On the impact of the TFP growth on the employment rate: does training on-the-job matter?

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Abstract

This paper seeks to gain insights on the relationship between growth and unemployment when considering heterogeneous agents in terms of skills. We allow for the possibility of training for unskilled employed workers and for the possibility of human capital depreciation for skilled unemployed workers. These features are introduced in an endogenous job destruction framework *à la* Mortensen and Pissarides (1998). We show that, when growth accelerates, a larger share of unskilled workers gets trained, increasing the incentives of firms to update the job-specific technology, rather than destroying it. The positive impact of growth on the employment rate is then magnified and the predicting ability of the model to reproduce the sensibility of employment with respect to growth too. When calibrated, the model manages to reproduce the aggregate capitalization effect estimated on the basis of OECD data. Furthermore, whereas for skilled and unskilled workers getting trained growth yields a reduction in the unemployment rates, for unskilled workers not getting trained growth fosters a rise in the unemployment rates.

Keywords: TFP growth, unemployment, training, human capital depreciation, capitalization, creative destruction effect

JEL: J23, J24, O33

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1 Introduction

Recent literature on the relationship between growth and employment does not manage to reproduce the elasticity of employment with respect to growth. This paper shows that, by introducing the possibility of training and human capital depreciation in a vintage framework a la Mortensen and Pissarides (1998), we greatly improve the predicting ability of the model to reproduce the sensibility of employment with respect to growth. The intuition behind this result is very simple: when growth accelerates, the opportunity cost of the training investment for workers is lower, shifting the human capital distribution to the right since more people get trained. This tends to increase the incentives of firms to update the job-specific technology. Therefore training magnifies the impact of growth on the employment rate.

The relationship between growth and employment has often been claimed to be ambiguous. Indeed, when growth accelerates, two contradictory effects arise. On the one hand, as Pissarides (1990) claims, an acceleration of growth improves the employment rate, because growth increases "freely" the expected profits and then provides incentives to open new jobs (the capitalization effect). On the other hand, Aghion and Howitt (1994) argue that growth fosters a "creative destruction" process inducing more job destruction and less job creation, yielding higher unemployment rates (creative destruction effect). Even if, theoretically the relationship seems ambiguous at the empirical level, the capitalization effect clearly overcomes the creative destruction effect (see Blanchard and Wolfers (2000), Pissarides and Vallanti (2007) or Tripier (2007)), fostering a positive relationship between growth and employment.

Recent theoretical works on the subject have tried to mimic this empirical finding. Pissarides and Vallanti (2007) present a vintage model a la Mortensen and Pissarides (1998) with a representative agent and underline the difficulty to reproduce the estimated size of the capitalization effect over employment. Using a panel of OCDE countries, Langot and Moreno-Galbis (2008) estimate the impact of growth on the employment rate of young and old workers. They find that the capitalization effect dominates the creative destruction one in the young workers' case whereas for old workers, the creative destruction effect is dominant. Nevertheless, using a standard calibration, they do not manage to reproduce the estimated elasticities.

Previous studies based on Mortensen and Pissarides (1998), seem then to miss an important aspect of the functioning of labor market. In all cases technological progress is embodied, so that all jobs are created at the technological frontier. However, once created, their productivity remains constant. Because wages increase at the same pace as the technological frontier, positions are decreasingly profitable. The firm can then decide to update the technology associated with a job or wait until the job becomes non profitable and destroy it. One of the main drawbacks of this traditional framework is that workers are not allowed to react when their positions loose profitability. A more realistic framework, should allow them to search on the job or to train themselves so as to improve their relative productivity. Michaud (2007) already shows that the predicting ability of the Mortensen and Pissarides (1998) model is considerably improved when introducing on the job search[1]. An alternative approach proposed in our paper consists in introducing human capital investment à la Ljunqvist and Sargent (2008)[2] in a vintage model a la Mortensen and Pissarides (1998).

By introducing the opportunity of training and the risk of human capital depreciation during the unemployment spell, our theoretical framework aims at increasing the probability that the expected profits of the firm net of renovation costs increase with growth (which will favor a capitalization effect). Training choices are highly dependent on the growth rate: an acceleration in growth decreases the opportunity cost of training, leading more workers to train and shifting then the human capital distribution to the right. Finally, note that the possibility of human capital depreciation during unemployment spells of skilled workers as well as the possibility for unskilled workers getting trained to become skilled, yields both skilled workers and unskilled workers getting trained to accept a lower wage. Firms' profits are then increased and the optimal destruction horizon is likely to be delayed, favoring technological updating. Both training and human capital depreciation tend to increase the positive impact of growth on the employment rate. The contribution of this paper consists then in proposing two illustrative mechanisms, training of workers and human capital depreciation, which should magnify the importance of the capitalization effect in a growth context.

We consider a one job-one firm vintage model à la Mortensen and Pissarides (1998) where wages are posted by firms and where we distinguish between the vintage of the machines and the human capital level of workers. Firms decide about the optimal destruction or renovation horizon associated to a machine-position. Workers may be skilled (high-productivity) or unskilled (low-productivity). Unskilled workers must decide if it is in their interest to train themselves. Those

[1]The main problem with this approach comes from the wage bargaining problem. Indeed, Shimer (2005b) shows that the standard solution of the wage bargaining process is not robust to the introduction of the on-the-job search assumption.

[2]Low-productivity workers are allowed to train themselves and skilled unemployed workers may suffer human capital depreciation.

that get trained profit from an increased human capital in case of technological updating. On the other hand, unemployed skilled workers see their human capital depreciated with probability π. We claim, that both mechanisms improve the model's ability to reproduce the impact of growth on employment estimated by Pissarides and Vallanti (2007). According to these authors a one percentage point increase in the growth rate should increase employment by 1.2 to 1.5 percentage points. However, in their paper, Pissarides and Vallanti are obliged to make the unrealistic assumption that all technological progress is disembodied and that $r + \delta = 0.05$ in order to be able to reproduce this semielasticity of growth on the employment rate.
Our numerical simulations show that, when using a standard calibration, an increase of one percentage point in the growth rate, yields a reduction of one percentage point in the aggregate unemployment rate. Moreover, whereas skilled and unskilled workers getting trained benefit from a reduction in their unemployment rates (capitalization effect) by 1.3 percentage points, unskilled workers not getting trained suffer from an increase in their unemployment rates.
Next section presents the main assumptions of the model. Section 3 describes the agents' behavior as well as the wage bargaining process. The equilibrium of the model is computed in section 4. Section 5 presents the numerical simulations and section 6 concludes.

2 The model's assumptions

We build a matching model based on Mortensen and Pissarides (1998) where the economy is populated by a continuum of firms and workers. Each firm employs only one worker. Human capital adopts then two values $h \in [\underline{h}; \overline{h}]$. Low skilled employed workers may decide to train themselves by paying a cost equal to σ, which varies among unskilled workers. Heterogeneity among workers is represented by the distribution $G(\sigma)$, over the support $[\underline{\sigma}, \overline{\sigma}]$. We will determine a critical cost level $\widetilde{\sigma}$ below which unskilled workers decide to train themselves and above which unskilled workers do not get trained. On the other hand, unemployed skilled workers may become unskilled with probability π. We will denote $X(\overline{h}, \sigma)$ all variables referring to skilled workers and $X(\underline{h}, \sigma)$ those referring to unskilled.
The productivity level of each worker is linked to their qualification, so that high skilled workers' productivity is larger than that of low skilled. We distinguish between the human capital level of the worker and the vintage of the technology she works with. Firms decide whether to update or not the vintage of the technology, whereas workers decide whether to train themselves or not

(this only applies to unskilled workers).

At each moment of time a mass $u(h,\sigma)$ for $h = \overline{h}, \underline{h}$ of unemployed workers and a mass $v(h)$ for $h = \overline{h}, \underline{h}$ of vacant jobs coexist on the labor market. Aggregate unemployment u is defined by $u = \sum_{h=\{\overline{h},\underline{h}\}} \int u(h,\sigma)dG(\sigma)$. Firms do not direct their open vacancy to a particular skill segment. Jobs and workers meet pairwise at a Poisson rate $M(u, v(h,\sigma))$. This function is assumed to be strictly increasing and concave, exhibiting constant returns to scale. Furthermore it satisfies the Inada conditions and $M(0, v(h,\sigma)) = M(u, 0) = 0$.

Under these assumptions and knowing that $M(u, v(h,\sigma))$ represents the number of matches per unit of time, we can represent the probability of filling a vacancy as $q(\theta(h,\sigma)) = M(u, v(h,\sigma))/v(h,\sigma) = M(u, v(h))/\theta(h)$. Equivalently, the probability of finding a job is given by $p(\theta(h,\sigma)) = M(u, v(h,\sigma))/u = M(\theta(h,\sigma), 1)$. The probability of filling a vacancy with a high skilled worker will be given by $q(\theta(h,\sigma))\frac{u(\overline{h},\sigma)}{u}$, with an unskilled not getting trained by $q(\theta(h,\sigma))\frac{u(\underline{h},\sigma>\tilde{\sigma})}{u}$ and with an unskilled getting trained by $q(\theta(h,\sigma))\frac{u(\underline{h},\sigma<\tilde{\sigma})}{u}$.

New jobs embody the most advanced known technology (the latest vintage). However, once created, their productivity remains constant for the rest of their life. Job creation commits the firm to the technology available at that date. A firm without a worker advertises a job vacancy at a cost $p(t)c$ per period, where $p(t) = e^{gt}$ is a common growth factor and g is the rate of productivity growth at the technological frontier (creation costs must grow at rate g to ensure the existence of a steady state with balanced growth). Across newly created jobs, match productivity thus grows at the exogenous rate $g = \dot{p}(t)/p(t)$ (new jobs always embody the most advanced known technology). Once the job is created at date τ its associated technology, $p(\tau)x(h)$ for $h = \overline{h}, \underline{h}$ does not change. The worker's productivity $x(h)$ may be modified in the unskilled workers' case if the worker decides to train herself. The opportunity cost of unemployment is represented by $p(t)b(h)$ for $h = \overline{h}, \underline{h}$. Because the outside option of employment increases in response to growth whereas the job's productivity remains constant (even in the unskilled worker's case, once their training is over and they have become skilled, their productivity does not longer change), the surplus associated with a match is decreasing over time.

Once the job is created two situations may arise. First, the firm can continue to produce with the same technology embodied in the job at the creation date. Secondly, the firm may decide to pay a fixed renovation cost to update the technology and continue producing with the same worker. Note though, that when employing an unskilled worker getting trained, a technological updating is also associated with a human capital increase.

The firm chooses optimally the scrapping and the renovation horizon associated with a given position. We denote as $T^R(\overline{h})$, $T^R(\underline{h},\sigma)$ and $T^R(\underline{h}^n)$ the optimal implementation horizon for skilled workers, unskilled workers getting trained and unskilled workers not getting trained, respectively. Note that training introduces an heterogeneity among labor market earnings of workers and then between the optimal renovation horizons. Similarly, we denote as $T(\overline{h})$, $T(\underline{h})$ and $T(\underline{h}^n)$ the corresponding scrapping horizon for the same workers, where $T(\underline{h}) = T(\underline{h}^n)$ since, in the absence of renovation, there is no training.

If the optimal scrapping time is above the optimal implementation time, the firm decides to update the technology rather than destroying it. Otherwise, if the scrapping horizon is below the renovation horizon, firms choose to destroy the job. To keep our representation as close as possible to Mortensen and Pissarides (1998) we will also assume that jobs might be destroyed by an exogenous shock with probability δ.

3 The agents' behavior

An open vacancy can remain empty or be filled and become productive. The associated asset value for each of these situations is represented by $V(t)$ if the vacancy is empty at the current date t. $J(\tau,t,h,\sigma)$ for $h=\overline{h},\underline{h}$ stands for the value of an existing job at date t which was created at time τ. Similarly, the value of employment in a job at date t which was created at time τ is represented by $W(\tau,t,h,\sigma)$ for $h=\overline{h},\underline{h}$, whereas the value of unemployment at date t is given by $U(t,h,\sigma)$. We consider the case where the optimal renovation time associated with a job occupied by a high skilled worker is below the optimal renovation time of a job occupied by a low-skilled worker.

3.1 Workers

The machine's or technology characteristics are summarized by the actual creation time, τ, and the current time, t. The worker's characteristics are summarized by the vector $\{h,\sigma\}$, where $h=\overline{h},\underline{h}$ stands for the human capital level and σ corresponds to the cost borne by a worker that wants to become skilled. Because σ varies across individuals, we may also interpret it as the inherent ability of each individual.

3.1.1 Employed workers

The skilled worker does not train, since she already has the top human capital level[3].

The asset value of an employed skilled worker:

$$rW(\tau,t,\overline{h},\sigma) = w^s(\tau,t,\overline{h},\sigma)\overline{h} - \delta(W(\tau,t,\overline{h},\sigma) - U(t,\overline{h},\sigma)) + \dot{W}(\tau,t,\overline{h},\sigma) \tag{1}$$

where $w^s(\tau,t,\overline{h},\sigma)$ stands for the wage earned at date t by a skilled worker operating a machine created at τ and δ corresponds to the exogenous job destruction rate.

For the unskilled worker, the situation is somewhat different. In order to improve her human capital (become skilled) in case of technological updating, the worker can decide to get trained. Depending on her inherent ability, this training may become more or less expensive. We show that there exist a threshold value $\widetilde{\sigma}$ such that if $\sigma < \widetilde{\sigma}$ the cost of training is lower than the expected returns of getting trained, so that the worker trains. Conversely, for very low ability levels, the training cost is too high ($\sigma > \widetilde{\sigma}$) and it is not in the interest of the worker to get trained.

We first consider the asset value associated with an unskilled employed worker getting trained ($\sigma < \widetilde{\sigma}$). We assume that the worker will only be able to benefit from an increase in his human capital if the firm renovates the technology, that is, after $T^R(\overline{h})$:

$$\begin{aligned} W(\underline{h},\sigma) &= \int_0^{T^{R(\underline{h})}} e^{-(r+\delta-g)s}[w(\underline{h},\sigma) - \sigma + \delta U(\underline{h},\sigma)]ds + e^{-(r+\delta-g)T^{R(\underline{h})}} W(\overline{h},\sigma) \\ &= \left(\frac{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}{r+\delta-g}\right)[w(\underline{h},\sigma) - \sigma + \delta U(\underline{h},\sigma)] + e^{-(r+\delta-g)T^{R(\underline{h})}} W(\overline{h},\sigma) \\ &= \frac{w(\underline{h},\sigma) - \sigma + \delta U(\underline{h},\sigma)}{r+\delta-g} + \frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}[W(\overline{h},\sigma) - W(\underline{h},\sigma)] \end{aligned} \tag{2}$$

where $\tau = t = 0$, $W(t,t,\underline{h},\sigma) \equiv p(t)W(\underline{h},\sigma)$, $W(t,t,\overline{h},\sigma) \equiv p(t)W(\overline{h},\sigma)$ and $U(t,\underline{h},\sigma) \equiv p(t)U(\underline{h},\sigma)$.

The worker decides to train herself if the expected gain from training compensates the cost:

$$\begin{aligned} \tilde{\sigma}\int_0^{T^{R(\underline{h})}} e^{-(r+\delta-g)s}ds &= e^{-(r+\delta-g)T^{R(\underline{h})}}[W(\overline{h},\sigma) - W(\underline{h},\sigma)] \\ \Leftrightarrow \frac{\tilde{\sigma}}{r+\delta-g} &= \frac{e^{-(r+\delta-g)T^R(\underline{h})}}{1-e^{-(r+\delta-g)T^R(\underline{h})}}[W(\overline{h},\sigma) - W(\underline{h},\sigma)] \end{aligned} \tag{3}$$

When the inherent ability of the worker is so low that the training cost σ overcomes $\widetilde{\sigma}$ the unskilled worker does not get trained. In this case, the asset value of the unskilled worker

[3]Even if the cost of training, σ, does not play any role, we leave it in the notation because any skilled worker can loose her human capital during an unemployment episode and become unskilled.

equals:

$$W(\underline{h},\sigma) = \int_0^{T^{R(\underline{h}^n)}} e^{-(r+\delta-g)s}[w(\underline{h},\sigma)+\delta U(\underline{h},\sigma)]ds + e^{-(r+\delta-g)T^{R(\underline{h}^n)}} W(\underline{h},\sigma) \quad (4)$$

$$= \left(\frac{1-e^{-(r+\delta-g)T^{R(\underline{h}^n)}}}{r+\delta-g}\right)[w(\underline{h},\sigma)+\delta U(\underline{h},\sigma)] + e^{-(r+\delta-g)T^{R(\underline{h}^n)}} W(\underline{h},\sigma) \quad (5)$$

$$= \frac{w(\underline{h},\sigma)+\delta U(\underline{h},\sigma)}{r+\delta-g} \quad (6)$$

3.1.2 Unemployed workers

An unemployed worker receives a flow of earnings[4] $p(t)bh$ for $h = \overline{h}, \underline{h}$ including unemployment benefits, leisure, domestic productivity, etc... and increasing with the technology frontier. A skilled job seeker comes into contact with a vacant slot at rate $p(\theta) = \theta q(\theta)$ and becomes unskilled with probability π.

The associated asset value is given by:

$$rU(t,\overline{h},\sigma) = p(t)b\overline{h} + \theta q(\theta)[W(t,t,\overline{h},\sigma) - U(t,\overline{h},\sigma)]$$
$$-\pi[U(t,\overline{h},\sigma) - U(t,\underline{h},\sigma)] + \dot{U}(t,\overline{h},\sigma) \quad (7)$$

Unskilled job seekers enter into contact with a vacancy at rate $p(\theta) = \theta q(\theta)$. The asset value of unemployment associated to unskilled workers is given by:

$$rU(t,\underline{h},\sigma) = p(t)b\underline{h} + \theta q(\theta)[W(t,t,\underline{h},\sigma) - U(t,\underline{h},\sigma)]$$
$$+\dot{U}(t,\underline{h},\sigma) \quad (8)$$

3.2 The wages

In the aim of simplicity the analysis presented along the lines of this paper considers the wage posting case[5], that is, workers have a bargaining power equal to zero and so, the asset value of unemployment equals the asset value of employment ($W(\tau,t,h,\sigma) = U(t,h,\sigma)$). Furthermore, according to Pissarides and Vallanti (2007), the rigid wage representation improves the elasticity of the employment rate with respect to the TFP growth rate.

[4]We impose unemployment benefits to be increasing in human capital so as to ensure a positive training effort even under the assumption of wage posting.

[5]The alternative hypothesis of bargained wages in the presence of interactions between the skilled and the unskilled segment implies that negotiated wages depend on the relationship between the labor market tightness of the two segments, which rends the analytical resolution of the model too complex.

The wage offered by firms to skilled workers is given by:

$$w(\tau, t, \overline{h}, \sigma)\overline{h} = p(t)\left[b\overline{h} - \pi\frac{b\overline{h} - b\underline{h}}{r + \pi - g}\right] \quad \forall\sigma \quad \Rightarrow w(\tau, t, \overline{h}, \sigma)\overline{h} = w(\tau, t, \overline{h})\overline{h} \tag{9}$$

This wage does not depend on the specific worker ability σ since it is independent from the worker's productivity. This implies that the wage earned in the skilled segment is the same for all workers.

A skilled worker is ready to accept a wage lower than her own unemployment benefit since she interiorizes the fact that if she becomes unemployed her human capital may depreciate and her unemployment benefit may then fall. Skilled wages decrease then with the probability π of human capital depreciation in case of unemployment. The higher the divergence between the unemployment benefit perceived by skilled and unskilled unemployed workers or, equivalently, the higher the divergence between human capital levels, the lower the wage skilled workers are ready to accept, since in case of unemployment the risk of becoming unskilled includes also the risk of suffering a sharp decrease in unemployment benefits.

Unskilled workers getting trained become skilled when the firm updates a technology. In the wage posting case we deduce, using the asset value of an employed worker that:

$$U(\underline{h}, \sigma) = \frac{w(\underline{h}, \sigma) - \sigma + \delta U(\underline{h}, \sigma)}{r + \delta - g} + \frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1 - e^{-(r+\delta-g)T^{R(\underline{h})}}}[U(\overline{h}, \sigma) - U(\underline{h}, \sigma)] \tag{10}$$

The asset value of unemployed workers equals:

$$U(\underline{h}, \sigma) = \frac{b\underline{h}}{r - g} \qquad U(\overline{h}, \sigma) = \frac{b\overline{h} + \pi U(\underline{h}, \sigma)}{r + \pi - g}$$

which implies:

$$U(\overline{h}, \sigma) - U(\underline{h}, \sigma) = \frac{b(\overline{h} - \underline{h})}{r + \pi - g}$$

Replacing these expressions in (10), we deduce the reservation wage associated to unskilled getting trained:

$$w(\tau, t, \underline{h}, \sigma) = p(t)b\underline{h} + p(t)\sigma - (r + \delta - g)\left(\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1 - e^{-(r+\delta-g)T^{R(\underline{h})}}}\right)\frac{p(t)b(\overline{h} - \underline{h})}{r + \pi - g} \tag{11}$$

The wage an unskilled worker is ready to accept increases with his own unemployment benefit as well as with the cost of getting trained in order to become skilled (the worker transfers this cost to the firm during the bargaining process). The larger the differential between the skilled and the unskilled unemployment benefits, the lower the wage the worker will be ready to accept since if she gets a job she will be able to train herself and become skilled. Then, in case of loosing the job she will get a larger unemployment benefit. On the other hand, if the probability of human capital depreciation π is high, the interest of being skilled is reduced and therefore the unskilled worker asks for a higher wage.

The wage[6] earned by unskilled workers getting trained depends on σ, implying that there is a continuum of wages varying with σ. The instantaneous profit obtained by the firm varies hence with the σ associated with the worker they hire.

When the unskilled worker does not get trained the wage equals:

$$w(\tau, t, \underline{h}, \sigma)\underline{h} \quad = \quad p(t)b\underline{h} \quad \forall\sigma \quad \Rightarrow w(\tau, t, \underline{h}, \sigma)\underline{h} = w(\tau, t, \underline{h})\underline{h} \tag{12}$$

To keep a coherent notation, we leave σ in the previous expression. Note though that, for workers not getting trained, i.e: having a $\sigma > \widetilde{\sigma}$, the training cost does not affect wages and so the labor market is not segmented: productivity and wages are homogenous.

3.3 The decision to train

The training effort is positive if the training cost is lower or equal to the returns from training. In the wage posting case $W(\tau, t, \overline{h}, \sigma) = U(t, \overline{h}, \sigma)$ and $W(\tau, t, \underline{h}, \sigma) = U(t, \underline{h}, \sigma)$, using equation (3) we find:

$$\begin{aligned} \tilde{\sigma} \int_0^{T^{R(\underline{h})}} e^{-(r+\delta-g)s} ds &= e^{-(r+\delta-g)T^{R(\underline{h})}} [W(\overline{h}, \sigma) - W(\underline{h}, \sigma)] \\ \tilde{\sigma} &= \frac{e^{-(r+\delta-g)T^R(\underline{h})}}{1 - e^{-(r+\delta-g)T^R(\underline{h})}} \frac{r+\delta-g}{r+\pi-g} b(\overline{h} - \underline{h}) \end{aligned} \tag{13}$$

As far as $\sigma \leq \tilde{\sigma}$, it is always in the interest of the worker to get trained. If $\sigma > \tilde{\sigma}$, the worker never gets trained. If she ever got trained she would ask for a wage above b, then no firm will hire her. Firms prefer to hire higher ability workers that ask for a lower wage.

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[6] In order to insure the positivity of wages for all workers having a training cost below $\tilde{\sigma}$ some constraints must be imposed on the parameters. Let's set wages to zero in expression (11):

$$b\underline{h} + \sigma > (r + \delta - g) \quad \frac{e^{-(r+\delta-g)T^R(\underline{h},\sigma)}}{1 - e^{-(r+\delta-g)T^R(\underline{h},\sigma)}} \Bigg) \frac{b(\overline{h} - \underline{h})}{r + \pi - g}$$

Then, there exist a lower value of σ, denoted $\underline{\sigma}$ such that

$$\underline{\sigma} = -b\underline{h} + \frac{r+\delta-g}{r+\pi-g} \quad \frac{e^{-(r+\delta-g)T^R(\underline{h},\underline{\sigma})}}{1 - e^{-(r+\delta-g)T^R(\underline{h},\underline{\sigma})}} \Bigg) b(\overline{h} - \underline{h})$$

The adopted calibration in the numerical simulations will be such that the minimum σ of the distribution will be above $\underline{\sigma}$.

We suppose a continuum of σ with a distribution function $G(\sigma)$. Given the critical $\widetilde{\sigma}$, the mass of trained workers is then given by $G(\widetilde{\sigma})$, whereas the number of unskilled worker who do not choose to train is given by $1 - G(\widetilde{\sigma})$. We will then have a group of workers that gets trained and a group of workers that does not train. Congestion problems guarantee that there will be posted vacancies for all ability levels.

3.4 Firms

3.4.1 Vacancies

When the firm opens a vacancy at date t it bears a cost $p(t)c$, whatever the type of worker, skilled or unskilled, required to fill the vacancy. On the other hand, there is a probability $q(\theta(h,\sigma))\frac{u(\overline{h},\sigma)}{u}$ that the vacancy gets filled with a σ-type skilled worker, $q(\theta(h,\sigma))\frac{u(\underline{h},\sigma>\tilde{\sigma})}{u}$ with a σ-type unskilled not getting trained and $q(\theta(h,\sigma))\frac{u(\underline{h},\sigma<\tilde{\sigma})}{u}$ with a σ-type unskilled getting trained.

We suppose then undirected search, that is, when the firm posts a vacancy it does not know what type of worker will fill the vacancy. Once the contact takes place, the firm will be able to recognize the type of worker.

The asset value of an empty vacancy is given by:

$$\begin{aligned} rV(t) &= -p(t)c + \dot{V}(t) + q(\theta)\left[\int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(t,t,\underline{h},\sigma)dG(\sigma)\right. \\ &\quad \left. + \int_{\tilde{\sigma}}^{\overline{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(t,t,\underline{h},\sigma)dG(\sigma) + \int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\overline{h},\sigma)}{u} J(t,t,\overline{h},\sigma)dG(\sigma)\right] \end{aligned} \tag{14}$$

Because firms open vacancies until all rents are exhausted, we obtain the following free entry condition:

$$\begin{aligned} \frac{p(t)c}{q(\theta)} &= \int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(t,t,\underline{h},\sigma)dG(\sigma) + \int_{\tilde{\sigma}}^{\overline{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(t,t,\underline{h},\sigma)dG(\sigma) \\ &\quad + \int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\overline{h},\sigma)}{u} J(t,t,\overline{h},\sigma)dG(\sigma) \equiv \overline{J} \end{aligned} \tag{15}$$

The asset value at date t of a job filled at date τ varies depending on whether the job is filled by a skilled or an unskilled worker. In the former case, if at date τ the worker filling the vacancy is an unskilled with $\sigma < \widetilde{\sigma}$, the firm interiorizes the fact that when updating the technology the worker will also benefit from a human capital increase. Furthermore, wages paid by the firm vary depending on the type of worker, since not all of them bear a training cost.

3.4.2 Jobs occupied by skilled workers

Let us start with the expected flow of profits when the vacancy is filled by a skilled worker. The firm chooses the optimal updating time so as to maximize the expected flow of profits taking into account the fact that the job may be destroyed with probability δ:

$$\begin{aligned} J(\tau,t,\overline{h},\sigma) &= \max_{T^{R(\overline{h})}} \int_t^{\tau+T^{R(\overline{h})}} e^{-(r+\delta)(s-t)}[p(\tau)x(\sigma)\overline{h} - w^s(\tau,s,\overline{h},\sigma)\overline{h}]ds \\ & \quad + e^{-(r+\delta)(\tau+T^{R(\overline{h})}-t)}[J(\tau+T^{R(\overline{h})},\tau+T^{R(\overline{h})},\overline{h}) - p(\tau+T^{R(\overline{h})})I(\overline{h},\sigma)] \quad (16) \end{aligned}$$

We assume that the productivity of a skilled worker depends inversely on his training effort σ (which is actually linked to the worker's ability) implying that $x = x(\sigma)$ with $x'(\sigma) < 0$. The updating time varies then with σ.

> **Property 1** *The equilibrium optimal renovation date of a job occupied by a skilled worker depends on the worker's training effort σ via the productivity $x(\sigma)$ and via the implementation cost, which is exogenously fixed. The renovation horizon increases with the updating cost (I) and decreases with the growth rate of technical progress (g).*

Proof: Given a stationary time path for future labor market tightness, we conjecture that the value of a new job is proportional to productivity on the technology frontier, i.e. $J(t,t,\overline{h},\sigma) = p(t)J(\overline{h},\sigma)$. Replacing in $J(t,t,\overline{h},\sigma)$ when $\tau = t = 0$ confirms this conjecture and yields our "job renovation rule":

$$\begin{aligned} J(\overline{h},\sigma) &= \max_{T^{R(\overline{h})}} \int_0^{T^{R(\overline{h})}} e^{-(r+\delta)s}\left[x(\sigma)\overline{h} - e^{gs}w(\overline{h})\right] ds \\ & \quad + e^{-(r+\delta-g)(T^{R(\overline{h})})}[J(\overline{h},\sigma) - I(\overline{h},\sigma)] \end{aligned} \quad (17)$$

The optimal renovation time is determined by maximizing (17), which leads to the FOC:

$$w(\overline{h}) = \frac{x(\sigma)\overline{h}}{e^{gT^{R(\overline{h})}}} - (r+\delta-g)(J(\overline{h},\sigma) - I(\overline{h},\sigma)) \quad (18)$$

Replacing $w(\overline{h},\sigma) = [b\overline{h} - \pi\frac{(b\overline{h}-b\underline{h})}{r+\pi-g}]$ in (17) yields:

$$I(\overline{h},\sigma) = x(\sigma)\overline{h}\int_0^{T^{R(\overline{h})}} e^{-(r+\delta)s}(1 - e^{g(s-T^{R(\overline{h})})})ds \quad (19)$$

■

Equation (19) determines $T^{R(\overline{h})}$, which can then be replaced in (17) so as to find the optimal value of $J(\overline{h}, \sigma)$.

In the aim of simplicity, we assume that renovation is always the preferred option for jobs occupied by skilled workers. This is guaranteed if $I(\overline{h}, \sigma) \to 0$. In this case, equation (19) implies that jobs are continuously updated since $T^{R(\overline{h})} \to 0$. The value function of a job occupied by a skilled-worker becomes then:

$$J(\overline{h}, \sigma) = \frac{x(\sigma)\overline{h} - w(\overline{h})}{r + \delta - g}$$

where the impact of an acceleration in growth is unambiguously positive (capitalization effect).

3.4.3 Job occupied by an unskilled worker getting trained

Similarly, the firm chooses the optimal updating time so as to maximize the expected flow of profits associated to the job filled by an unskilled worker getting trained:

$$\begin{aligned} J(\tau, t, \underline{h}, \sigma) &= \max_{T^{R(\underline{h},\sigma)}} \int_t^{\tau + T^{R(\underline{h})}} e^{-(r+\delta)(s-t)} [p(\tau) x \underline{h} - w^t(\tau, s, \underline{h}, \sigma)\underline{h}] ds \\ &\quad + e^{-(r+\delta)(\tau + T^{R(\underline{h})} - t)} [J(\tau + T^{R(\underline{h})}, \tau + T^{R(\underline{h})}, \overline{h}, \sigma) - p(\tau + T^{R(\underline{h})}) I(\underline{h}, \sigma)] \end{aligned} \quad (20)$$

In this case, the wage is a function of the specific worker ability (σ). We then have a σ-specific optimal updating time.

> **Property 2a** *The equilibrium optimal renovation date of a job occupied by an unskilled worker getting trained depends on his training effort σ via the wage, via the potential productivity that will be obtained in the skilled position and via the implementation cost, which is exogenously given. The optimal renovation time increases with the updating cost (I), decreases with the growth rate of technical progress (g) and may increase or decrease with the training cost*[7]*, (σ).*

Proof: Given a stationary time path for future labor market tightness, we conjecture $J(t, t, \underline{h}, \sigma) = p(t)J(\underline{h}, \sigma)$. For $t = \tau = 0$, we obtain

$$\begin{aligned} J(\underline{h}, \sigma) &= \max_{T^R(\underline{h})} \left\{ \int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s} [x\underline{h} - e^{gs} w(\underline{h}, \sigma)\underline{h})] ds \right. \\ &\quad \left. + e^{-(r+\delta-g)T^R(\underline{h})} [J(\overline{h}, \sigma) - I(\underline{h}, \sigma)] \right\} \end{aligned} \quad (21)$$

The FOC associated with the optimal renovation date is

$$w(\underline{h}, \sigma) = \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}} - (r + \delta - g)(J(\overline{h}, \sigma) - I(\underline{h}, \sigma)) \quad (22)$$

[7] The adopted calibration ensures that the renovation horizon increases with the training cost.

3.4.4 Job occupied by an unskilled worker not getting trained

If the job is filled by an unskilled worker not getting trained the expected flow of profits maximized by the firm is given by:

$$\begin{aligned} J(\tau, t, \underline{h}, \sigma) &= \max_{T^{R(\underline{h}^n)}} \int_t^{\tau+T^{R(\underline{h}^n)}} e^{-(r+\delta)(s-t)}[p(\tau)x\underline{h} - w^n(\tau, s, \underline{h}, \sigma)\underline{h}]ds \\ &\quad + e^{-(r+\delta)(\tau+T^{R(\underline{h}^n)}-t)}[J(\tau+T^{R(\underline{h}^n)}, \tau+T^{R(\underline{h}^n)}, \underline{h}, \sigma) - p(\tau+T^{R(\underline{h}^n)})I(\underline{h}, \sigma)] \end{aligned} \tag{23}$$

The optimal updating time is homogenous among this type of workers because neither their wages nor their productivity are σ-specific.

Property 2b *The equilibrium optimal renovation date of a job occupied by an unskilled worker not getting trained depends on his training effort only via the implementation cost, which is exogenously given. The optimal renovation time increases with the updating cost (I) and decreases with the growth rate of technical progress (g).*

Proof: Again, given a stationary time path for future labor market tightness, we conjecture that the value of a new job is proportional to productivity on the technology frontier, i.e. $J(t, t, \underline{h}, \sigma) = p(t)J(\underline{h}, \sigma)$. Replacing in (23) when $\tau = t = 0$ leads to:

$$\begin{aligned} J(\underline{h}, \sigma) &= \max_{T^{R(\underline{h}^n)}} \int_0^{T^{R(\underline{h}^n)}} e^{-(r+\delta)s}[x\underline{h} - e^{gs}b\underline{h}]ds \\ &\quad + e^{-(r+\delta-g)T^{R(\underline{h}^n)}}[J(\underline{h}, \sigma) - I(\underline{h}, \sigma)] \end{aligned} \tag{24}$$

The FOC is given by:

$$w(\underline{h}) = \frac{x\underline{h}}{e^{gT^{R(\underline{h}^n)}}} - (r+\delta-g)(J(\underline{h}, \sigma) - I(\underline{h}, \sigma)) \tag{25}$$

Replacing in (24) yields:

$$I(\underline{h}, \sigma) = x\underline{h} \int_0^{T^{R(\underline{h}^n)}} e^{-(r+\delta)s}(1 - e^{g(s-T^{R(\underline{h}^n)})}) \tag{26}$$

■

4 Equilibrium

4.1 Definition of the equilibrium

Definition 1 *The equilibrium of the economy is characterized by:*

$$
\begin{aligned}
& \textbf{Segmentation} \\
\widetilde{\sigma} &= (r+\delta-g)\left(\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}\right)\frac{b(\overline{h}-\underline{h})}{r+\pi-g} && (27)\\
& \textbf{Skilled-labor segment} \\
J(\overline{h},\sigma) &= \frac{x(\sigma)\overline{h}-b\overline{h}}{r+\delta-g} && (28)\\
w(\tau,t,\overline{h},\sigma)\overline{h} &= p(t)\left[b\overline{h}-\pi\frac{b\overline{h}-b\underline{h}}{r+\pi-g}\right] && (29)\\
& \textbf{Unskilled labor segment with training: for } \sigma\leq\widetilde{\sigma}\textbf{:} \\
w(\underline{h},\sigma) &= \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}}-(r+\delta-g)(J(\overline{h},\sigma)-I(\underline{h},\sigma)) && (30)\\
w(\underline{h},\sigma) &= b\underline{h}+\sigma-(r+\delta-g)\left(\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}\right)\frac{b(\overline{h}-\underline{h})}{r+\pi-g} && (31)\\
J(\underline{h},\sigma) &= \int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s}[x\underline{h}-e^{gs}w(\underline{h},\sigma)]ds+e^{-(r+\delta-g)T^{R(\underline{h})}}[J(\overline{h},\sigma)-I(\underline{h},\sigma)] && (32)\\
& \textbf{Unskilled labor segment without training: } \forall\sigma>\widetilde{\sigma}\textbf{:} \\
w(\underline{h}) &= \frac{x\underline{h}}{e^{gT^{R(\underline{h}^n)}}}-(r+\delta-g)(J(\underline{h})-I(\underline{h},\sigma)) && (33)\\
w(\underline{h}) &= b\underline{h} && (34)\\
J(\underline{h}) &= \int_0^{T^{R(\underline{h}^n)}} e^{-(r+\delta)s}[x\underline{h}-e^{gs}w(\underline{h})]ds+e^{-(r+\delta-g)T^{R(\underline{h}^n)}}[J(\underline{h})-I(\underline{h},\sigma)] && (35)\\
& \textbf{Free entry condition} \\
\frac{c}{q(\theta)} &= \int\left[\frac{u(\overline{h},\sigma)}{u}J(\overline{h},\sigma)+\frac{u(\underline{h},\sigma)}{u}J(\underline{h},\sigma)\right]dG(\sigma) && (36)
\end{aligned}
$$

4.2 The interaction between renovation, training decisions and growth

We start this section analyzing the interaction between the training decision (a worker's choice) and the renovation decision (a firm's choice). We then focus on the behavior of the marginal unskilled worker getting trained, the $\widetilde{\sigma}$-type worker.

> **Proposition 2:** There is a unique pair $(T^{R(\underline{h})},\widetilde{\sigma})$ satisfying simultaneously the training choices and the optimal renovation date.

Proof: The link between the renovation date and the training effort is given by the following relations:

$$\begin{aligned} b\underline{h} &= \frac{x\underline{h}}{e^{gT^{R(\underline{h},\tilde{\sigma})}}} - (r+\delta-g)(J(\overline{h},\widetilde{\sigma}) - I(\underline{h},\widetilde{\sigma})) \\ \widetilde{\sigma} &= (r+\delta-g)\left(\frac{e^{-(r+\delta-g)T^{R(\underline{h},\tilde{\sigma})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h},\tilde{\sigma})}}}\right)\frac{b(\overline{h}-\underline{h})}{r+\pi-g} \end{aligned}$$

In the first equation, the left hand-side corresponds to the wage of the $\widetilde{\sigma}$-worker, whereas the right hand-side corresponds to the optimal renovation date. The higher the value of $\tilde{\sigma}$ the lower the productivity if the worker becomes skilled (since $x'(\sigma) < 0$) and then the lower the value of the filled vacancy, $J(\overline{h},\widetilde{\sigma})$. This yields a higher renovation horizon. That is, the right hand side establishes a positive relationship between $\tilde{\sigma}$ and $T^{R(\underline{h})}$.
The second equation shows that there is a negative link between $T^{R(\underline{h})}$ and $\widetilde{\sigma}$ (since $\frac{\partial}{\partial T^{R(\underline{h},\tilde{\sigma})}}\left(\frac{e^{-(r+\delta-g)T^{R(\underline{h},\tilde{\sigma})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h},\tilde{\sigma})}}}\right) <$ 0). Then, we deduce that there is a unique intersection point $(T^{R(\underline{h})},\widetilde{\sigma})$ satisfying simultaneously these two equations ■
With $\tilde{\sigma}$ determined, the cost level σ of workers getting trained is not longer an unknown, since it is in the interest of all workers with a σ included in the interval $[0,\tilde{\sigma}]$ to get trained, where σ is supposed to be distributed between 0 and $\overline{\sigma}$. Then, it is enough to replace (11) in (22) in order to find the optimal renovation horizon for each σ.

4.2.1 The relationship between the renovation horizon and training decision, $(T^{R(\underline{h})},\tilde{\sigma})$

The optimal renovation horizon for firms varies depending on whether the unskilled worker gets trained or not. Does training advance or delay renovation? Let's compare the two FOC allowing to compute the renovation horizons for the last unskilled worker getting trained ($T^{R(\underline{h},\tilde{\sigma})}$) and for unskilled workers not getting trained ($T^{R(\underline{h^n})}$):

$$\begin{aligned} b\underline{h} &= \frac{x\underline{h}}{e^{gT^{R(\underline{h^n})}}} - (r+\delta-g)(J(\underline{h},\sigma) - I(\underline{h},\sigma)) \text{ for } \sigma > \tilde{\sigma} \\ b\underline{h} &= \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}} - (r+\delta-g)(J(\overline{h},\widetilde{\sigma}) - I(\underline{h},\widetilde{\sigma})) \text{ for } \sigma = \tilde{\sigma} \end{aligned}$$

The last worker getting trained, the $\tilde{\sigma}$-worker, receives the same wage as unskilled workers not getting trained. However, for $I(\underline{h},\sigma) = I\ \forall\sigma$, since $J(\overline{h},\widetilde{\sigma}) > J(\underline{h},\sigma > \tilde{\sigma})$, it is easy to verify that $T^{R(\underline{h},\tilde{\sigma})} < T^{R(\underline{h^n})}$. That is, the last trained unskilled worker is renovated before any of the unskilled worker not getting trained.
What about the relationship between the renovation horizon of workers with $\sigma < \tilde{\sigma}$ and $\sigma > \tilde{\sigma}$?

Corollary: For all $\sigma \leq \tilde{\sigma}$ the renovation horizon will necessarily be smaller for any of the workers getting trained than for workers not getting trained.

Proof: Let's start by simply replacing equation (22) in expression (21), which yields:

$$I(\underline{h}, \sigma < \tilde{\sigma}) = J(\overline{h}, \sigma < \tilde{\sigma}) - J(\underline{h}, \sigma < \tilde{\sigma}) + x\underline{h} \int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s}(1 - e^{g(s-T^{R(\underline{h})})}) \tag{37}$$

where the last term of the previous expression corresponds to equation (26) but with a different renovation horizon. Because we assume $I(\underline{h}, \sigma < \tilde{\sigma}) = I(\underline{h}, \sigma > \tilde{\sigma})$ and $J(\overline{h}, \sigma < \tilde{\sigma}) > J(\underline{h}, \sigma > \tilde{\sigma})$ [8] the integral term $\int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s}(1 - e^{g(s-T^{R(\underline{h})})})$ must necessary have an upper limit below the one of equation (26), i.e. $\int_0^{T^{R(\underline{h^n})}} e^{-(r+\delta)s}(1 - e^{g(s-T^{R(\underline{h^n})})})$, that is $T^{R(\underline{h})}$ must necessarily be below $T^{R(\underline{h^n})}$.■

Because firms renovate faster jobs occupied by unskilled getting trained than jobs occupied by unskilled not getting trained, the larger the share of workers getting trained in an economy, the more likely the capitalization effect will dominate when growth accelerates.

4.2.2 The relationship between growth and training decision, $(g, \tilde{\sigma})$

Without training, the relationship between employment and growth is less positive because capitalization occurs only in skilled positions where $I(\overline{h}, \sigma) = 0$. At the opposite, when training is possible, a rise in the growth rate leads workers to expect higher returns from the training decision. Then, the positive impact on the employment rate associated with the acceleration of the growth rate is magnified by a composition effect: a larger share of workers has the incentive to train and access to jobs where the capitalization effect exists.

Proposition 3: As far as $\delta \geq \pi$ an acceleration in the growth rate increases the share of unskilled workers getting trained.

[8] Note that, the minimum instantaneous profit obtained in a job occupied by a $\tilde{\sigma}$-worker equals $\underline{h}(x - b)$ which is below the profit associated with a high skilled position, $\overline{h}(x(\tilde{\sigma}) - b) + \pi \frac{b(\overline{h}-\underline{h})}{r+\pi-g}$ as far as productivity differentials compensate unemployment benefit differentials, which will be the case if $x(\tilde{\sigma}) \geq x$ (the adopted calibration ensures that this inequality is always satisfied). In the same way, the maximum profit associated with the $\sigma = 0$-worker equals $\underline{h}(x - b) + (r + \delta - g) \frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}} \frac{b(\overline{h}-\underline{h})}{r+\pi-g}$. When comparing to the profit associated with a high skilled position, $\overline{h}(x(\tilde{\sigma}) - b) + \pi \frac{b(\overline{h}-\underline{h})}{r+\pi-g}$, we realize that, since $\underline{h}(x - b) < \overline{h}(x(\tilde{\sigma}) - b)$, in order to ensure that the instantaneous profit associated with a skilled worker is larger than that of the most profitable low-skilled worker, it is enough to have $\pi > (r + \delta - g) \frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}$. This inequality is likely to be satisfied since the renovation horizon is very small for $\sigma = 0$. This ensures then that $J(\overline{h}, \sigma < \tilde{\sigma}) > J(\underline{h}, \sigma < \tilde{\sigma})$, $\forall \sigma \epsilon [0, \tilde{\sigma}]$.

Proof: We proceed by steps. First, because $b\underline{h} = \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}} - (r+\delta-g)(J(\overline{h},\widetilde{\sigma}) - I(\underline{h},\widetilde{\sigma}))$ is positive sloped in the $(T^{R(\underline{h})},\widetilde{\sigma})$ space and since $\tilde{\sigma} = \frac{e^{-(r+\delta-g)T^{R}(\underline{h})}}{1-e^{-(r+\delta-g)T^{R}(\underline{h})}}\frac{r+\delta-g}{r+\pi-g}b(\overline{h}-\underline{h})$ is negative sloped in the $(T^{R(\underline{h})},\widetilde{\sigma})$ space, we know that there exists a unique intersection point between both lines, determining the equilibrium $\tilde{\sigma}$. Second, we use a static comparative analysis to determine the impact of an acceleration of growth in each of these locus. Let's start with $b\underline{h} = \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}} - (r+\delta-g)(J(\overline{h},\widetilde{\sigma}) - I(\underline{h},\widetilde{\sigma}))$. For a given $\tilde{\sigma}$ and since $J(\overline{h},\tilde{\sigma}) = \frac{x(\tilde{\sigma})\overline{h}-b\overline{h}}{r+\delta-g}$ we find:

$$g\frac{\partial T^{R(\underline{h})}}{\partial g} = -T^{R(\underline{h})} - I < 0$$

so that the locus shifts downwards.

Concerning $\tilde{\sigma} = \frac{e^{-(r+\delta-g)T^{R}(\underline{h})}}{1-e^{-(r+\delta-g)T^{R}(\underline{h})}}\frac{r+\delta-g}{r+\pi-g}b(\overline{h}-\underline{h})$, for a given $T^{R(\underline{h})}$ an acceleration of growth yields an upward shift of the locus:

$$\frac{\partial\tilde{\sigma}}{\partial g} = b(\overline{h}-\underline{h})\left(\frac{\delta-\pi}{(r+\pi-g)^2}\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{(1-e^{(r+\delta-g)T^{R(\underline{h})}})^2}\right)+\frac{b(\overline{h}-\underline{h})(r+\delta-g)}{(r+\pi-g)}\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{1-e^{-(r+\delta-g)T^{R(\underline{h})}}}T^{R(\underline{h})} > 0 \text{ for } \delta > \pi$$

Unambiguously, the share of workers getting trained (determined by $\tilde{\sigma}$) increases then when growth accelerates. The impact on the renovation horizon depends on the relationship between the productivity in the skilled segment and the value of σ. More precisely, combining the total differentials with respect to g of $b\underline{h} = \frac{x\underline{h}}{e^{gT^{R(\underline{h})}}} - (r+\delta-g)(J(\overline{h},\widetilde{\sigma}) - I(\underline{h},\widetilde{\sigma}))$ and $\tilde{\sigma} = \frac{e^{-(r+\delta-g)T^{R}(\underline{h})}}{1-e^{-(r+\delta-g)T^{R}(\underline{h})}}\frac{r+\delta-g}{r+\pi-g}b(\overline{h}-\underline{h})$, yields:

$$\frac{\partial T^{R(\underline{h})}}{\partial g}\left(-\frac{g}{x'(\tilde{\sigma})}+\frac{b(\overline{h}-\underline{h})(r+\delta-g)^2e^{-(r+\delta-g)T^{R}(\underline{h})}}{(r+\pi-g)(1-e^{(r+\delta-g)T^{R}(\underline{h})})}\right) = \tag{38}$$

$$= \frac{1}{x'(\tilde{\sigma})}(T^{R}(\underline{h})+I(\underline{h},\widetilde{\sigma}))+b(\overline{h}-\underline{h})\left(\frac{\delta-\pi}{(r+\pi-g)^2}\frac{e^{-(r+\delta-g)T^{R(\underline{h})}}}{(1-e^{(r+\delta-g)T^{R(\underline{h})}})}\right)+$$
$$+\frac{b(\overline{h}-\underline{h})(r+\delta-g)e^{-(r+\delta-g)T^{R}(\underline{h})}}{(r+\pi-g)(1-e^{(r+\delta-g)T^{R}(\underline{h})})}$$

The term in brackets on the left hand side is unambiguously positive since $x'(\tilde{\sigma}) < 0$. The first term on the right hand side is negative whereas the two last terms are positive. The economic interpretation of these signs is quite intuitive. If the productivity in the skilled position decreases sharply with σ (so $1/x'(\sigma)$ is low), firms are incited to postpone renovation ($\frac{\partial T^{R}(\underline{h})}{\partial g} > 0$), since the potential gain of implementing a new technology is lower. On the other hand, if the decrease in the productivity of skilled positions is not very sensitive to the rise in σ, firms renovate faster when growth accelerates ($\frac{\partial T^{R}(\underline{h})}{\partial g} < 0$). ■

An acceleration in the growth rate fosters unambiguously a rise in the share of trained workers. In contrast, the pace of updating may increase or decrease depending on the relationship between the productivity of skilled positions and the ability of the workers becoming skilled, measured by the training effort σ. Note that in periods of high turbulence (high growth, g, associated with a high depreciation of human capital, such that $\delta < \pi$) the impact of growth on the share of unskilled getting trained may be ambiguous.

4.2.3 The capitalization vs. the creative-destruction effect

It is in the interest of the firm to renovate if and only if the renovation cost is lower than the optimal value it obtains without renovation. In this case, the optimal updating time T^R is shorter than the optimal scrapping time T (see the appendix A for the derivation of T):

$$I(\overline{h},\sigma) \leq J^*(\overline{h},\sigma) \quad \Leftrightarrow \quad T^R(\overline{h}) < T(\overline{h}) \quad \text{for a skilled worker}$$

$$I(\underline{h},\sigma) \leq J^*(\underline{h},\sigma) \quad \Leftrightarrow \quad T^R(\underline{h}^n) < T(\underline{h}^n) \quad \text{for an unskilled worker not getting trained}$$

$$I(\underline{h},\sigma) \leq J^*(\underline{h},\sigma) \quad \Leftrightarrow \quad T^R(\underline{h}) < T(\underline{h}) \quad \text{for an unskilled worker getting trained}$$

Updating a technology associated with a job is a necessary but not a sufficient condition for the capitalization effect to dominate over the creative destruction one. Even if it is in the interest of the firm to update a job rather than destroying it, this does not mean that the expected net profits increase when growth accelerates. Actually, even if it is in the interest of the firm to renovate, the labor cost effect induced by the rise in wages may overcome the actualization effect coming from the rise in productivity associated to updating. In this case, the profits of the firm, net of renovation costs, may fall when growth accelerates, leading to a reduction in the number of open vacancies.
For the skilled segment we assume that $I(\overline{h},\sigma) \to 0$, which implies that $T^R(\overline{h}) \to 0$. Renovation is then the preferred option ($T^R(\overline{h}) < T^(\overline{h})$) and the capitalization effect is always dominant when growth accelerates ($\frac{\partial J(\overline{h},\sigma)}{\partial g} > 0$).

Proposition 5 *For each $(\underline{h},\sigma)$ there exists a unique implementation cost $I^*(\underline{h},\sigma) > 0$, such that $\partial J(\underline{h},\sigma)/\partial g > 0$ for all $I(\underline{h},\sigma) < I^*(\underline{h},\sigma)$ and $\partial J(\underline{h},\sigma)/\partial g < 0$ for all $I(\underline{h},\sigma) > I^*(\underline{h},\sigma)$.*

Proof See appendix. ■

4.3 The labor market tightness

Replacing $J(t,t,\underline{h},\sigma > \tilde{\sigma}) = p(t)J(\underline{h},\sigma > \tilde{\sigma})$, $J(t,t,\underline{h},\sigma < \tilde{\sigma}) = p(t)J(\underline{h},\sigma < \tilde{\sigma})$ and $J(t,t,\overline{h},\sigma < \tilde{\sigma}) = p(t)J(\overline{h},\sigma < \tilde{\sigma})$ in the free entry condition allows us to verify that the labor market tightness is stationary:

$$\frac{c}{q(\theta)} = \int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(\underline{h},\sigma<\tilde{\sigma})dG(\sigma) + \int_{\tilde{\sigma}}^{\overline{\sigma}} \frac{u(\underline{h},\sigma)}{u} J(\underline{h},\sigma>\tilde{\sigma})dG(\sigma) + \int_{\underline{\sigma}}^{\tilde{\sigma}} \frac{u(\overline{h},\sigma)}{u} J(\overline{h},\sigma<\tilde{\sigma})dG(\sigma) \equiv \overline{J} \tag{39}$$

where we refer to this expression as the job creation rule. Whereas the optimal values associated to each type of filled vacancy are already known, the labor market tightness as well as the number of unemployed workers remains unknown. Combining this job creation rule with the equilibrium employment flow equations will allow us to find both, the labor market tightness and unemployment rates.

There is an externality induced by the undirected search process: when growth accelerates, the number of unskilled workers getting trained increases, which rises the expected return of the search process and stimulates the opening of new vacancies. Actually, the impact of a variation of growth on the labor market tightness depends on its heterogenous impact on the job values. If the capitalization effect is dominant in all jobs, then, a higher growth rate leads to more employment. In contrast, if the creative-destruction effect dominates for at least one type of job, the impact of growth on employment is ambiguous. The introduction of training allowing unskilled workers to access high quality jobs (skilled jobs), allows to magnify the positive impact of growth on employment: the highest ability workers can access to jobs where renovation costs are equal to zero and so the capitalization effect dominates[9].

4.4 The skilled and unskilled unemployment rates

Note first that, if it is optimal for firms to renovate positions occupied by unskilled the only source of job destruction will be the exogenous shock δ. In this case, exists from employment will be given by $\delta E(\underline{h}) + p(\theta)u(\underline{h})e^{-\delta T^{R(\underline{h})}}$, where $E(\underline{h})$ and $u(\underline{h})$ stand for the number of unskilled employed and unemployed workers of ability $\sigma < \tilde{\sigma}$ and $p(\theta)u(\underline{h})e^{-\delta T^{R(\underline{h})}}$ represents the fraction of job creation that survives to exogenous destruction and that is then renovated (becoming skilled). Entries to employment are given by: $p(\theta)u(\underline{h})$. The equilibrium flow equation is then equal to:

$$p(\theta)u(\underline{h}) = \delta E(\underline{h}) + p(\theta)u(\underline{h})e^{-\delta T^{R(\underline{h})}} \tag{40}$$

[9]
$$\frac{\partial J(\overline{h},\sigma)}{\partial g} = \frac{(x(\sigma)-b)\overline{h}}{(r+\delta-g)^2} > 0$$
The impact of growth is larger when σ is low.

Following the same reasoning we can establish the equilibrium flow equation for skilled workers:

$$p(\theta)u(\overline{h}) + p(\theta)u(\underline{h})e^{-\delta T^{R(\underline{h})}} = \delta E(\overline{h}) \tag{41}$$

To keep the population of skilled and unskilled workers constant, the external outflows and inflows from each population category must be equalized. More precisely, the number of skilled workers suffering a depreciation of their human capital must equal the number of unskilled workers whose positions are renovated and then, their human capital improved:

$$p(\theta)u(\underline{h})e^{-\delta T^{R(\underline{h})}} = \pi u(\overline{h}). \tag{42}$$

Finally, we normalize to one the total population associated to the ability level $\sigma < \tilde{\sigma}$ so that we find $P(\underline{h}) + P(\overline{h}) = 1$, where $P(\underline{h}) = u(\underline{h}) + E(\underline{h})$ and $P(\overline{h}) = u(\overline{h}) + E(\overline{h})$. We have thus four equations and four unknowns for an ability level $\sigma < \tilde{\sigma}$: $u(\underline{h})$, $u(\overline{h})$, $P(\underline{h})$, $P(\overline{h})$.
Combining equations (41) and (42) yields $(p(\theta) + \pi + \delta)u(\overline{h}) = \delta P(\overline{h})$. Similarly, from equation (40) we obtain: $\delta P(\underline{h}) = u(\underline{h})(p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta)$. We then have:

$$u(\underline{h}) = \frac{\delta P(\underline{h})}{p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta} \qquad u(\overline{h}) = \frac{\delta P(\overline{h})}{p(\theta) + \pi + \delta} \tag{43}$$

leading to the following unemployment rates:

$$u(\underline{h})/P(\underline{h}) = u^R(\underline{h}) = \frac{\delta}{p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta} \qquad u(\overline{h})/P(\overline{h}) = u^R(\overline{h}) = \frac{\delta}{p(\theta) + \pi + \delta} \tag{44}$$

If we want rather to compute the total population of skilled and unskilled workers as well as the number of employed and unemployed, we should use equation (42):

$$\frac{\delta p(\theta)e^{-\delta T^{R(\underline{h})}} P(\underline{h})}{p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta} = \frac{\delta \pi P(\overline{h})}{p(\theta) + \pi + \delta} \tag{45}$$

$$\frac{\delta p(\theta(\underline{h}))e^{-\delta T^{R(\underline{h})}} P(\underline{h})}{p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta} = \frac{\delta \pi (1 - P(\underline{h}))}{p(\theta(\overline{h})) + \pi + \delta} \tag{46}$$

$$P(\underline{h}) = \frac{\frac{\delta\pi}{p(\theta)+\pi+\delta}}{\frac{\delta p(\theta)e^{-\delta T^{R(\underline{h})}}}{p(\theta)(1-e^{-\delta T^{R(\underline{h})}})+\delta} + \frac{\delta\pi}{p(\theta)+\pi+\delta}} \qquad P(\overline{h}) = 1 - \frac{\frac{\delta\pi}{p(\theta)+\pi+\delta}}{\frac{\delta p(\theta)e^{-\delta T^{R(\underline{h})}}}{p(\theta)(1-e^{-\delta T^{R(\underline{h})}})+\delta} + \frac{\delta\pi}{p(\theta)+\pi+\delta}} \tag{47}$$

$$u(\underline{h}) = \frac{\delta}{p(\theta)(1 - e^{-\delta T^{R(\underline{h})}}) + \delta} \cdot \left(\frac{\frac{\delta\pi}{p(\theta)+\pi+\delta}}{\frac{\delta p(\theta)e^{-\delta T^{R(\underline{h})}}}{p(\theta)(1-e^{-\delta T^{R(\underline{h})}})+\delta} + \frac{\delta\pi}{p(\theta)+\pi+\delta}} \right) \tag{48}$$

$$u(\overline{h}) = \frac{\delta}{p(\theta) + \pi + \delta} \cdot \left(1 - \frac{\frac{\delta\pi}{p(\theta)+\pi+\delta}}{\frac{\delta p(\theta)e^{-\delta T^{R(\underline{h})}}}{p(\theta)(1-e^{-\delta T^{R(\underline{h})}})+\delta} + \frac{\delta\pi}{p(\theta)+\pi+\delta}} \right) \tag{49}$$

When renovation is not optimal for the firm there are two sources of job destruction, the exogenous shock and the endogenous decision of the firm. In the absence of renovation there is no incentive for the workers to get trained so there are no transitions from the unskilled population towards the skilled population. Then, we suppose that there are no skilled in the economy. In this case we only have labor flows between the employment and the unemployment situation and $P(\underline{h}) = 1$. The unemployment rate is given by:

$$U(\underline{h})/P(\underline{h}) = u^R(\underline{h}) \quad = \quad \frac{\delta}{\delta + p(\theta)(1 - e^{-\delta T^{R(\underline{h})}})} \tag{50}$$

5 Numerical simulations

The theoretical analysis shows that there is an equilibrium such that an acceleration in the growth rate will certainly improve the employment rate of skilled workers, for whom the capitalization effect will always be dominant under the hypothesis that $I(\overline{h}, \sigma) \rightarrow 0$. Results are more ambiguous when analyzing unskilled workers. Three possible situations may arise:

- If $I(\underline{h}, \sigma) > J^*(\underline{h}, \sigma|_{\sigma<\tilde{\sigma}}) > J^*(\underline{h}, \sigma|_{\sigma>\tilde{\sigma}})$, then renovation costs are higher than search cost whatever the type of unskilled worker considered. It is never in the interest of the firm to renovate and the creative destruction effect is always dominant when growth accelerates.

- If $J^*(\underline{h}, \sigma|_{\sigma<\tilde{\sigma}}) > I(\underline{h}, \sigma) > J^*(\underline{h}, \sigma|_{\sigma>\tilde{\sigma}})$, it is in the interest of the firm to renovate positions occupied by unskilled workers getting trained but not those occupied by unskilled not getting trained. The capitalization effect may then dominate when growth accelerates for workers getting trained whereas the creative destruction effect is dominant for those not getting trained.

- If $J^*(\underline{h}, \sigma|_{\sigma<\tilde{\sigma}}) > J^*(\underline{h}, \sigma|_{\sigma>\tilde{\sigma}}) > I(\underline{h}, \sigma)$, it is in the interest of the firm to renovate any job occupied by an unskilled worker.

In this section, we propose numerical experiments in order to test the ability of the model to reproduce the impact of growth on the employment rate estimated by Pissarides and Vallanti (2007) on the basis of OCDE data. The paper tries to contribute in this way to the existing discussion in the literature concerning the capacity of the matching model to explain the impact of productivity changes on the employment rate[10]. Our claim is that by introducing the possi-

[10]See Pissarides and Vallanti (2007) for the long-run relationship between TFP growth and employment, or Shimer (2005a), Hall (2005), Mortensen and Nagypal (2007), Hagedorn and Manovskii (2008), Costain and Reiter (2008) or Pissarides (2007) for the short-run relationship between productivity shocks and employment fluctuations.

bility of training, our model should be able to improve the performance of the matching models in what concerns the relationship between employment and growth.

Table 1: Baseline Parameters Values

Job productivity	$x = x_0$	$=$	1
Productivity parameters	$\mu = 2$		$\iota = 1$
Renovation parameters	$I_{UN} = 4$		$I_{UF} = 4$
Interest rate	r	$=$	.04
Matching elasticity	α	$=$	.5
Matching efficiency	m_0	$=$	0.1
Recruiting cost	c	$=$	.33
Exogenous separation rate	δ	$=$	.032
Outside option	b	$=$	.6
Probability of human capital depreciation	π	$=$	0.6

The numerical values of the parameters of our benchmark simulation are summarized in Table 1. A matching function of the Cobb-Douglas form is assumed: $M = m_0 u^{\alpha} v^{1-\alpha}$, where α is the elasticity with respect to unemployment and it is assumed to be equal to 0.5 (see Petrongolo and Pissarides (2001)). Concerning the exogenous destruction rate, we employ the estimations provided by Gomez-Salvador, Messina, and Vallanti (2004) for France (typical European country in terms of labor market functioning) for the period from 1992 to 2000. The estimated annual job destruction rates linked to exits from the manufacturing sector are estimated to be equal to 3.2%. We adopt this value. We set the outside option on the basis of the values estimated by Blanchard and Wolfers (2000), $b = .6$. The productivity in skilled positions is given by $x = x_0 \cdot \mu \cdot e^{-\iota * \sigma}$ where $x_0 = 1$, $\mu = 2$ and $\iota = 1$, so that the productivity in a skilled position is inversely proportional to the training cost.

Human capital levels of unskilled and skilled workers are given by $[\underline{h}, \overline{h}] \; \epsilon \; [0.6, 1]$. We suppose a continuum of training costs defined between 0 and 1, i.e. $\sigma \; \epsilon \; [\underline{\sigma}, \overline{\sigma}] = \sigma \; \epsilon \; [0, 1]$. The matching efficiency parameter, $m_0 = 0.1$ and the renovation costs associated with each type of worker (i.e. I_{UN} and I_{UF}) are set so as to reproduce the unemployment rate observed for a typical European economy (an aggregate unemployment rate around 8% for a growth rate equal to 2%, where unskilled not getting trained support an unemployment rate around 12% whereas skilled workers and unskilled getting trained support an unemployment rate around 7%) as well as the average unemployment duration (around two years[11]). With this calibration, the share of workers being skilled or unskilled getting trained increases by 9% as the growth rate evolves from 1% to 2%, attaining almost 90% of the total population at $g = 2\%$. Unskilled workers not getting trained represent a progressively decreasing proportion of the population. An acceleration of growth induces thus training of unskilled workers. Does this favor a decrease in the unemployment rate? The answer is provided by table 2, where we display the variation (in percentage points) of the unemployment rates associated to each workers' qualification, when the growth rate increases from 1 to 2%.

Table 2: The impact of growth on aggregate unemployment by qualification

	USK+UNT	UNT	USK	UN	UTOT
ΔU if g $\uparrow$ from 1% to 2%	-1.3560	-1.1929	-0.1631	+1.8741	-1.0768

USK=Skilled unemployed; UNT=Unskilled getting trained unemployed; UN=Unskilled unemployed; UTOT=Total unemploy

Table 2 reveals a key result: an increase of one percentage point in the growth rate (from 1% to 2%) decreases aggregate unemployment rate by 1.0768 percentage points, which corresponds well to the estimations of Pissarides and Vallanti (2007) and which confirms that, at the aggregate level, the capitalization effect is dominant. However, a detailed analysis by qualification level reveals that the relationship between unemployment and growth varies according not only to the skill level of the worker but also to the training decision. Actually, whereas the correlation between unemployment and growth arises as negative when considering skilled workers and unskilled getting trained, it becomes positive as soon as we focus on unskilled workers not getting trained. The smaller reduction in skilled unemployment with respect to unskilled is explained by the lower starting level of skilled's unemployment rate.

Figure 1, allows us to better understand the capitalization and the creative destruction effects by worker's skill and training decision. First of all, note that the aggregate labor market tightness increases with the growth rate, implying a dominant capitalization effect. This yields a reduction in the aggregate unemployment rate by around one percentage point. The unemployment rate

[11]Because there is a single labor market tension, we will have a single unemployment duration for all workers.

of unskilled workers not getting training continuously increases as growth rises, underlining the existence of a dominant creative destruction effect all along the growth path. Conversely, if the attention is focused on the joint unemployment rate of skilled and unskilled getting trained, we observe a continuously decreasing path. The capitalization effect is then dominant for this worker category.

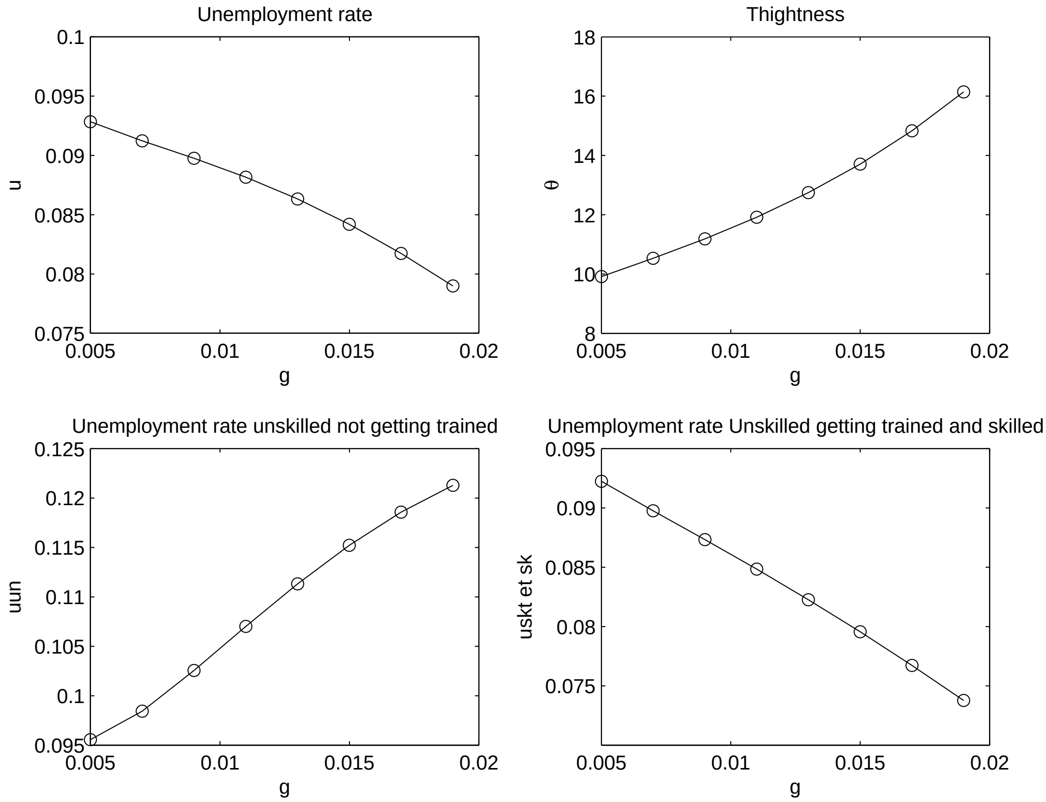
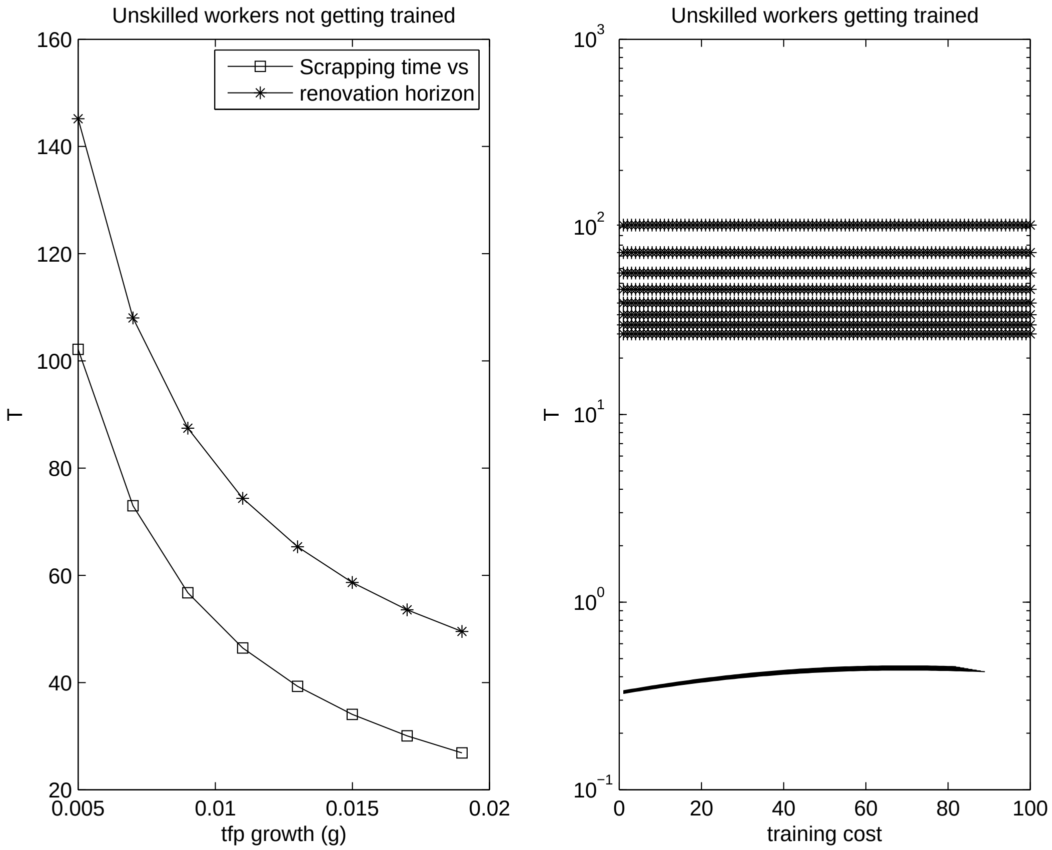


Figure 1: Capitalization versus creative destruction effect.

Figure 2, displays the renovation and the scrapping horizon for each worker category. First of all, note that, since the renovation cost associated with skilled positions is assumed to be zero, this type of jobs is continuously renovated, leading to a renovation horizon equal to zero (we do not represent this trivial result in the figure). Secondly, concerning unskilled not getting trained (first panel of figure 2) the renovation horizon is above the scrapping horizon, meaning that it is never in the interest of the firm to renovate this type of jobs. There is then a dominant creative destruction effect. Finally, when considering unskilled workers getting training, the renovation horizon is below the scrapping horizon for each growth level, meaning that it is in the interest of the firm to update the technology when growth accelerates rather destroying the job.



For unskilled workers getting trained: the starred lines stand for the scrapping horizon, which is the same for all training cost levels and decreases with the growth rate. The dark lines stand for the renovation horizon, which increases with the training cost (X-axis) and decreases with the growth rate. Note that the scale on the y-axis is not linear but logarithmic.

Figure 2: Unskilled workers getting trained: scrapping horizon and renovation horizon.

In sum, the capitalization effect dominates for skilled workers and for unskilled workers getting trained. In contrast, the creative destruction effect is dominant for unskilled not getting trained. Because, the proportion of this last type of worker on the total population decreases as growth accelerates, the relationship between growth and unemployment is negative. Actually, an increase in one percentage point in the growth rate decreases aggregate unemployment by around one percentage point, which corresponds well to the aggregate capitalization effect estimated by Pissarides and Vallanti (2007) who find an elasticity of around 1 to 1.5 percentage points. The introduction of training in a standard vintage model à la Mortensen and Pissarides (1998) allows a great improvement in its predicting ability of the relationship between growth and unemployment.

5.1 Sensitivity tests

The introduction of training for low-skilled workers, allowing those having the best ability endowment to become skilled in case of renovation, seems to solve the difficulties found by recent literature to numerically reproduce the estimated positive impact of growth on the unemployment rate.

In this section, we briefly show that, if instead of considering heterogeneous agents in terms of abilities (i.e. skilled versus unskilled but also between unskilled getting trained and unskilled not getting trained), we had considered homogeneous workers, our numerical simulations would have found difficulties in reproducing empirical estimations. Most precisely, we consider two possible scenarios: one where we assume that all workers have a human capital level equal to 0.6 (which stands for the human capital level of unskilled workers in our benchmark simulation), and another scenario where all workers have $h = 1$ (which stands for the human capital level of skilled workers).

The efficiency of the matching process is set ($m_0 = 0.26$) so as to reproduce an unemployment rate around 7% for high-skilled ($h = 1$), around 12% for low-skilled ($h = 0.6$), an unemployment duration of one year for high skilled and of two years and a half for unskilled, when $g = 2\%$. We leave all the other parameters at the same level as in the benchmark simulation. We now compare what would have been the evolution of the unemployment rate in case of an acceleration in the growth rate, if all agents were homogeneous in terms of human capital.

Table 3: The impact of growth on the unemployment rate

	$h = 0.6$	$h = 1$
ΔUnemployment rate	5.4696	4.1909

We realize that in both cases, the creative destruction effect is dominant, so that the unemployment rate increases by 5.5 percentage points when $h = 0.6$ and by 4.2 percentage points when

$h = 1$. In sum, eliminating skill and training heterogeneity from our original set up, prevents the model to numerically reproduce the estimated elasticity of unemployment with respect to growth.

6 Conclusion

The impact of growth on the employment rate has often been claimed to be ambiguous. Indeed, when growth accelerates, two contradictory effects arise. On the one hand, as Pissarides (1990) claims, an acceleration of growth improves the employment rate, because growth increases "freely" the expected profits and then provides incentives to open new jobs (the capitalization effect). On the other hand, Aghion and Howitt (1994) argue that growth fosters a "creative destruction" process inducing more job destruction and less job creation, yielding higher unemployment rates (creative destruction effect). At the empirical level, it seems though to be a common agreement on the fact that growth yields a reduction on the unemployment rates (see Blanchard and Wolfers (2000), Pissarides and Vallanti (2007) or Tripier (2007)). However, when calibrated, recent theoretical works, as Pissarides and Vallanti (2007) or Langot and Moreno-Galbis (2008), do not manage to reproduce the estimated negative elasticity of unemployment with respect to growth.

In this paper we show that, by introducing the possibility of training and human capital depreciation in a vintage framework a la Mortensen and Pissarides (1998), we greatly improve the predicting ability of the model to reproduce the sensibility of employment with respect to growth. The intuition behind this result is very simple: when growth accelerates, the opportunity cost of the training investment for workers is lower, shifting the human capital distribution to the right since more people get trained. This tends to increase the incentives of firms to update the job-specific technology. Therefore training magnifies the impact of growth on the employment rate.

When calibrated, our theoretical framework predicts that an increase of one percentage point in the growth rate reduces aggregate unemployment by more of one percentage point. This reduction comes mainly from the fall in the unemployment rate of unskilled workers getting trained and, in a minor measure, from the decrease in the skilled unemployment rate. Conversely, unskilled workers not getting trained will suffer a rise in their unemployment rate as growth accelerates.

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A The optimal scrapping time

When the updating cost is extremely high, it will not be in the interest of the firm to update the technology since it will be more profitable to directly offer a new vacancy. In this case, unskilled workers have no incentive to get trained while employed, since their position will never be updated. There will then be one scrapping time associated with skilled positions and one scrapping time associated with unskilled positions. The asset values of filled vacancies are respectively given by:

$$\begin{aligned} J(\tau,t,\overline{h},\sigma) &= \max_{T^{\overline{h}}} \int_t^{\tau+T^{\underline{h}}} e^{-(r+\delta)(s-t)}[p(\tau)x(\sigma)\overline{h} - w^s(\tau,t,\overline{h},\sigma)\overline{h}]ds \\ J(\tau,t,\underline{h},\sigma) &= \max_{T^{\underline{h}}} \left\{ \int_t^{\tau+T^{\underline{h}}} e^{-(r+\delta)(s-t)}[p(\tau)x\underline{h} - w(\tau,s,\underline{h},\sigma)\underline{h}]ds \right. \end{aligned}$$

Maximizing $J(\tau,t,\overline{h},\sigma)$ with respect to $T^{\overline{h}}$ yields the following FOC:

$$b\overline{h} - \pi\frac{b\overline{h} - b\underline{h}}{r+\pi-g} = \frac{x(\sigma)\overline{h}}{e^{gT^{\overline{h}}}} \tag{51}$$

The higher the probability of human capital depreciation, the lower will be the wage asked by skilled workers and, therefore, the optimal destruction horizon increases. Actually, everything that tends to reduce the wage earned by skilled workers will induce a rise in the optimal scrapping time (this will also increase the probability that the job gets renovated). Similarly, a productivity improvement tends to increase the optimal destruction horizon.

Once $T^{\overline{h}}$ is determined from (51), we can replace it in the asset value function of the firm leading to $J^*(\overline{h},\sigma) = \int_0^{T^{\overline{h}}} e^{-(r+\delta)s}[x(\sigma)\overline{h}(1-e^{g(s-T^{\overline{h}})})]ds$. The optimal asset value function is increasing in the worker's productivity and the optimal scrapping horizon, whereas it decreases with the growth rate.

The optimal scrapping time associated with a machine used by an unskilled worker[12] is given by:

$$b\underline{h} = \frac{x\underline{h}}{e^{gT^{\underline{h}^n}}}, \tag{52}$$

and it is increasing with the worker's productivity and decreasing with the growth rate and the worker's wage. Replacing in $J(\underline{h}, \sigma)$ yields: $J^*(\underline{h}, \sigma) = \int_0^{T^{\underline{h}^n}} e^{-(r+\delta)s}[x\underline{h}(1 - e^{g(s-T^{\underline{h}^n})})]$. This optimal value is again increasing in the worker's productivity and in the optimal scrapping horizon, while it decreases with the growth rate.

In sum, in this framework we have two scrapping times (one for skilled and another one for unskilled) and three renovation horizons (one for skilled, another one for unskilled getting trained and another one for unskilled not getting trained). A job occupied by a skilled worker is renovated rather than destroyed if the associated renovation horizon is below the scrapping horizon. Similarly, if the implementation horizon of a job occupied by an unskilled worker getting trained or by an unskilled not getting trained is below the scrapping horizon associated to unskilled positions, the job is renovated rather than destroyed. Since $T^{R(\underline{h}^n)} > T^{R(\underline{h})}$ we might find a situation where it is in the interest of the firm to renovate only positions occupied by unskilled workers getting trained, i.e.$T^{R(\underline{h}^n)} > T^{\underline{h}} > T^{R(\underline{h})}$.

B Proof of the proposition 5

- Unskilled workers not getting trained:

$$\begin{aligned}
\lim_{(I(\underline{h},\sigma), T^{R(\underline{h})}) \to (0,0)} J(\underline{h}, \sigma) &= \lim_{(I(\underline{h},\sigma), T^{R(\underline{h})}) \to (0,0)} [I(\underline{h}, \sigma) + \frac{x\underline{h} - e^{gT^{R(\underline{h})}} w(\underline{h}, \sigma)}{(r + \delta - g)e^{gT^{R(\underline{h})}}}] \\
&= \frac{x\underline{h} - w(\underline{h}, \sigma)}{r + \delta - g} \qquad (53)
\end{aligned}$$

which is increasing in g $(\frac{\partial J(\underline{h},\sigma)}{\partial g} > 0)$.

[12]Note that in the absence of renovation unskilled workers have no incentive to train since they know that they will never become skilled. Because no one gets trained, all unskilled workers are homogeneous.

Then, it is sufficient to prove that $\frac{\partial^2 J(\underline{h},\sigma)}{\partial g \partial I(\underline{h},\sigma)} < 0$ in order to have a critical $I^*(\underline{h},\sigma)$ from which the creative-destruction effect dominates the capitalization effect:

$$\frac{\partial J(\underline{h},\sigma)}{\partial I(\underline{h},\sigma)} = \frac{-1}{e^{(r+\delta-g)T^{R(\underline{h})}} - 1} < 0 \quad \text{if} \quad (r+\delta-g) > 0 \tag{54}$$

deriving by g leads to

$$\frac{\partial^2 J(\underline{h},\sigma)}{\partial I \partial g} = \frac{e^{(r+\delta-g)T^{R(\overline{h})}}(-T^{R(\underline{h})} + (r+\delta-g)\frac{\partial T^{R(\underline{h})}}{\partial g})}{(e^{(r+\delta-g)T^{R(\underline{h})}} - 1)^2} < 0 \tag{55}$$

since

$$\frac{\partial T^{R(\underline{h})}}{\partial g} = \frac{\int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s} e^{g(s-T^{R(\underline{h})})}(s - T^{R(\underline{h})})ds}{g\int_0^{T^{R(\underline{h})}} e^{-(r+\delta)s} e^{g(s-T^{R(\underline{h})})}ds} < 0$$

Because $\frac{\partial J(\underline{h},\sigma)}{\partial g} > 0$ when $I(\underline{h},\sigma) \to 0$ and $\frac{\partial^2 J(\underline{h},\sigma)}{\partial I \partial g} < 0$ for all $I(\underline{h},\sigma)$, there exists a unique $I^*(\underline{h},\sigma)$ from which the capitalization effect is dominated by the creative destruction effect (see figure 3).

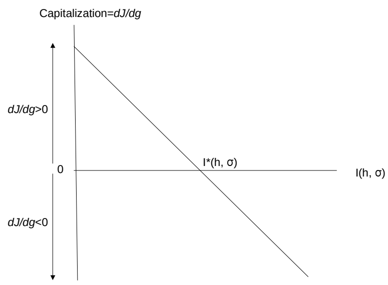

Figure 3: The capitalization vs. the creative destruction effect.

- For unskilled workers getting trained we follow a similar procedure. Replacing equation (22) in (21) and integrating leads to:

$$\begin{aligned} J(\underline{h},\sigma) &= \frac{x\underline{h}}{r+\delta}(1 - e^{-(r+\delta)T^{R(\underline{h})}}) - \frac{w(\underline{h},\sigma)}{r+\delta-g}(1 - e^{-(r+\delta-g)T^{R(\underline{h})}}) + \\ &+ e^{-(r+\delta-g)T^{R(\underline{h})}}(J(\overline{h},\sigma) - I(\underline{h},\sigma)) \\ J(\underline{h},\sigma) &= x\underline{h}[\frac{1 - e^{-(r+\delta)T^{R(\underline{h})}}}{r+\delta} - \frac{1 - e^{-(r+\delta-g)T^{R(\underline{h})}}}{(r+\delta-g)e^{gT^{R(\underline{h})}}}] + J(\overline{h},\sigma) - I(\underline{h},\sigma) \end{aligned}$$

Computing the limit of the previous expression when $(I(\underline{h},\sigma), T^{R(\underline{h})}) \rightarrow (0,0)$ yields:

$$\lim_{(I(\underline{h},\sigma),T^{R(\underline{h})})\rightarrow(0,0)} J(\underline{h},\sigma) \quad = \quad J(\overline{h},\sigma) + x\underline{h} * 0 = J(\overline{h},\sigma) = \frac{x\overline{h} - w(\overline{h},\sigma)}{r+\delta-g}$$

which is increasing in g ($\frac{\partial J(\underline{h},\sigma)}{\partial g} > 0$ for $\sigma < \tilde{\sigma}$). Again, it will be then sufficient to prove that $\frac{\partial^2 J(\underline{h},\sigma)}{\partial g \partial I(\underline{h},\sigma)} < 0$ for $\sigma < \tilde{\sigma}$, in order to have a critical $I^*(\underline{h},\sigma)$ from which the creative-destruction effect dominates the capitalization effect:

$$\frac{\partial J(\underline{h},\sigma)}{\partial I(\underline{h},\sigma)} = -e^{-(r+\delta-g)T^{R(\underline{h})}} < 0 \tag{56}$$

then,

$$\frac{\partial^2 J(\underline{h},\sigma)}{\partial I(\underline{h},\sigma)\partial g} \quad = \quad -e^{-(r+\delta-g)T^{R(\underline{h})}} T^{R(\underline{h})} - e^{-(r+\delta-g)T^{R(\underline{h})}}(-(r+\delta-g))\frac{\partial T^{R(\underline{h})}}{\partial g} < 0 \tag{57}$$

since $\frac{\partial T^{R(\underline{h})}}{\partial g} < 0$.■