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DUALISM AND MACROECONOMIC VOLATILITY*

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ABSTRACT

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This paper develops a simple macroeconomic model that shows that capital market imperfections together with unequal access to investment opportunities across individuals, can generate endogenous macroeconomic volatility, that is permanent fluctuations in aggregate GDP, investment and interest rates. Structural policies reducing the extent of dualism can therefore be a necessary condition for macroeconomic stabilization. Our model of the business cycle also offers a transparent rationale for countercyclical fiscal policies: since slumps are periods where some idle savings are not being used efficiently because of the limited debt capacity of potential investors, one natural strategy to foster recovery is to issue public debt during recessions in order to absorb those idle saving and finance investment subsidies and/or tax cuts for investors.

Key Words: Business Fluctuations, Fiscal Policy
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RESUME

DUALISME ET VOLATILITE MACROECONOMIQUE

Ce papier développe un modèle macroéconomique simple montrant comme la combinaison de contraintes de crédit et d'une inégalité d'accès aux opportunités d'investissements peut engendrer de la volatilité macroéconomique, c'est-à-dire des fluctuations permanentes du PIB, de l'investissement et des taux d'intérêt. Des politiques structurelles réduisant l'importance du dualisme peuvent donc être une précondition pour la stabilisation macroéconomique. Notre modèle du cycle économique conduit également à une justification transparente des politiques fiscales contre-cyclique: puisque les récessions sont des périodes où toute l'épargne disponible n'est pas utilisée par les investisseurs du fait de leur capacité limitée d'emprunt, l'émission de dette publique dans les récessions peut permettre d'absorber cet excès d'épargne et de subventioner l'investissement et donc d'accélérer la reprise.

Key Words: Fluctuations, Politique fiscale.

"The most important single fact about saving and investment activities is that in our industrial society they are generally done by different people and for different reasons. (...) For years there might tend to be too little investment, leading to deflation, losses, excess capacity and unemployment. For other years, there might tend to be too much investment, leading to periods of chronic inflation - unless prudent and proper public policies in the fiscal and monetary fields are followed"

P.A. Samuelson, Economics , pp. 196-198 (8th edition, 1970).

1 Introduction

The idea that the separation of savers and investors has implications for the short-run macroeconomic stability of an economy is an idea that goes back at least to Keynes and Harrod. This paper represents an attempt to answer this question within the framework of a simple macro model with explicit micro-foundations.

In our model there are two dimensions of separation between savers and investors: sheer physical separation, manifested in the fact that savers and investors are often different people in the sense that many people who save are in no position to directly invest in physical (as distinct from financial) capital; and a more market-based separation embodied in the constraints on the amounts investors can borrow from savers.

We will justify the borrowing constraints by invoking the usual excuse of asymmetric information, modeled here as an ex post moral hazard problem. By a careful choice of parameters we ensure that the solution to this problem takes the extremely simple form of a credit-multiplier. In other words, there is a constant $\nu < 1$ such that anyone who wants to invest an amount I must have assets of at least νI.[1]

There are a number of reasons why not all savers are also investors in the

[1] There is now a large body of evidence documenting that such borrowing constraints have large effects on investor behavior even in countries like the United States which have very well–developed capital markets: see for example Fazzari, Hubbard and Petersen (1988), Fazzari and Petersen (1993).

sense of being able to invest in physical capital. First, being able to invest may be crucially dependent on having certain skills, ideas and connections: for the vast majority of individuals, the idea of using one's savings to borrow additional funds in order to open a new factory or purchase capital equipment makes no sense at all. Second, unequal access to investment opportunities may just be an extreme consequence of the credit constraints themselves: many investments are indivisible and require a certain minimum investment. In a world where capital markets are imperfect, this means that the investor would have to put up a minimum amount of his own wealth; those who cannot afford this minimum amount will not be able to make direct investments in production and will have to put their money in a bank who then lend their savings to active investors. Third, distances may limit the ability of investors to invest: the investment might require the investor to live and work in the city, whereas he may have other reasons for wanting to live in a village. Investors may also be restricted by social distance: investing in most industries involves some degree of cooperation with investors in other industries and even with investors in other firms in the same industry. Therefore if the industrial sector is dominated by people from one social group, and people from this social group are known to cooperate better with their own (perhaps for the usual repeated-games reasons), outsiders may hesitate to invest in industry. Finally government regulations may restrict the ability of certain social groups to invest in certain assets.[2]

We model this physical separation of savers and investors by introducing an asset which we call *active investment* (e.g. starting a firm) which has the highest return of all assets and which is only accessible to already existing businesses and their owners and to a fixed fraction of the labor force. We measure the separation by a constant μ which measures the share of labor income going to those who can actively invest (say, the managerial elite).

The main contribution of the paper is the development of a tractable framework for studying the business cycle implications of this kind of separation between savers and investors. The first result in the paper tells us

[2]For example in colonial India, under the Punjab Land Tenancy act, the colonial administration in the Punjab restricted the right to own land to certain "peasant" castes

that a high degree of such separation leads the economy to fluctuate around its steady state growth path. More specifically, it is shown that under a condition which in our model turns out to take the very simple form of $\mu < \nu$, implying both a relatively high degree of physical separation of savers and investors and a poorly functioning capital market, the economy always converges to a cycle around its trend growth path unless μ is very small (or ν very large).

The logic behind the endogenous cycles is straightforward. Periods of slow growth are periods when savings are plentiful relative to the limited debt capacity of potential investors, which implies a low demand for savings and therefore low equilibrium interest rates. This in turn implies that the investors can retain a high proportion of their profits (since the interest rate and hence the debt-burden on investors is low) which allows them to rebuild their reserves and debt capacity and expand their investment. This, in turn, generates more profits and more investment till, eventually, planned investment runs ahead of savings forcing the interest rates to rise. Now the debt-burden on the investors is going to be higher, retained earnings will be lower, and investment will collapse, taking us back to a period of slower growth.

It is worth noting that this endogenous cycle is a product of two distinct forces. On the one hand high investment begets high profits and high investment. On the other, high investment pushes up interest rates and reduces future profits and investment. The first causes output to be positively serially correlated; the second generates a negative serial correlation. Depending on which of these forces dominates, the overall correlation of output may be positive or negative. In particular, we show that if credit constraints are severe and the fraction of active investors is low, then booms cannot last forever: in other words, the second force will eventually exhaust the debt capacity of investors and push the economy into recession.

It is also worth emphasizing that cyclical fluctuations in this world are always inefficient: the economy cycles because it has phases when savings are underutilized in the sense of being invested in an inferior asset. In this sense our model partially captures the Keynesian idea that slumps are associated with a liquidity trap.[3]

The cycle is however just one of three possibilities: if $\nu < \mu$, so that the degree of separation is low, we have a permanent boom: the economy converges to a stable growth path where all available savings are always invested in the high-return activity. While if either ν is very high or μ is very low, the economy converges to a permanent slump in the sense of permanently underutlizing its saving.

Of these three cases the highest trend growth rates correspond, not entirely surprisingly, to the permanent boom case; in this case the growth rate is exactly the Harrod-Domar growth rate. In both the other cases the trend growth rate is always lower than the Harrod-Domar growth rate but a priori, we cannot say which of the two is higher.

These three types of economies also differ in their response to shocks: where the degree of separation is low, i.e. in what we have called permanent boom economy, the full effect of a temporary productivity shock registers immediately in the output. When the degree of separation is higher, responses to shocks are more gradual and therefore there is more persistence in changes in output.

A permanent positive productivity shock raises the growth rate in a permanent boom economy and the new higher growth rate is achieved immediately. By contrast, in a permanent slump economy the full impact of the shock on the growth rate comes about only gradually. The effect in the cyclical economy turns out to be more complex than in either of the two previous cases because one has to take account of the effect of higher interest rates generated by the positive productivity shock on the length of the subsequent slump. It turns out that in some cases a positive productivity shock can actually lower the average growth rate by prolonging the slump.

The overall picture that emerges is that economies with less developed financial markets and a sharper physical separation between savers and investors will tend to be more volatile and to grow more slowly. For a number

[3]However since we do not have a notion of liquidity in our model, we cannot capture the Keynesian idea that in a liquidity trap the inferior asset that savers hold is also a very liquid asset.

of obvious reasons both of these dimensions of separation are likely to be greater in LDCs.[4] Therefore many LDCs and especially perhaps the more unequal LDCs, will suffer most from the problems discussed here.[5]

However there is at least some evidence that this kind of mechanisms based on functioning of the credit market are also relevant for understanding the business cycle properties of more developed economies. The case for a credit channel for the transmission of shocks to the U.S. economy has been argued for in various versions going back at least to the work of Haberler (1964)[6] and more recently in a number of papers by Bernanke and Gertler.[7] More directly relevant is the conclusion of Eckstein and Sinai (1982), after a careful examination of the US postwar evidence, that all US recessions since World War II have been preceded by investment booms, leading to high interest rates, deteriorating balance sheets for businesses and eventually credit crunches. Eckstein and Sinai then describe US recessions as times when debt repayments decrease and reliquidification of balance sheets occurs, leading to a subsequent boom, and so on. This informal story coincides exactly with the mechanism analyzed in our formal model.

Given the inefficiency of cyclical fluctuations in our model, there is a clearly a potential role for counter-cyclical macro policies. Since slumps are times where there are some idle savings that are not being used efficiently because of the limited debt capacity of potential investors, a natural strategy for recovery is to issue public debt in recessions in order to absorb those idle savings and finance investment subsidies and/or tax cuts for businesses. We show that under some conditions such policies can at the same time restore maximum growth and raise the savers' welfare.

[4]Some reasons why the extent of separation of savers and investors may be specially large in LDCs include: poor transportation facilities exaggerate physical distances and limit certain types of investment to those who live close enough to the natural locale for the investment; badly defined and poorly enforced property rights make loan transactions harder; social segmentation on the basis of caste and tribe, often acts to limit entry into specific industries; and high levels of social and economic inequality might mean that only a small fraction of the population has the wherewithal to be investors.

[5]See Gavin-Hausman (1995) for a recent IDB report documenting the positive correlation between underdevelopment, inequality, macroeconomic volatility and low average growth.

[6]Who, in turn, refers to Hawtrey and Wicksell.

[7]These papers and a number of others are surveyed in Bernanke and Gertler (1995).

Of course a different policy perspective on our model would be to argue that it stresses the importance of structural reforms aim at reducing the extent of separation of savers and investors. This is especially likely to be important in situations where the appropriate counter-cyclical policies are difficult to carry out (for example because the extent of separation is changing rapidly over time and in situations where counter-cyclical policies have adverse distributional consequences (the policies suggested by our model always help investors, but sometimes at the cost of the savers). Structural reforms may in such cases be a necessary condition for the success of stabilization policies.

Our model is far from being the first paper that emphasizes the role of the separation between savers and investors as a source of macro-economic volatility. This general point can be found in the works of Keynes (1936), Harrod (1947), Kaldor (1957) among others, but in their models the primary source of instability is the exogenous instability in the investment rate.

Goodwin's (1967) model of growth cycles is closer to ours, in that it also describes cycles as the outcome of a dynamic process with the imbalance between investors and savers changing endogenously over time, though Goodwin's mechanism operates through the equilibrium wage rate rather than the equilibrium interest rate (for some evidence that our mechanism is important in practice, see section 3.4). Despite this difference the two stories are entirely consistent with each other and share the common prediction that downturns in economic activity are caused by the decline of investors' investment capacity. Where the papers differ most is in the modeling style. Unlike ours, Goodwin's model has no explicit micro foundations and therefore does not allow for rigorous policy analysis and checks of robustness. In Appendix C, we show how our model can be easily extended so as to offer a micro-founded version of a Goodwin cycle.

Our work also is also related to the recent literature on the business cycle implications of credit market imperfections which follows on the work of Bernanke-Gertler (1989). In that paper they introduce credit constraints into an otherwise neo-classical framework and argue that such constraints lead to positive serial correlation in the deviations of output from the trend.

This is clearly one of the implications of our model as well. Bernanke-Gertler do not however analyze the counteracting effect of equilibrium interest rate and therefore do not consider the possibility of negative serial correlation.

Perhaps closest to our paper is Kiyotaki-Moore (1993) who build on the Bernanke-Gertler model and introduce the idea of looking at the effects of an endogenously changing price (the price of collateral in their model, the price of savings in ours).. Our paper differs from their model in giving an active role to the supply of savings (in our model it is the race between savings and investment which drives the business cycle) and also in emphasizing both the possibility of an endogenous cycle and the role for stabilizing policies. .

Finally, our focus on the interplay between credit imperfections and the dynamics of distribution relates this work to the literature on inequality, credit-market imperfections and growth (see, e.g., Banerjee-Newman (1993), Galor-Zeira (1993), Aghion-Bolton (1997) and Piketty (1997)). However the emphasis on short-run fluctuations directs our attention towards a very different kind of inequality than the one emphasized in this literature. We find that from the point of view of short run stability, it is inequality between savers and investors, rather than the inequality between the rich and the very poor (who neither invest nor save very much) that matters.

The paper is organized as follows. Section 2 outlines the basic framework. In section 3 the basic model is analyzed and the basic results about stability and the nature of the response to shocks are derived. Section 4 discusses the impact of government policies on volatility and average growth.

2 Basic Framework

The economy has one non-produced input (labor) and one produced good, which serves both as a capital input and as a consumption good. Each agent is endowed with 1 unit of labor at each period.

Technology: The production function for the one produced good is $F(K, L) = AK^{\beta}L^{1-\beta} = Y$. In order to focus our attention upon the capital market equilibrium, we assume that the labor market takes a very simple form: the wage rate is permanently equal to 1, and firms can always hire as much labor as they wish to at that price.[8] In equilibrium:

$$\begin{aligned}\frac{\partial F}{\partial L}=1 \;\Rightarrow\quad L &= ((1-\beta)A)^{1/\beta}K\\ \Rightarrow\quad Y &= \sigma K\\ \text{with}\ \sigma &= A((1-\beta)A)^{(1-\beta)/\beta})\end{aligned}$$

This is an example of what has been called an AK model and therefore will generate positive long-run growth. Also note that the labor and capital shares of final output are given by the usual formulas: $\frac{\partial Y}{\partial L}L = (1-\beta)\sigma K$ and $\frac{\partial Y}{\partial K}K = \beta\sigma K$.[9]

Investment Possibilities: Not everyone has a direct access to investments in production. Those who cannot invest directly in production (the "non-investors", or the "lenders", or "savers") can either lend at the current interest rate r to those who can invest in production (the "investors", or the "borrowers") or invest in a low-yield asset which yields a return σ_2, with $\sigma_2 < \sigma_1 = \beta\sigma$.[10] We will think of this asset as storage but it could as well be some other production technology (or some government bond, see section 4 below). In addition to firms' owners, the investors' class also includes a fixed fraction of the labor force (say, the managers who have sufficient knowledge about direct investment opportunities) , and the size of this group is measured by its share μ in total labor income. The case $\mu = 0$ is the case where all productive investment is carried out directly by businesses and their own-

[8]Therefore we are implicitly assuming that the size of the labor force grows at least as fast as the capital stock. It is sufficient to assume that the sum of (exogenous) population growth and labor productivity growth is at least equal to the Harrod-Domar growth rate (see below). The wage rate is fixed to 1 either for standard labor supply reasons (agents are prepared to sell their labor unit at any price higher or equal than 1), or because of efficiency-wage considerations (firms need to pay workers at least 1 in order to avoid shirking).

[9]Instead of assuming an unlimited labor supply at price 1, one could also assume a fixed labor supply L_0 and introduce an aggregate capital accumulation externality à la Frankel (1962) or Romer (1986) in order to obtain an AK model with steady-state long-run growth: if $Y = AK^{\beta}L^{1-\beta}K_a^{\gamma}$, where K_a is the aggregate capital stock (the effects of which are not internalized by individual firms), then if $\gamma = 1-\beta$ and $L \equiv L_0$, we have: $Y = \sigma K$, with $\sigma = AL_0^{1-\beta}$, and the capital share (resp. the labor share) of output is again equal to βY (resp. $(1-\beta)Y$). These alternative assumptions would leave our analysis of macroeconomic volatility unaffected.

[10]We assume the financial intermediation process between lenders and investors to be perfectly competitive, so that lenders always get the full return to their savings. This allows us to avoid dealing with the banking sector in what follows.

ers and the "household sector" can only lend its savings to the "business sector" (or invest in the low-yield asset). At the other extreme, the case $\mu = 1$ is the case where everyone can directly invest its own savings into the high-yield production activity and where there is thus no separation between savers and investors.

Capital market imperfection: Due to standard incentive-compatibility considerations, an investor with initial wealth W can invest at most $\frac{1}{\upsilon}W$, where $\frac{1}{\upsilon} > 1$ is a credit multiplier.[11] Credit constraints vanish as υ tends towards 0, while $\upsilon = 1$ is the polar case where the credit market collapses and investors can only invest their own wealth.

Determination of the interest rate: The interest rate is the one price in this model which is determined endogenously. The linearity of the production technology implies that the equilibrium interest rate will be exactly equal to the rate of return $\sigma_1 = \beta\sigma$ whenever planned investment in production is higher than aggregate savings, and it will drop down to σ_2 in the opposite case (see below).Our results can be extended to a more general model where the interest rate changes smoothly over time (see section 3.3. below).

The Timing of Events: The timing of events within in each period t is depicted in Figure 1 below:

See Figure 1

Borrowing and lending takes place at the *beginning* of the period (which we denote by t^-), at an interest rate which is determined as indicated above by the comparison between planned investments and savings at t^-.

Everything else occurs at the *end* of the period (which we denote by t^+): first, the returns to investments are realized; second, the repayment of debt from borrowers to lenders; fourth the consumption decisions and the savings decisions which in turn will determine the total amount of available savings at the beginning of next period (i.e. at $(t+1)^-$).

Savings Behavior: For simplicity, we assume a linear saving behavior: all agents save a fixed fraction $(1 - \alpha)$ of their total end-of-period wealth

[11] An explicit microeconomic derivation of this (constant) credit multiplier is developed in Appendix A.

and consume a fixed fraction α.[12] (see section 3.3 below for a discussion of alternative assumptions).

Now that the model has been fully laid out, we can analyze the dynamics of the underlying economy and in particularly try to understand why the inequality in investment opportunities between investors and non-investors, in other words the separation between savings and investments, can generate macroeconomic volatility:

3 The Mechanics of the Model

3.1 The Basic Dynamic Relationships

Let W_B^t and W_L^t denote the wealth of investors (borrowers) and non-investors (lenders) at the beginning of period $(t+1)$. Total savings from previous period t are by definition equal to:

$$S_t = W_B^t + W_L^t.$$

Total planned investments in the high-yield activity at the beginning of period $(t+1)$ are equal to:

$$I_{t+1}^d = \frac{W_B^t}{\nu}.$$

Note that:

1. the interest rate r_{t+1}in period $(t+1)$ is equal to $\beta\sigma = \sigma_1$ if $I_{t+1}^d > S_t$ (that is, whenever the investment capacity of investors is higher than aggregate savings) and to σ_2 if $I_{t+1}^d < S_t$ (that is, whenever the investment capacity of investors is lower than aggregate savings).

2. actual investment in the high-yield activity at date $(t+1)$ is equal to $\min(S_t, I_{t+1}^d)$, whereas investment in the low yield activity (with return σ_2) is equal to $S_t - \min(S_t, I_{t+1}^d)$.

[12] Such a saving behavior can be "rationalized" by standard finite-horizon saving models where substitution and income effects exactly cancel out and by bequest models with Cobb-Douglas "warm-glow" preferences that have been used by the recent theoretical literature on income distribution and credit constraints (see the references given in section 1).

3. current borrowing by investors is equal to the difference between actual investment in the high-yield activity and their initial wealth W_B^t.

During periods where investment demand is higher than aggregate savings, all savings are invested into the high-yield activity (with total return σ) and the growth rate of the economy takes its maximum value $g^* = (1-\alpha)\sigma$.[13] We will call these periods "booms". Conversely, during periods where investment demand is higher than aggregate savings, a positive fraction of aggregate savings is invested in the low-yield activity so that the growth rate is smaller than g^*. We will call these periods "slumps". We can now obtain the following equations describing the dynamic evolution of capital accumulation between two consecutive periods. During a boom (i.e. if $I_{t+1}^d > S_t$):

$$\begin{aligned} W_B^{t+1} &= (1-\alpha)[\mu(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma(W_B^t + W_L^t) - \beta\sigma W_L^t] \\ W_L^{t+1} &= (1-\alpha)[(1-\mu)(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma W_L^t] \end{aligned} \quad \text{(B)}$$

In other words, given that all available savings $(W_B^t + W_L^t)$ are invested in the high yield activity during a boom, total revenue from this activity is equal to $\sigma(W_B^t + W_L^t)$. A fraction $(1-\beta)$ of that revenue remunerates the labor force, with investors (resp. non-investors) representing a fraction μ (resp. $1-\mu$) of the total labor share of output. Second, whilst borrowers realize the yield rate of return $\beta\sigma$ on capital investment $(W_B^t + W_L^t)$ they must repay the high interest rate $r = \beta\sigma$ on the amount W_L^t they have borrowed from the lenders (hence the term $-\beta\sigma W_L^t$ in the RHS of the first equation, which corresponds to the term $\beta\sigma W_L^t$ in the RHS of the second equation).

Similarly, during a slump (i.e. if $I_{t+1}^d < S_t$):

$$\begin{aligned} W_B^{t+1} &= (1-\alpha)[\mu(1-\beta)\sigma \cdot \tfrac{1}{\nu} W_B^t + \beta\sigma\tfrac{1}{\nu} W_B^t - \sigma_2(\tfrac{1}{\nu} - 1) W_B^t] \\ W_L^{t+1} &= (1-\alpha)[(1-\mu)(1-\beta)\sigma \cdot \tfrac{1}{\nu} W_B^t + \sigma_2 W_L^t]. \end{aligned} \quad \text{(S)}$$

Namely, given that only the amount $\frac{1}{\nu} W_B^t$ can be invested in the high-yield activity during a slump, this activity will generate a revenue equal to $\sigma \cdot \frac{1}{\nu} W_B^t$. A fraction $(1-\beta)$ of that revenue remunerates labor (with borrowers (resp. lenders) getting a fraction μ (resp. $1-\mu$) of that revenue). The fraction β of

[13] g^* is the standard Harrod-Domar growth rate, i.e. the product of the savings rate by the output/capital ratio.

that revenue remunerates capital investment and thus accrues entirely to the borrowers. However, borrowers must pay the interest rate σ_2 on the amount $(\frac{1}{\nu}W_B^t - W_B^t)$ they borrow from the middle-class lenders. On the other hand, lenders realize the rate of return σ_2 on their wealth W_L^t, both by lending a fraction of it to wealthy borrowers and by investing the complementary fraction in the low-yield activity.[14]

Letting $q^t = \frac{S_t}{I_{t+1}^d}$ denote the ratio of savings over planned investments in the high-yield activity at the beginning of period $(t+1)$, simple manipulation of the above equations leads to:

$$\frac{1}{q^{t+1}} = \frac{\mu(1-\beta)}{\nu} + \beta \cdot \frac{1}{q^t} \tag{B'}$$

when $q^t \leq 1$ (i.e. when the economy is in a boom at the beginning of period $t+1$) and:

$$q^{t+1} = \frac{1}{\mu(1-\beta)\frac{\sigma}{\nu} + \frac{1}{\nu}\beta\sigma - (\frac{1}{\nu}-1)\sigma_2}[(\sigma - \sigma_2) + \sigma_2 q^t] \tag{S'}$$

when $q^t > 1$ (i.e. when the economy is currently experiencing a slump).

Equations (B′) and (S′) define a simple first-order difference equation which allows a complete characterization of the global dynamics of the economy.

3.2 Slumps and Booms

Note first that both curves (B′) and (S′) are monotonically increasing and intersect the 45^0 degree line only once (from above). Note also that the (B′)-curve lies always above the (S′)-curve at $q^t = 1$: $q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) > q^{t+1}(q^t = 1, r_{t+1} = \sigma_2)$.[15] Therefore there are only three possible cases corresponding to the three possible rankings between $1, s$ and b, where s and

[14] They lend the amount $(\frac{1}{\nu}W_B^t - W_B^t)$ to the high-yield investors and invest the remaining amount $W_L^t - (\frac{1}{\nu}W_B^t - W_B^t)$ in the low-yield activity.

[15] The intuition for this is quite straightfoward: borrowers benefit from lower interest rates at the expense of lenders; therefore the savings/investment demand ratio is lower next period if the current interest rate r_{t+1} is lower, for a given allocation of funds (if $q^t = 1$, all savings are invested in the high-yield activity, irrespective of whether $r_{t+1} = \sigma_1$ or σ_2). More formally: when $q^t = 1$,

$$q^{t+1}(q^t = 1, r_{t+1} = \sigma_2) = \frac{1}{\frac{\mu(1-\beta)}{\nu} + \frac{\beta}{\nu} - \left(\frac{1}{\nu} - 1\right)\frac{\sigma_2}{\sigma}}$$

b are the intersections of the (S$'$) and (B$'$) curves with the 45^0 line. These are depicted in Figures 2, 3, and 4 below.

Figure 2 corresponds to the case where $1 > b(> s)$. In this case, as is evident from the figure, the economy converges to a permanent boom with $q_t \to b$.

See Figure 2

The long-run growth rate for the economy is just the Harrod-Domar growth rate $g^* = (1-\alpha)\sigma$. Simple manipulation of equation (B$'$) yields:

$$b = \frac{\nu}{\mu}$$

The condition $b < 1$ is therefore equivalent to $\nu < \mu$. This necessary and sufficient condition is very intuitive: a permanent boom will occur when the fraction of the labor force that has direct access to investments in production is sufficiently large to compensate for the borrowing constraints faced by these investors. This condition ensures that in the long-run the debt capacity of investors will always be sufficient for all available savings to be absorbed and invested in the high-yield activity. In particular, the Harrod-Domar permanent boom regime will occur if credit constraints are negligible (ν close to 0) or if all agents have direct access to productive investments (μ close to 1). Conversely, permanent booms will never occur if $\mu = 0$, i.e. if business profits are the only investible funds. This is because if $\mu = 0$, then the investment capacity of the economy grows at rate $(1-\alpha)\beta\sigma$ while savings grow at the higher rate $(1-\alpha)\sigma$: the latter will therefore always be in excess supply after some finite time.[16] The condition $\mu > \nu$ also shows that both dimensions of separation between investors and savers (i.e. $\nu > 0$ and $\mu < 1$) are necessary in order to generate slumps in the long-run.

Figure 3 corresponds to the case where $1 < s(< b)$. Here, the economy always converges to a permanent slump (with $q^t \to s$),

See Figure 3

$$< q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) = \frac{1}{\frac{\mu(1-\beta)}{\nu} + \beta}.$$

($\sigma_1 = \beta\sigma > \sigma_2$ implies that $\beta < \frac{\beta}{\nu} - (\frac{1}{\nu} - 1)\frac{\sigma_2}{\sigma}$).

[16]If $\mu > 0$, then the investment capacity of the economy grows at rate $(1-\alpha)(\frac{\mu(1-\beta)\sigma}{\upsilon} + \beta\sigma)$, which is smaller than the savings growth rate iff $\mu < \nu$.

and with a rate of growth which is less than the Harrod-Domar growth rate $(1-\alpha)\sigma$. Simple manipulation of equation (S′) yields:

$$s = \frac{(\sigma - \sigma_2)\nu}{\mu(1-\beta)\sigma + \beta\sigma - \sigma_2}$$

The necessary and sufficient condition defining the permanent slump regime is then given by:

$$s < 1 \text{ , or equivalently, } \mu(1-\beta) + \beta < \nu + \frac{\sigma_2}{\sigma}(1-\nu)$$

This condition is also very intuitive: permanent slumps will tend to occur when the credit multiplier is small (ν high) and few people have direct access to investments in production (μ small). Note also that permanent slumps are more likely to occur is $\frac{\sigma_2}{\sigma}$ is high: a high $\frac{\sigma_2}{\sigma}$ ratio implies high debt repayment/profit ratios for investors and therefore makes it less likely that their debt capacity will ever be able to absorb all available savings (the same reasoning applies if the capital share β is low). The expression for s can also be used to compute the exact growth rate g_s associated to the permanent slump regime: in steady-state a fraction $1/s$ of aggregate savings is invested into the high-yield activity (with return σ), while the remaining fraction $1 - 1/s$ is invested into the low-yield activity (with return σ_2), so that the steady-state growth rate g_s is given by:

$$\begin{aligned} g_s &= (1-\alpha)(\frac{1}{s}\sigma + (1-\frac{1}{s})\sigma_2) \quad (< (1-\alpha)\sigma) \\ &= (1-\alpha)(\sigma_2 + \frac{\mu(1-\beta)\sigma + \beta\sigma - \sigma_2}{\nu}) \end{aligned}$$

This is always lower than the Harrod-Domar growth rate.

Figure 4 corresponds to the case where $s < 1 < b$. In this case the figure shows that the economy keep moving back and forth between booms and slumps and will eventually converge to a limit-cycle.

See Figure 4

The two conditions for this case are: $\beta + (1-\beta)\mu > \nu + \frac{\sigma_2}{\sigma}(1-\nu)$ and $\mu < \nu$. These two conditions imply that an economy with a very high degree of separation between savers and investors as well as an economy where both groups are extremely well integrated will not cycle: in the first case there will be a permanent slump and in the second a permanent boom.[17] It is only in the intermediate case where the separation is large but no too large that we observe short-run instability. Note that short-run instability is associated with the economy performing below its potential (as measured by growth rates), since slumps are phases where some of the capital is put at less than optimal use: the growth rate is equal to the Harrod-Domar growth rate during booms (when $q^t < 1$) but is strictly below the Harrod-Domar growth rate during recessions (when $q^t > 1$).

Note also that the limit-cycle need not be a two-cycle as depicted in figure 4. Figures 5 and 6 depict $n > 2$-cycles, with either a *debt-build-up phase* during which borrowers are getting poor relative to creditors (figure 5) or a *profit-reconstitution phase* during which borrowers are getting richer relative to creditors - both because creditors get the low rate σ_2 on their savings and because borrowings are low during slumps (figure 6). The relative length of boom (i.e. debt build-up) and slump (i.e profit reconstitution) phases will depend upon the basic parameters of the model: expansionary phases with debt-build-up will last longer when the slope of the (B′) curve at the point b is close to 1 (or equivalently, when β is close to 1); on the other hand the length of recessions phases will go up when the slope of (S′)-curve gets closer to 1 (in particular when ν and/or σ_2 are large).

Intuitively, during an expansionary phase, investment (and therefore borrowings) will go up since output and profits, and therefore the borrowing capacity of investors, are growing. And the higher the share of capital β in the revenue from the high-yield activity, the more will profits increase over time and therefore the longer will the investors' borrowing capacity keep on absorbing total savings. Hence the higher β, the longer the debt-build-up phases. On the other hand, the larger σ_2 and/or ν, the larger the ratio of savings over planned investment when the economy enters a recession; this together with the fact that investors must repay a higher interest rate σ_2 to

[17] In particular, a highly underdeveloped economy where businesses rely entirely on their own retained earnings to invest, will not cycle.

their lenders throughout the recession, implies that it will take longer before the borrowing capacity of investors can again absorb the totality of savings (at which point the economy can re-enter a boom).

See Figure 5: Prolonged debt-build-up. [18]

See Figure 6: Prolonged recession. [19]

3.3 The Effects of Shocks

How does such an economy react to shocks? In order to answer this question we first look at how the shocks move the two curves (B′) and (S′). We will focus on two kinds of shocks: shocks to σ, which are naturally interpreted as productivity shocks and shocks to σ_2 which may either be thought of as productivity shocks (for example if this asset is thought of as home production) or as policy shocks (for example if this asset is thought of as money, the shock could be a change in the variance of the inflation rate).

It is easy to see from equation B′ that the curve (B′) is unaffected by changes in σ and σ_2. This should be intuitive: an increase in σ during a boom increases both the return to investment and the return on savings exactly in the same proportion and therefore does not affect the distribution of wealth between the savers and investors (measured by q). A change in σ_2 has no effect because in a boom no one puts their money in the inferior asset.

A trivial calculation establishes that in the range in which the (S′) curve is relevant (i.e. as long as $q > 1$), an increase in σ and a fall in σ_2 lowers the (S′) curve. This too should be intuitive: in a slump an increase in σ benefits the investors but not the savers, while a fall in σ_2 hurts the savers while benefitting the investors.

[18] This case is more likely to arise when the slope of the (B′)-curve is sufficiently close to 1 at b, that is when β is close to 1.

[19] This case is more likely to arise when the slope of the (S′)-curve is sufficiently close to 1, i.e. when

$$\underbrace{\left|\sigma_2 - \mu(1-\beta)\frac{\sigma}{\nu} - \frac{1}{\nu}\beta\sigma + (\frac{1}{\nu} - 1)\sigma_2\right|}_{=\frac{1}{\nu}|\sigma_2 - (\mu(1-\beta)+\beta)\sigma|}$$

is sufficiently small.

It follows that if the economy is in a permanent boom, neither a temporary nor a permanent shock to productivity (i.e. σ going up) has any effect on the distribution of wealth between savers and investors. It does of course have an effect on the rate of growth: in periods where σ is higher the growth rate will be proportionally higher. However because there is no effect on q, the entire effect on the growth rate is registered in the period of the shock. In other words, if the shock is permanent the economy immediately goes to its steady state growth rate and if it is temporary, after the shock, the economy grows at a the same rate as before the shock, albeit starting at a higher *level* of G.N.P. Changes in σ_2 have no effect for obvious reasons. Figure 7a (resp. 7b) describes the dynamic effects of a permanent (resp. temporary) increase in σ on the growth rate when the economy is - and remains - in a permanent boom.[20]

See Figures 7a and 7b

The picture in a permanent slump economy is quite different. Note first that the steady state growth rate g_s is an increasing function of σ but a decreasing function of σ_2: that is, the positive effect of a higher σ_2 on the returns to the fraction of savings that are not invested in the high-yield activity is more than offset by the negative effect of a higher σ_2 on debt repayments and therefore on the steady-state debt capacity of investors. Taxing σ_2 and throwing the tax revenues away would actually increase the long-run growth rate![21]

The process of convergence to the new steady state is also quite different from that in a permanent boom economy. As can be seen from figure 8a, a permanent rise in σ or a permanent fall in σ_2 shifts the (S′) curve downward. As a result, q falls initially and then continues to fall as it converges towards its steady state level. Since, *ceteris paribus*, a fall in q raises the growth rate[22] by shifting the distribution of wealth towards the investors, the initial

[20]In this and the following figures, we assume for simplicity that the economy is initially at a steady-state.

[21]Alternatively, the government could impose interest rate ceilings so as to push the interest rate below σ_2 (see section 4 below).

[22]The growth rate in a slump period is given by $(1-\alpha)(\frac{1}{q}\sigma + (1-\frac{1}{q})\sigma_2)$.

rise in σ or fall in σ_2 will end up having both a *direct* and *indirect* positive effect on the growth rate, as shown in figure 8b.

See Figures 8a and 8b

Similarly, Figure 9a and 9b shows that the growth-enhancing effects of a one-period increase in σ or a one-period fall in σ_2 will also persist beyond the period of the shock. This is again due to the fact that the shock shifts the distribution of wealth in favor the investors (i.e. shifts q down) and this distributional effect only dies away gradually.

See Figures 9a and 9b

The effects of shocks in a cyclical economy are, for obvious reasons, more complex. In particular, an increase in σ has an ambiguous effect on average growth in this case: on the one hand it increases growth during booms, but on the other hand investors end up accumulating a higher debt burden during booms, which causes recessions to be more severe (i.e. with higher q ratios in recessions). The overall effect on average growth is unclear in general. However for the same reasons as in the permanent slump regime, an increase in σ_2 will have an unambiguously negative effect on average growth: a higher σ_2 leads to higher debt repayments during slumps and therefore both to lower growth during and also to longer periods of slumps.

The process of convergence to the new cycle after a permanent shock is potentially quite complicated (as the reader can readily verify by moving the (S′) curve in figures 4, 5 or 6) and defies a straightforward classification. However, what remains unambiguously true is that unlike in the permanent boom case, the growth rate will only gradually adjust to its new steady-state level.

The above analysis ruled out the possibility that the shock results in shifting the economy from one of our regimes to another. As is evident from the condition for the cycling case, $\beta + (1-\beta)\mu > \nu + \frac{\sigma_2}{\sigma}(1-\nu)$, high values of σ and low values of σ_2, tend to move the economy away from a permanent slump towards a cycle. This is not necessarily a good thing: it turns out that an increase in σ can actually *reduce* average growth by forcing the economy to shift from a permanent slump regime to a cyclical regime because at least over some open parameter interval, it is better to never have a boom than to have it some of the time.[23]

3.4 Robustness of our Results

Our results are derived in an exceedingly pared down model and it is legitimate to wonder whether these results would survive in a more realistic model. Here we make a number of comments emphasizing the directions in which the model can be extended without losing our basic results.

1. In the present version of the model, the equilibrium interest rate r fluctuates discontinuously between one high value and one low value. We have constructed a version of the model where investors can choose among a continuum of technologies and therefore where the equilibrium interest rate can take on a continuum of values. In this model, the interest rate changes smoothly over time and we still get cycles (see Appendix B).

2. If consumption-savings decisions were taken at the beginning of the period instead of the end, an expected fall in interest rates would reduce savings, limiting the fall in interest rates and thereby limiting the extent of fluctuation. However, if this were to eliminate all fluctuations, the interest elasticity of savings would have to be quite high, which it does not seem to be (Attanasio-Weber (1993)). The same comment applies to whether our result will change if savers are forward-looking.

3. Even if investors were long-lived instead of living for just one period, in our model they would have no reason to postpone investment since

[23]Namely, assume $\mu < \nu$ and consider $\sigma = \sigma^*$such that $\mu(1-\beta)+\beta = \nu + \frac{\sigma_2}{\sigma^*}(1-\nu)$. If $\sigma < \sigma^*$, the economy is in a permanent slump regime. As $\sigma \longrightarrow \sigma*$, the permanent slump growth rate increases: $g_s \longrightarrow g^* = (1-\alpha)\sigma$. But if σ goes above σ^*, then the economy exits the permanent slump regime and begins to cycle, that is, the average growth rate falls below its Harrod-Domar value. It follows that at least over some range $[\sigma^*, \sigma^* + \varepsilon]$, the average growth rate is decreasing function of σ (although there needs not exist any discontinuity in average growth rates). For the same reasons, the average growth rate is an increasing function of σ_2 at least over some range $[\sigma_2^* - \varepsilon, \sigma_2^*]$, in the region where an increase in σ_2 shifts the economy from a cyclical regime to a permanent slump.

investment earns a higher return than the cost of capital ($r \leq \sigma_1$). That is, slumps are indeed the best period to invest (interest rates are low), but investors have no reason to delay their investment until the next slump because they will always have more money to invest next period if they invest today.[24]

4. Our results clearly depend on the interest rate being endogenous and not given from outside, as it might be, for example, in a small open economy. However this seems a reasonable approximation to the reality in many economies (see Feldstein-Horioka (1980) for evidence showing that capital tends not to move across borders).

5. We show in appendix A that our results are also robust to allowing investors and savers to write long-term debt contracts. Intuitively a long term contract may allow borrowers to postpone a part of the repayment on their debt to periods when they expect to be relatively cash rich and thereby limit the variation in q^t. We show in the appendix that while this intuition is correct, the economy will continue to have cycles under conditions which are somewhat stronger than those assumed in our basic model.[25]

6. Two empirical predictions which emerge from our analysis are, first that the ratio of debt - obligations over cash-flow should peak towards to end of booms, and second that the real interest rate paid by firms should be strongly pro-cyclical. While the first prediction appears to be strongly supported by existing empirical evidence (e.g., see Bernanke-Gertler (1995)), the evidence on the latter prediction seems less clear. Indeed the existing interest rate data for most developed countries shows that the real interest rates on bonds of various maturities are only weakly

[24] On the other hand, forward-looking firms may tend to accumulate savings at a higher rate during booms (e.g. by cutting dividends) in order to expand their investment in slumps. This would also tend to reduce the amplitude of the fluctuations, but it should not eliminate all fluctuations unless the shareholders of the firm are extremely patient.

[25] Actually this result assumes that long-term contracts are no harder to enforce than short-term contracts - the conditions under which the economy cycles would be weaker if we were prepared to make the reasonable alternative assumption that long-term contracts are harder to enforce.

procyclical (nominal rates, on the other hand, are strongly pro-cyclical). This is potentially a serious criticism of our model since the basic mechanism driving our results relies on the interest rate being substantially lower in slumps - this is what allows investors to accumulate enough wealth to take the economy out of the slump. .

Note however that one must be careful in interpreting the evidence on real interest rates: for one, short rates always pass above long rates during booms,[26] which, given the fact that the share of short-term debt tends to go up at the end of booms,[27] *implies that the average interest rate paid by firms actually varies more strongly with the business cycle than either the long or the short rate.*[28] Second, the empirical results relate to risk-adjusted interest rates. Part of the reason why interests rates do not decline very much in slumps is probably that the risk premium goes up in slumps in order to adjust for the higher risk of default during slumps. Our model does not have defaults, which is why this effect never comes up. Introducing this effect into our model would indeed raise interest rates in slumps, but it would also lead to a certain fraction of the debt being wiped out through default, so that the rate of growth of the net wealth of investors in slumps may actually be *higher* when there are defaults than it would be in our model.

4 Policy Analysis

What can a government do in the context of our model in order to limit the extent of cyclical fluctuations and the length of suboptimal growth periods associated to slumps?

The ultimate source of the instability and the associated inefficiency highlighted in this paper is inequality in the access to the most rewarding investment opportunities - what we have called dualism. An obvious policy response would be to reduce the extent of dualism in the economy: if the

[26]On this point see, Stock and Watson (1997).

[27]See Friedman-Kuttner (1993a, 1993b) for evidence on the shifts in the composition of firms' debt portfolios along the business cycle.

[28]Friedman-Kutnner (1993a, 1993b) make a similar point.

government could improve access to credit (reduce ν) and/or access to direct investment opportunities in production (increase μ) so that $\mu > \nu$, then the economy would switch to a regime of permanent boom and no slump would ever occur in the long-run (figure 3). This argues for emphasizing policies which improve credit access, create infrastructure and human capital in areas where such things are missing and reduce barriers to entry. Therefore our model suggests that the issue of macroeconomic stabilization should not be examined separately from the issue of structural reforms: removing the institutional obstacles and rigidities hat separate savers and investors can promote growth, stability and equity at the same time.

It is worth stressing that the immediate policy prescription of this model is the improvement of access to investment opportunities for savers, which is not the same thing as the more broad-based promotion of equity emphasized, for example, in many of the recent papers on inequality and growth.[29] The difference comes from the fact that the people who have the most savings to invest are not necessarily poor and probably do not include the very poor: policies that targeted at savers may not promote overall equity at least in the short run.[30]

However, such structural policies may be difficult to implement (especially in the short-run), and in some cases they are just not feasible: governments cannot simply decide that access to credit and investment opportunities should be extended. Interestingly, our model also allows us to explore the effects of more conventional countercyclical macroeconomic policies. Our theory of business fluctuations describes slumps as periods where a positive fraction of savings are not being used efficiently ("idle" savings) because of the investors' limited borrowing capacity. Assuming that the structural parameters of the model cannot be changed, the obvious way to prevent the occurrence of recessions would be to transfer those idle savings from savers to investors whenever necessary. That is, if at the beginning of some period

[29] See Benabou (1996) for a survey.

[30] Nothing in our model would change if we added a class of poor agents in the economy who never save anything and therefore have no wealth whatsoever, although this would increase inequality as it is usually measured. This distinguishes our model from political economy models in which the have-nots often play a crucial role.

t the investment capacity $\frac{W_B^t}{\nu}$ of investors is smaller than the total amount $W_L^t + W_B^t$ of available savings (i.e. $q^{t-1} = \frac{W_L^t + W_B^t}{W_B^t/\nu} > 1$), then in order to achieve the Harrod-Domar rate of growth it is sufficient to redistribute wealth dW from savers to investors such that:

$$\frac{W_B^t + dW}{\nu} = W_L^t + W_B^t$$

This policy ensures that all available savings will be invested in the high-yield activity, and therefore that output will grow at rate $(1-\alpha)\sigma$. Moreover, note that such a growth-enhancing countercyclical policy does not necessarily entail negative distributive consequences for savers. First, the wealth transfer dW will boost the demand for investment credit and therefore raises the equilibrium interest rate from its depressed value σ_2 to its high value $\sigma_1 = \beta\sigma > \sigma_2$. Therefore the interest income of savers shifts from $\sigma_2 W_L^t$ to $\sigma_1(W_L^t - dW)$. In particular, if the required wealth transfer dW is small, (i.e. if the coming recession is not too severe or, more formally, if we start with q^{t-1}close to 1), then the interest income of savers is higher with the expansionary wealth transfer dW than without it, especially if σ_2 is small relative to σ_1.[31]

In addition, if the wage rate in this economy has an efficiency wage component (so that employed workers earn some rents) the wealth transfer dW can also benefit the non-investors because of its expansionary effects on the labor market. The explicit calculations for this case are given in a previous version of this paper.

In practice, a countercyclical wealth transfer policy needs not take the form of redistributive wealth taxation: the same goals can be achieved more easily through expansionary monetary and fiscal policies. One natural interpretation of expansionary monetary policy in the context of our model is that during periods where the limited borrowing capacity of investors forces the economy to enter in a recession, monetary authorities may decide to print money and give it to over-indebted businesses. Given the resulting increase

[31]The reason why private actors do not implement this wealth transfer themselves is obviously because in this perfectly competitive environment they do not internalize the aggregate effect of such transfers on the price of capital. Note also that this interest rate effect is not always strong enough to compensate the savers for the extra taxes they pay: for example, if $\nu = 1$, i.e. if the savers need to transfer all their wealth to investors in order to ensure Harrod-Domar growth, then their interest income will unambiguously fall.

in the price level, this is equivalent to a real transfer dW from savers to investors, and will have the same effects as described above.[32]

Our model also delivers a very natural interpretation of countercyclical fiscal policies: *since slumps are periods with idle savings, governments can promote recovery by issuing public debt in order to absorb those idle savings and finance investment subsidies (and/or tax cuts for businesses)*. That is, at the beginning of any period t where there are excess savings and the economy is about to enter a recession ($q^{t-1} > 1$), the government should issue new public debt dB and use the proceeds to finance investment subsidies and/or tax cuts dT for investors. So long as government bonds yield a return at least equal to σ_2, savers will be willing to lend their money to the government in order to finance this wealth transfer to investors. If public debt repayment at the end of the period is financed out of general tax revenues, then this countercyclical fiscal policy is equivalent to a direct wealth transfer dW from savers to investors. In fact, in the extreme case where the increase dB in public debt at t^- used to finance the investment subsidy and/or tax cut $dW = dB$ is paid back at t^+ by a tax hike $dT = \sigma_2 dB$ falling entirely on the labor income and/or interest income of savers, both policies are exactly equivalent. In particular, under the conditions described above, such a countercyclical fiscal policy can be in everybody's interest (including the savers) because of its expansionary effects on both interest and labor income.[33]

Finally, note that such a countercyclical transfer policy may have to be permanently sustained. In particular, when the government tries to prevent a

[32] Although this crude type of monetary policy is by no means unheard of, this is not the way modern central banks usually intervene (at least in western countries). Qualitatively, it is likely however that more standard interventions (such as lowering the discount rate below the equilibrium interest rate level) will also result into the same net real transfer from lenders to borrowers.

[33] If the government had a higher administrative and legal ability than lenders in terms of enforcing debt repayments, then an expansionary fiscal policy would be in the savers' interest even in the absence of the induced expansionary effects on interest and labor income: public debt could be paid back at t^+ by raising taxes only on investors, which in effect would amount to substitute public lending to investors for incentive-constrained private lending. However if the government faces the same incentive constraints as private lenders, this is however not feasible. For example, in the context of the simple credit market model described in Appendix A, a tax on investors' income would induce investors to default in case the tax is too high, and in any case would amount to a reduction in σ and therefore to a lower credit multiplier (unless the government can tax investors while making sure that they don't shirk on their other obligations).

recession at t^- by lowering the savings/investment ratio q^{t-1} from its laissez-faire value ($\dot{q} > 1$) to 1 (through the wealth transfer dW), the equilibrium interest rate goes up from $r_t = \sigma_2$ to $r_t = \sigma_1$, which in turn implies that the savings/investment ratio q^t next period will be higher than 1.[34] In other words, if the government stops intervening in period $t+1$, the economy will fall into a recession in the following period: the effect of the period-t counter-cyclical policy is then just to postpone the recession from period t to period $t+1$. In order to guarantee permanent Harrod-Domar growth rates (and to raise everybody's welfare under the conditions described above), the government will need to implement a permanent policy regime of wealth transfers from savers to investors at the beginning of each period (e.g. via investment subsidies). One alternative strategy would be to overshoot in period t^-, i.e. to implement a wealth transfer dW higher than the required minimum amount, so as to push q^{t-1} below 1 and thereby ensure that the boom will continue in period $t+1$. This would allow the government to achieve a permanently high growth rate through policy interventions that remain only periodic. Another option would be to enforce interest rate ceilings at period t^- together with the wealth transfer dW: if the government can ensure that the interest rate paid by investors does not go up all the way to σ_1, then the boom could be maintained for ever without further policy interventions.[35]

5 Conclusion

This paper tried to develop an analytical framework for reconsidering traditional questions in macroeconomics in a context in which the idea of a sep-

[34] As long as we were not in a permanent boom regime (in which case there is no policy issue), $q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) > 1$ (see section 3 and figures 2-4).

[35] In the case of persistent volatility (figure 4), we have: $q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) > 1$ and $q^{t+1}(q^t = 1, r_{t+1} = \sigma_2) < 1$ (see section 3). By continuity, it follows that there exists some interest rate $r^* \in]\sigma_2, \sigma_1[$ such that $q^{t+1}(q^t = 1, r_{t+1} = r^*) = 1$. If the government sets an interest rate ceiling less or equal to r^*, the economy will remain in a permanent boom. In the case of a permanent slump regime (figure 3), the government would need to set an interest rate ceiling below σ_2. Interest rate ceilings in the presence of excess demand for investible funds can obviously entail non-negligible costs: when it is enforced successfully we risk losing the benefits of the price as a screening device (in the model everyone is identical, but in the world some investors are better than others and the price mechanism plays important screening role).

aration between savers and investors is taken seriously. Although the model in this paper already generates a number of interesting results and insights, e.g. on the sources of macroeconomic volatility, on the dynamic response to aggregate shocks and finally on the optimal design of government policies, we see this work as no more than a very first step in a broader research agenda. In particular, we are already exploring two potential extensions of the above framework, first on the open economy implications of the same kind of separation assumptions, and second on the optimal design of government policy when both investment and consumption activities are subject to credit market imperfections in an otherwise similar economic environment to the one analyzed in this paper

Appendix A:
A Model of the Capital Market

i) Basic Model

In this sub-section we outline a simple microeconomic model of (imperfect) lending that generates a constant credit-multiplier $\frac{1}{\nu}$ of the kind assumed in the paper.

Consider a borrower who needs to invest $W + L = I$ in the high-yield technology, where W denotes his/her initial wealth and L his/her requested loan. The source of capital market imperfection is ex post moral hazard and costly state verification. Namely, once the return $\sigma(W + L)$ is realized, the borrower can either repay immediately and get a net income equal to $\sigma(W + L) - rL$, or he/she can stall. Stalling revenues away from the lender has a cost to the borrower (who has to keep ahead of the lender), and let this cost be a fixed proportion τ of total revenues. Finally, whenever the borrower defaults on his/her repayment obligation, the lender may still invest effort into debt collection. Specifically, assume that a lender who incurs a non monetary effort cost $L \cdot C(p)$ has probability p of collecting her due repayment $r \cdot L$.[36]

Anticipating a monitoring effort p from the lender, the borrower will decide not to (strategically) default if and only if:

$$\sigma_1(L+W) - rL \geq \sigma_1(1-\tau)(L+W) - prL,$$

or equivalently:

$$L + W \leq \frac{W}{1 - \frac{\sigma_1 \tau}{r(1-p)}}. \tag{A1}$$

[36]Here, we are implicitly assuming that debt repudiation is not verifiable by outsiders, so that the lender's ex post revenue cannot be made contingent upon whether default took place or not. This in turn explains why the lender cannot collect more than $r \cdot L$, even following a strategic default by the borrower. Alternatively, had we assumed that lenders can sign debt-contracts allowing them to collect everything in case of strategic default (and successful monitoring), then the credit-multiplier would always be a declining function of the interest rate (e.g. as in Holmstrom-Tirole (1995)) instead of being constant as in our model in this paper. As we argue below, having ν increase with r would not dramatically affect our analysis.

Now, turning to the choice of the optimal monitoring policy p, the lender will solve:

$$\max_p \{p \cdot rL - L \cdot C(p)\},$$

so her optimal choice of p is given by the first-order condition:

$$r = C'(p).$$

In the special case where $C(p) = -c \cdot \ln(1-p)$, we obtain:

$$r = \frac{c}{1-p},$$

so that the incentive-compatibility constraint (A1) becomes simply:

$$L + W = I \leq \frac{1}{\nu} \cdot W,$$

where $\nu = 1 - \frac{\sigma_1 \tau}{C}$ is indeed independent of the interest rate.

The case where the credit-multiplier $\frac{1}{\nu}$ is constant is clearly a knife-edged case, and departing from the monitoring cost function $C(p) = -c \cdot \ln(1-p)$ one can get this multiplier to increase or decrease with r. If $\frac{1}{\nu}$ increases with r, then the interest rate will increase during booms and then drop down to σ_2 as investment demand falls below savings (which in turn will happen sooner than before as debt-repayment obligations build up faster than in the case where ν is constant). On the other hand, if $\frac{1}{\nu}$ decreases with r, then the interest rate will decrease during booms which in turn will delay (and sometimes preclude[37]) the occurrence of recessions. For example, if $C(p) = \frac{1}{2}cp^2$, we obtain the first-order condition:

$$r(1-p) = r - \frac{r^2}{c},$$

so that the incentive constraint (A1) becomes:

$$L + W \leq \frac{1}{\nu(r)} \cdot W,$$

where:

$$\nu(r) = 1 - \frac{\sigma_1 \tau}{r - \frac{r^2}{c}}$$

[37]Namely, when the interest rate decreases sufficiently rapidly during booms.

is increasing in r whenever $r > \frac{c}{2}$, thus in particular when $\sigma_2 > \frac{c}{2}$.

On the other hand, if $C(p) = -\frac{c(1-p)^{1-\theta}}{1-\theta}$ with $\theta > 1$, we have:

$$\frac{W}{L+W} = \nu(r) = 1 - \tau r^{\frac{1-\theta}{\theta}} c^{\frac{1}{\theta}},$$

which is increasing in r, so that $\frac{1}{\nu(r)}$ decreases in r. $\Box$.

Remark: If credit-rationing was due to ex ante rather than ex post moral hazard (e.g. formalized as in Holmstrom-Tirole (1995)), then $\nu(r)$ would also increase with r, but one can show that it will not increase too rapidly when the marginal efficiency of effort, measured by the ratio of the increase in the probability of success over the effort cost required to achieve such an increase, is not too high. The occurrence of cyclical (or volatile) growth patterns will then be preserved.

ii) Long-term debt-contracts

In this section we investigate the effects of introducing long-term credit contracts into our basic model. Intuitively a long term contract may allow borrowers to postpone a part of the repayment on their debt to periods when they expect to be relatively cash rich and thereby limit the variation in q^t. Here we ask whether such intertemporal substitution can eliminate the cycle we get in our basic model.

Consider an extension of the above model where agents live for two periods instead of one period but each new generation is only born when the previous generation dies (i.e. it is a non-overlapping generation model). For simplicity we also assume: (a) that in the absence of long-term lending, the economy converges to a two-cycle; (b) that agents (borrowers and lenders) do not care about consumption smoothing and therefore may as well consume everything at the end of their two period life. During a current boom, young borrowers might then be interested in signing a two-period debt contract with their lending counterparts whereby a "current" rate $r_1 < \sigma_1$ would be paid at the end of the first period and a future rate $r_2 > \sigma_2$ would be paid at the end of the second period. Borrowers may also engage in further short-term borrowing during the second period.

The borrower's incentive constraint at the end of the first period will be:

$$\sigma_1(L_1 + W) - r_1 L_1 \geq \sigma_1(1-\tau)(L_1 + W) - p r_1 L_1, \tag{A2}$$

where L_1 is the long-term debt contracted at the beginning of period 1 and where $r_1(1-p) = c$ under the same monitoring technology as the one postulated in part (i) of this Appendix.

The above incentive-constraint can then be reexpressed as in (i) above, namely:

$$L_1 = \left(\frac{1}{\nu} - 1\right) W, \tag{A2'}$$

where $\nu = 1 - \frac{\sigma_1 \tau}{c}$.

The borrower's accumulated cash-net of current debt-repayment - at the end of period 1, will be equal to:

$$\widehat{W} = \sigma_1 W + (\sigma_1 - r_1) L_1.$$

However the borrower will not be able to invest up to the amount $\frac{\widehat{W}}{\nu}$ simply because $\widehat{W}$ does not reflect his true wealth at the beginning of period 2. Short-term lenders in period 2 will indeed take into account the existence of further outstanding debt-repayment obligations towards long-term lenders.

More formally, if L_2 denotes the additional amount to be borrowed short-term in period 2, the borrower's incentive constraint at the end of that period is written as:

$$\begin{aligned} & \sigma_1(\sigma_1 W + (\sigma_1 - r_1)L_1 + L_2) - r_2^* L_2 - r_2 L_1 \\ \geq \;& \sigma_1(1-\tau)(\sigma_1 W + (\sigma_1 - r_1)L_1 + L_2) - p_2 \cdot r_2^* L_2 - p_1 r_2 L_1, \end{aligned} \tag{A3}$$

where, if we stick to the same debt-monitoring technology as before:

$$r_2^*(1-p_2) = r_2(1-p_1) = c. \tag{A4}$$

Using (A4) to eliminate p_1, p_2, r_2 and r_2^* in the incentive-constraint (A3), we end up reexpressing (A3) as:

$$\sigma_1 W + (\sigma_1 - r_1)L_1 + L_2 \geq \frac{c}{\tau\sigma_1}(L_1 + L_2). \tag{A3'}$$

Now if $r_1 = 0$ (which corresponds to the best long-term contract candidate for maximizing the borrowing capacity L_2 and thereby possibly delaying the occurrence of a slump), the above incentive constraint becomes:

$$L_2 \leq \frac{\sigma_1 W + (\sigma_1 - \frac{c}{\tau\sigma_1})L_1}{\frac{c}{\tau\sigma_1} - 1}.$$

Using (A2′) to substitute for L_1, we get:

$$L_2 \leq \Omega \cdot W,$$

where

$$\Omega = \frac{1}{\nu}(\sigma_1 \frac{1}{\nu} - (\frac{1}{\nu} - 1)) = \beta\sigma\frac{1}{\nu} - (\frac{1}{\nu} - 1).$$

Whenever $\Omega < \sigma$, which in particular will be the case for β sufficiently small and ν sufficiently close to 1, then it will still be the case that investment capacity will grow at a lower rate than savings grows in a boom, so that even if we allow for long-term debt contracts the occurrence of slumps will remain unavoidable. Note however that allowing for long-term debt contracts may increase the borrower's investment capacity as compared to the pure short-term borrowing case and yet not to a sufficient extent that the occurrence of slumps can be avoided: for example, for ν close to 1,

$$\underbrace{\frac{1}{\nu}\sigma_1 W}_{\substack{\text{invstment capacity}\\ \text{under short-term}\\ \text{borrowing}}} \quad < \quad \underbrace{\Omega \cdot W}_{\substack{\text{investment capacity}\\ \text{under long-term}\\ \text{bargaining}}} \qquad \text{but still } \Omega < \sigma. \quad \square$$

Appendix B: Smooth Technology Choice

So far, we have assumed only two types of activities, a high-yield production activity with linear return rate σ on capital investment and which involves constrained borrowing with credit-multiplier $\frac{1}{\nu}$, and a low-yield (storage) activity with low linear return rate σ_2 and with implicitly unconstrained borrowing (i.e. corresponding to an infinite credit-multiplier).

A smoother version of the same framework would involve a continuum of production activities indexed by their return rates σ on capital investments. More advanced production technologies would require a higher monitoring effort from potential lenders, so that the credit-multiplier $\frac{1}{\nu}$ should be non-increasing in the technological level σ. To keep a close as possible to our basic model, we restrict attention to the case where the $\nu(\sigma)$ function has a "logistic" shape, as shown in Figure B.1 below.

See Figure B1

For given interest rate r, investors with initial wealth W will choose the production technology $\sigma = \sigma(r)$ that solves:

$$\max_{\sigma} \{\sigma \cdot \frac{1}{\nu(\sigma)} \cdot W - r(\frac{1}{\nu(\sigma)}W - W\},$$

that is:

$$\sigma(r) = \arg\max_{\sigma} \left\{ \frac{\sigma - r}{\nu(\sigma)} \right\} \tag{B1}$$

Total available savings at the beginning of period $(t+1)$ are equal to:

$$S_t = (1-\alpha)\sigma(r_t)K_t,$$

where K_t is the capital invested in period t.

Total investment demand at the beginning of period $(t+1)$ is given by:

$$I^d_{t+1} = \frac{1}{\nu_{t+1}} \cdot (1-\alpha)[\sigma r_t)(\beta + \mu(1-\beta))K_t - r_t(\frac{1}{\nu_t} - 1)K_t] = \frac{1}{\nu_{t+1}} \cdot W_{t+1}$$

where

$$\frac{1}{\nu_{t+1}} = \frac{1}{\nu(\sigma(r_{t+1}))}$$

is the credit multiplier in period $t+1$,and

$$\frac{1}{\nu_t} = \frac{1}{\nu(\sigma(r_{t+1}))}$$

is the credit multiplier through the previous period t, and W_{t+1} is the net disposable wealth of investors at the beginning of period $(t+1)$.

In equilibrium of this *smooth* "$I-S$" model, we have $I^d_{t+1} = S_t$,or equivalently:

$$\chi(r_{t+1}) = \beta + (1-\beta)\mu - \frac{(1-\chi(r_t))r_t}{\sigma(r_t)}, \tag{B2}$$

where $\chi(r) = \nu(\sigma(r))$ for all r, and $\sigma(r)$ is defined by (B1).

Equation (B2) characterizes the dynamic evolution of the economy. In particular for a suitable choice of the logistic curve $\nu(\sigma)$, one can again generate equilibrium cycles with periods of debt-build up [where productivity is high and the interest rate increases] followed by recessions [during which investment demand and interest rates fall].

We have tried to perform preliminary simulations based on the functional form:[38]

$$\nu(\sigma) = \frac{\exp(b\sigma - c)}{1+\exp(b\sigma - c)}.$$

For $\beta + (1-\beta)\mu = k \sim 0.375, b = 0.2$ and $c = 2.25$, we obtain (from (B1) and (B2)) the following relationship between r_t and r_{t+1} (see Figure B2),

See Figure B2

which accounts for the coexistence of "debt build-up" and "investment collapse" phases.

[38]The two parameters b and c are constrained in a some what unintuitive way: the maximum value that r_t can take with this functional form for ν is given by $\sigma = \frac{c}{b}$. This means that one value of r_t can generate two different values of σ_t if the range of σ is unconstrained. This is a serious problem because it means that the solution is not identifable. The constraint that we impose is that the maximum value that r_t takes is 1, which in turn implies that $\sigma = \frac{c}{b}$ at this point. Because $r \leq 1$ in all cases of interest, this effectively constrains $\sigma \leq \frac{c}{b}$. This is necessary to make $\sigma(r)$ a function. Given this constraint, we can solve for the relationship between b and c, which is $c = b+2$. Further, the exponential term is always less than or equal to 1 and hence ν is never greater than 0.5.

Appendix C: Comparison with the Goodwin Model

In this appendix, we show how our model can easily be extended in order to offer a micro-founded version of Goodwin(1967)'s model of growth cycles.

Assume that the interest rate is permanently equal to $r^* \in [\sigma_1; \sigma_2]$ (if world capital markets were perfectly integrated, r^*could simply be the world real interest rate). In order to generate fluctuations in the profit share, assume for simplicity a Leontieff production function: $Y = \sigma\ min(K, L)$. Assume that $\mu = 0$ (only business profits can be directly invested in production) and that the labor supply schedule is such that the wage rate v can take only two equilibrium values: if $v = v_1 = (1-\beta)\sigma$, then labor supply (in efficiency units) grows at a rate n from the previous period, and if $v = v_2 = (1-\beta')\sigma$ (with $\beta' < \beta$ and $v_2 > v_1$) then labor supply grows at a rate $n(1+a)$ (say that workers accept to work extra hours if they are paid a higher wage).

It follows that if $g_t (= K_t/K_{t-1}) < n$, then $v_t = v_1$, $W_t^B = g_{r^*} W_{t-1}^B$, with $g_{r^*} = (1-\alpha)(\beta\sigma + (\beta\sigma - r^*)(\frac{1}{v} - 1)$ and $g_{t+1} = g_{r^*}$. Conversely, if $g_t > n$, then $v_t = v_2$, $W_t^B = g'_{r^*} W_{t-1}^B$, with $g'_{r^*} = (1-\alpha)(\beta'\sigma + (\beta'\sigma - r^*)(\frac{1}{v} - 1)$ and $g_{t+1} = g'_{r^*}$.

Therefore if we assume $g'_{r^*} < n < g_{r^*} < n(1+a)$, there exists a 2-period cycle with $g_{2t} = g'_{r^*}, v_{2t} = v_1$ and $g_{2t+1} = g_{r^*}, v_{2t+1} = v_2$: booms contain the seeds of their own destruction because the implied rise in wage rates reduces businesses's future ability to invest (and conversely for recessions). Unlike in our model, the capital share of output is countercyclical (it fluctuates between β and β'). Note however that in both models, the net-of-debt-payments profit share of capital income (and of output) is countercyclical, and is the leading indicators of economic fluctuations.

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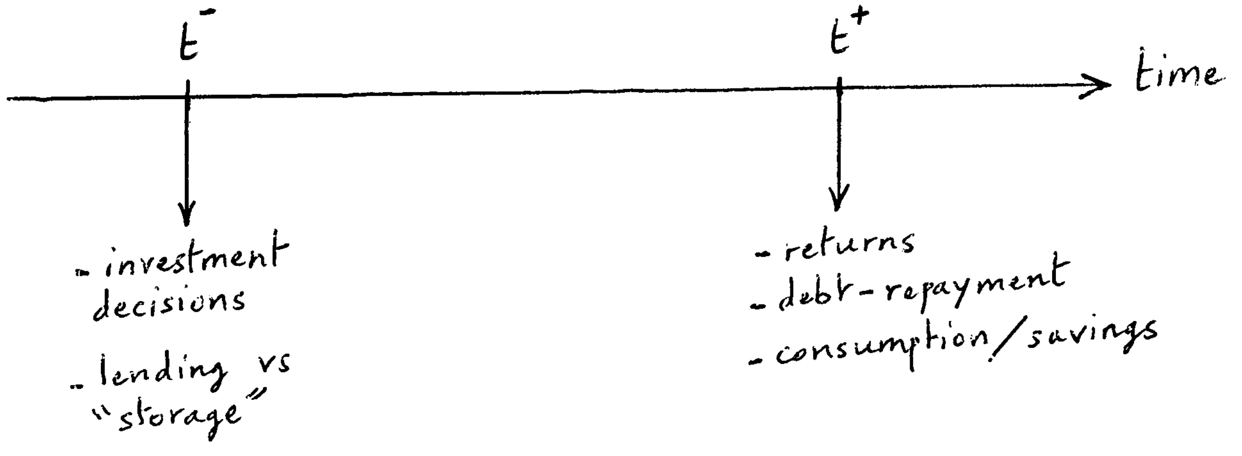


Figure 1 : The Timing

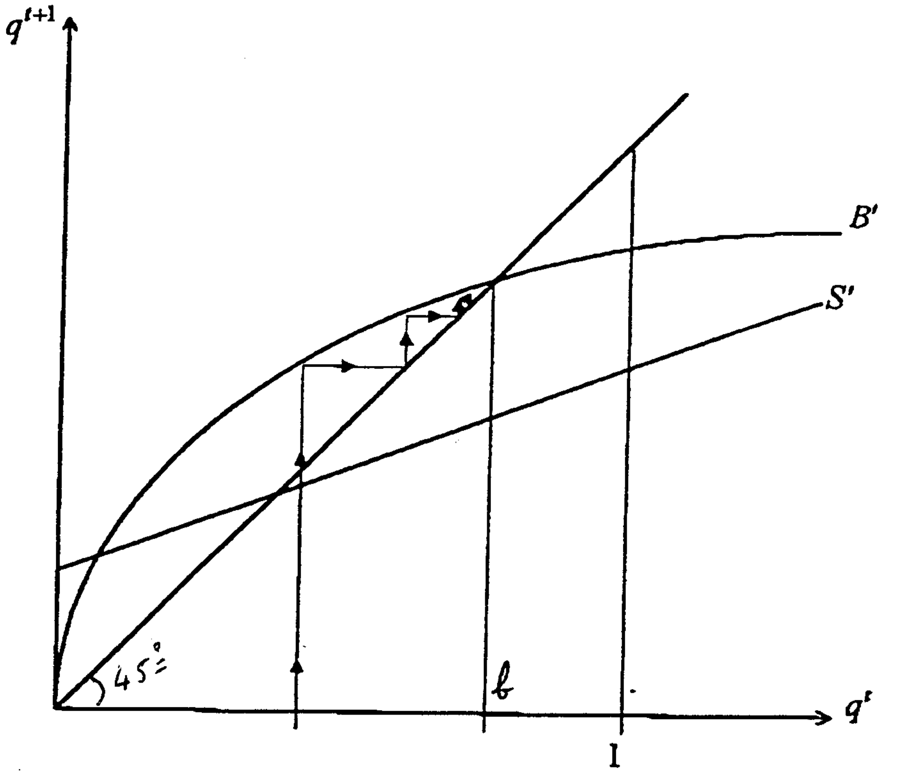


Figure 2 :
The permanent boom regime

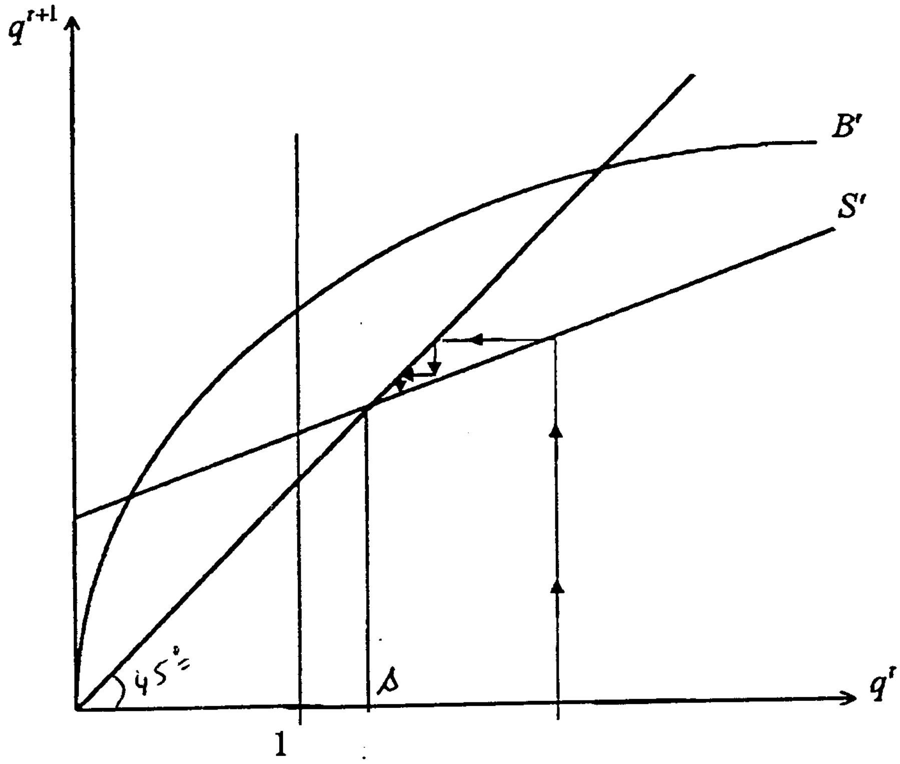


Figure 3:
The permanent slump regime

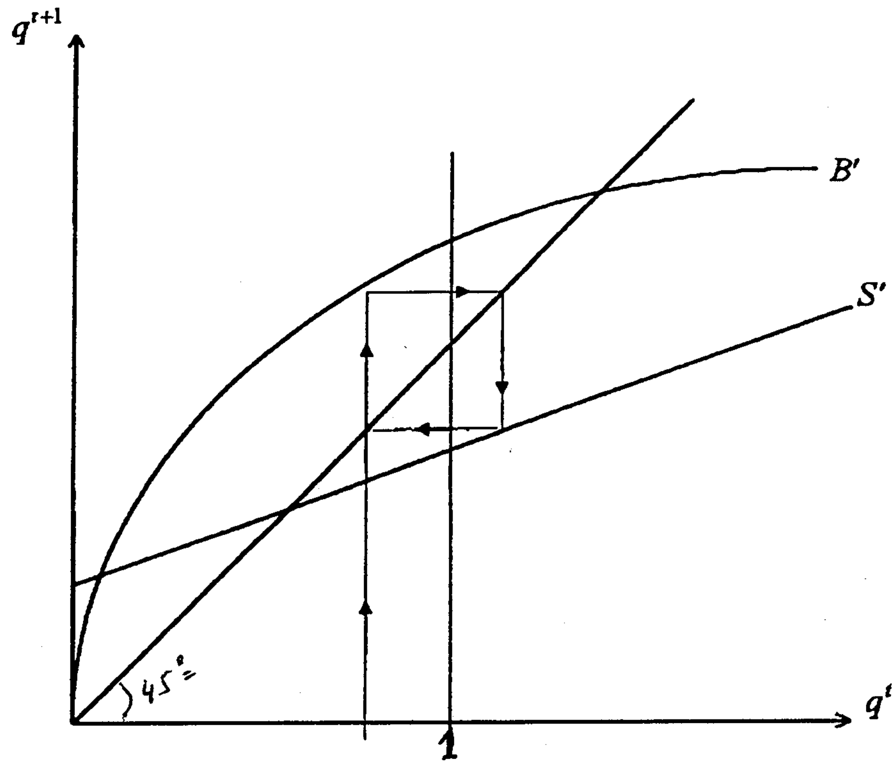


Figure 4:
The cycles regime

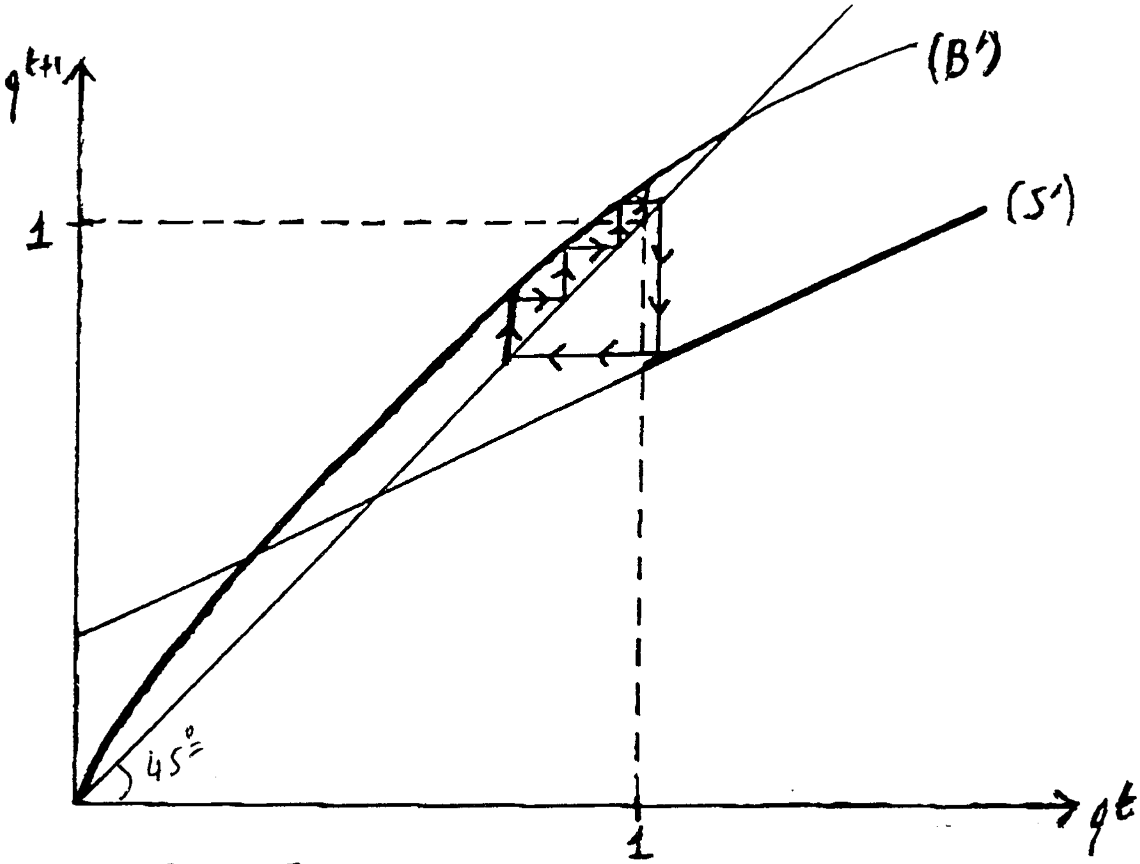


Figure 5: Prolonged boom (debt build-up)

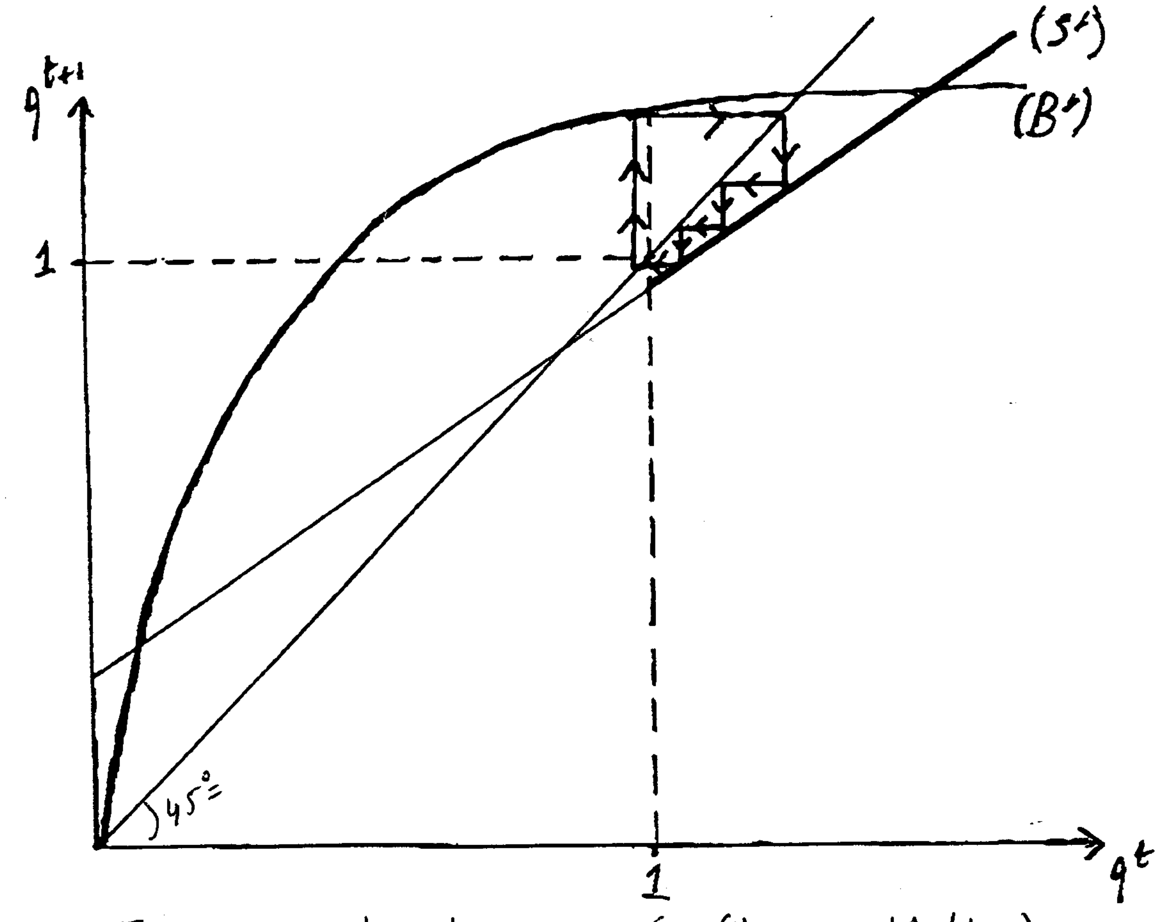


Figure 6: Prolonged recession (profit reconstitution)

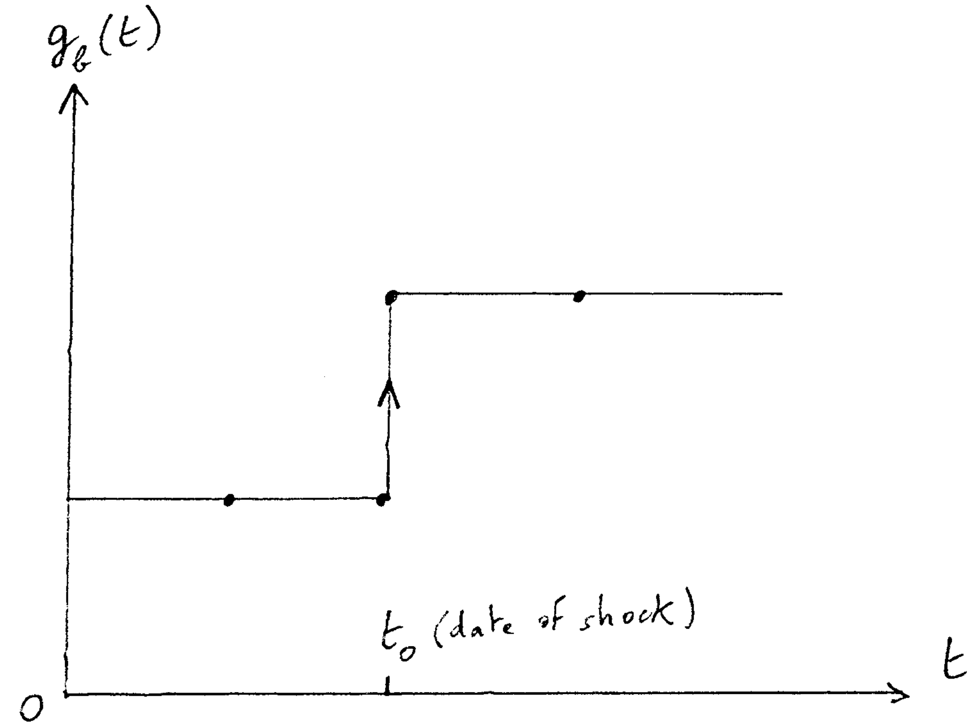


Figure 7a : Effect of a permanent increase in σ on a booming economy

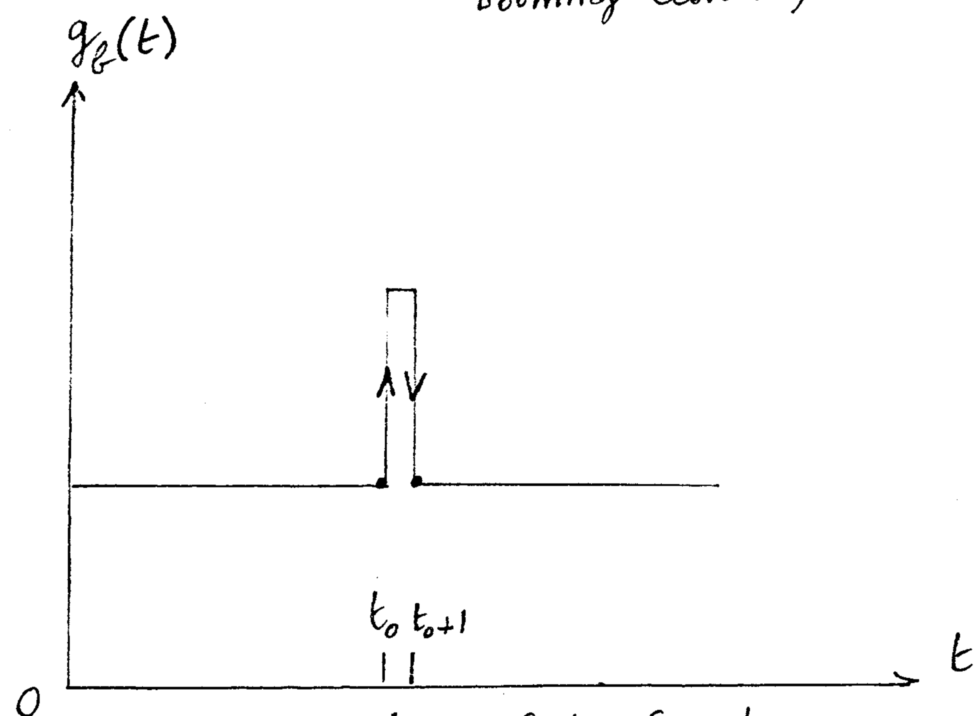
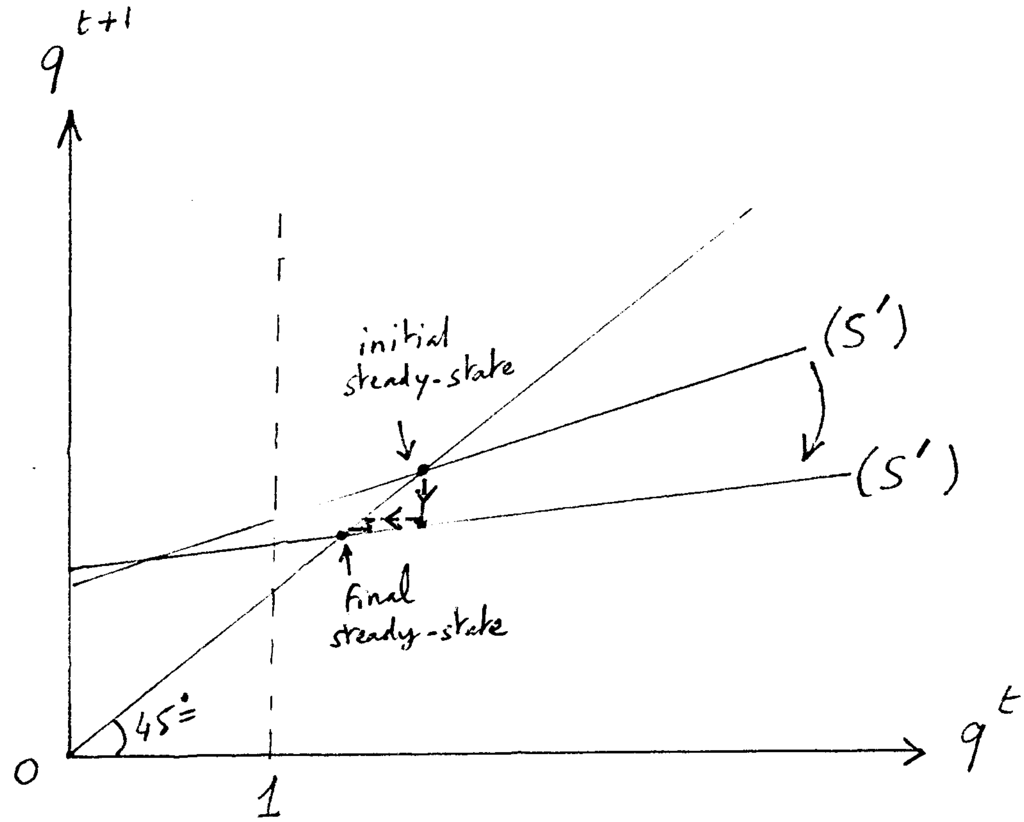
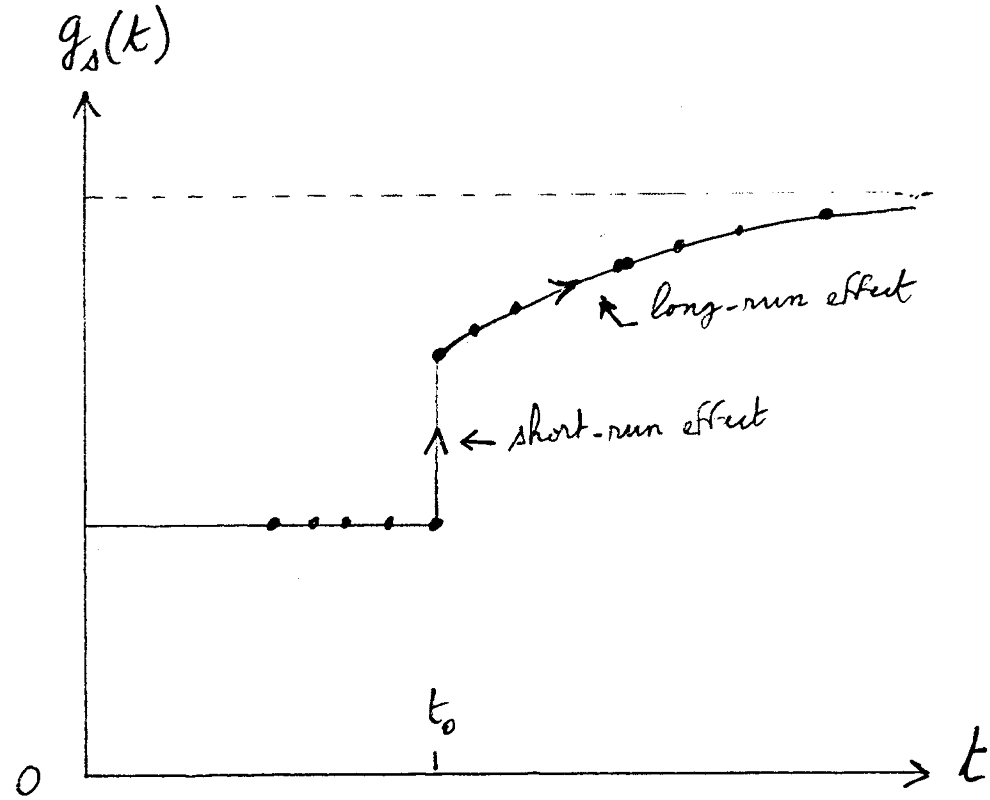


Figure 7b : Effect of a temporary increase in σ on a booming economy.

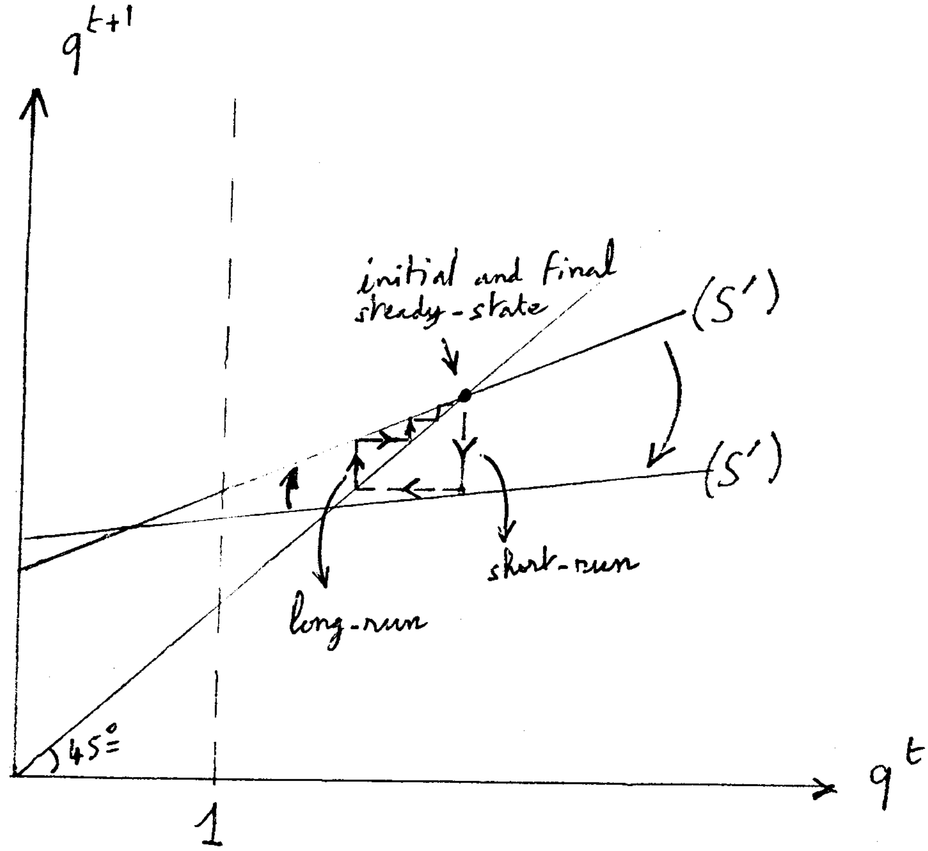


– Figure 8a –

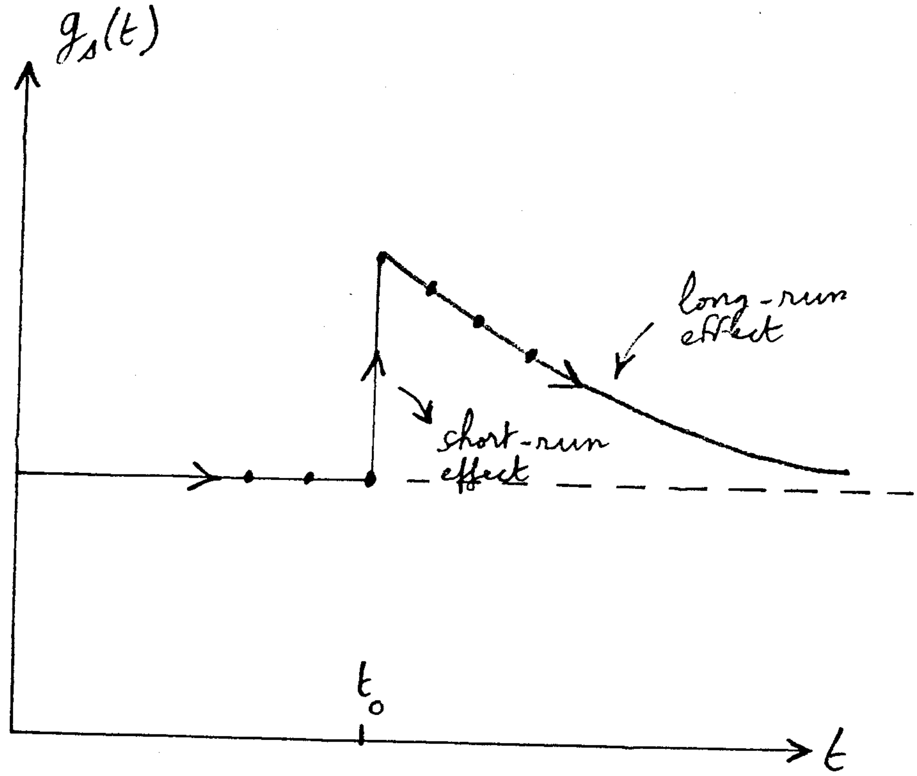


– Figure 8b –

Figure 8: Effect of a permanent increase in σ on economy in slump.



– Figure 9a –



– Figure 9b: $g_s(t)$ responds non-monotonically to temporary increase in σ, but it rem permanently above pre-shock steady-state level.

Figure 9: Effect of a temporary increase in σ on economy in slump.