THE CLASSICAL-MARXIAN EVOLUTIONARY MODEL OF TECHNICAL CHANGE

APPLICATION TO HISTORICAL TENDENCIES

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Version: March 16, 2009. Paper presented in the URPE Session, The Classical Approach to Technical Change and Capital Accumulation, of the ASSA meetings, New York, January 3-5, 1999.

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SUMMARY

This study is devoted to the classical-Marxian evolutionary model of technical change that we substitute for the neoclassical production function. Innovation is described as a random local process. The techniques of production actually used are selected according to their profitability. Under the assumptions of a rising labor cost and a temporary variation of the profile of innovation, it is possible to reproduce the historical trends of the variables over the three subperiods, 1869-1920, 1920-1960, and 1960-1992, in particular the successive decline, rise, and decline of the profit rate. This framework allows for a discussion of Marx's analysis of historical tendencies in capitalism. Trajectories *à la Marx* can be obtained, with a declining profit rate, a rising organic composition of capital, a constant rate of surplus value, and a rising mass of capital accumulated. These tendencies can be interpreted as the effect of the "difficulty" to find innovations economizing on both inputs, labor and capital, or even as the consequence of a gradual increase of this difficulty. The model can be used within meso or micro frameworks, where disequilibrium and heterogeneity are observed, or in the analysis of the world economy (the analysis of catching-up). The conditions of innovation can be treated endogenously.

Introduction

Central to the classical and Marxian analyses of technical change is the idea that capitalists choose among competing techniques of production, depending on their comparative profitability. A new technique is implemented if it increases the profit rate of the firm. This idea is common to Ricardo and Marx. It is also part of Sraffa's framework.[1] Although capitalists do not "maximize" their profit rate on the basis of a given production function, as within neoclassical models, they seek to obtain the best possible profit rate by choosing the most appropriate technology. The wage rate is an important parameter in this selection (see the reference to Marx on the next page).

This very simple principle should not be mistaken for a theory of technical change or innovation in general. Why does a firm or an economy generate new and better performing techniques whereas others do not? What determines the pattern of innovation? Why does technical change display favorable features in some periods, and not in others, etc.? All these issues relate to major aspects of the analysis of technical change. The choice of the most profitable techniques of production *per se* is in no way sufficient to answer these questions.

Nonetheless, many properties of technical change can be derived from the mere principle of the selection of the most profitable techniques, provided that it is embedded within an appropriate framework of analysis. It is the purpose of this paper to define such a model and to investigate its properties. There is no denying the fact, that this framework is, in a sense, reminiscent of the neoclassical production function, but with the significant difference that no such function is considered!

This model can be called the *classical-Marxian evolutionary model of technical change*[2], since it interprets the classical-Marxian analysis of technical change in a framework analogous to many evolutionary

1. P. Sraffa, *Production of Commodities by Means of Commodities*, Cambridge: Cambridge University Press (1960), ch. XII.
2. This is how Duncan Foley named the model we presented a few years ago in various papers (D. Foley, Simulating Long-Run Technical Change, Department of Economics, Barnard College, Columbia University, New York (1998); G. Duménil, D. Lévy, "A Stochastic Model of Technical Change, Application to the US Economy (1869-1989)", *Metroeconomica*, 46 (1995), p. 213-245).

models. It is difficult to devise a more straightforward approach to innovation. Innovations appear randomly in a vicinity of actual techniques. They are selected if the profit rates that they would yield at existing prices if they were implemented, are larger than prevailing rates. This process is repeated period after period in a stochastic dynamical model. This model is presented in section 1.

In spite of its simplicity, this framework of analysis yields several interesting theoretical and empirical applications. In section 2, we use what could be called the "*aggregate* classical-Marxian evolutionary model of technical change" to interpret the secular profile of the main variables accounting for technology and distribution in the US since the Civil War. Three periods can be distinguished, corresponding roughly to the late 19th century, the first half of the 20th century, and the second half of the 20th century. The model suggests an interpretation of these three periods as an effect of a steady variation of *the conditions of innovation*. The first and third periods can be characterized by unfavorable conditions of innovation and the downward trend of the profit rate, in sharp contrast with the intermediate period. The model can also be applied to the investigation of the catching-up of European economies and Japan with the US.

Section 3 is devoted to understanding Marx's analysis in Volume III of *Capital* concerning the specific properties of technical and distributional change in capitalism. Marx's tendency for the profit rate to fall is part of a broader system of laws including labor productivity, the composition of capital, the rate of surplus value, and accumulation. With specific assumptions concerning wages, the model allows for the derivation of these tendencies. Finally, we attribute the tendency for the profit rate to fall to the specific features of innovation — in general and within capitalism in particular. These features echo Marx's idea of the increasing composition of capital inherent to mechanization. They can be expressed in various forms, such as the "difficulty of innovating" or an intrinsic labor-saving capital-consuming "bias" of innovation. The assumption that this difficulty increases tendencially over time increases its consistency with Marx's overall picture of historical tendencies within capitalism.[3]

Section 4 abandons the global approach of the previous sections to concentrate on meso or micro mechanisms and disequilibrium. It

3. In this study, we leave aside discussion of the use of variables measured in terms of value (as in Marx's analysis) or prices (as in data bases). What is, for example, the relationship between the rate of surplus value and the ratio of profits to wages? What is the importance of the distinction between productive and unproductive labor?

1 - The impact of labor cost on technnical change

It is explicit in Marx's analysis that innovations are implemented depending on a comparison between the cost of the equipment and the cost of labor saved. In the following extract from a chapter of *Capital*, entitled *Machinery and Large-Scale Industry*, Marx compares the labor time embodied in the machine (which will be transferred to the product) to the labor time saved as a result of the use of the machine. However, he then explains that the capitalist only pays the value of the labor power. This is what matters in this comparison. Marx finally considers the actual wage which may diverge from the value of labor power. The reference to *competition* indicates a transition to an approach based on prices.

The use of machinery for the exclusive purpose of cheapening the product is limited by the requirement that less labor must be expended in producing the machinery than is displaced by the employment of that machinery. For the capitalist, however, there is a further limit on its use. Instead of paying for the labor, he pays only for the value of the labor-power employed; the limit to his using a machine is therefore fixed by the difference between the value of the machine and the value of the labor-power replaced by it. Since the division of the day's work into necessary labour and surplus labour differs in different countries, and even in the same country at different periods, or in different branches of industry; and further, since the actual wage of the worker sometimes sinks below the value of his labor power, and sometimes rises above it, it is possible for the difference between the price of the machinery and the price of the labour-power replaced by that machinery to undergo great variations, while the difference between the quantity of labour needed to produce the machine and the total quantity of labour replaced by it remains constant. But it is only the former difference that determines the cost to the capitalist producing a commodity, and influences his actions through the pressure of competition. (a)

The circulation of capital (the existence of capital stock and the progressive transfer of its value to the product) is not discussed in this extract. Using the framework of Volume II, it is clearly the profit rate which is at issue. This is explicit in volume III:

No capitalist voluntarily applies a new method of production, no matter how much more productive it may be or how much it might rise the rate of surplus value, if it reduces the rate of profit. (b)

(a) K. Marx, *Capital, Volume I*, New York: First Vintage Book Edition (1867), ch. 15, p. 515-516.

(b) K. Marx, *Capital, Volume III*, New York: First Vintage Book Edition (1894), ch. 15, p. 373.

shows that the model can be used in frameworks analyzing firms or industries, in which technology is heterogeneous. A sub-section introduces endogenous properties of innovation and technical change.

Section 5 is devoted to the discussion of the nature of this model. On what grounds can it be called Marxian and classical? How does it differ from the neoclassical production function? In what sense and to what extent can it find roots in evolutionary approaches?

1 - Modeling technical change

The model is presented in section 1.1. Section 1.2 uses this framework to discuss the features of innovation and technical change.

1.1 The basic model

We present in this section the simplest possible form of the model. Only one good exists and it is produced by a representative firm. At a given point in time, the production of one unit of this commodity requires a certain amount of itself, A, used as fixed capital, and a quantity of labor (also assumed homogeneous), L. Thus, a technique is denoted (A, L). The ratio of output to either one of the inputs is the productivity of this input. The productivity of capital is $P_K = 1/A$, and labor productivity is $P_L = 1/L$.

A new technique, (A_+, L_+), appears at each period. It can be compared to the existing technique by the rates, a and l, of saving on each input:

$$A_+ = A/(1+a) \text{ and } L_+ = L/(1+l) \tag{1}$$

If the new technique is adopted, a and l are also the growth rates of the two productivities:

$$\rho(P_K) = a \text{ and } \rho(P_L) = l \tag{2}$$

In panel (a) of diagram 1, the horizontal and vertical axes measure the quantity of the good and the quantity of labor used as inputs respectively. The existing technique, (A, L), is represented by the black dot (•). A new technique, (A_+, L_+), can be located on this diagram, and falls within any one of the four regions [1] to [4]. Within region [1] the amount of each input is reduced. Conversely, both inputs are increased in region [4]. Within regions [2] and [3], the amount of one input is reduced whereas the other is increased.

A similar image is displayed in panel (b), where the performances of the new technique are described in terms of variations, using the variables, a and l defined in equations 1. Thus, the two axes account respectively for the growth rates of capital and labor productivities (positive or negative).

Technical change can be decomposed into two distinct steps: innovation and selection. We will consider these steps successively:

1. New techniques result from R&D activities. We make the following assumptions: (1) the outcome of R&D is to a large extent unpredictable; (2) new techniques are devised on the basis of the existing technology, which is only modified gradually (innovation is local). Thus, innovation is modeled as a random process, which follows a *probability distribution*, $\pi(a, l)$, whose support is bounded and denoted as the *innovation set* (see panel (c)). Maintaining the actual technique is always a possibility, and the origin belongs to the innovation set.

2. The criterion used in the decision to adopt a new technique is whether it yields a larger profit rate at prevailing prices (including the wage rate). If the innovation falls within region [1] the result is obvious and independent of prices: Since the new technique saves on both inputs, it is adopted. If it falls in region [4], increased amounts of the two inputs would be required, and the new technique is rejected. A computation must be made in order to compare the profit rates of the old and new techniques whenever the innovation falls in regions [2] or [3]. We call the *selection frontier* the line which separates the adopted ($r_+ > r$) from the rejected techniques ($r_+ < r$). This line represents the points satisfying the condition $r_+ = r$. As shown in panel (c) of diagram 1, it is a downward sloping line crossing the origin. We denote as the *profitable innovation set*, Π, the subset of the innovation set which lies above this line. Only innovations falling in this region are selected.

The equation of the selection frontier can be determined as follows. Only one relative price is required in this model in which a single good is considered. It is the unit wage deflated by the price of the good ("labor cost" for short), denoted w. The corresponding

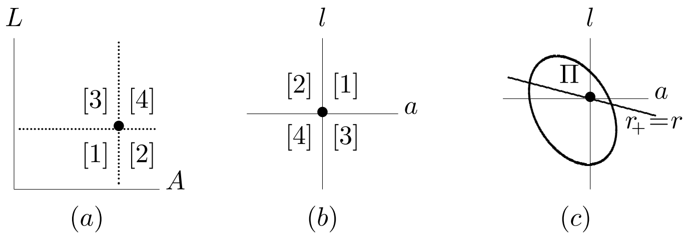

Diagram 1

profit rates are:

$$r = \frac{1 - Lw}{A} \; and \; r_+ = \frac{1 - L_+ w}{A_+} \tag{3}$$

If the innovation set is small, the profit rate, r_+ of the new technique can be developed linearly in the vicinity of the prevailing profit rate r:

$$r_+ = r\left(1 + \frac{\mu a + l}{\mu}\right) \tag{4}$$

where μ is the ratio of profits to wages, or the "rate of surplus value", with $\mu = (1 - Lw)/Lw$, and Lw is the *wage share*, later denoted as ω. The equation for the selection frontier is:

$$\mu a + l = 0 \tag{5}$$

The slope of this frontier is $-\mu$.

This framework defines a dynamical model that determines the technique in any period from the technique prevailing in the previous period. The labor cost, w, is the only exogenous variable. More generally, beginning with a technique (A_0, L_0), one can derive a sequence of techniques, A_t, L_t (with $t = 1, 2, \ldots$), from a given sequence of labor costs w_t (with $t = 0, 1, 2, \ldots$). We denote such a sequence as a *technical trajectory*. Formally, a *stochastic dynamical model* has been defined.

In the investigation of the properties of this model, it is useful to consider the *average values* of variables a and l. Considering only innovations which are selected, their average value corresponds to G, the center of gravity of the innovation set, as shown in diagram 2. When innovations are not retained because they are less profitable than the prevailing technique, the origin, O, continues to represent the technique used during the new period. Thus, the average value

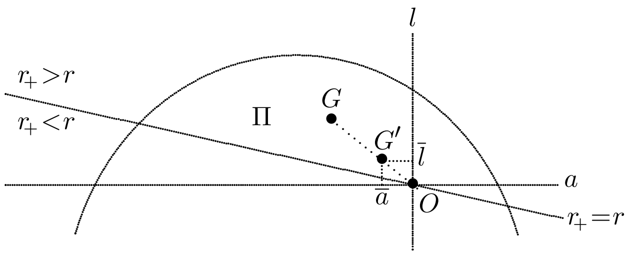

Diagram 2

of the random variable is a weighted average, G', of these two cases (located on GO). The coordinates of G' are denoted $\overline{a}$ and $\bar{l}$.[4]

1.2 The features of innovation and of technical change

Diagram 3 illustrates four types of properties of innovation:

1. Panels (a) and (b) show how the *difficulty* of innovating can be expressed in this model. In panel (a), finding profitable innovations is easy in comparison to the situation in panel (b), as a result of the reduction of the innovation set (a homothetical transformation centered in the origin).

2. Panels (c) and (d) suggest another interpretation of the difficulty of innovating. In these two diagrams the radius of the circle is the same, and the two centers are located on the first bisector. It is the location of the center, its distance from the origin, which accounts for the difficulty of innovating.

3. Panels (e) and (f) are devoted to the notion of *bias*. In panel (e), the circle is centered on the first bisector, and innovations economizing on each input are equally probable. There is, therefore, no bias. The converse is true of panel (f), where the circle has been shifted toward the upper left-hand side. Consequently, the probability of finding labor-saving capital-consuming innovations ($l > 0$ and $a < 0$) is larger ($\overline{a} \searrow$ and $\bar{l} \nearrow$).

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4. One has:

$$\overline{a} = \iint_{\Pi} a \, d\pi(a,l) \ and \ \bar{l} = \iint_{\Pi} l \, d\pi(a,l)$$

in which the integrals are limited to selected innovations, *i.e.*, the profitable innovation set Π.

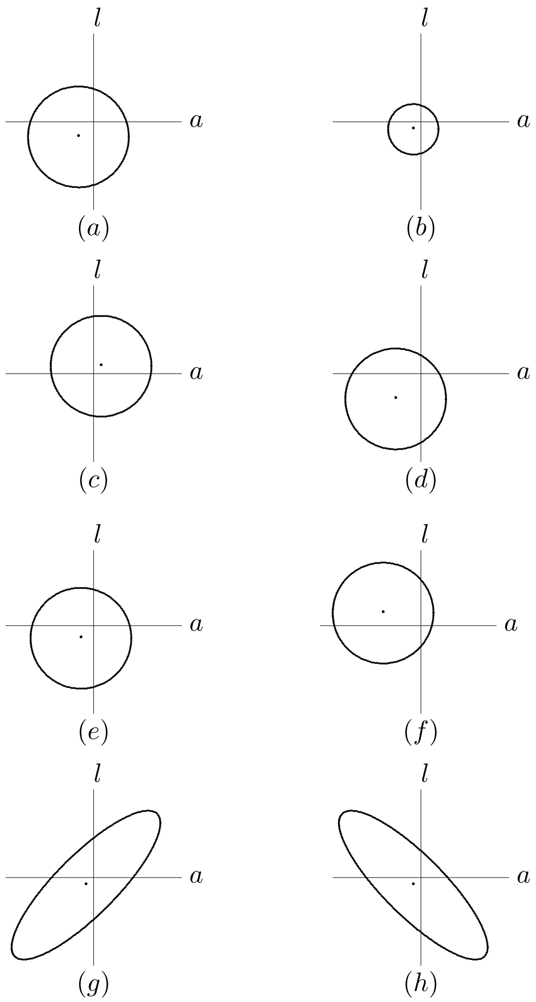

Diagram 3

4. Panels (g) and (h) describe two distinct patterns concerning the direction of variation of the two inputs when innovations occur. The circle has been replaced by an ellipse. In panel (g), the use of the two inputs tends to vary in the same direction. In panel (h), the use of one input tends to increase while the use of the other tends to diminish.[5]

5. All techniques in this model are represented by fixed coefficients. The

The pattern in panel (g) matches, for example, the complementary features of structures and labor (like an office environment), while panel (h) may correspond to the case of equipment and labor.

Obviously, these various features of innovation can be combined.

The characteristics of technical change, in an enterprise, industry or country, may also be influenced by the existence of competitors. Firms producing the same good tend to copy one another. New organizational and management patterns spread from one enterprise to another, from one industry to another. Countries that confront one another on the world market must adapt to their competitors' performances.

Catching-up represents an interesting special case of the above. The overall idea is that technical change in one country, the follower, is *influenced* by the technology of a more advanced country, the leader. For obvious reasons, switching immediatedly to the technology of the leader is impossible (assuming that it would be justified on account of the difference in wages). This was, in particular, true of competition between the European countries and Japan on the one hand, and the US on the other, after World War II.

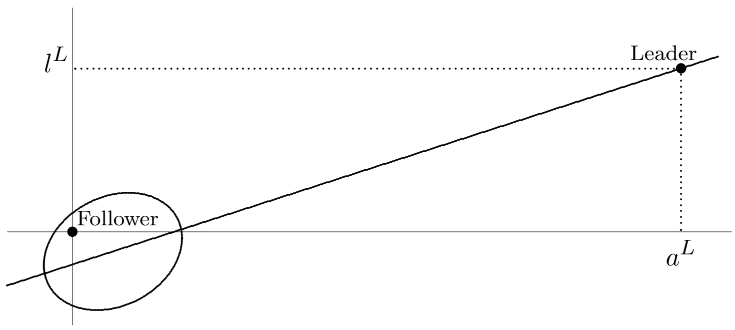

Diagram 4

The existence of a leader has an impact on the conditions of innovation. Innovations which tend to reproduce the technology of the leader are favored. This can be captured in the model by giving the innovation set a particular shape, for example an ellipse, whose main axis points toward the technique of the leader (diagram 4).

patterns of variation described in panels (g) and (h) are, however, evocative of the notions of complementary and substitutable factors.

The intersect of the axes represents the technology of the follower, (A, L). The technology, (A^L, L^L), in the leading country can be located in this plane by its two coordinates, (a^L, l^L), which measure the distance between the two technologies:

$$A^L = \frac{A}{1+a^L} \textit{ and } L^L = \frac{L}{1+l^L}$$

As is evident from the diagram, the leader dominates the follower on account of the higher productivities of both labor and capital[6], and the ellipse points toward the upper-right side.[7]

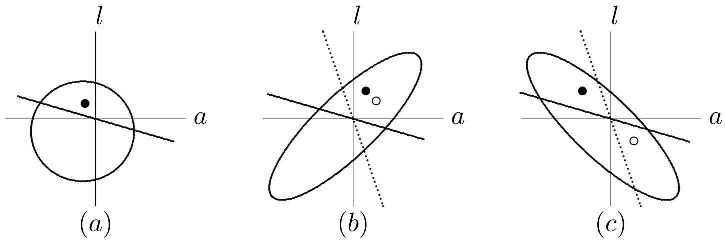

Diagram 5

We now turn to the analysis of the second step in the process of technical change: the selection of profitable innovations. Even on the basis of an unbiased pattern of innovation as in panel (e) of diagram 3, technical change will usually be biased as a result of the effect of distribution on the slope of the selection frontier. The profitable innovation set in panel (a) of diagram 5 is not symmetrical with respect to the first bisector, although the innovation set is symmetrical. Obviously, this bias may coexist with the bias in innovation as in panel

6. For example, for the productivity of capital: $a^L > 0 \Leftrightarrow \frac{1}{A^L} > \frac{1}{A}$.

7. The equation of the ellipse is:

$$bx^2 + 2cxy + dy^2 = 1 \textit{ with } x = a - \delta_a \textit{ and } y = a - \delta_l$$

The parameters are:

$$c = \frac{m}{1+m^2}\left(\frac{1}{R^2} - \frac{1}{R'^2}\right), \quad b = \frac{1}{1+m^2}\left(\frac{m^2}{R^2} - \frac{1}{R'^2}\right),$$

$$\textit{and } d = \frac{1}{1+m^2}\left(\frac{1}{R^2} - \frac{m^2}{R'^2}\right)$$

δ_a and δ_l are the coordinates of the center of the ellipse, $m = (l^L - \delta_l)/(a^L - \delta_a)$ is the slope of the main axis, and R and R' are half the lengths of the axes.

(f) of diagram 3. In an empirical study, we estimated the average annual growth rate of the capital-labor ratio in the United States over the period 1869-1992 at 1.39%, of which 0.89% could be attributed to the bias of the innovation set, and the remainder to the effect of distribution[8].

The size of the impact of distribution on technical change depends on the properties of innovation. Consider, for example, the two cases described in panels (g) and (h) of diagram 3. Two alternative selection frontiers are drawn in panels (b) and (c) of diagram 5. In panel (b), the average features [(•) or (◦)] of technical change depend only slightly on the slope of the selection frontier, *i.e.*, on distributional outcomes, but the converse is true in panel (c) of diagram 5.

2 - The historical trends of technology and distribution

The above model is capable of many applications. This section is devoted to the historical profile of technology and distribution. Section 2.1 provides an interpretation of the evolution of technology and distribution in the US since the Civil War. Section 2.2 shows how the catching-up of less "advanced" countries toward a leader modifies such patterns of evolution.

It is important to stress from the outset, that the model is only one tool among many. It cannot alone provide a comprehensive interpretation of any particular phenomenon. Take, for example, the actual features of technical and distributional change in the US: the model points to a set of basic observations, which must in turn be interpreted within a larger social and political framework. Similarly, in the disucussion of catching-up, the explanatory power of the model is real, but limited. In particular, it does not account for the reasons why one country did catch up, whereas another did not.

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8. G. Duménil, D. Lévy, "The Acceleration and Slowdown of Technical Progress in the US since the Civil War: The Transition Between two Paradigms", *Revue Internationale de Systémique*, 10 (1996), p. 303-321.

2.1 Secular trends in the US

Four variables are used in the analysis of the secular trends of technology and distribution in the US: labor cost, labor productivity, the productivity of capital, and the rate of profit (for the total private economy). *Labor cost*, w, is the total compensation per hour worked deflated by the Net National Product (NNP) deflator. *Labor productivity*, P_L, is NNP (in constant dollars) divided by the number of hours worked. The *productivity of capital*, P_K, is NNP divided by the net capital stock (equipment and structures). The profit rate is the ratio of *NNP minus labor remuneration* and the *net stock of fixed capital*.[9] A wage-equivalent for the self-employed is included within labor income.

Average Annual Growth Rates (% per Year)				
	1869-1920	1920-1960	1960-1997	*1869-1997*
$\rho(w)$	1.45	2.34	1.56	*2.01*
$\rho(P_L)$	1.29	2.51	1.53	*2.03*
$\rho(P_K)$	−0.97	0.85	−0.49	*0.03*
$\rho(r)$	−1.25	1.07	−0.58	*0.01*

The last column in the above table displays the average annual rate of growth of these variables over the entire period (1869-1997). It is clear from these figures that the four variables can be separated into two groups. Labor cost and labor productivity display a clear upward historical trend, whereas the trend of the profit rate is approximately horizontal, as is the case for the productivity of capital.

The evolution of each of the four variables around its trend conforms to a common pattern of fluctuation. Hence three subperiods can be distinguished in the table, with the breaks in 1920 and 1960:

1. Beginning with the Civil War and stretching up to the early 20th century, the growth rates of labor cost and labor productivity remain comparatively low (lower than the average for the entire period), while the productivity of capital and the profit rate display a downward trend.

9. Such a measure of the profit rate is appropriate in the analysis of technical and distributional change. To obtain the profit rate garnered by firms, it would be necessary to subtract taxes and interests. The measure of capital could also be made more precise, to include, in particular, inventories.

2. From the early 20th century to the 1950s, the growth rates of labor cost and labor productivity are higher (larger than the average for the entire period), and the trends of the productivity of capital and the profit rate trend upward. Thus, this intermediate period appears very favorable: technical progress is rapid and a comparatively large growth rate of labor cost coincides with a rising profit rate.

3. From the 1960s onward, the trends of the first period are reasserted. The similarity between the first and third periods is striking.

The notion of technical progress is ambiguous during the first and third periods since labor productivity rises and the productivity of capital declines. This observation recalls the importance of the simultaneous consideration of labor and capital in relation to output, not simply labor productivity.

The model of section 1 can easily account for such patterns of evolution. Considering the labor cost as exogenous, we interpret the succession of these three periods as the expression of a continuous transformation in the conditions of technical change. Using the terminology defined in section 1.2, we contend that the *difficulty* of innovating varied over time.

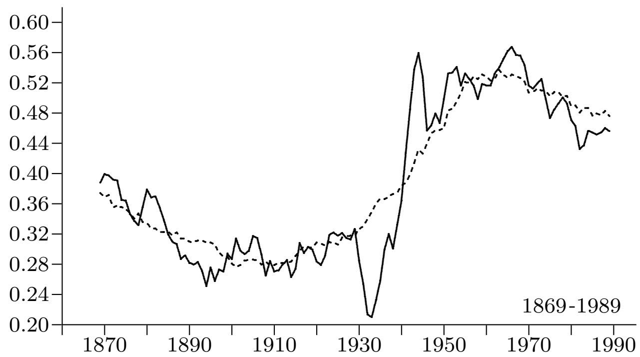
Our hypothesis is that innovation was relatively: difficult, then easy, and then difficult. Within the framework of panels (c) and (d) of diagram 3 (where innovation is unbiased, that is, the coordinates, δ_a and δ_l, of the center of the circle are equal), this is equivalent to saying that the innovation set was comparatively low (a large negative common value, δ, as in panel (d)) at the beginning of the period, moved progressively upward ($\delta \nearrow$), thus creating the favorable conditions prevailing in the intermediate period (as in panel (c)), and returned progressively to its original position ($\delta \searrow$ back to (d)).[10] A similar result can be obtained considering a transformation such as that between panels (a) and (b).

10. More specifically, we used the following analytical form (the derivative of a logistic function):

$$\delta(t) = \delta_0 + 4\delta_1\, exp\left(-\frac{t-\bar{t}}{\Delta}\right) \Bigg/ \left(1 + exp\left(-\frac{t-\bar{t}}{\Delta}\right)\right)^2$$

In this expression, $\bar{t}$ denotes the year in which the maximum value of $\delta(t)$ was reached, and Δ provides a measure of the duration of this movement. It is easy to verify that the curve is symmetrical with respect to $\bar{t}$.

Figure 1 The productivity of capital in the US (1869-1989)



Actual series: (——)
Model: (-----)

Figure 1 illustrates the ability of such a model to account for the evolution of the productivity of capital (for the period 1869-1989).[11] The actual series displays more fluctuations than the model because of short-term perturbations (notice, for example, the effect of the Great Depression). The other variables in the table above can be reproduced in a similar manner. *These results show that changes in labor cost together with gradual variation in the difficulty innovating, account convincingly for trends in the main variables associated with technical and distributional change in the US since the Civil War.* The reconstruction of the series in figure 1 was made without assuming any *a priori* bias in innovation. Other assumptions were made in other studies.[12]

The main results of the investigation thus far can be summarized as follows:

1. Technical change results from a random neutral innovation process, followed by the selection of techniques which appear to be the most

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11. This analysis is borrowed from G. Duménil, D. Lévy, "Complexity and Stylization: An Evolutionary Model of Technical Change in the US Economy", *in* R. Delorme, K. Dopfer (eds.), *The Political Economy of Diversity: Evolutionary Perspectives on Economic Order and Disorder*, Aldershot: Edward Elgar, 1994, p. 229-251.
12. G. Duménil, D. Lévy, "The Acceleration and Slowdown", *op. cit.* note 8.

profitable (the most able to allow for survival within competitive markets).

2. Labor productivity and wages evolve in concert, because of the effect of the wage share in the selection of new techniques.

3. Depending on the difficulty of innovating, rising labor costs may be associated with distinct patterns of variation of the productivity of capital and the profit rate: (1) If innovation is difficult, the two variables decline, (2) If it is comparatively easier, they rise.

4. Since the Civil War, the first configuration has prevailed twice, during the earlier and latter decades of this period. The second was observed from the early 20th century to the 1950s.

5. Overall, the secular trends of the variables correspond to a situation close to the boundary between the two cases above, with nearly horizontal trends in the profit rate and of productivity of capital.

The specific profile of the intermediate period relates, in our opinion, to the transformations of relations of production and class patterns at the turn of the century. They correspond to what has been called the *corporate revolution* and the *managerial revolution*. A new efficiency was achieved within large corporations due to the revolution in technology and organization, a revolution in management in the broad sense of the term.[13]

2.2 Catching up with the US

An important feature of technical change since World War II has been the propensity of European countries and Japan to catch up with the US. The effects of this catching-up combined with the decline of the profit rate in a complex pattern of events. In a sense, this tendency of the profit rate to decline can be described as a world phenomenon, but trends in technology in Europe and Japan were also historically specific during the first few decades of the postwar period, displaying differences among countries. As these countries

13. A.D. Chandler, *The Visible Hand. The Managerial Revolution in American Business*, Cambridge: Harvard University Press (1977); G. Duménil, D. Lévy, *The Economics of the Profit Rate: Competition, Crises, and Historical Tendencies in Capitalism*, Aldershot: Edward Elgar (1993); *La dynamique du capital. Un siècle d'économie américaine*, Paris: Presses Universitaires de France (1996).

were progressively converging toward the US economy, similar evolutions were observable in all countries. The overall picture is difficult to untangle.[14]

It is possible to illustrate this pattern of events using the framework of diagram 4. The results of two simulations are presented in figures 2 and 3:

1. We first assume that the leader has reached a smooth trajectory with a declining productivity of capital and a constant wage share. An assumption must be made concerning wages in the follower country. We arbitrarily assume that the wage share is equal to that of the leader. Figure 2 shows the patterns of evolution of the two productivities of capital. During a first phase, the productivity of capital of the follower rises, as a result of the favorable conditions created by the existence of the leader (from the point of view of the availability of new techniques, abstracting, in particular, from the effects of international competition[15]).

2. The realism of the picture is increased in figure 3 by using the actual evolution of technology in the US to represent the leader, and the actual series of labor cost in France, to denote the follower. A similar evolution results. Although the parameters accounting for the conditions of innovation in France have been determined more or less arbitrarily, the profile of the productivity of capital deriving from the simulation for France is not significantly different from the actual series (also plotted in the figure for comparison). In particular, the productivity of capital, as simulated, reaches its maximum in the early 1970s as in the actual series.

The model illustrates an intuitive property of catching-up. With a configuration such as that of diagram 4, the impact of labor cost is small in the economy of the follower as long as it remains at a considerable distance from the leader.

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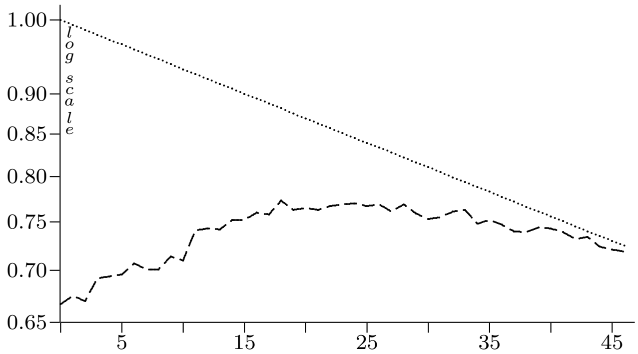
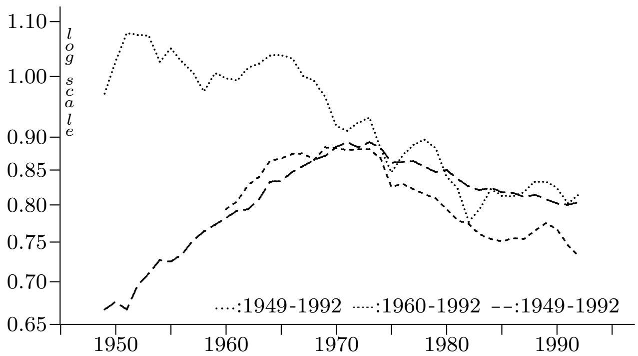
14. Robert Brenner locates mistakenly, in our opinion, the *cause* of the decline of the profit rate in the catching-up (R. Brenner, "The Economics of Global Turbulence", *New Left Review*, 229 (1998), p. 1-264).
15. Obviously excess exposure to international competition can kill the follower.

Figure 2 Catching-up in two fictitious countries: the productivities of capital of the leader and the follower



Leader: (········)
Follower: (— —)

Figure 3 France catching up with the US: the productivities of capital in the US and in France



US (actual series): (········)
France (actual series): (-----)
France (model): (— —)

The actual series for France have been multiplied by a constant, since the the two definitions are not coherent. Data for the USA are from the BEA, and for France, from the OECD.

3 - Marx's analysis of historical tendencies

The tendency for the profit rate to fall is only one component of a larger framework of analysis in which technology, distribution and accumulation are involved. Section 3.1 recalls the main features of Marx's presentation. In the remainder of this section, we use the framework of section 1 to interpret Marx's analysis. The simplest case, in which the rate of growth of the labor cost is exogenous, is discussed in section 3.2. Section 3.3 adds to the model a feedback relationship linking changes in labor cost to changes in the profit rate. Accumulation is introduced in section 3.4. Section 3.5 provides a brief synthesis of these results. Last, section 3.6 suggests an interpretation of Marx's thesis of a falling profit rate, associated with specific "unfavorable" conditions of innovation or their tendencial deterioration —in general and within capitalism in particular.

3.1 A system of tendencies

At least five "laws of motion" are considered by Marx in his famous analysis of Volume III of historical tendencies: (1) the diminishing value of use-values (the progress of labor productivity); (2) the rising value composition of capital; (3) the rising rate of surplus value; (4) the falling profit rate; (5) accelerated accumulation.

As is well known, Marx first addresses the issue of the falling profit rate under the assumption of a constant rate of surplus value: "[...] a gradual fall in the general rate of profit, given that the rate of surplus value, or the level of exploitation of labour by capital, remains the same"[16]. How can the profit rate decline whereas the rate of surplus value is constant? Marx's answer is straightforward: this is the effect of the rising composition of capital, the fact that more and more constant capital is required compared to variable capital. The assumption of a constant rate of surplus value is used by Marx to contend that the fall of the profit rate is not due to excessive wages, but to a given feature of technical change. This analysis sharply contrasts with Ricardo's analysis that locates the declining profitability of capital in the rise of the relative price of corn, and, thus, of the nominal wage and of the wage share. In a

16. K. Marx, *Capital, Volume III*, New York: First Vintage Book Edition (1894), ch. 13, p. 318.

contemporary formulation, Marx contends that the downward trend of the profit rate must not be interpreted as a *wage squeeze*.

This configuration is very relevant factually. In the account that we provided of the features of technical and distributional change in the US, a falling profit rate prevailed in the late 19th century and in the second half of the 20th century. During these two periods the share of wages, *i.e.*, the rate of surplus value[17], remained more or less constant.

As one progresses into the chapters of *Capital* devoted to the falling profit rate, it becomes clear that Marx is not content with the assumption of a constant rate of surplus value. The fall of the profit rate is said to be compatible with a rising rate of exploitation. At the end of chapter 14, one can read: "The tendential fall in the profit rate is linked with a tendential rise in the rate of surplus value [...]"[18].

Last, Marx was conscious of the link between the falling profit rate and accumulation: "A fall in the profit rate, and accelerated accumulation, are simply different expressions of the same process, [...]. In this way there is an acceleration of accumulation as far as its mass is concerned, even though the rate of this accumulation falls together with the rate of profit[19]". Thus, the rate of accumulation tends to fall with the profit rate, while the mass of capital accumulated rises: $\rho(K) = \frac{\Delta K}{K} \searrow$ and $\Delta K \nearrow$.

There is no denying the fact that Marx's analysis is also deficient in several respects. Five problem areas are discussed below:

1. Why would a declining profit rate be paralleled by a rising rate of exploitation? Marx is not explicit in this respect. Since labor productivity increases, capitalists can impose a larger rate of exploitation on the workers without lowering their real wage. But why is this tendency so strongly linked to the downward trend of the profit rate?

2. Although Marx insists repeatedly on the tendency of the composition of capital to rise, he is not very explicit concerning the origin of this tendency. Is mechanization a feature of technical change in general, not only within capitalism? Does such a mechanization always require the rise of the technical and organic compositions of capital? Marx repeatedly asserts that the perpetuation of capitalist relations of production impacts on the rythms of mechanization, but

17. Still abstracting from a number of difficulties.
18. K. Marx, *ibid.*, ch. 14, p. 347.
19. K. Marx, *ibid.*, p. 348.

the direction of this effect is not always the same. He sometimes contends that capitalists push the use of machinery even beyond purely technical requirements in order to control the workers. He sometimes points to the fact that exploitation (the low cost of labor) limits the incentive to employ more mechanized processes, since capitalists only pay a fraction of the labor time expended by the workers.[20]

3. The formalism in chapter 13 of Volume III of *Capital* is not really appropriate. The tendency for the profit rate to fall is presented within the framework used in Volume I to account for the theory of surplus value. Capital, $c+v$, is the sum of two flows. As is well known, the profit rate is written: $s/(c+v)$ or $s'/(1+\gamma)$, with s' denoting the rate of exploitation and γ, the organic composition of capital. This framework abstracts from the circulation of capital introduced (later) in Volume II. In Volume III, surplus value is designated as *profit*, π, and capital is actually a stock, the sum of three components: productive, commodity, and money capitals. Thus the profit rate should be: π/K. Within K, it should be possible to distinguish two components, one due to the financing of variable capital, and one to constant capital. In addition to the difficulties inherent to Marx's presentation, for practical reasons due to the availability of data, one must substitute the productivity of capital or its inverse, the capital-output ratio, for Marx's organic composition of capital. Marx's statements concerning the rise of the organic composition of capital can be translated into *a declining productivity of capital*.[21]

4. In his analysis of historical tendencies, Marx is reluctant to refer to wages, nominal or real. He only considers the rate of exploitation: "We entirely leave aside here the fact that the same amount of value represents a progressively rising mass of use-values and satisfactions, with the progress of capitalist production..."[22]. If labor productivity increases, a constant rate of surplus value results in a rising real wage. In other parts of his work, Marx quite explicitly refers to the movement of the real wage (see for example, the quotations at the beginning of this study, or the famous chapter 25 of Volume I of *Capital*).

5. It is also necessary to recall that Marx's description of the mechanisms leading to a diminished average profit rate is problematic.

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20. See K. Marx, *ibid.*, ch. 15, section IV.

21. Instead of $r = \frac{s'}{1+\gamma}$, we use $r = P_K(1-\omega)$.

22. K. Marx, *ibid.*, ch. 13, p. 325.

Marx's account is well known: (1) Individual producers may introduce a new technique on account of the incremental profit that it yields prior to its diffusion to all producers; (2) Once it is generalized to all producers and a uniform profit rate is reestablished, the average profit rate is diminished. In order to reach a conclusion concerning the comparison between the profit rate prevailing before the introduction of the new process of production and the eventual profit rate after its diffusion, one additional assumption must actually be made concerning distribution. Nobuo Okishio has shown, in his famous theorem, that the profit rate must rise if the real wage is maintained[23], *i.e.*, if capitalists absorb the entire advantage of the new improved conditions of production. The profit rate can decline only if the workers benefit from at least a portion of the progress accomplished, *i.e.*, if the real wage increases to an extent. It is therefore not possible to establish a falling profit rate under the assumption of a constant real wage rate.[24]

Overall, Marx's analysis of the historical tendencies of capitalism is fascinating. Its relevance is still obvious after more than a century. But it is also, in several important respects, deficient.

3.2 The falling profit rate with an exogenous growth rate of labor cost

In this section, we interpret historical tendencies as asymptotic trajectories of the dynamical model. This means that, under certain assumptions, beginning with any technique and any level of labor cost, the model converges toward a trajectory *à la Marx*. We use in turn two sets of assumptions:

1. *We first assume that the innovation set, the probability distribution, and the growth rate, ρ_w, of the labor cost are all given.*

The average features of technical change are described by, $\overline{a}$ and $\bar{l}$, the coordinates of G' (see diagram 2). They are functions of the innovation set, of the probability distribution (which is given), and of the slope of the selection frontier (the rate of surplus value), μ,

23. N. Okishio, "Technical Change and the Rate of Profit", *Kobe University Economic Review*, 7 (1961), p. 86-99.
24. Or a basic assumption must be abandoned. For example, one can assume that capitalists choose, for some reason, techniques which do not maximize the profit rate (A. Shaikh, "Marxian Competition versus Perfect Competition: Further Comments on the So-Called Choice of Technique", *Cambridge Journal of Economics*, 4 (1980), p. 75-83).

and, thus, of the wage share ω: $\overline{a} = \overline{a}(\omega)$ and $\bar{l} = \bar{l}(\omega)$. The following properties are intuitive: (1) The average growth rate of the productivity of capital, $\overline{a}(\omega)$, is a decreasing function of ω; (2) The average growth rate of labor productivity, $\bar{l}(\omega)$, is an increasing function of ω.

After substituting the average values of innovation, $\overline{a}$ and $\bar{l}$, for their stochastic values, a and l, into equations 2, a deterministic dynamical system is obtained for the two variables which describe technology, A and L (or equivalently P_K and P_L). Replacing L (or P_L) by the wage share $\omega = Lw$, the dynamical system can be written as:

$$\begin{aligned} \rho(\omega) &= \rho_w - \bar{l}(\omega) \\ \rho(P_K) &= \overline{a}(\omega) \end{aligned} \tag{6}$$

The first equation can be studied independently of the second.

The equilibrium value of the wage share, $\omega^\star$, is the solution of the following implicit equation:

$$\bar{l}(\omega^\star) = \rho_w$$

Since $\bar{l}(\omega)$ is a monotonically increasing function of ω, a unique fixed point, $\omega^\star$, exists, if ρ_w belongs to the interval $[\bar{l}(0), \bar{l}(1)]$. At the fixed point, the wage share is constant and, thus, the growth rate of labor productivity is also constant and equal to that of wages:

$$\rho(P_L) = \rho_w \tag{7}$$

In continuous time, the local stability of this fixed point is easy to prove.[25]

Consider now the second of the equations in 6. The fixed point of the first equation corresponds to an asymptotic trajectory in which the productivity of capital, P_K, and the profit rate, r, increase or diminish at the same constant rate:

$$\rho(P_K) = \rho(r) = \overline{a}(\omega^\star) \tag{8}$$

25. These properties are rather intuitive. If labor productivity grows at a slower rate than the exogenous labor cost, ($\bar{l}(\omega) < \rho_w$), a rising labor share follows. The rotation of the selection frontier provokes, in turn, a larger growth rate of labor productivity. Conversely, labor productivity growing faster than labor cost rotates the selection frontier toward a more vertical position, and initiates a decline in the growth rate of labor productivity. Equilibrium is reached when the two growth rates are equal.

Thus, the profile of the series over time can be derived from their initial values in period 0:

$$P_L = \frac{w}{\omega^\star} = P_L(0)e^{\rho_w t} \textit{ and } P_K = P_K(0)e^{\overline{a}(\omega^\star)t}$$

This allows for the derivation of the trajectories for r and the organic composition of capital, γ:

$$r = (1-\omega^\star)P_K \textit{ and } \gamma = \frac{1}{\omega^\star P_K}$$

The direction of the variation of the profit rate or of the productivity of capital along this trajectory is determined by the sign of $\overline{a}(\omega^\star)$ and depends on the exogenous growth rate of the cost of labor, the innovation set, and the probability distribution. This sign is discussed in section 3.6. Thus, trajectories *à la Marx* may obtain, but are subject to certain conditions.

2. *We now assume that the innovation set, the probability distribution, and the growth rate of the labor cost vary over time: the innovation set is gradually reduced as in panel (b) of diagram 3.*

We assume that this variation of the innovation set is a homothety centered in the origin, whose ratio is $1/t^\alpha$. Thus, the average values, $\overline{a}$ and $\overline{l}$, can be written:

$$\overline{a} = \frac{\overline{a}(\omega)}{t^\alpha} \textit{ and } \overline{l} = \frac{\overline{l}(\omega)}{t^\alpha}$$

in which the functions $\overline{a}(\omega)$ and $\overline{l}(\omega)$ are independent of time. The assumption made about the wage rate is: $\rho(w) = \frac{\rho_w}{t^\alpha}$.

With these assumptions, the system in 6 becomes:

$$\rho(\omega) = \frac{\rho_w - \overline{l}(\omega)}{t^\alpha}$$
$$\rho(P_K) = \frac{\overline{a}(\omega)}{t^\alpha}$$

The implicit equation for $\omega^\star$ is formally unchanged. Equations 7 and 8 become respectively:

$$\rho(P_L) = \frac{\rho_w}{t^\alpha} \textit{ and } \rho(P_K) = \rho(r) = \frac{\overline{a}(\omega^\star)}{t^\alpha}$$

The growth rates of the variables along their asymptotic trajectories diminish with time[26]:

$$P_L = \frac{w}{\omega^\star} = P_L(t_0)\left(\frac{t}{t_0}\right)^{\rho_w} \; and \; P_K = P_K(t_0)\left(\frac{t}{t_0}\right)^{\overline{a}(\omega^\star)} \quad if \; \alpha = 1$$

$$P_L = \frac{w}{\omega^\star} = P_L(t_0)\, exp\left(\frac{\rho_w}{1-\alpha}\left(t^{1-\alpha} - t_0^{1-\alpha}\right)\right)$$

$$and \quad P_K = P_K(t_0)\, exp\left(\frac{\overline{a}(\omega^\star)}{1-\alpha}\left(t^{1-\alpha} - t_0^{1-\alpha}\right)\right) \quad if \; \alpha < 1$$

It is interesting to compare the properties of the asymptotic trajectory with those obtained under the previous set of assumptions:

1. Again, the rate of surplus value is constant (ω is constant).
2. A productivity slowdown is observed, with $\rho(P_L) = \frac{\rho_w}{t^\alpha}$.
3. The condition required to obtain a downward trend of the profit rate is unchanged: $\overline{a}(\omega^\star) < 0$, with $\omega^\star$ still given by $\overline{l}(\omega) = \rho_w$.
4. What changes is the rapidity of the decline of the profit rate and of the productivity of capital. For example, if $\alpha = 1$, power trajectories are substituted for exponential trajectories.

The results obtained in this section under two different sets of assumptions (given conditions of innovation and a constant rate of growth real wages, or the gradual decline of these parameters at the same rate) are well in line with Marx's analysis at the beginning of chapter 13 of Volume III of *Capital*. Stable trajectories with a downward trend of the profit rate and a constant rate of surplus value can be reproduced under certain conditions.

3.3 Exploitation: A feedback effect of the profit rate on labor cost

We have already noted in section 3.1 that Marx is not explicit concerning the reasons for the coexistence of a declining profit rate and a rising rate of exploitation. The underlying idea could, in our opinion, be more adequately expressed by referring to the rate of growth of the real wage rate or labor cost.

A declining (or low) profit rate will strengthen the resistance of firms to any further rise of the labor cost. The recurrence of recessions, associated with a declining profit rate, forces down wage

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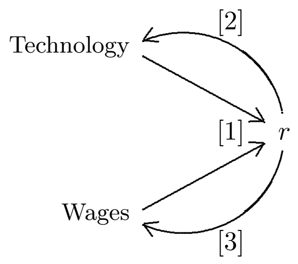
26. The first period corresponds to $t_0 > 0$. If $\alpha > 1$, the slowdown is too strong: Labor productivity tends toward a constant.

increases. Accumulation is slowed and unemployment increases during a structural crisis. The converse is true when the profit rate rises and is high: Accumulation is rapid, the labor market is tight, and this is a favorable environment for rising wages. Such a relationship between the trend of the profit rate and that of wages was clearly manifested during the 20th century, and this confirms that Marx's insight should be taken seriously.[27]



[1] Determination of the Profit rate

[2] Effect of the Profit Rate on the Selection of New Techniques

[3] Feedback Effect of the Profit Rate on the Rate of Growth of Wages

Diagram 6

As suggested by diagram 6, the relationships investigated in the previous sections can be supplemented by a feedback effect of the profit rate on wages. The first two arrows, [1], recall that the profit rate is determined, by definition, by technology and wages. The second arrow [2] denotes the effect of the profit rate on the selection of new techniques. The third arrow [3] represents the new relationship: the impact of the profit rate on the growth rate of the labor cost. Such a model can be fitted to the data.[28]

As suggested by historical observation, both the variation of the profit rate and its level can play a role in this relationship.[29] In

27. In the structural crisis of the 1970s, the decline of the profit rate slowed, or even stopped, the rise of wages, even if the share of wages was not considerably increased. Conversely, during the first half of the 20th century, the evolution of technology, favorable to the rise of the profit rate, allowed for a larger rate of growth of wages. (In spite of this increased growth rate of the labor cost, the profit rate still rose.)

28. G. Duménil, D. Lévy, *The Economics of the Profit Rate, op. cit.* note 13, ch. 15.

29. Consider, for example, the situation in the US, at the beginning of the 20th century. The low profitability of capital prolonged the slow growth of wages while the profit rate was already beginning to recover. In a similar manner, the effects of the high profit rates of the 1960s on wages were still felt in the 1970s, when the decline of the profit rate was already well established. A situation similar to that observed at the beginning of the century seems to prevail presently: a rising profit rate and continuing wage stagnation.

the first set of assumptions considered in the previous section, this feedback effect of the profit rate on wages can be modeled simply as follows:

$$\rho(w) = f + g\rho(r) + h\,log(1 + r) \qquad (9)$$

In this equation, parameter f accounts for an exogenous historical trend, and g and h, for the fluctuations of w around this trend.[30]

Equation 9 does not suggest that wages are not determined by the struggle between workers and capitalists. First, the secular growth rate of the labor cost remains exogenous. Second, this equation only accounts for an observable, rather stable, quantitative pattern in the outcome of this struggle. It is due to the fact that the effect of class struggle on wages depends to a considerable extent on underlying economic conditions. The model stresses the importance of the profit rate and its movement in the determination of this outcome of class struggle.

When we first introduced this model, the relationship between the movement of wages and the profit rate was not recognized as such. The standard analysis among the "left" linked the movement of wages to that of labor productivity, as if the relevant variable was the share of profits instead of the profit rate. Even, if it was not explicitly considered by Marx himself, the establishment of this relationship plays, in our opinion, a significant role in the restoration of the centrality of the profit rate to the analysis of capitalism.

Examining the properties of the asymptotical trajectories of our variables shows that the third term in equation 9, $h\,log(1 + r)$, plays an important role:

1. If this term is deleted ($h = 0$), the feedback of the variation of the profit rate on that of labor cost only impacts on the growth rates of variables.[31] The properties of the asymptotic trajectories are not changed.

2. If the second term is included, the feedback of the variation of the profit rate on that of labor cost stabilizes the profit rate at a certain level. A stationary state *à la Mills* obtains.[32] As shown in diagram 7,

30. The case $g = h = 0$ corresponds to the exogenous growth rate of labor cost of the previous section.

31. The equation accounting for $\omega^\star$ becomes: $\bar{l}(\omega^\star) = f + g\bar{a}(\omega^\star)$.

32. The equilibrium wage share is given by: $\bar{a}(\omega^\star) = 0$.

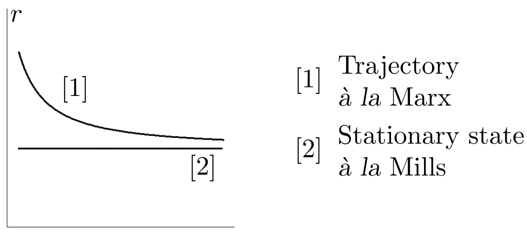


Diagram 7

the trajectory described by Marx could only be interpreted as a pre-asymptotic state, preliminary to the convergence of the profit rate toward its limit.

In spite of the feedback effect of the profit rate on the movement of wages, these models always lead to a stabilization of the share of wages, along the asymptotic trajectory, at a certain level. They are not compatible with a rising rate of surplus value except during pre-asymptotic stages.

3.4 Accumulation

The behavior of accumulation is also a component of the description of historical tendencies, and this connection is explicit in Marx's analysis.

A central aspect of the classical-Marxian analysis is that the rate of accumulation is a function of the profit rate. This is traditionally represented by the relationship between the growth rate of the capital stock, $\rho(K)$, and the profit rate[33]:

$$\rho(K) = sr \tag{10}$$

Beginning with a given stock of capital, the entire series of capital can be derived. As shown in diagram 8, new relationships must be introduced in diagram 6. The above expression of accumulation as a function of the profit rate is depicted by the arrow [4]. Output and employment can be derived from the capital stock, [5], and technology, [6]:

$$Y = KP_K \; and \; L = K\frac{P_K}{P_L}$$

33. In this long-term analysis, we abstract from business-cycle fluctuations.

The overall dynamics described in diagram 8 correspond to a model with four variables (P_L, P_K, w and r), to which three other variables are added (K, Y, and L). It goes without saying that this model emphasizes a number of relationships which are of primary importance, abstracting from other possible interactions of lesser influence. This model can be fitted to the data for the US economy.[34]

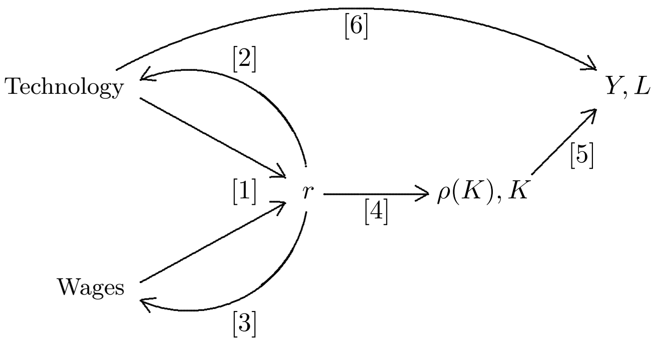


Diagram 8

Whether or not it is possible to recover Marx's statements concerning accumulation depends on the set of assumptions considered in section 3.2:

1. *Constant innovation set and growth rate of wages.*

Along a trajectory on which the profit rate declines, the growth rate of the capital stock also diminishes (equation 10). Its trajectory can be made explicit:

$$K = K(0)\, exp\left(\frac{sr(0)}{\overline{a}(\omega^\star)}(exp(\overline{a}(\omega^\star)t) - 1)\right)$$

On a trajectory *à la Marx*, one has $\overline{a}(\omega^\star) < 0$, and the capital stock tends toward a constant (see panel (a) of diagram 9). Since the productivity of capital declines, output must also decline. The amount of capital accumulated in each period also declines.

In spite of its simple and apparently basic characteristics, this first interpretation of Marx's analysis is not consistent with his views

34. G. Duménil, D. Lévy, *ibid.*.

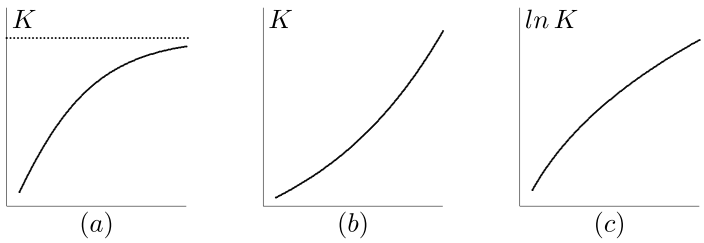


Diagram 9

concerning accumulation. It is clear that the eventual decline of output is inappropriate.

2. *The gradual reduction of the innovation set and of the growth rate of wages.*

It is also possible to determine explicitly the profile of the capital stock using the second set of assumptions with $\gamma = 1$[35]:

$$K = C_1\, exp\left(C_2 t^{\overline{a}(\omega^\star)+1}\right)$$

The case $\gamma < 1$ is more complex.

Since $\overline{a}(\omega^\star) > -1$, the capital stock rises indefinitely, as well as production and the amount of capital accumulated in each period. These profiles are described in panels (b) and (c) of diagram 9. Panel (b) illustrates the fact that the capital stock increases more and more ($\Delta K \nearrow$). The logarithm of the capital stock in panel (c) shows that the growth rate of the capital stock is gradually diminished.

Abstracting from the tendency for the rate of surplus value to rise, this second set of assumptions is in line with Marx's analysis. Therefore, his view of historical tendencies seems more consistent with the thesis of *a gradual increase in the difficulty of innovating* in the sense of panels (a) and (b) of diagram 3. The downward trend of the profit rate obtains in spite of the gradual reduction of the growth rate of the real wage, at the same rate as the difficulty of innovating increases. It is not possible to attribute the tendency for the profit rate to fall, in this model, to a wage squeeze: (1) the share of wages is constant; (2) the deterioration of the conditions of innovation is paralleled by a similar decline of the rate of growth of the real wage rate.

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35. With: $C_1 = K(t_0)\, exp\left(-C_2 t_0^{\overline{a}(\omega^\star)+1}\right)$ and $C_2 = \dfrac{sr(t_0)}{t_0^{\overline{a}(\omega^\star)}\left(\overline{a}(\omega^\star)+1\right)}$.

3.5 A summing up

Two basic sets of assumptions concerning technical change have been considered in the previous sections in order to discuss Marx's analysis of historical tendencies:

1. In a first group of models, the conditions of innovation are assumed constant. Three variants of this model have been discussed, that differ according to the assumed growth rate of the real wage.[36] The growth rate of the real wage can alternatively:

- be constant.
- respond to the variations of the profit rate. (A rising profit rate allows for a larger variation of the real wage, and a declining profit rate diminishes the capacity of the real wage to rise.)
- react to the variations of the profit rate as above *and* to its level. (A high profit rate is favorable to a rise in the real wage rate, and a low profit rate unfavorable to this increase.)

2. A second model assumes that the conditions of innovation are subject to a constant deterioration, and that the growth rate of the real wage diminishes at the same rate.

The results can be summarized as follows:

1. None of these models vindicates the tendency for the rate of surplus value to rise (a decline of the share of wages in the model). All assymptotic trajectories display a constant share of wages.[37]

2. A declining profit rate may prevail in each model under certain assumptions. However, in the third variant of the first group of models, due to the strong adjustment of the growth rate of real wages, the profit rate tends toward a constant.

3. Consider now accumulation and output. A problem with the two first variants of the first group is that the declining profit rate leads to

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36. These variants correspond to the number of terms conserved in equation 9: (1) only the first term; (2) the two first terms; (3) the three terms.
37. Tom Michl obtains trajectories with a declining profit rate and a rising rate of surplus value ("Biased Technical Chance and the Aggregate Production Function", *International Review of Applied Economics*, 13 (1999), p. 193-206). In his model, the growth rates of labor productivity and capital productivity, that we denote $\overline{l}$ and $\overline{a}$, are assumed constant, and positive and negative respectively. Thus, they do not respond to variations in wages. In our model, $\overline{l}$ and $\overline{a}$ are functions of wages, and the tendency for the rate of surplus value to rise, *i.e.*, the decline of the share of wages toward 0, results in a vertical selection frontier. In this situation, $\overline{a}$ is positive and the profit rate necessarily rises asymptotically.

the stagnation of the capital stock, which results in a declining output due to the falling productivity of capital. In the third variant, the growth rates of the capital stock and of output both tend to stabilize with the profit rate. Only the second set of assumptions, with the simultaneous deterioration of the conditions of innovation and of the growth rate of the real wage (at the same rate), allows for: (1) a declining profit rate; (2) an acceleration of accumulation as far as the mass investment in each period is considered, and a decline of the rate of accumulation with respect to the stock of capital; (3) a declining (but still positive) growth rate of output. As stated in section 3.4, this model is in line with Marx's insights in his analysis of historical tendencies, the tendency for the rate of surplus value to rise being the only exception.

3.6 The conditions of innovation: The roots of the tendency for the profit rate to fall

At the beginning of chapter 13 of Volume III of *Capital*, Marx presents the rise of the technical or organic composition of capital, in combination with a constant rate of surplus value, as the *cause* of the tendency for the profit rate to fall. As stated in section 3.1, Marx is, however, not clear concerning the origin of the rise of the composition of capital.

We interpret Marx's analysis of the *tendency* for the profit rate to fall as a thesis concerning the features of innovation. According to Marx, innovation displays certain conditions such that the profit rate will tend to decline, even if the growth of the labor cost remains moderate. For a given growth rate of the labor cost, the economy will enter into a trajectory *à la Marx*, if certain features become manifest.

We must therefore confront two questions: (1) What are these conditions? (2) Why do they prevail, in particular within capitalism?

Diagram 3 can assist in this discussion. We add the selection frontiers for a given wage share, as well as the center of gravity of the innovation set. (We abstract from the difference between G and G'). Thus, diagram 3 can be transformed into diagram 10. It is easy to locate visually on these panels the cases corresponding to a falling profit rate. Whenever, the coordinate, $\overline{a}$, of the center of gravity on the horizontal axis is negative, the profit rate falls along an asymptotic trajectory. It is clear that this configuration is observed for each panel in the right-hand column. The profit rate is more inclined to

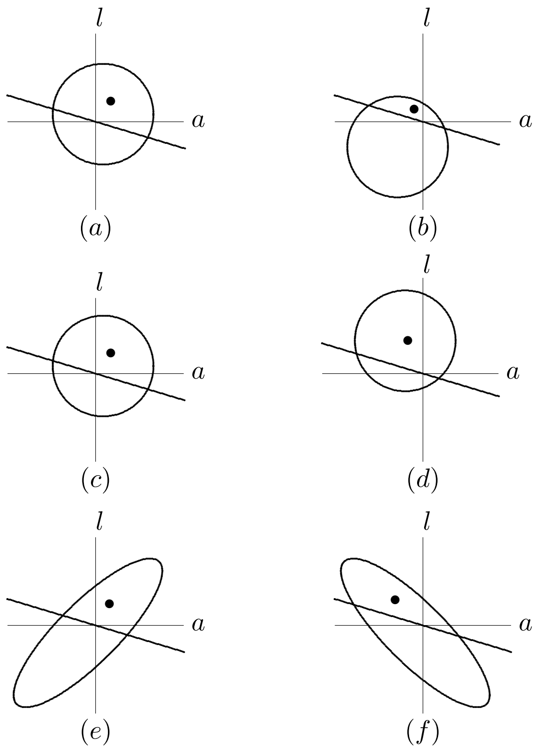

Diagram 10

decrease whenever: (1) the difficulty of innovating is larger, (a) → (b); (2) innovation is biased, (c) → (d); (3) inputs are substitutes, (e) → (f).

This discussion can be easily translated into the second set of assumptions of the model (the gradual reduction of the innovation set and of the growth rate of wages). As a result of the assumption of a homothetical transformation centered on the origin, the diagrams in the first column are unchanged with the exception that the scale of the axes is reduced over time.

The configurations described in panels (d) and (f) are quite reflective of Marx's insight concerning the composition of capital. Innovations can be found which diminish the productivity of capital (signaling heavy mechanization). Other cases are possible, but rare. The first configuration in panel (b) is interesting, since it signals that this propensity of innovations to display characteristics *à la Marx* can result from the *difficulty* of finding profitable innovations in general, independently of any *a priori* bias.

Consider this later case. Is it a property of capitalism in particular (which could be avoided within "socialism")? Obviously, R&D activities are intrinsically costly and risky. However, one interpretation could be that the limits set by private property within capitalism pose specific barriers to innovation, or at least some forms of it. These problems arise from the contradiction between the cost of R&D, and the difficulty of privately capturing the total profit from the innovation. Either patent legislation is too narrow, or it is protective and patents claims are too broad, making the diffusion of inventions or follow on innovations too costly. In the first case, R&D will be weak; in the second case, new innovations cannot spread rapidly. In this respect, private interest contradicts collective interest.[38]

Independently of the exact nature of the problem with technical change within capitalism, the tendency for the profit rate to fall points to some limitation of capitalism. A configuration such as that in panel (a) of diagram 10, characteristic of what we called the *intermediate period* in section 2.1, is favorable. Technical progress can be rapid, and wages can rise in concert with the profit rate. Conversely, Marx's analysis points to an unfavorable pattern, a kind of contradictory process — possibly increasing over time. Technical progress is paralleled by a decline of the profit rate which tends to diminish the workers chances of obtaining wage increases. Accumulation is slowed. The outcome is a structural crisis, following which the dynamic of the mode of production can only be restored as a result of important transformations. Overall, capitalism does "revolutionarize" technology and organization, but in a convulsive manner.

4 - A broader framework of analysis

In the previous sections, the model of section 1 is used within very simple frameworks of analysis. The economy is generally consid-

38. Note that what is at issue concerning the falling profit rate is *process innovation*, not *product innovation*. Product innovation is not a countertendency to the falling profit rate. *A priori* a new product results from any kind of technique, with a low or high composition of capital.

ered globally and in equilibrium. Only section 2.2 contrasts the features of technical change within two distinct economies. This framework also abstracts from traditional determinants of technical change, such as growth or competition. Obviously, nothing restricts the use of the model to such frameworks or forbids the consideration of other mechanism that affect technical change. It is the purpose of this section to sketch two such possible developments. Section 4.1 uses the model in a disagregated economy, where disequilibrium may prevail. Section 4.2 briefly suggests a number of developments concerning endogenous technical change.

4.1 Heterogeneity and disequilibrium

The model presented in section 1 can be used to account for the behavior of firms, industries, larger sectors of the economy, or the total economy. Significant heterogeneities may prevail and impact considerably on the functioning of the economy. A number of potentially important phenomena are *a priori* linked to the fact that decisions are actually made by individual agents in a decentralized manner and within the context of disequilibrium. Supply may differ from demand, and productive capacities are not necessarily fully utilized.

The heterogeneity of the economy may be crucial. An important aspect of the historical transformations described in section 2.1 is that the favorable profile of technical change observed during the intermediate period, was concentrated, in the US, within a given segment of the economy: large corporations backed up by the new finance. Far from affecting the economy uniformly, the corporate and managerial revolutions of the early 20th century left aside a large segment of the economy, composed of smaller firms still dependent on traditional technology and management.

Instead of the simple characterization of an average transformation of the conditions of innovation described in section 2.1, one can contemplate a model in which two sectors are considered. One sector evolved along the traditional lines of evolution, whereas new organization and technology prevailed in the emerging corporate managerial sector. The resulting new sector was more efficient. Consequently, two technologies and patterns of technical change must be described, even assuming for simplicity that wages are identical. This model generates two distinct technical trajectories. The total economy can be described as a weighted average of the two sectors, with changing weights mirroring the rise to dominance of the new sector, and the progressive elimination of the other. Such a model is studied in one of our recent papers.[39] Note that this heterogeneous character of technology is not merely a hypothetical extension of the analysis. It was, in our opinion, a key factor in the occurrence of the Great Depression.[40]

The model of section 2.2 only considers the impact of a leader on the conditions of innovation faced by a follower. It is, however, clear that the actual process is one of reciprocal interaction. Various countries compete on an international basis, and tend to borrow innovations from one another. The *catching-up* corresponds to the case in which a leader can be distinguished from a follower, and imitation denotes reciprocal interaction. Obviously, there would be nothing wrong with a model that takes into account a reciprocal influence of innovation sets.

Heterogeneous techniques also coexist among firms, within a given product line, in the same country. It is clear that the diffusion of innovations can also be treated in a framework such as that outlined above.

The consideration of individual agents in interaction opens our analysis to the field of microeconomics and disequilibrium. Elsewhere, we have presented in other works what we call *disequilibrium microeconomics* to be substituted for neoclassical microeconomics, and a general disequilibrium model.[41]

The framework of analysis in such *general disequilibrium models* can be briefly sketched as follows. A straightforward meaning is given to the notion of disequilibrium: markets do not clear, productive capacities are not fully utilized, etc. Decisions are decentralized. When production decisions are made, demand is still unknown. At the close of the market, inventories of unsold commodities may exist, and are transmitted to the next period. Rationing may occur. Prices are also decided by individual firms, and they are not necessarily uniform. The demands facing the various producers of the same good depend on their individual prices. The issuance of money by the banking

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39. G. Duménil, D. Lévy, “The Acceleration and Slowdown”, *op. cit.* note 8.
40. G. Duménil, D. Lévy, The Great Depression: A Paradoxical Event?, Cepremap, num. 9510, Paris (1995).
41. G. Duménil, D. Lévy, *The Economics of the Profit Rate, op. cit.* note 13; *La dynamique du capital, op. cit.* note 13.

system is endogenous to the model, and responds to the general level of activity and inflation. The demand for fixed capital (investment) follows from the accumulation of profits and new loans. Investment is also influenced by the capacity utilization rate and the profit rate of the various industries. Consumption is determined by wages, a fraction of profits devoted to consumption, and the stock of money held by potential consumers. Technology is heterogeneous (among the producers of the same good). Decisions are modeled in terms of *adjustment*, *i.e.*, reaction to disequilibrium. For example, any firm that produces and does not sell its output as expected, reduces production in the next period.

In such models, one can determine a classical long-term equilibrium with a uniform profit rate among industries (averaging the various techniques in each industry). It is usually stable. A short-term equilibrium also exists. It can be stable or unstable, and the economy remains generally in the vicinity of short-term equilibria. The succession of periods of stability and instability accounts for business-cycle fluctuations.

We studied a model in which two goods are produced, each by two firms, using the framework of section 1.[42] Each of the 4 firms is described by 7 variables: the capacity utilization rate, inventories, the price of output, the stock of capital and its growth rate, and the two technical parameters A and L. To this one must add the money stock, its growth rate and inflation. (The number of variables is 27.)

The properties of this model can only be investigated through a simulation approach. It appears that the model has several interesting properties:

1. It reproduces the usual properties obtained in other classical dynamical models, in particular, a tendency toward a uniform profit rate among industries.[43]

2. Tendencies such as those studied in sections 2.1 may prevail.

3. A number of additional results are observed. For example, the technical heterogeneity among firms can be maintained over time, or even increase. However, firms lagging behind tend to disappear since less capital flows into them.

42. G. Duménil, D. Lévy, "Complexity and Stylization", *op. cit.* note 11.
43. As usual these results are subject to conditions. For a discussion of these conditions, see G. Duménil, D. Lévy, *The Economics of the Profit Rate, op. cit.* note 13 and *La dynamique du capital, op. cit.* note 13.

Overall, the adoption of a disaggregated framework of analysis does not question the relevance of the aggregate analysis, but many industry- or firm-specific traits can be identified. Clearly, such analysis opens a broad research field for future investigation.

4.2 Endogenous technology and endogenous technical change

In the model used in this paper, the pattern of innovation (the innovation set) is given or varies exogenously, but technology is determined endogenously:

1. The profit rate, which is used as a criterion in the selection among new innovations, is an endogenous variable of the model. In a more complex model, as in section 4.1, the profit rate is a function of a broad set of circumstances: demand, competition, etc. All these circumstances will impact on the trajectory of technical change.
2. Although the random variables a and l are exogenous, the technique in one period is always derived from the technique prevailing in the previous period, and is, therefore, endogenous.

Moreover, in a vintage model[44], the average technology in a given year is a function of the rate of accumulation. If the growth rate of the fixed capital stock is large, the average technology is closer to the most recent technology embodied in the later investments.

There would be no difficulty in treating the *innovation set* itself as endogenous:

1. In a model in which the innovation set is a circle, the conditions of innovation are described by a set of parameters, the radius of the circle, the coordinates of the center, and the probability distribution. All of these parameters can be expressed as functions of time or of economic variables. For example, they can be modeled as functions of the growth rate of output (as in the Kaldor-Verdoorn Law), of the growth of the capital stock per worker (as in Kaldor's technical progress function), or of the accumulation of "human capital", if such a variable is introduced into the model.
2. In addition to the traditional sources of endogenous of technical change listed above, the model itself suggests new developments. For

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44. G. Duménil, D. Lévy, "Stylized Facts about Technical Progress since the Civil War: A Vintage Model", *Structural Change and Economic Dynamics*, 5 (1994), p. 1-23.

example, the entire innovation set, rather than just the profitable innovation set, can be linked to distribution. One can, for example, assume that R&D is oriented in specific directions by prevailing prices. Firms search along lines that are more likely to produce large gains. This could be dealt with in a model in which the innovation set is oriented in a direction perpendicular to the selection frontier, and then constantly redirected depending on the prevailing distribution of income.

Only empirical analysis can determine the relevance of such extension of the model.

5 - Classical-Marxian, evolutionary, and neoclassical perspectives

In what sense can the framework of section 1 be called *classical-Marxian*, when considered in isolation independently of the analysis of historical tendencies or disequilibrium microeconomics (for example, the allocation of capital as a function of comparative profit rates)? In a very simple and limited sense, the answer is straightforward: techniques of production are selected if they provide larger profit rates at prevailing prices[45]. The specificity of the neoclassical framework lies in the next component of the analysis: the production function. Neoclassical models assume that the set of techniques available, *i.e.*, the innovation set, can be described by a production function, and that firms maximize their profits along such functions.

Diagram 11 compares technical change in one period in our model, and using a production function. It clearly illustrates the limitation of technical change (its local features) in our model, in sharp contrast with the production function. The plane is (A, L) as in panel (a) of diagram 1. Consider first panel (a) of diagram 11. The dot represents the technique actually used in the current period. The line describes the set of technical combinations available for the next period with a Cobb-Douglas production function (with a shift factor). Depending on the variation of the real wage (such that $0 < w < \infty$),

45. Obviously, we abstract here from other features of the neoclassical framework which cannot be accepted (for example, innovation is not local).

technical change can be very large. Panel (b) illustrates the possibilities available in our approach, assuming a broad innovation set ($R = 0.4$). The doted line (········) is the image in (A, L), of a circular innovation set in (a, l). The tiny curve close to the dot depicts the positions of the centers of gravity (G') of the profitable innovation set for all possible values of the real wage. (Panel (c) simply enlarges the picture in panel (b).) Even if the real wage rate varies tremendously, the extent of technical change in one period is quite limited. In this framework, the effect of a decrease of wages on employment remains weak in the short run, in sharp constrast with the neoclassical model.

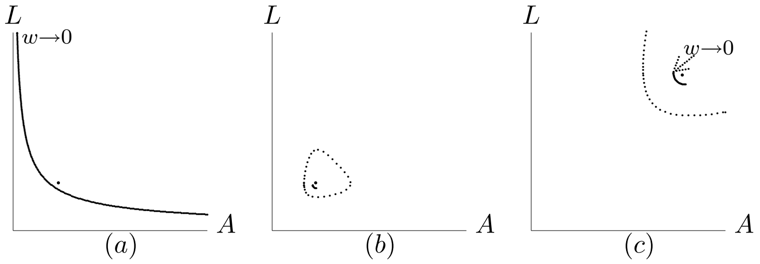


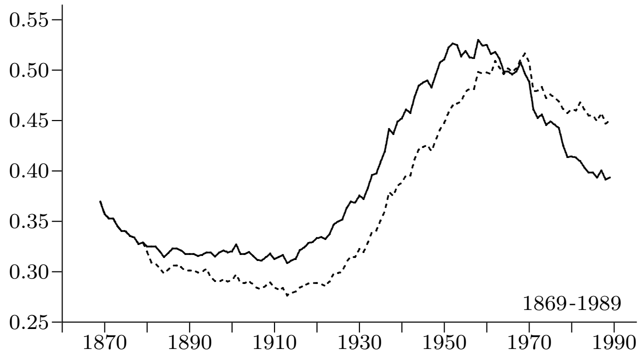
Diagram 11

The neoclassical framework incorporates the idea that production function evolves over time as a result of technical progress, allowing a number of parameters of the function to vary. This variation can be exogenous in the simplest models, or endogenous, as within *endogenous growth models*.

Although path-dependence can be incorporated in a neoclassical model within an endogenous-growth framework, it is typically excluded from the analysis. Conversely, it is easy to illustrate the path-dependence which prevails in our model by running simulations. Consider, for example, the investigation whose results are displayed in figure 1. We reran a similar simulation, conserving the actual values of the labor cost in 1869 and 1989, but assuming that the cost of labor grew at a constant rate throughout the period, *i.e.*, a pattern of evolution similar to that actuallyi observed. As shown in figure 4, the technology obtained toward the end of the period is significantly different. Not just the current value of labor cost but its entire trajectory matters.

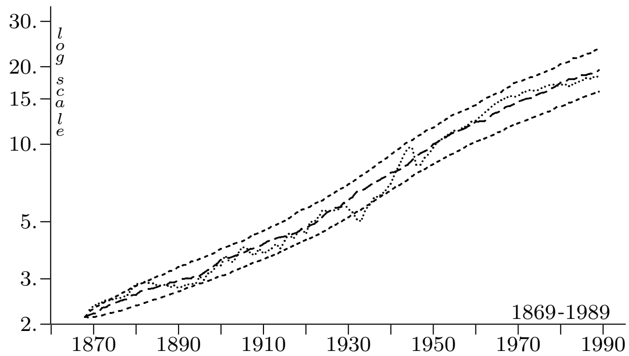
The modeling of technical change in the present paper is closer to evolutionary models. This explains why we refer to the model

Figure 4 Path-dependence: two simulations of the productivity of capital in the US for the same labor costs in the first and last periods, but two different patterns of evolution in between



Simulation 1: (——)
Simulation 2: (-----)

Figure 5 The impact of random variables: a set of 1000 runs for labor productivity in the US



Actual series: (·········)
Model: (— —)
Lower and upper bounds: (-----)

as a *classical-Marxian evolutionary model of technical change*. The framework of analysis used by Nelson and Winter is similar to our approach in several important respects.[46] Innovation is random and local. Techniques are selected depending on their profitability. The model also allows for wages to affect the choice of technology, without resorting to the neoclassical production function. There are also a number of differences. Nelson and Winter refer to *satisficing*: reducing the profit rate below a certain minimum triggers the adoption of new techniques. They also distinguish between innovation and imitation, in a manner which is significantly different from what we call *catching-up*.

The general disequilibrium model of section 4.1 is also "evolutionary" in several respects. Rationality is bounded (behaviors are *sensible* but distinct from neoclassical optimization): agents react to disequilibrium. Heterogeneity is crucial in the model. Several producers of the same good are considered, and they use different techniques. Technology and behaviors evolve only gradually.

There is no denying the fact that the classical notion of *economic law* is *a priori* alien to the evolutionary train of thought, or even contrary to one of its fundamental tenets. Between an excessively deterministic approach and total contingency, it is very difficult to find a satisfactory compromise. This problem is well known to Marxist economists. The *law* of the tendency of the profit rate to fall, and its host of countertendencies is probably the most famous example of this conflict.

The simulation presented in figure 1 provides an interesting illustration of this problem. Since innovation is a random process in this model, one may wonder to what extent the reconstruction of the series depends on the exact sequence of innovations randomly determined (within the innovation set). We reran our model 1000 times, for the same conditions of innovation and the same series of labor costs. Figure 5 presents the results of these simulations for labor productivity. The dotted lines mark the upper and lower bounds of a band within which lies 95% of the possible outcomes. As would be expected, the distance between these two lines increases with time. An interval of $\pm 20\%$ obtains in the last year. It is clear that the exact sequence of innovation impacts on the profile of the series, but the

46. R.R. Nelson, S.G. Winter, "Factor Prices Changes and Factor Substitution in an Evolutionary Model", *Bell Journal of Economics*, 6 (1975), p. 466-486; *An Evolutionary Theory of Economic Change*, Cambridge: Harvard University Press (1982).

same basic evolution is nevertheless observed. This is a form of what could be called *mild determination*. This is how we should always look at historical tendencies.