Stability versus Efficiency of the Banking Sector and Economic Growth*

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Abstract

The paper investigates, from the welfare and growth point of view, the existence of a trade-off between the stability and the efficiency of the banking system, studying the costs and benefits of regulatory programs. Welfare is considered in the context of an overlapping generation model with endogenous growth. There is horizontal differentiation and imperfect competition in the banking sector. Macroeconomic shocks affect the return on capital. We specify how deposit insurance may increase the number of deposits, welfare and growth. We characterize the conditions under which excess banking capacity may appear and how its reduction may improve welfare.

JEL classification numbers : 016
keywords: financial stability, growth

Stabilité ou efficacité du système bancaire et croissance.

résumé

Cet papier s’interroge sur l’existence d’un arbitrage du point de vue du bien-être et de la croissance entre la stabilité et l’efficacité concurrentielle du système bancaire, en étudiant les coûts et bénéfices de programmes réglementaires. Le bien-être est considéré dans le contexte d’un modèle à générations imbriquées avec croissance endogène. Il y a différenciation horizontale et concurrence imparfaite dans le secteur bancaire. Des chocs macroéconomiques affectent le rendement du capital. On spécifie comment une assurance sur les dépôts peut augmenter l’épargne, l’investissement, la croissance et le bien-être. On caractérise les conditions sous lesquelles il peut y avoir excès de capacité bancaire et comment cette réduction peut améliorer le bien-être.

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1 Introduction

Over the last 15 years, banking markets have been deregulated in a number of countries around the world with a view to achieving more efficiency. As banking markets were seen as partly characterized by collusive behavior, liberalization has been targeted at fostering innovation and efficiency, by increasing competition and reducing oligopoly rents. An additional objective was to put an end to the so-called “financial repression”, where the level of interest rates to depositors were maintained artificially low by governments. However, the US Savings and Loans debacle in the 1980s, the Scandinavian banking crisis in the early 1990s and structural problems in the Japanese banking system have indicated that deregulation and increased competition may be socially costly if the banking system becomes more fragile.¹

There are two ways to circumvent the problem of instability. First, deregulation may be complemented by a safety net, in the form of deposit insurance. However, leaving incentive problems for banks aside, deposit insurance may imply a diversion of resources from more productive use, which may more than offset its benefits. This is particularly relevant, if one consider that, in the recent past, public expenditure to meet deposit insurance claims and recapitalize banks have been sizeable, amounting to nearly 3% of annual GDP in the US, while even more substantial amount have been spent in Norway, Sweden and Finland, and the ultimate cost to taxpayers is also expected to be large in Japan. Deposit insurance should not be implemented at all costs and it is one of the aims of the paper to measure the overall effect of deposit insurance on welfare and on economic growth.

Second, instability may be limited by a continuous reliance on prudential supervision based on licensing and ownership control, as well as risk management requirements. Such instruments remain useful even when banking markets are deregulated. It is often argued that, in deregulated markets, the risk of instability is only transitory due to the “regime shift” in the regulatory environment which may create incentives for institutions to increase risk-taking, triggering price wars and lending mania. But the deregulation of imperfectly competitive markets may also contribute to a more permanent accumulation of excess capacity, as it is currently estimated to be the case in most European countries.

To counter that evolution, in the countries that liberalized their banking market, the shift away from regulation has never been fully completed. However, the authorities in charge of banking supervision have never been offered with clear guidelines regarding the appropriate scope of deregulation. The second aim of the paper is therefore to investigate under which circumstances it might be necessary, in order to reduce instability and maximise welfare and growth, to avoid that too many firms enter the market, to promote bank mergers or to facilitate the exit of some institutions from the market.

Regarding more specifically the second point, Canadian history offers an illustrative example where an oligopolistic banking system turned out to be more efficient (or less inefficient) than the U.S. system with respect to stability and consumer’s welfare in the long run. During the period 1925-1980, interest rates paid on deposits were higher in Canada than in the US, and interest rates charged on loans were quite similar in the two countries (Bordo, Rockoff and Redish [1994]). No bank failure has been registered in Canada since 1924. By contrast, over 9000 failures of mostly small banks occurred in the US between 1930 and 1933. Although banks also benefited from the absence of

unit-banking regulation and the smaller size of the banking system allowed them to organize implicit deposit insurance, the main factor explaining the stability of the Canadian banking system is, according to Bordo, Rockoff and Redish [1996], that the Canadian federal government favored mergers and banking concentration during the period 1900-1925. Mergers are a substitute to bankruptcy which limits bankruptcy costs during times of financial distress. They also decrease competition, increase margins, but also the expected return on deposits by lowering the probability of bankruptcy for the subsequent periods. They helped banks to achieve their efficiency level as well as regional diversification and therefore to increase depositors’ welfare. Hence, restrictions to entry may have improved welfare and achieved stability even in the absence of deposit insurance, as it was the case in Canada up to 1966 (Carr, Mathewson and Quickley [1995]). One should acknowledge, however, that both the U.S. and the Canadian banking systems where affected by two different inefficient regulations from 1925 to 1980, so that the Canadian experience may have not been so efficient in absolute terms. What is more, the U.S. banking system dramatically changed since 1980 (Berger, Kashyap and Scalise [1995]), and the Canadian banking system experienced severe difficulties at the same time.

The trade-off between competitive efficiency and stability of the banking sector has an impact on the long run growth of an economy. Large (and possibly low frequency) macro-economic shocks may lead to a breakdown of the financial system, which affects the average growth rate over a decade or more. As detailed by Friedman and Schwartz [1965] and Bordo et al. [1996], distrust of depositors and recurrent bank runs over the period 1850-1925 in Canada and during the 1930s in the US may have had a long run negative impact on the efficiency of the collection of savings as households attempted to convert deposit into currency. As financial autarky generally implies a less efficient allocation of savings than intermediated savings, the lack of depositors confidence due to the threat of a failure of the banking system may be detrimental to long run economic growth. Over the period starting from the last bank failure in Canada till the eve of World War II (1925-1938), the average annual growth of GDP per head of Canada was above 0.5% over the growth rate in the United States, where a breakdown of the financial system happened during the Great Depression. Nowadays, in several less developed countries, depositors’ lack of confidence in the financial system is an acute problem which inhibits the collection of savings and may contribute to the persistence of poverty traps (Fry [1995]).

The present paper tackles this general issue on the ground that the overall social cost of banks failures are higher than the costs of failures in other industries. This is due to the existence of negative externalities of bank failures, leading to systemic risk.

The model introduces horizontal differentiation among banks on the deposit market, which differ in terms of location, range of services offered and pattern of relationships.

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2This distrust of banks can be measured by the ratio of cash/deposits which exhibit a negative correlation with growth rates (Friedmann and Schwartz [1965]).

3Authors’ calculations based on data from Maddison [1995], p.206-209.

4Loss rates defined as the ratio of total losses ultimately experienced by depositors of the failing banks in a given year to total deposits during that year were higher than 40% in Canada for 4 out of 65 years between 1880 and 1925 (Bordo et al.[1996]). According to Gendreau and Prince [1986], direct costs of bankruptcy in large US banks during the 1929-1933 period amounted to 6% of liabilities and where higher than the costs of bankruptcy of non-financial firms. Regarding the issue of indirect failure costs, Rajan [1996] gives a measure of the value of relationships: in 1984, client firms of Continental Illinois Bank incurred average abnormal stock returns of -4.2% during the bank’s impending insolvency. See also Berger, Kashyap and Scalise’s [1995] calculations.
with customers. The choice of households between deposits and an alternative “storage technology” allows us to take into account the effect of distrust through changes in the deposit/currency ratio.\footnote{This storage technology could be modelled as currency, with its supply growing exogenously, as in Williamson [1987].} We extend Matutes and Vives [1996] “perfect Bayesian equilibrium” definition to an oligopolistic banking sector in a general equilibrium model with overlapping generations. The deposit contract is efficient but there is imperfect competition between depositors not only on the liability side, but also on the asset side. We study regulatory programs (deposit insurance, reduction of excess capacity) with regard to their efficiency when a large exogenous macro-economic shock is likely to occur, but cannot be diversified away (real world examples are the explosion of speculative bubbles on asset prices).

First of all, we exhibit conditions for a trade-off between the efficiency and the stability of the banking system. Due to imperfect competition in the banking activity, intermediaries can apply a margin on interest rates, which has an adverse effect on saving and investment. But a smaller number of banks implies higher expected profits and a lower risk of default for each bank. This trade-off occurs if an increase in competitive efficiency for deposits and credit triggers a substantial increase in expected instability, as measured by the probability of bank failures. It has an impact on the equilibrium between intermediated saving and investment, hence on welfare and growth.\footnote{Besanko and Thakor [1992] study the effects of barriers to entry in the banking sector in a partial equilibrium model, which does not take into account the effects on economic growth. An innovation of our model is to endogenise the probability of bank failures, whereas this is an exogenous parameter in their model. Chan et alii [1992] also showed that competition reduces charter value and may therefore induce “excessive” risk-taking.} In particular, we show that households’ complete loss of confidence in banks leads to a poverty trap.

Deposit insurance may be socially desirable if it eliminates the costs of banks failures, in the sense that the cost of funding the program does not exceed its benefit, for specified conditions. A similar increase in welfare may obtain from a reduction of excess capacity, the latter being defined as the existence of a too large number of banks which may potentially increase the instability of the banking system.

Section 2 of the article describes the behavior of firms, households and financial intermediaries, including the equilibrium with free entry. Section 3 analyses the possible growth paths. Section 4 considers the impact of deposit insurance on welfare. The effects of reducing excess capacity in the banking system is assessed in section 5. A last section briefly concludes the paper.

\section{The model}

\subsection{Firms}

The technology exhibits constant returns to scale with respect to capital $k_t$ and labor $N$ and the production function has a Cobb-Douglas specification. Capital entirely depreciates in one period. Population $N$ is constant. We have $y_t = u_t \cdot a_t \cdot k_t^\alpha \cdot N^{1-\alpha}$, where $u_t$ represents a macro-economic shock affecting technology, which cannot be diversified. It is identically and independently distributed on $[u_t, \tilde{u}]$ from period to period, with an
expectation equal to unity \((E_{t-1}[u_t] = 1)\). The productivity term \(a\) introduces a positive externality, depending on aggregate private capital \(K_t\) as in Romer [1986], so that \(a_t = A \cdot K_t^{1-\alpha}\).

Aggregate output is denoted \(Y_t\). This simple specification of the technology can be understood as a reduced form of more complex endogenous growth models. Firms are price takers on the final good market. At date \(t-1\), entrepreneurs choose capital and labour for production at date \(t\) by maximizing expected profits, taking into account the expectation of the macroeconomic shock \(u_t\) that will hit the economy at the next period:

\[
(N^*_t, k'_t) \in \text{ArgMax} \quad E_{t-1} \left[ u_t \cdot a_t \cdot k_t^{\alpha} \cdot N_t^{1-\alpha} - w_t \cdot N_t - R_t \cdot k_t \right]
\]  

(1)

Wage earners’ expected income is \(E_{t-1}[w_t]\), and the expected return on capital is \(E_{t-1}[R_t]\). Labor market is perfectly competitive. Ex ante factor demands are functions of marginal productivities:

\[
E_{t-1}[R_t] = a_t \cdot \alpha \cdot k_t^{\alpha-1} \cdot N_t^{1-\alpha} \quad \text{(2)}
\]

\[
E_{t-1}[w_t] = a_t \cdot (1 - \alpha) \cdot k_t^{\alpha} \cdot N_t^{-\alpha} \quad \text{(3)}
\]

Once the shock is realised, wage and the return on capital are determined by realised marginal productivities:

\[
R_t = u_t \cdot a_t \cdot \alpha \cdot k_t^{\alpha-1} \cdot N_t^{1-\alpha} \quad \text{(4)}
\]

\[
w_t = u_t \cdot a_t \cdot (1 - \alpha) \cdot k_t^{\alpha} \cdot N_t^{-\alpha} \quad \text{(5)}
\]

### 2.2 Households’ behavior

A simple model of overlapping generations is considered. The population of each generation is of fixed size and lives for two periods. The welfare of future generations is not taken into account in the agent’s utility function. The population is a continuum of mass \(N\) spread on a circle of length 1 in order to formalize spatial differentiation. In the first period, each agent offers one unit of labor and saves a fraction of her income. The utility function depends on each period’s consumption in a linear fashion, so that households are risk neutral.\(^7\) Households have no direct access to financial markets, and they cannot set up a business by themselves. They decide upon the amount of savings \(S_t\) and its allocation between a riskless asset \((1 - b_t)\) and a risky asset \((b_t)\), taking into account the expected net return of each asset.

\[
(S_t, b_t) \in \text{Argmax} \quad (w_t - S_t) + \frac{E_t[b_t \cdot (R_{t+1}^{IF} - \delta \cdot l) + (1 - b_t) \cdot v] \cdot S_t}{1 + \rho} 
\]  

\(w\) is the real wage, \(\rho\) is the subjective rate of time preference, \(v\) is the riskless asset’s return, \(R_{t+1}^{IF}\) is the random return of a deposit with a financial intermediary. In order to make such a deposit, agents face a ‘transport’ cost which is expressed as a linear function of the distance \(l\) between the financial intermediary and the agent, with a fixed distance coefficient \(\delta\). This hypothesis represents the effects of horizontal differentiation between financial intermediaries. Various interpretations of this effect can be given. There \(^7\)A minimum consumption level constraint can be included.
is an opportunity cost of time spent to go to the bank. More fundamentally, financial intermediaries differentiate themselves by the nature of services offered to depositors, such as the size of their automated teller machines (ATM) networks, the possibilities for consumption credit, the quality of service, etc. Differentiation is taken here as given. The horizontal differentiation representation is similar to that in Salop [1979]; n financial intermediaries are located at a distance $1/n$ of each other on the circle where households are uniformly distributed. Given the utility function specified above, an agent will save all her income if the expected return on savings is larger than the rate of time preference, i.e. if $\max \left[ v, E_t \left( R^{IF}_{t+1} \right) - \delta \cdot l \right] > 1 + \rho_t$ this condition is assumed to hold. The individual propensity to save does not depend on the interest rate ($S_t = w_t$).

### 2.3 Oligopolistic Banking Equilibria without Entry

We proceed in two steps. First, we determine equilibria for a given number of banks. In the following subsection, we allow free entry to pin down the number of banks. The equilibrium sequence follows broadly the one applied by Matutes and Vives [1996, p.189] in their duopoly model with horizontal differentiation, which they also described as the perfect Bayesian equilibria of a game with Bayesian depositors having point prior beliefs. At date $t$, Depositors are endowed with ex ante identical and prior beliefs about the probability to have the principal and interests on deposits actually being paid back by any bank. This identical probability of success for banks is denoted $p_t$. It describes an instantaneous and perfect correlation between banks failures as well as between depositors expectations, which may happen during depositors panics. We retained the assumption of symmetrical beliefs as we intend to stress the confidence in the intermediation sector as a whole. This is not an explicit model of runs, but a model of bank failures.

Households cannot observe ex post (at $t + 1$) banks’ return from lending to firms. This hypothesis of an infinite cost of monitoring allows us to introduce Diamond’s [1984] framework so that deposits are optimal debt contracts. Households know the probability distribution of the ex ante return. A bank $i$ offers an interest rate on deposits $r_{i,t}$ and incurs an endogenous non-pecuniary bankruptcy cost, as in Diamond [1984]. This bankruptcy cost corresponds to the time spent by intermediaries in justifying the low return, the cost of finding a new management for the bank, or it can be associated to the loss of reputation. If the bank’s income cannot repay the debt, bankruptcy is declared. In this case, we suppose that the remaining value of the bank is not paid back to depositors but lost in bankruptcy costs borne by depositors, as in Matutes and Vives [1996]. Households expect to lose their deposits with a probability $1 - p_t$, so that the ex ante expected return on deposits is given by $E_t[R^{IF}_{t+1}] = p_t \cdot r_{i,t}$. Banks are aware of households’ expectations and determine the rate of interest on deposits accordingly. Households then decide to deposit their savings in the nearest bank, if the expected return net of transport costs to $v$ exceeds the return on the storage technology $(p_t \cdot r_{i,t} - \delta \cdot l \geq v)$. Financial intermediaries may collect the savings from all depositors, or from some of them, or none at all. If a sufficient amount of savings has been collected by banks, they lend to firms and pay operating costs.

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$^8$Formally, the optimal contract is such that the bank incurs an endogenous non-pecuniary penalty so that it is indifferent to pay back a constant deposit rate $r_i$ to depositors. If it decides to reimburse at an inferior rate $z$, the bank will incur a penalty $\phi(z) = \max(r_i - z, 0)$.

$^9$An alternative model where banks can influence depositors’ expectations with their choice of the rate of interest on deposits is possible but would lead to complications we wish to avoid here.
costs. The financial intermediary can invest in productive projects, without asymmetric information.\footnote{As Modigliani and Miller’s theorem applies to the credit market, any kind of financial contract contingent on \textit{ex post} state realisation can be chosen, as long as the reservation level of profits of firms or of financial intermediaries is satisfied. For simplicity, we assumed that the contract between banks and firms specifies \textit{ex post} state contingent returns. Nonetheless, an intermediary margin on the expected return is taken \textit{ex ante} by banks when supplying funds to firms.} Macroeconomic risk cannot be diversified away, and is entirely borne by banks, which are the only suppliers of capital. Therefore, $R_{t+1}$ is the random return for one unit of capital invested by any bank. In equilibrium, households’ expectations are rational, so that the probability of success is equal to the probability of positive profits of banks. The interaction between this probability of success and the deposit rate defines the equilibria for a given number of banks.

In the event of success, banks repay depositors in the second period and consume the remaining surplus. In the event of failure of the banking system, depositors (who now belong to the old generation) and banks receive no income. The younger generation also faces risky wages.

Before computing the optimal program for a banker’s decision, we specify the demand for deposit for a given bank $i$. We consider here an equilibrium with an incomplete collection of savings. Banks do not compete directly on the deposit market, a situation we can label as “pure” local monopolies.\footnote{See Appendix 1 for the equilibrium with touching markets.} Therefore, we assume that the equilibrium number of banks ($n_t$) is such that the distance between two banks is always strictly larger than $2l_{i,t}$, where $l_{i,t}$ is the distance between the marginal depositor and the nearest bank $i$ ($2l_{i,t} \leq \frac{1}{n_t}$). Depositors’ expectations are supposed to be such that $p_t \cdot r_{i,t} > v$, so that some households deposit in banks. The marginal lender is indifferent between putting her saving in the bank and storing it. Its distance to the nearest bank is given by:

$$l_{i,t} = \frac{p_t \cdot r_{i,t} - v}{\delta}$$ \hspace{1cm} (7)

Deposits with bank $i$ are then $2l_{i,t}$ times the amount of individual saving. Taking account of the depositors’ density $N$, the amount $d_{i,t}$ deposited with bank $i$ is equal to:

$$d_{i,t} = 2 \cdot N \cdot l_{i,t} \cdot w_t$$ \hspace{1cm} (8)

Intermediated savings increase when the transport cost $\delta$, or when the return on the alternative asset $v$ decreases, or when the expected return on bank deposits $p_t \cdot r_{i,t}$ rises.

The presence of non-pecuniary externalities $\phi$ implies that risk neutral banks maximize expected profits unconditional to success (Diamond [1984]). Fixed cost $C_t$ in the banking activity requires a minimum size for deposits $d_{i,t}$ to operate. Bank $i$ lends to firms all their available funds in the risky projects ($k_{i,t+1} = d_{i,t}$), because their return is higher than the return on the safe asset (the storage technology). Banks have no other mean of finance than deposits. We suppose that competition on the credit and deposit markets is imperfect and determined in a Cournot-Nash equilibrium with $n$ given players (banks). On the credit side, there is no horizontal differentiation but the elasticity of the demand for credit is the inverse of $1 - \alpha$\footnote{Mixing spatial differentiation and interest rate elasticity effects on the credit demand in general equilibrium leads to technical complications beyond to the scope of this paper (See Bensouïd and De Palma [1995, p.170] for a partial equilibrium study). Introducing Cournotian competition on the credit side gives room for the persistence of the incomplete collection of savings by banks with free entry, in a}. On the deposit side, there is horizontal differentiation.
but the elasticity of the demand for deposits is zero. Each bank solves the following problem:

\[
(d_{i,t}, k_{i,t+1}) \in \text{Argmax } E_t[R_{i,t+1}] \cdot k_{i,t+1} - r_{i,t} \cdot d_{i,t} - C_t
\]

subject to

\[
d_{i,t} = 2 \cdot N \cdot w_t \cdot p \cdot \frac{r_{i,t} - (v/p)}{\delta}
\]

\[
E_t[R_{i,t+1}] = \alpha \cdot a \cdot k_{i,t+1}^{\alpha-1} \cdot N^{1-a}
\]

This choice is made under a resource equilibrium constraint, taking into account the deposit demand for bank \( i \) and the aggregate credit demand. One may notice that the probability of bankruptcy will appear in the marginal condition as a factor that raises the relative return of the riskless asset \( v/p_t \).

The best response function of a bank gives the first order condition of the above program. As usually done, we only study symmetric equilibria, where the interest rates are the same for all banks. We therefore omit the index \( i \) in what follows, as it is unnecessary. They are given by:

\[
r_t = \left( 1 + \frac{\alpha-1}{n} \right) E_t[R_{t+1}] + \frac{v}{p_t}
\]

The rate of interest on deposits is an average of the uncertainty-corrected return on the storage technology \( (v/p_t) \) and of the credit interest rate less a mark-up depending on the elasticity of the firms’ demand for funds \( (1/(\alpha - 1)) \) and the number of competing banks because of Cournotian competition in the credit market. A rise in the probability of success \( p_t \), which measures households’ confidence in the banking system, decreases the deposit rate and then increases the imperfect competition margin.

A rational expectation equilibrium imposes that households anticipate the actual probability of success:

\[
p_t = \Pr(R_{t+1} \cdot k_{t+1} - r_t \cdot d_t - C_t \geq 0) = 1 - F\left( r_t + \frac{C_t}{k_{t+1}} \right)
\]

We define \( c_t = \frac{C_t}{k_{t+1}} \) as the ratio of the fixed cost with respect to deposit. There exist two main options for the stream of fixed costs. A first possibility is to keep them constant over time, while the economy is growing. But the most common assumption in endogenous growth models is to have costs indexed on a growing variable. This captures the idea of growing costs from organisation, so that fixed costs do not become negligible in the infinite horizon.\(^{13}\)

manner similar to Williamson [1987]. In the case of perfect competition on the credit market, free entry of banks fills “holes”, i.e. new banks enters in areas where households do not deposit; therefore free entry equilibria are always of the “touching markets” variety.

\(^{13}\)It is important to notice that endogenous growth models with monopolistic competition do not collapse if the fixed intermediation cost is not indexed on a growing factor. If it is the case, the gradual decrease of the ratio of the intermediation cost with respect to production creates a specific dynamic which converges only in the infinite horizon to perfect competition in the intermediation sector, for a maximal growth rate (Gali [1995]). We retained the indexation of the intermediation cost, as it seems to be the case in the real world, where the secular “falling rate of profit” in the banking industry predicted by Gali’s assumption has not been observed in major developed countries. On the contrary, the deposit market remains concentrated in developed countries (except when a specific regulation inhibits concentration as in the U.S.), so that Gali’s result is not asymptotically relevant when it is applied to the banking sector.
We keep the option open until section four, where we assume for simplicity that the 
fixed cost of a given bank is proportional to the share of aggregate output financed by 
that bank \( C_t = f \frac{E_t[R_{t+1}]}{n} \), which is also a rather realistic assumption. The maximisation 
problem solved by banks is therefore similar to the firms’ problem. They are not able 
to take into account the aggregate output externality. As a consequence, they do not 
attempt to affect aggregate output and to modify the fixed cost.

For a given number of banks, one needs to specify the distribution of risk in order to 
solve the system consisting of the two preceding equations. This determines the number 
of “short run equilibrium” values for the probability of success and the interest rate on 
deposits \( (p_t^*, r_t^*) \). The amount of deposits collected by one bank \( d_t^* \) and of credit \( k_t^* \) can then be computed immediately. For a given number of banks and when deposit markets 
are not touching, several cases are possible: multiple equilibria, a unique equilibrium or 
none, as in Matutes and Vives [1996] duopoly model. Banks perceived as “low risk” by 
depositors will have, for a given interest rate, larger markets and hence a lower probability 
of default, which reinforces the initial confidence. Because of the fixed cost, a minimum 
size is required for any banking activity. There always exists an equilibrium without banks, 
which corresponds to a poverty trap for a zero probability of success of banks, where 
rational expectations are also self-fulfilling. Increasing returns in banking reinforce the 
possibility of multiple equilibria. One self-fulfilling mechanism may be characterized by 
the perception of a lower risk in banking by depositors, the increase in the intermediation 
margin and the decrease in the probability of bankruptcy associated with a bigger bank. 
But the standard monopolistic competition “long run equilibrium” with free entry exhibits 
a smaller set of equilibria as presented in the next section.

2.4 Banking Equilibrium with Free Entry

Knowing the equilibrium for a given number of banks \( (p_t^*, r_t^*) \), we now suppose that there 
is free entry in the banking sector. The number of banks, \( n_t^{**} \), is determined by a zero 
profit condition. Ignoring the integer constraint, the zero profit condition for banks allows 
to pin down the number of banks, as:

\[
    n_t^{**} = \frac{(1 - \alpha) \cdot E_t[R_{t+1}]}{2 \cdot c_t + \frac{\alpha}{p_t^*} - E_t[R_{t+1}]} = n_t^* \left( E_t[R_{t+1}], p_t^{**, v, c_t} \right)
\]

(15)

A decrease in marginal productivity for the final good sector and a rise in the inter/ 
mediation cost or in the relative return of the alternative asset diminish the number of 
banks and aggravate imperfect competition. This new equation is to be added to the 
system of the preceding section in order to solve the long run equilibrium with free 
entry \( (p_t^{**}, r_t^{**}, n_t^{**}) \) (the ** superscripts refer to equilibrium values with free entry). For 
\( p \in ]0, 1[ \):

\[
    r_t^{**} = E_t[R_{t+1}] - c_t
\]

(16)

\[
    p_t^{**} = 1 - F \left( E_t[R_{t+1}] \right)
\]

(17)

The zero expected profit condition (where profit is a linear function of the shock) 
implies that the probability of negative profit should be equal to the probability that the 
\textit{ex post} return equals its expectation. When the random variable is symmetrically
distributed, the usual free entry condition implies a rather high probability of default, equal to 1/2. As a consequence, whatever the ex ante expectation on the default probability \( p_t \in [0, 1] \), banks’ behavior and the free entry condition will determine ex post a probability of success \( p_t^\ast \). Apart from the “distrust” equilibrium \( (p_t = 0 \Rightarrow p_t^\ast = 0) \), there exists only one other long-run equilibrium, defined by the above equations. The condition for a partial collection of deposit with respect to the full collection over the whole circle is given by \( 2l_t^\ast n_t^\ast < 1 \) with \( l_t^\ast = \frac{2p_t^\ast - v}{\delta} \).\(^{14}\)

### 3 Three growth regimes

At the macro-economic level, the aggregate growth rate requires to take into account the productive externality, so that \( E_t [R_{t+1}] = \alpha \), as we normalize the population size to unity \( (N = 1) \), households’ density at each point of the circle becomes 1. Under the previous assumptions regarding the non-availability of other sources of funding outside banks and the total depreciation of capital in one period, the capital stock is equal to investment in that period, as well as to intermediated savings. Three regimes are possible. A first equilibrium corresponds to the absence of banks and therefore to the non-intermediation of savings. Investment is null and the growth rate is zero. This defines a poverty trap. A second type of equilibrium has banks collecting deposits as local monopolies. A third equilibrium is characterized by total intermediation of aggregate saving, which corresponds to the intermediation of all aggregate saving and to a maximum growth rate.

It is interesting to assess the relationship between welfare and growth in each of these regimes. There are four sources of inefficiency in this model. First, there is the lack of intergenerational exchange, which appears in overlapping generation models when the utility of finitely lived agents does not include bequest motives. Second, there is the productive externality in the production function which introduces a wedge between the social and private returns to investment. Third, there is the distortion due to imperfect competition in the banking sector, which affects the level of saving and investment. Finally, the expectation coordination problem between households is at the root of the multiple equilibria problem.

- **The poverty trap**

It may come from households’ distrust towards the banking sector. If agents expect a probability of success \( p_t = 0 \), the zero growth equilibrium appears whatever the levels of expected productivity for firms and the levels of intermediation cost. The self-fulfilling prophecy mechanism is the following. Depositors have an anticipation of a zero probability of success. As a consequence, nobody will make deposits to the banks, whatever the interest rate offered. The amount of intermediated saving is null, and no bank can operate. Since intermediation is necessary for investment, there is no growth.

A second possibility of a poverty trap exists, when markets are non touching. The condition is not so much based on expectations, but rather, on technology. When \( l_t^\ast \leq 0 \), the productivity factor \( \alpha \) is not high enough to have agents go to the bank considering the level of intermediation costs and the default risk: \( \alpha < c_t + (v/p_t^\ast) \). No bank is active and growth is zero.

\(^{14}\)The case of full collection of savings is available from the authors upon request.
Local monopolies obtain when \( 0 < 2n_t^{**} l_t^{**} < 1 \) and \( n^{**} > 0 \). This supposes that capital productivity \( A \alpha \) is such that \( v/p_t^{**} + c_t < A \alpha < \min[v/p_t^{**} + 2c_t, H(c_t, v/p_t^{**})] \). The existence of a long term equilibrium where aggregate saving is not entirely intermediated by banks is made possible by imperfect competition in banking activity. Without it, a bank could always enter and take control of the market share left by other banks, until all banks are in competition for the marginal saver (this is what happens when markets are touching). When markets are not touching, the random growth rate (as wages \( w_t = u_t A(1 - \alpha) \) are random) is given by:

\[
G_t = \frac{K_{t+1}}{K_t} = \frac{w_t}{K_t} \cdot 2l_t^{**} \cdot n_t^{**} = u_t \cdot A \cdot (1 - \alpha) \cdot 2 \left[ 1 - F(A \alpha) \right] \cdot (A \alpha - c_t) - v \cdot \frac{A \cdot \alpha \cdot (1 - \alpha)}{2 \cdot c_t + \frac{v}{1 - F(A \alpha)} - A \cdot \alpha} = G_t \left( A, c_t, v, p^{**}, \alpha \right) = G_t
\]

Financial intermediation determines the growth rate through the number of banks \( n^{**} \) and the market share of each bank measured by \( 2 \cdot l_t^{**} \). The growth rate is constant. As in all “AK” endogenous growth model, it depends positively on capital productivity. Imperfect competition in the banking sector introduces three explanatory factors: population density \((1/L)\), intermediation cost ratio \( c_t \) and \( v/p^{**} \), the return on the alternative asset, augmented by the risk of a failure of the banking system. In the general case, the intermediation cost ratio is more precisely defined by the following implicit equation:

\[
c_t = \frac{C_t}{2 \cdot l_t^{**} \cdot w_t}
\]

where \( l_t^{**} = l_t^{**}(c_t) \). If the fixed intermediation cost \( C_t \) is a constant, then the intermediation cost ratio decreases over time as real wage and the size of collected deposit increases over time. It implies that the extent of imperfect competition decreases between intermediaries, so that there is a market extension effect on the deposit side and a lower cost of capital for firms. Therefore, the expected growth rate increases over time, up to the growth rate determined by the full collection of savings (described in what follows). In the other polar case where fixed intermediation costs are indexed on a growing factor, for example \( C_t = f \cdot \frac{E_t[Y_{t+1}]}{n} \), it turns out to be simply proportional to \( A \):

\[
c_t = f \cdot \frac{E_t[Y_{t+1}]}{nk_{t+1}} = f \cdot A
\]

One may express the relationship between the welfare of a generation and the growth rate of the economy in this regime. A measure of welfare for a population of heterogeneous agents (with respect to the return they obtain on their savings, which depends on their location) is to sum over the individual utilities for a representative generation. Households save their whole income in the first period. The return on
savings depends on the transaction cost they incur. Noting $U_t^n$ the sum of individual utilities of households born at date $t - 1$, dividing by the capital stock of the first period, and substituting $\delta \cdot l^* = p^{**} \cdot r^{**} - v$, one obtains the following relationship between welfare (normalised by capital $K_t$) and growth:

$$
\frac{U_t^n}{K_t} = \frac{1}{1 + \rho} \cdot w_t \cdot 2 \cdot n^{**} \cdot \left\{ \int_{\rho}^{2n^{**}} \left[ p^{**} \cdot r^{**} - \delta \cdot i \right] di + \int_{2n^{**}}^{1} [v] di \right\} 
$$

$$
= \frac{w_t \cdot 2 \cdot n^{**}}{(1 + \rho) \cdot K_t} \cdot \left\{ p^{**} \cdot r^{**} \cdot l^{**} - \delta \cdot \frac{(l^{**})^2}{2} + v \cdot \left( \frac{1}{2} \cdot n^{**} - l^{**} \right) \right\} 
$$

$$
= \frac{G_t}{(1 + \rho)} \cdot \left\{ \frac{p^{**} \cdot r^{**} - \frac{p^{**} \cdot r^{**} - v}{2}}{G_t \cdot K_t \cdot (1 - \alpha)} \cdot (1 - \alpha) \cdot A \cdot v \right\} 
$$

$$
= \frac{1}{1 + \rho} \cdot \left[ \frac{p^{**} \cdot r^{**} - v}{G_t \cdot K_t \cdot (1 - \alpha)} \cdot (1 - \alpha) \cdot A \cdot v \right] 
$$

- Complete collection of saving

There is complete collection of saving when $l^{**} = 1/2n^{**}$, which sets a lower bound to capital productivity $A\alpha > H(\alpha, v/p^{**})$. Since all saving is collected, the saving-investment equality gives the following growth rate: $G_t = \frac{w_t}{K_t} = u_t \cdot A \cdot (1 - \alpha)$. Since the individual saving behavior does not depend on interest rates, neither does aggregate collected saving and growth when markets are touching.\textsuperscript{15} But imperfect competition affects the returns on savings and welfare. The relationship between welfare (which depends on second period returns on savings) and growth (which is independent of these returns) is given in the appendix.

In the remaining part of the paper, we assume for simplicity that the fixed cost is such that $C_t = f \cdot \frac{E_t[R_{t+1}]}{n}$, so that the intermediation costs ratio is a constant $c$. Gali’s [1995] assumption of non-indexed fixed cost is nonetheless possible in our setting, but leads to an increased complexity of the mathematical results of the model. The welfare trade-off between stability and efficiency investigated in sections 4 and 5 would be only altered at the margin by the ratio $c_{t+1}/c_t$ stating the increase of competition between date $t$ and date $t + 1$.

4 Deposit insurance

One possible way to increase welfare is to introduce deposit insurance. As indicated in the preceding section, there are two types of free entry equilibria, corresponding to two types of ex-ante expectations of depositors. On the one hand, a self-fulfilling confidence crisis is such that $p_t = 0 \Rightarrow p_t^{**} = 0$. On the other hand, a perfect-foresight equilibrium is characterised by $p_t \in [0, 1] \Rightarrow p_t^{**} = 1 - F(E_t[R_{t+1}])$.

\textsuperscript{15}The effect on growth of a “mark-down” of financial intermediaries on the amount of savings of a representative household which depends positively on the interest rate on deposit has already been dealt with in several papers (see, for example, Berthélemy and Varoudakis [1996]). In this paper, we stress another effect. Households are heterogenous in terms of location, so that intermediated savings is a function of the number of depositors. The two effects can be mixed, if one assumes that individual savings function depends on interest rate, but our point would be less clear.
To eliminate systemic risk, one may introduce a deposit insurance fund which ensures that depositors are paid back with certainty in the case of failure of the bank (i.e., when $R_{t+1} \leq E_t [R_{t+1}]$ in the case of free entry in the banking industry). In order to avoid losing profitable investment opportunities by investing a large fraction of deposits in the safe asset, the deposit insurance fund may decide to tax the young generation to make up for the difference between the realized value of $R$ and the promised value of $r$.\(^{16}\) Taxation imposes that the fund be run by the government. In addition, we focus on the case where the deposit insurance fund is a “pay-as-you-go” system, designed to ensure, ex post (i.e. when the macroeconomic shock is realized), that the “old” generation receives $r$ with certainty. In other words, we require premia paid to the deposit insurance fund to be fair and no other resources to be made available to the government, which only organizes the eventual transfer between generations.\(^{17}\)

The overall impact of deposit insurance results from two opposite effects: (i) an increase in the number of deposits collected by banks (market extension effect) since the probability of bank failure is now equals to zero; (ii) a reduction in overall saving as a consequence of taxation. In that sense, deposit insurance may be viewed as a way to reduce autarky, i.e. to induce depositors to invest their savings in bank deposit. However, (ii) means that if deposit insurance is too costly and reduce overall investment too significantly, its introduction will not be welfare improving.

With free entry, the young generation has therefore to make up for the difference $E_t [R_{t+1}]-R_{t+1}$, through the deposit insurance fund, when banks are going bankrupt, i.e. for $R_{t+1} \leq E_t [R_{t+1}]$. Let $D_t$ be aggregate deposits of the old. Obviously, the tax proceeds are bounded by the aggregate income of the young generation:

$$
\left( E_t [R_{t+1}] - u \cdot E_t [R_{t+1}] \right) D_t \leq w_{t+1} \Rightarrow \\
\left[ 1 - \left( \frac{1-\alpha}{\alpha} \right) \right] \cdot E_t [R_{t+1}] \leq u \cdot E_t [R_{t+1}] = \bar{u} \cdot \alpha \cdot A
$$

This condition implies that the lower bound of the macroeconomic return $u \cdot E_t [R_{t+1}]$ should not be too low, else depositors are not insured against all the states of nature. When deposit markets are not “touching”, aggregate collected saving with deposit insurance provides the expression of the “growth factor with deposit insurance” $G_l^I$ as it is equal to investment of the next period:

$$
G_l^I = \frac{K_{t+1}}{K_t} = \frac{1}{K_t} \cdot \frac{2 \cdot |l^s \cdot n^s|}{L} \left\{ u_t - D_{t-1} \cdot \mathbf{1}_{\{R_t < E_{t-1}(R_t)\}} \cdot [E_{t-1}(R_t) - R_d] \right\}
$$

$$
= \frac{2 \cdot |l^s \cdot n^s|}{L} \cdot \left\{ u_t \cdot A \cdot (1-\alpha) - \mathbf{1}_{\{R_t < E_{t-1}(R_t)\}} \cdot [E_{t-1}(R_t) - R_d] \right\}
$$

where $\mathbf{1}_{\{R_t < E_{t-1}(R_t)\}}$ equals 1 when $R_t < E_{t-1}(R_t)$, and 0 otherwise. On the one hand, individual income and savings can be lowered by the deposit insurance tax, depending

\(^{16}\)Another mechanism would be to introduce a deposit insurance scheme when the same generation is taxed before the crisis arises (when young) with their savings invested in the safe storage technology. When they are old, they receive their deposit (augmented with the return) for sure and the excess resources of the insurance fund. Nonetheless, this scheme is less efficient than a transfer between generations, once the crisis is known. When a swift action is necessary to avoid the risk of a breakdown of the banking system, wage earners have generally to contribute for savers.

\(^{17}\)Matutes and Vives (1996), who also consider the existence of a deposit insurance fund, do not impose this constraint in a partial-equilibrium framework.
on the realization of $R_t$ in the previous period. Deposit insurance introduces additional randomness in first period income and then on the growth rate in order to eliminate randomness on the second period income. This policy may improve welfare as uncertainty is related to bankruptcy cost of banks in the second period. On the other hand, as the probability of bank failure is now always equal to zero ($p = 1$), there is an extension of the market share of each bank and the number of bank increases. The market share of a single bank is now equal to $2 \cdot l^* = 2 \cdot (\frac{R_t - v}{\delta}) = 2 \cdot (E_{t-1}(R_t) - c_t - v) / \delta \geq 2 \cdot l^{**}$. This is akin to the “market extension effect” of Matutes and Vives [1996], which only works for non-touching markets ($l^{**} \leq 1/2n^{l*}$ holds in that case). The number of banks under free-entry and deposit insurance is higher than without deposit insurance:

$$n^{l*} = \frac{(1 - \alpha) \cdot E(R)}{2 \cdot c_t + v - E(R)} \geq n^{**} \quad (25)$$

The number of banks is higher with deposit insurance than without. The joint effect of an increase of $l^{**}$ and $n^{l*}$ is that the whole circle is more likely to be covered. The overall effect of deposit insurance on welfare depends on the probability of being taxed. Hence, the expected welfare for a newly born generation at date $t - 1$ is given by its expected consumption at date $t + 1$:

$$E_{t-1} \left[ \frac{U_t^*}{K_{t-1}} \right] = \frac{1}{1 + \rho} \cdot 2 \cdot n^{l*} \cdot \left[ E_{t-1} [u_t \cdot (1 - \alpha) \cdot A] - \int_{u_k E(R)}^{E_{t-1} [R_t] - R_t} (E_{t-1} [B_t] - R_t) \cdot dF(R) \right]$$

$$\cdot \left\{ \int_0^{l^{**}} \left[ l^{**} - \delta \cdot i \right] \cdot di + \int_{l^{**}}^{1/2n^{l*}} \left[ 1 - \delta \cdot i \right] \cdot v \cdot di \right\} \quad (26)$$

where the last term in curly brackets is similar to (21) for $p^{**} = 1$.

As the economy is growing, the welfare of the generation born at date $t - 1$ depends on the stock of capital financed by the savings of the existing generation at this date.

Welfare in the case of deposit insurance is obviously subject to a trade-off between the cost and the benefit of deposit insurance.

We exhibit a numerical case where welfare maximization suggests to introduce this kind of deposit insurance (as the most usual belief is that it is inefficient). It is convenient to use $R = A \cdot \alpha \cdot (1 - m + X)$ where $X$ follows a beta distribution $\beta(a, b)$ on $[0, 1]$ with a mean $m = E[X] = a/(a + b) \in [0, 1]$. In that case, $[A \cdot \alpha - u \cdot A \cdot \alpha] \cdot D_t = m \cdot D_t \leq \omega_{t+1}$, or $m \leq (1 - \alpha) \cdot A$. The full insurance constraint is therefore easily satisfied for a beta distribution, since $m < 1$. Regarding the other parameters, we provide here an example where $\rho = 0$, $A = 3.7$, $\alpha = 0.36$, $c_t = 0.22$, $v = 0.95$, $\delta = 0.975$, $a = 2$, $b = 0.05$. Numerical simulations show that this is an example where welfare is higher with deposit insurance ($E_{t-1} \left[ \frac{U_t^*}{K_{t-1}} \right] / K_{t-1} = 1.1624$) than without deposit insurance ($U_{t-1}^* / K_{t-1} = 0.4581$). Due to the “market extension effect” the equilibrium number of banks controls a market share which covers entirely the circle. It decreases with respect to the case of free entry without deposit insurance ($n^{l*} = 2.6042$ to be compared to $n^* = 5.0084$). The conclusion is that deposit insurance will increase welfare if the equilibrium probability of success without deposit insurance is low, which depends on the distribution of the macroeconomic shock.
5 Reduction of excess capacity

We investigate now cases where free entry may lead to the accumulation of excess capacity. This appears when a too large number of banks increase the overall level of risk in the economy, hence reduce depositors’ welfare. Such a situation may lead to favour the exit of the market by some institutions, to raise barriers to entry through licensing, or to promote bank mergers. In that case a smaller number of banks can lower the probability of default of banks, as well as the expected bankruptcy costs borne by depositors, hence increase welfare. However, it is important to notice that such a policy does not eliminate the self-fulfilling equilibrium related to a confidence crisis (the case for \( p = 0 \)), whereas a full deposit insurance does, if it can be implemented. In this framework, the government could determine the number of banks that maximizes welfare. Public authorities would make an arbitrage between stability of the banking system and competitive efficiency,\(^{18}\) since imperfect competition implies the existence of oligopoly rents in banking activity.

When banks are local monopolies, using (22), one gets:\(^{19}\)

\[
\frac{U^n_t}{K_t} = \frac{1}{(1 + \rho)} \cdot \left[ \left( \frac{p \cdot r - v}{2} \right) \cdot G_t + u_t \cdot (1 - \alpha) \cdot A \cdot v \right]
= \frac{u_t}{(1 + \rho)} \cdot \left[ 2 \cdot \left( \frac{p \cdot r - v}{\delta \cdot L} \right) \cdot n \cdot (1 - \alpha) \cdot A \right]
\left[ (p \cdot r - v) \cdot n \cdot \left( \frac{p \cdot r - v}{\delta L} + v \right) \cdot (1 - \alpha) \cdot A \right]
= u_t \cdot W(n, r, p) \cdot \frac{2 \cdot [(1 - \alpha) \cdot A]^2}{(1 + \rho) \cdot (\delta \cdot L)^2}
\]  

(27)

With \( W \) defined below. The number of banks that maximises the expected welfare (for a given stock of capital at date \( t - 1 \)) obtains under the constraints defining an equilibrium for a given number of banks. It is therefore such that:

\[
n \in \text{Arg max} W(n, r, p) = \text{Arg max} \left[ n \cdot (p \cdot r - v)^2 + v \cdot \delta L \right] \cdot n \cdot (p \cdot r - v)
\]  

(28)

subject to:

\[
r = \frac{1}{2} \cdot \left\{ E_t[R_{t+1}] \cdot \left[ 1 + \frac{\alpha - 1}{n} \right] + \frac{v}{p} \right\}
\]  

(29)

\[
p = 1 - F(r + \alpha_t)
\]  

(30)

\[
1 \leq n \leq \frac{L}{2 \left( \frac{\alpha \cdot r^{* \cdot r^{*} - v}}{\delta} \right)}
\]  

(31)

The first constraint can be written in order to provide an explicit expression of the number of bank \( n \) (it is lower than the number of banks in the case of free entry):

\[
n(r, p) = \frac{(1 - \alpha) \cdot E_t[R_{t+1}]}{E_t[R_{t+1}] - 2 \cdot r + \frac{v}{p}} \leq n^* = \frac{(1 - \alpha) \cdot E_t[R_{t+1}]}{2 \cdot c_t - E_t[R_{t+1}] + \frac{v}{1 - F(E_t[R_{t+1}] )}}
\]  

(32)

\(^{18}\)The overall economic efficiency is indeed a combination of stability and “competitive” efficiency.

\(^{19}\)The case of touching markets is available from the authors upon request.
$n$ is increasing monotonically in the rate of interest on deposits for a given probability of bankruptcy.

The inequality constraints imply that (i) at least one bank do exist, (ii) banks have at least zero profit (the case for free entry is a corner solution) and (iii) even in the case of the highest number of banks (free entry), banks are “local monopolies” in the deposit market.

$W(r, p)$ is an increasing function in both arguments ($W_r > 0, W_p > 0$). The first order condition can be written as:

$$W_r - f(r + c_t) \cdot W_p = 0$$

(33)

An increase in the deposit interest rate implies an arbitrage between the rise in households’ welfare due to a higher return on saving and a reduction due to the increased instability of the banking system measured by an increased probability of default. Such a phenomenon is not always the rule in this model. In many cases, the increase in competitive efficiency dominates the decrease in expected stability ($W_r > f(r + c_t) \cdot W_p$) for the range of possible deposit rates. In this case, a corner solution with free entry in the banking sector is welfare maximizing. Excess capacity appears when small shifts of deposit rates change widely the probability of default of banks, i.e. when the distribution of risk is concentrated at the point $r + c_t$.

For the same parameters as in the preceding section, welfare in the case of free entry is measured by $U_a/K_t = 0.4581$ (which corresponds to $n^{**} = 5.0084$ banks). An interior solution exists which maximises welfare ($U_a/K_t = 0.4870$) for a lower number of banks ($n = 4.8148$). The growth rate for a number of intermediaries which maximises welfare turns out to be higher than with free entry (6.74% vs. 0.55%). Nonetheless, the number of banks which maximises growth is different from the one which maximises welfare. Finally, in this particular case, welfare with a full deposit insurance is higher than welfare with entry regulation (although a combination of the two regulatory program is also possible). In the general case, however, not only a full deposit insurance cannot always be implemented if the lower bound on the return is too low, but also the expected cost of deposit insurance may exceed its expected benefit, namely the reduction of bankruptcy costs associated with systemic risk in the banking system.

6 Conclusion

The paper shows that long term confidence in the banking sector can improve the efficiency of the financial intermediation sector, investment and growth. Public intervention in the banking activity can decrease the bankruptcy costs through lowering the probability of bankruptcy by two means: on the one hand, deposit insurance and, on the other hand, licensing or the removal of excess capacity. Nonetheless, both policies require a careful assessment of their costs and benefit on welfare. It turns out that they are not always welfare maximising, even when bankruptcy costs do exist. Excess capacity appears if and only if the marginal changes on deposit rates implies large changes on default risk and on stability of the banking system. By reducing capacity, increased stability may more than offset losses due to a rise in imperfect competition margins. A higher level of stability may even be achieved in the absence of deposit insurance, as stressed by Carr and al. [1995].
References


7 Appendix

7.1 Equilibrium with total saving collection

We now relax the assumption of non-overlap in potential deposit markets. The depositor who is indifferent between bank $i$ located in $l_i = i \cdot \frac{L}{n}$ and bank $i + 1$ located at a distance $L/n$ from bank $i$, is at $l_{i,i+1}$ with $\frac{iL}{n} < l_{i,i+1} < \frac{(i+1)L}{n}$:

\[
p \cdot r_i - \delta \cdot \left( l_{i,i+1} - \frac{i \cdot L}{n} \right) = p \cdot r_{i+1} - \delta \cdot \left[ \frac{(i+1)}{n} - l_{i,i+1} \right] \Rightarrow (34)
\]

\[
l_{i,i+1} = \frac{1}{n} \left( i + \frac{1}{2} \right) + \frac{1}{2 \cdot \delta} \cdot (p \cdot r_i - p \cdot r_{i+1}) (35)
\]

Saving collected by a bank is then:

\[
d_{i,t} = N \cdot w_t \cdot (l_{i,i+1} - l_{i-1,i}) (36)
\]

\[
d_{i,t} = N \cdot w_t \cdot \left[ \frac{1}{n} + \frac{1}{\delta} \cdot \left( p \cdot r_i - \frac{p \cdot r_{i-1} + p \cdot r_{i+1}}{2} \right) \right] (37)
\]

The bank’s maximization program becomes:

\[
(d_{i,t}, k_{i,t+1}) \in Arg\ max E_t[\Pi_i] = E_t[R_{i,t+1}] \cdot k_{i,t+1} - r_{i,t} \cdot d_{i,t} - c_{Ut} (38)
\]

\[
s.c. k_{i,t+1} = d_{i,t} (39)
\]

\[
d_{i,t} = N \cdot w_t = \frac{p}{\delta} \left\{ r_i - \left[ \frac{r_{i-1} + r_{i+1}}{2} - \frac{1}{n \cdot p} \right] \right\} (40)
\]

\[
E_t[R_{i,t+1}] = \alpha \cdot a \cdot k_{i,t+1}^{\alpha-1} \cdot N^{1-\alpha} (41)
\]

At the symmetric Nash equilibrium, where credit and deposit rates are identical for all banks, the expression for the intermediation margin is different from the case with non-touching markets. The role of the alternative asset has vanished since depositors’ outside option is no longer storage but rather a deposit with the next nearest bank.

\[
r_i = E_t[R_{t+1}] = \left[ 1 + \frac{\alpha - 1}{n} \right] - \frac{1 \cdot \delta}{n \cdot p} (42)
\]

Equilibrium is realized when depositors’ expectations are correct. The probability of success is the following:

\[
p = 1 - F \left( r_i + \frac{c_t}{d_{i,t}} \right) (43)
\]

\[
d_{i,t} \text{ corresponds to } \frac{1}{n} \text{th of total saving:}
\]

\[
d_i = N \cdot w_t \cdot \frac{1}{n} = \frac{N \cdot w_t}{n} (44)
\]

Free entry in banking determines the number of banks:

\[\text{As in Williamson [1987], we do not consider the possibility for one bank to capture the entire market in a location game followed by a pricing decision. This effect, made possible by a linear specification of transport costs with respect to distance, disappears when costs are quadratic.}\]
From which one can deduct the following equilibrium values (for $N = 1$):

\[ n^{**} = \frac{A \cdot \alpha \cdot (1 - \alpha) + \delta \cdot \frac{1}{p}}{c_t} \]

\[ r_{i,t}^{**} = E_t[R_{t+1}] - c_t \]

\[ p^{**} = 1 - F(E_t[R_{t+1}]) = 1 - F(E_t[R_{t+1}]) \]

\[ r^{**} = 1/2n^{**} \]

\[ d_{i,t} = w_t/n^{**} \]

\[ n^{**} = \frac{(1 - \alpha) E_t[R_{t+1}] + \delta \cdot \frac{1}{p^t}}{c_t} \]

Under free entry, compared to the case of non-touching markets, the number of banks is the only variable which is modified.

Local monopolies appear when some household do not put their saving in a bank, i.e. $0 < l^{**} < 1/2n^{**}$, and $n^{**} > 0$, which means that capital productivity $A \cdot \alpha$ is bounded: $c_t + (v/p^{**}) < A \cdot \alpha < \min (2 \cdot c_t + (v/p^{**}), H(c_t, v/p^{**}))$, where $H$ is the larger solution of the second order equation in $A \cdot \alpha$ associated to the equation $l^{**}n^{**} = L/2$.

The growth rate is given by the savings and investment equality, so that the growth factor is: $G = (1 - \alpha)A$. The relationship between welfare (which depends on second period returns on savings) and growth (which is independent of these returns) is given in the appendix.

\[ \frac{U_t}{K_{t-1}} = \frac{G w_t \cdot 2 \cdot n^{**}}{K_t \cdot (1 + \rho)} \cdot \left\{ \int_0^{1/(2n^{**})} [p^{**} \cdot r^{**} - \delta \cdot i] di \right\} \]

\[ = \frac{G^2 \cdot 2 \cdot n^{**}}{(1 + \rho)} \cdot \left\{ p^{**} p_{p}^{**} \frac{1}{2n^{**}} - \delta \cdot \left( \frac{1}{2n^{**}} \right)^2 \right\} \]

\[ = \frac{G^2}{1 + \rho} \left\{ p^{**} p_{p}^{**} - \frac{\delta}{4n^{**}} \right\} \]

### 7.2 Excess capacity with touching markets

When banks are no longer local monopolies ($l^{*} < 1/2n^{*}$), welfare maximization amounts to choosing $n$ such that:

\[ n \in Arg \max W = p \cdot r - \frac{\delta}{4 \cdot n} \]

under the following constraints:

\[ r_{i} = E[R_{t+1}] \cdot \left[ 1 + \frac{\alpha - 1}{n} \right] - \frac{\delta}{p \cdot n} \]

\[ p = 1 - F(r_{i} + c_{t}) \]

\[ \frac{1}{2l} \leq n < n^{**} \]
The first constraint gives $n$ as an explicit function of $r$:

$$
n = \frac{A \cdot \alpha \cdot (1 - \alpha) + \frac{\delta}{1 - \tilde{F}(r_i + c_t)}}{A \cdot \alpha - r}
$$

(58)

$n$ increases monotonously with the rate of interest, for a given default probability. An equivalent program is:

$$
r \in \text{Arg max} \ W = [1 - F(r + c_t)] \cdot r - \frac{\delta}{4 \cdot n(r)}
$$

(59)

under the following constraints:

$$
\frac{v}{p} \leq r \leq A \cdot \alpha - c_t
$$

(60)

$$
\frac{A \cdot \alpha \cdot (1 - \alpha) + \frac{\delta}{1 - \tilde{F}(r_i + c_t)}}{A \cdot \alpha - r} \geq \frac{\delta}{2 \{[1 - F(r_i + c_t)] \cdot r - v\}}
$$

(61)