

REPORT ON M. ALLAIS'
SCIENTIFIC WORK

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A B S T R A C T

The report reviews briefly the main scientific contributions in economics of Maurice ALLAIS, with specific emphasis on theory.

Keywords : Maurice ALLAIS, equilibrium, welfare, uncertainty

JEL code : 020, 131

RAPPORT SUR L'OEUVRE SCIENTIFIQUE
DE M. ALLAIS

R E S U M E

Le rapport évalue brièvement les principales contributions scientifiques en économie de Maurice ALLAIS, en portant l'accent sur la théorie.

Mots clés : Maurice ALLAIS, équilibre, bien-être, incertitude

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Maurice ALLAIS is without any doubt one of the most outstanding economists of his generation. He has had in particular a decisive direct and indirect influence from 1945 onward, on the development of economic research in France, and its subsequent opening to the international scientific community. Being initially an engineer himself, he has been teaching economics at the Ecole des Mines in Paris until the seventies, and managed to attract around him after World War II a brilliant cadre of young economists or graduates, such as Marcel BOITEUX, Gérard DEBREU, Jacques LESOURNE, Edmond MALINVAUD, André NATAF ... The impact of Maurice ALLAIS on the state of our discipline in France and at the international level, has been indeed tremendous, not only through his own great scientific achievements but also those of his students or disciples.

1. An anecdote circulates among French economists, claiming that the respective careers of two students of ALLAIS, i.e. M. BOITEUX and G. DEBREU, were in fact determined by chance after World War II. Here was a single fellowship for an extended visit to American universities, and the two candidates agreed on flipping a coin in order to decide who would go. DEBREU won and what followed is known ; BOITEUX, while making deep contributions to economic theory on his way (see, e.g., 1956),¹ went on to head eventually one of the largest French public firm, Electricité de France. It is doubtful indeed that a coin alone could ever have had such dramatic consequences : the respective intellectual bends of the two men were probably already evolving in rather different directions when the event occurred. Yet the story is revealing of the scientific stature of M. ALLAIS and of the surprisingly large range of his interests, since he was able to gather around him quite a

few of the best minds of the time, be they attracted by the most abstract theory or by more directly practical matters.

M. ALLAIS believed in fact strongly that, in the social sciences as in the natural sciences, abstract theory should always be confronted to the facts, and moreover that theoretical models should be constructed essentially to give answers to practical questions. This led him to analyse an impressive series of economic problems, and not to hesitate in numerous instances to apply himself sophisticated theory to the study of economic, institutional, or social policy issues of his time. His works on the French public coal mines (A5, 1953), on the effects of competition in the steel and coal industries in the Common Market (A11, 1957), on the economics of energy (A13, 1961), on investment and pricing policies in the transportation sectors in which there are important fixed costs (A15, 1965), to mention only a few, were authoritative and influential applications of the theory of "maximum social efficiency" that he had developed independently during the war (A1, 1943, 1952), and in particular, of efficient marginal cost pricing in situations in which returns to scale were not necessarily decreasing or constant. His recommendations on French public coal mines have been in fact followed partly in the sixties, when cheaper imported coal began to be substituted to inefficient mines. His case study of costs and benefits of mining exploration in Algerian Sahara is another striking example of his skill to blend successfully theory and practical considerations (A10, 1957), by taking into account both the expected gain and the probability of ruin induced by the project. The study had a significant impact and was awarded two important prizes from the operations research international community.²

M. ALLAIS made also recommendations on general policy and institutions, both at the national and international levels, through a series of articles and books aimed at a larger audience. He argued forcefully for the benefits of free trade among nations as a means to promote wealth and welfare (e.g. A3, 1946 ; A23, 1972). He believed, however, that the efficient operation of free markets had to be sustained by appropriate institutions and social organization. He has been throughout a firm advocate of the construction of a federal union and of the adoption of a single currency in

Europe (see, e.g., A12, 1959 ; A17, 1965). At the national level, he argued that the main opposition to the proper functioning of free markets and private property, came from the fact that these were perceived as generating a socially unacceptable distribution of income, in particular too many "unwarranted" gains accruing to the owners of capital goods or of financial claims and debts : pure rents or capital gains were viewed as "unfair", because they did not appear to depend upon the activity of the owner. M. ALLAIS advocated accordingly sweeping fiscal and monetary reforms, by proposing to suppress all taxes on "warranted" gains (e.g. income taxes), and to tax at a uniform annual rate (2 %) all durable (capital) goods as well as expenses on consumption goods. On the monetary side, the same arguments led him to recommend to control the growth rate of the money supply so as to guarantee price stability, but also a complete indexation of all financial securities in order to avoid unwarranted shifts of wealth between lenders and debtors (see e.g. A19, 1966 ; A28, 1977).

2. Beyond his involvement in worldly affairs, what M. ALLAIS is perhaps most famous for – and, may be, what makes him so unique – is the formidable task he assigned to himself during World War II : to reconstruct the economic theory of his time on a rigorous, scientific basis. The result of this ambitious effort is an impressive series of books that appeared during or immediately after the war. He presented in these books a general and rigorous analysis of the workings and the efficiency of competitive markets, of capital, money and interest, the scope and depth of which is truly outstanding in view of the state of the discipline just before the war, and which stand comparison with the classical (and independent) texts of P.A. SAMUELSON's Foundations of Economic Analysis (1947) or of J. HICKS' Value and Capital (1939). The first of these books was the monumental Traité d'Economie Pure (about 900 pages), first published during the war under the quite significant title A la Recherche d'une Discipline Economique (Toward an Economic Discipline) (A1, 1943), and then in 1952. This was followed shortly by a shorter expository piece, Economie Pure et Rendement Social (Economic

Theory and Social Efficiency) (A2, 1945). And then again, another monumental work (two volumes, about 800 p.) Economie et Intérêt (A4, 1947). Writing about these contributions, P.A. SAMUELSON stated indeed (1983) "Professor Allais ... had largely worked out on his own during the Occupation of Paris the fundamentals of advanced economics." ... "Maurice Allais is a fountain of original and independent discoveries. ... Had Allais' earliest writings been in English, a generation of economic theory would have taken a different course".

The Traité d'Economie Pure put forward a rigorous, and surprisingly advanced for his time, analysis of the general equilibrium of a competitive closed nonmonetary economy. M. ALLAIS proved there independently the first and second welfare theorems, which were to become after the war the two pillars of modern welfare theory. The techniques he used were differentiable calculus, and more precisely, the characterization of a constrained maximum through Lagrange multipliers, an important methodological innovation brought systematically into economics at about the same time by P.A. SAMUELSON in his Foundations (1947). M. ALLAIS showed that what he called a state of "maximum social efficiency" – i.e. a situation in which trying to improve the welfare of a group of consumers while meeting all feasibility, in particular technological constraints, would necessarily harm others – would be characterized by the equality of marginal rates of substitution or of transformation across consumers and producers. He deduced from this fact that a competitive equilibrium is efficient and that conversely, one could associate to a state of maximum social efficiency, a set of prices and a distribution of income such that the state would be achieved as a general competitive equilibrium ³. M. ALLAIS showed furthermore that these results could be extended beyond the (by now standard) case in which production and utility functions are concave, to simple frameworks in which there are what he called "undifferentiated sectors", with few firms producing a single good or service, and increasing returns to scale. He stressed that efficiency still required marginal cost pricing in such cases, and that the deficits appearing in the sectors with increasing returns should not be viewed as evils but should be financed through taxation. As noted above, he used extensively that sort of analysis later on when making applied policy

recommendations on the proper way to administer specific public sectors such as the transportation sectors, in which there are significant fixed costs.

M. ALLAIS was careful enough to emphasize that such results did not justify a laissez faire attitude. He argued that actual economies involved significant inefficiencies due to far from perfect competition, and that the equilibrium distribution of income could be unacceptable in actual private ownership economies, on political or moral grounds. His theoretical analysis of the Traité clarified a point that was a little obscure in previous writings, namely that there exists a continuum of efficient states with different distributions of income whenever there are several consumers. He advocated strongly that distributional issues be clearly separated from efficiency considerations : if the distribution of income had to be modified, this should be done through taxation without distorting the competitive mechanism. M. ALLAIS emphasized also that a socialist economy would have to face the same sort of issues as a capitalistic economy : in order to achieve efficiency, a planning institution would have to reintroduce prices in some form or another. All these points are now part of mainstream economic theory. The amazing fact was M. ALLAIS' outstanding ability to construct by himself, during wartime, the rigorous foundations of modern general equilibrium and welfare economics, with so limited access to the literature.

The Traité d'Economie Pure did not present any formal analysis of the existence of a competitive equilibrium. This task was to be undertaken soon by G. DEBREU, who read and was deeply influenced by the book in 1948, as testified by a text of his that was reproduced in part in the preface of the 1952 edition of the Traité. The book contains nonetheless a pathbreaking analysis of the stability of a competitive equilibrium in the original version of Walras' tâtonnement, in which price adjustments are supposed to take place successively in one market after another. M. ALLAIS proved stability of equilibrium under assumptions which, taken together, are essentially the same as gross substitutability, and through an argument that uses in fact Lyapounov stability method (1907). This discovery occurred well before K. ARROW and L. HURWICZ' innovative study of the issue (1958). M. ALLAIS's priority is in particular established by T. NEGISHI in his survey

(1962, p. 656). ⁴

The Traité d'Economie Pure dealt explicitly, although at a somewhat abstract level, with an intertemporal economy that evolved over a finite number of periods, through an indexation of commodities by their date of availability. M. ALLAIS noted that the study of a competitive economy in which all transactions take place at a single date is then formally equivalent to the analysis of an intertemporal economy, provided that prices are in the latter case properly interpreted as discounted prices. He considered also in the Traité a given demographic structure, generations of individuals, and made explicitly the simplifying assumptions of no bequests, and of no possibility of storing commodities. It did not take long for M. ALLAIS to face squarely the complex problems posed by the introduction of time in the theory of general competitive equilibrium.

M. ALLAIS undertook indeed in his other monumental work published right after World War II, Economie et Intérêt (A4, 1947, 2 vol., 800 p.), a systematic theoretical analysis of intertemporal allocation, investment and money in a closed competitive economy under conditions of perfect foresight. This study contained, among many important insights on the mechanisms at work in a dynamic (monetary or nonmonetary) competitive economy, an amazing collection of original discoveries.

According to M. ALLAIS himself, the main contribution of Economie et Intérêt was to provide for the first time a rigorous proof of the fact that, in a nonmonetary economy with a constant population, a stationary state in which welfare (or in some simple cases, output per capita) was maximum – M. ALLAIS called such a state a "capitalistic optimum", or a "maximum of social productivity" – was characterized by a zero real rate of interest. To reach that conclusion, M. ALLAIS relied heavily on the analysis of efficient states he had constructed earlier in the Traité. He asked then whether or not welfare or in some simple cases, output per capita, could be maximized along a stationary state (which he called a "permanent regime"), by adjusting the amount and the composition of initial capital equipment. M. ALLAIS showed indeed, through differentiable calculus, that such a capitalistic optimum

involved a real rate of interest equal to 0. His work provided in 1947 apparently the first rigorous proof of what has become known in modern mainstream economics as the Golden Rule, which is generally attributed to T. SWAN (1956) and E. PHELPS (1961).⁵ M. ALLAIS generalized later the arguments of Economie et Intérêt, in the first Walras-Bowley lecture of the Econometric Society (A14, 1962), to the case of a growing economy, by showing that optimality required then equality of the real rate of interest and of the rate of growth.

Chapter VIII of Economie et Intérêt (pp. 229-422) contains a wealth of insightful observations on the liquidity services of money, the role of banks in the creation of money in closed and open economies. M. ALLAIS noted in particular there that economic agents usually hold positive amounts of money although there are other short term safe assets yielding a positive interest. He explained the phenomenon by the presence of costs of exchanging money for interest bearing bonds. By considering an individual who received a constant stream of money income, and who invested in bonds at discrete and equal intervals of time at a fixed cost, M. ALLAIS was able to prove through a simple analytical argument (A4, pp. 238-241) that the resulting mean demand for money should be proportional to the square root of mean income, and inversely proportional to the square root of the rate of interest on bonds. The interest elasticity of the transaction demand for cash was thus established rigorously in 1947, somewhat before the well known contributions of W. BAUMOL (1952) and J. TOBIN (1956) on this important topic.

M. ALLAIS should also be credited for the formal introduction in Economie et Intérêt, of overlapping generations in the study of dynamic economic processes, eleven years before P.A. SAMUELSON (1958). There are in the main text many informal remarks about the welfare of different generations, in particular of "old" people versus "young" ones. But in Appendix 2 of the book (A4, pp. 640-755), M. ALLAIS proceeded to a detailed mathematical analysis of a specific model, the structure of which is indeed close to the familiar one. The households sector was composed of overlapping generations of identical individuals who live for two periods, supply inelastically one unit of labour when young, and consume a single good in both

periods of their lives – M. ALLAIS chose their utility function to be Cobb-Douglas for analytical tractability (A4, pp. 641-642). On the production side, production of the consumption good could be achieved directly from labour and land, again with a Cobb-Douglas technology. M. ALLAIS introduced an additional complication, by assuming that the productive service of land could be replaced by a production (capital) good, produced from labour one period earlier with constant return to scale. He assumed perfect foresight and, for the most part, no bequest propensity (the no bequest assumption was relaxed in the last part of the appendix, see A4, pp. 748-755). Most of the analysis in the appendix dealt with a nonmonetary economy, but money, banks were considered explicitly in the last part (A4, pp. 728-748). M. ALLAIS focussed on the study of stationary competitive equilibria, in particular on their efficiency properties, the positivity of the real rate of interest, as a function of the Government's net wealth and of various assumptions about private or public ownership of land. Although the analytical treatment was intricate and encumbered with complicated formulas and diagrams, the overlapping generations structure was clear and played without any doubt a major role in shaping the arguments of the main text of Economie et Intérêt. E. MALINVAUD (1986, 1987) established recently ALLAIS' priority, and gave a simplified account of his model and of some of his results. He concluded (1987, p. 105) that "if they had been placed side by side, ALLAIS' appendix and SAMUELSON's independent article would have complemented each other ...".⁶

3. Another outstanding contribution of M. ALLAIS, which is this time well known, concerns the theory of decision making among risky prospects, i.e. in situations in which economic agents are able to assign well defined, subjective or objective, probabilities to the variables (states of nature) describing their unknown environment (A6, 1953 ; A7, 1953). After the work of von NEUMANN and O. MORGENSTERN (1944), the standard way to deal with the issue, had been (and in many cases, still is) to portray agents as maximizing an index that is linear in probabilities (expected utility hypothesis). The attitude of M. ALLAIS toward the issue has been characteristic of his

constant view that theory should be confronted to facts. On the occasion of a meeting that he organized on this topic in 1952, M. ALLAIS conducted a series of experiments in order to test the empirical relevance of the hypothesis, the results of which were partially reported in (A7, 1953).⁷ These experiments showed that actual behaviour violated systematically the expected utility hypothesis. This fact, which is known to economists as the ALLAIS paradox, has generated an increasing amount of research work, both empirical and theoretical, especially in recent years.

The version of the paradox that is most famous, was described by M. ALLAIS himself in (A7, 1953) – see also the discussion in M. MACHINA (1982, pp. 287-293). Individuals were asked to rank a pair of risky prospects, a_1 , which yields 1 M (million francs) with certainty, and a_2 , which yields 5 M with probability 0.10, 1 M with probability 0.89, and nothing with probability 0.01. The same individuals were asked to rank also another pair of risky prospects, a_3 , which yields 5 M with probability 0.10 and nothing with probability 0.90, and a_4 , which yields 1 M with probability 0.11 and nothing with probability 0.89. M. ALLAIS found that a large majority of subjects preferred a_1 in the first pair, and a_3 in the second. These choices, however, violated the expected utility hypothesis. Moving from a_1 to a_4 , or from a_2 to a_3 , consists of lowering the probability of winning 1 M by 0.89, and raising the probability of winning nothing by the same amount. An expected utility maximizer who prefers a_1 in the first pair, should therefore prefer a_4 in the second pair and conversely, since expected utility then changes by the same amount when moving from a_1 to a_4 , or from a_2 to a_3 .

The fact that a large percentage of individuals were found to choose a_1 over a_2 and a_3 over a_4 in the above experiment, made the ALLAIS paradox a key example of the systematic violation in actual settings of the expected utility hypothesis. These experimental violations have been found again and again in a variety of contexts (H. RAIFFA (1968), D. MORRISSON (1967), M. ALLAIS and O. HAGEN (A32, 1979)). These findings have been confirmed and extended by a host of experiments conducted by psychologists (see e.g. W. EDWARDS (1954), D. KAHNEMAN and A. TVERSKY (1979) ; for more information, see the discussion in M. MACHINA (1982, pp. 280-292)).

There has been a parallel and increasing amount of theoretical research work, in order to account for the experimental evidence. M. ALLAIS, and subsequently others, have emphasized the need to bring into the picture not only the mean of the distribution of utiles that is generated by a risky prospect, as it is done in the expected utility hypothesis, but also other features of this distribution such as its dispersion, its lower bound or the probability of ruin. It is perhaps fair to say that these theoretical developments have not yet led to a new and accepted paradigm. But the increasing activity on the issue, especially in recent years, gives hopes that a convincing synthesis may not lie too far ahead. On this important matter, again, M. ALLAIS was a precursor.

4. M. ALLAIS has made also quite a number of impressive, although less known, contributions to the theory of economic dynamics. There were first a series of papers in the fifties showing the possibility of limit cycles in macroeconomic models of the economy, by using the theory of nonlinear difference equations (A8, 1954 ; A9, 1956), only shortly after J. HICKS (1950) and R. GOODWIN (1951) made their pathbreaking studies of this topic. The key ingredient of M. ALLAIS' analysis was in these papers a demand for money that depended nonlinearly (it had in fact a logistic form) on lagged expenditures, along the lines explored independently by P. CAGAN at about the same time (1956). Interestingly enough, another contribution of M. ALLAIS, "Explication des cycles économiques par un modèle nonlinéaire à régulation retardée" (Explanation of economic cycles by a nonlinear monetary model with lagged relations), in Les Modèles Dynamiques en Econométrie, CNRS Editions, Colloques Internationaux vol. 62, Paris, 1956, pp. 259-308, contained the innovative observation that to be realistic, a theory of business cycles should make the stationary state stable for very small perturbations, and involve a cycle that would be attracting when perturbations from the stationary state are a little larger, with the dynamics being globally explosive if the perturbations are more important. M. ALLAIS provided indeed

in the above contribution, conditions ensuring such a configuration, in a nonlinear macroeconomic model. A. LEIJONHUFVUD's corridor, and modern business cycles theory, were already anticipated by M. ALLAIS in 1956 ...

M. ALLAIS pursued later systematically his nonlinear approach to the demand for money in a series of important contributions, and applied it with a remarkable success to the prediction of the demand for real balances for a wide range of countries and of time spans, including severe hyperinflations (A18, 1966 ; A20, 1969 ; A22, 1972 ; A25, 1974). M. ALLAIS' formulation involved a very interesting model of expectation formation, that relied upon an original distinction between "physical" time and "psychological" time (the idea appeared already in his contribution to the 1956 CNRS volume referred to above). He called in fact his theory relativistic, in view of its obvious connections with relativity theory. M. ALLAIS' basic postulate was that economic behaviour (in his model, the velocity of money) was the same in psychological time at all times and places. In particular, expectations were supposed in effect to be formed in psychological time as a weighted average of past observations, with a constant "rate of forgetfulness", an universal constant. M. ALLAIS assumed however that the correspondence between physical and psychological time scales, depended on the rate of change of the observed variable : the higher this rate, the faster (relatively) physical time appeared to go. Thus in physical time, expectations were also weighted averages of the past, but the weights were dependent upon the actual rate of change of observations : the higher this rate, the faster the past was forgotten in physical time. M. ALLAIS gave an analogous and symmetric treatment of discount rates and of intertemporal preferences in psychological and physical time scales ; he argued furthermore that discount rates and rates of forgetfulness were in fact equal in psychological time. As mentioned above, beyond the broad intuitive appeal of the whole approach, M. ALLAIS' assumptions appear to have led to quantitative models that fit the facts remarkably ⁸.

From 1967 on, M. ALLAIS became more and more dissatisfied with the neoclassical paradigm, and embarked on an important theoretical work, which culminated in the publication in 1981 of La Théorie Générale des Surplus (The General Theory of Surpluses), 716 p. (A33), and which may be interpreted as an effort to construct rigorous foundations for the theory of the disequilibrium dynamics of a capitalistic, decentralized economy. The key concept there is the notion of "surplus distribuable" (potential surplus), that M. ALLAIS worked out during the war in the Traité (A1, 1947). Moving from one feasible state to another generates individual welfare gains or losses. M. ALLAIS proposed to measure these gains or losses by the amount of an always desired commodity that should be given to, or taken from, each individual to generate the same gain or loss. The sum of all positive or negative corresponding variations of the commodity, would then measure the global potential surplus corresponding to the move. A Pareto optimum, or a second best optimum when additional constraints are imposed, is then a feasible allocation from which no positive potential surplus can be generated, given all constraints. M. ALLAIS proposed in (A33, 1981) to view the dynamic evolution of a decentralized economy as a succession of moves from one feasible state to another, which generate at each stage a positive global surplus. M. ALLAIS' arguments are complicated and his General Theory of Surpluses has not perhaps been studied and exploited as it should have. But it does seem provide the foundations for a general theory of the dynamics of a capitalistic economy out of equilibrium, in which returns to scale may be increasing and exchanges do take place at (nonnecessarily unique) prices out of equilibrium.

M. ALLAIS is indeed one of the most powerful and brilliant economists of his generation. Despite the fact that his earliest writings were written in French during or right after World War II, he has had a tremendous impact on the profession, directly through his outstanding contributions to economic theory and issues of economic policy, and indirectly through his influence on

his students or disciples. As this brief summary showed, M. ALLAIS' works contain an amazing set of independent discoveries on fundamental questions in economics. As it is often the case for original inventors, M. ALLAIS' style of writing is not always easy to penetrate. It is thus quite likely that other gems wait to be rediscovered, for the benefit of all, in the monumental production of one of the great thinkers of our time.

REFERENCES

Listed below are the references to authors other than M. ALLAIS given in the text.

- ARROW, K.J., and L. HURWICZ, "On the Stability of the Competitive Equilibrium, I", Econometrica 26 (1958), 522-552.
- BAUMOL, W., "The Transactions Demand for Cash : An Inventory Theoretic Approach", Quarterly Journal of Economics 66 (1952), 545-556.
- BOITEUX, M., "Sur la Gestion des Monopoles Publics astreints à l'Equilibre Budgétaire", Econometrica 24, (1956), 22-40.
- BOITEUX, M., de MONTBRIAL, T. and B. MUNIER, (eds.) Marchés, Capital et Incertitude, Economica, Paris, 1986.
- CAGAN, P., "The Monetary Dynamics of Hyperinflation", in M. FRIEDMAN (ed.), Studies in the Quantity Theory of Money, University of Chicago Press, Chicago, 1956, 25-117.
- CAGAN, P., "Allais' Monetary Theory. Interpretation and Comment", Journal of Money, Credit and Banking 1 (1969), 425-432.
- DARBY, M., "Comment on Allais' Restatement of the Quantity Theory", American Economic Review 60 (1970), 444-446.
- DEBREU, G., Theory of Value, John Wiley and Sons, New York, 1959.
- DREZE, J., "Some Postwar Contributions of French Economists to Theory and Public Policy", American Economic Review 46 (1964), 1-64.
- EDWARDS, W., "The Theory of Decision Making", Psychological Bulletin 51 (1954), 380-417.
- GOODWIN, R., "The Nonlinear Accelerator and the Persistence of Business Cycles", Econometrica 19 (1951), 1-17.
- HICKS, J., Value and Capital, Oxford University Press, Oxford, 1939.
- HICKS, J., A Contribution to the Theory of the Trade Cycle, Oxford University Press, Oxford, 1950.

- KAHNEMAN, D. and A. TVERSKY, "Prospect Theory : An Analysis of Decision under Risk", Econometrica 47 (1979), 263-291.
- KEYNES, J.M., The General Theory of Employment, Interest and Money, Harcourt Brace, New York, 1936.
- LANGE, O., "The Foundations of Welfare Economics", Econometrica 10 (1942), 215-228.
- LYAPOUNOV, A., "Problème Général de la Stabilité du Mouvement", Annales de la Faculté des Sciences de Toulouse 9 (1907), 27-474.
- MACHINA, M., "Expected Utility Analysis without the Independence Axiom", Econometrica 50 (1982), 277-323.
- MALINVAUD, E., "Maurice Allais, Précurseur Méconnu des Modèles à Générations Renouvelées", in BOITEUX, M., de MONTBRIAL, T., and B. MUNIER (eds.), 1986, 91-104.
- MALINVAUD, E., "The Overlapping Generations Model in 1947", Journal of Economic Literature 25 (1987), 103-105.
- MEADE, J., An Introduction to Economic Analysis and Policy, Oxford University Press, Oxford, 1937.
- MORISSON, D., "On the Consistency of Preferences in Allais' Paradox", Behavioral Science 12 (1967), 373-383.
- NEGISHI, T., "The Stability of a Competitive Economy : A Survey Article", Econometrica 30 (1962), 635-669.
- NEUMANN, von, J., and O. MORGENSTERN, Theory of Games and Economic Behaviour, Princeton University Press, Princeton, 1944.
- PHELPS, E., "The Golden Rule of Accumulation : A Fable for Growthmen", American Economic Review 51 (1961).
- RAIFFA, H., Decision Analysis : Introductory Lectures on Choice under Uncertainty, Addison-Wesley, Readings, Mass., 1968.
- SAMUELSON, P., Foundations of Economic Analysis, Harvard University Press, Cambridge, 1947.

- SAMUELSON, P., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money", Journal of Political Economy 66 (1958), 467-482.
- SAMUELSON, P., "A Chapter in the History of Ramsey's Optimal Feasible Taxation and Optimal Public Utilities Prices", in Economic Essays in Honour of Jorgen Gelting, 1983, 164-165.
- SCADDING, J.L., "Allais' Restatement of the Quantity Theory of Money : A Note", American Economic Review 61 (1971), 151-154.
- SWAN, T., "Economic Growth and Capital Accumulation", Economic Record 32 (1956).
- TOBIN, J., "The Interest Elasticity of Transactions Demand for Cash", Review of Economics and Statistics 38 (1956), 241-247.

REFERENCES TO M. ALLAIS' WORKS

- A1. A la Recherche d'une Discipline Economique, 1st ed., Ateliers Industria, Paris, 1943. 2nd ed., Traité d'Economie Pure, Imprimerie Nationale, Paris, 1952.
Charles Dupin Prize, Academie des Sciences Morales et Politiques, Paris, 1954.
- A2. Economie Pure et Rendement Social, Sirey, Paris, 1945.
- A3. Abondance ou Misère, Librairie de Medicis, Paris, 1946.
- A4. Economie et Intérêt, Imprimerie Nationale, Paris, 1947.
- A5. La Gestion des Houillères Nationalisées et la Théorie Economique, Imprimerie Nationale, Paris, 1953.
J. Dutens Prize, Académie des Sciences Morales et Politiques, Paris, 1959.
- A6. "L'extension des théories de l'équilibre économique général et du rendement social au cas du risque", Econometrica 21 (1953), 269-290. English version in Theory and Decision (1987).
- A7. "Le comportement de l'homme rationnel devant le risque. Critique des postulats et axiomes de l'école américaine", Econometrica 21 (1953), 503-546.
- A8. "Illustration de la théorie des cycles économiques par des modèles monétaires non linéaires", Econometrica 22 (1954), 116-120.
- A9. "Explication des cycles économiques par un modèle non linéaire à régulation retardée", Metroeconomica 8 (1956), 4-83.
- A10. "Methods of Appraising Economic Prospects of Mining Exploration over Large Territories : Algerian Sahara Case Study", Management Science 3 (1957), 285-347.
Lanchester Prize of John Hopkins University and of the American Society of Operations Research, 1958.

- A11. "Le système des prix et la concurrence dans le marché commun de la Communauté Européenne du charbon et de l'acier", Bulletin SEDEIS (1957).
- A12. L'Europe Unie. Route de la Prospérité, Calmann-Lévy, Paris, 1959.
- A13. Les Aspects Essentiels de la Politique de l'Energie, Imprimerie Nationale, Paris, 1961.
- A14. "The influence of the capital-output ratio on real national income", Econometrica 30 (1962), 700-728 (Walras Bowley lecture of the Econometric Society). Reprinted in Readings in Welfare Economics, American Economic Association, Vol. XII (1969), 682-714.
- A15. "Programme d'investissement et d'exploitation optimum d'une infrastructure de transport", in Essays in honour of Professor J.L. Meij, Bedrijfseconomische Verkenningen, G. Delwel, La Haye, 1965.
- A16. "The role of capital in economic development", in The Econometric Approach to Development Planning, North Holland, Amsterdam, and Rand Mac Nally, Chicago, 1965.
- A17. "Toward an Integrated Atlantic Community", in NATO in Quest of Cohesion, Hoover Institutions Publications, Praeger, 1965.
- A18. "A restatement of the quantity theory of money", American Economic Review 56 (1966), 1123-1156.
- A19. "L'impôt sur le capital", Droit Social 29 (1966), 465-544.
- A20. "Growth and inflation", Journal of Money, Credit and Banking 3 (1969), 355-426. Discussion : 427-462.
- A21. "Les théories de l'équilibre économique général et de l'efficacité maximale, impasses récentes et nouvelles perspectives", Revue d'Economie Politique 81 (1971), 331-409.
- A22. "Forgetfulness and interest", Journal of Money, Credit and Banking (1972), 40-71.
- A23. La Libéralisation des Relations Economiques Internationales, Gauthier-Villars, Paris, 1972.

- A24. "Inequality and civilizations", Social Science Quarterly (1973).
- A25. "The psychological rate of interest", Journal of Money, Credit and Banking (1974), 285-331.
- A26. "Le concept de monnaie, la création de monnaie et de pouvoir d'achat par le mécanisme du crédit et ses implications", in Essais en l'Honneur de Jean Marchal, Cujas, Paris, 1975.
- A27. "Theories of general economic equilibrium and maximum efficiency", in G. Schwodiauer (ed.), Equilibrium and Disequilibrium in Economic Theory, Reidel, Dordrecht, 1977.
- A28. L'impôt sur le Capital et la Réforme Monétaire, Hermann, Paris, 1977.
- A29. "On the concept of probability", Rivista Internazionale di Scienze Economiche e Commerciali 11 (1978), 937-956.
- A30. "Foundations of a positive theory of choice involving risk and criticism of the postulates and axioms of the American School", pp 25-145 in [A32].
- A31. "The so-called Allais' paradox and rational decisions under uncertainty", pp 434-681 in [A32].
- A32. Expected Utility Hypothesis and the Allais' Paradox, M. Allais and O. Hagen (eds.), Reidel, Dordrecht, 1979.
- A33. "La théorie générale des surplus", Economie et Sociétés 15, 1981, 716 p.
- A34. "Frequency, probability and chance", in Foundations of Utility and Risk Theory with Applications, B.P. Stigum and F. Wenstop (eds.), Reidel, Dordrecht, 1983. Published also in French in Journal de la Société de Statistique de Paris, 1983.
- R. Blanché Prize, Académie des Sciences Morales et Politiques, Paris, 1983.
- A35. "The foundations of the theory of utility and risk", in Progress in Decision Theory, O. Hagen et F. Wenstop (eds.), Reidel, Dordrecht, 1984.

- A36. "The concepts of surplus and loss, and the reformulation of the general theory of economic equilibrium and maximum efficiency", in Foundations of Economics, M. Baranzini and R. Scazzieri (eds.), Blackwell, Oxford, 1985.
- A37. "The credit mechanism and its implications", in Essays in Honour of K.J. Arrow, G. Feiwel (ed.), Mc Millan, New York, 1986.
- A38. "Determination of Cardinal utility according to an intrinsic invariant model", in Recent Developments in the Foundations of Utility and Risk Theory, L. Daboni et al. (eds.), Reidel, Dordrecht, 1986.
- A39. "The Allais Paradox" and "Economic surplus and the equimarginal principle", in The New Palgrave. A Dictionary of Economic Theory and Doctrine, Mc Millan, London and New York, 1987.
- A40. "The general theory of random choices in relation to the invariant cardinal utility function and the specific probability function. The (u, θ) model. A general overview", in Risk, Decision and Rationality, B. Munier (ed.), Reidel, Dordrecht, 1987.
- A41. Cardinal Utility and General Random Choice Theory, M. Allais (ed.), Reidel, Dordrecht, forthcoming.

F O O T N O T E S

* CNRS and CEPREMAP, 142, rue du Chevaleret, 75013 PARIS. I greatly benefited while preparing this appraisal from the excellent surveys of ALLAIS' works written in French by B. MUNIER, and T. de MONTBRIAL, in M. BOITEUX, T. de MONTBRIAL and B. MUNIER (eds.), Marchés, Capital et Incertitude, Economica, Paris, 1986.

1. References to works by authors different from ALLAIS are listed at the end of this appraisal. References to articles or books of ALLAIS are denoted (A1), (A2), etc ... : they are gathered separately afterward. For a complete bibliography of ALLAIS, in French, see the above mentioned book edited by M. BOITEUX, T. de MONTBRIAL and B. MUNIER.

2. The review of the contributions of French economists by J. DREZE (1964) makes clear the important influence of the works of M. ALLAIS on a whole generation of applied economic studies in France.

3. G. DEBREU acknowledged the importance of ALLAIS' contribution by writing in his Theory of Value (1959, pp. 96-97) : "The most interesting treatments of [the optimum] problem by the calculus have been given by O. LANGE (1942) and by M. ALLAIS (A1, 1943, Chapter 4, Section E)". He also indicated there that the concept of utility frontier, i.e. the representation of the set of attainable states of an economy and of the Pareto frontier in the space of the households' utilities, which is generally attributed to P.A. SAMUELSON (1947, Ch. 8), can be in fact found in M. ALLAIS' Traité (A1, 1943, Ch. 4, Section E).

4. ALLAIS' proof was not entirely correct, as the Lyapounov function he used was not differentiable everywhere. He may be forgiven, perhaps, for having overlooked this technical point ...

5. M. ALLAIS wrote in the introduction of Economie et Intérêt (A4, p. 19) that he had thought in December 1940 to have been the first to have discovered that sort of result, but that he found after the war a brief allusion to the issue in J.M. KEYNES (1936, Chap. XVI) and a thorough verbal discussion of the matter in J. MEADE (1937, Chap. III). See also M. ALLAIS (A14, 1962, p. 725).

6. The complexity of Appendix 2 of Economie et Intérêt might explain why ALLAIS' priority failed to have been noticed earlier by his students. E. MALINVAUD (1987) remarks that he is "not the only one whose attention has failed". Indeed M. ALLAIS thanks, in (A4, p. 633), G. DEBREU for having checked the mathematics of the appendices ... Yet the overlapping generations structure is quite clear on the first two pages of Appendix 2 (A4, pp. 641-642).

7. This article in French was a summary of a longer contribution presented at the 1952 meeting and published in 1953 in the proceedings. An English translation of the longer contribution was published in (A30, 1979).

8. There has been some controversies about the reasons underlying this empirical success. Some authors worried about possible automatic correlations between the aggregates constructed by M. ALLAIS, others about a possible circularity of the argument. The structure of M. ALLAIS' reasoning is often complex, and the matter may not be easily settled. See P. CAGAN (1969), M. DARBY (1970), J. SCADDING (1971) and ALLAIS' reply in American Economic Review (June 1975, pp. 454-464).