

# Testing for hysteresis : unemployment persistence and wage adjustment

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**Abstract**

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**Testing for hysteresis : unemployment persistence and wage adjustment**

*Abstract*

This paper proposes a new testing strategy for unemployment hysteresis as the joint restriction of a unit-root in the unemployment rate and no-effect of the level of unemployment in the Phillips wage equation. The relevant test statistics are derived when this joint restriction is imposed during estimation and when a sequential two steps testing strategy is adopted. The empirical application leads to rejection of the null hypothesis of wage hysteresis for most of our sample of OECD countries. We get an interesting contrast between the "core European countries" – rejecting hysteresis, but not unemployment non-stationarity – and the scandinavian ones where unemployment appears to be stabilized despite a lack of the wage correction.

**Keywords :** hysteresis, Phillips effect, unit root, covariates, Wald test.

**Test d’hystérèse : persistance du chômage et ajustement des salaires**

*Résumé*

Le document introduit une nouvelle stratégie de tests de l’hypothèse d’hystérèse du chômage comme la restriction jointe d’une racine unitaire dans le processus du taux de chômage et de l’absence d’un effet "Phillips" dans l’équation d’ajustement des salaires. Les statistiques de test sont dérivées à la fois pour l’hypothèse jointe et pour une stratégie séquentielle de test en deux étapes. On discute aussi la distribution non standard des t-statistiques calculées équation par équation. Une application empirique permet de rejeter le modèle salarial d’hystérèse pour une majorité des 17 pays de l’OCDE que l’on étudie. Il apparaît un contraste intéressant entre les pays européens du "coeur" de la CEE et les pays scandinaves.

**Mots-clés :** Hystérèse, effet Phillips, racine unitaire, covariable, test de Wald.

**JEL classification :** C12, C22

# 1 Introduction

During the eighties, unemployment hysteresis has resulted from the almost general failure of the unemployment rates to return back to the low levels experienced in the pre-oil-shock era. One lesson from the nineties is the persistence of high unemployment rates in some OECD countries, especially the continental European ones. Assessing the hysteresis hypothesis still remains one important issue. For instance in order to evaluate what will be the ultimate real cost of disinflation and European nominal convergence.

In the standard linear representation,<sup>1</sup> hysteresis – or sometime, full hysteresis – implies a unit root in unemployment rate dynamics. This is associated with a lack of adjustment through the wage rate which takes place when the level of unemployment rate disappears from the Phillips curve. Since Blanchard and Summers (1986, 1987), a large strand of literature has been devoted to identifying and assessing the alternative channels of hysteresis. However, while testing for hysteresis, the respective implications of the assumptions for unemployment rate dynamics and for the Phillips curve are always considered separately.

Assessing hysteresis through the univariate unemployment rate dynamics lies on testing for an unit root in its autoregressive representation. Generally, unit root tests, when applied to post WWII series, fail to reject the null hypothesis, with possibly some doubts for the US (see Mitchell 1993,

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<sup>1</sup>See however Amable and alii (1995) for nonlinear alternatives.

Roed 1996). However, the lack of power of these tests questions these results. However, alternative empirical strategies, as taking the stationarity as the null hypothesis (Leslie, Pu, Wharton 1995, Roed 1996) have not so far resulted in a more clear cut evidence.<sup>2</sup> Jaeger and Parkinson (1994) let hysteresis work through an influence of past cyclical unemployment on the current natural rate. Their empirical findings only support the hypothesis for three of the OECD major countries, but not for the US. Finally, structural changes -especially a shift in the mean- may provide a better alternative to unit root (Bianchi and Zoega, 1998). However, the paper of Bianchi and Zoega essentially underline the lack of power of univariate unit root tests.

An alternative strategy evaluates the robustness of a negative and correcting effect of the level of unemployment rate in a wage-Phillips curve. Typically, Coe (1988) finds that using the long term unemployment rate or various moving average of the actual rate as a proxy for the natural rate offsets the level effect of this rate in aggregate wage equations.

Recent studies consider the joint dynamics of unemployment and wage in a multivariate framework. They generally accept the *a priori* characterization of the unemployment rate as a non stationary variable to be embedded in a multivariate setting. They adopt a cointegration-VECM approach in order to focus on the relative contributions of alternative disturbances on the unemployment persistence (Dolado and Jimeno 1997, Jacobson, Vredin, Warne 1997, Andersen and Hylleberg, 1998). This focus leaves unexplored

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<sup>2</sup>In order to increase power, Song and Wu (1997), (1998) used panel data tests. Nevertheless, these tests impose strong restrictions on the joint dynamics of unemployment rates and they largely ignore national features.

an intermediate strategy, embedding the unemployment rate dynamics in a multivariate setting through testing jointly for the implication of hysteresis for unemployment dynamics and for wage equations<sup>3</sup>. The purpose of this paper is to propose and implement such a strategy, exploiting cross-equation restrictions and cointegration information. It proposes various statistics which nest the usual univariate approach, *i.e.* the standard unit root tests and the single auxiliary regression based tests. Beyond the statistical inference, our approach allows to locate potential sources of hysteresis, through the unemployment dynamics and a negative correcting effect in a wage equation.

Section 2 presents the restrictions of interest from a simple benchmark model and introduces the associated statistics. Section 3 presents comparative results based on various OECD countries. The last section concludes.

## 2 Hysteresis as a joint hypothesis

Rather than testing separately for the order of integration in the unemployment rate and for a negative “Phillips” effect in the wage equation, we choose to test for hysteresis as a joint restriction of a zero coefficient for the level of (lagged) unemployment rate in both an autoregressive and a wage equation. To motivate this approach, we will refer to the simple, but very influential, model of Blanchard and Summers (1986, 1987).

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<sup>3</sup>We will depart here from the tradition initiated by Sargan (1964), which relates the level of wages (rather than their changes) to the unemployment rate. See Layard and alii (1991) as a major reference for this approach, and Manning (1993) for providing consistent microfoundations for the Phillips-like specification, we retain here.

## 2.1 A simple framework : hysteresis and wage dynamics

Assuming monopolistic competition and constant return of labor, Blanchard and Summers get the following log-linear labor demand of a representative firm:

$$n_i = m - w - a(w_i - w) \quad (1)$$

where  $m$  stands for aggregate money supply.  $w_i$  and  $w$  denote the individual and the average nominal wage rate, respectively. Wage are set by monopoly unions before the observation of money supply. The union maximizes the wage rate with respect to an employment constraint. When controlled by insiders, the union aims at maintaining their employment in expectation. With straightforward notations, one gets

$$En_i = n_{i-1} \quad (2)$$

Alternatively, if unions take care about outsiders, one gets:

$$En_i = n_{i-1} + b(l - En) \quad b > 0 \quad (3)$$

where  $l - En$  is the expected unemployment rate, for a given labor supply  $l$ . It is worth noting that (3) nests the previous one (2), when  $b = 0$ . Wage maximization under (3) yields the wage setting equation at symmetric equilibrium:

$$w = Em + \frac{b}{1+b}l - \frac{1}{1+b}n_{-1}$$

and the employment equation

$$n = m - Em + \frac{1}{1+b}n_{-1} + \frac{b}{1+b}l$$

Denoting by  $u \equiv l - n$ , the unemployment rate, we get

$$\Delta u = Em - m - \frac{b}{1+b}u_{-1} \quad (4)$$

and the Phillips – wage – equation writes as

$$\Delta w = Em - m_{-1} - \frac{b}{1+b}u_{-1} \quad (5)$$

In this simple model, the money innovation ( $\varepsilon = m - Em$ ) is the only source of disturbance. Equations (4)–(5) make clear that full hysteresis occurs when unions do not care about average employment, as the level unemployment rate  $u_{-1}$  drops out from both the unemployment (4) and the wage (5) equations. Indeed, it appears a cross-equations restriction through the autoregressive parameter

$$\begin{aligned} \Delta u &= (\rho - 1)u_{-1} - \varepsilon \\ \Delta w &= (\rho - 1)u_{-1} + \Delta m - \varepsilon \end{aligned}$$

where  $\rho = (1 + b)^{-1}$ . Assuming a pure insider effect, we have  $b = 0$  and thus  $\rho = 1$ . Conversely, if unions take care about outsiders,  $b > 0$  and thus  $\rho < 1$ . In this case, the unemployment rate follows a stationary process and a negative correcting effect appears in the wage equation.

Though stated by Blanchard and Summers (1986, 1987) (see also Dolado and Jimenez 1996), this cross-equations restriction has not been used yet as a way of testing for hysteresis. Within the simple framework, the coefficient affecting  $u_{-1}$  in both equations is the same under both the null and the alternative hypotheses. Hysteresis thus implies only one cross-equations

restriction. This restriction can be tested by a proper generalization of the ADF statistics called the FADF (see Fève and Hénin, 1998). However, this specification is not robust with respect to small departures from the basic model, such that non constant returns of labor.. We will thus adopt a more general framework, relaxing the assumption that the coefficient of  $u_{-1}$  in (4) and (5) is the same under both the null and the alternative hypotheses.

## 2.2 The econometric model and test statistics

Allowing for a non unitary labor demand elasticity  $\lambda$ , the system (4)–(5) can be restated as

$$\Delta u_t = -\frac{\lambda b}{1 + \lambda b} u_{t-1} + \varepsilon_{1t} = (\rho_1 - 1) u_{t-1} + \varepsilon_{1t} \quad (6)$$

$$\omega_t = -\frac{b}{1 + \lambda b} u_{t-1} + \varepsilon_{2t} = (\rho_2 - 1) u_{t-1} + \varepsilon_{2t} \quad (7)$$

where  $\omega_t = \Delta(w_t - p_t)$  is the real wage adjustment. Assuming full *ex ante* indexation on expected price,  $\omega_t$  would differ from  $\Delta(w_t - p_t)$  only by the price surprise. The disturbances  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  are composite of money – or demand – and of productivity – or real – shocks. The coefficients  $\rho_1 = (1 + \lambda b)^{-1}$  and  $\rho_2 = [1 + b(\lambda - 1)](1 + \lambda b)^{-1}$  differ as long as  $\lambda \neq 1$ . From the cross equations restriction, equations (6) and (7) can be written as:

$$\Delta u_t = \lambda(\rho - 1) u_{t-1} + \varepsilon_{1t}$$

$$\omega_t = (\rho - 1) u_{t-1} + \varepsilon_{2t}$$

where  $\rho = \rho_1 = (1 + \lambda b)^{-1}$ . It follows that when  $b = 0$ ,  $\rho_1 = \rho_2 = 1$  whatever the value of  $\lambda$ . In this “pure insider” case, the unemployment rate vanishes in



equations (6) and (7). For  $\lambda = 0$ , labor demand is inelastic with respect to the wage rate. Then, the unemployment process displays a unit root ( $\rho_1 = 1$ ) but there still exists a “Phillips” effect in the wage equation ( $\rho_2 - 1 < 0$ ). Hence, except for the previous restriction  $b = 0$  to be tested, the parameters  $\rho_1$  and  $\rho_2$  are not necessarily equal.

We now develop our empirical restrictions. The statistical inference based on equations (6) and (7) can be conducted as follows. Hysteresis resulting from an “insider effect” ( $b = 0$ ) introduces the joint restriction  $\rho_1 = \rho_2 = 1$  in equations (6) and (7). Thus, the hypothesis of interest is  $b = 0$  as in this case both  $\rho_1$  and  $\rho_2$  are equal to one. Under the alternative hypothesis  $b > 0$ ,  $\rho_1 < 1$  and  $\rho_2 < 1$  but they can differ. Thus, we propose to build a joint test of these twin restrictions

$$\begin{cases} H_o : \rho_1 = \rho_2 = 1 \\ H_a : \rho_1 \neq 1, \rho_2 \neq 1 \end{cases}$$

Given a consistent estimate  $V(\hat{\rho}_1, \hat{\rho}_2)$  of the covariance matrix of  $\rho_1$  and  $\rho_2$ , the associated Wald statistics, denoted  $W_{(1)}$ , is given by

$$W_{(1)} = (R_1 \hat{\rho} + r_1)' V(\hat{\rho}_1, \hat{\rho}_2)^{-1} (R_1 \hat{\rho} + r_1) \quad (8)$$

where

$$R_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad r_1 = (-1, -1)' \quad \text{and} \quad \hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)'$$

Alternatively, a sequential strategy can be adopted, testing in a first step

$$\rho_1 = \rho_2$$

$$\begin{cases} H_o : \rho_1 = \rho_2 \\ H_a : \rho_1 \neq \rho_2 \end{cases}$$

using a Wald statistics, denoted  $W_{(2)}$ , defined by substituting in (8)  $R_1$  and  $r_1$  by

$$R_2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \text{ and } r_2 = (0, 0)'$$

In a second step, conditionally on  $\rho_1 = \rho_2$ , we test for the restriction  $\rho_1 = \rho_2 = \rho$

$$\begin{cases} H_o : \rho = 1 \\ H_a : \rho \neq 1 \end{cases}$$

This can be done using the  $t$ -statistics  $(\hat{\rho} - 1) / \sigma_{\hat{\rho}}$ , where  $\sigma_{\hat{\rho}}$  is the standard error of  $\hat{\rho}$  (see Fève and Hénin 1998, Fève, Hénin and Jolivaldt 1998).

In order to provide the limiting distributions of these various statistics, we assume for simplicity that the following conditions hold:

### Assumptions

**A1:** the sequence  $\{\varepsilon_{1t}, \varepsilon_{2t}\}$  is *iid* and normally distributed

**A2:**  $E(\varepsilon_{1t} / \underline{u}_{t-1}) = 0$  and  $E(\varepsilon_{2t} / \underline{u}_{t-1}) = 0$  where  $\underline{u}_{t-1} = \{u_{t-1}, u_{t-2}, \dots, u_0\}$

**A3:**  $\sup_t E(\varepsilon_{1t}^4) < \infty$  and  $\sup_t E(\varepsilon_{2t}^4) < \infty$

**A4:**  $u_0 = 0$

Assumption A1 is introduced in order to simplify the asymptotic distribution of the statistics. It can easily be relaxed in order to provide a more operational framework. Assumption A2 imposes that the error terms are orthogonal to the past values of the regressor. Assumption A3 insures integrability conditions and A4 fixes the initial condition to zero. We do not exclude contemporaneous correlation between the two error terms. The covariance matrix is given by:

$$\Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \equiv \sigma_2^2 \begin{pmatrix} \eta^2 & \tau\eta \\ \tau\eta & 1 \end{pmatrix}$$

where  $\tau = \sigma_{12} (\sigma_1 \sigma_2)^{-1} < 1$  is the correlation parameter between the two disturbances and  $\eta^2 = \sigma_1^2 \sigma_2^{-2}$  is the variance ratio. The asymptotic theory for the Wald statistics are derived using the "local-to-unity" asymptotics (see Phillips, 1987). Local departure from the null  $\rho_1 = \rho_2 = 1$  can be expressed as  $\rho_1 = 1 - c_1 T^{-1}$ ,  $\rho_2 = 1 - c_2 T^{-1}$ . These two local departures are used in order to take into account for the fact that the two roots are necessarily equals under the alternative hypothesis. The null hypothesis holds when  $c_1 = c_2 = 0$  and "locally" as  $T \rightarrow \infty$ . Using the diffusion process representation, we define the functional  $B_1^{c_1}(r)$  generated by the stochastic differential equation  $dB_1^{c_1}(r) = -c_1 B_1^{c_1}(r) + dB_1(r)$ . We can then derive the asymptotic distribution of the Wald statistics under the local alternative and under the null hypothesis.

**Proposition 1** *Under the local alternative  $c_1 \neq 0$ ,  $c_2 \neq 0$ , as  $T \rightarrow \infty$*

$$W_{(1)}^{c_1 c_2} \Rightarrow X_3^{c_1} (\xi_1 + \{X_1^{c_1}\}^2 + \{X_2^{c_1}\}^2 + \xi_2 X_1^{c_1} + \xi_3 X_2^{c_1})$$

$$W_{(2)}^{c_1 c_2} \Rightarrow \varkappa_1 X_3^{c_1} (\varkappa_2 X_1^{c_1} + \varkappa_3 X_2^{c_1} + c_2 - c_1)^2$$

and under the null  $c_1 = c_2 = 0$ , as  $T \rightarrow \infty$

$$W_{(1)} \Rightarrow X_3 (X_1^2 + X_2^2)$$

$$W_{(2)} \Rightarrow \varkappa_1 X_3 (\varkappa_2 X_1 + \varkappa_3 X_2)^2$$

where  $X_1^{c_1} = \int_0^1 B_1^{c_1}(r) dB_1 / \int_0^1 \{B_1(r)\}^2$ ,  $X_2^{c_1} = \int_0^1 B_1^{c_1}(r) dB_2 / \int_0^1 \{B_1^{c_1}(r)\}^2$ ,  $X_3^{c_1} = \left( \int_0^1 \{B_1^{c_1}(r)\}^2 \right)$ . The parameters of nuisance are given by:  $\xi_1 = (c_1^2 + \eta^2 c_2^2 - 2\eta\tau c_1 c_2)(1 - \tau^2)^{-1/2}$ ,  $\xi_2 = -2c_1$ ,  $\xi_3 = -2(c_2\eta - c_1\tau)(1 - \tau^2)^{-1/2}$ ,  $\varkappa_1 = (1 + \eta^2 - 2\eta\tau)\eta^{-2}(1 - \tau^2)^{-1}$ ,  $\varkappa_2 = (\eta - \tau)$  and  $\varkappa_3 = (1 - \tau^2)^{1/2}$ .

For given local alternatives  $c_1 \neq 0, c_2 \neq 0$ , the two Wald statistics depend on the correlation parameter  $\tau$  and variance ratio  $\eta$  through the nuisance parameters  $\xi_1, \xi_2, \xi_3, \varkappa_1, \varkappa_2$  and  $\varkappa_3$ . However, under the null, the Wald statistics  $W_{(1)}$  for the joint test  $\rho_1 = \rho_2 = 1$  is free of nuisance parameters. In this case, the asymptotic distribution of  $W_{(1)}$  reduces to:

$$W_{(1)} \Rightarrow \frac{\int_0^1 B_1(r)dB_1}{\int_0^1 B_1^2(r)} + \frac{\int_0^1 B_1(r)dB_2}{\int_0^1 B_1^2(r)}$$

*i.e.* the sum of the square for the *t-ratio* statistic in the unemployment equation and in the wage equation. Conversely, the Wald statistics  $W_{(2)}$  for  $\rho_2 = \rho_1 = \rho$  is still affected by nuisance parameters under the null and we cannot provide a general interpretation of the statistics. One exception concerns the particular restriction  $\eta = \tau < 1$  where the asymptotic distribution of  $W_{(2)}$  is a chi-squared type

$$\begin{aligned} W_{(2)} &\Rightarrow \frac{1 - \eta^2}{\eta^2} \frac{\left(\int_0^1 B_1(r)dB_2\right)^2}{\int_0^1 B_1^2(r)} \\ &\Rightarrow (1 - \eta^2)\eta^{-2}\chi^2(1) \end{aligned}$$

Other simplifications of the asymptotic distributions exist, but they always correspond to specific restrictions on the parameters of the covariance matrix of the residuals.

### 2.3 Empirical implementation and single equation related tests

The testing strategy and the associated asymptotic distributions of the statistics have been obtained assuming that  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  are *iid*. A practical

implementation of this strategy imposes to relax this assumption. We thus consider the following “augmented” version of the model.

$$\Delta u_t = a_1 + (\rho_1 - 1)u_{t-1} + a_{uu}(L)\Delta u_t + a_{uw}(L)\omega_t + e_{1t} \quad (9)$$

$$\omega_t = a_2 + (\rho_2 - 1)u_{t-1} + a_{wu}(L)\Delta u_t + a_{ww}(L)\omega_t + e_{2t} \quad (10)$$

where the  $a_{ii}(L)$  are lag polynomials in  $\Delta u_t$  and  $w_t$  with the restriction  $a_{ii}(0) = 0$ . The order of these polynomials can be selected according to information and autocorrelation criteria. Empirical estimates  $\hat{\rho}_1$ ,  $\hat{\rho}_2$  and  $\hat{\Omega}$  allows to compute the two Wald statistics  $W_{(1)}$  and  $W_{(2)}$ . Furthermore, single equation tests may be performed on (10) and (11). Hysteresis hypothesis imposes the restriction  $\rho_1 = 1$  in (9) and it can be tested using the t-statistics  $\hat{t}_{\rho_1} = (\hat{\rho}_1 - 1)/\hat{\sigma}_{\rho_1}$ , where  $\hat{\sigma}_{\rho_1}$  is the standard error for  $\hat{\rho}_1$ . This statistics is distributed according to Hansen’s (1995) CADF (Covariate Augmented Dickey Fuller) distribution, *i.e.* a mixture of a normal and an ADF distribution, depending upon a nuisance parameter.<sup>4</sup> As the restriction on wage adjustment, hysteresis hypothesis requires  $\rho_2 = 1$  in (7) and thus in (10). It can be tested again using the t-statistics  $\hat{t}_{\rho_2} = (\hat{\rho}_2 - 1)/\hat{\sigma}_{\rho_2}$ , where  $\hat{\sigma}_{\rho_2}$  is the standard error for  $\hat{\rho}_2$ . This is a standard approach to test for a “Phillips effect” in a wage equation. However, this framework makes clear that assuming a standard distribution would lead to an invalid inference under the null hypothesis of hysteresis. The problem comes from equation (7) or (10) being an unbalanced regression under the null. This problem has been already noted by Mankiw and Shapiro (1985) while testing for the permanent

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<sup>4</sup>This nuisance parameter measures the correlation between the ADF residual and the contribution of additional regressors.

income hypothesis<sup>5</sup>. The asymptotic distribution of the t-statistics  $\hat{t}_{(\rho_2-1)}$  is derived by Fève and Hénin (1998) as the AR-OLS (AR for auxiliary regression). It corresponds again to a – possibly non convex – combination of a normal and an ADF distribution and depends on two nuisance parameters<sup>6</sup>, the correlation  $\tau$  and variance ratios  $\eta$ . As critical values of the AR-OLS generally exceed the one of the student distribution, the common practice of inferring about Phillips effect on wage equation using student test is biased in favor of the hypothesis of a Phillips curve.

### 3 Empirical results

We now apply this strategy to OECD countries. Results will be presented in three steps : the first one, for usual ADF and stationarity tests, the second one for tests based on unconstrained bivariate model and the third one for tests of hysteresis based on constrained bivariate model.

#### 3.1 Basic data and stationarity tests

Quarterly data are taken from the OECD “Business sector data base” and cover the period 1970-1 to 1996-4. The unemployment series are defined consistently with the ILO definition. The labor compensation per worker (not corrected for hours) was taken for nominal wages. Prices are measured as CPI. The theoretical properties of our test statistics are derived only for a bivariate system, precluding any covariate other than changes in wage. We thus

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<sup>5</sup>See Elliott and Stock (1994) for a study following an approach close to the present contribution.

<sup>6</sup>See Fève and Hénin (1998).

impose a very parsimonious specification of the wage equation. We retain as the real wage change  $\omega_t$  the difference between the growth rate in nominal compensation per worker and the long run component of inflation rate (CPI), this long run component being extracted using the Hodrick–Prescott filter<sup>7</sup>. Although visual examination does not preclude the possibility of a deterministic drift in the unemployment rate for some countries, we *a priori* discard such specifications for the unemployment process, as lacking of economic significance.

Wage equations often include dummy variables, which are more rarely introduced in unemployment equations. Consistently with our system approach we impose the same dummies in the two equations. Those dummies are introduced in a parsimonious way. They are : *i*) three seasonal components to account for residual seasonality in OECD data and *ii*) institutional or political events documented in sources external to our study, either the OECD secretariat (Coe, 1988), either the IMF commentaries to the IFS database.<sup>8</sup> We check that none of our conclusions was affected by the inclusion of these dummies.

Results for unit root (denoted ADF) and stationarity (denoted KPSS) tests are reported in table 1. The number of lags  $k$  reported in this table is selected from the subsequent bivariate models in order to allow for comparison, but it was checked that our conclusion are not affected when the lag

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<sup>7</sup>We retain the usual value of 1600 for the weight with quarterly data.

<sup>8</sup>Exceptional wage push episodes documented by OECD affect Australia (74.3), Switzerland (69.2), France (68.2), UK (74.3, 75.1), Italy (70.3), Japan (73.4, 74.3, USA (70.1). Statistical changes documented by IMF affect Switzerland (93.4), Denmark (88.1, 88.2), Spain (81.2). We also introduce a dummy for German reunification (91.1).

order is selected from equation (9).

**Table 1 - Unemployment unit root and stationarity test**

Country	k	ADF	KPSS	Dummies
Australia	1	-0.005 (-1.47)	5.19	74.3
Austria	6	0.004 (0.42)	1.61	
Canada	2	-0.029 (-1.88)	2.17	
Denmark	2	-0.29* (-3.10)	3.59	88.1-88.2
Finland	5	-0.004 (1.28)	1.12	
France	1	-0.003 (-0.82)	5.56	
Germany	6	-0.001 (-0.13)	1.64	91.1
Greece	6	-0.003 (-0.15)	1.17	
Ireland	6	-0.007 (-1.62)	1.53	
Italy	4	-0.004 (-0.22)	2.3	70.3-93.2
Japan	5	-0.90 (-0.024)	1.66	73.4-74.3
Norway	3	-0.264 <sup>+</sup> (-2.82)	3.04	
Spain	1	-0.003 (-0.48)	5.51	81.2
Sweden	2	-0.169 (-1.91)	1.60	
Switzerland	6	0.006 (0.87)	1	69.2-93.4
United Kingdom	5	-0.012 (-1.42)	1.31	74.3-75.1
USA	1	-0.045* (-2.94)	1.69	70.1

Critical values at 5% : ADF : -2.91, KPSS : 0.45

The ADF column reports the point estimates of  $\rho_1 - 1$

and the associated t-statistics in parentheses

a superscript + refers to a statistics significant at 10% ( a \*, at 5%)

These results strongly support the presence of a unit root in the unemployment dynamics, which is consistent with the hysteresis hypothesis. For each of the 17 countries under study, the KPSS statistics exceeds the 5% critical value of 0.45. The ADF tests rejects the null of a unit root only for Denmark and the USA at 5% (for the critical value of -2.91) and marginally, for Norway. We obtain point estimates of the autoregressive parameter  $\rho$  that exceed one for Austria and Switzerland. These results are in the line of previous studies.



### 3.2 Results from the bivariate unconstrained system

The core of our inference strategy lies on the estimation of the two equations (9)–(10), together with the covariance matrix of the residuals. For each country, we select the number of lags for  $\Delta u_{t-i}$  and  $\omega_{t-i}$  in each equations, using both information and  $k$ -max criteria. The hysteresis hypothesis implies restrictions on the coefficients  $\rho_1$  and  $\rho_2$  of  $u_{t-1}$  which can be tested separately or jointly, but always involves non standard distribution of the statistics.

The restriction  $\hat{\rho}_1 = 1$  in the unemployment equation (9) can be tested using the t-statistics. This statistics follows Hansen’s CADF distribution and depends on a nuisance parameter. In the first column of table 2, we report: first the point estimate of  $\hat{\rho} - 1$ , secondly the associated t-statistics and third the  $p$ -value for this test statistics according to CADF distribution.<sup>9</sup> Our results do not support the stationnarity of unemployment rate. Using  $\omega_{t-i}$ ,  $i = 1, \dots, k$  as covariates, the CADF statistics does lead to a more frequent rejection of the null than the ADF test. We thus conclude that accounting for past wage changes as covariates in the single equation Hansen’s strategy do not provide gains in power.<sup>10</sup>

The second column in table 2 reports the tests of the “Phillips” effect of  $u_{t-1}$  in the wage equation. We report successively the point estimate  $\hat{\rho}_2 - 1$ , the t-statistics and the  $p$ -values.<sup>11</sup> The AR-OLS test yields to rejection of

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<sup>9</sup>The p-values are computed from 20.000 replications using the estimated nuisance parameter.

<sup>10</sup>Our results do not exclude possible gains in power using CADF test for the unemployment rate using either another covariate, either contemporaneous or leading wage changes.

<sup>11</sup>The p-values are again computed from 20.000 replications of the AR-OLS statistics

the null hypothesis of the absence of a feedback from unemployment on the real wage changes for 13 of the 17 countries, except for the U.K. and the three Scandinavian countries, Finland, Norway and Sweden.

**Table 2 - Single equation and joint tests**

Country	CADF	AR OLS	WALD(1)
Australia	-0.010 (-0.97/0.22)	-0.138** (-2.61/0.01)	7.22 (0.14)
Austria	-0.001 (-0.05/0.94)	-1.60* (-1.88/0.05)	3.55 (0.46)
Canada	-0.019 (-1.43/0.54)	-0.090** (-2.48/0.01)	8.47 <sup>+</sup> (0.09)
Denmark	-0.002 (-1.04/0.71)	-0.033* (-2.33/0.02)	5.62 (0.24)
Finland	-0.005 (-0.51/0.88)	-0.011 (-0.66/0.33)	0.68 (0.91)
France	-0.003 (-0.86/0.78)	-0.079** (-4.78/0.01)	23.30** (0.01)
Germany	-0.001 (-0.15/0.93)	-0.171** (-4.06/0.01)	18.15** (0.01)
Greece	-0.003 (-1.42/0.54)	-0.034** (-2.32/0.01)	8.42 <sup>+</sup> (0.09)
Ireland	-0.004 (-1.75/0.38)	-0.014** (-2.79/0.01)	8.95 <sup>+</sup> (0.07)
Italy	-0.019 (-1.06/0.70)	-0.164** (-3.12/0.01)	12.06* (0.02)
Japan	-0.005 (-0.18/0.93)	-0.901** (-3.57/0.01)	13.37** (0.01)
Norway	-0.018 (-1.06/0.70)	-0.083 (-1.14/0.18)	2.35 (0.65)
Spain	-0.007 (-1.47/0.52)	-0.058** (-4.47/0.01)	20** (0.01)
Sweden	-0.013 (-0.69/0.83)	-0.009 (-0.61/0.34)	1.28 (0.82)
Switzerland	0.002 (0.51/0.98)	-0.041** (-2.75/0.01)	8.43 <sup>+</sup> (0.09)
United Kingdom	-0.006 (-1.01/0.72)	-0.034 (-1.10/0.19)	2.32 (0.65)
USA	-0.037 (-2.40/0.13)	-0.053 (-2.33/0.02)	14.04** (0.01)

$k$  lag numbers and dummies as in table 1

a superscript + refers to a statistics significant at 10% (\* at 5%) (\*\* at 1%)

As far as hysteresis is concerned, we get two opposite results in the majority of the cases: the unemployment rate appears to be non stationary, although the equilibrating Phillips effect is operative. A more specific test of the model presented in section 2 may be performed using the Wald statistics  $W_{(1)}$  of the joint restriction  $(\rho_1 = 1, \rho_2 = 1)$  against the alternative  $(\rho_1 \neq 1, \rho_2 \neq 1)$ . In the third column of table 2, we report the value of  $W_{(1)}$  and its  $p$ -value, computed from 20.000 replications, as the percentage of the simulated using the estimated nuisance parameters.

simulated values exceeding the sample value. Hysteresis as a joint hypothesis is now rejected for 10 countries (over 17) at 10%, for 6 countries at 5% for 5 countries at 1%.

The cases of strong rejection include not only the US, where we get univariate evidence for rejection, but also Japan and four representatives of the "European model" of unemployment persistence: Germany, France, Spain and Italy. Our results clearly oppose this model to the case of Scandinavian countries, Denmark, Finland, Sweden and Norway, where the hypothesis is not rejected although the unemployment rate was found stationary according to univariate ADF tests<sup>12</sup>. The lack of a Phillips wage correcting effect in Norway and Sweden (or in Denmark) is clearly responsible for this results. The joint test  $W_{(1)}$  thus does not appear as a way of improving the power of the ADF statistics, but rather as another view of the hysteresis hypothesis, as associated to the failure of adjustment through wages. Comparative evidences on hysteresis must be further obtained from the two-step sequential strategy.

### 3.3 Evidences from the constrained model

The twin restrictions  $\rho_1 = \rho_2 = 1$  can be tested sequentially. First, the restriction  $\rho_1 = \rho_2$  can be tested using the Wald statistics  $W_{(2)}$ . Second, we test for the hypothesis  $\rho = 1$ , conditionally on  $\rho = \rho_1 = \rho_2$ , using the FADF  $t$ -statistics.

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<sup>12</sup>In Finland while the null of a unit root in unemployment rate was not rejected by the ADF, unemployment persistence is lower than in the core European countries, although we find no trace of a Phillips effect.

In column one of table 3, we report the Wald statistics  $W_{(2)}$  and its  $p$ -value obtained through simulation. The hypothesis  $\rho_1 = \rho_2$ , associated with an unitary labor demand elasticity ( $\lambda = 1$ ), is strongly rejected for the core European countries (Germany, France, Spain, Italy) as well as for Japan or Switzerland. At the opposite, neither the US, nor the "Scandinavian" countries (Finland, Norway, Sweden) reject this restriction.

**Table 3 - Sequential approach**

<b>Country</b>	<b>WALD(2)</b>	<b>FADF</b>
Australia	5.94* (0.02)	-0.014 (-1.24/0.67)
Austria	3.52 (0.47)	-0.004 (-0.35/0.91)
Canada	3.30 <sup>+</sup> (0.08)	-0.039* (-3.26/0.02)
Denmark	4.84* (0.04)	-0.003 (-1.06/0.72)
Finland	0.12 (0.73)	-0.006 (-0.75/0.77)
France	20.57** (0.01)	-0.009 (-1.96/0.31)
Germany	18.50** (0.01)	-0.015 (-2.06/0.26)
Greece	4.33* (0.04)	-0.004 (-1.87/0.35)
Ireland	3.90*(0.05)	-0.006 <sup>+</sup> (-2.74/0.07)
Italy	7.55** (0.01)	-0.024 (-2.05/0.26)
Japan	12.01** (0.01)	-0.60 (-1.25/0.64)
Norway	0.75 (0.40)	-0.032 (-1.26/0.64)
Spain	16.47** (0.01)	-0.010 (-1.74/0.27)
Sweden	0.04 (0.85)	-0.001 (-0.01/0.95)
Switzerland	7.92** (0.01)	-0.002 (-0.57/0.86)
United Kingdom	0.77 (0.40)	-0.007 (-1.24/0.67)
USA	0.29 (0.60)	-0.041** (-3.35/0.01)

$k$  lag numbers and dummies as in table 1

a superscript + refers to a statistics significant at 10% (\* at 5%), (\*\* at 1%)

According to the sequential strategy, when the restriction of a common coefficient  $\rho$  is not rejected, the constrained model can be considered as a relevant representation of the data, thus allowing to test  $\rho = 1$  using the FADF  $t$ -statistics. Results reported on last column of table 3 are more supportive of the null hypothesis. The FADF column reports successively

the point estimates of  $\rho - 1$ , the associated t-statistics and the p-value. The FADF consistently reject the null hypothesis against the alternative of a common autoregressive parameter  $\rho < 1$  only for US. Evidences for Canada is less clear cut, as the support for a common alternative is weaker. Evidence for Ireland is, to some extent, similar. We get an interesting contrast between the core European countries where the self correcting coefficient ( $\rho_1 - 1$ ) in the unemployment equation is decisively weaker than the wage correcting coefficient ( $\rho_2 - 1$ ) and the Scandinavian case where the failure of the wage correction mechanism results in a non significant estimate of the common ( $\hat{\rho} - 1$ ) coefficient in the constrained model. Countries like Australia and Greece appears as intermediate cases for the constrained model.

## 4 Concluding remarks

The use of new test statistics<sup>13</sup> permit to perform various test of unemployment hysteresis, as a mechanism resulting from the lack of wage adjustment. When combining results univariate and multivariate representations, we get a much clearer view on the mechanism of hysteresis.

Univariate unit root tests reject hysteresis only for the US and Denmark. Further tests confirm for the US the alternative of unemployment stabilization through wage adjustment, while they undermine the evidence against hysteresis in Denmark, due to the weakness of the "Phillips" effect.

For the core European countries, Japan and marginally Canada, hysteresis, as related to a wage rigidity mechanism, is rejected. The lack of evidence

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<sup>13</sup>Except ADF, KPSS and Hansen's (1995) CADF.

from ADF against a unit root may be related either to a lack of power of this test for this sample span, either to the occurrence of too large and persistent shocks over the period.

Scandinavian countries exhibit an interesting contrast with this core European case : while labor market institutions apparently insulate wages from unemployment pressure, they are likely to provide alternative adjustment mechanisms that prevent the unemployment rate to follow a non stationary process.<sup>14</sup>

Of course, our results stem from a small and parsimonious model and have to be confronted to the ones from structural and more elaborated models. Moreover, another line of further research should take into account the consequences of possible institutional or structural breaks. However, our results illustrate the possibility and the interest of enriching unit root tests with auxiliary hypotheses in order to gain both more statistical power and further economic understanding.

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<sup>14</sup>See Calmfors and Nymo en (1990) for a discussion of wage formation and employment policies in the Nordic countries.

## Appendix

The covariance matrix can be written as  $\Omega = PP'$ , using a Choleski decomposition, whose elements are given by:  $P_{11} = \sigma_2\eta$ ,  $P_{12} = 0$ ,  $P_{21} = \sigma_2\tau$  and  $P_{22} = \sigma_2(1 - \tau^2)^{1/2}$ . Then the residuals from the regression equations can be written as  $\varepsilon_{1t} = \sigma_2\eta e_{1t}$  and  $\varepsilon_{2t} = \sigma_2\tau e_{1t} + \sigma_2(1 - \tau^2)^{1/2}e_{2t}$ , where  $e_{1t}$  and  $e_{2t}$  are two independent  $iid\mathcal{N}(0,1)$ . Using this decomposition, we have

$$\begin{aligned} \frac{1}{T} \sum u_{t-1}\varepsilon_{1t} &\implies \sigma_2^2\eta^2 \int_0^1 B_1^{c_1} dB_1(r) \\ \frac{1}{T^2} \sum u_{t-1}^2 &\implies \sigma_2^2\eta^2 \int_0^1 \{B_1^{c_1}\}^2 dr \\ \frac{1}{T} \sum u_{t-1}\varepsilon_{2t} &\implies \sigma_2^2\eta\tau \int_0^1 B_1^{c_1} dB_1(r) + \sigma_2^2\eta(1 - \tau^2)^{1/2} \int_0^1 B_1^{c_1} dB_2(r) \end{aligned}$$

where  $B_2$  is independent of  $B_1$  and  $B_1^c$ . The OLS estimates of  $\rho_1$  and  $\rho_2$  are given by  $\hat{\rho}_1 = 1 + \sum u_{t-1}\Delta u_t / \sum u_{t-1}^2$  and  $\hat{\rho}_2 = 1 + \sum u_{t-1}\omega_t / \sum u_{t-1}^2$ . We use the following notations:  $X_1^{c_1} = \int_0^1 B_1^{c_1}(r)dB_1 / \int_0^1 \{B_1(r)\}^2$ ,  $X_2^{c_1} = \int_0^1 B_1^{c_1}(r)dB_2 / \int_0^1 \{B_1^{c_1}(r)\}^2$  and  $X_3^{c_1} = \left(\int_0^1 \{B_1^{c_1}(r)\}^2\right)$ . The two OLS estimates admit the following asymptotic distributions under the local alternatives  $c_1 \neq 0$  and  $c_2 \neq 0$ , as  $T \rightarrow \infty$ :

$$\begin{aligned} T(\hat{\rho}_1 - 1) &\Rightarrow -c_1 + X_1^{c_1} \\ T(\hat{\rho}_2 - 1) &\Rightarrow -c_2 + \tau\eta^{-1}X_1^{c_1} + (1 - \tau^2)^{1/2}\eta^{-1}X_2^{c_1} \end{aligned}$$

The covariance matrix of the two OLS estimates is given by:  $V(\hat{\rho}_1, \hat{\rho}_2) = \hat{\Omega} (\sum u_{t-1}^2)^{-1}$ , where  $\hat{\Omega}$  denotes the estimate of the covariance matrix of the

residuals. The consistency of  $\hat{\rho}_1$  and  $\hat{\rho}_2$  implies  $\hat{\Omega} \rightarrow \Omega$ . The two Wald statistics can be thus constructed using the inverse of  $V(\hat{\rho}_1, \hat{\rho}_2)$ , which admits the following asymptotic distributions:

$$\begin{aligned} (T^2V(\hat{\rho}_1, \hat{\rho}_2))^{-1} &\Rightarrow \sigma_2^2 \eta^2 X_3^{c_1} \Omega^{-1} \\ &\Rightarrow \frac{1}{1 - \tau^2} \begin{pmatrix} 1 & -\eta\tau \\ -\eta\tau & \eta^2 \end{pmatrix} X_3^{c_1} \end{aligned}$$

The two Wald statistics can be formulated using the following restrictions  $R\rho + r = 0$ , where  $R$  is  $2 \times 2$  matrix,  $r$  a  $2 \times 1$  vector and  $\rho = (\rho_1, \rho_2)'$ . The Wald statistics are thus given by:

$$W_{(i)}^{c_1 c_2} = T(R_i \hat{\rho} + r_i)' (T^2V(\hat{\rho}_1, \hat{\rho}_2))^{-1} T(R_i \hat{\rho} + r_i) \quad i = 1, 2$$

and their asymptotic distributions can be deduced from the ones of  $T(\hat{\rho}_1 - 1)$ ,  $T(\hat{\rho}_2 - 1)$  and  $(T^2V(\hat{\rho}_1, \hat{\rho}_2))^{-1}$ .



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