

Tendering, Auctions and Preparation Costs

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This paper draws upon Waterson (1990) but some of the material written there is not explored in detail in the present paper, whilst other aspects are greatly extended. The opportunity to correct some of the analysis has also been taken. In drafting the paper we have benefitted greatly from advice, discussion and written comments provided by former colleagues of Canoy at the University of Amsterdam, in particular Roald Ramer, Claus Weddepohl and Dave Furth, and comments from Norman Ireland. Thanks are due also to Brian Beavis, Graham Loomes, Bob Sugden, Peter Hart, Simon Domberger and participants in the Warwick Summer Workshop 1991, a staff seminar at the University of Bristol and the 1992 EARIE conference in Stuttgart. Canoy is grateful to the NWO and the Tinbergen Institute for funding his work, and we are both grateful to Reading University for providing research facilities. An earlier version of the paper appeared as Chapter 5 in the PhD-thesis of Canoy.

SUMMARY

TENDERING, AUCTIONS AND PREPARATION COSTS

The main purpose of the paper is to examine the effects on the nature of equilibrium, and on comparisons between equilibria, of introducing bid preparations costs into various auction and tendering settings. In a model with private values and risk neutral players, we characterize the equilibria for the English auction, the Dutch open auction and the sealed-bid auction. We show that the revenue equivalence theorem still holds if bid preparation costs are introduced. Subsequently, we introduce asymmetry and we show that the revenue equivalence theorem holds as well under asymmetry, but not under both asymmetry and bid preparation costs.

RÉSUMÉ

DES COUPS D'ENTRÉE DANS LES ENCHÈRES

L'objet de cet article est d'examiner les effets sur les équilibres de l'introduction des coups d'entrée dans les enchères. Nous considérons les enchères hollandaises et anglaises qui peuvent être ouvertes ou secrètes. On utilise le modèle à valeurs privées indépendantes avec des acheteurs potentiels qui sont neutres envers le risque. On montre que le théorème d'équivalences des revenus reste valide si on introduit soit les coups d'entrée soit une asymétrie. Finalement, on montre que le théorème d'équivalences des revenus ne reste pas valide si on introduit des coups d'entrée et une asymétrie dans le même temps.

Mots clef: enchères, coups d'entrée, équivalences des revenus.

Key words: auctions, bid preparation costs, revenue equivalence.

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Tendering, Auctions and Preparation Costs

1. Introduction

The main purpose of this paper is to examine the effects on the nature of equilibrium, and on comparisons between equilibria, of introducing bid preparation costs into various auction and tendering settings. It is well known that there are similarities between auctions and competitive tendering, more specifically that under a range of circumstances first-price tendering is isomorphic to a Dutch auction.¹ Let us discuss two established facts from the auction literature.

ESTABLISHED FACT 1 (McAfee & McMillan 1987a)

Expected revenue for the seller is independent of the type of auction.

As usual, the auctions under consideration are the English (open) auction, the Dutch (open) auction, the first-price sealed-bid auction and the second-price sealed-bid auction. The underlying assumptions of the result are:

A1 Bidders are risk neutral.

A2 Independent-private values for the bidders.

A3 The bidders are symmetric.

A4 Payment is a function of bids alone.

It has been shown that revenue equivalence fails under risk aversion (see e.g. McAfee & McMillan 1987b). Equally so, there is no revenue equivalence if Assumption A2 is replaced by affiliated values (Milgrom & Weber 1982). The purpose of this paper is to relax Assumptions A3 and in particular A4, while maintaining Assumptions A1 and A2. We show that if *either* Assumption A3 or Assumption A4 is relaxed that revenue equivalence maintains to hold for the English and Dutch auctions. However, if *both* assumptions are relaxed at the same time revenue equivalence fails to hold and the English auction gives the seller the highest revenue. With this we provide a counter-example against the widely felt belief that asymmetry alone is sufficient to destroy revenue equivalence (see e.g. McAfee and McMillan 1987a). We relax Assumption A4 by introducing bid preparation costs. This is different from McAfee & Mcmillan (1987b), who look at an incentive compatible direct mechanism in which the seller determines payment functions that depend on the reported value of the bidders. Finally, we show that if Assumption A4 is relaxed that the seller has to reveal its information if it is risk averse and the seller can exploit the possibility of revealing if the bidders are incompletely informed about the possibility to reveal or not. If the seller is risk-neutral and it announces its revealing/concealing strategy (which may be contingent on

¹ For surveys of the literature, see eg. McAfee and McMillan, 1987a; Milgrom, 1987; Bulow and Roberts, 1989; see also the influential Milgrom and Weber, 1982.

the number of actual bidders showing up) then we re-establish the following result.

ESTABLISHED FACT 2 (McAfee & McMillan 1987b)

If the number of bidders is stochastic, revealing or concealing the actual number of bidders does not influence expected revenue of both the seller and the bidders.

The paper is organized as follows. In section 2 we introduce preparation costs into the various auctions settings with symmetric players. In the presence of bid preparation costs the game for the bidders is a two-stage decision procedure. In the first stage potential bidders decide upon entry after which they decide upon bidding. We solve for the Bayesian Nash equilibria of this game. In section 3 we consider an example in which there is asymmetry either in the distribution of the valuations or in the bid preparation costs. We show that introducing both asymmetry and preparation costs destroys revenue equivalence to the seller. Section 4 concludes.

Throughout our analysis we adopt four crucial assumptions which are appropriate to particular economic settings of the general problem (and are less appropriate to alternative settings). The first assumption is that the number of potential bidders is fixed exogenously. Here it is important to distinguish between the short and long run. If the government announces a procurement programme for a new jet fighter, say, then it is likely to know full well that there are only two plausible bidders/consortia to supply it.² If they had announce fifteen or twenty years ago that they would be procuring this sort of plane, then another airplane manufacturer might have developed the expertise to make a plausible bid. But in the short run, the number of bidders is definitely exogenous. The extent of time differentiating the short and long run obviously depend upon the particular setting. In letting a contract for waste collection, if the fact of the contract is unexpected with say one month to prepare bids, then the number of plausible bidders is likely to be exogenously determined. But if it is known six months in advance that a contract will be forthcoming, then treating the number as endogenous seems more reasonable. The preparation of a "select list of tenderers", widely used in certain circles, is tantamount to fixing exogenously the number of potential bidders.

Second, we assume private values. Again this is inappropriate for some settings (eg. the purchase of government securities). However, in bidding to supply a good or service, it seems more natural to assume that whilst the supplier has a good idea of its own costs in doing so (and therefore a good idea of the value of winning the contract, for any particular bid), its information about the contract's value to rivals is limited to a distribution of possible values. In the government securities case, there is a well-developed resale market, which is what gives rise to the common values. When bidding to supply a good or service,

² This was the case in the competition concluded in April 1991 in the US, won by the YF-22 of Lockheed, Boeing and General Dynamics.

it is implicit that you will engage in supply yourself, if you win the bid (unlike von Ungern-Stenberg's 'Swiss Auctions', 1991). Indeed, in certain settings, this is made explicit, and substantial personnel changes ex post can lead to the contract being declared void. Hence even an "affiliated values" setting (Milgrom and Weber, 1982) may be inappropriate. This assumption of course means there is no potential "winner's curse".

Third, we assume risk neutrality. While this is obviously a simplification, it is theoretically appropriate for a shareholder-controlled firm, the shareholders of which have diversified portfolios. They would then want to encourage management to behave in a risk-neutral way. It would also be appropriate for a profit maximizing firm engaged in a wide range of activities. It is therefore a relatively appropriate assumption for cases where a firm bids for contracts, whilst being less appropriate to consumers bidding for an object. Furthermore, the players face only risk, not uncertainty, by assumption.

Fourth, we assume that each potential bidder receives the private information about its own valuation of the object for sale (or its own costs in supply), before having to incur the bid preparation cost. Again, this is an assumption more suited to the tendering context than some others. At its simplest, it can be thought of as the potential bidder reading an outline "invitation to tender" or receiving a document inviting tenders or a telephone call inviting interest, then, knowing his or her own capabilities, being able to assess whether it is a task at which they are adept or not. The bidder then uses this information, together with some considerations examined below, in deciding whether to incur the full bid preparation costs. To put it another way, the costs of discovering their own capabilities are small relative to the bid preparation costs. Again, there are contexts where this assumption may be undesirable.

In order that we may treat auctions and tendering as equivalent, there is one further assumption we must make. The auctioneer starts bidding at a low value in the English auction, or will take bidding down to that value in the Dutch auction. We may set this value arbitrarily at zero. In tendering, the bidders themselves choose values. We have to assume the presence of a minimum price which they are able to bid or, if selling a service, a maximum price beyond which the organiser will not purchase. Failing this assumption, certain additional complexities may ensue (cf. Milgrom and Weber 1982).

In sum, our set of assumptions is particularly appropriate for contests amongst firms to supply a relatively unusual and complex good for which it is costly to communicate your ability to produce.

Given these assumptions, the appropriate equilibrium conception is Bayesian Equilibrium. We consider only pure strategy equilibria.

2. Preparation Costs in Auctions

The standard analysis of auctions makes the following (often implicit) assumption, "Transactions costs, including the cost of thinking, calculating, deciding and bidding, are negligible" (Smith, 1987, p.139).

Unlike in the standard auction, transaction (hereafter, preparation) costs are non-zero. There are relatively few existing models which treat such bidding costs seriously. Two early papers by Samuelson (1985) and by French and McCormick (1984) take the results of auction theory and graft onto them the idea of bid preparation costs. In both papers, the number of bidders is endogenous, and equilibrium is assumed to arise where the expected profit of the marginal bidder, after subtracting bid preparation costs, is zero. More sophisticated and extensive analyses are provided by Harstad (1990 and 1991). In a common values auction, he takes the case of endogenous bidder numbers, and he assumes that bid preparation costs have to be incurred before bidders discover their private information values. McAfee & McMillan (1988) consider a monopsonist that wants to buy one indivisible good from one of potentially many sellers. The buyer has to incur search costs to communicate with a seller. So, unlike in our paper, the total costs of the buyer depend on the number of sellers with which it communicates. The real decision in McAfee & McMillan (1988) is how many sellers to consult, whereas in our paper the real issue (for the seller) is which type of auction to choose. Whatieu (1989) is closest in conception to the present paper, as we shall note in later sections, but his purpose is more limited and we believe his analysis to be flawed in the Dutch auction case. Riley and Samuelson (1981) have a small section on preparation costs. Our Proposition 1 was already established by them. In common with all the literature cited in this paragraph, we assume bid preparation costs are a fixed amount for each potential bidder.

2.1 ENGLISH AUCTION

Assume there are n bidders in the auction room and the cost of putting in a bid is c . While second-price tendering is not often observed, a priori there is no reason why the organizer of a tender should not allocate the tender on the basis of a second-price auction. There is a two-stage decision-making process, first whether to enter at all and second, what bid to make. Once the decision to enter the auction room by paying the fee has been made (and so the cost becomes sunk), the auction can be analysed as a standard English auction and the dominant strategy is to bid up to the individual's valuation. Thus the perfect Bayesian equilibrium for the two-stage decision can be analysed from the outset, since each bidder will know how many bidders are active once he puts in a bid. The real decision concerns whether to enter the auction or not. The alternative of a second-price sealed-bid auction is not formally considered here since it is logically equivalent to the English auction (Vickrey 1961, McAfee & McMillan, 1987a).

Potential bidders' individual valuations, v_i , are identically and independently distributed as $F(v_i) \in C^1$ and monotonically increasing in v_i . Assume that each player knows the value of his or her own draw. The entry decision of the players can be represented by finding critical valuations, $x_1, \dots, x_n$, below which a particular player will decide not to enter the bidding. In other words player n chooses x_n , knowing that the other players face a similar choice. Probabilities are denoted by P .

Consider player 1's decision-making procedure. The crucial question is whether the realization of the random draw, $\underline{v}_1$, is above the critical level, or: $\underline{v}_1 > x_1$. Define $Y_i(m)$ as the i^{th} order statistic with m

variables, so that $Y_1(n-1) = \max \{v_2, \dots, v_n\}$, which we abbreviate with Y_1 . Using independence and symmetry we get $F_{Y_1}(v) = F^{n-1}(v)$ with density $f_{Y_1}(v) = (n-1)F^{n-2}(v)f(v)$.

Now the probability that player 1 has the highest valuation is:

$$P(\underline{v}_1 > Y_1) = F_{Y_1}(\underline{v}_1) \quad (1)$$

The players conjecture that the other players will employ a similar rule for entry decisions. We know that at the lower bound of the support the expected profit for a players must be zero. As explained above, in the English auction, determination of the equilibrium bids is straightforward and independent of x_i . Suppose players $2, \dots, n$ play strategy $x^* = (x^*, \dots, x^*)$. Now expected return $E(H_1(v_1))$, for simplicity denoted as $H_1(v_1)$, from entering for player 1 at a certain realization $v_1 = \underline{v}_1$ is:

$$\underline{v}_1 F_Y(x^*) - c \quad \text{if } \underline{v}_1 < x^* \quad (2a)$$

$$\underline{v}_1 F_Y(x^*) + \int_{x^*}^{\underline{v}_1} (\underline{v}_1 - z)(n-1)F^{n-2}(z)f(z)dz - c \quad \text{if } \underline{v}_1 \geq x^* \quad (2b)$$

In (2a), since $\underline{v}_1 < x^*$, player 1 knows that it can only win the contest if the other players do not enter the auction. In that case it receives $\underline{v}_1$. In (2b) there is also a chance that player 1 wins the contest if other players enter. Finding a critical value x_1 , given x^* , means finding the critical v_1 for which the expected pay-off in (2) is zero, so that player 1 is indifferent between entering and not entering.

Proposition 1 (Riley and Samuelson 1981)

The unique equilibrium entry threshold in the English auction is the solution of:

$$x^* F^{n-1}(x^*) = c.$$

Proof

Setting (2a) equal to zero at $\underline{v}_1 = x_1$ gives $x_1 = c/F^{n-1}(x^*)$. Since this has to hold for all players the assertion follows. The equilibrium is unique because $F(x)$ is monotonically increasing in x .

For $F(\cdot)$ uniform $[0,1]$ the equilibrium becomes $x_i = c^{\frac{1}{n}}$. Notice that as the number of players goes to infinity, players only enter if $v_i \rightarrow 1$. Also, the probability that nobody enters is $(c^{\frac{1}{n}})^n = c$, i.e., independent of n . Although more players potentially enter, their individual probability of entering decreases as n increases.

3.2 DUTCH AUCTION

The problem here is more complex even in seemingly simple cases. This is basically because, in a two-stage procedural framework, namely choosing whether to enter and then if entering, how much to bid, there is no unique dominant bid for the second stage. The models which follow illustrate the point.

Each player realizes the others are potential players. Each will incur a cost of c in entering the auction. There are two possibilities. You may know whether other players have entered or not (we call this case "Dutch (open) auction"). Alternatively, you may not be aware that the other player has not tendered (which we call "sealed-bid auction"). This leads to two separate cases of the two-stage game, where the first stage, whether to enter or not, depends upon the expected pay-offs from the second stage.

Take the case where players know how many players are in, after having paid fee c (i.e., player 1 is let into the auction tent, and may discover there is no one else there). In the case where there is no one else, the optimal bid clearly is zero. But if other players are present, this provides a signal regarding the other player's valuations. The optimal bids of the players should take account of the fact that the other players have drawn a valuation that was high enough to enter the contest. This point seems to be missed by Whatieu (1989), whose result for the Dutch auction oversimplifies matters.

Notice that we allow for the possibility that only one player shows up, whereas in some contexts sellers may elect to postpone the tender until (at least) two players enter the contest. Even so, a postponement can be identified with an undesirable (and therefore costly) delay. In addition, if the players have an equal chance of winning the contest, postponement of the tender will lead to the expected pay-off being negative, unless the player that shows up receives some sort of compensation.

It is clear that the bids are optimally a function of critical values x as well as valuations, so suppose:

$$b_i = B(v_i; x)$$

We will write $b_i = B(v_i)$, since the x 's are parameters rather than variables here. For the derivation of the bidding function B we assume that the x 's are the same for all players. Recall that $Y1(m-1) = \max\{v_2, \dots, v_m\}$. By abuse of notation we abbreviate with $Y1$. Expected pay-off from entering if player 1 knows it is not the only bidder is:

$$P(b_1 > Y1) = F_{Y1}(B^{-1}(b_1))$$

$$\text{Hence expected return becomes } H_1 = (v_1 - b_1)F_{Y1}(B^{-1}(b_1)) - c \quad (3)$$

The first order condition (FOC) is:

$$(\partial F / \partial b_1)(v_1 - b_1) = F_{Y1}(B^{-1}(b_1)).$$

We apply the same method as McAfee and McMillan (1987a). Using the Nash condition we may write $B^{-1}(b_1) = v_1$ and using the Envelope Theorem the FOC becomes:

$$(v_1 - b_1)(1/b')f = F.$$

This is a non-homogeneous first order differential equation of the form:

$$b' + u(v)b = w(v)$$

where $u(v) = f/F$ and $w = (f/F)v$. The solution of this is:

$$b(v) = \exp(-\int u dv)[A + \int w \exp(\int u dv) dv]$$

The expression can be simplified by using the second requirement for b to be an equilibrium. The expected profit at the lower bound of the support must be zero, which eliminates the constant A . Now

$\exp(-\int u dv) = \exp(-\int f/F dv) = F(x)/F(y)$ and similarly $\exp(\int u dv) = F(y)/F(x)$.

Proposition 2 (see also McAfee and McMillan 1987a)

In the Dutch Auction the equilibrium bid with m bidders is equal to:

$$b^*(y) = 1/F_{Y1}(y) \int_x^y z dF_{Y1}(z).$$

Proof

Follows from solving the differential equation.

It is important to notice that, unlike in McAfee and McMillan (1987a), b^* is the equilibrium bid if there are m *active* bidders. There were n potential bidders, so we abused notation by using $F(\cdot)$ and $f(\cdot)$ for $m-1$ rather than for $n-1$ (other) bidders.

Let $F(\cdot)$ be uniform $[0, 1]$. Notice that we consider the case that player 1 knows that there are $m-1$ other players present, so player 1 knows that all $m-1$ bidders draw from $[x, 1]$. The equilibrium bid becomes:

$$b(y) = [y(m-1) + x]/m, \quad (4)$$

Giving the equilibrium bids we can solve for the equilibrium entry thresholds.

Proposition 3

In the Dutch Auction the equilibrium threshold x^* is the solution of:

$$x^* F^{n-1}(x^*) = c.$$

Proof

Take expected return from entering at $y_1 = x_1$ and assume all other players set $x = x^*$. Clearly, since all players draw from the same continuous distribution the only way a player can make a profit at the lower bound of the support if it is the only bidder. Expected return then becomes: $H_1 = x_1 F_{Y1}(x^*) - c$. Set this equal to zero and realize again that it has to hold for all players. It follows that Proposition 1 applies.

Although equilibrium entry values are the same for the English and Dutch auction, this does not necessarily imply that the revenue equivalence properties continue to hold (see eg. Philips 1988; Milgrom and Weber 1982; Riley 1989).

The expected revenue from the symmetric English auction described earlier is zero unless at least two players enter, for otherwise the item is knocked down at the second highest bid (or not at all). The density of the second order statistic is:

$$f_{Y2}(v) = m(m-1)(1-F(v))F(v)^{m-2}f(v)$$

so that expected revenue to the seller is:

$$R_E = \int_x^1 zm(m-1)(1-F(z))F(z)^{m-2}f(z)dz \quad (5)$$

The expected revenue from the symmetric Dutch auction with complete information likewise is zero, unless at least two players enter. In the latter case, as in McAfee & McMillan (1987a), each bidder will assume that it has the highest valuation. The players can do so, because even though losing is not costless here, at the bidding stage the costs are already sunk. Each bidder will estimate how far below his own valuation the next highest valuation is on average and then submits a bid that is that amount below its own valuation. For the seller this means that the expected highest bid is exactly equal to expected second highest valuation, which was the expected return from the English auction. This leads to the following result:

Proposition 4

With strictly positive preparation costs, the seller's expected revenue maintains the same for the Dutch and the English auction.

Proof

Follows from above.

The result is the same as in Riley and Samuelson (1981), R&S from now on, who consider reservation prices for the seller. They show that if the reservation value is the same for all bidders, revenue equivalence follows. For the bidders the situation with a reservation price is strategically similar to the setting of an entry fee, since the bidding costs become sunk after entering. The difference is that in case of a reservation price bidders have to pay the reservation price rather than the lower bound of the support of the valuations. This in turn influences their expected return, but not their bidding strategy. For the seller the difference is more important, since it may be optimal to set both a reservation price and an entry fee (see also Milgrom and Weber 1982).

There is another thing worth investigating here. Suppose the seller can set c . What would be the optimal c ? This question was also considered by R&S in the case of the reservation price. There are two variants. If one interprets the model as a reservation price model, the seller does not gain anything in particular, i.e. it sets x but receives nothing. R&S look at this case for 2 players and uniform distribution. Their conclusion is that the optimal x is equal to $\frac{1}{2}$. Although their display of the bidding strategy in the example is flawed,³ this remains without consequence here. From (5) it follows that expected revenue in the example is equal to:

³ According to R&S the bidding strategy is $b(v) = \frac{1}{2}v + \frac{1}{8v}$, but that does not account for the fact that the bidders realize that the other bidder draws from $[x, 1]$, rather than from $[0, 1]$, so it should be $b(v) = \frac{1}{2}v + \frac{1}{4v}$.

$$[1/3 + 2/3 x][1 - x]^2 = 1/3 - x^2 + 2/3 x^3$$

The optimal x is zero here, not $1/2$. This can be explained from the fact that R&S assume that in case of one bidder the bid is not zero but the "reservation price" x (which makes sense), in which case the term $2x^2(1-x)$ should be added after which the optimum is indeed $1/2$. The optimum $1/2$ is also obtained if we assume instead that the seller can keep the entry fee. The expected "entry fee revenue" is $2c(1-x)^2 + c2x(1-x) = 2c(1-x)$ which (in equilibrium) is equal to $2x^2(1-x)$. Some conclusions:

Observation 1

It does not matter for the seller whether it receives c or whether it sets a reservation price.

Suppose now we are in a tendering situation where the bidders need to incur preparation costs. Assume that the seller does not receive anything (which makes sense for this case). It seems optimal for the seller now that $c=0$. Suppose initially that $c=1/4$ so that $x=1/2$ and revenue of the seller is $1/6$. If it cannot manipulate the costs, is it better for the seller if it announces a reservation price R in addition to the fee? If it does the entry threshold x changes. Rather than receiving $\underline{y}$ if you are the only bidder you receive $\underline{y} - R$, yielding an equilibrium threshold of:

$$x^* = 1/2 R + 1/2(R^2 + 1)^{1/2} \quad (6)$$

Observation 2

The seller can increase its expected revenue if it sets a positive reservation price.

Proof

Using a numerical approximation the optimal values for $c=1/4$ are (H is expected profit):

$R^* \approx 0.317$; $x^* \approx 0.683$; $H^* \approx 0.2165 > 1/6$. For other values of c the same was obtained: if $c=0$ $R=1/2$ and $x=1/2$; if c increases R decreases (to zero) and x increases (to one).

Normalizing the reservation price of the seller to 0 is not that innocent, since it may be optimal (as in the numerical example) to have a reservation price that is at the interior of the support of the players. Since the relative gain of setting $R > 0$ is high for 2 players the effect may be not so dramatic for higher bidder numbers, but it is bound to be existent anyway. The above result does not imply that the previous results are meaningless, since the bidding strategies do not depend on the existence of a reservation price (see also Milgrom and Weber 1982). Also, it is not clear whether setting such a reservation price is always feasible. Finally, if the seller keeps the costs c (which is might if it the costs would be the entry costs in an auction) it may set a reservation price in addition. In the above mentioned example, the results are that the seller sets $R=1/2$ for $c=0$ and $R=0$ for $c=1/4$. But what if $c > 1/4$? We have a Santa Claus result:

Observation 3

If the seller can keep the entry costs, there are parameter values such that it has to set a negative reservation price.

Proof

Invert (6) to obtain $R = (x^2 - c)/x$. The expected return to seller, is:

$$\begin{aligned} 1/3 - x^2 + 2/3x^3 + 2x(1-x)R + 2c(1-x) = \\ 1/3 + x^2 - 4/3x^3 \end{aligned}$$

This expression optimizes to $x = 1/2$, so that $R = 1/2 - 2c$. For $c > 1/4$, R becomes negative. ■

The intuition of the seller playing Santa Claus is that it wants to avoid the situation of 1 active bidder. In presenting them a bonus R it achieves that both bidder are more keen to enter.

3.3 FIRST-PRICE SEALED-BID AUCTION

For the sealed-bid auction the analysis is the same as in the Dutch auction. The only difference is that players have to bid without the knowledge of the number of actual bidders. This only effects the distribution so that Propositions 2 and 3 also apply for this case. However because of the different distribution the equilibrium bids differ from the Dutch auction. For the uniform $[0, 1]$ case the equilibrium bid becomes:

$$b(\underline{v}) = (n-1)/n [\underline{v} - x^n/\underline{v}^{n-1}]. \quad (7)$$

Using the same arguments as in Propositions 1 and 3, it follows that the equilibrium entry decision is the same as before, namely the solution of $xF^{n-1}(x^*) = c$. McAfee & McMillan (1987b) show that the expected winning bid in case of a stochastic number of bidders is not effected by revealing the number or not. Since in the sealed-bid auction the number of bidders is effectively stochastic the same applies and revenue equivalence follows. Notice that a bidder does not calculate the conditional probability that the other bidders are active given that it is active itself. If all bidders would do so the expected pay-off would be the same, but the strategies cannot be equilibrium strategies. In doing so bidders must put a mass on bidding zero. Hence the probability that all active bidders bid zero is strictly positive. But then bidders can improve if they bid ε instead.

Proposition 5

In the presence of bid preparation costs the expected return of the seller is the same for all auctions.

Proof

Follows from McAfee & McMillan (1987b).

Observe that for $x=0$ the expression (7) is equivalent to (4). It is important to observe that players always bid according to (7) in the sealed-bid because n is fixed here. In the Dutch auction case (4), the m reflected the observed number of bidders. Let us compare (4) with (7) when the number of active bidders is equal to n . Clearly, if all potential bidders turn out to be active ones, the seller would want to reveal the number of bidders ex post to avoid the lower bids of (7). If there is only one player it wants to avoid the zero bid of the Dutch auction, so that it wants to conceal the number of bidders. The one player - not knowing it is the only active bidder - will bid according to (7) whereas otherwise it would bid zero. So there exists a critical number m^* such that the seller wants to reveal ex post for higher bidder numbers. However, the seller has to realize that the players do anticipate to the possibility of revealing so that they update their number of potential bidders. Suppose that the bidders correctly interpret the revealing decision of the seller. We show that the seller can do no better than to reveal always or conceal always. Both actions do not provide extra information to the bidders.

Proposition 6

Revealing and concealing the number of bidders yield the same expected outcome to the seller. Moreover, any combination of the two (contingent on the number of bidders) yield the same expected pay-off.

Proof

Suppose the seller decides to reveal if the number of actual sellers exceeds m^* and to conceal in the other cases. The bidders are assumed to be able to calculate m^* , implying that if the seller conceals the bidders know that $m \leq m^*$. So they will update according the number of potential players to m^* . For any higher number than m^* the strategy gives the same revenue as the Dutch auction. For a lower number of bidders fully revealing gives Dutch auction profits. Now suppose there were only m^* players in the first place. We know that revealing and concealing would give the same revenue, so that applying any differentiated revealing strategy is revenue neutral in expectation sense. The same argument holds if the seller applies a mixed strategy. As long as the randomization is known to the bidders they update their beliefs about the actual number of players. ■

So the seller is ex ante indifferent between revealing and concealing or anything in between. However, ex post, i.e. after it observes the actual number of bidders, the seller is not indifferent anymore. As argued above, it wants to reveal for high bidder numbers. If the bidders correctly perceive the revealing strategy of the seller, the seller is indifferent between revealing and concealing. But how realistic is it to assume that bidders correctly perceive the strategy of the seller? This would only be realistic if the bidders can actually calculate a unique equilibrium strategy of the seller, which in this case it cannot. There is a continuum of pure strategy equilibria here. It implies that the bidder fails to predict the action of the seller with probability one (given the continuity of the number of equilibria). Assuming pure strategies, if the bidder "overestimates" m^* , the seller gains. What happens though if the bidder "underestimates" m^* ? In

the most extreme case the seller always conceals, but the bidders think the seller only conceals if there is one bidder, in which case the bidders bid zero and the seller loses. Unlike in non-cooperative game theory there is a solution here. To avoid indeterminacy the seller should always reveal or announce its $R(m)$ (if that is credible). In non-cooperative game theory this would be considered as unallowed communication but here it is perfectly feasible. Moreover, if the seller is risk averse it wants to avoid the fluctuation in profits that follow from the indeterminacy of the outcome. If players think a sealed-bid auction is played, whereas there is a possibility of revealing, the seller can exploit this by applying the optimal m^* strategy. This assumption is not realistic in many classical games between firms, but often in auctions or tendering there is not a long-term relationship between the seller and its buyers.

Proposition 7

If the bidders are imperfectly informed about the possibility of the seller to reveal, the seller's expected profit increases if it has the possibility to reveal.

Proof

If bidders do not update (or imperfectly) their beliefs about the number of potential players, the seller's expected revenue increases. It can reveal for high bidder numbers and reveal for low bidder numbers. ■

This finding puts Established Fact 2 of McAfee & McMillan (1987b) into perspective. Under risk aversion and zero preparation costs McAfee and McMillan showed that the first-price sealed-bid auction dominates the Dutch auction. Here the same holds under risk-neutrality and imperfect information.

3. Asymmetries

The following extension of the model relaxes Assumption A3 of the benchmark model. In other words we allow for asymmetries, either in the distribution of the players' valuations or in the preparation costs. To keep the model tractable we have to confine ourselves to the two-players case with F the uniform distribution here.⁴ This will not be problematic here, since we mainly want to show by an example that certain views that are held in the literature are not necessarily correct. Without providing a formal proof McAfee & McMillan (1987a) state that the asymmetry destroys revenue equivalence. We show in a counter-example that this need not be the case. We concentrate on the English and the Dutch auction, since the sealed-bid auction produces non-existence of the threshold equilibrium.

ENGLISH AUCTION

Let us assume that player 1 draws from $[0,1]$, but player 2 draws from $[0,u]$. Player 1's expected profit

⁴ With n players and asymmetry we cannot solve the differential equation anymore so that the solution of the equilibrium bids in the Dutch auction is unclear.

of entering becomes:

$$\underline{v}_1 x_2 / u - c \quad \text{if } \underline{v}_1 < x_2 \quad (8a)$$

$$(\underline{v}_1^2 + x_2^2) / 2u - c \quad \text{if } \underline{v}_1 \geq x_2 \quad (8b)$$

which leads to the best response function:

$$\begin{aligned} x_1 &= [2uc - x_2^2]^{1/2} && \text{for } x_2 \leq (uc)^{1/2} \\ &= cu/x_2 && \text{for } x_2 > (uc)^{1/2} \end{aligned} \quad (9)$$

The best response for player 2 is:

$$\begin{aligned} x_2 &= [2c - x_1^2]^{1/2} && \text{for } x_1 \leq c^{1/2} \\ &= c/x_1 && \text{for } x_1 > c^{1/2} \end{aligned} \quad (10)$$

By comparison of the various conditions involved in equations (9) and (10), the only admissible solution for $u > 1$ is the case where $x_2 = c/x_1$ and $x_1 = (2uc - x_2^2)^{1/2}$, which gives the equilibrium:

$$x_1 = c^{1/2} [u + (u^2 - 1)^{1/2}]^{1/2} \quad (11)$$

$$x_2 = c^{1/2} [u + (u^2 - 1)^{1/2}]^{-1/2} \quad (12)$$

For x_1 and x_2 to be in the relevant ranges we need $u + (u^2 - 1)^{1/2} \leq 1/c$. For u beyond this value the equilibrium is just $x_1 = 1$, $x_2 = c$, which corresponds with the intuition that if the difference between two players is too large, the disadvantaged player (here player 1) will drop out.

DUTCH AUCTION

Let us assume again that player 1 draws from $[0, 1]$ and player 2 from $[0, u]$ $u > 1$. Now the asymmetry implies that the equilibrium bids will depend on both entry decisions, while remaining linear in v .

So take $b_2 = B(v_2)$ and B linear, so that v_2 has support $[x_2, u]$, it follows that the optimal bids are:

$$\begin{aligned} b_1^* &= \frac{1}{2}(\underline{v}_1 + \frac{2}{3}x_2 + \frac{1}{3}x_1) \\ b_2^* &= \frac{1}{2}(\underline{v}_2 + \frac{2}{3}x_1 + \frac{1}{3}x_2) && \text{if } \underline{v}_2 \leq 1 + \frac{1}{3}(x_2 - x_1) \\ &= \frac{1}{2}(1 + \frac{2}{3}x_2 + \frac{1}{3}x_1) && \text{if } \underline{v}_2 > 1 + \frac{1}{3}(x_2 - x_1) \end{aligned} \quad (13)$$

If player 2 draws a valuation that would yield a bid higher than the upper bound of the support of the bid

of player 1, it bids this upper bound. Clearly, player 2 does not put in a high bid if it is sure to win with a lower bid. This means that there is a mass point at this upper bound. For the polar case of $c = x_1 = x_2 = 0$, it implies that there is a mass point at $\frac{1}{2}$. Since there is no such mass point in the English auction case, one feels that revenue equivalence will be destroyed. This was indeed the intuition of McAfee & McMillan (1987a). However, it turns out that revenue equivalence still holds for this polar case. To see this let us decompose the $[0, u]$ case into two separate case. With probability $1/u$ player 2 draws from $[0, 1]$ and with $(u-1)/u$ player 2 draws from $[1, u]$. In the first case we are back to the symmetric case and revenue equivalence checks. If player 2 draws from $[1, u]$ then, by (13), it will bid $\frac{1}{2}$ and the expected price will therefore also be $\frac{1}{2}$. In the English Auction player 2 is sure to win too so that the expectation of the minimum is equal to the expectation of the bid of player 1 which is also $\frac{1}{2}$.

Proposition 8

Let $c=0$ and $n=2$. If players draw uniform from $[0, 1]$ and $[0, u]$ respectively, there is revenue equivalence for the seller.

proof

From (13) it follows that if $\underline{v}_2 \leq 1$ then $b_1^* = \frac{1}{2}\underline{v}_1$ and $b_2^* = \frac{1}{2}\underline{v}_2$ yielding revenue equivalence as before. If $\underline{v}_2 > 1$, then the English auction gives an expected price of $\frac{1}{2}$, since that is the expected value of the lowest bid, i.e. player 1's bid. In the Dutch auction $b_2 = \frac{1}{2}$, i.e. expected prices are the same and thus expected revenue is the same. ■

So we provided a counter-example for the assertion made by McAfee and McMillan (1987a), which was partially based on examples provided by Maskin & Riley (1985), M&R henceforth. M&R provide an example in which revenue equivalence breaks down under asymmetry, but their example is degenerate in the sense that they assume distributions are discrete. They assume that players draw 0 with probability $1-p$ and some high value v_H (or w_H respectively) with probability p . If $v_H = w_H$ revenue equivalence checks, but if you increase w_H the expectation of the minimum bid remains the same, whereas the expectation of the maximum bid increases. For this reason M&R conclude that the Dutch auction does better for this example.⁵ Our counter-example shows that their result typically depends on the distributions being discrete.

Let us now determine the equilibrium entry thresholds. If player 2 draws a valuation equal to its threshold, assuming it is lower than the threshold of the disadvantaged player 1, it can only win if it is the only player. This will lead to a best response of

$$x_2 = c/x_1 \tag{14}$$

⁵ M&R do not formally prove this assertion nor do they provide a bidding equilibrium function for this case.

For player 1 things are different. Since we can conjecture that $x_1 > x_2$, at the threshold valuation player 1 still has a positive probability of winning if player 2 shows up. Player 1's expected revenue at its entry threshold becomes:

$$x_1 * [x_2/u] + [x_1 - b_1(x_1)] * P(b_1(x_1) > b_2(v_2)) * [(u-x_2)/u] - c \quad (15)$$

Now $x_1 - b_1(x_1) = \frac{1}{3}(x_1 - x_2)$ and $P(b_1(x_1) > b_2) = \frac{2}{3}(x_1 - x_2)/(u - x_2)$. Combining (14) and (15) to solve for the equilibrium leads to:

$$x_1 = c^{1/2} [\frac{1}{4}(9u-5) + ((9u-5)^2/16 - 1)^{1/2}]^{1/2} \quad (16)$$

$$x_2 = c^{1/2} [\frac{1}{4}(9u-5) - ((9u-5)^2/16 - 1)^{1/2}]^{1/2} \quad (17)$$

It is immediately obvious that $u=1$ induces $x_1 = x_2 = c^{1/2}$. It is important to observe that if one compares (16) with the English auction (11), it turns out that the x_1 of the Dutch auction is always higher. This in turn means that the probability that both enter is lower in the Dutch Auction.

The sealed-bid asymmetric case is rather complicated. Using similar methods as in the Dutch auction case, one obtains equilibrium bids:

$$b_1 = A_1 * \frac{1}{2}(\underline{v}_1 - x_1^2/\underline{v}_1)$$

$$b_2 = \frac{1}{2}(\underline{v}_2 - A_2/\underline{v}_2)$$

where $A_1 = (x_2^2 + A_2)/2c$ and $A_2 = \{2c^2 - (x_1 x_2)^2 - 2c[(c^2 - (x_1 x_2)^2)^{1/2}]\}/x_1^2$. For $x_1 = x_2 = c^{1/2}$ the expression reduces to expression (7). Now $b_1(x_1)=0$ therefore player 1 can only win if it is the only player, yielding the best response $x_1 = uc/x_2$, but then A_2 is not defined anymore, so that there is non-existence of a threshold equilibrium.

ASYMMETRY IN PREPARATION COSTS

Let us consider asymmetric bid preparation costs. It turns out that the result parallels the result from asymmetry in valuations.

Let $c_1 > c_2$ and both players draw from $[0,1]$. Consider the English auction first. From (2) the best responses can be determined, leading to the equilibrium:

$$x_1 = [c_1 + (c_1^2 - c_2^2)^{1/2}]^{1/2} \quad (18)$$

$$x_2 = [c_1 - (c_1^2 - c_2^2)^{1/2}]^{1/2} \quad (19)$$

Naturally, if $(c_1 - c_2)^{1/2} > c_1$ player 2 always enters and if $c_1 + (c_1 - c_2)^{1/2} > 1$ player 1 never enters. In the cases with asymmetry, there are many parameter constellations for which joint expected pay-off is higher for the players than in the symmetric case. This is so, because there is an advantaged player, so that more often only one player shows up, extracting all the surplus. Clearly, in the case of only one competitor, the seller is more likely to postpone the tender, the more asymmetry there is.

Similarly, in the Dutch auction equilibrium becomes:

$$x_1 = \{ \frac{1}{4}(9c_1 - 5c_2) + [(9c_1 - 5c_2)^2/16 - c_2^2]^{1/2} \}^{1/2} \quad (20)$$

$$x_2 = \{ \frac{1}{4}(9c_1 - 5c_2) - [(9c_1 - 5c_2)^2/16 - c_2^2]^{1/2} \}^{1/2} \quad (21)$$

Using numerical approximations, it turns out that the same result as in the case of asymmetric distributions emerges. The English auction always gives the lower x_1 .

Let us return to the asymmetric distributions now. It turns out that the revenue equivalence is not robust to the inclusion of both preparation costs and asymmetries. The English auction gives the auctioneer the minimum of the two valuations (if both players submit bids). Because of the asymmetries (which leads to discontinuities) we have to decompose the bids of player 2 into three parts. In part A player 2 draws a valuation lower than x_1 , so that player 1 is the sure winner if it draws a valuation higher than x_1 , leading to expected revenue for this subcase of:

$$R_A(EA) = P(x_2 < v_2 < x_1) * E[v_2] = [(x_1 - x_2)/(u - x_2)] * [(x_1 + x_2)/2]$$

In subcase B player 2 draws from $[x_1, 1]$. Now the expected return can be obtained from the previous section, yielding:

$$R_B(EA) = [(1 - x_1)/(u - x_2)] * [\frac{1}{3} + \frac{2}{3}x_1]$$

In subcase C player 2 draws from $[1, u]$, making him a sure winner. The expected return is the expected value of the bid of player 1:

$$R_C(EA) = [(u - 1)/(u - x_2)] * [(1 + x_2)/2]$$

For the total revenue we just add up $R_A + R_B + R_C$ and multiply by the probability that both players enter, yielding:

$$R(EA) = [(1-x_1)*(u-x_2)/u]*[R_A(EA)+R_B(EA)+R_C(EA)] \quad (22)$$

The analysis of the Dutch auction is similar. The bids of player 2 are decomposed again into three parts. Let us assume first that player 2 draws its valuation from $[x_2, 1 + \frac{1}{3}(x_2-x_1)]$, implying (from (13)) that it bids $b_2 = \frac{1}{2}(1 + \frac{2}{3}x_2 + \frac{1}{3}x_1)$. Let us call this case A. The probability that A occurs is:

$$P_A = (1 + \frac{1}{3}(x_2-x_1) - x_2)/(u-x_2)$$

There are two subcases. In subcase A1 player 2 draws from $[\frac{2}{3}x_1 + \frac{1}{3}x_2, \frac{1}{3}x_1 + \frac{2}{3}x_2]$, implying that if both players enter player 1 wins for sure, since the support of the bid of player 1 is $[\frac{2}{3}x_1 + \frac{1}{3}x_2, \frac{1}{2} + \frac{1}{3}x_1 + \frac{1}{6}x_2]$. The expected revenue is then equal to the expected bid of player 1 multiplied by the probability that player 2 draws from $[\frac{2}{3}x_1 + \frac{1}{3}x_2, \frac{1}{3}x_1 + \frac{2}{3}x_2]$ or:

$$R_{A1}(DA) = [\frac{1}{3}(x_1-x_2)/(\frac{1}{2}-\frac{1}{3}x_2-\frac{1}{6}x_1)]*[\frac{1}{4} + \frac{5x_1}{12} + \frac{1}{3}x_2]$$

In case A2 both players draw from $[\frac{2}{3}x_1 + \frac{1}{3}x_2, \frac{1}{2} + \frac{1}{3}x_1 + \frac{1}{6}x_2]$. The expectation of the maximum of two valuations that are uniformly distributed over $[a, b]$ is equal to:

$$\int_a^b 2z(z-a)/(b-a)^2 dz = \frac{2}{3}b + \frac{1}{3}a.$$

Thus the expected revenue for this subcase becomes:

$$R_{A2}(DA) = [1 - (\frac{1}{3}(x_1-x_2)/(\frac{1}{2}-\frac{1}{3}x_2-\frac{1}{6}x_1))]*[\frac{1}{3} + \frac{1}{3}(x_1+x_2)]$$

In case B player 2 draws his valuation from $[1 + \frac{1}{3}(x_2-x_1), u]$, yielding a bid of $\frac{1}{2} + \frac{1}{3}x_1 + \frac{1}{6}x_2$ and a sure win for player 2. Expected revenue becomes:

$$R_B(DA) = [\frac{1}{2} + \frac{1}{3}x_2 + \frac{1}{6}x_1].$$

Total revenue becomes the some of the revenues from the subcases, multiplied by the appropriate probabilities and with the probability that both players enter. Notice that this last multiplication is necessary since the probability that both players enter can be different in both auctions.

$$R(DA) = [(1-x_1)*(u-x_2)/u] * \{[P_A]*[R_{A1}(DA) + R_{A2}(DA)] + [1-P_A]*[R_B(DA)]\} \quad (23)$$

Comparing $R(EA)$ with $R(DA)$, using numerical approximations, it turns out that the English auction gives the seller a higher revenue than the Dutch auction. This can be explained from the fact that the probability that both enter is lower in the Dutch auction. With this we showed that the introduction of preparation costs and asymmetries destroys revenue equivalence. Another point is that the asymmetric case does not necessarily give a lower revenue than the symmetric case. The fact that more often only one player shows up can be compensated by the fact that player 2 draws from a wider support than in the symmetric case. In particular if c is low the asymmetric case will give a higher revenue.

4. Conclusion and discussion

Enough has been said to demonstrate the falsity a proposition which, whilst not universally held, nevertheless appear to be widely believed. The introduction of bidding costs into bidding models is not something which can be done without potentially substantial modifications to the outcomes (cf. eg. French and McCormick, 1984). This paper showed that the revenue equivalence only holds in the symmetric case and with the number of bidders being common knowledge. In the presence of bidding costs, any asymmetry in the distribution of the valuations or the preparation costs destroys the revenue equivalence theorem. Furthermore, if the number of bidders is not known and bidders are imperfectly informed about the possibility to reveal, it depends on the actual number of bidders, which of the auctions will be best for the seller.

References

- BULOW, J. AND ROBERTS, J. (1989) "The simple economics of optimal auctions", *Journal of Political Economy*, 97, 1060-90.
- CANOY, M. (1993) *Bertrand meets the fox and the owl - essays on the theory of price competition*, PhD thesis, Thesis Publishers, Tinbergen Institute/University of Amsterdam.
- FRENCH, K.R. AND MCCORMICK, R.E. (1984) "Sealed bids, sunk costs and the process of competition", *Journal of Business*, 57, 417-41.
- HARSTAD, R.M. (1990) "Alternative common-values auction procedures: revenue comparisons with free entry", *Journal of Political Economy*, 98, 421-9.
- HARSTAD, R.M. (1991) "Auction with endogenous bidder participation", *Working Paper*, Virginia Commonwealth University, Richmond.
- MASKIN, E. AND RILEY, J.G. (1985) "Auction Theory with Private Values", *American Economic Review, Papers and Proceedings*, 75-2, 150-155.
- MCAFEE, R.P. AND MCMILLAN, J. (1987a) "Auctions and bidding", *Journal of Economic Literature*, 25, 699-738.
- MCAFEE, R.P. AND MCMILLAN, J. (1987b) "Auctions with a Stochastic Number of Bidders", *Journal of Economic Theory* 43, 1-19.
- MCAFEE, R.P. AND MCMILLAN, J. (1988) "Search Mechanisms", *Journal of Economic Theory* 44, 99-123.
- MILGROM, P.R. (1987) "Auction Theory" in T. Bewley (ed) *Advances in Economic Theory*, Cambridge University Press.
- MILGROM, P.R. AND WEBER R.J. (1982) "A theory of auctions and competitive bidding", *Econometrica* 50, 1089-1122.
- PHILIPS, L. (1988) *The economics of imperfect information*, Cambridge, Cambridge University Press.
- RILEY J.G. AND SAMUELSON W.F. (1981) "Optimal Auctions" *American Economic Review*, 71, 381-392.
- RILEY, J.G. (1989) "Expected revenue from open and sealed bid auctions", *Journal of Economic Perspective*, 3, 41-50.
- SAMUELSON, W.F. (1985) "Competitive bidding with entry costs", *Economic Letters*, 17, 53-57.
- SMITH, V.L. (1987) "Auctions" in J. Eatwell, M. Milgate and P. Newman (eds) *The New Palgrave: A Dictionary of Economics*, Basingstoke, Macmillan.
- VON UNGERN-STERNBERG, T. (1991) "Swiss Auctions", *Economica* 58, 341-357.
- VICKREY, W. (1961) "Counterspeculation, auctions and competitive sealed tenders", *Journal of Finance*, 16, 8-37.
- WATERSON, M. (1990) "Competitive Tendering and Equilibrium", Mimeo, University of Reading, July.
- WHATIEU, L. (1989) "Auctions with submission costs", *Cahiers de la Faculte des Sciences Economiques et Sociales de Namur*, Namur, Belgium.