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Public good congestion and the optimal number of immigrants

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Immigration optimale en présence d'une congestion de biens publics

Résumé : Ce document considère un modèle de croissance avec des immigrants qui ne possèdent pas de capital. Le revenu des nationaux et leur bien-être augmentent avec le nombre d'immigrants. Un bien public est alors introduit dans le modèle. Les immigrants ne contribuent pas à son financement mais diminuent la qualité du service public en augmentant l'encombrement du bien public. Nous calculons le nombre d'immigrants et la consommation de bien public optimaux. Quand le poids de cette dernière augmente dans l'utilité des nationaux, le nombre optimal d'immigrants devient plus élevé. A l'équilibre les nationaux investissent trop. Aussi, le Gouvernement doit, non seulement restreindre le nombre d'immigrés, mais aussi taxer le capital afin d'amener l'équilibre à coïncider avec l'optimum.

Mots clefs : Effets d'encombrement, Immigration, Bien public
Classification JEL : F22, H41, O41

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Abstract: This paper considers a growth model with migrants who do not own capital. The income of domestic residents and their welfare increase with the number of migrants. Then, a public good is introduced. Migrants do not contribute to its financing but induce congestion effects and decrease the quality of public service. We compute the optimal number of migrants and consumption of public good. When the weight of public consumption increases in the utility of domestic residents, the optimal number of migrants increases. At the equilibrium domestic residents invest too much. Thus, the Government must, not only restrain the number of migrants, but also tax capital to lead the equilibrium to the optimum.

Keywords : Congestion, Immigration, Public good
JEL Classification : F22, H41, O41

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Introduction

An interesting and important question often raised in policy making is: does the inflow of illegal migrants result in the decline of the welfare of domestic residents and if so, why does this happen? When migrants and domestic labour are complementary or imperfect substitutes, migrants do not directly compete with local workers. Therefore, in this case we can reasonably expect that the welfare of domestic residents will increase with the increase in the number of migrants. This paper uses a simple dynamic neoclassical model to show that this result is also valid when migrants and domestic labour are perfect substitutes. More precisely, it shows that the welfare of domestic residents is an increasing and unbounded function of the number of migrants.

The above result regarding the relation between migrants and welfare is robust. However, it is also unrealistic as all governments and in particular those of developed countries attempt to limit the number of immigrants. They use numerous methods to discourage and prevent foreigners from illegally entering and working within their national boundaries. If any of the industrialized countries allowed free mobility of labour then many migrants would immediately enter the country and become employed. Thus, migrants are taken on a quota basis, which means the number of immigrants (legal and illegal) is much lower than the number of foreigners who wish to immigrate.

To link theory and reality consistent with policy making we introduce a public good. The Government sets the spending on this good, which is financed by contributions from domestic residents. However, migrants who do not contribute to the financing cannot be excluded from the use of the public good. Thus, for a given level of spending, the service provided by the public good decreases with the number of migrants, because the quality of the public good decreases because of congestion. In this scenario, although migrants make a positive contribution to national income, they will reduce the welfare of the domestic residents when too many of these migrants are present in the economy, by causing a decline in the quality of public services and goods. Migrants are assumed not to own capital in their host country. So, either they will consume their whole income, or they will remit part of it to their country of origin. Under these assumptions we can compute the optimum number of migrants, which is positive, that is, it lies between zero and infinity.

We show that market equilibrium without taxes will not maximise the intertemporal welfare of domestic residents. There are two reasons for such suboptimality. First, if foreigners can enter the country freely, their number will probably be very high, the quality of public services will deteriorate and the welfare of domestic residents will decline. Secondly, domestic residents invest under a perfect forecast of future wages that they consider as given. However, a social planner will notice that wages, which are also the payments made to migrants, increase with capital. Moreover, an increase in wages will increase the number of migrants, which will directly reduce the welfare of domestic residents. Thus, the social planner will invest less than in the market equilibrium, to reduce the transfers to migrants and to limit the number of foreigners who want to migrate into the country.

We assume that the social planner is able to choose the levels of fixed capital, public good consumption, and the number of migrants working in the country. Then, we can show that the optimal spending on public good decreases and the optimal number of migrants increases if the preference of domestic residents for the public good decreases. This result is consistent with the stylised fact that countries, such as the United States, who have a weak welfare system and provision of public good and who partly exclude migrants from the benefits of these system and good, welcome migrants much more than countries, such that those of Continental Europe, who have a generous welfare system and provision of public good. For instance *The Economist* (June 3rd 2006) quotes Kathleen Newland of the Migration Policy Institute in Washington, DC: “The big difference in the way Europeans and Americans look at immigration springs from the fact that America protects its welfare system from immigrants but leaves its labour market open, while the EU protects its labour market and leaves its welfare system open”. Then, *The Economist* pursues “The result is that in America political debate centres on illegal immigration, and there is no sense that legal immigrants impose burdens on others. In Europe things are different. There, even legal immigrants are often seen as sponging on others through welfare receipts; and the fact that some have taken jobs which would not otherwise have been done so cheaply is forgotten".

The Government is a provider of public goods and redistributes income. This paper only considers the first function. Much of the theoretical and empirical literature focuses on the link between migration and tax-transfer policy. Moreover, instead of setting the relative preferences for the public and the private good exogenously, it derives the taste of domestic residents for redistributive policies from their own interest and from their before-tax income. A good example of this literature is a paper by Razin, Sadka and Swagel (2002). The authors build a political economy model. Then, they estimate on a panel of 11 European countries from 1974 to 1992 several equations relating the tax rate on labour income or the social transfers per capita to the skewedness of before-tax income, the share of immigrants out of the total population, and other variables that are of no concern to us. They find that a larger share of low-skill migrants in the population leads to a smaller tax rate and per capita social transfers. This econometric result rests on the assumption that the number of migrants is exogenous. However, the Government has strong influence on this number (and on the degree to which migrants benefit of public goods and transfers) and the number of migrants should instead depend on variables similar to those appearing in the labour tax equation. A conclusion of our analysis is that the optimal spending on public good and the optimal number of migrants are negatively correlated and depend on a common cause, which is the relative taste of domestic residents for the private and for the public good. The econometrics of Razin, Sadka and Swagel is consistent with this negative correlation.

We can also show that a Government can guide market equilibrium to the optimum provided it sets a tax on capital and immigration controls (the setting of the number of working permits, the search of illegal migrants and deportations) at adequate levels. The model presented is general enough to accommodate other types of reasons for not allowing free entry to migrants. For example this model can be re-written as a model in which there is a taste for discrimination.

Our definition of an optimum has two important and distinguishing features. First, it only considers the welfare of domestic residents, and gives no weight to the welfare of the migrants. Secondly, it considers the intertemporal welfare computed at time 0, which is the sum of the discounted utilities of current and future periods. Previous papers limited their investigation to the consumption and utility of domestic residents only in the steady state. Although, this investigation has some interest it can be misleading: a policy, which maximises long run utility will differ from a policy, which maximises intertemporal welfare, because the latter policy takes into account the utility of the agents along the transition path.

The paper assumes a very sharp difference between domestic residents and immigrants. This difference is consistent with the concept of transitory migrants, such as Dominican maids in the US or Filipino maids in Hong Kong. So, the fate of the migrants is not included in the welfare function of the social planner, which considers them only as an opportunity for extracting a rent. The fact that migrants will stay for a limited period of time also justifies the assumption that they do not invest in the country (they consume a part of their wages and they send the other part to their country of origin). However, many migrants become permanent residents and later citizens. They may have planned that when they decided to migrate, or they could have taken the decision of permanently settling in the country several years after their arrival. The assumptions of this paper do not apply to these cases. The definition of an optimal policy of immigration in this situation is also very different from the definition we give in this paper. It is an open question as to what stage of his "integration" a migrant would be considered as a domestic resident by the social planner. Note that this definition covers all migrants who are temporary and in transition (both legal and illegal).

The model used in this paper is presented in section 1 and its optimum solution is investigated in section 2. The market equilibrium solution is analysed in section 3. The calibration of the model is given in section 4. Section 5 presents and comments on a series of simulations. In the conclusion, we will show that, even if migrations and migration policies have strong redistributive effects, which have been widely analysed in the economic literature, they also may have important effects on the aggregate welfare of domestic residents.

1. The model

A dynamic model is written in discrete time and assumes that agents hold perfect foresight. A variable in the current period is identified by the absence of an index. A variable with lag (lead) of one period is identified by the index –1 (+1). We consider a closed one-good economy, except for the foreign migrants who enter the country. We assume that domestic and foreign workers are perfect substitutes. The production technology is Cobb Douglas. Current domestic output Y is

(1) $$Y = AK_{-1}^{\beta}(1+M)^{1-\beta}, \text{ with } 0<\beta<1 \text{ and } A>0$$

K_{-1} represents the quantity of fixed capital, which is available during the period and which is wholly owned by domestic residents. The quantity of domestic labour is constant over time and normed to 1. M is the number of migrants working in the country. There is no technical progress and capital does not depreciate. These hypotheses simplify the model without altering the essential message of the paper.

National workers and migrants are paid the same wage rate w, which is equal to their marginal productivity. Setting lower wages for migrants would require the introduction of mechanisms that justify this difference and would not bring much insight to the problems we are investigating. We have

(2) $$w = (1-\beta)A\left(\frac{K_{-1}}{1+M}\right)^{\beta}$$

The undifferentiated good produced in the economy can be transformed without cost and in a ratio of one to one in private consumption good, public consumption good or capital good. Domestic residents consume c in terms of private good and g in terms of public good. The public good (hospitals, schools) is made available freely to all by the Government. Migrants cannot be excluded from the consumption of the public good. The reasons for this non exclusion are humanitarian (it is difficult to refuse the admission to a hospital of a very sick person whatever his legal situation) and practical (refusing to provide health care to migrants will increase the risk of epidemic in the country). However, migrants do not pay taxes and do not contribute to the financing of the public good. Equation (3) sets the equality between national income and the sum of investment, the consumption of domestic resident and the cost of the public good.

(3) $$Y - wM = K - K_{-1} + c + g$$

If we use equations (1) and (2) this equality can be rewritten as

(4) $$A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M) = K - K_{-1} + c + g$$

We have to be more precise with our definition of the public good. Actually, this good is a set of elementary goods. Each of them has an indivisible size and can serve the whole population of domestic residents without congestion. However, congestion problems arise when the number of users becomes higher than the population of domestic residents that is by the entry of migrants. At a more formal level, we assume that each elementary public good has a unit cost in term of the undifferentiated good produced in the economy. So, g represents the number of these goods and their total cost, which is wholly financed by domestic residents. The *service* created by these goods is $g\left[1-\left(M/\overline{M}\right)^{d}\right]$, with $\overline{M} > 0$, $d \geq 2$. When there are no migrants, the public good service is maximum and equal to g. When the number of migrants increases, congestion effects appear and the provision of public good declines. It becomes 0 for a number of migrants equal to (or greater than) $\overline{M}$. The assumption that the indivisibility threshold in the public good, after which it manifest congestion effects, coincides with the size of the population of domestic residents, is strong. However, it simplifies the exposition of the paper. We will discuss the extensions to the cases of a threshold smaller or larger than the size of the domestic population in the appendix. In the second case, we will obtain results quite similar to those we obtain in the main body of the paper. However, in the first case the results we will get will be much less precise and attractive. It is also to simplify the exposition of the paper that we have assumed that the indivisibility threshold is exogenous to the model.

The current utility of domestic residents, u, is a function of the consumption of private good and of public good service. We assume that these two goods are indispensable in the utility function. Thus, we will assume a CES utility function, with an elasticity of substitution, $1+\sigma$, smaller than 1.

(5) $$u=\left\{b^{1/(1+\sigma)}c^{\sigma/(1+\sigma)}+\overline{b}^{1/(1+\sigma)}\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}\right\}^{(1+\sigma)/\sigma}$$, with $b,\overline{b}\geq 0$, $b+\overline{b}=1$ and $-1<\sigma<0$

In the rest of the paper we will use the parameters $a=b^{1/(1+\sigma)}$ and $a'=\overline{b}^{1/(1+\sigma)}$, which both are positive. The welfare of domestic residents, computed at time 0, U, is the sum of their discounted utilities at this time and in all future periods. We have

(6) $$U=\sum_{t=0}^{\infty}\left[u_t^{1-\theta}/(1-\theta)\right](1+\rho)^{-t}, \ \theta>0 \ \rho>0$$

where θ is the inverse of the intertemporal utility of domestic residents, and ρ the discount rate.

Finally, the number of migrants cannot be larger than the number of foreigners who wish to enter and work in the country. This is an increasing function of the wage rate, with elasticity $\delta\geq 0$. We could also have made the number of foreigners who wish to immigrate, a function of the quantity of public good available in the country. However, this extension would not have added much to our analysis. We have

(7) $M\leq M_0 w^{\delta}$, where $M_0>0$ is a scale parameter

2. The optimum of the model

In this section we will assume that all decisions for all periods are taken at time 0 by the social planner. The social planner sets the values of the fixed capital, the consumption of private good by domestic residents, the quantity of public good available in the economy and the number of migrants, to maximise the intertemporal welfare of domestic residents (equations 5 and 6). This maximisation is made under the national budget constraint (4). We have the program

$$(8)\quad \underset{K_t, c_t, g_t, M_t}{Max}\; U \equiv \sum_{t=0}^{\infty} \frac{\left\{ a c_t^{\sigma/(1+\sigma)} + a'\left[g_t \left[1 - \left(M_t / \overline{M} \right)^d \right] \right]^{\sigma/(1+\sigma)} \right\}^{(1-\theta)(1+\sigma)/\sigma}}{1-\theta} (1+\rho)^{-t}$$

$$A\left(\frac{K_{t-1}}{1+M_t} \right)^{\beta} (1+\beta M_t) = K_t - K_{t-1} + c_t + g_t$$

$t \geq 0$, $K(-1)$ given

The definition of an optimum makes two important assumptions. First, the social planner cannot pay migrants less than their marginal productivity. Secondly, the social planner can freely set the number of migrants. Thus, this number must be less than the number of people who wish to enter the country and work, which implies that inequality (7) must be non binding in all periods. Assuming that the social planner can freely set the number of migrants is to give it strong empowerment and the ability to control the legal situation of any worker without cost.

We use the equation of the program to eliminate c_t from the objective function, which now becomes

(9)

$$\underset{K_t, g_t, M_t}{Max}\; U \equiv$$

$$\sum_{t=0}^{\infty} \frac{\left\{ a\left[A\left(\frac{K_{t-1}}{1+M_t} \right)^{\beta} (1+\beta M_t) - (K_t - K_{t-1}) - g_t \right]^{\sigma/(1+\sigma)} + a'\left[g_t \left[1 - \left(M_t / \overline{M} \right)^d \right] \right]^{\sigma/(1+\sigma)} \right\}^{(1-\theta)(1+\sigma)/\sigma}}{1-\theta} (1+\rho)^{-t}$$

$A\left(\frac{K_{t-1}}{1+M_t} \right)^{\beta} (1+\beta M_t)$ represents the national income of the country (the income of domestic residents) in period t. It increases with the number of migrants and tends to infinity with this number[1]. It is a convex function of the number of migrants for $M_t < 1/\beta$ and a concave function for

[1] This result can be found in Berry and Soligo (1969) and in Borjas (1999).

$M_t > 1/\beta$. The term $g_t\left(1-\left(M_t/\overline{M}\right)^d\right)$ represents the public good service in period t. It decreases with the number of migrants at a more than quadratic rate.

The introduction of a public good from which migrants cannot be excluded is important for the model and for policy making. If the public good provides no utility to domestic residents ($b = a = 1$ and $\overline{b} = a' = 0$), then the optimal number of migrants in period t is infinity and the optimal spending on public good is 0. More precisely we have the proposition

Proposition 1. If the public good provides no utility to domestic residents and if fixed capital is always set at its optimal level, then an increase in the number of illegal migrants increases the welfare of domestic residents. The welfare of domestic residents tends to infinity when the number of migrants is increased indefinitely[2 3].

Proof. Let us assume that the economy is initially in a steady state. Then, according to equation (4), national consumption is equal to national income

$$c = A\left(\frac{K_{-1}}{1+M}\right)^{\beta}\left[1+\beta M\right]$$

Let us assume that the number of migrants increases by dM. If the amount of fixed capital is kept unchanged the consumption of domestic residents immediately increases and permanently stays at this higher value

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[2] It can easily be shown that these results are still valid for a general production function $Y = F(K, 1+M)$, which is twice continuously differentiable, increasing and concave in its arguments, with constant return to scales and such that $F_1^{'}(k,1) \underset{k\to 0}{\longrightarrow} \infty$.

[3] Hazari and Sgro (2003) and Moy and Yip (2006) complement this proposition by proving that for a general neo classical utility function, long run national consumption may decrease or increase with the number of migrants. Moy and Yip (2006) found that for a Cobb Douglas production function, long run national consumption always increases with the number of migrants. Palivos (2006) showed that, for a general neo-classical production function, long run national consumption increases when the number of migrants increases by a small amount starting from 0. However, these results bear on comparisons between the initial and final values of the consumption of domestic residents and not on their intertemporal welfare computed along the dynamic path.

$$\frac{dc}{dM} = \beta(1-\beta)Y\frac{M}{(1+M)^2}dM > 0$$

Thus, the intertemporal welfare of domestic residents increases. Moreover, their consumption tends to infinity with the number of migrants. Consequently, if the social planner makes the optimal adjustment in capital accumulation, the intertemporal welfare will increase by even more.■

A consequence of Proposition 1 is that, without a public good, the social planner will never constrain the number of migrants, which will be equal to the number of people who wish to enter the country. Thus, migration will never be rationed. The introduction of a public good will imply that, for reasonable values of parameters, there exists a finite optimal number of migrants, which is lower than the number of foreigners who wish to enter and work in the country.

Now, we can explain why we have assumed that the elasticity of substitution in the preferences of domestic residents is smaller than 1. If this elasticity were larger than 1 that is if $\sigma > 0$, then, the optimal number of migrants in any period would be infinity, national income and the consumption of private good by domestic residents would also be infinity, and the public good service would be 0, whatever the spending on it. So, the optimal spending on public good would also be 0.

We have to maximise the welfare function given in equation (9) with respect to three variables. We will decompose this problem into three one dimension problems. For each of them the welfare function is maximised with respect to one of its arguments, the two other being set at arbitrary values. Each time we will give the first order condition associated to the decision made in period t. We will continue to use the convention of identifying a variable in this period by the absence of index and a variable with a lag (lead) of one period by the index –1 (+1).

2.1. The optimal number of migrants

We have the following lemma.

Lemma 1. Let us set the spending on public good g *and the amount of fixed capital* K *and* K_{-1} *at arbitrary levels such that* $0 < g < A(K_{-1})^{\beta} - (K - K_{-1})$. *Then, the optimal number of migrants in period* t *conditional on these values is the root of the equation*

$$\begin{aligned}(10)\quad & \frac{aA\beta(1-\beta)}{1+M}\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})-g\right]^{-1/(1+\sigma)}\left(\frac{K_{-1}}{1+M}\right)^{\beta} = \\ & a'dg^{\sigma/(1+\sigma)}\frac{M^{d-2}}{\overline{M}^{d}}\left[1-\left(\frac{M}{\overline{M}}\right)^{d}\right]^{-1/(1+\sigma)}\end{aligned}$$

This root is unique. It increases from 0 to a value less than $\overline{M}$ *when the spending on public good increases from 0 to* $AK_{-1}^{\beta} - (K - K_{-1})$.

Proof. See the appendix.■

National income is equal to the sum of the consumption of private good and of public good. When the spending on public good is increased, domestic residents have to reduce their consumption of private good. However, they will also increase their (national) income by admitting more migrants. Of course this last decision will reduce the public good service by inducing congestion effects.

2.2. The optimal spending on public good

We have the following lemma.

Lemma 2. Let us set the number of migrants M *and the amount of fixed capital* K *and* K_{-1} *at arbitrary levels such that* $0 \le M < \overline{M}$. *Then, the optimal spending on public good in period* t *conditional on these values is the root of the equation*

$$(11)\quad a\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})-g\right]^{-1/(1+\sigma)} = a'g^{-1/(1+\sigma)}\left[1-\left(\frac{M}{\overline{M}}\right)^{d}\right]^{\sigma/(1+\sigma)}$$

This root is unique. It increases with the number of migrants, is equal to $\bar{b}\left[AK_{-1}^{\beta}-\left(K-K_{-1}\right)\right]$*, when* $M=0$*, and converges to* $A\left(\frac{K_{-1}}{1+\overline{M}}\right)^{\beta}\left(1+\beta\overline{M}\right)-(K-K_{-1})$ *when* M *tends to* $\overline{M}$*. It decreases with the preference of domestic residents for the public good.*

Proof. See the appendix.

When the number of migrants becomes higher, the national income of domestic residents increases. However, the quality of the service provided by the public good deteriorates. So, domestic residents will react by spending more on the public good.

2.3. The optimal number of migrants and spending on public good

Before maximising the objective function of the social planner in the last dimension, which is the amount of fixed capital, we will combine the results of the two last paragraphs. Let us set the amount of fixed capital K and K_{-1} at arbitrary levels. Conditionally on these values, the optimal decision of the social planner with respect to the number of migrants and the spending on public good, M and g, are defined implicitly by equations (10) and (11). We easily deduce from these equations the relation

(12) $$g=\left[1-\left(\frac{M}{\overline{M}}\right)^{d}\right]\frac{A\beta(1-\beta)}{1+M}\left(\frac{K_{-1}}{1+M}\right)^{\beta}\frac{\overline{M}^{d}}{dM^{d-2}}$$

In this expression g is a decreasing function of M, which tends to infinity when $M=0$ and which tends to 0 when M tends to $\overline{M}$.

The optimal policy in period t (conditional on the amounts of fixed capital) is determined by this equation and equation (11). We can see that this policy is unique. Moreover, we can easily notice that the optimal spending on public good decreases and the optimal number of migrants increases, if the preferences of domestic residents for the public good decreases.

Of course, the validity of these results for the optimum solution of the model has not been rigorously proved. If the preference for the public good changes, so will the amount of fixed capital and we cannot assume it to be exogenously given. However, the simulations of paragraph 5.5 will confirm that, at the optimum of the model, the consumption of public good decreases and the number of migrants increases when the preference for the public good decreases.

2.4. The optimal capital at the end of period t

We have the following lemma.

Lemma 3. Let us set the number of migrants M and M_{+1}, the quantity of public good g and g_{+1} and the capital at the end of periods $t-1$ and $t+1$, K_{-1} and K_{+1}, at arbitrary positive levels such that $M, M_{+1} < \overline{M}$. Then, the optimal capital at the end of period t, is given by the equation

$$\text{(13)} \quad \left\{ \frac{ac^{\sigma/(1+\sigma)} + a'\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}}{ac_{+1}^{\sigma/(1+\sigma)} + a'\left[g_{+1}\left[1-\left(M_{+1}/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}} \right\}^{(1-\theta)(1+\sigma)/\sigma-1} \left(\frac{c_{+1}}{c}\right)^{1/(1+\sigma)} = \frac{1}{1+\rho}\left[1+\beta A\left(\frac{K}{1+M_{+1}}\right)^{\beta}\frac{1+\beta M_{+1}}{K}\right]$$

Proof. See the appendix.■

Now, we can put the three first order conditions of the maximisation of the welfare of domestic residents all together. We get a model, which in each period includes 8 equations and one inequality. The model also includes 8 endogenous variables appearing with their current values: c, g, K, M, u, U, w and Y. One endogenous variable appears with a lag K_{-1} and three endogenous variables appear with a lead c_{+1}, g_{+1} and M_{+1}.

3. The market equilibrium of the model

All the equations given in section 1 are still valid for the market equilibrium. We will introduce a tax on capital income of rate $t \geq 0$. Thus, firms can borrow at interest rate r, but households will receive an after-tax return $(1-t)r$. Firms maximise their profit in each period under the constraint of their production function (1) and under the condition that the wage rate and the interest rate, w and r, are given. Thus, workers are paid at their marginal productivity and we get equation (2). Moreover the marginal productivity of capital is equal to the interest rate

$$(14) \qquad r = \beta A \left(\frac{1+M}{K_{-1}} \right)^{1-\beta}$$

The tax revenue of the Government is distributed to domestic residents as lump sum transfers $T = trK_{-1}$. Domestic residents maximise their intertemporal welfare under their budget constraints. Their income includes their after-tax capital income $(1-t\ )r\ K_{-1}$, their wages income $w\ L$, the lump sum transfers they receive from the Government T, minus the lump sum taxes financing the cost of the public good g. They save a part of this income $K - K_{-1}$ and they consume the rest c. Thus, they have to solve the program

$$(15)\ \underset{K_t, c_t}{Max}\, U \equiv \sum_{t=0}^{\infty} \frac{\left\{ ac_t^{\sigma/(1+\sigma)} + a'\left[g_t\left[1-\left(M_t/\overline{M}\right)^d\right]\right]^{\sigma/(1+\sigma)} \right\}^{(1-\theta)(1+\sigma)/\sigma}}{1-\theta}(1+\rho)^{-t}$$

$$(1-t_t)r_t K_{t-1} + w_t L - g_t + T_t = K_t - K_{t-1} + c_t$$

$$t \geq 0,\ K(-1) \text{ given}$$

The first order condition of this program is

$$(16)\ \left\{ \frac{ac^{\sigma/(1+\sigma)} + a'\left[g\left[1-\left(M/\overline{M}\right)^d\right]\right]^{\sigma/(1+\sigma)}}{ac_{+1}^{\sigma/(1+\sigma)} + a'\left[g_{+1}\left[1-\left(M_{+1}/\overline{M}\right)^d\right]\right]^{\sigma/(1+\sigma)}} \right\}^{(1-\theta)(1+\sigma)/\sigma-1} \left(\frac{c_{+1}}{c} \right)^{1/(1+\sigma)} = \frac{1+(1-t_{+1})r_{+1}}{1+\rho}$$

We can notice that domestic residents perfectly foresee the path that the wage rate will follow in the future. However, they consider this path as given and they differ in this respect from the social planner, which considers future wages as a function of capital accumulation. So, domestic residents do not take into account that investment drives wages up and increases the transfer to migrants.

Moreover, high wages will attract more migrants who will consume the public good although they do not pay for it. We will show later that domestic residents will invest too much, at least when capital income is not taxed. A sufficiently high taxation of capital will prevent this overinvestment from occurring. Palivos (2006) gives an excellent discussion of this problem. He shows that the equilibrium path of the economy (without taxation of capital income) may converge to an inefficient steady state, which means that domestic residents would be better off if a part of the fixed capital was simply scrapped.

We will assume in the definition of the market equilibrium that the Government cannot directly and freely set the number of migrants working in the country. However, it controls the number of job permits, it can search for illegal migrants with more or less intensity and deport those who are caught. The difficulties to legally immigrate and the harassment and possible deportation of illegal migrants can be interpreted and modelled as a reduction in the wages migrants receive, bur not in the wages they cost their employer. Thus, we introduce an anti-immigration tax of rate $\tau \geq 0$. This 'tax' has no return for the Government. Actually, searching and deporting migrants has a cost, which increases with the intensity of searching. However, we have preferred assuming that this cost is zero, so as not to complicate the model. Finally, the number of migrants working in the country is

(17) $M = M_0 \left[w(1-\tau) \right]^{\delta}$

In each period the model includes 8 equations. It also includes 8 endogenous variables appearing with their current values: c, K, M, r, u, U, w and Y. One endogenous variable appears with a lag K_{-1} and three endogenous variables appear with a lead c_{+1}, r_{+1} and M_{+1}. The model also includes three policy economic variables, which are the two tax rates, t and τ and the spending on the public good g.

We will assume now that the Government wants to use fiscal policy to set the economy in the optimum state, which was computed in section 2. To do that, first it must set the spending on public good at the optimum level, which was computed in this section. Then, it will use the two taxes, which were introduced in the model. We have the proposition.

Proposition 2. There exists a unique level for the anti-immigration tax and the capital income tax such that the economic equilibrium coincides with the optimum. The values of those taxes for $t \geq 0$*, are given by the expressions*

$$\tau_t = 1 - \left(\hat{M}_t / M_0\right)^{1/\delta} / \hat{w}_t \tag{18}$$

$$t_{t+1} = \frac{(1-\beta)\hat{M}_{t+1}}{1+\hat{M}_{t+1}} \tag{19}$$

where the variables appearing with a ^ on the right-hand sides are set at their optimum values.

Proof. The difference between the optimum and the equilibrium model is that the first includes equations (11), (12) and (13) and the second has equations (14), (16) and (17). Let us set the values of the endogenous variables of the equilibrium model at their optimal values. Equation (14) determines the interest rate. Then, equations (16) and (17) give the values of the two tax rates, which are given in the proposition.■

This paper does not consider any strategic interaction between domestic residents and the Government (or the social planner). In section 2, the social planner makes all the decisions and there is no place for such interactions. In section 3, domestic residents make the investment and private consumption decisions and assume that the current and future values of their environment are given. This assumption makes sense because each domestic resident is too small to change his environment. Then, we assume that the social planner sets public consumption on its optimal path. It also sets the paths of two taxes, which leads domestic residents and foreign migrants to choose to follow the optimal path of the economy. So, this paper excludes several strategic problems. For instance, we could consider that the Government cannot tax capital, but that it can control the number of migrants. Then, it would determine this number by maximising the welfare function of domestic residents conditionally on their behaviour that is on the first order condition of their program (16). We could also compute under the same conditions the optimal path followed by the consumption of public good. We have not done that in this paper. Another question that we do not investigate is the time consistency of Government policy.

Remarks. Assume that the taxation of capital income is zero. Equation (16) shows that in the steady state we have $r=\rho$, that is that the interest rate is equal to the consumers' discount rate. Then, equation (14) shows that the capital intensity of the production process, $K/(L+M)$, is independent of the number of migrants and of any parameter of the migration equation (17). This result was obtained by Palivos (2006, figure 1 and equation 2.13 of his paper).

4. Calibration

The model was calibrated on the steady state of its market equilibrium version, with taxes set at zero and a number of migrants equal to 10% of the population of domestic residents.

The main endogenous variables and parameters were set at the following values

$r=0.1$; $M=0.1$; $Y=10$; $c/g=4$; $\beta=0.3$, $\delta=1.5$; $\theta=0.5$; $d=2.5$; $\sigma=-0.5$

We also assumed that equations (10) and (11) are satisfied, which means that the number of migrants and the quantity of public good of the steady state are optimal, conditionally on the amounts of fixed capital.

Then the values of the other variables and parameters were computed

$\rho=r=0.1$

$K=\beta Y/r=30$

$w=(1-\beta)Y/(L+M)=6.3636$

$A=YK^{-\beta}(L+M)^{-1+\beta}=3.3720$

$g=(Y-wM)/[1+c/g]=1.8727$

$$\overline{M}=\left[M^{d}+\frac{g(1+M)dM^{d-2}}{A\beta(1-\beta)[K/(1+M)]^{\beta}}\right]^{1/d}=0.9398$$

$$\frac{a}{a'} = \frac{g^{-1(1+\sigma)}\left[1-\left(0.1/\overline{M}\right)^{d}\right]^{\sigma/(1+\sigma)}}{\left[A(K/1.1)^{\beta}(1+0.1\beta)-g\right]^{-1/(1+\sigma)}} = 16.0593$$

$$b = \frac{(a/a')^{1+\sigma}}{1+(a/a')^{1+\sigma}} = \quad 0.8003$$

$$a = b^{1/(1+\sigma)} = 0.6405$$

$$a' = (1-b)^{1/(1+\sigma)} = 0.0399$$

$$c = g(c/g) = 7.4909$$

$$M_0 = M / w^{\delta} = 0.0062$$

$$u = \left\{ ac^{\sigma/(1+\sigma)} + a'\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)} \right\}^{(1+\sigma)/\sigma} = 9.367$$

$$U = u(1+1/\rho)/(1-\theta) = 67.2952$$

In the simulations of the next section, we will convert the relative change in the welfare of the domestic residents as equivalent changes in the private consumption in the steady state relative to its reference value. We have

$$\frac{dU}{U} = \frac{ac^{\sigma/(1+\sigma)}}{ac^{\sigma/(1+\sigma)} + a'\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}} \frac{dc}{c} = 0.4\frac{dc}{c}$$

5. Simulations[4]

5.1. Market equilibrium for several values of the immigration function

This series of simulations uses the market equilibrium model without taxes. We start from the reference steady state, then we assume that the immigration function (7), more precisely its scale parameter M_0, is permanently changed. In simulations 0 (1, 2 and 3) parameter M_0 is multiplied by 0 (1, 2 or 3). The welfare of domestic residents is given in the following table

[4] The model was simulated with the software Dynare, which was run under MATLAB. Dynare was developed by Michel Juillard, and can be downloaded from the website: http://www.cepremap.cnrs.fr/dynare.

Simulation	Welfare	dc/c %
0	67.3034	0.03
1=Reference	67.2952	0.00
2	67.2306	-0.24
3	67.0674	-0.85

Graphs 1 show the paths followed by the main economic variables. In the long run, the interest rate is equal to the consumers' discount rate. Then, the factor prices frontier determines the wage rate as a function of the interest rate and of the parameters of the production function. So, in the long run the costs of labour and of capital are equal to their reference values for all simulations. This is consistent with a result by Palivos, quoted above, which is that the long run equilibrium value of the capital-labour ratio is independent of the number of migrants.

If the number of foreigners who wish to migrate in the country increases, its adjustment to its new long run value is almost instantaneous. The increase in employment in the country decreases the wage rate. Then, fixed capital increases smoothly, which progressively drives the wage rate up, until it converges to its reference value.

The capital accumulation process drives the private consumption down in the short run. Then, this consumption increases and reaches in the long run a value higher than its reference level. The current utility of domestic residents follows a similar movement, although, its increase is slowed down by the congestion on the use of the public good induced by the higher number of migrants.

The progressive increase in the wage rate after its initial fall leads to a progressive increase in the number of migrants over time. However, this effect is small, and most of the increase in the number of migrants occurs at the time of the shock.

Graphs 1 also plot the effects of a decrease of parameter M_0 (in the specific case when no foreigner wishes to migrate to the country). These effects are exactly opposite to the effects of an increase of this parameter.

Finally, we can see that the welfare of domestic residents decreases when the number of migrants increases. We already noticed that an increase in the number of migrants leads to a decrease in the current utility of domestic residents in the short run and an increase in the long run. Now, we have the supplementary result that the first effect dominates. Thus, in the market equilibrium, the least foreigners want to enter the country, the better its domestic residents are. As, we will see, this does not mean that the optimal policy for the Government is to forbid all foreign immigration.

5.2. Market equilibrium with socially optimal investment policy

We will make the same simulations as in last paragraph, except that we will now assume that, when the immigration function is changed, the Government also decides to set a socially optimal investment policy. The number of migrants will still be equal to the number of foreigners who want to settle in the country, and the consumption of public good will remain at its reference value. So, we just have to substitute equation (13) to equation (16) in the model. Equation (21) gives the tax rate which has to be applied to capital income to induce domestic residents to make the socially optimal investment decision.

The welfare of domestic residents is given in the following table

Simulation	Welfare	dc/c %
0	67.3034	0.03
1	67.3106	0.06
2	67.2856	-0.04
3	67.1873	-0.40

An interesting result, different from the ones we got in previous paragraph, is that some immigration is good for the welfare of the domestic residents. A second result, which is not a surprise, is that the welfare of domestic residents is higher under the optimal investment policy than under the market equilibrium investment policy, as soon as some immigration takes place.

Graphs 2 give the paths followed by the main economic variables. The optimal investment policy progressively drives the level of fixed capital down (this decrease is a little weaker when the number of migrants is higher). This movement allows for an increase in the consumption of domestic residents in the short term and induces a decrease of it in the long run. The current utility of domestic residents follows the same trend, which dominates the effect of the changes in the congestion of the public good. The taxation rate of capital income is zero when there are no migrants: then there is no rent to extract from them and the equilibrium and optimal paths of capital are identical. This taxation rate increases with the number of foreigners who want to enter the country. However, higher taxation is insufficient to prevent capital from (slightly) increasing with the number of migrants.

We can see on Graphs 1 and 2 that the *long run* consumption of domestic residents increases with the number of migrants. This result is consistent with the conclusion reached by Moy and Yip (2006) and quoted above. Long run consumption is *lower* on Graphs 2, when the social planner sets the level of fixed capital (to a lower level than its market equilibrium value).

Wages are lower on the optimum path than in market equilibrium (for the same number of migrants and if this number is positive). The graphs also show that wages decrease with the number of migrants.

5.3. Market equilibrium with socially optimal investment policy and spending on public good

We will make the same simulations as in last paragraph, except that we will assume that when the immigration function is changed and the Government decides to set a socially optimal investment policy, it will also decide to set the consumption of public good to its optimal level. The number of migrants will still be equal to the number of foreigners who want to settle in the country. All we have to do is to stop considering the consumption of public good exogenous and to introduce equation (11) in the model.

The welfare of domestic residents is given in the following table

Simulation	Welfare	dc/c %
0	67.3061	0.04
1	67.313203	0.07
2	67.2888	-0.02
3	67.1946	-0.37

We get a higher welfare as in last paragraph, which is unsurprising because now the setting of the consumption of public good is optimal. As in the previous paragraph, we find that some immigration is good for the welfare of domestic residents.

Graphs 3 show that the quantity of public good increases with the number of migrants, which is consistent with the results of paragraph 2.2.

5.4. Social optimum

We will make the same four simulations as in last paragraph. But now, we will assume that the Government will set investment, public gross consumption and the number of migrants to their optimal levels, starting from the reference steady state.

The results of these simulations only differ by the immigration tax, which increases with the number of foreigners who want to enter the country. The welfare of domestic residents is 67.313244 ($dc/c = 0.07\%$).

Graph 4 shows that, as in the previous paragraph fixed capital smoothly decreases over time and converges to a value smaller than its reference value. The wage rate follows a similar trend. In the short run the number of migrants decreases: the cost of labour is still high and it is not in the interest of the country to accept too many foreigners. However, as the wage rate decreases over time, the country accepts more and more migrants, and their long run number is equal to its reference value.

5.5. Effects on the social optimum of a change in the preference for the public good

These series of simulations are run with the optimum model. Simulation 1 is the same as simulation 1 of the previous paragraph. In simulation 2 the economy starts in the reference steady state. Then, parameter $1-b$, which measures the relative preference for the public good, is decreased permanently by 0.1.

The welfare of domestic residents increases, from simulation 1 to simulation 2 from 67.3132444 to 67.359015 (dc/c increases from 0.07% to 0.24%).

The results of the simulation are given in Graphs 5. If we compare the second simulation to the first we can see that 1) The consumption of public good decreases, which is unsurprising ; 2) The optimal number of migrants increases ; so we confirm the result of paragraph 2.3 that a decrease in the national preference for the public good increases the number of accepted immigrants and decreases the anti-immigration tax ; we remind that the number of migrants becomes infinite when domestic residents have no preference for the public good, (parameter $1-b$ becomes 0), and then their welfare becomes infinite ; 3) The consumption of private good by domestic residents increases, which is still unsurprising ; 4) Fixed capital and current utility does not change much ; 5) The wage rate decreases, because the higher number of migrants is an incentive to extract more rent from them.

6. Conclusion

Since the seminal paper by Berry and Soligo (1969), most of the economic literature has dismissed the economic importance of migrations for national income and the aggregated welfare of domestic residents. It bases this evaluation on the theoretical result that, if the initial number of migrants is zero, the entry of a small number of migrants will have a second order effect on national income and welfare. Thus, economists have preferred to focus on the effects of migrations on income distribution: the raise in profits and in the wages of skilled workers, the deterioration of the conditions of unskilled workers and the split in public opinion between those who favour and those who are against allowing more migrants in the country.

Our simulations confirm that migrations and economic policies taking migrations into accounts may have limited effects on the welfare of domestic residents, even when they change much in the other features of the economy. However, these effects are not always negligible. For instance, we computed that the gain in welfare resulting from the optimum policy is equivalent to an increase in the steady state consumption by 0.07%, which is pretty low. However, we took as reference an economic equilibrium where very few foreigners *wished* to migrate into the country. If we take as reference the economic equilibrium where the number of these foreigners represents 30% of the active population of the country (a reasonable assumption), the increase in welfare is pushed to 0.85+0.07=0.92%, which is a huge gain. This gain does not wholly result from immigration controls. If there are no controls, but if the Government sets fixed investment and the consumption of public good to optimal levels, the gain in welfare is still equivalent to an increase in steady state consumption of 0.85-0.37=0.48%.

References

Berry R. Albert and Ronald Soligo (1969) - "Some Welfare Aspects of International Migration", *Journal of Political Economy* 77, 778–794.

Borjas Georges (1999) – “The Economic Analysis of Immigration”, *Handbook of Labor Economics*, volume 3A, edited by Orley Ashenfelter and David Card, North Holland, 1697-1760.

Hazari Bharat R. and Pasquale M. Sgro (2003) – The Simple Analytics of Optimal Growth with Illegal Migrants”, *Journal of Economic Dynamics and Control* 28(1), 141-151.

Moy Hon Man and Chong K. Yip (2006) – “The Simple Analytics of Optimal Growth with Illegal Migrants: A Clarification”, *Journal of Economic Dynamics and Control*, forthcoming.

Palivos Theodore (2006) – “Optimal Growth with Illegal Immigrants: The Competitive Equilibrium Case”, mimeographed, Department of Economics, University of Macedonia, April.

Razin Assaf, Efraim Sadka and Phillip Swagel (2002) – “Tax Burden and Migration: A Political Economy Theory and Evidence”, *Journal of Public Economics* 85, 167-190.

Appendix

Proof of Lemma 1. We have to minimise with respect to M the function

$$v(M)=a\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})-g\right]^{\sigma/(1+\sigma)}+a'\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}$$

This function is defined on the range $0\leq M<\overline{M}$. We have $v(0)=a\left[AK_{-1}^{\beta}-(K-K_{-1})-g\right]^{\sigma/(1+\sigma)}+a'g^{\sigma/(1+\sigma)}>0$ and $v\left(\overline{M}\right)=+\infty$. The derivative of $v(M)$ is

$$v'(M)=\frac{\sigma M}{1+\sigma}\left[z(M)-y(M)\right]$$

with $z(M)\equiv\dfrac{aA\beta(1-\beta)}{1+M}\left[A\left(\dfrac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})-g\right]^{-1/(1+\sigma)}\left(\dfrac{K_{-1}}{1+M}\right)^{\beta}$

and $y(M)\equiv a'dg^{\sigma/(1+\sigma)}\dfrac{M^{d-2}}{\overline{M}^{d}}\left[1-\left(\dfrac{M}{\overline{M}}\right)^{d}\right]^{-1/(1+\sigma)}$

This derivative is zero for $M=0$. Its second root is the solution of the equation $z(M)=y(M)$. $y(M)$ is an increasing function, with $y(0)=0$ and $y(\overline{M})=+\infty$. If the spending on public good g increases, then $y(M)$ decreases. $z(M)$ is a positive decreasing function, with $z(0)>0$. If the spending on public good g increases, then $z(M)$ increases. Thus, the equation has a unique root, which is increasing with g. This root converges to zero with g. It converges to a value less than $\overline{M}$ when g tends to $AK_{-1}^{\beta}-(K-K_{-1})$.■

Analysis of the case when the indivisibility threshold in the public good differs from the size of the population of domestic residents.

a) *Congestion starts for a number e of users smaller than the population of domestic residents:* $0\leq e<1$.

Then, the contribution of the public good service in the utility function U is

$$a'\left[g\left[1-\left(\frac{1-e+M}{\overline{M}}\right)^d\right]\right]^{\sigma/(1+\sigma)} \text{, with } \overline{M}>1-e$$

The utility function is defined for $0 \le M < \overline{M}-(1-e)$. The adjustments to the proof of Lemma 1 are

$$v(0)=a\left[AK_{-1}^{\beta}-(K-K_{-1}-g)\right]^{\sigma/(1+\sigma)}+a'\left[g\left[1-\left(\frac{1-e}{\overline{M}}\right)^d\right]\right]^{\sigma/(1+\sigma)}>0 \text{, and}$$

$v(\overline{M}-(1-e))=+\infty$. Then,

$$y(M)\equiv a'dg^{\sigma/(1+\sigma)}\frac{(1-e+M)^{d-1}}{M\overline{M}^d}\left[1-\left(\frac{1-e+M}{\overline{M}}\right)^d\right]^{-1/(1+\sigma)}$$

$$v'(0)=-\frac{\sigma}{1+\sigma}a'dg^{\sigma/(1+\sigma)}\frac{(1-e)^{d-1}}{\overline{M}^d}\left[1-\left(\frac{1-e}{\overline{M}}\right)^d\right]^{-1/(1+\sigma)}>0$$

So, when there are no migrants in the country, a small arrival of foreign workers has a negligible (second order) effect on national income, and decreases the public service available to domestic residents (by increasing the congestion effect). So, the welfare of domestic residents decreases.

We also have $y(0)=y(1-e+\overline{M})=+\infty$. Then, the equation $y(M)=z(M)$ may have no solution or an even number of solutions. In the second case, the best solution will determine a positive optimal number of migrants if the associated utility of domestic residents is higher than when there are no migrants in the country. Moreover, the optimal number of migrants can jump from a positive value to zero when the spending on public good or the amount of fixed capital changes by a small amount.

b) *Congestion starts for a number* e *of users larger than the population of domestic residents:* $e \ge 1$.

Then, the contribution of the public good service in the utility function U is

$a'g^{\sigma/(1+\sigma)}$, if $0 \le M \le e-1$

$$a'\left[g\left[1-\left(\frac{1-e+M}{\overline{M}}\right)^{d}\right]\right]^{\sigma/(1+\sigma)}, \text{ if } M \geq e-1$$

The utility function is defined for $0 \leq M < \overline{M}-(1-e)$.

Then, if $M \leq e-1$, migrants do not induce any congestion effect, and the utility of domestic residents increases with the number of migrants, as national income.

If $M \geq e-1$, the adjustments to the proof of Lemma 1 are $0 < v(e-1) < v(0)$ and $v(\overline{M}-(1-e)) = +\infty$. We also have $v'(e-1) < 0$. The function $y(M)$ increases from zero to infinity when M increases from $e-1$ to $\overline{M}-(1-e)$. Thus, the optimal number of migrants is positive and larger than $e-1$. Finally, the results that we obtain in this case are very similar to those of Lemma 1.

Proof of Lemma 2. We have to minimise with respect to g the same objective function as in the proof of lemma 1, now denoted $v(g)$. This function is defined on the range $0 < g < A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})$. We have $v(0) = v\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})\right] = +\infty$. The derivative of $v(g)$ is

$$v'(g) = \frac{\sigma}{1+\sigma}\left[-z(g)+y(g)\right]$$

with $z(g) \equiv a\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M)-(K-K_{-1})-g\right]^{-1/(1+\sigma)}$

and $y(g) \equiv a' g^{-1/(1+\sigma)}\left[1-\left(\frac{M}{\overline{M}}\right)^{d}\right]^{\sigma/(1+\sigma)}$

$y(g)$ is a positive decreasing function, with $y(0) = +\infty$. If the number of migrants M increases, then $y(g)$ increases. $z(g)$ is an increasing function with

$z(0) = a\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M) - (K-K_{-1}) - g \right]^{-1/(1+\sigma)} > 0$, and

$z\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M) - (K-K_{-1}) \right] = +\infty$. If the number of migrants increases, then $z(g)$ decreases. If the preference of domestic residents for the public good decreases (a increases and a' decreases), then $z(g)$ increases and $y(g)$ decreases.

Thus, equation $v'(g)$ has a unique root, which is increasing with the number of migrants. This root belongs to the range on which function $v(g)$ is defined. It is equal to $\bar{b}\left[AK_{-1}^{\beta} - (K-K_{-1})\right]$, when $M = 0$. It converges to $A\left(\frac{K_{-1}}{1+\overline{M}}\right)^{\beta}\left(1+\beta\overline{M}\right) - (K-K_{-1})$ when M tends to $\overline{M}$.■

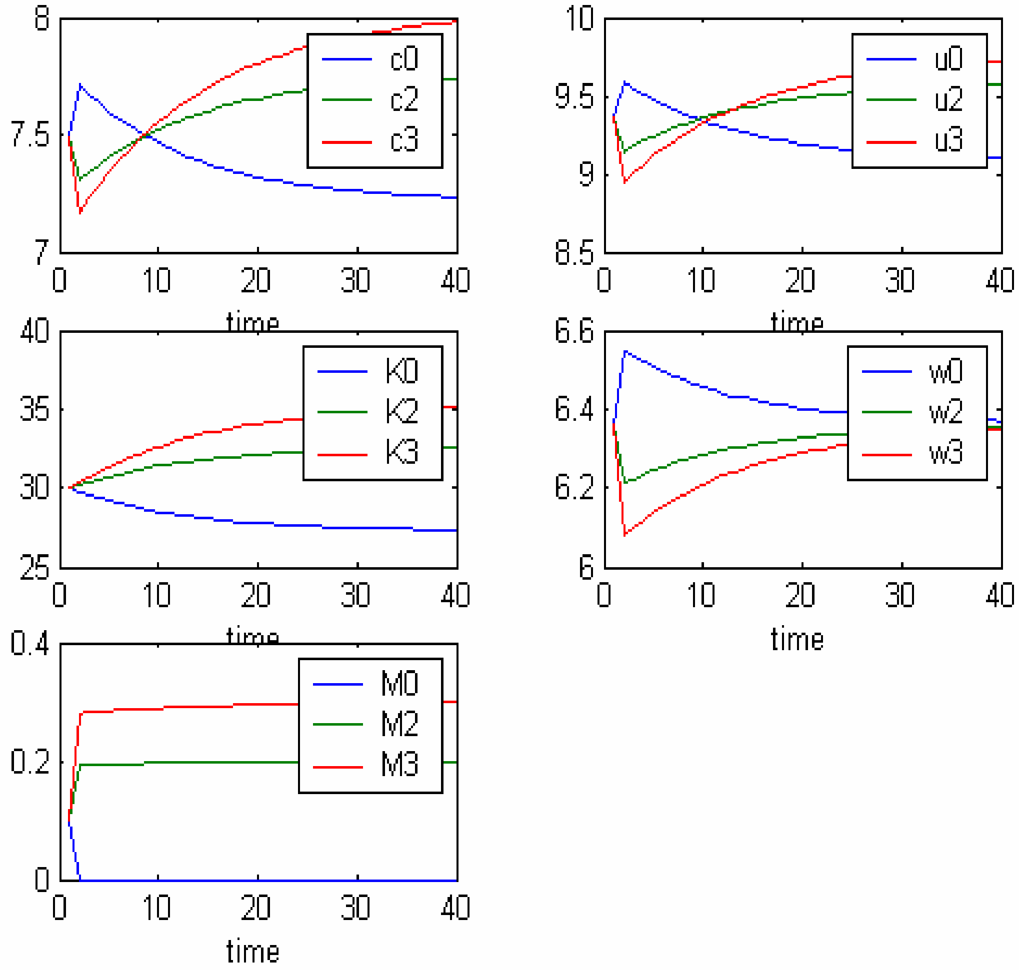
Proof of Lemma 3. To compute the optimal capital at the end of period t, K, we have to maximise the objective function appearing in equation (9) with respect to this variable. After the elimination of the useless terms, the expression to maximise is

$$\left\{ a\left[A\left(\frac{K_{-1}}{1+M}\right)^{\beta}(1+\beta M) - (K-K_{-1}) - g \right]^{\sigma/(1+\sigma)} + a'\left[g\left[1-\left(M/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)} \right\}^{(1-\theta)(1+\sigma)/\sigma} +$$

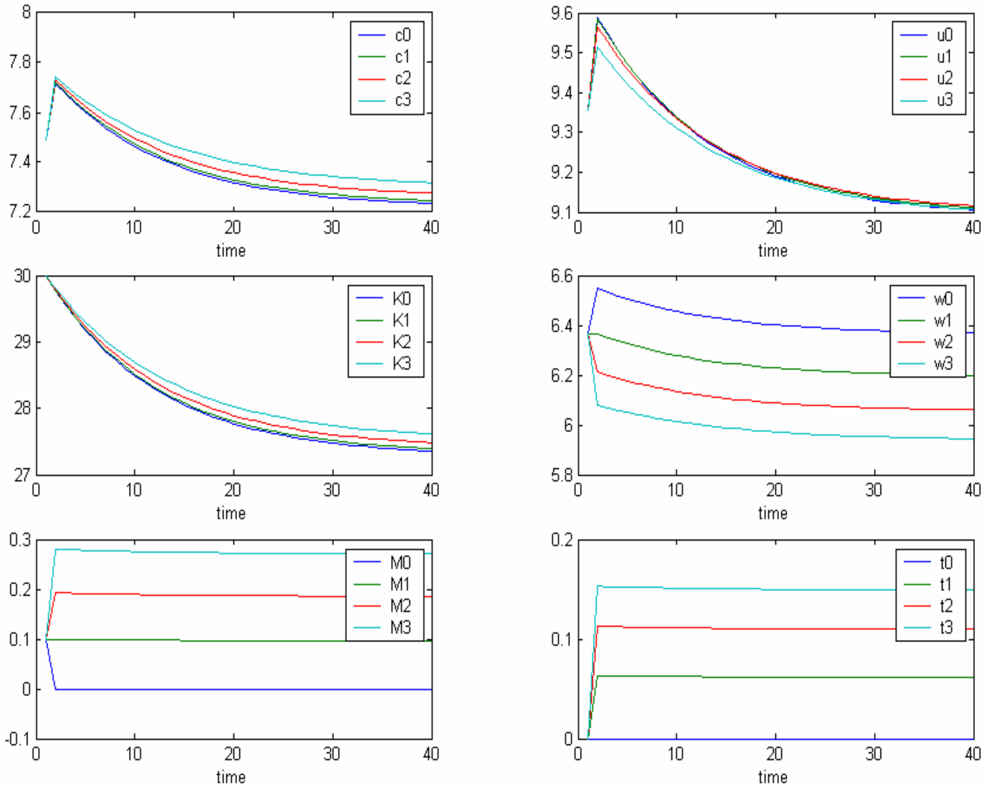
$$\frac{1}{1+\rho}\left\{ a\left[A\left(\frac{K}{1+M_{+1}}\right)^{\beta}(1+\beta M_{+1}) - (K_{+1}-K) - g_{+1} \right]^{\sigma/(1+\sigma)} + a'\left[g_{+1}\left[1-\left(M_{+1}/\overline{M}\right)^{d}\right]\right]^{\sigma/(1+\sigma)} \right\}^{(1-\theta)(1+\sigma)/\sigma}$$

The first-order condition of this program is equation (13).■

GRAPHS 1



GRAPHS 2



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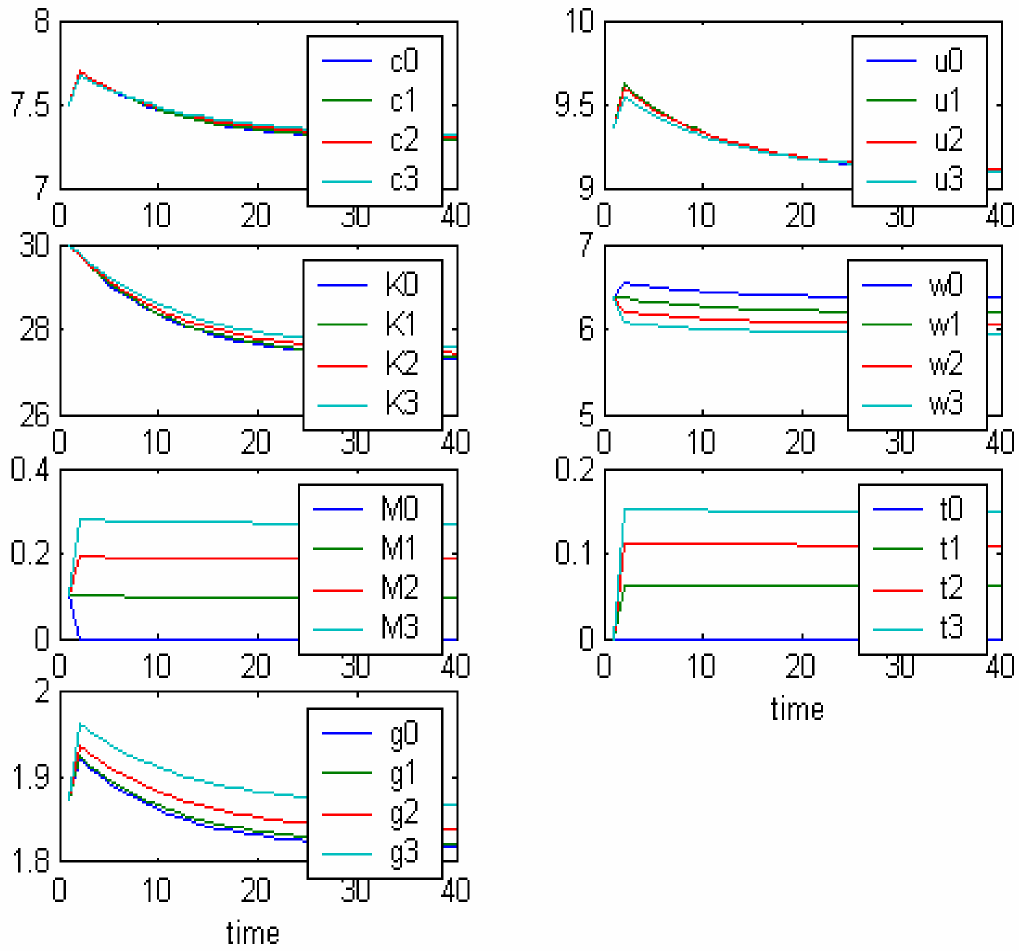
GRAPHS 3



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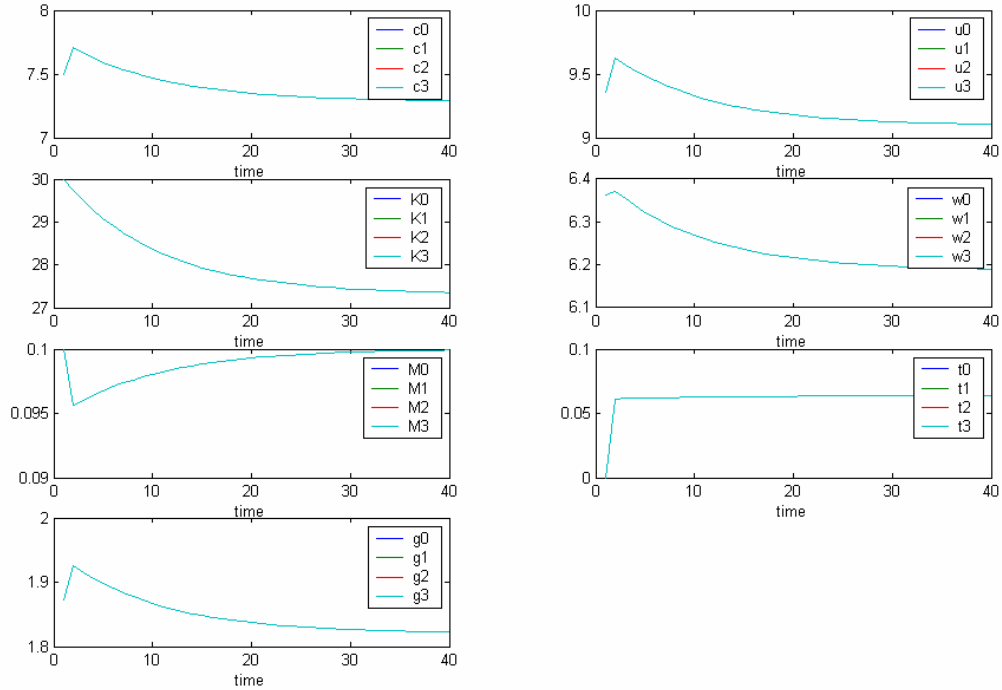
GRAPHS 4



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GRAPHS 5