

Life Cycle Equilibrium Unemployment

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Abstract

This paper extends the job creation-job destruction approach to the labor market to take into account a deterministic finite horizon. As hirings and separations depend on the time over which to recoup investment costs, the life-cycle setting implies age-differentiated labor market flows. Whereas the search effort of unemployed workers presents an age-decreasing profile, the age-dynamics of the separation rate can be either decreasing or increasing. Worker heterogeneity in the context of the undirected search implies an intergenerational externality, which is not eliminated by the Hosios condition. We show that age-specific policies are necessary to reach the first best allocation.

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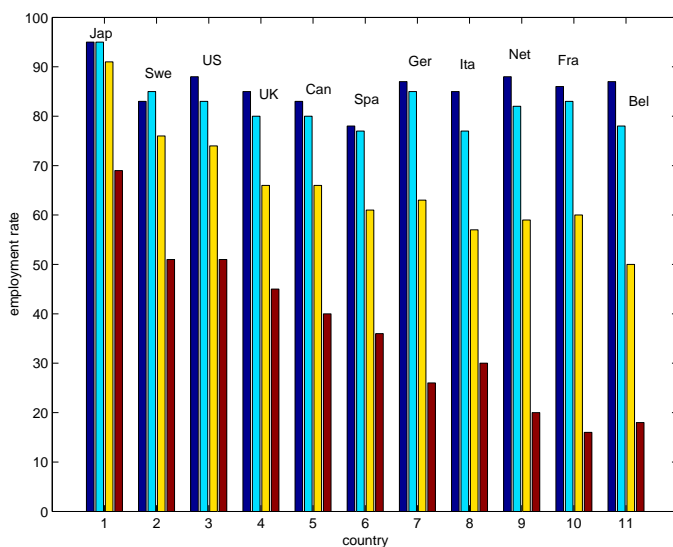
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1 Introduction

It is now well known that the low employment rate of older workers accounts for half of the European employment gap (see OECD [2006]). Moreover, low older worker employment rates, especially before the early retirement age, are a matter of concern in many countries in the context of an aging population. A first strand of the empirical literature which attempts to explain these features emphasizes the negative role played by labor market institutions (specific insurance programs) on the job-search decisions of older unemployed workers (see for instance Blondal and Scarpetta [1998]). A second strand gives greater importance to skill obsolescence, arguing that older workers suffer from technological progress. Under wage stickiness, this gives firms incentives to send older workers into early retirement (see for instance Hellerstein, Newmark and Troske [1999]).¹

Figure 1: Employment rates from age 30 to 64 for OECD Countries



Ranking of effective retirement age
(from the highest to the lowest)

In each country, each bar refers to employment rates of the age groups : 30 - 49 (first bar on the left), 50 - 54, 55 - 59 and 60 - 64 (last bar on the right)

Source: OECD data for 1995 (authors' calculation).

However, something is missing in this picture. Figure 1 shows that the fall in the employment rate of older workers is steeper when the retirement age gets closer, whatever the country considered. Two country groups emerged very clearly in the mid-nineties: those with high employment rates for workers aged 55-59 (Canada, Great Britain, Japan, the United States and Sweden) and those

¹This point has already been put forward by Lazear [1979], from a theoretical standpoint.

which experience a huge decrease in employment rates at these ages, around 25 points with respect to the 50-54 age group (Belgium, France, Italy and the Netherlands). As documented by Gruber and Wise [1999], the second group of countries was characterized by a normal retirement age of 60 (versus 65 in the first group). This suggests that the retirement age could affect the employment rate of older workers prior to this age: the later the retirement age, the higher the employment of older workers before 60.

In this paper, we investigate the influence of the distance to retirement on labor market job flows. We propose a canonical labor market equilibrium model with matching frictions and Nash wage bargaining, along the lines of Mortensen and Pissarides [1994], characterized by a deterministic exogenous age at which workers exit the labor market, typically an exogenous retirement age. We aim to show that this canonical theoretical setting naturally emphasizes the influence of impending retirement on both job creation and job destruction, as endogenous hirings and separations depend on the expected duration of jobs. Surprisingly enough, the implications of a deterministic finite horizon for workers in this extensively-used framework have not yet been addressed. From this point of view, our paper fills a gap.

In our model, the only heterogeneity across workers comes from the distance to retirement. Fundamentally, the expected sum of production flows generated by older workers is lower, due to the shorter career duration. This horizon effect explains why unemployed older workers search less: as for any investment, the shorter the payoff, the lower the return. Concerning job separations, the horizon effect has an ambiguous impact. If new opportunities are easier to reach inside the firm, through labor hoarding, then the job surplus of older workers is penalized by their shorter horizon, and it leads to higher separation rates for older workers. If new opportunities are easier to reach outside the firm, through searching for a new job, as older workers have a lower unemployment value, their job surplus is then higher and there are lower separation rates for older workers. We then show that the age-profile of job separations ultimately depends on the persistence of the productivity shocks, the bargaining power of workers and the search intensity of unemployed workers. The age-dynamics of the search intensity is then key for both the age-profile of job creations and job separations. As it is highly age-decreasing, the natural prediction of our model is a U-shaped profile for the separation rates over the life-cycle. Overall, the search effort decisions explain why older workers simultaneously experience low hiring rates and high separation rates, whereas both the hiring and the separation rates are high for the younger workers. This is partially at odds with Menzio, Telyukova and Visschers [2010] who have recently shown on US data that the flows from employment to unemployment are age-decreasing over the life cycle. We argue that it is the specificity of the older worker labor market to generate a specific unemployment status characterized by a low search intensity, consistently with our model's predictions, that explains unemployed older workers' claim to be retired. Once this population of workers is included in the unemployment pool, the proportion of individuals flowing from employment to unemployment continuously increases until retirement.

The life-cycle equilibrium unemployment theory intrinsically implies a heterogeneity across workers. Since Davis [2001], it has been well known that heterogeneity can have important implications for the efficiency of the matching allocation when there is only one matching function. We assume in this paper that the search is undirected, which is then crucial for the normative implications of the horizon effect, whereas the positive implications discussed above are independent of this assumption². Although firms could implement age-directed recruitment policies, as age is an observable characteristic, there is indeed legislation prohibiting age-discrimination in the US and European countries as far as vacancies are concerned. On the other hand, as job productivity is assumed to be not age-specific³, a search equilibrium with separate markets does not exist in our economy. We therefore assume that firms cannot *ex-ante* age-direct their search, that is vacancies cannot be targeted at a specific age group. However, as we observe ex-post discrimination against older workers (Neumark [2001]), we assume that once the contact with a worker is made, according to the observed productivity of the job-worker pair, the firm can make the choice of not recruiting the worker⁴. The non-directed search assumption explains why the Hosios condition is not enough to restore the social optimality of the life-cycle equilibrium. The impact of each generation of unemployed workers on the average return on vacancies, named hereafter an intergenerational externality, makes the internalization of the search costs for the other generations imperfect through the ex-post Nash wage bargaining. The job search value is therefore overestimated for older workers and underestimated for younger workers at the market equilibrium. We emphasize that the horizon heterogeneity across workers implies that there is a life-cycle specificity of the efficiency gap created by the non-directed search, leading to an intrinsically suboptimal labor market tightness. Moreover, we show that there are too many separations and too high a search intensity for older workers at the equilibrium. The opposite is true for younger workers. We then derive some age-specific policies in order to restore the social optimality of the decentralized life-cycle equilibrium. They share the same objective of keeping the older workers away from the search sector, either by subsidizing their jobs and/or by taxing their search effort. The latter policy is enough to make both the separation and the search effort margins efficient, as the tax on the job search intervenes at the roots of the inefficiency centered on the search value. This emphasizes the crucial role played by the search effort of unemployed workers in reconciling the equilibrium job separations with their optimal level.

In the final section, we extend the life cycle equilibrium unemployment theory to endogenous

²See Chéron, Hairault and Langot [2007] for an analysis with directed search.

³Technology diffusion and standardization, training programmes and worker experience are likely to make age requirements irrelevant to most jobs, except maybe for jobs requiring the use of the most recent wave of technology.

⁴From our point of view, there is a crucial difference between advertising for a job vacancy on the one hand and hiring and wage contracting on the other hand. The former decision is made public ex ante, before meeting the worker. The latter decisions are taken on the basis of private information once the worker has been met: other individual characteristics can be put forward to justify age-differences, which can be hardly proven as age-discrimination.

retirement decisions. We firstly show that a natural implication of the search frictions is that unemployed workers retire before employed workers. Separations after the retirement age of the unemployed workers are then actually retirement decisions: as the outside opportunity is no longer unemployment, the inefficiencies, especially the intergenerational externality, conveyed by the search process, disappear and these retirement decisions are optimal. On the other hand, the decisions on the search effort, job creation and separation before the retirement age of unemployed workers are consistent with those analyzed in our benchmark model with an exogenous retirement age: especially, the intergenerational externality makes them suboptimal, for the same reason that it explains why unemployed workers retire too late: the search costs are underestimated for older workers at the equilibrium.

Section 2 discusses related literature. Section 3 presents the benchmark model, whereas Section 4 shows the age-dynamic properties of the equilibrium. Section 5 deals with the social efficiency of the equilibrium and presents some optimal age-policies. Section 6 presents the endogenous retirement extension of the benchmark model and discusses the robustness of the main conclusions. A final section concludes.

2 Related literature

The horizon effect has already received some empirical support and some theoretical foundation based on the job search theory (Seater [1977], Ljungqvist and Sargent [2008] and Hairault, Langot and Sopraseuth [2010]). Our paper is the first to examine the horizon effect in the canonical matching model *à la* Mortensen-Pissarides. We then propose a life-cycle equilibrium unemployment theory with persistent productivity shocks, endogenous separations and search effort, then extended to the retirement decision. This generalizes the model proposed in Cheron, Hairault and Langot [2008], which analyzes the age-dependent employment protection in a context where the participation decisions are neglected (search effort and retirement age are exogenous) and the productivity shocks purely transitory.

We believe that this is a natural starting point for a life-cycle approach to the labor market flows before incorporating additional features. Adopting a more quantitative approach, some papers use a simplified version of Mortensen and Pissarides [1994], but incorporate some additional heterogeneity across ages, especially in human capital and/or in labor market institutions. Along these lines, Ljungqvist and Sargent [2008] or Cheron, Hairault and Langot [2009] explain the differences between the US and French labor market performance. More recently, Menzio, Telyukova and Visschers [2010] have added, beyond human capital accumulation, on-the-job search in order to better match the observed wage and labor market transition age-profile on US data. On the other hand, Bettendorf and Broer [2003] take into account capital accumulation. By contrast, our paper rests on the canonical matching model and adopts a more analytical approach in or-

der to derive the basics of the horizon effect. Moreover, we show that the Mortensen-Pissarides model, only modified by the existence of a retirement age, generates labor market flows that can be considered, at least at the qualitative level, as consistent with reality.

From a normative point of view, our analysis is related to the literature incorporating ex-ante heterogeneity into undirected search models. In Davis [2001], Acemoglu [2001] or Bertola and Caballero [1994], heterogeneous jobs are allocated by a non-directed matching function to homogeneous workers. In this context, even if the Hosios condition is satisfied, there is an excessive relative supply of bad jobs because firms obtain only a fraction of the extra-surplus associated to upgrading the job quality, whereas incurring all the additional costs. Let us notice that Davis [2001] shows that the result is symmetric (excessive supply of low skilled workers) if workers are heterogeneous and invest in human capital. Blazquez and Jansen [2008], following Albrecht and Vroman [2002], are the first to analyze the inefficiency of the allocation when two-sided heterogeneity is introduced in the canonical Mortensen-Pissarides model. Their paper extends the seminal paper of Sattinger [2003], where the meeting rate was fixed. We share with Blazquez and Jansen [2008] the failure of the ex-post wage bargaining to internalizing search costs, leading to inefficient job creation decisions. But, contrary to Blazquez and Jansen [2008], the directed search is not incentive-compatible from the viewpoint of the bad-type workers, the older workers in our model, as the latter would suffer from a lower probability of contact for the same expected productivity draws if they specialized their search to possible older worker-specific jobs. On the other hand, the shortcoming of the Hosios condition in the case of heterogeneity also concerns the search and participation decisions of workers. This point has been already discussed in Shimer and Smith [2001] in a search model with a fixed number of participants. Albrecht, Navarro and Vroman [2010] have recently extended this result to an endogenous participation margin.

Our paper is the first to address the efficiency issue of the heterogeneity created by the life-cycle matching model when the search is undirected. We analyze in an original and unified framework all the efficiency gaps implied by the wrong evaluation of the job surplus traditionally put forward in the literature without life-cycle features. We emphasize the specificity of the intergenerational externality created by the age heterogeneity of workers: aging naturally leads workers to move in the distribution of these heterogeneous characteristics. Overall, our paper is the first to focus on the normative implication of age heterogeneity, which is shown to imply a specific efficiency gap due to both the horizon heterogeneity across workers and the aging process.

3 Job creation and job destruction when the horizon is finite

Let us consider an economy *à la* Mortensen - Pissarides [1994] (MP hereafter). Labor market frictions imply that there is a costly delay in the process of filling vacancies, and endogenous job destructions closely interact with job creations. Unlike the large literature following MP, we

consider a life cycle setting characterized by a deterministic age at which workers exit the labor market, interpreted as the retirement age.

3.1 Model environment

We consider a discrete time model and assume that at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labor force growth in the economy. We denote the worker's age i and T the exogenous age at which workers exit the labor market: they are both perfectly known by employers. There is no other heterogeneity across workers. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

3.1.1 Shocks

Firms are small and each has one job. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. At the beginning of each age, a new productivity level is drawn with probability $\lambda \leq 1$ in the distribution $G(\epsilon)$, with $\epsilon \in [0, 1]$. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. For age $i \in [2, T - 1]$, employed workers are faced with layoffs when their job becomes unprofitable. The firms decide to close down any jobs whose productivity is below an (endogenous) productivity threshold (productivity reservation) denoted R_i . Workers, contacted when they were $i - 1$ years old, whose productivity is below the reservation productivity R_i , are not hired, as the productivity value is revealed after firm and worker have met.

3.1.2 Worker flows with non age-directed search

We assume that firms cannot *ex-ante* age-direct their search and that the matching function embodies all unemployed workers. Let v be the number of vacancies, u the number of unemployed workers, and e_i the endogenous search effort for a worker of age i . Let us define the effective unemployment rate $\tilde{u} = \sum_{i=1}^{T-1} e_i u_i$. The matching function gives the number of contact, $M(v, \tilde{u})$, where M is increasing and concave in both its arguments, and with constant returns-to-scale. From the perspective of a firm, the contact probability is $q(\theta) \equiv \frac{M(v, \tilde{u})}{v} = M(1, \theta^{-1})$ with $\theta = \frac{v}{\tilde{u}}$ the labor market tightness accordingly defined. The probability for unemployed workers of age i of being employed at age $i + 1$ is then defined by $jc_i \equiv e_i p(\theta)[1 - G(R_{i+1})]$ with $p(\theta) \equiv \frac{M(v, \tilde{u})}{\tilde{u}} = M(\theta, 1)$ the contact probability of the effective unemployed worker. Note that the hiring process is then age-differentiated even if firms are not allowed to age-direct their search. Similarly, we define the job destruction rate for an employed worker of age i as $jd_i = \lambda G(R_i)$.

For any age i , the flow from employment to unemployment is then equal to $\lambda G(R_i)(1 - u_{i-1})$. The other workers who remain employed $(1 - \lambda G(R_i))(1 - u_{i-1})$ can renegotiate their wage. The age-dynamic of unemployment is then given by:

$$u_{i+1} = u_i [1 - e_i p(\theta)(1 - G(R_{i+1}))] + \lambda G(R_{i+1})(1 - u_i) \quad \forall i \in [1, T - 1] \quad (1)$$

for a given initial condition $u_1 = 1$. The overall level of unemployment is $u = \sum_{i=1}^{T-1} u_i$, so that the average unemployment rate is $u/[T - 1]$.

3.2 Intertemporal values of firms and workers

Firms. Any firm is free to open a job vacancy and engage in hiring. c denotes the flow cost of recruiting a worker and $\beta \in [0, 1]$ the discount factor. Let V be the expected value of a vacant position and $J_i(\epsilon)$ the value of a job filled by a worker of age i with productivity ϵ :

$$V = -c + q(\theta)\beta \sum_{i=1}^{T-1} \left(\frac{e_i u_i}{\tilde{u}} \int_0^1 J_{i+1}(x) dG(x) \right) + (1 - q(\theta))\beta V$$

Vacancies are determined according to the expected value of a contact with an unemployed worker. This value is the average of the expected job-worker pair values (hiring values) over the age distribution of unemployed workers. The expected value of a contact for the firm then depends on the age distribution of the unemployed workers, as uncertainty in the hiring process arises not only from productivity, but also from the age of workers. We will show in Section 5 that heterogeneity across ages in hiring values will imply the existence of intergenerational externalities in the search process.

For a bargained wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a filled job by a worker of age i , $\forall i \in [1, T - 1]$, is defined by:

$$J_i(\epsilon) = \max \left\{ \epsilon - w_i(\epsilon) + \beta \left[\lambda \int_0^1 J_{i+1}(x) dG(x) + (1 - \lambda) J_{i+1}(\epsilon) \right]; V \right\} \quad (2)$$

Workers. Values of employed (on a job of productivity ϵ) and unemployed workers of any age i , $\forall i \in [1, T - 1]$, are respectively given by:

$$\mathcal{W}_i(\epsilon) = \max \left\{ w_i(\epsilon) + \beta \left[\lambda \int_0^1 \mathcal{W}_{i+1}(x) dG(x) + (1 - \lambda) \mathcal{W}_{i+1}(\epsilon) \right]; \mathcal{U}_i \right\} \quad (3)$$

$$\mathcal{U}_i = \max_{e_i} \left\{ b - \phi(e_i) + \beta \left[e_i p(\theta) \int_0^1 \mathcal{W}_{i+1}(x) dG(x) + (1 - e_i p(\theta)) \mathcal{U}_{i+1} \right] \right\} \quad (4)$$

with $b \geq 0$ denoting the instantaneous opportunity cost of employment and $\phi(\cdot)$ the convex function capturing the disutility of the search effort e_i . Let us denote the elasticity of this disutility $\omega(e_i) \equiv \phi'(e_i) \frac{e_i}{\phi(e_i)}$, with $\omega(e_i) > 1$ because $\phi(\cdot)$ is a convex function.

3.3 Wage bargaining

The rent associated with a job is divided between the employer and the employee according to a wage rule. Following the most common specification, wages are determined by the Nash solution to a bargaining problem:

$$w_i(\epsilon) = \arg \max_w (\mathcal{W}_i(\epsilon) - \mathcal{U}_i)^\gamma J_i(\epsilon)^{1-\gamma}$$

For a given bargaining power of the workers γ , considered as constant across ages, the global surplus generated by a job $S_i(\epsilon) \equiv J_i(\epsilon) + \mathcal{W}_i(\epsilon) - \mathcal{U}_i - V$, is divided according to the following sharing rule:

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma S_i(\epsilon) \quad \text{and} \quad J_i(\epsilon) = (1 - \gamma) S_i(\epsilon) \quad (5)$$

and the equilibrium wage rule solves:

$$w_i(\epsilon) = \gamma \epsilon + (1 - \gamma) \left[b - \phi(e_i) + e_i p(\theta) \gamma \beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) \right] \quad (6)$$

The higher the job surplus generated by the search process, the higher the reservation wage, the higher the bargained wage. On the other hand, it is possible to give another expression for the wage in terms of the search costs saved by the firms.

$$w_i(\epsilon) = \gamma [\epsilon + c \theta e_i \tau_i] + (1 - \gamma) [b - \phi(e_i)] \quad (7)$$

where τ_i is defined as follows:

$$\tau_i \equiv \frac{\int_{R_{i+1}}^1 S_{i+1}(x) dG(x)}{\sum_{i=1}^{T-1} \left(\frac{e_i u_i}{u} \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) \right)} \quad (8)$$

As in the MP model, the way that market tightness enters the wage equation is through the asymmetry between firms and workers in the search process. If the worker has a competitive advantage in the search process, implying $p(\theta)e > q(\theta)$, i.e. $\theta e > 1$, then the evaluation of the search cost is larger than c in the wage equation. In our life-cycle model, there is an additional asymmetry due to the undirected search process. For the workers, the search cost saved when the match is formed is proportional to their individual hiring value. On the other hand, for the firms, this cost is proportional to the average hiring value, as the result of the non-directed search assumption. This is why the bargained wage depends on τ_i , which gives the value of a worker hired at age i relative to the average value of a job according to the age distribution of unemployed workers. Workers with a higher hiring value than average can capture a larger fraction of these hiring costs than workers with a lower hiring value. Despite the assumption of undirected search, implying a homogenous search cost for each match, wages are age-specific.

3.4 The labor market equilibrium

Definition 1. *Assuming a free-entry condition, leading to the zero-profit condition $V = 0$, the labor market equilibrium with search frictions in a finite-horizon environment is defined by:*

$$\frac{c}{q(\theta)} = (1 - \gamma)\beta \sum_{i=1}^{T-1} \left(\frac{e_i u_i}{\tilde{u}} \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) \right) \quad (9)$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)]\beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) - (1 - \lambda)\beta S_{i+1}(R_i) \quad (10)$$

$$S_i(\epsilon) = \max\{\epsilon - R_i + (1 - \lambda)\beta[S_{i+1}(\epsilon) - S_{i+1}(R_i)]; 0\} \quad (11)$$

$$\phi'(e_i) = \gamma p(\theta)\beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) \quad (12)$$

$$u_{i+1} = u_i [1 - e_i p(\theta)(1 - G(R_{i+1}))] + \lambda G(R_{i+1})(1 - u_i) \quad (13)$$

The terminal conditions are $e_{T-1} = 0$ and $R_{T-1} = b$. The initial condition is given by u_1 .

Equation (9) derives from the zero-profit condition $V = 0$, and shows that the labor market tightness depends on the average job surplus over the unemployed workers of different ages. Equation (10) defines the productivity threshold R_i from the condition $S_i(R_i) = 0$. As in MP, a crucial implication of this rule is that the job destruction is optimal, not only from the firm's point of view but also from that of the worker. $S_i(R_i) = 0$ indeed entails $J_i(R_i) = 0$ and $\mathcal{W}_i(R_i) = \mathcal{U}_i$. Equation (11) is the job surplus definition using the expression of the productivity threshold. Equation (12) is the first order condition on the search effort of the unemployed worker. At the end of the life cycle, we have $e_{T-1} = 0$ because at the age T , agents are retired. Finally, equation (13) is the unemployment age-dynamic.

4 Age-dynamics of job creations and job destructions

Given the homogenous labor market tightness θ , solving for the age-dynamics of the productivity thresholds R_i and of the search intensity e_i determines the age-dynamics of job creations and job destructions. We first investigate the age-dynamics of the job surplus, which is key for both the age-profile of R_i and e_i .

4.1 The job surplus heterogeneity across ages and the horizon effect

Combining equations (2), (3) and (4), the job surplus can be written at the equilibrium of the labor market as follows:

$$S_i(\epsilon) = \max \left\{ \begin{array}{l} \underbrace{\epsilon - b + \phi(e_i)}_{\text{Current surplus}} \\ + \underbrace{[\lambda - \gamma e_i p(\theta)] \beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x)}_{\text{New opportunities}} + \underbrace{(1 - \lambda) \beta S_{i+1}(\epsilon)}_{\text{Capitalization}}; 0 \end{array} \right\} \quad \forall i < T - 1$$

with the terminal condition $S_{T-1}(\epsilon) = \max \{ \epsilon - b; 0 \}$.

The job surplus differs across ages, although there is no heterogeneity across workers, except the distance to retirement. The main difference comes from the larger sensitivity of the job surplus to the current surplus for the older workers. Considering the oldest workers, just before the retirement age, it is obvious that only the current surplus matters. The fundamental feature of older workers is that the remaining time on the labor market is shorter. It has two deep implications for the job surplus of older workers. Firstly, the capitalization value, which is measured by the sum of any current productivity flows on the expected remaining career time, is particularly low. The older the worker, the lower the capitalization component: as for any investment, the longer the payoff, the higher the return⁵. Secondly, older workers have less time to have new opportunities through new productivity draws. These new opportunities can be obtained inside the firm with a probability λ , yielding some value to the labor hoarding strategy. A share γ of these new opportunities can also be obtained outside the firm by an unemployed worker with a probability $e_i p(\theta)$, giving some value to the search strategy. The comparison between the arrival rate of productivity draws within the firm and the arrival rate of job offers outside the firm determines the relative value of the labor hoarding and of the search strategies. If $\lambda > \gamma e_i p(\theta)$, the search process is too slow and the bargaining power of the workers too small relative to the probability of having a new opportunity within the firm: the longer horizon of younger workers gives them a higher labor hoarding value. But, it can be the case that the persistence is so high that the search strategy becomes a better strategy than labor hoarding. This may reverse the implication of the horizon differentials in the production sector. Older workers are less likely to leave their jobs, simply because there is less time for them to reap the benefit of finding a better job in the search sector, making a shorter horizon synonymous with a lower reservation wage.

Overall, the life-cycle equilibrium unemployment framework creates heterogeneity in the job surplus generated at any age. This heterogeneity is perfectly expected at any age and propagated through generations by all forward-looking decisions: the expectations of aging impact the set

⁵Note that this is all the more true in that the shock is highly persistent (λ low).

of productivity profitable at each age. Firstly, aging reduces the horizon in the future time and then the value of any particular current productivity draw. Secondly, there is a loss/gain associated with the dynamics of the expected value of the new opportunities. This dynamic is the combination of the capitalization effect on any future profitable productivity draw and of a selection effect on the set of new opportunities: aging by reducing the horizon is a loss through the capitalization effect, but aging can also lead to more or less strictness in the selection of the new opportunities, either inside or outside the firm, according to the age-dynamic of the productivity thresholds. This selection effect is more stringent when the age-dynamics of reservation productivity is increasing ($R_i < R_{i+1}$), reinforcing the capitalization effect to make the expected value of new opportunities (the expected job surplus) age-decreasing. On the other hand, it is less stringent for age-decreasing reservation productivity ($R_i > R_{i+1}$), which can compensate for the capitalization effect.

The aging expectations explain why the solution for the job surplus is highly dependent on the age-dynamics of the productivity thresholds, which in turn depend on the job surplus. Solving for the age-dynamics of job creations and job destructions implies postulating the existence of a given age-profile for the productivity thresholds in order to derive its implications for the job surplus and search effort and then characterizing the conditions under which this age-profile exists.

4.2 Age-increasing productivity thresholds: the low persistence case

We first consider the case with age-increasing productivity thresholds.

Proposition 1. *If $R_i < R_{i+1}$, then the job surplus, the expected job surplus and the productivity reservation are the solution to:*

$$S_i(\epsilon) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \max\{\epsilon - R_{i+j}; 0\} \quad (14)$$

$$ES_{i+1} \equiv \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j I(R_{i+j}) \quad (15)$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)] \beta ES_{i+1} \quad (16)$$

with the terminal conditions $R_{T-1} = b$ and $e_{T-1} = 0$.

Proof. Straightforward using equations (10) and (11) and noticing that $I(R_{i+j}) \equiv \int_{R_{i+j}}^1 (x - R_{i+j}) dG(x) = \int_{R_{i+j}}^1 (1 - G(x)) dx$. \square

When the age-profile of the productivity thresholds is increasing, this implies that the current reservation productivity becomes unprofitable at the next age: $S_{i+1}(R_i) = 0$. The job surplus

is lower for the older workers due to this more stringent selection effect. This reinforces the capitalization effect, due to a summation over a shorter horizon, to make the older workers' surplus lower.

Proposition 2. *If $R_i < R_{i+1}$, then $e_i > e_{i+1}$.*

Proof. From equation (15), we deduce that $ES_i - ES_{i+1} = [I(R_i) - I(R_{i+1})] + \beta(1 - \lambda)(ES_{i+1} - ES_{i+2})$. As $ES_{T-2} > ES_{T-1}$, it is then straightforward that $ES_i > ES_{i+1}, \forall i$. Then, using equation (12), we obtain $e_i > e_{i+1}$, given that $\phi''(\cdot) > 0$. \square

As the expected job surplus is age-decreasing when the productivity thresholds are age-increasing, the return on searching for the unemployed workers is lower for older workers: the search effort is then age-decreasing. As the retirement age gets closer, the return on job-search investments decreases because the horizon over which workers can recoup their investment is reduced on the one hand and because fewer jobs are profitable due to a more severe selection effect on the other hand. This contributes to generating age-decreasing hiring rates, whereas job separations are age-increasing.

Proposition 3. *If $\lambda > \gamma p(\theta)e_i$, $R_i < R_{i+1}$, $\forall i < T - 1$. A sufficient condition ensuring $R_i < R_{i+1}, \forall i < T - 1$, is then $\lambda > \gamma$.*

Proof. Consistently with equations (12) and (16), the solution for the productivity reservation can be written as follows:

$$\gamma p(\theta)R_i = \gamma p(\theta)b - \underbrace{[\gamma p(\theta)\phi(e_i) + (\lambda - \gamma p(\theta)e_i)\phi'(e_i)]}_{\equiv \Upsilon(e_i)}$$

R_i is then age-increasing if and only if

$$\begin{aligned} \gamma p(\theta)(R_{i+1} - R_i) &= \Upsilon(e_i) - \Upsilon(e_{i+1}) > 0 & \forall i < T - 2 \\ \gamma p(\theta)(R_{T-1} - R_{T-2}) &= \Upsilon(e_{T-2}) > 0 \end{aligned}$$

We have $R_i < R_{i+1}, \forall i < T - 2$, implying that $e_i > e_{i+1}$ (Proposition 2), if $\Upsilon'(x) > 0, \forall x \in [e_{T-2}, e_1]$. As $\Upsilon'(x) = (\lambda - \gamma p(\theta)x)\phi''(x)$, the condition $\lambda > \gamma p(\theta)x, \forall x \in [e_{T-2}, e_1]$ implies that $\Upsilon'(x) > 0$. $\lambda > \gamma p(\theta)e_i, \forall i \leq T - 2$, then implies $R_i < R_{i+1}$. On the other hand, for $i = T - 2$, the terminal restriction is given by $\Upsilon(e_{T-2}) > 0 \Leftrightarrow \lambda > \left(\frac{\omega(e_{T-2})-1}{\omega(e_{T-2})}\right) \gamma p(\theta)e_{T-2}$ which is a less restrictive condition than $\lambda > \gamma p(\theta)e_{T-2}$. This is why, $\forall i < T - 1$, $\lambda > \gamma p(\theta)e_i$ implies that $R_i < R_{i+1}$. Finally, because $e_i p(\theta) < 1$, a sufficient condition for $R_i < R_{i+1}, \forall i < T - 1$, is then $\lambda > \gamma$ \square

Proposition 3 states that older workers face more job separations, that is $R_i < R_{i+1}$, when the persistence of the productivity level is low enough to make the value of future opportunities

unambiguously better inside the firm than outside. This is the case because the arrival rate of new draws inside the firm is higher than the arrival rate of job offers. Due to the shorter horizon of older workers, giving up part of the option value of receiving a better offer by leaving their jobs is less costly.

4.3 Age-decreasing productivity thresholds: the high persistence case

Let us now turn to the case with continuous age-decreasing productivity thresholds.

Proposition 4. *If $R_i > R_{i+1}$, then the job surplus, the expected job surplus and the productivity reservation are solution to:*

$$S_i(\epsilon) = P_i(T) \max\{\epsilon - R_i; 0\} \quad \text{with } P_i(T) = \sum_{j=0}^{T-1-i} \beta^j (1 - \lambda)^j \quad (17)$$

$$ES_{i+1} = \int_{R_{i+1}}^1 S_i(x) dG(x) = P_{i+1}(T) I(R_{i+1}) \quad (18)$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)] \beta ES_{i+1} - (1 - \lambda) \beta S_{i+1}(R_i) \quad (19)$$

with the terminal conditions $R_{T-1} = b$ and $e_{T-1} = 0$.

Proof. Straightforward using equations (10) and (11) and noticing that $I(R_{i+1}) \equiv \int_{R_{i+1}}^1 (x - R_{i+j}) dG(x) = \int_{R_{i+1}}^1 (1 - G(x)) dx$. \square

When the reservation productivity profile is age-decreasing, a job opened at a given age remains profitable at older ages until retirement, as long as the level of productivity is unchanged. This implies that the capitalization effect depends on the degree of persistence as in MP, but also, more interestingly, on the horizon of the workers until retirement through the variable $P_i(T)$. Younger workers yield a higher job surplus through the age-dynamics of the capitalization effect since $P_i(T) > P_{i+1}(T)$. However, when the productivity thresholds are age-decreasing, the selection effect is more stringent for younger workers: fewer jobs are profitable, although the distance to retirement is longer. This is why the expected job surplus is not necessarily lower for older workers, as the capitalization effect and the selection effect go in the opposite direction, as can be deduced from equation (18):

$$ES_i - ES_{i+1} = \underbrace{[P_i(T) - P_{i+1}(T)] I(R_i)}_{\text{capitalization age-dynamics}} - \underbrace{P_{i+1}(T) \int_{R_{i+1}}^{R_i} [1 - G(x)] dx}_{\text{selection age-dynamics}} \quad (20)$$

Given that $P_i(T) > P_{i+1}(T)$ and $R_i > R_{i+1}$, $ES_i \leq ES_{i+1}$. The expected surplus is age-decreasing when the effect of the capitalization age-dynamics dominates the effect of the selection age-dynamics.

Proposition 5. *If $R_i > R_{i+1}$, then $e_i \leq e_{i+1}$. If the effect of the capitalization age-dynamics dominates the effect of the selection age-dynamics, then $e_i > e_{i+1}$.*

Proof. Straightforward using equation (12) and (20). \square

The search effort of older workers is lower when the expected return on searching for a job is age-decreasing. When $R_i > R_{i+1}$, it is the case only when the effect of the capitalization age-dynamics overcompensates for the effect of the selection age-dynamics. If $R_i > R_{i+1}$, the job separations are age-decreasing, whereas the age-dynamics of the job creations are indeterminate.

Proposition 6. *If the search effort e_i is age-decreasing, then $R_i > R_{i+1}$, $\forall i < T - 2$, provided that the terminal condition $R_{T-2} > R_{T-1}$ holds. A necessary condition ensuring $R_i > R_{i+1}$, $\forall i < T - 1$, is $\lambda < \gamma$.*

Proof. Combining equations (12) and (19) leads to:

$$\gamma p(\theta) R_i = \gamma p(\theta) b - \Upsilon(e_i) + \gamma p(\theta) \beta (1 - \lambda) P_{i+1}(T) (R_{i+1} - R_i)$$

Then, R_i is age-decreasing if and only if

$$\begin{aligned} R_{i+1} - R_i &= \frac{\Upsilon(e_i) - \Upsilon(e_{i+1}) + \gamma p(\theta) \beta (1 - \lambda) P_{i+2}(T) (R_{i+2} - R_{i+1})}{\gamma p(\theta) [1 + \beta (1 - \lambda) P_{i+1}(T)]} < 0 \quad \forall i < T - 2 \\ R_{T-1} - R_{T-2} &= \frac{\Upsilon(e_{T-2})}{\gamma p(\theta) [1 + \beta (1 - \lambda)]} < 0 \end{aligned}$$

If the terminal restriction is satisfied ($R_{T-1} - R_{T-2} < 0 \Leftrightarrow \Upsilon(e_{T-2}) < 0$), by backward iterations, it is sufficient to determine the restriction which ensures $R_{i+1} - R_i < 0$, given that $R_{i+2} - R_{i+1}$ is negative. The terminal condition is: $\lambda < \left(\frac{\omega(e_{T-2}) - 1}{\omega(e_{T-2})} \right) \gamma p(\theta) e_{T-2}$, which is more restrictive than the condition $\lambda < \gamma p(\theta) e_{T-2}$. If e_i is age-decreasing and $\Upsilon'(x) < 0$, $\forall x \in [e_{T-2}, e_1]$, i.e. $\lambda < \gamma p(\theta) e_i$, $\forall i \leq T - 2$, then $\Upsilon(e_i) - \Upsilon(e_{i+1}) < 0$ and $R_{i+1} - R_i < 0$, $\forall i < T - 2$. Consistently with the age-decreasing dynamics of e_i , the condition $\lambda < \gamma p(\theta) e_i$ is the most restrictive for $i = T - 2$, implying that the terminal condition is enough to ensure that $R_i > R_{i+1}$. A necessary condition for $R_i > R_{i+1}$, $\forall i < T - 1$, is then $\lambda < \gamma$. \square

Proposition 6 shows that a high degree of persistence ($\lambda < \gamma$ at least) implies that the search sector gives more chances than the production sector of having new opportunities. When the persistence is particularly high, the shorter horizon of older workers may lead to fewer separations from their jobs. However, this conclusion is valid only if the search value is lower for older workers. This is the case if the capitalization effect dominates the selection effect, i.e. the search effort is age-decreasing. There are indeed two opposite forces induced by the age-decreasing productivity thresholds, which could lead to a self-inconsistency. Firstly, if the reservation productivity is lower and lower as workers age, the selection process is less and less stringent at the end of the working life, and the new opportunities in the search sector are easier and easier to reach for older workers. This dynamic selection effect, ceteris paribus, implies that the older workers are fired more often, which is not internally consistent. Secondly, as the horizon over which the next reservation productivity will be capitalized is longer in the case of younger workers, the

age-profile of the search value may be decreasing, the capitalization effect overcompensating for the dynamic selection effect. This case ensures that the search value of older workers is lower, and then that there are fewer job separations for older workers. In the following, we will consider the case where the capitalization effect dominates the selection effect, as it ensures that the age-decreasing profile of R_i exists when the persistence is high.

4.4 Discussion

Model predictions. In the two previous sections, the age-profiles of job creation and separation have been characterized. Whatever the case considered, the horizon effect is at the origin of heterogeneous flows across ages independently of any other factors. The shorter horizon of the older workers unambiguously leads to a lower capitalization effect over any profitable jobs. This explains why the job separation rates may be higher and the job creation rate weaker when retirement is imminent. The shorter horizon also reduces the number of new opportunities in the case of bad shocks. If the persistence of the shock is weak, this feature reinforces the implications of the capitalization effect. However, if the persistence is high, searching for a new job is a better strategy than labor hoarding, especially for younger workers with a longer horizon. In that case, there are fewer older worker job separations. The degree of persistence relative to the bargaining power of workers then appears a crucial feature determining the age-profile of job separation.

However, the dynamics of the search intensity over the life cycle are also key for that of the separation rate. Proposition 3 naturally predicts that the age-profile of the separation rate is age-increasing at the end of the life-cycle: as the search intensity is low for these ages, an age-increasing is consistent with almost any values of the persistent level. This prediction implies that the life-cycle matching model may help to explain why countries experience a drop in their employment rate at the end of the working life in Figure 1 and why it occurs at different ages. The important dimension of the model is indeed the retirement age. Only the distance between the current age and the retirement age matters according to a horizon effect. The biological age does not matter in itself. These results suggest that the observed low employment rate of near-to-retirement people cannot be considered as a reason for not postponing the age of retirement. The reasoning is completely reversed: retirement postponement is actually likely to increase the employment rate of these workers, thereby contradicting the widespread view that the low employment rate of older workers makes any extension of the retirement age pointless⁶.

Can the age-dynamics of the job separation rates be continuously increasing over the life cycle? Proposition 3 presents the maximal shock duration able to ensure an age-increasing separation rate: λ must be higher than γ . For instance, for a realistic value of γ equal to 0.5, the expected duration of a shock must be lower than 2 years. Such values of persistence are not necessarily

⁶See Hairault, Langot and Sopraseuth [2010] for such policy experiments.

unrealistic. Indeed, it is assumed that the shocks uniformly hit the workers, whatever their age, in order to focus on the horizon effect. In that sense, these shocks drive worker flows, and not job flows as in Mortensen and Pissarides [1994]. The persistence of these job-worker pair shocks is lower than the persistence of the job-technology pair shocks. Indeed, there exists some empirical evidence of a low persistence of the shocks that hit job-worker pairs⁷. For instance, in Ljungqvist and Sargent [2008], the earnings are the product of the wage by unit of human capital by the level of the human capital. In this context, they consider that the average time between wage draws is 1.9 years within a skill occupation, whereas the average duration for a skill occupation is 1.435 years.

However, it is certainly more relevant to believe that the reservation productivity displays a U-shaped age-profile. If $\lambda < \gamma$, a continuous age-increasing profile for the productivity thresholds is no longer ensured. In that case, the age-profile of the search effort becomes determinant. As the younger workers are likely to intensively search for a job, the reservation productivity age-dynamics may be U-shaped.

Proposition 7. *A U-shaped reservation productivity age-dynamic is such that $R_i > R_{i+1}$ for $i < \hat{i}$ and $R_i < R_{i+1}$ for $i \geq \hat{i}$. The age \hat{i} is defined by the condition $\lambda = \gamma p(\theta) e_{\hat{i}}$. A sufficient condition for the existence of this equilibrium is $\lambda < \gamma p(\theta) e_i$ for $i < \hat{i}$ and $\lambda > \gamma p(\theta) e_i$ for $i > \hat{i}$, provided that the search effort e_i is age-decreasing.*

Proof. See Appendix C. □

The intuition of Proposition 7 is straightforward⁸. Whereas the reservation productivity is age-increasing for older workers until retirement, the age-profile can be inverted below a given age \hat{i} if the search value dominates the labor hoarding value when considering younger workers with higher search intensity. Below this age ($i < \hat{i}$), searching for a job dominates labor hoarding ($\lambda < \gamma p(\theta) e_i$) and the productivity thresholds are age-decreasing. Above this age ($i > \hat{i}$), the opposite is true ($\lambda > \gamma p(\theta) e_i$) and the productivity thresholds are age-increasing. Over the life-cycle, the age-dynamics of the reservation productivity is then U-shaped when the search intensity of younger workers is high enough to invert the age-increasing profile featuring older ages. In that case, the job creations are unambiguously age-decreasing after \hat{i} , whereas this age-profile is indeterminate before \hat{i} , as result of the two opposite dynamics on R_i and e_i .

What are the facts? Menzio, Telyukova and Visschers [2010] have recently shown on US data that both the flows from employment to unemployment and the flows from unemployment to employment are age-decreasing. More precisely, the job creation rate is fairly flat over the life

⁷The average duration of a job for a given worker is lower than the average duration of an equipment in a firm.

⁸Though intuitive, Proposition 7 implies some complexity to be proved due to the inversion of the age-dynamics after a given age \hat{i} . See Appendix C.

cycle, with a large decline after 50, whereas the age-profile of the job separation rate has a more pronounced downward slope over the life-cycle. These results on job separations are partially at odds with those shown in a previous paper (Chéron, Hairault and Langot [2009]), which are characterized by a U-shaped profile over the life cycle on US and French data (Labor Force Survey): the job separation rate would increase at the end of the working life before retirement. The opposite result on the separation flows at the end of the working life relies on the difference in the pool of unemployed workers considered. Even more than for the other ages, we believe that the unemployment occupation definition is too restrictive when based on the self-declaration of looking for a job⁹: it is the specificity of the older worker labor market to generate a specific unemployment status characterized by such a low search intensity that unemployed people claim to be retired, as the probability of becoming employed again is very weak. This is in line with our model predictions. They are actually unemployed workers with a particularly low search intensity, and not retired people as they claim to be.

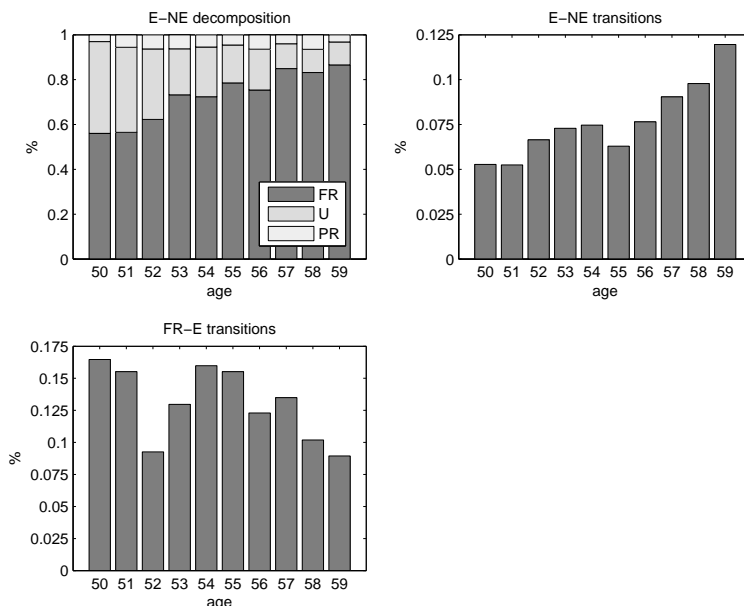
Using the Health and Retirement Survey (HRS) allows us to illustrate this point by focusing on the older worker's status. In HRS data, there is a clear distinction between unemployed, partially and fully retired workers. Partially retired people are looking for a job, and then can unambiguously be associated with unemployed workers, whereas the fully retired workers declare themselves as not searching. For people between 50 and 60, before the early age of eligibility to Social Security, the "fully retired" position becomes dominant among the non-employed people. More particularly, it is striking to see in Figure 2 that the structure of job separations is highly biased toward full retirement. As people age, the proportion of the retirement status grows as that of the unemployed falls. The proportion of those leaving work to become partly retired remains stable at a low level. For instance, 56% of people of age 50 leaving employment declare themselves to be fully retired. This proportion jumps to 85% at around 60, when workers are closer to the age at which it is possible to claim the SS benefits.

Are these workers still participating despite their claim to be fully retired? We believe that they may be considered as unemployed workers with a particularly low search intensity. Figure 2 shows the probability of working for individuals "retired" at different ages. What emerges is that although fully retired individuals declare themselves as not looking for a job, and certainly despite their low search effort, there is a significant fraction of them who will work in the future. Once the "fully retired" workers are included in the pool of workers searching for a job, Figure 2 shows that the separation rates from employment to unemployment change dramatically¹⁰: they increase almost continuously from age 50 to age 60. In that case, our model can simultaneously

⁹Blanchard and Diamond [1990] suggest being less restrictive concerning the unemployment definition: one half of the new matches does not come from this set of unemployed people, showing that workers not reporting "looking for a job" do search in fact because some of them obtain a job.

¹⁰Leaving aside this population leads to an age-decreasing separation rate as shown by Menzio, Telyukova and Visschers [2010].

Figure 2: The full retirement status into question



Source: HRS data for 1995 (authors' calculation). E-NE decomposition is the structure of the separations. E-NE transitions refer to separations from employment to (unemployment+fully retirement). FR-E transitions are job findings from fully retired people.

replicate the age-decline of the job creation and the age-increase of the separation rate when the retirement age is approaching¹¹.

This interpretation of the fully retired status for the workers in their fifties is debatable and a more comprehensive empirical analysis would be needed to conclude¹². We acknowledge that retirement and unemployment with low search effort are hardly distinguishable. However, it is beyond doubt that the end of the working life intrinsically generates this mixed status between retirement and unemployment. From our point of view, a key point is that the job separations leading to this mixed status are strongly affected by the distance to retirement. In other words, an increase in the "real" retirement age would eliminate most of these separations and increase the search intensity for a given age¹³, as the distance to retirement would be higher. Section 6, by making the retirement age endogenous, will help us to shed more light on these interactions between the decisions underlying retirement and labor market flows.

¹¹Menzio, Telyukova and Visschers [2010] explain the age-decreasing separation dynamics they observe by taking into account the higher human capital of older workers in order to compensate for the horizon effect.

¹²This is beyond the scope of this paper to provide with such empirical analysis.

¹³See Hairault, Langot and Sopraseuth [2010] for such counterfactual experiments.

5 Optimal allocation and age-policy

The previous sections highlighted economic rationales sustaining the age heterogeneity of job separations and job creations. It remains to show that this behavior is at odds with social optimality. The existence of an additional externality, namely an intergenerational externality, could lead to reconsidering the efficiency result obtained in an infinite horizon when the Hosios condition holds. Specific policies, in particular designed by age, should then be implemented to restore social optimality.

5.1 The efficient allocation

The problem of the planner is to determine the optimal allocation of each worker between the production and the search sectors, and the optimal investment in the search sector. The per-unemployed worker social value in the search sector and the per-employed worker social value in the good sector are respectively given by:

$$Y_i^s = b - \phi(e_i^*) - ce_i^*\theta^* + e_i^*p(\theta^*)\beta \int_0^1 Y_{i+1}(x)dG(x) + (1 - e_i^*p(\theta^*))\beta Y_{i+1}^s \quad (21)$$

$$Y_i(\epsilon) = \max \left\{ \epsilon + \lambda\beta \int_0^1 Y_{i+1}(x)dG(x) + (1 - \lambda)\beta Y_{i+1}(\epsilon); Y_i^s \right\} \quad (22)$$

where $ce_i^*\theta^*$ represents the total cost of vacancies in efficiency units per-unemployed worker. The job surplus for the planner is defined by $S_i^*(\epsilon) \equiv Y_i(\epsilon) - Y_i^s$:

$$S_i^*(\epsilon) = \max \left\{ \epsilon - b + \phi(e_i^*) + ce_i^*\theta^* + [\lambda - e_i^*p(\theta^*)]\beta \int_0^1 S_{i+1}^*(x)dG(x) + (1 - \lambda)\beta S_{i+1}^*(\epsilon); 0 \right\}$$

The planner's decisions R_i^* , e_i^* and θ^* are the solution to¹⁴:

$$\max_{\{e_i^*, R_i^*, \theta^*\}} \sum_{i=1}^{T-1} u_i^* Y_i^s \quad s.t. \quad S_i(R_i^*) = 0 \quad \forall i \in [2, T-1]$$

At each age, the social allocation between the two sectors is governed by the arbitration condition: $S_i(R_i^*) = 0$. On the other hand, the non-segmented search process implies choosing the labor market tightness that maximizes the social value of search per-unemployed worker averaged over age groups.

Definition 2. Let $\eta(\theta^*) = 1 - \frac{\theta^* p'(\theta^*)}{p(\theta^*)}$, the maximum value of steady-state social welfare is

¹⁴When shocks are persistent, this intertemporal planner problem is not equivalent to maximizing the production net of vacancy costs averaged over age class, as a worker of age $i + j$ is not representative of the future of any worker of age i .

reached when θ^* , $\{R_i^*\}_{i=2}^{T-1}$ and $\{e_i^*\}_{i=1}^{T-1}$ solve:

$$\frac{c}{q(\theta^*)} = [1 - \eta(\theta^*)]\beta \sum_{i=1}^{T-1} \left(\frac{e_i^* u_i^*}{\tilde{u}^*} \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x) \right) \quad (23)$$

$$R_i^* = b - \phi(e_i^*) - ce_i^* \theta^* - [\lambda - e_i^* p(\theta^*)]\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x) - (1 - \lambda)\beta S_{i+1}^*(R_i^*) \quad (24)$$

$$S_i^*(\epsilon) = \max\{\epsilon - R_i^* + (1 - \lambda)\beta[S_{i+1}^*(\epsilon) - S_{i+1}^*(R_i^*)]; 0\} \quad (25)$$

$$\phi'(e_i^*) = -c\theta^* + p(\theta^*)\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x) \quad (26)$$

$$u_{i+1}^* = u_i^* [1 - e_i^* p(\theta^*)(1 - G(R_{i+1}^*))] + \lambda G(R_{i+1}^*)(1 - u_i^*) \quad (27)$$

with the terminal conditions $e_{T-1}^* = 0$ and $R_{T-1}^* = b$.

The equation (23) is similar to the equation (9) obtained in the decentralized equilibrium, on condition that the worker share of employment surplus (γ) is now replaced by the elasticity relative to the unemployment in the matching function ($\eta(\theta^*)$). The equation (24) shows that a job is destroyed when the expected profit from the marginal job (the current product plus the option value for expected productivity shocks) fails to cover the social return on the unemployed worker. For the social planner, the return on an additional age i unemployed worker is reduced by the cost of vacancy per age i “efficient” unemployed worker, which is equal to $c\theta e_i$. Finally, the equation (26) shows that the marginal cost of the search effort is equalized to the social return on the search process for unemployed workers. The first return on this activity is the surplus associated with the transition from unemployment to employment for a worker aged i ($p(\theta^*)\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x)$). Moreover, the planner takes into account the fact that more effort devoted to the search from a worker aged i reduces the likelihood for the other workers of finding a job. In order to keep the job opportunities per unemployed worker constant, the planner must raise the number of vacancies: this cost is measured by $c\theta^*$ in the equation (26). Hence, this implies that an unemployed worker must search only if the expected value of the transition from unemployment to employment is higher than this congestion cost. More formally, we have:

$$e_i^* \geq 0 \quad \text{if and only if} \quad p(\theta^*)\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x) \geq c\theta^* \quad (C)$$

Let us note that the condition (C) is always satisfied in the MP model¹⁵: the search costs are lower than the expected hiring value for an unemployed worker. In our model, whereas the search costs are shared by all the unemployed workers, the expected hiring value for a specific age can be lower than its average value. The efficient allocation can then be such that the optimal search effort is nil before the retirement age if the condition (C) does not hold. In this case, the

¹⁵Indeed, in the MP model with homogenous agents, the free entry condition is simply $\frac{c}{q(\theta)} = (1 - \eta(\theta)) \int_R^1 [1 - G(x)] dx \Leftrightarrow c\theta = (1 - \eta(\theta))p(\theta) \int_R^1 [1 - G(x)] dx$. For $\eta(\theta) \in]0, 1[$, we deduce that in the MP model $c\theta < p(\theta) \int_R^1 [1 - G(x)] dx$.

arbitration of the social planner is simply to reallocate these unemployed workers to the home production sector. In the following, we assume for simplicity, and without loss of generality, that the condition (C) will be satisfied, $\forall i < T - 1$.

Proposition 8. *If $\lambda > p(\theta^*)e_i^*$, $\forall i < T - 1$, then $R_i^* < R_{i+1}^*$ implying that $e_i^* > e_{i+1}^*$ and $\tau_i^* > \tau_{i+1}^*$.*

Proof. See Appendix A. □

Proposition 8 restates at the social optimum the condition under which the job separations are age-increasing and the implication of this dynamics for the search intensity over the life cycle. This proposition emphasizes that higher (lower) job destruction (creation) rates for older workers are not only an equilibrium outcome but also an efficient age-pattern of labor market flows when persistence is low enough. Because of their shorter horizon, older workers must be fired more and hired less. On the other hand, for the same reasons as in the market equilibrium allocation, it is optimal to have an age-decreasing separation dynamic when the persistence is high enough (Proposition 9). Let us emphasize that the individual hiring value τ_i^* , defined by the counterpart of the equation (8) at the social optimum, is still age-decreasing when the search intensity e_i^* is age-decreasing, i.e. when the capitalization effect dominates the selection effect.

Proposition 9. *If the search effort e_i^* is age-decreasing, then $\tau_i^* > \tau_{i+1}^*$ and $R_i^* > R_{i+1}^*$, $\forall i < T - 2$, provided that the terminal condition $R_{T-2}^* > R_{T-1}^*$ holds. A necessary condition ensuring $R_i^* > R_{i+1}^*$ is then $\lambda < p(\theta^*)e_i^*$, $\forall i < T - 1$.*

Proof. See Appendix B. □

Although the optimal and equilibrium labor market flows share the same age-profile, this does not imply that their equilibrium levels are consistent with the efficient allocation.

5.2 The inefficiency of the labor market equilibrium: the intergenerational externality

Traditionally, the equilibrium unemployment framework is known to generate search externalities which take the decentralized equilibrium away from the efficient allocation. However, when the elasticity relative to vacancies in the matching function is equal to the bargaining power of workers (Hosios condition), social optimality can be reached. Does this result still hold when life-cycle features are considered?

It has been well-known since Davis [2001] that a single matching function when there is some heterogeneity across workers or jobs can create some additional inefficiencies. Traditionally, it is assumed that there are heterogenous jobs with different productivity, whereas workers are

homogenous¹⁶. This is exactly the symmetric case to ours where the jobs are identical and the workers differ across ages. Beyond this difference, it must be emphasized that there is a life-cycle specificity of the efficiency gap created by the worker heterogeneity: aging naturally leads workers to move in the distribution of this heterogenous characteristics. This specificity implies that the job creation is intrinsically not efficient. To easily unveil this property, let us assume that the search effort is constant over the age and normalized to unity ($e_i = 1, \forall i$) and that the destruction rate is exogenous, denoted by s . Then, the equilibrium and the first best allocations are defined as follows:

Equilibrium	First best
$S_i = y - b - \frac{1}{1-\gamma} c\theta\tau_i + c\theta\tau_i + \beta(1-s)S_{i+1}$	$S_i^* = y - b - \frac{1}{1-\eta(\theta^*)} c\theta^*\tau_i^* + c\theta^* + \beta(1-s)S_{i+1}^*$
$\frac{c}{q(\theta)} = \beta(1-\gamma) \sum_{i=1}^{T-1} \left(\frac{u_i}{\bar{u}} S_{i+1} \right)$	$\frac{c}{q(\theta^*)} = \beta[1-\eta(\theta^*)] \sum_{i=1}^{T-1} \left(\frac{u_i^*}{\bar{u}^*} S_{i+1}^* \right)$
$u_{i+1} = u_i [1 - p(\theta)] + s(1 - u_i)$	$u_{i+1}^* = u_i^* [1 - p(\theta^*)] + s(1 - u_i^*)$
$\tau_i = \frac{S_{i+1}}{\sum_{i=1}^{T-1} \left(\frac{u_i}{\bar{u}} S_{i+1} \right)}$	$\tau_i^* = \frac{S_{i+1}^*}{\sum_{i=1}^{T-1} \left(\frac{u_i^*}{\bar{u}^*} S_{i+1}^* \right)}$

The Hosios condition $\gamma = \eta(\theta^*)$ allows the market and the planner to agree on the search returns of an age- i worker ($\frac{1}{1-\gamma} c\theta\tau_i = \frac{1}{1-\eta(\theta^*)} c\theta^*\tau_i^*$). But they still disagree on the valuation of the search costs ($c\theta\tau_i$ for the market versus $c\theta^*$ for the planner), and then on the job surplus. Indeed, the ex-ante search costs are sunk for the market, and only the ex-post search costs saved by an age-specific hiring are accounted for through the wage bargaining process. On the contrary, for the planner, only the ex-ante search costs, independent of the worker's age, matter. As each individual hiring value differs from the average hiring value ($\tau_i \neq 1$), the social search costs are not properly internalized at the equilibrium. If the matched worker is young (old), the market over(under)-estimates the search costs. Nevertheless, these age-specific gaps could lead to an efficient vacancy decision, as this latter decision depends on the average surplus over all unemployed workers. Given the expressions of the age-specific job surplus in the simplified model, this is the case if and only if:

$$\sum_{i=1}^{T-1} \frac{u_i}{\bar{u}} \left[\tau_i + \sum_{j=1}^{T-i-1} \tau_{i+j} \beta^j (1-s)^j \right] c\theta = \sum_{i=1}^{T-1} \frac{u_i^*}{\bar{u}^*} \left[1 + \sum_{j=1}^{T-i-1} \beta^j (1-s)^j \right] c\theta^* \quad (28)$$

As long as workers are heterogenous in terms of distance to retirement, the condition (28) is never verified and the job creation is not socially efficient. The distortion does not vanish because the summation over ages is truncated by the retirement of older workers, making the job duration different across ages. By contrast, this is why the condition (28) is satisfied when all the jobs exogenously last only one period ($s \rightarrow 1$). This is also the case in a model with some job persistence but without any heterogeneity in the expected duration of the match, typically in an infinite-life environment (see Appendix E). In our life-cycle framework, only

¹⁶See for example Acemoglu [2001], Bertola and Caballero [1994] or Ljungqvist and Sargent [2000].

the initial distortions at the hiring stage of all the jobs compensate each other, because, by definition of τ_i , we have $\sum_{i=1}^{T-1} \frac{u_i}{u} \tau_i = 1$: the overestimation of the search costs induced by the hirings of younger workers is fully compensated for by the underestimation of these costs induced by the hirings of older workers. This is no longer the case over all the job duration, the future of younger workers being not compensated for. Moreover, as this future is aging, the average job surplus is underestimated at the equilibrium, due to the age-decreasing profile of τ_i . There are lower incentives to post vacancies at the market equilibrium than for the planner ($\theta < \theta^*$). In a nutshell, the intergenerational externality is specific, not only because it derives from an endogenous heterogeneity due to the distance to retirement, but also because it implies an inefficient job creation decision *per se*.

The comparison between the equilibrium and the first best allocations in the simplified model shows that they differ in terms of the valuation of the job surplus. This difference identically exists in the complete model with endogenous separation and search effort. Moreover, as these latter decisions are based on the individual job surplus, they are necessarily distorted, even in the case of a non-distorted job creation¹⁷. To show this result, it is more convenient to rewrite equations (24) and (26) as follows:

$$R_i^* + \lambda\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x) dG(x) + (1-\lambda)\beta S_{i+1}^*(R_i^*) = b - \phi(e_i^*) + \frac{1}{1-\eta(\theta^*)} c e_i^* \theta^* \tau_i^* - c e_i^* \theta^* \quad (29)$$

$$\phi'(e_i^*) = \frac{1}{1-\eta(\theta^*)} c \theta^* \tau_i^* - c \theta^* \quad (30)$$

and to compare them to their equilibrium counterparts:

$$R_i + \lambda\beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x) + (1-\lambda)\beta S_{i+1}(R_i) = b - \phi(e_i) + \frac{1}{1-\gamma} c \theta e_i \tau_i - c \theta e_i \tau_i \quad (31)$$

$$\phi'(e_i) = \frac{1}{1-\gamma} c \theta \tau_i - c \theta \tau_i \quad (32)$$

As the evaluation of the search costs is not the same for the market ($c\theta\tau_i$ in equations (31) and (32)) than for the planner ($c\theta^*$ in equations (29) and (30)), the Hosios condition no longer achieves efficiency of the separation and search decisions. The key point is again that the individual hiring value through the wage bargaining cannot capture the age-independent size of the search costs. When $\tau_i < 1$, for the older workers, the search cost is under-estimated: the reservation productivity and the search effort are then both too high at the equilibrium. The opposite is true for younger workers. It is worth re-emphasizing this efficiency gap by considering the equilibrium wage:

$$w_i(\epsilon) = \gamma(\epsilon + c\theta e_i) + (1-\gamma)(b - \phi(e_i)) + \gamma c \theta e_i (\tau_i - 1) \quad (33)$$

whereas the optimal wage would be:

$$w_i^*(\epsilon) = \eta(\theta^*)(\epsilon + c\theta^* e_i^*) + (1-\eta(\theta^*))(b - \phi(e_i^*)) + c\theta^* e_i^* (\tau_i^* - 1) \quad (34)$$

¹⁷In the particular case of purely idiosyncratic shocks ($\lambda = 1$), as jobs last only one period, the vacancy decision is not distorted *per se*.

If $\gamma = \eta(\theta^*)$, the gap between the equilibrium wage and the optimal wage is:

$$w_i^*(\epsilon) - w_i(\epsilon) = (1 - \gamma)c\theta^*e_i^*(\tau_i^* - 1)$$

For the younger (older) workers $\tau_i^* > 1$ ($\tau_i^* < 1$), implementing the optimal wage would lead to a higher (lower) wage. As vacancy investments are made in function of the representative worker, when the contact is a younger (older) worker, the job surplus is higher (lower) than expected. The optimal evaluation of this ex post gain (loss) is $c\theta^*e_i^*(\tau_i - 1)$ (see equation (34)). At the equilibrium, only a fraction γ of this value is captured by the worker (see equation (33)), leading to a wrong evaluation of the individual search value. Should the bargaining power of workers γ be equal to one, this inefficiency would be eliminated. This result is in line with Davis [2001]: there exists a tension between the condition for an efficient mix of jobs and the condition for an efficient total of jobs.

5.3 The optimal policy

Labor market policies designed by age may allow firms and workers to internalize the intergenerational externality. In order to specifically deal with this externality, we assume that the Hosios condition is satisfied ($\eta(\theta^*) = \gamma$). To restore optimality, we propose introducing age-specific taxes or subsidies. As different decisions are distorted, from the job creation to the job separation by passing through the search effort, different instruments may be necessary. First, we assume that the search effort is exogenous in order to show that it is possible to restore optimality on the job creation and separation through a unique tax on filled jobs; secondly, we reintroduce an endogenous search effort: we then show that a unique tax on the search effort is enough to restore optimality on all decisions.

5.3.1 The exogenous search effort and employment policy

When the search effort is exogenous, the age-specific externality cannot be partially compensated for by the age-specific search intensity: an old worker has the same probability of meeting a firm as a young worker. The equilibrium search value can be influenced only through a change in the surplus of a filled job. Let us consider an age-specific tax/subsidy (a_i) on jobs once they are filled. This policy, aiming at eliminating the gap between the equilibrium and optimal search values, or equivalently between the equilibrium and the optimal reservation productivity, can be

derived from the comparison of the equations (35) and (36):

$$\underbrace{R_i + \lambda\beta \int_{R_{i+1}}^1 S_{i+1}(x)dG(x) + (1-\lambda)\beta S_{i+1}(R_i)}_{\text{Lowest worker value inside the production sector}} = \underbrace{b - a_i + \frac{1}{1-\gamma}c\theta\tau_i - c\theta\tau_i}_{\text{Worker value outside the production sector}} \quad (35)$$

$$\underbrace{R_i^* + \lambda\beta \int_{R_{i+1}^*}^1 S_{i+1}^*(x)dG(x) + (1-\lambda)\beta S_{i+1}^*(R_i^*)}_{\text{Lowest worker value inside the production sector}} = \underbrace{b + \frac{1}{1-\eta(\theta^*)}c\theta^*\tau_i^* - c\theta^*}_{\text{Worker value outside the production sector}} \quad (36)$$

Proposition 10. *An optimal age-sequence for employment subsidies, denoted $\{a_i\}_{i=1}^{T-1}$, solves $a_i = c\theta^*(1 - \tau_i^*)$, where θ^* and R_i^* are defined by the efficient allocation.*

Proof. Straightforward by comparing equations (35) and (36). \square

It is noticeable that the optimal policy rule is independent of the degree of persistence, as the relative hiring value τ_i^* is always age-decreasing, under the conditions of Propositions 8 and 9.

Corollary 1. *Whatever the level of persistence λ , the optimal age-dynamics of employment subsidies are increasing with age, $a_i^* < a_{i+1}^* \quad \forall i \in [0, T-1]$, and there exists a threshold age \tilde{i} such that $a_i \leq 0 \quad \forall i \in [0, \tilde{i}]$ and $a_i \geq 0 \quad \forall i \in [\tilde{i}, T-1]$.*

Proof. Whatever the productivity threshold age-dynamics, the expected surplus is age-decreasing under the conditions of Propositions 8 and 9. We have necessarily $\tau_i^* > \tau_{i+1}^*$ implying $a_i^* < a_{i+1}^*$. Because $\tau_1^* > 1$ and $\tau_{T-1}^* < 1$, there exists $a_i^* = c\theta^*(1 - \tau_i^*) = 0$ \square

This implies subsidizing the employment of older workers more, and even taxing the employment of younger workers (for $i \leq \tilde{i}$). By equalizing the equilibrium job surplus with its optimal counterpart at each age, the averaged surplus over all the unemployed workers is then at its optimal value, leading to efficient job creations. The investment in vacancies taken without any information concerning the future match is now consistent with the planner's decision, as there is a compensatory age-dependent subsidy/tax once the job is filled by a worker characterized by a given distance to retirement.

5.3.2 Endogenous search effort and unemployment policy

Let us consider the general case with endogenous search effort. Actually, it is fairly intuitive that making this decision optimal, as it implies correcting for the value of the search cost internalized by the unemployed workers, can also benefit the job separation and job creation decisions. Let us consider a subsidy conditional on the level of the search effort, $s_i e_i$. The instantaneous utility

of an unemployed worker is then $b - \phi(e_i) + s_i e_i$. Let us consider again the employment subsidy a_i . In this case, the equilibrium conditions are:

$$R_i + \lambda\beta \int_{R_{i+1}}^1 S_{i+1}(x)dG(x) + (1 - \lambda)\beta S_{i+1}(R_i) = b - a_i - \phi(e_i) + s_i e_i + \frac{1}{1 - \gamma} c\theta e_i \tau_i - c\theta e_i \tau_i \quad (37)$$

$$\phi'(e_i) = s_i + \frac{1}{1 - \gamma} c\theta \tau_i - c\theta \tau_i \quad (38)$$

whereas the optimal allocation satisfies equations (29) and (30). The equation (38) shows that the marginal cost of the search effort is equal to its marginal return net of the subsidy s_i .

Proposition 11. *An optimal age-sequence for employment and search effort subsidies, denoted $\{a_i^*, s_i^*\}_{i=1}^{T-1}$, solves $a_i^* = 0$ and $s_i^* = -c\theta^*(1 - \tau_i^*)$.*

Proof. Straightforward by comparing equations (37) and (38) with equations (29) and (30). \square

Proposition 11 shows that only one instrument is needed to reach the optimal allocation. The reason is intuitive: the search policy intervenes upstream from the employment policy, at the roots of the inefficiency centered on the search value. Restoring the optimality of the search effort decision implies that the search value coincides with its optimal level, which then makes the separation and the vacancy decisions optimal.

Corollary 2. *Whatever the level of persistence, the optimal age-dynamics of search effort subsidies is characterized by $s_i^* > s_{i+1}^* \quad \forall i \in [0, T - 1]$, and there exists a threshold age \tilde{i} such that $s_i \geq 0 \quad \forall i \in [0, \tilde{i}]$ and $s_i \leq 0 \quad \forall i \in [\tilde{i}, T - 1]$.*

Proof. See the proof of Corollary 1. \square

This suggests that it is optimal to tax the search effort of the older workers (and to subsidize the younger worker's search effort). It is then optimal to discourage the older workers from searching for jobs. As for the employment policy, the objective is to keep the older workers away from the search sector. This result gives some theoretical arguments for the existence of pre-retirement schemes in some European countries. In France, for instance, older unemployed workers aged more than 57 receive generous benefits owing to the fact that they do not have to search for a job. It must be emphasized that our result validates this policy and its particular age threshold only because these workers are near to retirement (at 60 in France).

On the other hand, the size of the tax/subsidy correcting for the efficiency gap gives some insights on the quantitative importance of the intergenerational externality. The job surplus is close to zero for older workers ($\tau_o^* = 0$), implying that the tax is of the same magnitude as the search costs ($s_o = -c\theta^*$). It is possible from a kind of back to envelope calculation to determine the subsidy that must be given to the youngest workers: let us assume that the population

consists of three age groups of the same size and that the expected surplus of each population is proportional to their horizon, leading to $\tau_y^* = 2$ for the younger workers, the prime-age worker being representative of the average surplus¹⁸. Then, the optimal policy for younger workers is $s_y = c\theta^*$. Overall, the correction induced by the policy is then of the same order as the labor market frictions, which implies that the magnitude of the intergenerational externality is quite significant in the search framework¹⁹.

6 Endogenous retirement ages

Until now, the retirement age was exogenous. In this section, we propose making the extensive margin of the labor market participation endogenous through the retirement decision. We then show that our main results are robust to the introduction of this additional margin.

6.1 Accounting for an age-increasing retirement value

In traditional models of retirement choices (Sheshinski [1978], Heckman [1976]), households accumulate financial or social security assets allowing them to quit the labor market before an exogenous death D . After the retirement age, they use these assets to smooth their consumption paths. In our model, without any saving behavior, it seems to be heroic to derive a rational behavior for the retirement age²⁰, as long as no exogenous heterogeneity across ages is considered, for instance an increasing disutility in working²¹. A simplifying assumption consists of introducing exogenous earnings $\psi(i)$, assumed to increase with retirement age ($\psi'(\cdot) > 0$ and $\psi''(\cdot) < 0$). Once workers are retired, and we assume that retirement is an absorbing state, these earnings are constant. These exogenous earnings can be considered as a reduced form allowing our life-cycle matching model to easily capture the age-increasing earnings generated by accumulated assets, either on the financial market or through the social security system. Accounting for a retirement

¹⁸Let us notice these numbers depend on the population structure, which gives the relative scarcity of the younger workers. More younger workers in the population would imply that they are closer to the average group. This would lead to a subsidy lower than proportional to the search costs whereas the tax on older workers would remain unchanged.

¹⁹Given the optimal choices of the unemployed workers, implying $e_y > 0$ and $e_o = 0$, and using a quadratic search cost function, the total subsidies received by the younger unemployed workers are $s_y^* e_y^* = 2 \left(0.5 + \frac{\gamma}{1-\gamma} \right) (c\theta^*)^2$. Using the Pissarides [2009] values ($c = .356$, $\theta = .72$, $\gamma = \eta = .5$ and $b = .7$), we then deduce that $s_y e_y = 0.196$, corresponding to a 28.1% increase in the unemployment income ($b + s_y e_y$).

²⁰See, for example, Hairault, Langot and Sopraseduth [2010] for a model with endogenous retirement choices, endogenous saving and accumulation of social security wealth.

²¹In that case, it would create another age heterogeneity across workers, which would be at odds with our objective to focus on the endogenous heterogeneity created by the finite-life hypothesis.

value leads to modifying the value functions as follows:

$$\mathcal{W}_i(\epsilon) = \max \left\{ w_i(\epsilon) + \beta \left[\lambda \int_0^1 \mathcal{W}_{i+1}(x) dG(x) + (1 - \lambda) \mathcal{W}_{i+1}(\epsilon) \right]; \mathcal{B}_i \right\} \quad (39)$$

$$\mathcal{B}_i = \max_{e_i} \left\{ b - \phi(e_i) + \beta \left[e_i p(\theta) \int_0^1 \mathcal{W}_{i+1}(x) dG(x) + (1 - e_i p(\theta)) \mathcal{B}_{i+1} \right]; \mathcal{A}_i \right\} \quad (40)$$

$$\mathcal{A}_i = \sum_{j=0}^{D-i} \beta^j (b + \psi(i)) \quad (41)$$

where \mathcal{B}_i is the non-employment value and \mathcal{A}_i the retirement value. Employed and unemployed workers now have the outside opportunity to retire when the retirement value \mathcal{A} dominates the job search value.

6.2 Retirement decisions and job flows

The retirement occupation is chosen when the worker no longer has incentives to participate either as employed or as unemployed. Firstly, there is a maximal retirement age in the economy above which no jobs are opened: all workers are then retired after this age.

Proposition 12. *If the function $\psi(\cdot)$ is such that $\psi(T^e) = 1 - b$ for $T^e < D$, there exists a maximum age of retirement T^e for the employed workers. From this age on, the job surplus is never again positive, even for the highest productivity level. The retirement decision T^e is optimal.*

Proof. If $\psi(T^e) = 1 - b$, as, by assumption, $\psi(\cdot)$ is increasing with age, we have $\psi(D) > 1 - b$, and then $S_D(1) = \max\{1 - b - \psi(D); 0\} = 0$. This leads to $S_{D-1}(1) = \max\{1 - b - \psi(D-1); 0\} = 0$ because $\int_0^1 S_D(x) dG(x) = 0$. By backward induction, we find that all job surpluses are nil above T^e . T^e is such that $\psi(T^e) = 1 - b$ implying that $S_{T^e-1}(1) = 1 - b - \psi(T^e) > 0$. Identically, for the planner, we have $S_{T^e}^*(1) = 0$ and $S_{T^e+j}^*(1) < 0 \quad \forall j > 0$, implying $\psi(T^{e*}) = 1 - b$ and then $T^e = T^{e*}$. \square

At age T^e , the workers (even) at the highest level of productivity choose to retire. There are no more profitable jobs occupied by older workers after T^e , and unemployed workers are then obviously also retired. Proposition 12 shows that the retirement age is independent of the labor market equilibrium. This explains why the planner chooses the same optimal retirement age as the agents in the decentralized economy.

Before this age T^e , there can be flows from employment to retirement if the unemployment value is dominated by the retirement value, but also flows between employment and unemployment as long as unemployed workers choose to participate. The key age to determine is then that of retirement for unemployed workers, which happens when the unemployment value converges to the retirement value.

Proposition 13. *All unemployed workers retire at T^u before T^e . Even if the Hosios condition is satisfied, the intergenerational externality implies that this retirement age is not optimal.*

Proof. The equilibrium retirement age for the unemployed worker, T^u , solves $\mathcal{A}_{T^u} = \mathcal{U}_{T^u}$, implying, as $\mathcal{A}_{T^u+1} > \mathcal{U}_{T^u+1}$, that $\psi(T^u) = -\phi(e_{T^u}) + e_{T^u}p(\theta)\gamma\beta \int_0^1 S_{T^u+1}(x)dG(x)$. T^u is strictly inferior to T^e . Indeed, we know that there is no job surplus after T^e . $T^u = T^e$ would imply that $\psi(T^u) = -\phi(e_{T^u})$, which has no solution. Moreover, the retirement decision is not optimal as the equilibrium decision solves $\psi(T^u) = -\phi(e_{T^u}) + \frac{1}{1-\gamma}c\theta e_{T^u}\tau_{T^u} - c\theta e_{T^u}\tau_{T^u}$ whereas the optimal retirement age is given by $\psi(T^{u*}) = -\phi(e_{T^{u*}}) + \frac{1}{1-\eta(\theta^*)}c\theta^* e_{T^{u*}}\tau_{T^{u*}}^* - c\theta^* e_{T^{u*}}^*$. Even if $\gamma = \eta(\theta^*)$, the comparison between these two equations shows that $T^u \neq T^{u*}$ because $c\theta e_{T^u}\tau_{T^u} \neq c\theta^* e_{T^{u*}}^*$. \square

Once search frictions are considered, it is optimal that unemployed workers retire before the employed workers. However, the existence of the intergenerational externality makes the retirement age of the unemployed workers suboptimal. At the end of the life cycle, the older workers under-estimate the search costs, which leads them to retire too late, compared to the first best allocation.

Proposition 14. *After the retirement age T^u , job separations are optimal retirement decisions and are age-increasing.*

Proof. See Appendix D.1 \square

Job separations after T^u are retirement decisions. Our framework generates a continuous distribution of the retirement age. There is a minimal retirement age, given by the first age at which the unemployed workers choose to become retired. After this minimal age, the continuous separation process feeds the retirement pool until the employed workers even with the maximal productivity level choose to retire at T_e . These job separations are optimal because the outside option after T^u depends on the same exogenous function $\psi(\cdot)$ as that of the planner: if there are no longer workers in the search process, there are no externalities, neither the traditional aggregate search externality nor the intergenerational one.

Proposition 15. *Before T^u , the age-dynamics of job creations and separations are qualitatively the same as in the benchmark model with an exogenous retirement age T .*

Proof. See Appendix D.2 and D.3. \square

Proposition 15 sheds some light on the previous results obtained in the exogenous retirement case. Before T_u , job separations and job creations are characterized by the horizon effect and distorted by the intergenerational externality, as before T in the exogenous case. It is worth emphasizing that the search intensity decreases as the retirement age T_u is approaching: these

workers are unemployed, and not yet retired. The endogenous retirement case reinforces the idea that the existence of these discouraged unemployed workers is intrinsic to the proximity to the retirement age.

7 Conclusion

Because the horizon of older workers is shorter, firms and workers invest less in job-search and labor-hoarding activities at the end of the life cycle. This result shows that the retirement age is the key factor that governs the employment rate of older workers. Countries with a low retirement age would also suffer from a depressed employment rate for older workers at a relatively early age. This may explain why countries with a retirement age of around 60 such as France and Belgium also have a lower employment rate for workers aged between 55 and 59 than those with a retirement age of 65, such as Sweden and the United States (Figure 1).

Age-policies are necessary to reach the optimal level of output when the search process is not age-directed. At equilibrium, due to an intergenerational externality, there are indeed not enough (too many) job destructions for younger (older) workers. This is why it is optimal to subsidize the employment of the older workers. Concerning the labor supply, we show that it could be optimal to discourage the search effort of older workers. For a given retirement age, these results give some rationale for the age policies implemented in some European countries, such as pre-retirement for unemployed workers.

It must be emphasized that all these results are independent of the age-profile of human capital. Traditionally, this issue is at the heart of the life cycle approach, especially when the aim is to provide a quantitative analysis of the age-profile of wages. This paper leaves aside this dimension to focus on the basics of the life-cycle equilibrium unemployment theory. On the one hand, in the case of specific human capital, as it is not transferable to a new job, it is quite intuitive that the horizon effect is still the only dimension that determines the relative hiring value across ages. On the other hand, the expected hiring value of older workers can be higher than that of younger workers if their higher level of general human capital dominates their shorter horizon. In that case, the age-dependent policy could be reversed. But this case is quite unrealistic, as the age-profile of general human capital accumulation is typically considered as hump-shaped with a peak at about age forty (Kotlikoff and Gokhale [1992]). We believe that our normative results are robust to the introduction of human capital²². However, we acknowledge that taking into account human capital along the lines of Menzio, Telyukova and Visschers [2010] could be useful for a more quantitative analysis of the life cycle equilibrium unemployment framework. This is left for further research.

²²In a previous version of this paper, all these results are explicitly shown.

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A Proof of Proposition 8

If $R_i^* < R_{i+1}^*$, $\forall i < T - 1$, then the solutions for the surplus, the productivity reservation and the search effort are:

$$S_i^*(\epsilon) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \max\{\epsilon - R_{i+j}^*; 0\} \quad (42)$$

$$R_i^* = b - \phi(e_i^*) - ce_i^* \theta^* - [\lambda - e_i^* p(\theta^*)] \beta ES_{i+1}^* \quad (43)$$

$$\phi'(e_i^*) = -c\theta^* + p(\theta^*) \beta ES_{i+1}^* \quad (44)$$

Let us denote $I(y) = \int_y^1 (x-y) dG(x) = \int_y^1 (1-G(x)) dx$, with $I'(x) < 0$, and $ES_i^* = \int_0^1 S_i^*(x) dG(x) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j I(R_{i+j}^*)$.

Let us start by showing that $R_{i+1}^* > R_i^*$ implies $e_{i+1}^* < e_i^*$. As $ES_i^* - ES_{i+1}^* = [I(R_i^*) - I(R_{i+1}^*)] + \beta(1-\lambda)(ES_{i+1}^* - ES_{i+2}^*)$, and given that $I'(x) < 0$ and $ES_{T-2}^* > ES_{T-1}^*$, it is then straightforward that $ES_i^* > ES_{i+1}^*$, and then that $\tau_i^* > \tau_{i+1}^*$. If the condition (C) is satisfied and using equation (44), we then obtain $e_{i+1}^* < e_i^*$, given that $\phi''(\cdot) > 0$.

It remains to show that $R_i^* < R_{i+1}^*$, $\forall i < T - 1$, if $\lambda > p(\theta^*) e_i^*$. Consistently with equations (43) and (44), the solution for the productivity reservation can be written as follows:

$$p(\theta^*) R_i^* = p(\theta^*) b - \lambda c \theta^* - \underbrace{[p(\theta^*) \phi(e_i^*) + (\lambda - p(\theta^*) e_i^*) \phi'(e_i^*)]}_{\equiv \mathcal{Z}(e_i)}$$

If the condition (C) is satisfied, R_i^* is then age-increasing if and only if

$$\begin{aligned} p(\theta^*) (R_{i+1}^* - R_i^*) &= \mathcal{Z}(e_i^*) - \mathcal{Z}(e_{i+1}^*) > 0 \quad \forall i < T - 2 \\ p(\theta^*) (R_{T-1}^* - R_{T-2}^*) &= \lambda c \theta^* + \mathcal{Z}(e_{T-2}^*) > 0 \end{aligned}$$

We have $R_i^* < R_{i+1}^*$, $\forall i < T - 2$, implying that $e_i^* > e_{i+1}^*$, if and only if $\mathcal{Z}'(x) > 0$, $\forall x \in [e_{T-2}^*, e_1^*]$. As $\mathcal{Z}'(x) = (\lambda - p(\theta^*) x) \phi''(x)$, the condition $\lambda > p(\theta^*) e_i^*$, $\forall i \leq T - 2$, is enough to ensure $R_i^* < R_{i+1}^*$, $\forall i < T - 2$. On the other hand, for $i = T - 2$, the terminal restriction is given by

$\lambda c \theta^* + \mathcal{Z}(e_{T-2}^*) > 0 \Leftrightarrow \lambda > \left(\frac{\omega(e_{T-2}^*) - 1}{\omega(e_{T-2}^*) \left(1 + \frac{c \theta^*}{\phi'(e_{T-2}^*)} \right)} \right) p(\theta^*) e_{T-2}^*$ where $\omega(e_{T-2}^*)$ is the elasticity of the search cost function at the age $T - 2$. This inequality is a less restrictive condition than $\lambda > p(\theta^*) e_{T-2}^*$. This is why, $\forall i < T - 1$, $\lambda > p(\theta^*) e_i^*$ implies that $R_i^* < R_{i+1}^*$.

B Proof of Proposition 9

If the $R_i^* > R_{i+1}^*$, $\forall i < T - 1$, then the solutions for the surplus, the reservation productivity and the search effort are:

$$S_i^*(\epsilon) = P_i(T) \max\{\epsilon - R_i^*; 0\} \quad \text{with } P_i(T) = \sum_{j=0}^{T-1-i} \beta^j (1 - \lambda)^j \quad (45)$$

$$R_i^* = b - ce_i^* \theta^* - \phi(e_i^*) - [\lambda - e_i^* p(\theta)] \beta ES_{i+1}^* - (1 - \lambda) \beta P_{i+1}(T) (R_i^* - R_{i+1}^*) \quad (46)$$

$$\phi'(e_i^*) = -c\theta^* + p(\theta^*) \beta ES_{i+1}^* \quad (47)$$

where $ES_i^* = \int_0^1 S_i^*(x) dG(x) = P_i(T) I(R_i^*)$, with $I(y) = \int_y^1 (1 - G(x)) dx$ and then $I'(x) < 0$.

Let us start by showing that $R_{i+1}^* < R_i^*$ implies $e_{i+1}^* < e_i^*$ only if the capitalization effect dominates the selection effect. As $ES_i^* - ES_{i+1}^* = [P_i(T) - P_{i+1}(T)] I(R_i^*) - P_{i+1}(T) \int_{R_{i+1}^*}^{R_i^*} [1 - G(x)] dx$, given that $[P_i(T) - P_{i+1}(T)] > 0$ and $R_{i+1}^* < R_i^*$, $ES_i^* \leq ES_{i+1}^*$. Using equation (47), we then deduce that $e_i^* \leq e_{i+1}^*$, given that $\phi(\cdot) > 0$, $\phi'(\cdot) > 0$ and $\phi''(\cdot) > 0$. If the capitalization effect dominates the selection effect, i.e. $[P_i(T) - P_{i+1}(T)] I(R_i^*) > P_{i+1}(T) \int_{R_{i+1}^*}^{R_i^*} [1 - G(x)] dx$, then $ES_i^* > ES_{i+1}^*$ implying that $e_i^* > e_{i+1}^*$ and $\tau_i^* > \tau_{i+1}^*$.

The equations (46) and (47) lead to:

$$p(\theta) R_i = p(\theta) b - \lambda c \theta^* - \mathcal{Z}(e_i^*) + p(\theta^*) \beta (1 - \lambda) P_{i+1}(T) (R_{i+1}^* - R_i^*)$$

If the condition (C) is satisfied, R_i^* is age-decreasing if and only if

$$\begin{aligned} R_{i+1}^* - R_i^* &= \frac{\mathcal{Z}(e_i^*) - \mathcal{Z}(e_{i+1}^*) + p(\theta^*) \beta (1 - \lambda) P_{i+2}(T) (R_{i+2}^* - R_{i+1}^*)}{p(\theta^*) [1 + \beta (1 - \lambda) P_{i+1}(T)]} < 0 \quad \forall i < T - 2 \\ R_{T-1}^* - R_{T-2}^* &= \frac{\lambda c \theta^* + \mathcal{Z}(e_{T-2}^*)}{p(\theta^*) [1 + \beta (1 - \lambda)]} < 0 \end{aligned}$$

If the terminal restriction is satisfied ($R_{T-1}^* - R_{T-2}^* < 0 \Leftrightarrow \lambda c \theta^* + \mathcal{Z}(e_{T-2}^*) < 0$), by backward induction, it is sufficient to determine the restriction which ensures $R_{i+1}^* - R_i^* < 0$, given that $R_{i+2}^* - R_{i+1}^*$ is negative. The terminal condition is $\lambda < \frac{\omega(e_{T-2}^*)^{-1}}{\omega(e_{T-2}^*) \left(1 + \frac{c\theta^*}{\phi'(e_{T-2}^*)}\right)} p(\theta^*) e_{T-2}^*$, which is more restrictive than the condition $\lambda < p(\theta) e_{T-2}^*$. If e_i^* is age-decreasing and $\mathcal{Z}'(x) < 0$, $\forall x \in [e_{T-2}^*, e_1^*]$, i.e. $\lambda < p(\theta^*) e_i^*$, $\forall i \leq T - 2$, then $\mathcal{Z}(e_i^*) - \mathcal{Z}(e_{i+1}^*) < 0$ and $R_{i+1}^* - R_i^* < 0$, $\forall i < T - 2$. Consistently with the age-decreasing dynamics of e_i^* , the condition $\lambda < p(\theta^*) e_i^*$ is the most restrictive for e_{T-2}^* , implying that the terminal condition is enough to ensure that $R_i^* > R_{i+1}^*$.

C The U-shaped reservation productivity dynamics

C.1 The match surplus

If there exists an age \hat{i} such that $R_i > R_{i+1}$ for $i < \hat{i}$ and $R_i < R_{i+1}$ for $i \geq \hat{i}$, the job surplus is given by:

$$S_i(\epsilon) = \begin{cases} \max \{(\epsilon - R_i) + \beta(1 - \lambda) [S_{i+1}(\epsilon) - S_{i+1}(R_i)]; 0\} & \text{for } i < \hat{i} \\ \max \{(\epsilon - R_i) + \beta(1 - \lambda) S_{i+1}(\epsilon); 0\} & \text{for } i \geq \hat{i} \end{cases}$$

Forward iterations lead to the following expressions of the surplus, for $i \in [\hat{i}; T - 1]$

$$S_i(\epsilon) = \sum_{j=0}^{T-1-i} [\beta(1 - \lambda)]^j \max \{\epsilon - R_{i+j}; 0\}$$

whereas for $i < \hat{i}$, we have

$$\begin{aligned} S_i(\epsilon) &= \max \left\{ \left(\sum_{j=0}^{\hat{i}-i-1} [\beta(1 - \lambda)]^j \right) (\epsilon - R_i) + [\beta(1 - \lambda)]^{\hat{i}-i} [S_{\hat{i}}(\epsilon) - S_{\hat{i}}(R_i)]; 0 \right\} \\ &= P_i(\hat{i}) \max \{\epsilon - R_i; 0\} + \mathcal{V}_i(\hat{i}, \epsilon) \end{aligned} \quad (48)$$

The U-shaped dynamics for the R_i implies the presence of the additional term $\mathcal{V}_i(\hat{i}, \epsilon)$ when considering the age-decreasing dynamics before \hat{i} . On the other hand, after \hat{i} , the age-increasing dynamics is unchanged.

C.2 The age-dynamics of the reservation productivity after the age \hat{i}

For $i \geq \hat{i}$ we have

$$R_{i+1} - R_i = \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta)}$$

Because e_i is age-decreasing (see Proposition 2), and if $\Upsilon'(x) > 0$, $\forall x \in [e_{T-2}, e_{\hat{i}}[$, i.e. $\lambda > \gamma p(\theta) e_i$, $\forall i > \hat{i}$, then $-\Upsilon(e_{i+1}) + \Upsilon(e_i) > 0$ which ensures that $R_i < R_{i+1}$, $\forall i \geq \hat{i}$.

C.3 The age-dynamics of the reservation productivity before the age \hat{i}

The dynamics of the reservation productivity is given by:

$$\begin{aligned} R_{i+1} - R_i &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta) [1 + \beta(1 - \lambda) P_{i+1}(\hat{i})]} \\ &\quad + \frac{\gamma p(\theta) (1 - \lambda) \beta [P_{i+2}(\hat{i}) (R_{i+2} - R_{i+1}) + (\mathcal{V}_{i+1}(\hat{i}, R_i) - \mathcal{V}_{i+2}(\hat{i}, R_{i+1}))]}{\gamma p(\theta) [1 + \beta(1 - \lambda) P_{i+1}(\hat{i})]} \\ &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta) [1 + \beta(1 - \lambda) P_{i+1}(\hat{i})]} \\ &\quad + \Phi_{1,i}(R_{i+2} - R_{i+1}) + \Phi_{2,i}(\mathcal{V}_{i+1}(\hat{i}, R_i) - \mathcal{V}_{i+2}(\hat{i}, R_{i+1})) \end{aligned} \quad (49)$$

with $\Phi_{j,i} > 0$ for $i = 1, 2$. In the case where the reservation productivities are continuously age-decreasing, a sufficient condition to ensure the existence of this sequence is $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$. In the U-shaped case, it is necessary to solve the continuation values after \widehat{i} , ie. $\mathcal{V}_{i+1}(\widehat{i}, R_i)$ and $\mathcal{V}_{i+2}(\widehat{i}, R_{i+1})$, in order to determine the sufficient condition ensuring that $R_{i+1} < R_i$. Using the definition of $\mathcal{V}_{i+1}(\widehat{i}, R_i)$, we obtain:

$$\begin{aligned} \mathcal{V}_{i+2}(\widehat{i}, R_{i+1}) &= [\beta(1-\lambda)]^{\widehat{i}-i-2} [S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2})] \\ \mathcal{V}_{i+1}(\widehat{i}, R_i) &= [\beta(1-\lambda)]^{\widehat{i}-i-1} [S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})] \\ \Rightarrow \mathcal{V}_{i+1}(\widehat{i}, R_i) - \mathcal{V}_{i+2}(\widehat{i}, R_{i+1}) &= [\beta(1-\lambda)]^{\widehat{i}-i-2} \{[\beta(1-\lambda) - 1](S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})) + \Delta\} \\ \text{with } \Delta &= [S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})] - [S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2})] \end{aligned}$$

where the first term is unambiguously negative, whereas the sign of Δ is a priori indeterminate. Using the definition of $S_i(\epsilon)$ for $i > \widehat{i}$, we have:

$$\begin{aligned} S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2}) &= \sum_{j=0}^{T-\widehat{i}-2} [\beta(1-\lambda)]^j \left(\max\{R_{i+1} - R_{\widehat{i}+j}; 0\} - \max\{R_{i+2} - R_{\widehat{i}+j}; 0\} \right) \\ S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1}) &= \sum_{j=0}^{T-\widehat{i}-1} [\beta(1-\lambda)]^j \left(\max\{R_i - R_{\widehat{i}+j}; 0\} - \max\{R_{i+1} - R_{\widehat{i}+j}; 0\} \right) \end{aligned}$$

There exists some ages $\widehat{i} + p$, $\widehat{i} + n$ and $\widehat{i} + m$ from which the marginal jobs, respectively R_i , R_{i+1} and R_{i+2} , are closed after \widehat{i} . The associated productivity thresholds are such that $\max\{R_i - R_{\widehat{i}+p}; 0\} = 0$, $\max\{R_{i+1} - R_{\widehat{i}+n}; 0\} = 0$ and $\max\{R_{i+2} - R_{\widehat{i}+m}; 0\} = 0$. As $R_i > R_{i+1} > R_{i+2}$, we have $p > n > m$. From the previous expressions, we deduce that:

$$\begin{aligned} S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2}) &= \left(\sum_{j=0}^{\widehat{i}+m-1} [\beta(1-\lambda)]^j \right) (R_{i+1} - R_{i+2}) \\ &\quad + [\beta(1-\lambda)]^m \sum_{j=0}^{n-m} [\beta(1-\lambda)]^j (R_{i+1} - R_{\widehat{i}+m+j}) \\ S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1}) &= \left(\sum_{j=0}^{\widehat{i}+n-1} [\beta(1-\lambda)]^j \right) (R_i - R_{i+1}) \\ &\quad + [\beta(1-\lambda)]^n \sum_{j=0}^{p-n} [\beta(1-\lambda)]^j (R_i - R_{\widehat{i}+n+j}) \end{aligned}$$

The surplus gap between two ages comes from the initial difference of the productivity thresholds, which persists after \widehat{i} , plus the additional advantage of having a longer job duration for the

younger worker. Thus, we deduce that:

$$\begin{aligned} \Delta &= \left(\sum_{j=0}^{\widehat{i}+n-1} [\beta(1-\lambda)]^j \right) (R_i - R_{i+1}) - \left(\sum_{j=0}^{\widehat{i}+m-1} [\beta(1-\lambda)]^j \right) (R_{i+1} - R_{i+2}) \\ &\quad + \underbrace{[\beta(1-\lambda)]^m \left(\sum_{j=0}^{p-n} [\beta(1-\lambda)]^{j+n-m} (R_i - R_{\widehat{i}+n+j}) - \sum_{j=0}^{n-m} [\beta(1-\lambda)]^j (R_{i+1} - R_{\widehat{i}+m+j}) \right)}_{\Omega} \end{aligned}$$

Ω gives the relative value associated with the additional durations of the different jobs ($p-n$ versus $n-m$). If $p-n \equiv \tau \leq n-m$, we have²³:

$$\Omega = \widetilde{\Omega} - [\beta(1-\lambda)]^{m+\tau+1} \sum_{j=0}^{n-m-\tau-1} [\beta(1-\lambda)]^j (R_{i+1} - R_{\widehat{i}+m+\tau+1+j})$$

where the gap between Ω and $\widetilde{\Omega}$ is unambiguously negative, given that $\widetilde{\Omega}$ is defined as follows:

$$\begin{aligned} \widetilde{\Omega} &= [\beta(1-\lambda)]^m \left\{ \left(\sum_{j=0}^{\tau} [\beta(1-\lambda)]^j \right) (R_i - R_{i+1}) - \sum_{j=0}^{\tau} [\beta(1-\lambda)]^j (R_{\widehat{i}+n+j} - R_{\widehat{i}+m+j}) \right. \\ &\quad \left. + [1 - [\beta(1-\lambda)]^{\tau}] \left[-R_i \left(\sum_{j=0}^{\tau} [\beta(1-\lambda)]^j \right) + \sum_{j=0}^{\tau} [\beta(1-\lambda)]^j R_{\widehat{i}+n+j} \right] \right\} \end{aligned}$$

Let us note that the two last terms in the brackets are negative. Finally, the following expression can be deduced:

$$\mathcal{V}_{i+1}(\widehat{i}, R_i) - \mathcal{V}_{i+2}(\widehat{i}, R_{i+1}) = -\Psi_{1,i+1} + \Psi_{2,i+1}(R_i - R_{i+1}) - \Psi_{3,i+1}(R_{i+1} - R_{i+2}) \quad (50)$$

where $\Psi_{j,i+1} > 0$ for $j = 1, 2, 3$. The term $-\Psi_{1,i+1}$ captures all the negative components isolated at each step of the solution computation of $\mathcal{V}_{i+1}(\widehat{i}, R_i) - \mathcal{V}_{i+2}(\widehat{i}, R_{i+1})$. Using equation (50), we then obtain:

$$\begin{aligned} R_{i+1} - R_i &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\left(\gamma p(\theta) [1 + \beta(1-\lambda) P_{i+1}(\widehat{i})] \right) (1 + \Phi_{2,i} \Psi_{2,i+1})} \\ &\quad + \frac{-\Phi_{2,i} \Psi_{1,i+1}}{1 + \Phi_{2,i} \Psi_{2,i+1}} + \frac{\Phi_{1,i} + \Phi_{2,i} \Psi_{3,i+1}}{1 + \Phi_{2,i} \Psi_{2,i+1}} (R_{i+2} - R_{i+1}) \end{aligned}$$

where $\Phi_{x,y} > 0, \forall x, y$. A sufficient condition ensuring that $R_{i+1} - R_i < 0$ is that $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$. If e_i is age-decreasing and $\Upsilon'(x) < 0, \forall x \in [e_{\widehat{i}-1}, e_1]$, i.e. $\lambda < \gamma p(\theta) e_i, \forall i \leq \widehat{i} - 1$, then $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$ and $R_{i+1} - R_i < 0, \forall i < \widehat{i} - 1$, provided that the terminal condition $R_{\widehat{i}-1} > R_{\widehat{i}}$ holds.

²³The condition $p-n \leq n-m$ implies that $R_{\widehat{i}+j+2} - R_{\widehat{i}+j+1} \geq R_{\widehat{i}+j+1} - R_{\widehat{i}+j}, \forall j \geq 0$. Given that $e_i > e_{i+1}$, this implies that $\Upsilon''(e_i) \leq 0$, which is verified for a large class of the function ϕ .

At the age \widehat{i} , the age-dynamics of the reservation productivity changes. Indeed, for $i = \widehat{i} - 1$, we have $\gamma p(\theta)R_{\widehat{i}-1} = \gamma p(\theta)b - \Upsilon(e_{\widehat{i}-1}) - \gamma p(\theta)(1 - \lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})$, and for $i \geq \widehat{i}$, we have $\gamma p(\theta)R_i = \gamma p(\theta)b - \Upsilon(e_i)$. We then deduce that:

$$\begin{aligned} R_{\widehat{i}} - R_{\widehat{i}-1} &= \frac{\Upsilon(e_{\widehat{i}-1}) - \Upsilon(e_{\widehat{i}}) + \gamma p(\theta)(1 - \lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} \\ R_{\widehat{i}+1} - R_{\widehat{i}} &= \frac{\Upsilon(e_{\widehat{i}}) - \Upsilon(e_{\widehat{i}+1})}{\gamma p(\theta)} \end{aligned}$$

The first equation shows that the gap between $R_{\widehat{i}}$ and $R_{\widehat{i}-1}$ depends, through $S_{\widehat{i}}(R_{\widehat{i}-1})$, on the sequence of the future reservation productivities. Nevertheless, note that, if $R_{\widehat{i}+1} > R_{\widehat{i}-1}$, then the solution for $S_{\widehat{i}}(R_{\widehat{i}-1})$ is simply $R_{\widehat{i}-1} - R_{\widehat{i}}$.

Let us consider a first order approximation around $e_i = e_{\widehat{i}}$, leading to $\Upsilon(e_{\widehat{i}}) - \Upsilon(e_{\widehat{i}+1}) \approx \gamma p(\theta)(e_{\widehat{i}+1} - e_{\widehat{i}})^2 \phi''(e_{\widehat{i}})$ and $\Upsilon(e_{\widehat{i}-1}) - \Upsilon(e_{\widehat{i}}) \approx -\gamma p(\theta)(e_{\widehat{i}-1} - e_{\widehat{i}})^2 \phi''(e_{\widehat{i}})$. This implies that:

$$\begin{aligned} R_{\widehat{i}} - R_{\widehat{i}-1} &\approx \frac{-\gamma(e_{\widehat{i}-1} - e_{\widehat{i}})^2 p(\theta) \phi''(e_{\widehat{i}}) + \gamma p(\theta)(1 - \lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} \\ R_{\widehat{i}+1} - R_{\widehat{i}} &\approx \frac{\gamma(e_{\widehat{i}+1} - e_{\widehat{i}})^2 p(\theta) \phi''(e_{\widehat{i}})}{\gamma p(\theta)} \end{aligned}$$

Then, for $e_{\widehat{i}-1} = e_{\widehat{i}} + \delta$ and $e_{\widehat{i}+1} = e_{\widehat{i}} - \delta$, we obtain $R_{\widehat{i}+1} - R_{\widehat{i}-1} \approx \frac{\gamma p(\theta)(1 - \lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} > 0$ and then $S_{\widehat{i}}(R_{\widehat{i}-1}) = R_{\widehat{i}-1} - R_{\widehat{i}}$. Then, we deduce that:

$$R_{\widehat{i}} - R_{\widehat{i}-1} \approx \frac{-\gamma(e_{\widehat{i}-1} - e_{\widehat{i}})^2 p(\theta) \phi''(e_{\widehat{i}})}{\gamma p(\theta)[1 + \gamma p(\theta)\beta(1 - \lambda)]} < 0$$

The terminal condition $R_{\widehat{i}-1} > R_{\widehat{i}}$ holds.

D Solution when the retirement age is endogenous

D.1 After the age T^u

If the productivity thresholds are age-increasing after T^u ($\widetilde{R}_i < \widetilde{R}_{i+1}$), the agents choose between employment and retirement occupations. Then, the surplus is given by

$$\begin{aligned} S_i(\epsilon) &= \max \left\{ \epsilon - b - \psi(i) + \beta \left(\lambda \int_0^1 S_{i+1}(x) dG(x) + (1 - \lambda) S_{i+1}(\epsilon) \right); 0 \right\} \\ &= \max \left\{ \epsilon - \widetilde{R}_i + \beta(1 - \lambda) S_{i+1}(\epsilon); 0 \right\} = \sum_{j=0}^{T^e - i} [\beta(1 - \lambda)]^j \max \left\{ \epsilon - \widetilde{R}_{i+j}; 0 \right\} \end{aligned}$$

After T^u , the decision rule concerning the separation becomes $\mathcal{W}_i(\widetilde{R}_i) = \mathcal{A}_i \Leftrightarrow S_i(\widetilde{R}_i) = 0$. Because the outside opportunity is identical for the planner and for the market, the job surplus at the equilibrium corresponds to its optimal value. Separations are then optimal. It is possible to

show that they are unambiguously age-increasing. The dynamic of the reservation productivity is given by:

$$\tilde{R}_i = b + \psi(i) - \lambda\beta \int_{\tilde{R}_{i+1}}^1 S_{i+1}(x)dG(x) \quad (51)$$

with the terminal conditions $\tilde{R}_{T^e-1} = b + \psi(T^e - 1)$ and $1 = b + \psi(T^e)$. Then, we deduce that the gap between \tilde{R}_i and \tilde{R}_{i+1} evolves as follows:

$$\tilde{R}_{i+1} - \tilde{R}_i = \underbrace{\psi(i+1) - \psi(i)}_+ + \lambda\beta \underbrace{\left[\int_{\tilde{R}_{i+1}}^1 S_{i+1}(x)dG(x) - \int_{\tilde{R}_{i+2}}^1 S_{i+2}(x)dG(x) \right]}_+$$

with the terminal condition $\tilde{R}_{T^e-1} - \tilde{R}_{T^e-2} = \psi(T^e - 1) - \psi(T^e - 2) + \lambda\beta \int_{\tilde{R}_{T^e-1}}^1 S_{T^e-1}(x)dG(x)$. As $\psi(T^e - 1) - \psi(T^e - 2) > 0$, we have $\tilde{R}_{T^e-1} > \tilde{R}_{T^e-2}$ and then $\tilde{R}_{i+1} > \tilde{R}_i$, for $T^u \leq i < T^e - 1$.

D.2 Before age T^u : the case of age-increasing reservation productivity

If $R_i < R_{i+1}$, for $i < T^u$, we have $S_{i+1}(R_i) = 0$ and then, $\forall i \leq T^e$,

$$S_i(\epsilon) = \max \{ \epsilon - R_i + \beta(1 - \lambda)S_{i+1}(\epsilon); 0 \} = \sum_{j=0}^{T^e-i} [\beta(1 - \lambda)]^j \max \{ \epsilon - R_{i+j}; 0 \}$$

For $i < T^u$, the reservation productivity is:

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)]\beta \int_{R_{i+1}}^1 S_{i+1}(x)dG(x)$$

Using the FOC on the search effort, implying $\frac{\phi'(e_i)}{\gamma p(\theta)} = \beta \int_{R_{i+1}}^1 S_{i+1}(x)dG(x)$, we have:

$$\gamma p(\theta)(R_{i+1} - R_i) = \Upsilon(e_i) - \Upsilon(e_{i+1})$$

We have $R_i < R_{i+1}$, implying that e_i is age-decreasing (see proposition 2) if $\Upsilon'(x) > 0$, $\forall x \in [e_{T^u}, e_1]$. As $\Upsilon'(x) = (\lambda - \gamma p(\theta)x)\phi''(x)$, the condition $\lambda > \gamma p(\theta)e_i$, $\forall i \leq T^u$, implies that $R_i < R_{i+1}$. Nevertheless, because $e_i p(\theta) < 1$, a sufficient condition for $R_i < R_{i+1}$ is then $\lambda > \gamma$. By comparison with the case where the retirement age is exogenous and homogenous among workers (see Proposition 3), we obtain the same sufficient condition. Note that, unlike the case with exogenous retirement, the search effort just before the retirement age of the unemployed workers is not zero. Then, the terminal condition, which gives the gap between R_{T^u} and R_{T^u-1} , is the same as the condition giving the gap between R_i and R_{i+1} , $\forall i < T^u$. Finally, using the definition of T^u , we have $R_{T^u} = \tilde{R}_{T^u}$, ensuring that the sequence of productivity reservation is continuous.

D.3 Before age T^u : the case of age-decreasing reservation productivity

If $R_i > R_{i+1}$, the match surplus is for $i < T^u$:

$$\begin{aligned}
S_i(\epsilon) &= \max \left\{ \epsilon - b + \phi(e_i) + \beta \left(\lambda \int_0^1 S_{i+1}(x) dG(x) + (1 - \lambda) S_{i+1}(\epsilon) \right) \right. \\
&\quad \left. - \gamma e_i p(\theta) \beta \int_0^1 S_{i+1}(x) dG(x); 0 \right\} \\
&= \max \left\{ \left(\sum_{j=0}^{T^u-i-1} [\beta(1 - \lambda)]^j \right) (\epsilon - R_i) + [\beta(1 - \lambda)]^{T^u-i} [S_{T^u}(\epsilon) - S_{T^u}(R_{T^u-1})]; 0 \right\} \\
&= P_i(T^u) \max \{ \epsilon - R_i; 0 \} + \mathcal{V}_i(T^u, \epsilon)
\end{aligned}$$

This case with age-decreasing reservation productivity is equivalent to the U-shaped case before \hat{i} , as can be shown by comparing the last equation with equation (48) (Appendix C), when \hat{i} is replaced by T^u . It can be derived that a sufficient condition ensuring that $R_{i+1} - R_i < 0$ is $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$. If e_i is age-decreasing and $\Upsilon'(x) < 0$, $\forall x \in [e_{T^u-1}, e_1]$, i.e. $\lambda < \gamma p(\theta) e_i$, $\forall i \leq T^u - 1$, then $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$, ensuring $R_i > R_{i+1}$, $\forall i < T^u - 1$. As in the case with exogenous retirement age T , $\lambda < \gamma p(\theta) e_i$ is a sufficient condition, and $\lambda < \gamma$ a necessary condition, for $R_i > R_{i+1}$, $\forall i < T^u - 1$, provided that the terminal condition $R_{T^u-1} > R_{T^u}$ holds.

At age T^u , there is a change in the law of motion of the reservation productivity, implying:

$$\begin{aligned}
\tilde{R}_{T^u+1} - \tilde{R}_{T^u} &= \psi(T^u + 1) - \psi(T^u) + \lambda \beta \left[\int_{\tilde{R}_{T^u+1}}^1 S_{T^u+1}(x) dG(x) - \int_{\tilde{R}_{T^u+2}}^1 S_{T^u+2}(x) dG(x) \right] \\
R_{T^u} - R_{T^u-1} &= \frac{-\Upsilon(e_{T^u}) + \Upsilon(e_{T^u-1}) + \gamma p(\theta) (1 - \lambda) \beta S_{T^u}(R_{T^u-1})}{\gamma p(\theta)}
\end{aligned}$$

The optimal retirement age T^u is such that $\psi(T^u) = -\phi(e_{T^u}) + e_{T^u} p(\theta) \gamma \beta \int_{\tilde{R}_{T^u+1}}^1 S_{T^u+1}(x) dG(x)$, implying that

$$\begin{aligned}
\tilde{R}_{T^u+1} - R_{T^u-1} &= \underbrace{\psi(T^u + 1) + \phi(e_{T^u-1}) - e_{T^u-1} p(\theta) \gamma \beta \int_{\tilde{R}_{T^u}}^1 S_{T^u}(x) dG(x)}_{\phi(e_{T^u-1}) - e_{T^u-1} \phi'(e_{T^u-1}) < 0} \\
&\quad - \beta \left[\underbrace{\lambda \int_{\tilde{R}_{T^u+2}}^1 S_{T^u+2}(x) dG(x) - \left(\lambda \int_{\tilde{R}_{T^u}}^1 S_{T^u}(x) dG(x) + (1 - \lambda) S_{T^u}(R_{T^u-1}) \right)}_{\lambda(ES_{T^u+2} - ES_{T^u}) - (1 - \lambda) S_{T^u}(R_{T^u-1}) < 0} \right]
\end{aligned}$$

Under the assumption that the capitalization effect dominates the selection effect, implying $ES_{T^u+2} < ES_{T^u}$, a sufficient condition for $\tilde{R}_{T^u+1} - \tilde{R}_{T^u-1} > 0$ is that $\psi(T^u + 1) > -\phi(e_{T^u-1}) + e_{T^u-1} \phi'(e_{T^u-1})$. Using the definition of the retirement age of the unemployed workers, this restriction is always satisfied if the function $\psi(\cdot)$ is sufficiently increasing, i.e. such that $\psi'(T^u) >$

$e_{T^u} \phi''(e_{T^u})(e_{T^u-1} - e_{T^u})$. Then, under this restriction, we deduce that $S_{T^u}(R_{T^u-1}) = R_{T^u-1} - R_{T^u}$, implying that

$$R_{T^u} - R_{T^u-1} = \frac{-\Upsilon(e_{T^u}) + \Upsilon(e_{T^u-1})}{\gamma p(\theta)[1 + \beta(1 - \lambda)]}$$

Then, if e_i is age-decreasing and $\Upsilon'(x) < 0, \forall x \in [e_{T^u}, e_{T^u-1}]$, ie. $\lambda < \gamma p(\theta) e_i$, for $i = T^u - 1, T^u$, then $-\Upsilon(e_{T^u}) + \Upsilon(e_{T^u-1}) < 0$, ensuring that the terminal condition $R_{T^u-1} > R_{T^u}$ holds. Compared with the case where the retirement age is exogenous (see Proposition 6), this terminal condition leads to a less restrictive condition on λ . Finally, let us notice that the definition of T^u implies that $R_{T^u} = \tilde{R}_{T^u}$, ensuring the continuity of the reservation productivity sequence.

E Heterogeneity in an infinite-life environment

Let us assume that there is productivity heterogeneity across workers ($y(j)$, $j = 1, \dots, N$) in an infinite-life environment along the line of Davis [2001]:

Equilibrium	First best
$S_j = \frac{y(j) - b - \frac{1}{1-\gamma} c \theta \tau_j + c \theta \tau_j}{1 - \beta(1-s)}$	$S_j^* = \frac{y(j) - b - \frac{1}{1-\eta(\theta^*)} c \theta^* \tau_j^* + c \theta^*}{1 - \beta(1-s)}$
$\frac{c}{q(\theta)} = \beta(1-\gamma) \sum_{j=1}^N \left(\frac{u_j}{u} S_j \right)$	$\frac{c}{q(\theta^*)} = \beta[1 - \eta(\theta^*)] \sum_{j=1}^N \left(\frac{u_j^*}{u^*} S_j^* \right)$
$u_j = \frac{s}{s+p(\theta)}$	$u_j^* = \frac{s}{s+p(\theta^*)}$
$\tau_j = \frac{S_j}{\sum_{j=1}^N \left(\frac{u_j}{u} S_j \right)}$	$\tau_j^* = \frac{S_j^*}{\sum_{j=1}^N \left(\frac{u_j^*}{u^*} S_j^* \right)}$

In this case, all jobs have the same expected duration. The condition of equivalence between the equilibrium and the first best allocations becomes: $\sum_{j=1}^N \frac{u_j}{u} \tau_j = 1$, which is obviously verified, given the definition of τ_j .