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Private Information : An Argument for a Fixed Exchange Rate System

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Abstract

In a two-country model, the paper considers reputational equilibria for monetary policies in the case where the central banks have some private information. It is shown that a fixed exchange rate system may lead, in both countries, to lower inflation biases than a flexible exchange rate system. No exogenous costs (like "political costs") of leaving the fixed exchange rate system are required for such a result to hold. The reason is that private information makes a money supply rule more difficult to sustain through reputational forces than an exchange rate rule.

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1 Introduction

One of the argument in favor of a fixed exchange rate system which has been developed, in the literature is that, by pegging its exchange rate, a country can improve the credibility of its monetary policy. For, this permits this country to tie its monetary policy to the monetary policy of a foreign central bank which has better credibility and therefore a lower expected inflation rate[1].

There are however two problems with that argument. The first is that the leader of the fixed exchange rate system could loose. When the exchange rate is flexible, a country which unexpectedly increases the money supply in order to improve employment bears the additional inflationary cost due to the implied depreciation of the exchange rate. Such a cost is not present when the exchange rate is fixed. This creates a better trade-off between inflation and employment under a fixed exchange rate system than under a flexible one. The central bank is then more tempted to create monetary surprises. This, in turn, leads to higher expected inflation[2]. As shown by von Hagen (1992), however, this negative effect of a fixed exchange rate could disappear in a repeated game framework where reputational forces are at work. Thus, the leader of the fixed exchange rate system could also gain.

The second problem with the credibility argument of a fixed exchange rate system is that it requires the presence of some exogenous costs, due for example to the existence of "political" costs, of leaving the system[3]. This is true in the one shot game as well as in the repeated game. In the one-shot game there is always an incentive for the country which pegs the exchange rate to abandon that peg, in the absence of these exogenous costs. Furthermore, even in the repeated game with reputational equilibria, some exogenous costs of leaving the fixed exchange rate system are also required. As also underlined by von Hagen (1992), without such costs the incentives that the country which pegs the exchange rate faces when it has to decide on whether to renege on the announced rule or not, would be the same in the fixed exchange

[1]See Giavazzi and Pagano (1988) and Giavazzi and Giovannini (1989). The argument has been developed in the context of the European Monetary System with Germany as the leader. Thus, other European countries, by tying their monies to the Deutsche Mark could import the credibility of the Bundesbank which was considered to have a better credibility than the cental bank of other European countries. The argument has been cast in the framework developed by Kydland and Prescott (1977) and Barro and Gordon (1983a).

[2]Rogoff (1985) introduced such an argument to show the possibility of a "counterproductive" international cooperation.

[3]See Melitz (1988) and Giavazzi and Giovannini (1989) for a discussion on this issue.

rate system as in the flexible exchange rate system. Consequently, the fixed exchange rate system could not help to sustain better rules than the flexible exchange rate system through reputational forces.

These arguments in the literature were developed in the context of a model where there was no private information problem. But, as underlined by Canzoneri (1985) in a closed economy framework, private information considerations are likely to be important for the issue of credibility of monetary policy and of its proposed remedies. For, if the central banks possesses some private information on some shock of the economy, then the public may be unable, even ex post, to tell whether some rule has been followed or not. And this might give an incentive to the central bank to cheat.

In the present paper, such private information considerations will be introduced in a two-country framework, in which we will study reputational equilibria in the repeated game. The main result we will obtain is that, in both countries expected inflation may be lower (and the corresponding bias smaller) under a fixed exchange rate system than under a flexible exchange rate system, even when *no* exogenous costs of leaving the fixed exchange rate system are introduced into the analysis. The intuitive reason underlying such a result is that an exchange rate rule is not subject to the same private information problem as a money supply rule would be in a flexible exchange rate system. For, even if there is some forecast error made by the central bank when predicting the money demand shock, for example, such a forecast error does not prevent the public to know whether the exchange rate rule has been followed or not. The differential impact of private information on the two systems will be enough to change the incentives of the central bank in the required direction. This is why we will not have to rely on the presence of exogenous costs of leaving the exchange rate system.

To develop that argument, however, we will have, first, to complete the formal analysis of reputational equilibria under private information for monetary policy that was introduced by Canzoneri (1985). For simplicity, this will be more conveniently done in a closed economy frawework, the application to the two-country model used later on being then rather straightforward. Furthermore, the extension of the analysis that we make might be valuable for its own sake. For, we relate the marginal condition given by Canzoneri (1985) under private information, which is shown to be actually only necessary but not always sufficient, to the condition obtained when information is symmetric. In the case where the forecast error made by the central bank follows a uniform probability distribution, we give a necessary and sufficient condition for a money supply rule to be a reputational equilibrium.

Section 2 develops the extended analysis of reputational equilibria for monetary policy under private information in a closed economy framework. Section 3 introduces private information in a two-country model and considers reputational equilibria, both under a flexible exchange rate system and under a fixed exchange rate system. The main argument on the possible superiority of the fixed exchange rate system is then developed. Section 4 concludes.

2 Reputational equilibria under private information

In order to compare exchange rate rules to money supply rules in the two-country model of the next section, we will need, in the private information case as well as in the symmetric information case, necessary and sufficient conditions for a given monetary policy rule to be a reputational equilibrium.

Canzoneri (1985) has adapted the reputational equilibrium approach of Barro and Gordon (1983b) to the case of private information and gave a condition for the best monetary policy rule to be sustainable. This condition stated that no marginal deviation from that rule should be profitable. However, for own purpose, this analysis has to be completed in two directions. First, even when the best rule cannot be sustained as a reputational equilibrium, other rules, which are nonetheless less inflationary and therefore better than discretion, could be sustained. We will therefore look for conditions under which such rules are sustainable through reputational forces under private information. Second, the marginal condition given by Canzoneri is actually only a necessary condition. As we will see, even when this condition is satisfied it may still be possible for a non-marginal deviation to be profitable. We will therefore look for conditions which are both necessary and sufficient. Then, we will also be able to compare these conditions to the ones obtained in the symmetric information case. For simplicity, in this section, we will consider the issue in a closed-economy framework, as in Canzoneri (1985). The extension to the two- country case of the next section will be straightforward.

In order to simplify the analysis we make, at some point, the assumption of a uniform probability distribution for the forecast error of the central bank. This will actually rule out reversions to more inflationary episodes corresponding to reversions to discretionary policy, which would occur with other distributions even if the rule is being followed by the central bank, as

Canzoneri (1985) emphasized. Although this may eliminate some interesting aspects of the introduction of private information in the model, this is not really damaging from our point of view. For, our argument will use the property that there is a loss implied by the existence of private information. In the model, this loss could come from two sources. One is that, under private information, the conditions of sustainability are different from those under symmetric information. This is the point we will emphasize. When these conditions are more restrictive, it becomes more difficult to sustain some monetary policy rule under private information, which involves a cost. The other reason why the existence of private information may be costly would consist in the reversion to inflationary periods, which would occur with other distributions than the uniform distribution. For simplicity, this is not taken into account here. However, it is likely that introducing this point into the analysis would only add to the loss implied by private information and therefore would not change our basic argument and result.

The model is a standard one in the literature on the subject. We have

$$y_t = (1 - \alpha)\, n_t \tag{1}$$

$$w_t - p_t = -\alpha n_t \tag{2}$$

$$m_t = p_t + y_t + x_t \tag{3}$$

All variables are in logarithms. Equation (1) is a production function relating output y_t to employment n_t. Equation (2) is obtained by equating the marginal productivity of labor to the wage rate, where w_t is the nominal wage and p_t the output price. Equation (3) equates money supply to money demand through a simple quantity equation. In this equation, x_t is a zero-mean serially independent random variable which represents a money demand shock. The variable x_t is not known in period t at the time the central bank chooses its money supply m_t, but it is publicly known ex post at the end of period t after monetary policy has been set (because all variables y_t, p_t and m_t are assumed to be observable ex post). In period t before choosing m_t, the central bank has a forecast ξ_t of x_t. we can write

$$x_t = \xi_t + \varepsilon_t \tag{4}$$

where ε_t is the forecast error made by the central bank, which is an unserially correlated and independently distributed random variable.

In the case of private information, it is assumed that the forecast ξ_t is private information of the central bank. This means that the wage setters do not know ξ_t even ex post and will therefore be unable, at the end of period t, to decompose x_t into its components ξ_t and ε_t.

¿From (1), (2) and (3) we get the reduced form

$$n_t = m_t - w_t - x_t \tag{5}$$

$$p_t = w_t + \alpha\left(m_t - w_t - x_t\right) \tag{6}$$

The nominal wage w_t is predetermined. It is assumed that it is set by the private sector at the end of period $t-1$ in order to minimize the squared deviation n_t^2 of employment from a desired level, normalized to zero in the model. From (2), this gives, with rational expectations :

$$w_t = E_{t-1}p_t \tag{7}$$

which implies

$$E_{t-1}n_t = E_{t-1}y_t = 0 \tag{8}$$

where E_{t-1} is the expectation operator, conditional on information available at the end of period $t-1$, which is assumed to contain all variables (except the money demand forecast) up to period $t-1$.

¿From (5) and (8) we get $w_t = E_{t-1}m_t$. Therefore (5) and (6) can also be written :

$$n_t = m_t - E_{t-1}m_t - x_t \tag{5'}$$

$$p_t = E_{t-1}m_t + \alpha\left(m_t - E_{t-1}m_t - x_t\right) \tag{6'}$$

The central bank, who is assumed to represent social preferences, has both an employment target and an inflation rate target, which for simplicity is taken here to be zero. We have the following period t loss functions :

$$\Lambda_t = \left(n_t - \tilde{n}\right)^2 + \chi\pi_t^2 \qquad , \qquad \chi > 0, \quad \tilde{n} > 0 \tag{9}$$

where π_t is the inflation rate $p_t - p_{t-1}$, χ represents the relative weight between the two objectives and where $\tilde{n}$ is the level of employment desired by the central bank. It is assumed that this level is higher than the level desired by the private sector (we have $\tilde{n} > 0$). (As underlined in the literature

(Barro and Gordon (1983a)), the too low level of employment desired by the private sector is due to the existence of some distortions which, for example, could come from the presence of taxes). This creates an incentive for the central bank to unexpectedly increase the money supply in order to raise employment above the level desired by the private sector.

Under discretion, the central bank, when it determines monetary policy, takes as given the nominal wage w_t and minimizes the expected loss $E_t^c \Lambda_t$, where E_t^c is the expectation operator conditional on information available in period t to the central bank when it takes its decision on monetary policy (the superscrit "c" is for "central bank"). Therefore E_t^c (contrary to E_t, defined above as being conditional on information available at the end of period t by wage setters, at the moment they choose the wage rate w_{t+1} for period $t+1$) does not contain x_t in its information set but only its forecast ξ_t. Using (5)-(9), we easily obtain the following solutions under discretion :

$$n_t^d = -\varepsilon_t \tag{10a}$$

$$\pi_t^d = \frac{\widetilde{n}}{\chi\alpha} - \alpha\varepsilon_t \tag{10b}$$

where d is the index for "discretion". From (6) and (10b), this corresponds to a money supply growth rate μ_t, defined by $\mu_t = m_t - m_{t-1}$, which is equal to

$$\mu_t^d = \frac{\widetilde{n}}{\chi\alpha} + \xi_t + p_{t-1} - m_{t-1} \tag{11}$$

We see that the predicted money demand shock ξ_t is fully neutralized by an equal increase in the money supply.

This gives the expected loss under discretion :[4]

$$E\Lambda_t = E_t^c \Lambda_t^d = \left(1 + \frac{1}{\chi\alpha^2}\right)\widetilde{n}^2 + \left(1 + \chi\alpha^2\right)\sigma_\varepsilon^2 \tag{12}$$

where σ_ε^2 is the variance of ε_t. In (12), the last term corresponds to the loss implied by the forecast error ε_t.

As it has been emphasized in the literature (Kydland and Prescott (1977) and Barro and Gordon (1983a)), there is an inflation bias, expected inflation being equal to $\widetilde{n}/\chi\alpha$. Such a biais involves an expected loss $\widetilde{n}^2/\chi\alpha^2$ without

[4]We have $E\Lambda_t = EE_t^c\Lambda_t$ which is equal to $E_t^c\Lambda_t$ because the forecast errors ε_t are white noise.

any gain : expected employment, according to (8), stays at the level desired by the private sector.

It would therefore be better for the monetary authorities to commit to a rule, where the objective would be to maintain the expected inflation rate at its desired level. Such a rule can be written

$$E_t^c \pi_t^o = 0 \tag{13}$$

when the index "o" represents this "optimal" rule. Using (9), the corresponding rule for the money supply is given by

$$\mu_t^o = \xi_t + p_{t-1} - m_{t-1} \tag{14}$$

Under such a rule we get

$$n_t^o = -\varepsilon_t \tag{15a}$$

$$\pi_t^o = -\alpha\varepsilon_t \tag{15b}$$

$$E\Lambda_t^o = E_t^c \Lambda_t^o = \tilde{n}^2 + \left(1 + \chi\alpha^2\right)\sigma_\varepsilon^2 \tag{15c}$$

The inflation bias and its corresponding loss have disappeared, while the money supply continues to respond to the forecast of money demand shocks in order to offset them.

More generally we will consider the rules

$$E_t^c \pi_t^r(\gamma) = \gamma \tag{16}$$

where r is the index for "rule" and where γ is the targeted expected inflation rate. Such a target corresponds to the money supply growth rate rule:

$$\mu_t^r(\gamma) = \gamma + \xi_t + p_{t-1} - m_{t-1} \tag{17}$$

and the expected loss is given by

$$E\Lambda_t(\gamma) = E_t^c \Lambda_t(\gamma) = \tilde{n}^2 + \chi\gamma^2 + \left(1 + \chi\alpha^2\right)\sigma_\varepsilon^2 \tag{18}$$

Call γ_d the value of γ which would give discretion. We have from (10b) :

$$\gamma_d = \frac{\tilde{n}}{\chi\alpha} \tag{19}$$

When we have $0 \leq \gamma < \gamma_d$ the rule γ improves upon discretion. The lower γ is in this interval, the better the rule is.

As Barro and Gordon (1983b) have shown in a model without money demand shocks, such a rule may be obtained as a reputational equilibrium in the repeated game. In fact, if information were symmetric, the same analysis could be applied here. The central bank in period $t-1$ announces the rule γ before wage are set by the private sector. If, in period t, the rule is followed by the central bank, then the wage setters continues to expect the rule for period $t+1$. If, on the contrary, in period t, the central bank deviates from the rule, then the wage setters at the end of period t expect that discretion will hold in $t+1$. This "punishment" will be assumed here to last only one period. (This means that at the end of period $t+1$ the wage setters will expect the rule to be followed in period $t+2$).

The central bank, in period t, minimizes the expected value of its discounted loss, that is it minimizes $E_t^c \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}$, where $\delta\,(0 < \delta < 1)$ is the discount rate. The central bank therefore "cheats" in period t if the gain of deviating from the rule is larger than the cost of the "punishment". This cost is equal to the discounted loss which results from having the inflationary discretionary solution instead of the rule in period $t+1$. Therefore the central bank does not renege in period t if and only if we have

$$E_t^c \Lambda_t(\gamma) - E_t^c \Lambda_t^{ch}(\gamma) \leq \delta \left[E_t^c \Lambda_{t+1}^d - E_t^c \Lambda_{t+1}(\gamma) \right] \tag{20}$$

where $\Lambda_t(\gamma)$ represents the loss when the rule is expected by the wage setters and followed by the central bank, while $\Lambda_t^{ch}(\gamma)$ represents the loss when the rule is expected by the private sector but the central bank cheats in period t. ¿From (12) and (18) we get

$$E_t^c \Lambda_{t+1}^d - E_t^c \Lambda_{t+1}(\gamma) = \chi \left(\gamma_d^2 - \gamma^2 \right) \tag{21}$$

This simply represents the loss due to the difference between expected inflation rates under the rule and under discretion (when in both cases expectations of the private sector are fulfilled).

In order to calculate the expected gain from cheating, call $\Lambda_t(\gamma, \mu_t)$ the period t loss of the central bank when the rule γ was expected, in period $t-1$, by the wage setters to hold in period t but that, in period t, the central bank deviates and unexpectedly chooses the money supply growth rate μ_t (instead of $\mu_t^r(\gamma)$ as expected). From (5), (6) and (9) we get :

$$E_t^c \Lambda_t\left(\gamma, \mu_t\right)=\left(\mu_t-\mu_t^r(\gamma)-\tilde{n}\right)^2+\chi\left[\alpha\left(\mu_t-\mu_t^r(\gamma)\right)+\gamma\right]^2+\left(1+\chi \alpha^2\right) \sigma_{\varepsilon}^2 \tag{22}$$

This gives, using (19) :

$$\frac{\partial E_t^c \Lambda_t\left(\gamma, \mu_t\right)}{\partial \mu_t}=2\left[-\chi \alpha\left(\gamma_d-\gamma\right)+\left(1+\chi \alpha^2\right)\left(\mu_t-\mu_t^r(\gamma)\right)\right] \tag{23}$$

Equating this derivative to zero, we get the cheating money supply growth rate $\mu_t^{ch}(\gamma)$:

$$\mu_t^{ch}(\gamma)=\mu_t^r(\gamma)+\frac{1}{\alpha \eta}\left(\gamma_d-\gamma\right) \tag{24}$$

Where η is a parameter given by

$$\eta=1+\frac{1}{\chi \alpha^2}>1 \tag{25}$$

¿From (18), (22) and (24) we then obtain

$$E_t^c \Lambda_t(\gamma)-E_t^c \Lambda_t^{ch}(\gamma)=\frac{\chi}{\eta}\left(\gamma_d-\gamma\right)^2 \tag{26}$$

We see that when the inequality $\gamma<\gamma_d$ holds, there is always an incentive to raise employment by unexpectedly increasing the money supply. Using (21) and (26), condition (20) indicates that the rule γ is a reputational equilibrium if and only if we have $\Gamma^s \leq \gamma \leq \gamma_d$, where Γ^s (where s means "symmetric information") is given by

$$\Gamma^s=\frac{1-\delta \eta}{1+\delta \eta} \gamma_d \tag{27}$$

Γ^s is the rule with the lower expected inflation rate which can be sustained as a reputational equilibrium. If δ is close enough to 1 ($\delta \geq 1/\eta$) we have $\Gamma^s \leq 0$ and therefore the optimal rule $\gamma=0$ is reputational equilibrium. In the case $\delta<1/\eta$, we have $\Gamma^s>0$ and the optimal rule $\gamma=0$ cannot be sustained. Then Γ^s represents the best rule which can be sustained through reputational forces.

The above analysis is standard in the literature and represents the benchmark case of symmetric information, which would occur if ξ_t were known to the wage setters. However, when there is private information the above analysis has to be modified. For, at the end of period t, the wage setters cannot tell whether the rule $\mu_t^r(\gamma)$has been followed or not, because they do not know the forecast ξ_t, or equivalently the forecast error ε_t. When there is private information, the behavior of the central bank should be judged by wages setters according to variables which are observed by them. Canzoneri

(1985) proposed to modify the previous analysis in such a way. Thus, define $\widetilde{\mu}_t^r(\gamma)$ as the value obtained by substituting x_t to ξ_t in the expression (17) giving $\mu_t^r(\gamma)$. We have $\widetilde{\mu}_t^r(\gamma) = \mu_t^r(\gamma) + \varepsilon_t$. The wage setters observe the value of $\widetilde{\mu}_t^r(\gamma)$ because they observe x_t. Then, consider the following reputational mechanism where as long as we have $\mu_t \leq \widetilde{\mu}_t^r(\gamma) + \overline{\varepsilon}$, the wage setters, at the end of period t, continue to expect the rule γ to be followed by the central bank next period ; but when we have $\mu_t > \widetilde{\mu}_t^r(\gamma) + \overline{\varepsilon}$, there is a one-period reversion to the discretionary solution, in period $t+1$. The constant $\overline{\varepsilon}$ is present in order to take into account the fact that there could be a forecast error ε_t which could make the money supply growth rate differ from $\widetilde{\mu}_t^r(\gamma)$ even if the rule is being followed by the central bank. It is only when the money supply growth rate exceeds $\widetilde{\mu}_t^r(\gamma)$ by the threshold value $\overline{\varepsilon}$ that wage setters "punish" the central bank by returning to discretion next period. (The value of the parameter $\overline{\varepsilon}$ will be optimally chosen and its choice will be discussed below in the case of the uniform distribution).

As we have $\widetilde{\mu}_t^r(\gamma) = \mu_t^r(\gamma) + \varepsilon_t$, the inequality $\mu_t > \widetilde{\mu}_t^r(\gamma) + \overline{\varepsilon}$ can also be written $\varepsilon_t < \mu_t - \mu_t^r(\gamma) - \overline{\varepsilon}$. Therefore, the probability of reversion to discretion is equal to $F(\mu_t - \mu_t^r(\gamma) - \overline{\varepsilon})$ where $F(\cdot)$ is the cumulative distribution function of the random variable ε_t.

The rule is sustainable if and only if there is no incentive in period t for the central bank to deviate from that rule, given that, in period $t-1$, the wage setters expected the rule to be followed. First, as in Canzoneri (1985), consider a marginal increase in the money supply above the rule. The expected marginal gain of such a marginal cheating is equal to - $\partial E_t^c \Lambda_t(\gamma, \mu_t) / \partial \mu_t$ evaluated at $\mu_t = \mu_t^r(\gamma)$. Making $\mu_t = \mu_t^r(\gamma)$ in the expression (23), gives a marginal gain equal to $2\alpha\chi(\gamma_d - \gamma)$.

The expected cost of this marginal cheating is, on the other hand, equal to $\delta\partial\left[F(\mu_t - \mu_t^r(\gamma) - \overline{\varepsilon})\left[E_t^c \Lambda_{t+1}^d - E_t^c \Lambda_{t+1}(\gamma)\right]\right] / \partial \mu_t$.

Using (21) and making $\mu_t = \mu_t^r(\gamma)$, this is equal to $\delta f(-\overline{\varepsilon})\chi(\gamma_d^2 - \gamma^2)$, where $f(\cdot)$ is the derivative of $F(\cdot)$ and therefore is the density function of the probability distribution of ε_t.

Writing that the expected marginal gain is not higher than the expected marginal cost, and using the inequality $\gamma < \gamma^d$, we therefore get the inequality:

$$\delta f(-\overline{\varepsilon})(\gamma_d + \gamma) \geq 2\alpha$$

which can be written

$$\gamma \geq \Gamma^P$$

where Γ^P (where the index "P" means "Private information") is given by[5]

$$\Gamma^P = \frac{2\alpha}{\delta f(-\overline{\varepsilon})} - \gamma_d \tag{28}$$

In the case $\Gamma^P < \gamma_d$, all γ such that $\Gamma^P \leq \gamma \leq \gamma_d$ satisfy the marginal condition at $\mu_t^r(\gamma)$. In the case $\Gamma^P \geq \gamma_d$ only γ_d can be a reputational equilibrium[6].

In order to have $\Gamma^P < \gamma_d$ and therefore rules other than discretion sustainable as a reputational equilibrium, we must have, according to (28), the inequality

$$f(-\overline{\varepsilon}) > \frac{\alpha}{\delta\gamma_d} \tag{29}$$

This indicates that the density function has to be larger, at some point, than $\alpha/(\delta\gamma_d)$.

Take the case of a uniform distribution for ε_t, which will be considered in the rest of our analysis. Such a distribution, of support $[-\theta, +\theta]$, is represented in figure 1. Then condition (29) becomes[7]

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[5] As was already emphasized by Canzoneri (1985), a larger value of γ_d, and therefore a greater amount of distortions, actually lowers Γ^P and, consequently, makes it easier to satisfy this marginal condition. The reason of such a "paradoxical" result is that when γ_d increases, the punishment in case of reneging becomes harsher because this punishment consists in returning to the discretionary solution with expected inflation γ_d. This reduces the incentive to renege. It can be seen that this effect, going through a harsher punishment, actually dominates the effect going through a larger incentive to deviate which results from the fact that the expected marginal gain also increases with γ_d. For, as we have seen, the expected marginal gain is proportional to $\gamma_d - \gamma$, while the expected marginal cost due to the punishment is proportional to $\gamma_d^2 - \gamma^2$.

There is no such "paradox" in the symmetric information case, as (27) indicates. The intuitive reason for the difference between the two cases is, basically, that, as in the symmetric information case we consider a deviation from $\mu_t(\gamma)$ to $\mu_t^{ch}(\gamma)$, we actually have to take the integral of these marginal deviations in the interval $\left[\mu_t(\gamma), \mu_t^{ch}(\gamma)\right]$. This is why the linear terms $\gamma_d - \gamma$ of the expected marginal gain gives rise to the quadratic term $(\gamma_d - \gamma)^2$ which appears in (26). Then, in the symmetric information case, the effect on the expected gain, on the contrary, dominates the effect on the punishment, which stays proportional to $\gamma_d^2 - \gamma^2$.

[6] No rule γ where $\gamma > \gamma_d$ can be a reputational equilibrium. For, there would be no "punishment" in case of a deviation because γ_d would be better than the rule γ.

[7] The density of the uniform distribution is not differentiable at -θ and at $+\theta$. However, in the previous analysis, we actually only needed the right hand ride derivative of $F(-\overline{\varepsilon})$, which is equal to $1/(2\theta)$ in the interval $[-\theta, +\theta[$ and zero otherwise.

$$\theta < \frac{\delta\gamma_d}{2\alpha} \tag{30}$$

This implies that the variance of ε_t, which increases with θ, must not be too large. In other words, a too large amount of private information makes it impossible to sustain a rule better than discretion through reputational forces.[8]

In the uniform distribution case, when inequality (30) is satisfied, then condition (29) could be verified with any $\overline{\varepsilon}$ belonging to the interval $[-\theta, +\theta]$. For any $\overline{\varepsilon}$ in this interval, $f(-\overline{\varepsilon})$ stays equal to $1/(2\theta)$ and Γ^P is therefore independent of the value of $\overline{\varepsilon}$ in this interval. We have from (28) :

$$\Gamma^P = \frac{4\alpha\theta}{\delta} - \gamma_d \tag{31}$$

When the rule is followed, the probability of a reversion to discretion is given by $F(-\overline{\varepsilon})$, which is equal to $(1/2\theta)(\overline{\varepsilon} - \theta)$. But reversions to more inflationary episodes involve costs. As a consequence, the optimal value of $\overline{\varepsilon}$, in the case of a uniform distribution, is unambiguously equal to θ. From now on, when we consider the uniform distribution case, we will therefore take $\overline{\varepsilon} = \theta$. As it has alrealy been underlined at the beginning of this section, the probability of reversion to discretion will be equal to zero.

The marginal condition, considered in Canzoneri (1985), that no marginal increase of the money supply should be profitable to the central bank, is however only a necessary condition. For, this condition does not rule out the possibility that non marginal deviations from the rule could be profitable. Therefore, even if the marginal condition is satisfied, i.e. even when $\Gamma^P \leq \gamma \leq \gamma_d$ holds, the rule γ may still be non sustainable. This clearly appears if we consider the neighborhood of $\sigma_\varepsilon = 0$. In the case $\sigma_\varepsilon = 0$ there is no forecast error and we therefore have symmetric information because ξ_t, which is then equal to x_t, becomes public information. If there is continuity at $\sigma_\varepsilon = 0$, we should obtain almost the same condition of sustainability as in the symmetric information case and, therefore, approximately, all rules belonging to the interval $[\Gamma^s, \gamma_d]$ would be sustainable. This does not appear in the marginal condition we have considered. Such a marginal condition is

[8]This result does not depend on the use of a uniform distribution. For example, in the case of a normal distribution where $f(x) = 1/\left(\sigma\sqrt{2\pi}\right) \exp\left[-\left(1/2\sigma^2\right)x^2\right]$ the maximum value of $f(x)$ occurs at the origin $x = 0$. Therefore, from (29), there would exist some $\overline{\varepsilon}$ making $\Gamma^P < \gamma_d$ if and only if we had $1/\left(\sigma\sqrt{2\pi}\right) > \alpha/(\delta\gamma_d)$, which gives $\sigma < (\delta\gamma_d)/\left(\alpha\sqrt{2\pi}\right)$. The variance of ε_t should not be too large.

actually always satisfied for $\overline{\varepsilon}$ close to zero in that limit case. For, when σ_ε goes to zero, the density function $f(0)$ goes to infinity. Therefore Γ^P, given by (28), goes to $-\gamma_d$ and, consequently, any γ such that $0 \leq \gamma \leq \gamma_d$ satisfies the marginal condition. But, as we have indicated, only those rules satisfying $\gamma \geq \Gamma^s$ would be reputational equilibria.

In order to find necessary and sufficient conditions for a rule to be a reputational equilibrium under private information, we will therefore have to consider any deviation from the rule, whether marginal or non marginal. To simplify the analysis we will limit ourselves to the case of a uniform distribution. The expected gain implied by such a deviation from the rule from $\mu_t^r(\gamma)$ to $\overline{\mu}_t$ is equal to the integral $-\int_{\mu_t^r(\gamma)}^{\overline{\mu}_t} \left(\partial E_t^c \Lambda_t(\gamma, \mu_t) / \partial \mu_t\right) d\mu_t$ of the expected marginal gains . According to (23), the expected marginal gain is a linear and decreasing function of μ_t, which is represented by the straight line (D) in figure 2. In this figure, the intersection of (D) with the horizontal axis gives the cheating money supply growth rate $\mu_t^{ch}(\gamma)$ under symmetric information previously considered and given by (24). The (signed) area under (D) represents the expected gain from a deviation from the rule $\mu_t^r(\gamma)$.

The corresponding expected cost is equal to the integral $\int_{\mu_t^r(\gamma)}^{\overline{\mu}_t} \delta \left[\partial F\left(\mu_t - \mu_t^r(\gamma) - \overline{\varepsilon}\right) / \partial \mu_t\right] \left[E_t^c \Lambda_{t+1}^d - E_t^c \Lambda_{t+1}(\gamma)\right] d\mu_t$ of the expected marginal costs. Using (21) the marginal cost is equal to $\delta \chi \left(\gamma_d^2 - \gamma^2\right) f\left(\mu_t - \mu_t^r(\gamma) - \overline{\varepsilon}\right)$, which is proportional to the density function and is represented in figure 2 as the curve (C) in the uniform distribution case where we have taken $\overline{\varepsilon} = \theta$, as explained above. The expected cost of the deviation is equal to the area under (C).

The marginal condition at $\mu_t = \mu_t^r(\gamma)$ previously considered, is satisfied when at $\mu_t = \mu_t^r(\gamma)$ the expected marginal gain is lower than the expected marginal cost. In figure 2, this means that point A cannot be above point B.

There are two cases. When the (signed) area under (D) is always smaller that the area under (C), then no deviation, whether marginal or non marginal, can be beneficial to the central bank and, therefore, the rule γ is a reputational equilibrium. This always occurs when we have $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) \leq 2\theta$, as in figure 2. This also occurs when $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) > 2\theta$ holds and when we are in the case where the area of the dashed triangle is smaller than the dotted area, as in figure 3a. Then the marginal condition at $\mu_t = \mu_t^r(\gamma)$ is also sufficient. However, in the case of figure 3b where the area of the dashed triangle is larger than the dotted area, a deviation toward $\mu_t^{ch}(\gamma)$ would be beneficial because the expected gain would be larger than the expected cost. The reason is that, after the threshold value $\mu_t^r(\gamma) + 2\theta$, the marginal cost becomes zero. Therefore, marginal costs are not always increasing[9]. This explains why a local first order condition may not be sufficient.

The case of figure 3b has therefore to be ruled out. As, in such a case we have $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) > 2\theta$, this implies that the probability of a reversion toward discretion next period is equal to one when the central bank follows $\mu_t^{ch}(\gamma)$ (we have $F\left(\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) - \overline{\varepsilon}\right) = 1$). In that case, the problem facing the central bank, when we want to know whether it is profitable to cheat by following the policy $\mu_t^{ch}(\gamma)$, is identical to the one under symmetric information. For, in both cases, when there is a deviation from $\mu_t^r(\gamma)$ to $\mu_t^{ch}(\gamma)$, this will trigger a reversion toward discretion next period with probability 1. This implies that the necessary and sufficient condition required to rule out such a deviation will be the inequality $\Gamma^s \leq \gamma$ (note that we have already assumed $\gamma < \gamma_d$). Therefore, when we have $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) > 2\theta$, the rule γ will be sustainable if and only if we have $\max\left(\Gamma^s, \Gamma^P\right) \leq \gamma$.

In the case of figure 2 where we have $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) \leq 2\theta$, the deviation to $\mu_t^{ch}(\gamma)$ is not profitable to the central bank. But at $\mu_t^{ch}(\gamma)$ the probability of a reversion is lower than (or equal to) 1. Therefore, a fortiori, such a deviation would not be profitable if this probability were equal to 1, which would happen in the symmetric information case. This implies that, when $\mu_t^{ch}(\gamma) - \mu_t^r(\gamma) < 2\theta$ holds, the condition $\gamma \geq \Gamma^s$ would necessarily be satisfied whenever we have $\gamma \geq \Gamma^P$. This means that the inequality $\Gamma^s \leq \Gamma^P$ holds in that case. As, in figure 2, the condition $\Gamma^P \leq \gamma$ was both necessary and sufficient, we can still write the necessary and sufficient condition for $\gamma < \gamma^d$ to be sustainable as the inequality $\max\left(\Gamma^s, \Gamma^P\right) \leq \gamma$.

To sum up, we have shown that, under private information, and when the distribution of the forecast error ε_t is uniform, a necessary and sufficient condition for a rule γ, with $\gamma < \gamma_d$, to be a reputational equilibrium is that we have

$$\max\left(\Gamma^s, \Gamma^P\right) \leq \gamma \tag{32}$$

Call Γ the best sustainable rule, which is the rule with the lowest expected inflation rate. Noting that γ_d is always a reputational equilibrium, we have

$$\Gamma = \min\left[\max\left(\Gamma^s, \Gamma^P\right), \gamma_d\right] \tag{33}$$

We have $\Gamma^s < \Gamma^P$ if and only if we have

[9] This property does not depend on the use of a uniform distribution and will hold with other distributions, as for example the normal distribution.

$$\theta > \frac{\delta\gamma_d}{2\alpha}\frac{1}{1+\delta\eta} \tag{34}$$

This means that, in order to have a loss due to private information, the variance of ε_t (i.e. the amount of private information) has to be large enough[10].

3 Exchange rate pegging and private information

We will now consider a two-country version of the previous model and show that the existence of private information can make a pegged exchange rate regime better than a flexible exchange rate regime. The model used is a rather standard one and is taken from Canzoneri and Henderson (1991).

Equations (1), (2) and (3) are the same as in the chosed economy model:

$$\begin{array}{lcl} y_t = (1-\alpha)\, n_t & ; & y_t^* = (1-\alpha)\, n_t^* \\ w_t - p_t = -\alpha n_t & ; & w_t^* - p_t^* = -\alpha n_t^* \\ m_t = p_t + y_t + x_t & ; & m_t^* = p_t^* + y_t^* + x_t^* \end{array}$$

where a star is attached to variables of country 2.

We will need demand for outputs equations. These are increasing functions of the respective real exchange rate and decreasing function of the real interest rates. We have

$$y_t = \lambda\beta q_t - \nu\left[(1-\beta)\, r_t + \beta r_t^*\right] + \varphi\left[(1-\beta)\, y_t + \beta y_t^*\right]$$

$$y_t^* = -\lambda\beta q_t - \nu\left[(1-\beta)\, r_t^* + \beta r_t\right] + \varphi\left[(1-\beta)\, y_t^* + \beta y_t\right]$$

$$\nu > 0 \quad , \quad \lambda > 0 \quad , \quad 0 < \varphi < 1 \quad , \quad 0 < \beta < 1$$

In these expressions q_t is the real exchange rate defined by

$$q_t = e_t + p_t^* - p_t \tag{35}$$

where e_t is the nominal exchange rate (the price of one unit of country 2 currency in terms of units of country 1 currency) ; r_t and r_t^* are the real interest rates defined by

[10] Note that, from (30) (33) and (34), we will obtain $\Gamma = \Gamma^P \leq \gamma_d$ when θ belongs to the interval $\left[(\delta\gamma_d/2\alpha)\left(1/\left(1+\delta\eta\right)\right), (\delta\gamma_d/2\alpha)\right]$ and is therefore neither too small nor too large.

$$r_t = i_t - E_t\left(p_{I,t+1} - p_{I,t}\right)$$
$$r_t^* = i_t^* - E_t\left(p_{I,t+1}^* - p_{I,t}^*\right)$$

where i_t and i_t^* are the nominal interest rates and $p_{I,t}$ and $p_{I,t}^*$ are the price indices (all variables are in logarithms except interest rates):

$$p_{It} = (1-\beta)\,p_t + \beta\left(p_t^* + e_t\right) \qquad ; \qquad p_{It}^* = (1-\beta)\,p_t^* + \beta\left(p_t + e_t\right)$$

where β is the share of imported goods in spending.

Finally, we have the uncovered interest-rate parity :

$$i_t = i_t^* + E_t e_{t+1} - e_t$$

As before, the nominal wages are predetermined in period $t-1$ by the private sector at a level which minimizes the squared deviation of employment around some desired level, normalized at zero in the model. This leads to

$$w_t = E_{t-1}p_t \quad ; \quad w_t^* = E_{t-1}p_t^*$$

which implies

$$E_{t-1}n_t = 0 \qquad ; \qquad E_{t-1}n_t^* = 0 \tag{36}$$

Solving the model and excluding speculative bubbles (which implies $E_t q_{t+1} = 0$), we obtain the following reduced form

$$n_t = m_t - E_{t-1}m_t - x_t \quad ; \quad n_t^* = m_t^* - E_{t-1}m_t^* - x_t^* \tag{37}$$

$$p_t = E_{t-1}m_t + \alpha\left(m_t - E_{t-1}m_t - x_t\right) \quad ; \quad p_t^* = E_{t-1}m_t^* + \alpha\left(m_t^* - E_{t-1}m_t^* - x_t^*\right) \tag{38}$$

$$p_{It} = E_{t-1}m_t + (\alpha+\rho)\left(m_t - E_{t-1}m_t - x_t\right) - \rho\left(m_t^* - E_{t-1}m_t^* - x_t^*\right) \tag{39a}$$

$$p_{It}^* = E_{t-1}m_t^* + (\alpha+\rho)\left(m_t^* - E_{t-1}m_t^* - x_t^*\right) - \rho\left(m_t - E_{t-1}m_t - x_t\right) \tag{39b}$$

$$q_t = \frac{\rho}{\beta}\left[m_t - E_{t-1}m_t - x_t - \left(m_t^* - E_{t-1}m_t^* - x_t^*\right)\right] \tag{40}$$

where coefficient ρ is given by

$$\rho = \frac{\beta\,(1-\alpha)\left[1-(1-2\beta)\,\varphi\right]}{2\beta\lambda + (1-2\beta)^2\,\nu} > 0 \tag{41}$$

The central banks want to stabilize both employment levels and inflation rates. Their period t loss fonctions (which are also assumed to represent social preference) are :

$$\Lambda_t = (n_t - \widetilde{n})^2 + \chi \pi_{It}^2 \tag{42a}$$

$$\Lambda_t^* = (n_t^* - \widetilde{n}^*)^2 + \chi \pi_{It}^{*2} \tag{42b}$$

As before, we assume that, because of the existence of distortions, the desired employment levels of the monetary authorities are greater than the respective desired levels of the private sectors. We therefore have $\widetilde{n} > 0$ and $\widetilde{n}^* > 0$. The values of $\widetilde{n}$ and $\widetilde{n}^*$ can be different and, without loss of generality, we take the case $\widetilde{n} \geq \widetilde{n}^*$, which means that country 2 has less distortions and, as we will see, is therefore less inflationary than country 1 under discretion when the exchange rate is flexible. The inflation rates π_{It} and π_{It}^* are defined in terms of price indices : $\pi_{It} = p_{It} - p_{I,t-1}$ and $\pi_{It}^* = p_{It}^* - p_{I,t-1}^*$.

As in the closed economy model, the money demand shocks x_t and x_t^* are forecasted in period t by their respective central banks before decisions on monetary policy are made. These forecasts are assumed to be private informations of the respective central banks. We have

$$x_t = \xi_t + \varepsilon_t \qquad ; \qquad x_t^* = \xi_t^* + \varepsilon_t^*$$

where, the forecast errors ε_t and ε_t^* are assured to follow the same probability distribution, which will be a uniform distribution with support $[-\theta, +\theta]$.

We will compare two kinds of systems. In each of these systems, each central bank has announced some rule and, as in the closed economy case, we will examine whether such rules can be sustained as a reputational equilibrium or not. In each system, we will determine the best rule among the set of sustainable rules. Then, in order to compare the two systems, we will only have to compare these best rules.

We now have to be more precise about the two systems, about the rules that the central banks have announced they will follow, and about the reputational games involves in each system. The framework is actually similar to that of von Hagen (1992). The present analysis may therefore be considered as an extension of von Hagen (1992) to the case of private information.

The first system is a "flexible exchange rate regime", which will be denoted by (FL). In this system, each central bank has a money supply rule which targets the expected inflation rate. We have

$$E_t^c \pi_{It}^r = \gamma \tag{43a}$$

$$E_t^{c*} \pi_{It}^{*r} = \gamma^* \tag{43b}$$

where, as in section 2, E_t^c (or E_t^{c*}) is the expectation operator conditional on information avaible to the central bank of country 1 (or country 2) in period t before deciding on its policy. Thus E_t^c, or E_t^{c*}, contains the forecast ξ_t, or ξ_t^* (but not x_t, or x_t^*) in its information set.

¿From (39), the corresponding rules for the money supply growth rates are

$$\mu_t^r(\gamma) = \gamma + \xi_t + p_{I,t-1} - m_{t-1} \tag{44a}$$

$$\mu_t^{r*}(\gamma^*) = \gamma^* + \xi_t^* + p_{I,t-1}^* - m_{t-1}^* \tag{44b}$$

The reputational game is similar to the one described in the closed economy context. We consider $\widetilde{\mu}_t^r(\gamma) = \mu_t^r(\gamma) + \varepsilon_t$ and $\widetilde{\mu}_t^{r*}(\gamma^*) = \mu_t^{r*}(\gamma^*) + \varepsilon_t^*$. The values of $\widetilde{\mu}_t^r(\gamma)$ and $\widetilde{\mu}_t^{r*}(\gamma^*)$ are known to the wage setters at the end of period t because the money demand shocks x_t and x_t^* are known. When, in period t, we have $\mu_t > \widetilde{\mu}_t^r(\gamma) + \overline{\varepsilon}$, where $\overline{\varepsilon}$ is a parameter to be chosen, then there is a one period reversion, in period $t+1$, to the equilibrium where the central bank of country 1 follows (and is expected to follow) discretionary policy. In the same way, reversion to discretion in period $t+1$ for country 2 occurs when we have $\mu_t^* > \widetilde{\mu}_t^{r*}(\gamma^*) + \overline{\varepsilon}$. The set of rules γ and γ^* is a reputational equilibrium when there is no incentive for any central bank to deviate from its rule, given the equilibrium strategy of the other country (the two central banks are assumed not to cooperate). Consider for example the incentives of country 1. When country 2 follows its equilibrium strategy we always have $m_t^* = E_{t-1}m_t^* + \xi_t^*$ because the central bank always offsets the forecasted demand shock. (This occurs in a non-punishment as well as in a punishment period). Therefore from (39a), we have

$$p_{It} = E_{t-1}m_t + (\alpha + \rho)(m_t - E_{t-1}m_t - x_t) + \rho\varepsilon_t^* \tag{45}$$

Noting that (37a) is the same as (5') and comparing (45) to (6'), we see that the analysis of the incentives of country 1 to cheat becomes similar to that of the closed economy case, with coefficient $\alpha+\rho$ replacing coefficient α. This difference is due to the effect of the implied exchange rate depreciation on the price level. Note however that we have the additional term $\rho\varepsilon_t^*$ in (45), which comes from the forecast error of the central bank of country 2. The presence of this additional term, however, does not change the corresponding analysis, and the results obtained in section 2 remain valid[11].

Call γ_{FL} and γ_{FL}^* the expected inflation rates under discretionary policy, in countries 1 and 2 respectively. We have from (19):

$$\gamma_{FL} = \frac{\widetilde{n}}{\chi\left(\alpha+\rho\right)} \tag{46a}$$

$$\gamma_{FL}^* = \frac{\widetilde{n}^*}{\chi\left(\alpha+\rho\right)} \tag{46b}$$

Again, consider the uniform distribution case for the forecast errors (with $\overline{\varepsilon}$ optimally chosen equal to the parameter θ of that distribution). From the closed economy analysis, we obtain that the best rules γ and γ^* (which are also those with the lowest expected inflation rates) that can be reputational equilibria among those considered are equal to Γ_{FL} and Γ_{FL}^* for country 1 and country 2, respectively, where Γ_{FL} and Γ_{FL}^* are given by

$$\Gamma_{FL} = \min\left[\max\left(\Gamma_{FL}^s, \Gamma_{FL}^P\right), \gamma_{FL}\right] \tag{47a}$$

$$\Gamma_{FL}^* = \min\left[\max\left(\Gamma_{FL}^{*s}, \Gamma_{FL}^{*P}\right), \gamma_{FL}^*\right] \tag{47b}$$

where we have

$$\Gamma_{FL}^s = \frac{1-\delta\eta_0}{1+\delta\eta_0}\gamma_{FL} \tag{48a}$$

$$\Gamma_{FL}^{*s} = \frac{1-\delta\eta_0}{1+\delta\eta_0}\gamma_{FL}^* \tag{48b}$$

$$\Gamma_{FL}^P = \frac{4\left(\alpha+\rho\right)\theta}{\delta} - \gamma_{FL} \tag{49a}$$

[11] The presence of this term only adds a stochastic term in ε_t^* to the expression giving the price level and therefore only changes the coefficient of σ_ε^2 in the expected losses, in the equations analogous to equation (22) in the closed economy case. However, the terms in σ_ε^2 of the expected losses do not play any role in the analysis. For, only the difference between the expected losses as in equation (20), or the derivative of these expected losses with respect to μ_t (as in equation (23)), matter when we consider the incentives of central banks to cheat. But, then, the terms in σ_ε^2 cancel out or disappear.

$$\Gamma_{FL}^{*P} = \frac{4(\alpha+\rho)\theta}{\delta} - \gamma_{FL}^{*} \tag{49b}$$

and where the coefficient η_0 is given by

$$\eta_0 = 1 + \frac{1}{\chi(\alpha+\rho)^2} > 1 \tag{50}$$

The second system is a "fixed exchange rate system", in which one country, country 1 (the more inflationary under discretion), pegs the exchange rate. The other country, country 2, which is the less inflationary under discretion and the "leader" of this fixed exchange rate system, has a money supply rule aimed at targeting the expected inflation rate, as in the flexible exchange rate regime. We have

$$e_t = \overline{e}_t \tag{51a}$$

$$E_t^{c*}\pi_{It}^{*r} = \gamma^* \tag{51b}$$

In (51), $\overline{e}_t$ is the level of the exchange rate which, in period $t-1$, country 1 has announced that it will peg in period t. For simplicity of exposition, it is assumed that this level is such that the expected inflation rates are the same in both countries (we have $E_{t-1}\pi_{It} = E_{t-1}\pi_{It}^*)$[12]. Therefore, when each country follows its announced rule, the expected inflation rate is γ^* in both countries.

This system, where one country has an "exchange rate rule", will be denoted by (E). From (35) and (38), it can be seen[13] that $e_t = \overline{e}_t$ implies

$$m_t - E_{t-1}m_t - x_t = m_t^* - E_{t-1}m_t^* - x_t^* \tag{52}$$

Substituting this equality into (39b) gives

$$p_{It}^* = E_{t-1}m_t^* + \alpha(m_t^* - E_{t-1}m_t^* - x_t^*) \tag{53}$$

[12]¿From the definition of price indices, we can write $\pi_{It} - \pi_{It}^* = e_t - e_{t-1} - (1-2\beta)(q_t - q_{t-1})$ where q_t is the real exchange rate given by (35). Therefore, as we have $E_{t-1}q_t = 0$, the equality $E_{t-1}\pi_t = E_{t-1}\pi_{It}^*$ gives $\overline{e}_t = e_{t-1} - (1-2\beta)q_{t-1}$.

[13](35) implies $q_t - E_{t-1}q_t = p_t^* - E_{t-1}p_t^* - (p_t - E_{t-1}p_t)$. Using (38) and (40), this gives $[(\rho/\beta)+\alpha]\,[m_t - E_{t-1}m_t - x_t - (m_t^* - E_{t-1}m_t^* - x_t^*)] = 0$.

Comparing (53) to (6') we see that, as long as country 1 pegs the exchange rate, the analysis of the incentives for country 2 is the same as in the closed economy case. Note that we have coefficient α instead of the coefficient $\alpha+\rho$ that we had in the flexible exchange rate system. For these is no more effect on the price level going through exchange rate depreciation.

Therefore, from the point of view of the incentives of country 2, the system is sustainable if and only if we have

$$\Gamma^*_{E,2} \leq \gamma^* \leq \gamma_E \tag{54}$$

where $\Gamma^*_{E,2}$ is the rule with the lowest expected inflation rate for which there is no incentive for the central bank of country 2 to renege on. From the analysis in the closed economy context, we get (in the uniform distribution case for ε_t) :

$$\Gamma^*_{E,2} = \min\left[\max\left(\Gamma^{*s}_E, \Gamma^{*P}_E\right), \gamma_E\right] \tag{55}$$

where we have

$$\Gamma^{*s}_E = \frac{1-\delta\eta_1}{1+\delta\eta_1}\gamma_E \tag{56}$$

$$\Gamma^{*P}_E = \frac{4\alpha\theta}{\delta} - \gamma_E \tag{57}$$

$$\gamma_E = \frac{\widetilde{n}^*}{\chi\alpha} \tag{58}$$

$$\eta_1 = 1 + \frac{1}{\chi\alpha^2} > \eta_0 > 1 \tag{59}$$

Note that γ_E would be the expected inflation rate in the fixed exchange rate system where country 2 would follow a discretionary policy and country 1 would peg the exchange rate. In order to simplify the analysis, we will restrict our attention to the case where country 1 (the more inflationary country in the flexible exchange system under discretion) would not loose by participating to such a system[14]. This can be written

[14] As noted before, however, this fixed exchange rate system would not be sustainable in the one shot game, unless some exogenous cost of leaving the system is imposed. Otherwise there will be an incentive for country 1 to unexpectedly deviate from the announced peg.

Also, as emphasized in the literature (Rogoff (1985) and Giavazzi and Giovannini (1989)), we have $\gamma_E > \gamma^*_{FL}$. Therefore, in the one-shot game, country 2 would loose.

As we will show, results can be quite different in the repeated game under private information we consider.

$$\gamma_E \leq \gamma_{FL} \tag{60}$$

So far, only the point of view of country 2, the leader of the pegged exchange rate system, has been examined. Now, consider that of country 1. In period t , the central bank of country 1 decides on whether it will peg the exchange rate during that period. If it does not, then a one period punishment of a return to the case where it follows discretionary policy (with expected inflation rate γ_{FL}) will take place in period $t+1$. In any period, each country decides on it policy of following the rule or not without knowing what the other country does, i.e. theses choices are simultaneously made. Therefore, at the non cooperative equilibrium, where each central bank assumes that the other central bank follows its equilibrium strategy, the central bank of one country does not respond, during this period, to any deviation from the rule of the central bank of the other country. Thus, when considering country 2 choice, we have supposed that the exchange rate was pegged by country 1. And here, when considering the choice of country 1, country 2 will be supposed to follow its rule for the period (which is γ^* if it did not deviate last period and γ_E if it deviated[15]).

Consider the incentives that, in the reputational game, the central bank of country 1 faces in order to decide whether to peg the exchange rate or not, given that the wage setters expected that it will peg the exchange rate. A crucial point is that the wage setters can observe ex post whether the rule has been followed or not, because both the targeted exchange rate $\overline{e}_t$ and the exchange rate e_t are observable. This was not true for the money supply rule in the flexible exchange rate system. For, as the forecasted money demand shock ξ_t is private information of the central bank, the targeted money supply rule $\mu_t^r(\gamma)$ was not observable ex post by wage setters. Therefore, as in the case with no private information, any deviation, even marginal, from the peg will be considered by the wage setters as a deviation from the announced rule, and will be consequently punished by a return to discretion under a flexible exchange rate next period.

As we have indicated, the level of the pegged exchange rate $\overline{e}_t$ is such that the expected inflation rates are the same in the two countries. Therefore, for country 1, deciding on whether to peg the exchange rate is equivalent to

[15] Although, with a uniform distribution for ε_t, the probability of a reversion to discretionary policy for country 2, with expected inflation rate γ_E, is zero when countries follow their equilibrium strategies, we still have to specify what country 1 will do in case of this zero probability event.

deciding on whether to follow, in the flexible exchange rate system, a money supply rule which gives an expected inflation rate equal to that of country 2. This inflation rate is equal to γ^* when we are in a period where country 2 is not punished. It is equal to γ_E when we are in a period where country 2 is punished. Consider, for example, the case where country 2 follows the rule γ^*. From (39a) and (52) we can see that the exchange rate is pegged at the preannounced rate $\overline{e}_t$ if and only if the central bank of country 1 follows the money supply rule[16].

$$\mu_t^{E,r} = \gamma^* + \xi_t + p_{I,t-1} - m_{t-1} + \varepsilon_t - \varepsilon_t^* \tag{61}$$

But from (39a) and (52) we can also write an equation analogous to (53) for country 1

$$p_{It} = E_{t-1}m_t + \alpha\left(m_t - E_{t-1}m_t - x_t\right)$$

This last equation is the same as (61) and, therefore, in order to know whether $\mu_t^{E,r}$ is sustainable, the closed economy analysis of section 2 applies. There is still a slight difference due to the additional stochastic term $\varepsilon_t - \varepsilon_t^*$ in (61) but this term actually does not play any role in the analysis of the sustainability of the rule[17], and therefore the results would be the same as if we had considered the rule $\mu_t^r(\gamma)$, given by (44a) where γ^* has been substituted to γ.

The important point we have emphasized is that, for country 1, there is now no private information problem in the choice of whether to follow that rule $\mu_t^{E,r}$ or not. For, any deviation, even marginal from $\mu_t^{E,r}$ will show up as a breaking of the exchange rate rule $e_t = \overline{e}_t$, and will be punished by a return to discretion under the flexible exchange rate system. Therefore, from the closed economy analysis under symmetric information (and with a uniform distribution), country 1 will peg the currency, when country 2 follows the rule γ^*, if and only if we have the inequality :

$$\Gamma_{FL}^s \leq \gamma^* \leq \gamma_{FL} \tag{62}$$

[16]¿From (39a) we get $E_{t-1}m_t = \gamma^* + p_{I,t-1}$, where we have used the fact that $e_t = \overline{e}_t$ implies $E_{t-1}\pi_{It} = \gamma^*$. But (52) implies $m_t = E_{t-1}m_t + \xi_t + \varepsilon_t - \varepsilon_t^*$, which gives the above expression for $\mu_t^{E,r}$.

[17]The reason is the same as the one given, in the flexible exchange rate case, for the presence of the additional stochastic term $\rho\varepsilon_t^*$ in (45) (see footnote 11). Only the terms in σ_ε^2 in the expected losses are changed. These terms do not affect the incentives to cheat.

This inequality indicates that country 1 does not renege on the peg when we are in a period where country 2 is not punished for having cheated last period, and therefore follows the rule γ^*. We should also add the condition that country 1 does not renege on the peg in a period where country 2 is punished for having cheated last period, and where therefore its inflation rate is γ_E (even if with a uniform distribution this would be a probability zero event). It can be seen, however, that such a condition will always be satisfied if all other conditions hold[18].

The fixed exchange rate system considered is sustainable if and only if both inequalities (54) and (62) hold simultaneously. Using (55) and (60) these two inequalities are equivalent to the inequalities.

$$\Gamma^s_{FL} \leq \gamma_E \tag{63a}$$

$$\Gamma_E \leq \gamma^* \leq \gamma_E \tag{63b}$$

where the lowest (and best) sustainable inflation rate is Γ_E given by

$$\Gamma_E = \min\left[\max\left(\Gamma^s_{FL}, \Gamma^{*s}_E, \Gamma^{*P}_E\right), \gamma_E\right]$$

But, as we have $\eta_1 > \eta_0$ and $\gamma^*_{FL} < \gamma_E$, we always have $\Gamma^{*s}_E < \Gamma^s_{FL}$. Therefore we can write

$$\Gamma_E = \min\left[\max\left(\Gamma^s_{FL}, \Gamma^{*P}_E\right), \gamma_E\right] \tag{64}$$

Having found necessary and sufficient conditions for each system to be sustainable, we can now consider the issue of whether the fixed exchange rate system could yield a lower expected inflation rate bias than the flexible exchange rate system for both countries. It can easily be seen that, in order for such a result to hold, the amount of private information (represented here by the parameter θ of the uniform distribution) has to be neither too low nor too large. To see that, consider first the limit case $\theta = 0$ where there is no private

[18]Inequality (54) implies $\gamma^* \leq \gamma_E$. But using (26), the gain for country 1 to deviate from the peg is equal to $(\chi/\eta_0)\left(\gamma_{FL} - \gamma^*\right)^2$ in a non punishment period (for country 2) and to $(\chi/\eta_0)\left(\gamma_{FL} - \gamma_E\right)^2$ in a punishment period (for country 2). This incentive is therefore larger in the first case. On the other hand, the cost of not following the peg is the same in both cases and is equal to the cost of reverting to the discretionnary solution γ_{FL} instead of following the rule γ^* during the next period (as there is a one period punishment for country 2, in both cases the rule γ^* is expected to hold next period if country 1 does not deviate). Therefore, if country 1 does not renege on the peg in non-punishment period for country 2, it also does not renege on the peg in a punishment period for country 2.

information (as we have $\varepsilon_t = 0$ and therefore $\xi_t = x_t$ with probability 1, ξ_t is known to wage setters at the end of the period). Then, as also underlined in section 2, we would have $\Gamma_{FL} = \Gamma^s_{FL}$, $\Gamma^*_{FL} = \Gamma^{*s}_{FL}$ and $\Gamma_E = \Gamma^s_{FL}$. The expected inflation rate in the fixed exchange rate system will therefore not be strictly lower for country 1, and would be higher for country 2. This point has been emphasized by von Hagen (1992). This occurs because, in the absence of private information, the incentives of country 1 of reneging on the exchange rate rule are the same as those of reneging on the money supply rule. In fact, our main point is that it is the existence of private information and the fact that this private information does not affect the incentives of country 1 of following the exchange rate rule that permits a fixed exchange rate to lead to a smaller inflation bias than a flexible exchange rate, for both countries.

The amount of private information cannot be too large however. For, making θ going to infinity we get $\Gamma_{FL} = \gamma_{FL}, \Gamma^*_{FL} = \gamma^*_{FL}$ and $\Gamma_E = \gamma_E$. When the amount of information is too large, only the discretionary policies can be sustainable. The reason is that any marginal deviation from the money supply rule could be thought to be due to a forecast error, ($\overline{\varepsilon}$, which is equal to θ, becomes large) and there is always an incentive for the central bank to maginally increase the money supply growth rate above the level specified by the rule. But, in that case, the fixed exchange rate system gives a higher inflation bias than the flexible exchange rate system for country 2 (we have $\gamma^*_{FL} < \gamma_E$).

Therefore the issue becomes that of knowing whether the fixed exchange rate system can produce lower inflation biases for both countries when the parameter θ takes some intermediate values. The answer is actually positive, as Proposition 1 shows.

Proposition 1 *There exists some set of parameter values such that the fixed exchange rate system leads to lower expected inflation biases than the flexible exchange rate system, for both countries.*

More precisely, there exists $\underline{\delta}, \underline{k}, \overline{k}(\delta)$,$\underline{\theta}(\delta, \tilde{n}/\tilde{n}^*)$and $\overline{\theta}(\delta, \tilde{n}/\tilde{n}^*)$ *such that when the parameters* δ, $\tilde{n}/\tilde{n}^*$ *and* θ *verify the inequalities*

$$\delta > \underline{\delta} \tag{i}$$

$$\underline{k} < \frac{\tilde{n}}{\tilde{n}^*} < \overline{k}(\delta) \tag{ii}$$

$$\underline{\theta}(\delta, \tilde{n}/\tilde{n}^*) < \theta < \overline{\theta}(\delta, \tilde{n}/\tilde{n}^*) \tag{iii}$$

then we have $\Gamma_E < \Gamma_{FL}, \Gamma_E < \Gamma^*_{FL}, \Gamma_{FL} > 0$ and $\Gamma^*_{FL} > 0$. This set of parameters is non empty. For, we have $0 < \underline{\delta} < 1; 1 < \underline{k} < \overline{k}(\delta)$ when (i) holds ; and $\underline{\theta}(\delta, \widetilde{n}/\widetilde{n}^*) < \overline{\theta}(\delta, \widetilde{n}/\widetilde{n}^*)$ when (i) and (ii) hold.

The proof is given in the Appendix.

Note that the inequalities $\Gamma_{FL} > 0$ and $\Gamma^*_{FL} > 0$ are required in order for the optimal rule (which corresponds to a zero expected inflation rate) not to be sustainable in the flexible exchange rate system for any country.

Proposition 1 indicates that there is no need to introduce exogenous costs (like "political" costs) of leaving the fixed exchange rate system to obtain this kind of result. The introduction of private information can do it. As we have emphasized, the crucial point which underlies this result is that while the money supply rule is affected by a private information problem, this is not the case for the exchange rate rule. As a consequence, there is less incentive for the central bank of country 1, which has to peg the currency, to renege on the exchange rate rule than to renege on a money supply rule in the flexible exchange rate system. Although this property makes the result possible, it is not sufficient. For we also have to consider the behavior of country 2, the leader of the fixed exchange rate system which, in both systems, has a money supply rule. And this gives rise, in both systems, to a private information problem. Therefore, additional conditions have to be written. As Proposition 1 indicates, it can nonetheless be shown that all these conditions can be simultaneously satisfied for some parameter values, verifying the inequalities given in Proposition 1.

The role of an inequality like (iii) concerning parameter θ has already been emphasized. From condition (ii), the distortions of country 2 relatively to those of country 1 have to be low enough (i.e. $\widetilde{n}/\widetilde{n}^*$ has to be large enough) in order to make the fixed exchange rate better for country 1. But these relative distortions cannot be too small either (i.e. $\widetilde{n}/\widetilde{n}^*$ cannot be too large). Otherwise, in the fixed exchange rate system, it would not be possible to sustain, through reputational forces, an expected inflation rate which would be lower than the one that country 2 could obtain in the flexible exchange rate system. For this last kind of reason, the discount factor δ cannot be too low, as (i) indicates : a large enough discount factor, which makes punishment in the future appear strong enough, is necessary in order for reputational forces to have the required effect.

Proposition 1 compares the expected inflation rates in the two systems. One may however want to know which system is better in that case. For that we have to compare the expected loss functions in the two systems. From (9) we can write, for country 1

$$E_t^c \Lambda_t = \left(E_t^c n_t - \widetilde{n}\right)^2 + \chi \left(E_t^c \pi_{It}\right)^2 + E_t^c \left(n_t - E_t^c n_t\right)^2$$

$$+\chi E_t^c \left(p_{It} - E_t^c p_{It}\right)^2$$

When the announced rules are followed by the central banks we have, in both systems, $m_t - E_{t-1} m_t = \xi_t$. Using (37) this implies $E_t^c n_t = 0$ and $n_t - E_t^c n_t = -\varepsilon_t$. In the flexible exchange rate system we have from (45) :

$$p_{It}^{FL} - E_t^c p_{It}^{FL} = -\left[\alpha \varepsilon_t + \rho \left(\varepsilon_t - \varepsilon_t^*\right)\right]$$

In the fixed exchange rate system, from (39a) and (52) are can write

$$p_{It} = E_{t-1} m_t + \alpha \left(m_t^* - E_{t-1} m_t^* - x_t^*\right)$$

which yields

$$p_{It}^E - E_t^c p_{It}^E = -\alpha \varepsilon_t^*$$

As $\varepsilon_t + \varepsilon_t^*$ and $\varepsilon_t - \varepsilon_t^*$ are uncorrelated (we have $E\varepsilon_t^2 = E\varepsilon_t^{*2}$), we get

$$E\Lambda_t^{FL} = E_t^c \Lambda_t^{FL} = \widetilde{n}^2 + \chi \left(E_t^c \pi_{It}^{r,FL}\right)^2 + \alpha^2 \sigma_s^2 + (\alpha + 2\rho)^2 \sigma_D^2$$

$$E\Lambda_t^E = E_t^c \Lambda_t^E = \widetilde{n}^2 + \chi \left(E_t^c \pi_{It}^{r,E}\right)^2 + \alpha^2 \sigma_s^2 + \alpha^2 \sigma_D^2$$

where we have defined

$$\sigma_s^2 = E\left(\frac{\varepsilon_t + \varepsilon_t^*}{2}\right)^2 \qquad ; \qquad \sigma_D^2 = E\left(\frac{\varepsilon_t - \varepsilon_t^*}{2}\right)^2$$

We have the same corresponding expressions for the expected losses of country 2.

In these expressions, only the second and fourth terms differ in the two systems. The second terms corresponds to the inflation biases and have been considered in Proposition 1. Under the conditions stated, the corresponding loss is smaller in the fixed exchange rate system. And it is the existence of private information which made such a result possible. The fourth term is also smaller in the fixed exchange rate system. This occurs because, as indicated by (61), the fixed exchange rate system allows the money supply to respond to the asymetric components of the money demand forecast errors (through non-sterilized interventions). Such a superiority of the fixed exchange rate system has been underlined in the literature[19]. The important point is that this property is unrelated to the existence of private information and would occur even if ξ_t and ξ_t^* were public information. Therefore private information gives an additional advantage to the fixed exchange rate system only through its effect on expected inflation, which was the focus of Proposition 1.

4 Conclusion

We have compared two systems. In the flexible exchange rate regime each country has a money supply rule which targets an expected inflation rate. In the other system, one country pegs the exchange rate while the other, the leader of the system, still has a money supply rule. In both systems we have considered reputational equilibria where countries have annouced their respective rules and where, in case of reneging, there is a one period reversion to the respective discretionary policy.

We have shown that by introducing private information (the forecasts of money demand shocks by the central banks) the fixed exchange rate system could yield a lower inflation bias than the flexible exchange rate system, for both countries. A crucial point was that, in order to obtain such a result, we did not have to introduce some exogenous costs of leaving the exchange rate system, due for example, to the existence of "political costs" of abandoning the peg. This is at variance with the existing literature where some kinds of exogenous costs were necessary to obtain that result.

Here, the role which was played by these exogenous costs in changing the incentives of the country which pegs the exchange rate, is now played by the differential impact that the existence private information has on these incentives. For, in a flexible exchange rate system with money supply rules, if a central bank marginally cheats by marginally increasing the money supply above the level implicity required by the rule, in order to (marginally) raise employment, the wage setters will not really know whether this is due to such a marginal cheating or whether this is caused by a money demand which is lower than expected. The reason is that the money demand forecast is a private information of the central bank. The larger the amount of private information (i.e. the larger the variance of the forecast error) the

[19]See the literature on optimal foreign exchange intervention, as, for example, Turnovsky (1983).

less the wage setters will be inclined to consider that the central bank has cheated and, consequently, the more the central bank will be tempted to cheat in that way. This could prevent some rules to be sustainable. When we consider the fixed exchange rate system, however, this is very different. For, there is no ambiguity as to whether the rule has been followed or not. Money demand forecast errors do not interfere with the observability of the announced exchange rate rule. Therefore the "marginal cheating" which was possible with the money supply rule is no more possible with the exchange rate rule. This makes the exchange rate rule easier to sustain as a reputational equilibrium. This property is at the root of our result of the superiority of the fixed exchange rate system.

Appendix

Proof of Proposition 1

The fixed exchange rate system is sustainable and dominates the flexible exchange rate system if and only if (63a) holds and if we have the inequalities:

$$\Gamma_E < \Gamma_{FL} \tag{A1}$$

$$\Gamma_E < \Gamma^*_{FL} \tag{A2}$$

$$\Gamma_{FL} > 0 \tag{A3}$$

$$\Gamma^*_{FL} > 0 \tag{A4}$$

where Γ_E, Γ_{FL} and Γ^*_{FL} are given by (64), (47a) and (47b) respectively. In addition, we also have assumed $\gamma_E \leq \gamma_{FL}$

Note that we have $\Gamma_E \leq \gamma_E$. But, in order for (A2) to be satisfied, we should have $\Gamma_E < \gamma_E$. For, if we had $\Gamma_E = \gamma_E$ this would imply $\Gamma_E > \gamma^*_{FL}$ (because we have $\gamma_E > \gamma^*_{FL}$) and therefore $\Gamma_E > \Gamma^*_{FL}$ (because we have $\Gamma^*_{FL} \leq \gamma^*_{FL}$). From (64), the condition $\Gamma_E < \gamma_E$ implies

$$\gamma_E > \Gamma^s_{FL} \tag{A5}$$

Inequality (A1) implies

$$\Gamma^s_{FL} < \Gamma^P_{FL} \tag{A6}$$

For, if (A6) did not hold we would get (from (47a) and the fact that we always have $\Gamma^s_{FL} < \gamma_{FL}$) $\Gamma_{FL} = \Gamma^s_{FL}$. Then there are two cases. In the case $\max\left(\Gamma^s_{FL}, \Gamma^{*P}_E\right) \geq \gamma_E$, we have from (64), $\Gamma_E = \gamma_E$. Then $\gamma_E \geq \gamma_{FL}$ would imply $\Gamma_E \geq \Gamma_{FL}$. In the case $\max\left(\Gamma^s_{FL}, \Gamma^{*P}_E\right) < \gamma_E$, we get from (64) $\Gamma_E \geq \Gamma^s_{FL}$ and therefore $\Gamma_E \geq \Gamma_{FL}$. Therefore, in any case, (A1) would not be satisfied.

When (A6) holds we can write

$$\Gamma_{FL} = \min\left(\Gamma^{P}_{FL}, \gamma_{FL}\right) \tag{A7}$$

But, as we have $\gamma_{FL} > \gamma^{*}_{FL}$, we have $\Gamma^{*s}_{FL} < \Gamma^{s}_{FL}$ and $\Gamma^{*P}_{FL} > \Gamma^{P}_{FL}$ (this last inequality represents the "paradox" underlined in section 2). Therefore, inequality (A6) also implies

$$\Gamma^{*s}_{FL} < \Gamma^{*P}_{FL} \tag{A8}$$

which in turn implies

$$\Gamma^{*}_{FL} = \min\left(\Gamma^{*P}_{FL}, \gamma^{*}_{FL}\right) \tag{A9}$$

Using (A7) and (A9), inequalities (A3) and (A4) are satisfied if and only if we have, respectively

$$\Gamma^{P}_{FL} > 0 \tag{A10}$$

$$\Gamma^{*P}_{FL} > 0 \tag{A11}$$

But, as we have $\Gamma^{*P}_{FL} > \Gamma^{P}_{FL}$, inequality (A11) is implied by (A10) and can therefore be dropped as being redundant.

In order for inequality (A2) to be satisfied we must have

$$\Gamma^{*P}_{E} < \gamma^{*}_{FL} \tag{A12}$$

For, suppose (A12) did not hold, i.e. suppose we had $\Gamma^{*P}_{E} \geq \gamma^{*}_{FL}$. As, from (49b) and (57) and the inequality $\gamma_{E} > \gamma^{*}_{FL}$, we have $\Gamma^{*P}_{FL} > \Gamma^{*P}_{E}$, this would imply $\Gamma^{*P}_{FL} > \gamma^{*}_{FL}$ and therefore, from (A8), $\Gamma^{*}_{FL} = \gamma^{*}_{FL}$. This would give $\Gamma^{*}_{FL} \leq \Gamma^{*P}_{E}$. But the inequality $\gamma^{*}_{FL} < \gamma_{E}$ would also give $\Gamma^{*}_{FL} \leq \gamma_{E}$. Then, from (64), we would get $\Gamma^{*}_{FL} \leq \Gamma_{E}$, and (A2) would not hold.

Using $\gamma^{*}_{FL} < \gamma_{E}$, inequality (A12) implies $\Gamma^{*P}_{E} < \gamma_{E}$. Using $\Gamma^{s}_{FL} < \gamma_{E}$ (from (A5)), we therefore get from (64) :

$$\Gamma_{E} = \max\left(\Gamma^{s}_{FL}, \Gamma^{*P}_{E}\right) \tag{A13}$$

Conditions (A6) and (A12) have been shown to be necessary in order to obtain (A1) and (A2) respectively but they are not sufficient. But we are actually looking for sufficient conditions, in order to show that there exists some set of parameter values such that the fixed exchange rate systems dominates the flexible exchange rate system.

We will add the two following conditions

$$\Gamma_E^{*P} < \Gamma_{FL}^P \tag{A14}$$

$$\Gamma_{FL}^s < \gamma_{FL}^* \tag{A15}$$

Conditions (A6), (A12), (A10), (A14) and (A15) (together with (60) which was assumed to hold), can be shown to be sufficient in order to obtain (63a), (A1), (A2), (A3) and (A4). First, (63a) is implied by (A15) and the inequality $\gamma_{FL}^* < \gamma_E$. Second, it has already been shown that (A6) and (A10) imply (A3) and (A4). Third, from (A7), (A9) and (A13), inequality (A1) will hold if and only if we have

$$\Gamma_{FL}^s < \Gamma_{FL}^P \tag{A16a}$$

$$\Gamma_{FL}^s < \gamma_{FL} \tag{A16b}$$

$$\Gamma_E^{*P} < \Gamma_{FL}^P \tag{A16c}$$

$$\Gamma_E^{*P} < \gamma_{FL} \tag{A16d}$$

and inequality (A2) will hold if and only if we have

$$\Gamma_{FL}^s < \Gamma_{FL}^{*P} \tag{A17a}$$

$$\Gamma_{FL}^s < \gamma_{FL}^* \tag{A17b}$$

$$\Gamma_E^{*P} < \Gamma_{FL}^{*P} \tag{A17c}$$

$$\Gamma_E^{*P} < \gamma_{FL}^* \tag{A17d}$$

(A16a) is equivalent to (A6). (A16b) is always true. (A16c) is identical to (A14). (A16d) is implied by (A12) and the inequality $\gamma_{FL}^* < \gamma_{FL}$.

(A17a) is implied by (A6) and the inequality $\Gamma^P_{FL} < \Gamma^{*P}_{FL}$ (which is implied by $\gamma^*_{FL} < \gamma_{FL}$). (A17b) is identical to (A15). (A17c) is implied by (A14) and the inequality $\Gamma^P_{FL} < \Gamma^{*P}_{FL}$. (A17d) is identical to (A12).

It remains to show that all these sufficient conditions (A6), (A10), (A12), (A14) and (A15), together with (60), can be simultaneously satisfied for some parameter values.

Note, first, that, using (46a) and (58), condition (60), i.e. the inequality $\gamma_E < \gamma_{FL}$, can be written

$$\frac{\tilde{n}}{\tilde{n}^*} > \frac{\alpha+\rho}{\alpha}$$

¿From the expression giving $\Gamma^s_{FL}, \Gamma^P_{FL}$ and Γ^{*P}_E, inequalities (A6), (A10), (A12), (A14) and (A15) can be written respectively :

$$\theta > \frac{\delta}{2\left(\alpha+\rho\right)}\frac{1}{1+\delta\eta_0}\gamma_{FL} \equiv \theta_1$$

$$\theta > \frac{\delta}{4\left(\alpha+\rho\right)}\gamma_{FL} \equiv \theta_2$$

$$\theta < \frac{\delta}{4\alpha}\left(\gamma^*_{FL}+\gamma_E\right) \equiv \theta_3$$

$$\theta > \frac{\delta}{4\rho}\left(\gamma_{FL}-\gamma_E\right) \equiv \theta_4$$

$$\frac{\tilde{n}}{\tilde{n}^*} < k_1$$

Where k_1 is defined by $k_1 = \left(1+\delta\eta_0\right)/\left(1-\delta\eta_0\right)$ when $\delta < \dfrac{1}{\eta_0}$, and $k_1 = +\infty$ when $\delta \geq 1/n_0$

All these conditions can be written

$$\underline{\theta} < \theta < \overline{\theta} \tag{A18}$$

where $\overline{\theta} = \theta_3$ and $\underline{\theta} = \max\left(\theta_1, \theta_2, \theta_4\right)$

$$\frac{\alpha+\rho}{\alpha} < \frac{\widetilde{n}}{\widetilde{n}^*} < k_1 \tag{A19}$$

The two conditions (A18) and (A19) are possible if and only if we have $\underline{\theta} < \overline{\theta}$ and $(\alpha+\rho)/\alpha < k_1$.

This last inequality yields

$$\underline{\delta} \equiv \frac{1}{\eta_0}\frac{\rho}{2\alpha+\rho} < \delta$$

As we have $\eta_0 > 1$, this gives $\underline{\delta} < 1$.

The condition $\underline{\theta} < \overline{\theta}$, which is equivalent to $\theta_1 < \theta_3$, $\theta_2 < \theta_3$ and $\theta_4, < \theta_3$, gives

$$\frac{\widetilde{n}}{\widetilde{n}^*} < (1+\delta\eta_0)\frac{(\alpha+\rho)(2\alpha+\rho)}{2\alpha^2} \equiv k_2$$

$$\frac{\widetilde{n}}{\widetilde{n}^*} < \frac{(\alpha+\rho)(2\alpha+\rho)}{\alpha^2} \equiv k_3$$

$$\frac{\widetilde{n}}{\widetilde{n}^*} < \frac{(\alpha+\rho)^2+\alpha\rho}{\alpha^2} \equiv k_4$$

Note that k_2, k_3 and k_4 are always greater than $(\alpha+\rho)/\alpha$. Therefore let us define

$$\underline{k} = \frac{\alpha+\rho}{\alpha} \qquad , \qquad \overline{k} = \min(k_1, k_2, k_3, k_4)$$

We have $1 < \underline{k} < \overline{k}$

Then Proposition 1 follows.

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