors très proches

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de la retraite, parce qu'il est impossible de mettre en place un schéma incitatif efficace à cet horizon très court.

### **Abstract**

This paper shows that optimal unemployment insurance contracts are age-dependent. Older workers have only a few years left on the labor market prior to retirement. This short horizon implies a more decreasing replacement ratio. However, there is a sufficiently short distance to retirement for which flat unemployment benefits can be the optimal contract. It is the result of the inability to reconcile both incentives and insurance for the soon-to-be-retired unemployed workers. We show that the unemployment benefit agency could take advantage of the retirement period to tax pensions in order to optimize the trade-off between insurance and incentives at the end of working life.

Keywords: Unemployment insurance, Retirement, Recursive contracts, Moral Hazard

JEL Classification: C61, J64, J65

# 1 Introduction

In many European countries, older workers receive more generous unemployment benefits before retirement (see OECD (2006) for more details on all these programs). In some countries (Belgium, Finland, France, Germany and the United Kingdom), older people on unemployment benefits are exempt from the general eligibility requirement of having to look for work after a certain age. This was also the case previously in Austria and the Netherlands. In some countries (Ireland, Sweden), job-search requirements for people on unemployment benefits are less demanding for older people than for younger people. The “unemployment tunnel” leading to early exit from the labor market operates as a result of a general exemption from job-search requirements for older unemployed individuals. This allows them to remain on unemployment benefits until they reach the official retirement age. For these reasons, unemployment benefits are often considered as early retirement or pre-retirement schemes (Gruber and Wise (1998)).

In this paper, we analyze the economic rationale behind this policy. Since the seminal work of Shavell and Weiss (1979), it has been recognized that the optimal unemployment benefits should be such that the replacement ratio decreases with the unemployment spell. Providing incentives to promptly find a new job derives from the assumption that the agent’s search intensity (the unemployed worker) cannot be observed by the principal (the unemployment insurance agency). Due to the weakness of the search effort when the retirement age is getting closer, we first show that this insurance contract does not succeed in encouraging older workers to search for a job. This is why this paper advocates the introduction of other instruments in order to improve the terms of the trade-off between insurance and incentives at the end of the working life. Hopenhayn and Nicolini (1997) have already proposed introducing an increasing wage tax after re-employment together with a decreasing sequence of unemployment benefits. With this wage-tax, the principal provides a smoother consumption profile while leaving some incentives to search for a job. However, faced with the short horizon of older workers once reemployed, we propose that the unemployment insurance agency takes advantage of the retirement period to tax pensions in order to better reconcile insurance and incentives at the end of working life. This proposition provides some foundation for an integration of unemployment and retirement schemes, allowing the agency to increase older workers’ welfare. It thus supports the idea of a unified insurance system recently put forward by Stiglitz and Yun (2005)<sup>1</sup>.

The originality of our paper is to study the characteristics of the optimal unemployment insurance for older workers. Young and old workers are intrinsically characterized by different expected time horizons on the labor market. The existence of a retirement date intrinsically creates a sharp decrease in the search intensity just before this age. To the extent that there are search frictions on the labor market, the return on jobs is determined

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<sup>1</sup>Stiglitz and Yun (2005) propose in a very different framework that unemployed workers can borrow against future pensions. In the case of incomplete financial markets, this provides both insurance and effective incentives to all unemployed workers. There is nothing specific to older workers in their analysis.

by their expected duration: the time to retirement is then key to understanding older workers' transitions from unemployment to employment<sup>2</sup>. The unemployment insurance agency is then faced with this intrinsic low search intensity, which is even nil for seniors close enough to retirement: intuitively, unemployed workers who are one period away from retirement certainly do not search as this activity is costly and with a zero return. This situation can happen even earlier and depends on the horizon prior to retirement necessary to recoup the search costs<sup>3</sup>.

In this paper, we extend the framework of Hopenhayn and Nicolini (1997) by introducing a definitive exit rate from the labor market, i.e. retirement. More precisely, unemployed workers face a given probability of retiring. This probability is interpreted as a measure of distance to retirement. Workers with a higher probability of retiring correspond to older workers while workers with a smaller probability of retiring correspond to younger workers in our analysis. Thus, to find the optimal unemployment insurance for workers of different ages, we look for the optimal unemployment insurance for different probabilities of retiring. It must be emphasized that agents do not age in the model, since the probability of leaving the labor market is independent of the time spent in the labor market. This allows us to compute very easily the optimal contracts for any value of this probability. Taking into account agents' aging in parallel with the unemployment spell would add an unnecessary complexity. We believe that our simple theoretical framework captures the quintessence of the retirement deadline effect on the optimal unemployment benefit contract. Moreover, it is consistent with the choice of not considering any other state variables. Indeed, for the sake of simplicity, we omit financial asset accumulation, even though it is known that the optimal contract is quite sensitive to this assumption<sup>4</sup>. The no saving assumption could be even worse in the case of the older workers, who hold more financial assets than younger workers on average. However, in a non funded Social Security system, older workers have rather accumulated more claims on Social Security. Contrary to financial assets, these assets are totally illiquid and not available for smoothing income before retirement. Implementing a pension tax would allow the principal to act as a bank account: workers could borrow against their future pension to finance consumption during an unemployment episode. This is why our policy proposal is likely to perform better in countries relying on pay-as-you-go pension systems.

We show in this framework that there is a specific design for the optimal unemployment insurance for older workers. Firstly, the proximity to retirement implies proposing a highly decreasing profile of unemployment benefits in order to motivate unemployed

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<sup>2</sup>This retirement deadline effect has already received empirical support (Hairault et al. (2008)) and theoretical foundations based on the job search theory (Seater (1977) Ljungqvist and Sargent (2008) and Hairault et al. (2008)).

<sup>3</sup>This intrinsic feature of the oldest workers will be endogenously delivered as the corner solution of the optimal search problem, given that the search probability function we consider in this paper does not satisfy the Inada conditions.

<sup>4</sup>Shimer and Werning (2008) have recently shown that a insurance schedule that decreases with unemployment duration performs worse than a constant sequence in an economy where saving is allowed.

workers to search for a job again. Secondly, for unemployed older workers close enough to the retirement age, this policy becomes inefficient. A highly decreasing profile would imply high unemployment benefits in the first periods of the unemployment spell in order to compensate for providing less insurance (less consumption smoothing). The cost of this highly decreasing profile would be too high for the unemployment insurance agency, which then prefers to provide these soon-to-be retired unemployed workers with the flat unemployment benefit profile, although it means that they do not search at all. The agency does not succeed in reconciling the insurance with the incentive objectives, making the recommendations by Shavell and Weiss (1979) irrelevant. In that sense, search exemption for older workers and early retirement schemes can be viewed as optimal<sup>5</sup>.

This is why imposing a tax on the future job along the lines of Hopenhayn and Nicolini (1997) could be particularly efficient in the context of older workers. It could both make the profile of unemployment benefits less decreasing and preserve incentives for search thanks to the increasing profile of the wage taxes with the unemployment spell. It then allows the principal to set some inactive unemployed older workers to work. However, faced with the short re-employment duration, we show that a policy that makes the pension level dependent on the length of past unemployment spell allows the agency to reach a better mix between incentives and insurance, at least for workers near retirement. Firstly, the short horizon before retirement decreases the job finding probability, and so limits the efficiency of the wage tax. Even in the special case where the search intensity is nil, the perfect risk-sharing allocation is both available and efficient thanks to the pension tax: workers can borrow against their future pension to finance consumption during a unemployment spell. Secondly, the pension tax is able to give more incentives as the re-employment value is free of tax. Increasing the wage tax more in order to compensate for the low job finding probability leads to decreasing this probability even more. This is not the case for the pension tax, as higher taxes do not decrease the instantaneous value of employment. Finally, the short job duration before retirement makes the budgetary return and the search incentives provided by the wage tax very low; conversely, this short horizon give more strength to the pension tax policy. In contrast, for younger workers, as their optimal search intensity is higher and the future pension taxes are far remote and then highly discounted, the wage tax *à la* Hopenhayn and Nicolini (1997) is more efficient than the pension tax. In a nutshell, the closer the retirement age, the more efficient the pension tax relative to the wage tax.

For a calibration based on workers older than 50 in the French labor market, we propose a quantitative evaluation of these different insurance contracts; we especially aim at measuring the gains associated with the integration of the unemployment and retirement agencies. We show that this policy allows unemployed workers to achieve a smoother consumption profile, but also to search for a new job, when the proximity to retirement makes the other policies inefficient. Overall, for unemployed workers at 5 years prior to

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<sup>5</sup>Cremer et al. (2004) have followed a different route to reach the same conclusion: early retirement would be an ingredient of an optimally designed redistributive policy in a world of asymmetric information.

retirement, this policy leads to savings of about 40% of the existing unemployment insurance cost. Let us note that a tax only on the re-employment wage leads to reducing by 34% the total cost of unemployed older workers. This contrasts greatly with a decreasing unemployment insurance (UI) policy, which only lowers this cost by 3%. This shows that introducing a tax on wages and especially on pensions is particularly useful in the case of older workers near to retirement in order to reconcile incentives and insurance when the search activity is dramatically lowered by the short horizon on the labor market.

The paper is organized as follows. Section 2 describes the model and the two tax contracts. Section 3 proposes a quantitative evaluation. Section 4 concludes.

## 2 The model

Our objective is to determine the optimal timing of the UI benefits for older workers in a repeated moral-hazard environment. The first-best solution is not reachable, due to informational asymmetries: the search effort level is the agent's own private information and the planner has no way of monitoring this effort level. The second-best allocation is such that the principal (the UI agency) minimizes the expected discounted cost of the unemployment insurance, subject to two constraints: *(i)* to provide a certain expected lifetime utility level to the agent when she becomes unemployed, and *(ii)* to enforce the incentive compatibility constraints implying that the agent makes her own optimal decisions for search effort given the optimal timing of UI benefits. We assume that the agency has potentially three instruments: unemployment benefits, wage taxes and pension taxes. The level of these policy variables depends on the previous unemployment duration of the agent. Our benchmark is the case without optimal policy (constant unemployment benefits policy). We then compare the optimal contract *à la* Hopenhayn and Nicolini (non-stationary UI and taxes on re-employment wage), denoted  $P_e$ , with a contract, denoted  $P_r$ , where there are pension taxes, in addition to non-stationary UI.

The originality of our analysis is to introduce a finite horizon for workers. For simplicity, we present a model where workers are characterized by a given probability of retiring  $\lambda_w$  which determines the expected horizon of the working life. Once retired, agents face a probability of dying  $\lambda_r$ . We choose this stochastic structure for computational reasons. This reduces the dimension of the discrete state variables. With age as a deterministic and discrete state variable, the computational burden would have dramatically increased.

### 2.1 The agent

In this section, we present the behavior of the (unemployed) agent characterized by her preference and probability of retiring. If she finds a job in period  $\tau$ , she is employed from  $\tau + 1$  until retirement, as jobs are permanent until retirement<sup>6</sup>. An employed worker

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<sup>6</sup>Assuming that employment is permanent simplifies the analysis and is consistent with the literature. Hopenhayn and Nicolini (2009) relax this assumption.

is assumed to receive a constant wage  $w$ , a retiree a constant<sup>7</sup> pension  $p$ . The agent's preferences are given by:

$$E \sum_{\tau=0}^{\infty} \beta^{\tau} [u(c_{\tau}) - a_{\tau}]$$

where  $\beta < 1$  denotes the intertemporal discount factor,  $c_{\tau}$  consumption at time  $\tau$ ,  $a_{\tau}$  the job search intensity and  $E$  the expectation operator. The instantaneous utility function  $u(\cdot)$  is increasing, twice differentiable, strictly concave with  $u'(0) = \infty$ . We will assume that:

$$u(c_{\tau}) = \frac{c_{\tau}^{1-\sigma}}{1-\sigma}$$

where  $\sigma > 0$  is the coefficient of relative risk aversion. The probability of receiving a job offer depends on the level of search intensity. This probability is given by an exponential distribution, as in Hopenhayn and Nicolini (1997):

$$\pi(a) = 1 - \exp(-\psi a)$$

with  $\psi > 0$ . This hazard function is increasing, strictly concave, twice differentiable. Note that this function does not satisfy the Inada conditions. More particularly, we have  $\pi'(0) = \psi < +\infty$ . This means that the return on searching can be dominated by the cost of searching. This function is a parsimonious way of introducing fixed costs in the job search activity<sup>8</sup>. For a given set of parameters defining the environment and the preferences of the agents, we will show that the probability of being in the zero-search corner solution is increasing with the proximity to retirement. This function is then key for accounting for the specificity of older workers' job search.

We denote  $t$  the length of the previous unemployment spell. In the two contracts ( $P_e$  and  $P_r$ ), the level of the unemployment compensation is  $b(t)$  after  $t$  period of unemployment. With the contract  $P_e$ , unemployed worker pays taxes on wage  $\tau^e(t)$  when she becomes employed. As in Hopenhayn and Nicolini (1997), we assume that this tax depends on the length of the past unemployment spell  $t$ . With the contract  $P_r$ , the tax payment is deferred until the period of the retirement. The pension tax  $\tau^r(t)$  is also assumed to depend on the length of the past unemployment spell  $t$ .

The value functions of the employed worker ( $V^e$ ), the retiree ( $V^r$ ) and the unemployed

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<sup>7</sup>Pensions are not indexed on previous wages as in real life, since we do not consider all the life cycle history of workers. We then leave aside the fact that retirement payments are already conditional on unemployment. However, this effect is not important in most countries as the indexation rules make the pension quasi-independent of the unemployment spells. For instance, in France, the pension calculation is based on the best 25 years.

<sup>8</sup>In an equilibrium model of the labor market, the same result could be obtained when a fixed cost, corresponding for instance to training costs, is paid by the worker just after the hiring decision (McMillan and Rothschild (1994)). From an empirical point of view, this assumption is supported by the observed importance of both fixed costs of working and search costs for explaining participation decisions (see for instance Blundell et al. (1998)).

worker ( $V^u$ ) are the solution to the following Bellman equations:

$$P_e : \begin{cases} V_e^e(\tau^e(t)) = u(w - \tau^e(t)) + \beta[(1 - \lambda_w)[V_e^e(\tau^e(t))] + \lambda_w V_e^r] \\ V_e^r = u(p) + \beta(1 - \lambda_r)V_e^r \\ V_e^u(t) = \max_{a_e(t)} \begin{cases} u(b_e(t) - a_e(t) + (1 - \lambda_w)\pi(a_e(t))\beta V_e^e(\tau^e(t)) \\ +(1 - \lambda_w)(1 - \pi(a_e(t)))\beta V_e^u(t + 1) \\ +\lambda_w\beta V_e^r \end{cases} \end{cases}$$

$$P_r : \begin{cases} V_r^e(\tau^r(t)) = u(w) + \beta[(1 - \lambda_w)V_r^e(\tau^r(t)) + \lambda_w V_r^r(\tau^r(t))] \\ V_r^r(\tau^r(t)) = u(p - \tau^r(t)) + \beta(1 - \lambda_r)V_r^r(\tau^r(t)) \\ V_r^u(t) = \max_{a_r(t)} \begin{cases} u(b_r(t) - a_r(t) + (1 - \lambda_w)\pi(a_r(t))\beta V_r^e(\tau^r(t)) \\ +(1 - \lambda_w)(1 - \pi(a_r(t)))\beta V_r^u(t + 1) \\ +\lambda_w\beta V_r^r(\tau^r(t)) \end{cases} \end{cases}$$

With the contract  $P_e$ , the tax is collected during the re-employment spell. This makes the instantaneous value of employment dependent of the level of the wage tax. With the contract  $P_r$ , taxes are paid once retired, whatever the job search outcome. On the other hand, the probability  $\lambda_w$  determines the influence of the tax on the intertemporal values. The closer the retirement age, the lower (higher) the influence of the wage (pension) tax on the employment value. The optimal search effort, whatever the contract, is given by:

$$1 \geq \beta\pi'(a_x(t))(1 - \lambda_w) [V_x^e(\tau^x(t)) - V_x^u(t + 1)] \quad x = e, r \quad (1)$$

with equality if  $a_x(t) > 0$ . The right hand side of equation (1) states that, for a given gap between the employment and the unemployment value functions, the incentives to search decrease with the probability of retiring ( $\lambda_w$ ). The return on the job search effort is then reduced when the distance to retirement goes down, whatever the policy considered. Moreover, as the retirement age gets closer, the gap between employment and unemployment value functions narrows, since they depend on the same retirement value, except when a pension tax is allowed.

When retirement is tomorrow, the agent does not engage in any investment, as she will retire in the next period. This intrinsic feature of the oldest workers is endogenously delivered as the corner solution of the optimal search problem, given that the search probability does not satisfy the Inada conditions. In our stochastic aging framework, this situation occurs when  $\lambda_w \rightarrow 1$ . Overall, the probability of having the corner solution  $a = 0$  is increasing with the proximity to retirement. In the same way, it is intuitive that older workers search less than younger workers<sup>9</sup>. It will be the specific role of the optimal policy to offset these effects by increasing the job value near retirement.

## 2.2 The principal

We consider a risk-neutral planner (the principal) providing the risk-averse agent with an optimal contract. As in Hopenhayn and Nicolini (1997), the optimal unemployment insur-

<sup>9</sup>In Appendix A, we propose formal proofs of these assertions in the simplified case without any active policies.



ance contract  $P_e$  consists of a sequence of unemployment benefits  $B_e = \{b_e(1), b_e(2), \dots, b_e(T)\}$  and taxes  $\mathcal{T}^e = \{\tau^e(1), \tau^e(2), \dots, \tau^e(T)\}$ . With the optimal unemployment insurance contract  $P_r$ , the principal can transfer incomes from the retirement period and the contract is defined by the 2 vectors of instruments  $B_r = \{b_r(1), b_r(2), \dots, b_r(T)\}$ , and  $\mathcal{T}^r = \{\tau^r(1), \tau^r(2), \dots, \tau^r(T)\}$ . Given these contracts, the agent maximizes her intertemporal utility by choosing a sequence of search effort.

The objective of the principal is to minimize its total expenditures, under two constraints: (i) a given expected utility  $V_x(1)$  for a newly unemployed worker (the promise-keeping constraint), and (ii) an incentive-compatibility constraint:

$$C_x(V_x(t)) = \min_{\mathcal{F}_x} \left\{ \begin{array}{l} b_x(t) + \beta(1 - \lambda_w)(1 - \pi(a_x(t)))C_x(V_x^u(t+1)) \\ -(1 - \mathbb{I}_{x=r})[\beta(1 - \lambda_w)\pi(a_e(t))G^e(\tau^e(t+1))] \\ -\mathbb{I}_{x=r} \left[ \begin{array}{l} \beta(1 - \lambda_w)\pi(a_r(t))G^e(\tau^r(t+1)) \\ +\beta\lambda_w G^r(\tau^r(t+1)) \end{array} \right] \end{array} \right\}$$

subject to

$$V_x(t) \leq (1 - \mathbb{I}_{x=r})V_e^u(t) + \mathbb{I}_{x=r}V_r^u(t) \quad (\mu_x)$$

and

$$1 \geq \beta\pi'(a(t))(1 - \lambda_w)[V_x^e(\tau^x(t)) - V_x^u(t+1)] \quad (\nu_x)$$

where  $\mathcal{F}_x \equiv \{b_x(t), a_x(t), V_x^u(t+1), \tau^x(t)\}$ , for  $x = e, r$  and  $\mathbb{I}_{x=r}$  the indicator function which is equal to 0 if the contract is  $P_e$  and 1 if the contract is  $P_r$ . The functions  $G^e(\tau^e(t+1))$  and  $G^x(\tau^r(t+1))$ , for  $x = e, r$  are the discounted values of taxes during the re-employment and retirement periods for the wage tax and the pension tax respectively. They are defined as follows:

$$G^e(\tau^e(t+1)) = \frac{\tau^e(t)}{1 - \beta(1 - \lambda_w)} \quad (2)$$

$$G^e(\tau^r(t+1)) = \frac{\beta\lambda_w \frac{\tau^r(t)}{1 - \beta(1 - \lambda_r)}}{1 - \beta(1 - \lambda_w)} \quad (3)$$

$$G^r(\tau^r(t+1)) = \frac{\tau^r(t)}{1 - \beta(1 - \lambda_r)} \quad (4)$$

With the contract  $P_r$ , taxes are collected during the retirement period. This is why the discounted value of these taxes  $G^e(\tau^r)$  during the re-employment period is relatively high when considering workers close to retirement: the higher  $\lambda_w$ , the higher  $G^e(\tau^r)$ . On the other hand, wage taxes are collected during the re-employment spell, which can be short when considering older workers: the higher  $\lambda_w$ , the lower  $G^e(\tau^e)$ .

The first order conditions of the principal problem for the two contracts are respectively:

$$\begin{array}{ll}
\text{contract } P_e : & \text{contract } P_r : \\
1 = \mu_e u'(b_e(t)) & 1 = \mu_r u'(b_r(t)) \\
C'_e(V_e^u(t+1)) = \mu_e - \nu_e \frac{\pi'(a_e(t))}{1-\pi(a_e(t))} & C'_r(V_r^u(t+1)) = \mu_r - \nu_r \frac{\pi'(a_r(t))}{1-\pi(a_r(t))} \\
C'_e(V_e(t)) = \mu_e & C'_r(V_r(t)) = \mu_r \\
\frac{C_e(V_e^u(t+1))-G^e(\tau^e(t+1))}{V_e^e(\tau^e(t))-V_e^u(t+1)} = -\nu_e \frac{\pi''(a_e(t))}{\pi'(a_e(t))} & \frac{C_r(V_r^u(t+1))-G^e(\tau^r(t+1))}{V_r^e(\tau^r(t))-V_r^u(t+1)} = -\nu_r \frac{\pi''(a_r(t))}{\pi'(a_r(t))} \\
\frac{1}{u'(w-\tau^e(t))} = \mu_e + \nu_e \frac{\pi'(a_e(t))}{\pi(a_e(t))} & \frac{1}{u'(p-\tau^r(t))} = \mu_r + \nu_r \frac{\pi'(a_r(t))}{\pi(a_r(t))} \mathcal{Z} \\
& \text{with } \mathcal{Z} = \frac{\beta(1-\lambda_w)\pi(a_r(t))}{\beta(1-\lambda_w)\pi(a_r(t))+1-(1-\lambda_w)\beta}
\end{array}$$

The first three conditions with respect to  $b_x(t)$ ,  $V_x^u(t+1)$  and  $V_x(t)$  are exactly the same, whatever the contract considered. Altogether, they lead the unemployment benefits to be decreasing with the unemployment duration when assuming an interior solution (see Appendix B). Moreover, using the fourth and the fifth conditions relative to the optimality of the search intensity and the tax, it can be shown that the wage and pension taxes are increasing with the unemployment duration, implying that the search intensity is increasing during the unemployment spell (see Appendix B).

Beyond these similarities, the two contracts have some specific characteristics, which deserve to be emphasized. The fourth and the fifth conditions differ according to the tax policy considered. Four differences must be emphasized. (i) First, as the wage is higher than the pension, taxing during the employment state, when the marginal utility is lower, makes the insurance transfer less costly for the agent (fifth condition). *Ceteris paribus*, this entices the principal to implement lower pension taxes than wage taxes. (ii) However, the employment taxes have a perverse disincentive impact on instantaneous value of re-employment, whereas pension taxes do not introduce this distortion. Overall, the pension tax is then more incentive-compatible and could then help to stimulate the search effort more than the wage tax, eventually leading some older unemployed workers to search for a job. This explains why the pension tax may be higher as the impact of the participation constraint in the last condition on the tax is lower ( $\mathcal{Z} < 1$  in the fifth condition). (iii) Thirdly, the proximity to retirement reinforces the relative efficiency of the pension tax, as it makes the re-employment duration very short. According to the value functions of the worker and of the taxes received by the agency, a wage tax then provides a low budgetary return for the agency and a low incentive effect on the agent, leading to a lower search intensity (fourth condition). Conversely, considering younger workers makes the pension tax less efficient as both the budgetary revenues and the incentives are highly discounted in this case. In a nutshell, the higher  $\lambda_w$ , the more efficient the pension tax. (iv) Finally, in the limit case where the search effort is nil,

when retirement is imminent, the transfers would never be paid back through a wage tax. This is a big difference relative to the pension tax, which will be paid independently of the search process outcome. Even if the search effort is equal to zero, it is possible with the pension tax contract  $P_r$  to smooth consumption across unemployment and retirement episodes. In this case ( $\nu = 0$ ), unemployment benefits and pension taxes are constant:  $C'_r(V_r^u(t+1)) = C'_r(V_r^u(t)) \Rightarrow V_r^u(t+1) = V_r^u(t)$  and  $b_r(t) = b_r(t+1) = \tilde{b}$ , where  $\tilde{b} \neq b$ . Using the first and last first optimality conditions for the contract  $P_r$ , we have:

$$\frac{1}{u'(p - \tau^r(t))} = \frac{1}{u'(\tilde{b})} \Rightarrow \tau^r(t) = p - \tilde{b} \equiv \tilde{\tau}^r$$

It is possible to smooth the consumption across the unemployment and retirement states, even without unemployment-employment transitions. This limit case emphasizes the superiority of the pension tax when retirement is imminent. To go beyond these intuitions, we propose in the next section some numerical simulations based on a calibrated version of the model.

### 3 Integrating SS and UI programs?

In this section, we analyze the quantitative performances of integrating the SS and UI programs, i.e. of considering a pension tax indexed on the unemployment spell. Firstly, along the lines of Hopenhayn and Nicolini (1997), the first motive is to cope with the intrinsic difficulty of reconciling insurance and incentives when only decreasing unemployment benefits are considered. Secondly, the benefit from taking into account a wage tax as in Hopenhayn and Nicolini (1997) can be limited by the short employment spell before retirement, at least for unemployed workers close enough to retirement. Since there is no closed form solution, we resort to numerical simulations<sup>10</sup> in order to show these results.

#### 3.1 Calibration

The model is calibrated on a monthly basis and based on the French economy as far as the institutions or the features of the labor market are concerned. We set the discount factor  $\beta$  to 0.993. Following Hopenhayn and Nicolini (1997), the coefficient of relative risk aversion equals  $\sigma = 0.5$ . The expected duration of retirement is set to 20 years. A retiree then dies with probability  $\lambda_r = (1/(20 \times 12))$ . The number  $n$  of expected years prior to retirement is the key parameter and will be changed in order to measure how unemployment insurance affects the search behavior as individuals get closer to retirement ( $\lambda_w = 1/(n \times 12)$ ).

We normalize the wage  $w$  at 100, so that the unemployment benefit equals the replacement rate. The latter  $\bar{b}$  is set at 50, which is the average replacement ratio for individuals

<sup>10</sup>Appendix D presents a short computational description.

eligible for unemployment insurance (as computed by the French unemployment insurance agency). The pension level is calibrated to  $p = 70$ , which is consistent with the replacement ratio observed for French retirees in the late 1990s for an individual in the private sector with an earning history corresponding to the average wage profile (Charpin, 1999; COR, 2001).

We choose to calibrate the search efficiency  $\psi$  on the seniors. It is important to take into account the fact that this efficiency can go down during the life cycle due to a lower labor demand for older workers: older workers could suffer from skill obsolescence due to the technological progress (see for instance Hellerstein, Newmark and Troske [1999]). This is why we consider in the following only workers at less than 10 years to retirement. The search efficiency  $\psi$  is then set at 0.0045 so as to replicate the average unemployment spell for individuals aged 50-55 (between 10 and 5 years prior to retirement, which is at 60 in France), who are not yet exempt from job-search requirements (11 months according to the French unemployment insurance agency).

Given this calibration, the benchmark model with constant unemployment benefits implies that the individuals who are 4 years and 5 months away from retirement have a zero search effort. It must be emphasized that this result is very close to the situation we observe in France where the unemployed workers from 55 onwards choose to be exempted from job search requirements when they are eligible for this exemption<sup>11</sup>.

### 3.2 The intrinsic limit of the traditional optimal UI contracts for older workers

We present in this section some preliminary results without the tax instruments. This allows us to show the intrinsic limits of the traditional policy with only decreasing unemployment benefits, and so the relative ability of the two tax policies to better circumvent the weakness of the job search due to the proximity to retirement.

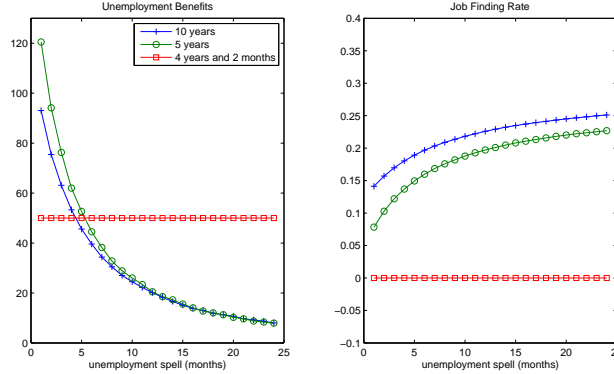
As in Shavell and Weiss (1979), the unemployment insurance agency can choose to only optimize the profile of unemployment benefits as a function of the length of the unemployment spell, without implementing taxes, either on employment or on retirement ( $\tau^e(t) = \tau^r(t) = 0, \forall t$ ). The shorter the horizon on the labor market, the steeper the unemployment benefit profile, at least until a threshold age (Figure 1). As the individual gets closer to the retirement date, individuals search less due to the distance effect. More search incentives then require a more severe punishment to individuals who do not find a job. The higher replacement ratio for newly unemployed individuals compensates for a steeper fall for longer unemployment spells.

For workers relatively far from the retirement age, this policy is quite efficient as the cost (circle line in Figure 4) is significantly reduced relative to the constant UI policy (triangle line in Figure 4). But for workers closer to retirement, this policy fails to reduce the

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<sup>11</sup>All unemployed workers are eligible from 57 onwards. Only those with enough years of contributions to the Social Security are eligible as soon as 55.

Figure 1: Downward sloping unemployment benefits



costs, because it does not succeed in encouraging older unemployed workers to search for a new job. This incentive policy succeeds in inducing only a little more search effort for individuals at the end of the working life. The zero search effort appears for individuals who are 4 years and 2 months away from retirement (Figure 1), against 4 years and 5 months without decreasing UI policy. For those individuals, UI benefits are the same from one period to the next, and, by consequence, there are no cost savings allowed by the policy.

The short distance prior to retirement intrinsically limits the effectiveness of decreasing unemployment benefits. A sharper decline in unemployment benefits would be needed to motivate those workers to look for a job. This is actually inefficient because the insurance agency would pay particularly high benefits today and promise particularly low benefits to individuals who will retire soon. The incentives consistent with the promise-keeping are not enough to encourage these individuals to search<sup>12</sup>.

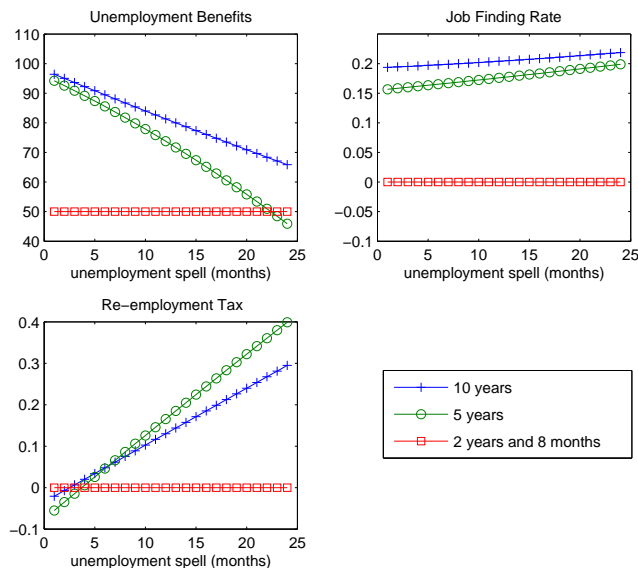
### 3.3 Introducing a wage tax or a pension tax?

The previous analysis provides an economic rationale behind the unconditional unemployment benefits until retirement available to older workers in some European countries. An alternative view to this "giving-up" policy is to introduce another instrument, a wage tax along the lines of Hopenhayn and Nicolini (1997), or a pension tax, as suggested in this paper.

**More incentives for job search of older workers.** We first explore the design of the insurance contract with a wage tax after re-employment. Figure 2 displays the unemployment insurance scheme for individuals who differ in terms of distance to retirement. Whatever the horizon, the unemployment benefits display a flatter profile compared with the case without tax on wages (Figure 1), as the principal now has two policy instruments

<sup>12</sup>A more formal intuition of this point can be given by inspecting the first-order conditions at period 1 of the unemployment spell in Appendix C.

Figure 2: Optimal insurance with a re-employment tax



to induce unemployed workers to put effort into the search process and smooth the unemployed worker's consumption. By taxing future wages, individuals are encouraged to look for a job while the slow decline in unemployment benefit ensures a smoother consumption. This is particularly true for younger workers. As can be seen in Figure 2, a shorter horizon makes consumption smoothing less effective.

Figure 3: Optimal insurance with a tax on pension

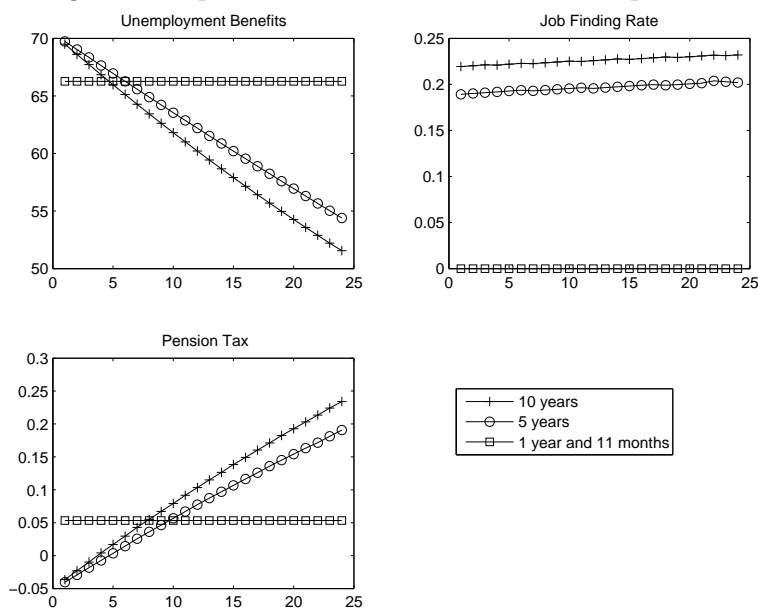


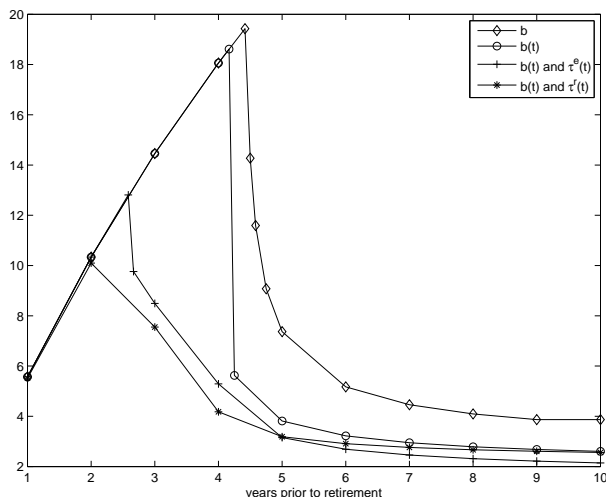
Figure 3 shows the results for the optimal contract integrating unemployment insurance and pension systems. As in the case of the wage tax, the pension tax increases with the length of unemployment spells: unemployed individuals who quickly find a job are

even rewarded with a subsidy on the pension that will be paid over a 20 year period on average. It must be emphasized that the carrot dimension is more crucial than the stick in the case of the pension tax. As expected, the pension tax can smooth consumption more easily for workers at 5 years before retirement than at 10 years, contrary to the wage tax. We can also observe that the search effort at this short horizon is particularly stimulated by the pension tax. On the other hand, whatever the horizon, the search effort is more increased through the action of the pension tax, as the disincentive effect of the tax on the instantaneous employment value is lower. This allows the pension tax policy to encourage more older workers to search for a job. Only individuals who are at less than 1 year and 11 months away from retirement are now characterized by constant optimal unemployment benefits, compared to 2 years and 8 months for the wage tax. But it must be emphasized that the pension tax is strictly positive (around 5%) for these workers, although their search effort remains nil. By taxing the retirement pension, the planner can improve consumption smoothing by transferring income from a period where individuals are better-off (retired) to a period where they suffer from a low income (unemployed): the unemployment benefit is constant, but higher than without pension tax (67% in Figure 3 versus 50% in Figure 2). Overall, for workers near to retirement, the pension tax seems to strengthen Hopenhayn and Nicolini (1997)'s strategy of considering the after-unemployment history.

**Reducing the costs of UI program.** To confirm this latter intuition, it remains to compare the cost of each unemployment insurance policy according to the horizon of a unemployed worker until retirement (Figure 4). Let us first emphasize that the cost evaluated at different horizons displays the same profile, whatever the policy considered. The cost tends to increase sharply when older workers with low search intensity are considered. For each policy, it peaks at the distance to retirement (the distance threshold) from which the search intensity becomes nil. For older unemployed workers, i.e. those at a shorter distance to retirement, the search intensity remains naturally nil, but it is fairly intuitive that the cost is reduced as the unemployment spell is mechanically reduced by the shorter distance to retirement. On the other hand, considering workers at longer distance to retirement than this distance threshold decreases the cost borne by the unemployment insurance as these workers do search for a new job. This is why the size of the cost reduction induced by a given policy highly depends on its capacity to delay this distance-to-retirement threshold: the closer to retirement this threshold, the shorter the inactive unemployment spell and the lower the associated cost. The cost is particularly high when the unemployed workers do not search at quite long distances to retirement. When the unemployment benefits are flat, older workers cease to search when they are at 4 years and 5 months from retirement: it costs about 20 monthly wages. Figure 4 shows that the differences between the policies considered are particularly significant from this distance to retirement onwards. Whatever the tax policy considered, the reduction in costs is significant relative to the unemployment insurance policy. Imposing a tax on the future

job along the lines of Hopenhayn and Nicolini (1997) or on the future pension as proposed in this paper is particularly efficient in the context of older workers. Figures 2 (with wage tax) and 3 (with pension taxes) show that the introduction of pension taxes significantly lowers the number of years prior to retirement at which individuals cease to look for a job (1 year and 11 months in Figure 3 and 2 years and 8 months in Figure 2 versus 4 years and 2 months in Figure 1). The introduction of taxes on wages and pensions would decrease the cost of unemployment insurance at 4 years to retirement by more than 70% and 80% respectively. Let us emphasize that only decreasing the unemployment benefits throughout the unemployment spell would lead to any cost reduction at this horizon. These results are as noticeable as the fact that the costs of constant unemployment benefits are five times higher at 4 years than at 10 years to retirement.

Figure 4: Cost of UI



Moreover, integrating UI and SS programs appears as a more efficient strategy at these older ages. The proximity to retirement allows the agency to provide the agent with a better consumption smoothing than the wage tax policy, which is limited by the low re-employment duration. The pension tax is more efficient for unemployed workers sufficiently close to retirement (at less than 5 years to retirement), because there are positive taxes even without re-employment. For individuals who are at the very end of their working life (1 year and 11 months away from retirement and less), none of the policies mentioned above succeeds in encouraging a positive search effort. But the policy integrating the UI and the Social Security yields a better consumption smoothing, which allows the principal to reduce the sum of revenues provided to the unemployed workers, net of the taxes collected during the retirement period: at 1 year and 11 months, the cost is 9.81 monthly wages whereas it is 9.96 for the other policies.

In contrast, the wage tax is more efficient than the pension tax when considering unemployed workers at more than 5 years to retirement: at these horizons, as expected, the pension tax is less effective, and the cost reduction converges to that provided by the policy without any taxes *à la* Shavell and Weiss (1979).



Table 1: Total cost reduction

age	unemployed older individuals	Cost per worker in terms of monthly wage			
		constant UI	downward sloping UI	tax on wage	tax on pension
55	13650	7.3672	3.8101	3.1506	3.1849
56	11950	18.0587	18.0587	5.2891	4.1768
57	22300	14.4578	14.4578	8.4924	7.5505
58	32150	10.3359	10.3359	10.3359	10.0962
59	13250	5.571	5.571	5.571	5.5152
$\Delta$ Cost (%)			-3	-34	-40

Note: Considering the flows into unemployment at different ages (second column), the cost per worker for each policy (columns from 2 to 5) is used to calculate the total cost of older workers' unemployment and so the reduction yield by each policy relative to the constant UI policy (last line).

Finally, the reduction in the cost of the insurance provided to older workers by integrating unemployment insurance and Social Security may be magnified by taking into account the number of unemployed older workers. Table 1, Column 2, shows that the flows of new unemployed workers<sup>13</sup> are particularly high at 55 and after 57 in France, respectively at 5 and 3 years prior to retirement. Considering these flows, we use the cost per worker yield by our simulations for each policy to calculate the total cost of older workers' unemployment and so the reduction yield of each policy. Overall, the total cost reduction allowed by the policies introducing taxes after the unemployment spell is much higher than that induced by the downward sloping UI contract. The latter cuts costs only by 3%, which contrasts with the reduction of 34% implied by the re-employment tax, and especially with the decrease of 40% achieved by an additional tax on pensions. These figures must be appreciated with regard to the younger worker case: as can be seen in Figure 4, for our calibration, the contribution of the tax policies relative to the contract with only decreasing unemployment benefits is much more limited in the case of workers at more than 5 years before retirement<sup>14</sup>. This illustrates the specific return on implementing the policy integrating unemployment insurance and social security systems for unemployed workers at the end of the working life.

<sup>13</sup>We consider the flows at different ages into the stock of unemployed workers exempt from the general eligibility requirement of having to look for work ("Dispense de recherche d'emploi"). More precisely, we average these flows over the years 2006 and 2007. Data come from the French Department of Labor (Labarthe and Merlier (2008) and Labarthe and Merlier (2009)).

<sup>14</sup>It is possible to increase the superiority of the wage tax policy at these younger ages by considering a higher risk aversion parameter for instance, as already shown in Hopenhayn and Nicolini (1997). The key point is that the increase in the return on tax policy when considering older workers remains quite similar to the benchmark calibration.

## 4 Conclusion

The existence of specific insurance programs for older workers in many European countries which leads them to retire early from the labor market is often viewed as responsible for the low employment rate at these ages. We show that the short distance to retirement implies strong specificities which can justify relinquishing active unemployment policies. Whereas the optimal strategy of the unemployment agency is to propose benefits decreasing with the unemployment spell for older workers who have to wait several years prior to retirement, the optimal contract becomes completely flat when retirement is imminent: it results from the inefficiency of such contracts when the horizon of both the agent and the principal is very short. Some countries (France, Ireland, Sweden) implement a general exemption from job-search requirements for older unemployed individuals. This paper provides an economic rationale for this looser job-search requirement for older workers, at least as long as conventional unemployment insurance contracts *à la* Shavell and Weiss (1979) are proposed.

On the other hand, we show that the inactivity of older workers could be reduced by introducing a pension tax. This tax is the appropriate tool to offset the effects of the short expected job duration at the end of the working cycle. Moreover, this is the only contract which is welfare-improving when going back to work is no longer incentive-compatible. It could also be the case when the very low labor demand for some older workers makes their job search inefficient.

More generally, this paper puts forward the view that the retirement age is a key institution that governs both search behavior and optimal unemployment benefits provided by the unemployment agency. This paper emphasizes that the age issue cannot be reduced to the biological age: the social age defined by the distance to the retirement age is the key dimension for positive as well as for normative analysis.

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## A The case without active policy

When unemployment benefits are constant, there are no state variables and the unemployment value  $V^u$  is time-invariant, as well as the search intensity. The optimal behaviors are summarized by:

$$\begin{aligned} V^e &= \frac{u(w) + \lambda_w \beta V^r}{1 - \beta(1 - \lambda_w)} \\ V^u &= \frac{u(b) - a + (1 - \lambda_w)\pi(a)\beta V^e + \lambda_w \beta V^r}{1 - \beta(1 - \lambda_w)(1 - \pi(a))} \\ 1 &\geq \pi'(a)\beta(1 - \lambda_w)(V^e - V^u) \end{aligned}$$

with equality if  $a > 0$ .

**Proposition 1.** *When  $a > 0$ , the search intensity is decreasing with  $\lambda_w$ .*

*Proof.* Differentiating the equation system leads to:

$$\begin{aligned} \frac{dV^e}{d\lambda_w} &= -\beta \frac{V^e - V^r}{1 - \beta(1 - \lambda_w)} \\ \frac{dV^u}{d\lambda_w} &= -\beta \frac{[\pi(a)V^e + (1 - \pi(a))V^u] - V^r}{1 - \beta(1 - \pi(a))(1 - \lambda_w)} + \frac{(1 - \lambda_w)\pi(a)\beta \frac{dV^e}{d\lambda_w}}{1 - \beta(1 - \pi(a))(1 - \lambda_w)} \\ \frac{da}{d\lambda_w} &= \frac{\pi'(a)}{\pi''(a)(1 - \lambda_w)(V^e - V^u)} \left[ V^e - V^u - (1 - \lambda_w) \left( \frac{dV^e}{d\lambda_w} - \frac{dV^u}{d\lambda_w} \right) \right] \end{aligned}$$

Because we always have  $V^e > V^r > V^u$ , it is possible to show that  $V^e - V^u - (1 - \lambda_w) \left( \frac{dV^e}{d\lambda_w} - \frac{dV^u}{d\lambda_w} \right) > 0$ , implying that  $\frac{da}{d\lambda_w} < 0$ , given that  $\pi''(a) < 0$ .  $\square$

The probability of having the corner solution  $a = 0$  is then increasing with the proximity to retirement.

**Proposition 2.** *The probability that  $a = 0$  increases with  $\lambda_w$ . From a distance threshold onwards ( $\lambda_w > \tilde{\lambda}_w$ ), the search effort is nil.*

*Proof.* Assuming that  $a = 0$ , for  $b > 0$ , we have

$$\left. \begin{aligned} V^u &= \frac{u(b) + \lambda_w V^r}{1 - \beta(1 - \lambda_w)} \\ V^e &= \frac{u(w) + \lambda_w V^r}{1 - \beta(1 - \lambda_w)} \end{aligned} \right\} \Rightarrow V^e - V^u = \frac{u(w) - u(b)}{1 - \beta(1 - \lambda_w)}$$

Using these value functions, the following inequality must be satisfied to be consistent with  $a = 0$ :

$$1 > \psi \left[ \frac{\beta(1 - \lambda_w)}{1 - \beta(1 - \lambda_w)} \right] [u(w) - u(b)] \quad (5)$$

For a given set of parameters  $\{b, w, \beta, \psi\}$ , this inequality is verified more easily the higher  $\lambda_w$  is. There exists a distance to retirement  $\tilde{\lambda}_w$  (the distance threshold hereafter) such that the condition (5) is just binding, i.e. is binding for  $a = 0$ :

$$\psi \left[ \frac{\beta(1 - \tilde{\lambda}_w)}{1 - \beta(1 - \tilde{\lambda}_w)} \right] [u(w) - u(b)] = 1$$

Considering a value of  $\lambda_w$  superior to  $\tilde{\lambda}_w$  necessarily leads to inequality (5).  $\square$

## B The dynamics of UI and pension taxes (contract $P_r$ )

First, let us note that the optimal contract  $P_e$  has exactly the same properties as in Hopenhayn and Nicolini (1997). On the other hand, considering the first order conditions in the case of the contracts  $P_r$ , it is possible to show that, assuming an interior solution, unemployment benefits decrease with the unemployment duration, whereas pension taxes are dependent on the unemployment duration.

From the FOC, we have:

$$C'_r(V^u(t)) = x(t) \frac{1}{u'(p - \tau^r(t))} + (1 - x(t)) C'_r(V_r^u(t + 1)) \quad (6)$$

with  $x(t) = 1 - \beta(1 - \lambda_w)(1 - \pi(a_r(t))) < 1$ . Because

$$\frac{1}{u'(p - \tau^r(t))} - C'_r(V_r^u(t + 1)) = \nu \frac{\pi'(a_r(t))}{1 - (1 - \pi(a_r(t)))\beta(1 - \lambda_w)} \Rightarrow \frac{1}{u'(w - \tau^r(t))} > C'_r(V^u(t + 1))$$

we deduce, using the fact that  $C'_r(V_r^u(t))$  is an average between  $C'_r(V_r^u(t + 1))$  and  $\frac{1}{u'(w - \tau^r(t))}$ , that

$$\frac{1}{u'(p - \tau^r(t))} > C'_r(V_r^u(t)) > C'_r(V_r^u(t + 1)) \Rightarrow b(t) > b(t + 1)$$

Forward iterations of equation (6) lead to:

$$C'_r(V_r^u(1)) = \left[ \sum_{j=0}^{T-1} \left( \prod_{i=0}^j (1 - x(i)) \right) x(j + 1) \frac{1}{u'(p - \tau^r(j + 1))} + \prod_{j=1}^T (1 - x(j)) C'_r(V_r^u(T)) \right]$$

If we assumed constant pension taxes,  $a(t)$  would increase with  $t$  because  $V_r^u(t)$  is decreasing. Then, we would have:

$$0 < \prod_{j=1}^T (1 - x(j)) < (1 - x(1))^T$$

Because  $C'_r(V_r^u(t))$  is bounded, we have:

$$\lim_{T \rightarrow \infty} \prod_{j=1}^T (1 - x(j)) C'_r(V_r^u(T)) = 0$$

Moreover, provided that pension taxes are constant:

$$\lim_{T \rightarrow \infty} \sum_{j=0}^{T-1} \left( \prod_{i=0}^j (1 - x(i)) \right) x(j+1) \frac{1}{u'(p - \tau^r(1))} = \frac{1}{u'(p - \tau^r(1))}$$

implying that

$$C'_r(V_r^u(1)) = \frac{1}{u'(p - \tau^r(1))}$$

which is not consistent with the envelope condition, which implies that

$$\frac{1}{u'(p - \tau^r(t))} > C'_r(V_r^u(t)) > C'_r(V_r^u(t+1))$$

Then, we deduce that the pension tax  $\tau^r(t)$  is dependent on the unemployment history.

## C The limit of the contract with decreasing UI

Assume that there are no taxes ( $\tau^e(t) = \tau^r(t) = 0$ ). In this case, the optimal unemployment insurance contract  $P_u$  is reduced to a sequence of unemployment benefits  $B_u = \{b_u(1), b_u(2), \dots, b_u(T)\}$ . The first order conditions of the problem are:

$$\begin{aligned} 1 &= \mu_u u'(b_u(t)) \\ C'_u(V_u^u(t+1)) &= \mu_u - \nu_u \frac{\pi'(a_u(t))}{1 - \pi(a_u(t))} \\ C'_u(V_u^u(t)) &= \mu_u \\ \frac{C_u(V_u^u(t+1))}{V^e - V_u^u(t+1)} &= -\nu_u \frac{\pi''(a_u(t))}{\pi'(a_u(t))} \end{aligned}$$

Firstly, let us define the natural upper bound of unemployment value  $\text{sup}(V_u^u(2))$  above which the agent does not search. We have  $a_u(1) > 0$  if and only if:

$$V_u^u(2) < V^e - \frac{1}{\psi\beta(1 - \lambda_w)} \equiv \text{sup}(V_u^u(2))$$

Consistent with Proposition 2 in Appendix A, the higher  $\lambda_w$  (older individuals), the lower the probability that this condition holds. Secondly, let us focus on the relationship between the values of  $b_u(1)$  and  $V_u^u(2)$  resulting from the optimal trade-off between incentives and insurance. Let us consider the promise-keeping at period 1:

$$V_u^u(1) = u(b_u(1)) - a_u(1) + \beta \{ (1 - \lambda_w) [\pi(a_u(1))V^e + (1 - \pi(a_u(1)))V_u^u(2)] + \lambda_r V^r \} \quad (7)$$

As  $a_u(1)$  is a function of  $V_u^u(2)$ , the promise-keeping constraint defines an implicit relationship between  $b_u(1)$  and  $V_u^u(2)$  such that  $b_u(1) = \Gamma(V_u^u(2), \lambda_w)$  with  $\frac{\partial \Gamma}{\partial V_u^u(2)} < 0$  and  $\frac{\partial \Gamma}{\partial \lambda_w} > 0$ , assuming that the values for  $V_u^u(t)$ ,  $\forall t \geq 3$ , are given.

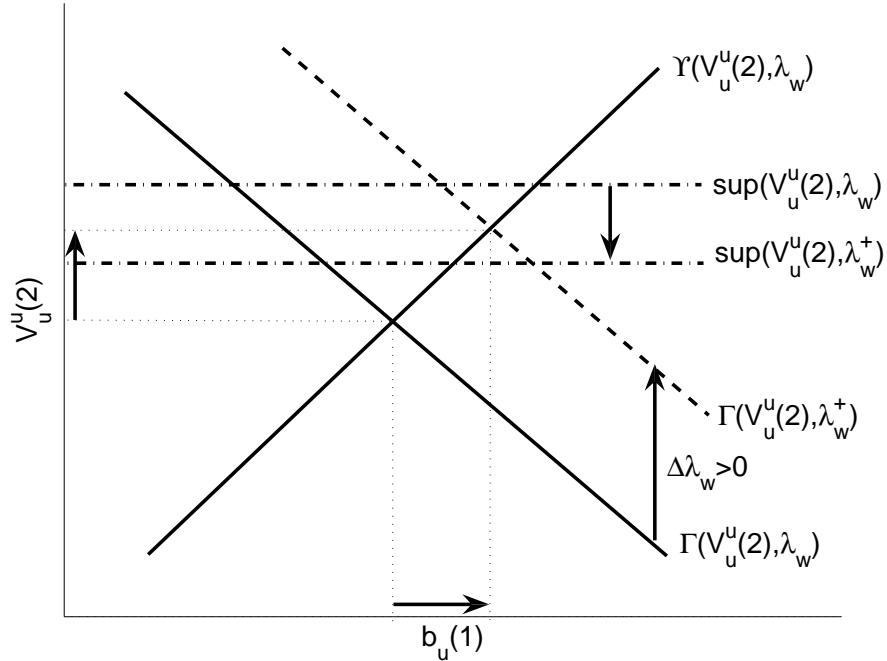
Moreover, assuming an interior solution, it leads to:

$$\frac{1}{u'(\mathcal{B}(V_u^u(2)))} = \frac{1}{u'(b_u(1))} - \frac{C_u(V_u^u(2))}{V^e - V_u^u(2)} \quad (8)$$

where  $b_u(2) \equiv \mathcal{B}(V_u^u(2))$  is obtained using the definition of the value function. Note that  $\mathcal{B}'(V_u^u(2)) > 0$ . This equation allows us to define another relationship between  $b_u(1)$  and  $V_u^u(2)$  such that  $b_u(1) = \Upsilon(V_u^u(2))$  with  $\frac{\partial \Upsilon}{\partial V_u^u(2)} > 0$  and  $\frac{\partial \Upsilon}{\partial \lambda_w} = 0$ , if we also assume that the values for  $V_u^u(t), \forall t \geq 3$ , are given.

Overall, equations (7) and (8) determine the values of  $b_u(1)$  and  $V_u^u(2)$ , which optimize the trade-off between incentives and insurance. Is this solution compatible with a binding incentive-compatibility constraint, i.e.  $V_u^u(2) < \text{sup}(V_u^u(2))$ ?

Figure 5: Trade-off between insurance and incentive



When  $\lambda_w$  increases, the combination of equations (7) and (8) leads simultaneously to higher levels for  $b_u(1)$  and  $V_u^u(2)$ , whereas the natural upper bound of unemployment value  $\text{sup}(V_u^u(2))$  decreases (Figure 5). There is a distance threshold  $\tilde{\lambda}_w$  such that the optimal level for  $V_u^u(2)$  corresponds to  $\text{sup}(V_u^u(2))$ , implying that, for all horizons shorter than  $\tilde{\lambda}_w$ , there is no interior solution (the equation (8) no longer holds), i.e. the optimal contract is such that the unemployment benefits are flat and the search effort is null.

## D The computational methodology

The computational strategy draws on Sargent and Ljungqvist (2000), with an approximation of the  $i$ th iterate  $C_i(V^u)$  of the cost function  $C(V^u)$  using Chebyshev polynomials. The numerical procedure consists of the following steps:

1. Choose a contract (wage or pension tax). Define a grid on the tax rate.



2. Choose a probability of retirement  $\lambda_w$
3. Choose upper and lower bounds for  $V^u$ , so that  $V$  and  $V^u$  will be understood to reside in the interval  $[\underline{V}^u, \overline{V}^u]$ .
4. Choose a degree  $n$  for the approximator, a Chebyshev polynomial, and a number  $m \succeq n + 1$  of nodes or grid points.
5. Generate the  $m$  zeros of the Chebyshev polynomial on the set  $[1, -1]$ .
6. By a change of scale, transform Chebyshev nodes to corresponding points  $V^u$  in  $[\underline{V}^u, \overline{V}^u]$ .
7. Choose initial values of the  $n + 1$  coefficients in the Chebyshev polynomial. Use these coefficients to define the function  $C_i(V^u)$  for iteration number  $i = 0$ .
8. Look at the optimal search effort choice (equation (1)). If equation (1) holds with equality, it determines the level of search effort  $a$ , otherwise,  $a = 0$ .
9. Compute  $\tilde{C}_i(V^u)$ , the objective of the principal, as a function of  $C_i(V^u)$ , search effort, taxes, unemployment benefits as described in Section 2.2.
10. For each point  $V^u$  and each tax rate, use a numerical minimization program to find  $C_{i+1}(V^u) = \text{Min } \tilde{C}_i(V^u)$ .
11. Using these  $m$  values of  $C_{i+1}(V^u)$ , compute new values of the coefficients in the Chebyshev polynomials with least squares.
12. Return to step 7, using the new values of the coefficients in the Chebyshev polynomials, and iterate to convergence.

For a contract and a retirement probability, the algorithm defines the sequence of unemployment benefits and tax rates.