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VARIABLE LABOR ADJUSTMENT COSTS AND AGGREGATE NONLINEAR DYNAMICS

Simulation based estimation and testing with US data

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COUTS D'AJUSTEMENT VARIABLES ET DYNAMIQUE NON-LINEAIRE SUR LE MARCHE DU TRAVAIL

Estimation par simulation et test sur données américaines

Fabrice Collard, Patrick Fève et Corinne Perraudin

Résumé

Ce papier étudie un modèle d'appariement sur le marché du travail, où l'environnement des entreprises est incertain. Dans un premier temps, nous adoptons une approche descriptive afin de caractériser certaines propriétés essentielles des heures travaillées aux Etats–Unis. Nous mettons en évidence une persistance et des non–linéarités significatives dans les fluctuations des heures. Nous proposons alors un modèle théorique potentiellement apte à expliquer ces résultats. Nous estimons et testons le modèle à l'aide d'une méthode d'inférence indirecte. Les résultats indiquent que le modèle est capable de reproduire le comportement observé des heures travaillées tant du point de vue de la persistance que des non–linéarités.

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Abstract

This paper investigates a model of labor-market search, where firms operate under uncertainty. The paper starts from a descriptive approach with U.S. hours worked data. Efficiency units display significant nonlinearities and specification tests allow to select a flexible non-linear model. We assess the ability of the structural model to mimic these empirical features. Since the reduced form induces untractable likelihood function, we investigate the model performances on the basis of an auxiliary model using a simulation based estimation and testing method. The results shows that our simple theoretical representation of the labor market is able to match the behavior of US efficiency unit of hours worked along a various dimensions, i.e. persistence and non-linearities.

Keywords: labor market search, asymmetries, nonlinear dynamics, simulation based estimation methods, minimum weighted residuals method.
Mots clés: Appariement sur le marché du travail, asymétries,dynamique non–linéaire, Estimation par simulation, méthodes de projection.
JEL Classification: C51, E32

1 Introduction

The existence of asymmetries within the business cycles is an old idea (see Mitchell [1927], Keynes [1936]). Recent developments in time series analysis furnish new tools to investigate the existence of such asymmetries and non–linearities. Neftci [1984] finds significant evidences for asymmetries in US postwar quarterly unemployment — *i.e.* contractions are steeper than expansions. Sichel [1993] shows that US postwar quarterly unemployment, real GNP, and industrial production are characterized by deepness asymmetry —*i.e.* troughs are further below trend than peaks are above. Nonlinear time series models and related statistical inference are needed to describe the mechanisms of asymmetric processes (see for example Hamilton [1989] for a discrete state Markov process formulation and Luukkonen and Teräsvirta [1991] for linearity tests). Rothman [1991] and Burgess [1993] emphasize that employment process exhibits significant asymmetries. Thus, the existence of an asymmetric behavior of macroeconomic aggregates seems to be a relevant assumption and should be incorporated in economic theories. Nevertheless, estimation of deep parameters underlying these models is rarely done and structural approaches to nonlinearities, such as non–linear fully specified rational expectations models, are not formally estimated and tested from the data.

> "To make much progress at all, I think we will have to re–emphasize the importance of structural estimation if we want to identify and estimate nonlinearities potentially responsible for irregularity" (Day [1992], pp 14)

There exists various theoretical models potentially able to reproduce nonlinearities and asymmetries on the labor market. Thus, because the standard linear–quadratic dynamic labor demand models (see Sargent [1978]) cannot provide any explanation of such phenomena, most of models concentrate on the specification of asymmetric adjustment costs (see Hussey [1988], Chang and Stefanou [1988], Pfann and Verspagen [1989], Pfann and Palm [1993], Jaramillo, Schiantarelli and Sembenelli [1993] and Pfann [1996] among others). The majority of these empirical studies is concerned with estimation of the structural parameters, based on Euler equations using the Generalized Method of Moments (GMM hereafter). Potential asymmetries are then deduced from the estimated values of adjustment costs parameters. The model's performance is thus not evaluated from its fit, but using a global specification test based on the Euler residuals.

In this paper, we depart from both of these theoretical specifications and quantitative evaluation methods. On a modeling point of view, we use a search model on the labor market. The reduced form is not that difficult to compute, even if a non–negativity constraint is introduced in the dynamic program. The non–linearities stem from the externalities inherent to the matching process. Moreover, the potential of the model in terms of nonlinearities can easily be illustrated in some particular cases. We do not use GMM to estimate and test the structural model. This is because we are able to compute the reduced form of the model. Indeed, in such a case, simulation based estimation methods (— e.g. Indirect Inference and Efficient Method of Moments (EMM) proposed respectively by Gallant and Tauchen [1992] and Gouriéroux, Monfort and Renault [1993].) perform better in small sample. The general consensus of simulation experiments is that the GMM criterion function leads to an "over rejection", important biases on the estimates and slow improvements as the sample size increases (see Tauchen [1986], Kocherlakota [1990] and the special issue of the Journal of Business & Economic

Statistics [1996] on GMM). Conversely, Chumacero [1996] presents various Monte Carlo experiments and finds that EMM outperforms GMM in each case. Parameters estimates are more unbiased and the gain in efficiency is substantial. Nevertheless, even if simulation based–inference has more attractive features, there exists actually very few studies which employ these methods to estimate and test nonlinear dynamic rational expectation models (see Bansal, Gallant, Hussey and Tauchen [1995], Tallarini and Zhang [1997]). The reason is that such models induce the use of numerical integration and thus large computational costs to obtain the solution within the estimation procedure. Numerical failures often occur and estimation is computationally demanding (see Bansal et al. [1995], p 273). The computational cost is not due to the estimation method but comes essentially from the calculations of the reduced form of the model. Thus, in order to reduce these costs, we employ a minimum weighted residuals method (Judd [1992]), which is one of the more accurate and less time consumming[1].

The empirical methodology adopted here can be viewed as a "general to specific" approach. The paper starts from a descriptive approach, that is the use of linear and nonlinear time series models to investigate the dynamic properties of the data. Given some key empirical features of the U.S. hours worked data, we propose a theoretical model. The model is parsimonious, in the sense that it describe paths of hours worked from a set of a few number of structural parameters. In this respect, the specification of the impulse source is as simple as possible. The propagation mechanism and the nonlinear dynamics come from a linear variable adjustment cost parameter,

[1]Bansal et al. [1995] and Tallarini and Zhang [1997] choose to implement the parameterized expectations algorithm proposed by den Hann and Marcet [1990] in connection with the simulation based estimation method. The use of a minimum weighting residuals method reduces significantly the computational costs. Although it relies on numerical integration, it does not require large sample size recursive stochastic simulations.

where the matching process implies an externality on this cost parameter. In order to fully combine the descriptive and the structural approaches, we use a simulation based estimation method. The idea of Indirect Inference is to estimate the deep parameters of a complicated structural model such that estimated auxiliary parameters obtained from simulated data are as close as possible to the ones obtained from the actual data. Because the auxiliary model is chosen in order to fit well the data, this implies a large number of auxiliary parameters. Thus, the performances of the model can be evaluated on the basis of its ability to match these auxiliary parameters from a few number of structural parameters. A global specification test can then be conducted. Further, in order to locate some model's failures, we use a simple diagnostic test on each auxiliary parameter. Finally, the dynamic properties and the induced nonlinearities may be evaluated using impulse response functions for various size of the shocks.

Our results are in favor of our structural model, since it is able to match a descriptive non–linear auxiliary model. The diagnostic test confirms this result, despite some difficulties to fully capture all non–linearities. The local dynamic properties of the structural model are very close to the one displayed by the auxiliary model.

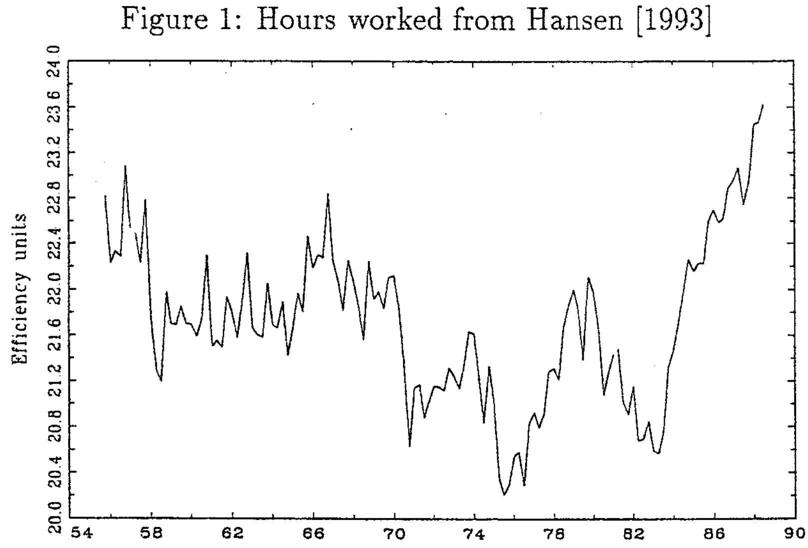
The paper is organized as follows. Section 2 presents a descriptive approach of nonlinearity and discusses the time series properties of US hours worked. In section 3, we specify a structural model and we discuss the solution methods. Section 4 then presents the Indirect Inference and discusses the practical implementation of this method. Empirical results are reported in section 5. Finally, section 6 concludes the paper.

2 A descriptive approach of nonlinearity

We first adopt a descriptive approach in order to characterize the time series properties of hours worked. This empirical investigation starts with the description of the data and further looks to asymmetric behavior through various statistics and estimated nonlinear models.

The empirical analysis is based on the US labor input series constructed by Hansen [1993]. It consists in efficiency hours, which means that hours are corrected for the fact that they are not all of the same quality. Contrary to hours worked series, efficiency units series displays only a very slight downward trend, which is not significant (see figure 1). The sample runs from the third quarter of 1955 to the second quarter of 1988 (132 observations). In

Figure 1: Hours worked from Hansen [1993]



spite of the lack of a significant deterministic trend, we check whether hours worked contain non stationary stochastic component. We thus perform various unit root tests since they provide mixed results: while the ADF test concludes to accept the null hypothesis of a unit root, the Phillips–Perron

Z statistic allows us to reject it. A level–stationarity test (Kwiatkowski, Phillips, Schmidt and Shin [1992]) provides result in favor of a stationary representation[2]. In order to study non–linearities, we first must specify a linear representation of the variable. We use an autoregressive model as benchmark model. The lag order is selected using AIC and BIC criteria and the Lyung-Box statistic. The results indicate that four lags for both level and log are needed. We choose between level and log representation using the Box-Cox transformation. The transformed autoregressive model is estimated using maximum likelihood (ML). The estimated value of the transformation parameter is close to zero and the likelihood ratio test rejects the level representation.

In order to reproduce cyclical asymmetries and non–linearities in hours worked, threshold autoregressive (TAR) models may be appropriate since they are characterized by different dynamics (see Tong [1990]). However the TAR model is a two-regimes model and can be too restrictive if we consider that aggregated behavior leads to smooth changes. Then the two-regimes TAR model can be extended by considering a smooth transition between regimes. The Smooth Transition Auto–Regressive (STAR) model is specified as follows :

$$nh_t = \pi_0 + \sum_{i=1}^{p} \pi_i nh_{t-i} + (\tilde{\pi}_0 + \sum_{i=1}^{p} \tilde{\pi}_i nh_{t-i}) \times F(nh_{t-d}) + \varepsilon_t \qquad (1)$$

where $\varepsilon_t \simeq iid(0, \sigma^2)$ and $F(.)$ is a continuous function bounded by zero and one. Instead of considering a heaviside function as in the TAR model, Teräsvirta [1994] considers two continuous transition functions. The logistic function $F(nh_{t-d}) = (1 + \exp\{-\gamma[nh_{t-d} - c]\})^{-1}$ and the exponential func-

[2]Using the Andrews and Monahan [1992] formula to compute the long run variance, the test statistics are equal respectively to 0.00389 and 0.00012 for log and level of hours worked, to be compared to the critical value of 0.463 for a 5% significance level.

tion $F(nh_{t-d}) = (1 - \exp\{-\gamma^*[nh_{t-d} - c^*]^2\})$. The parameter γ denotes the speed of transition from one regime to the other. It is worth noting that when $\gamma \to \infty$ in the logistic function, $F(.)$ tends to the heaviside function, and the model tends to a TAR model. The case of the logistic function is interesting since it allows parameters in the autoregressive model (1) to vary monotonically with nh_{t-d}. Applied to the modelling of business cycles indicators, the LSTAR (logistic STAR) model describes situations where contraction and expansion phases of an economy may have rather different dynamics, and transition (change in dynamics) from one to the other may be smooth. The LSTAR model is also able to generate cyclical asymmetric realizations as the process follows an AR(p) process when $nh_{t-d} > c$ and switches to another AR(p) in the next period if $nh_{t+1-d} \leq c$ (for discussion see Teräsvirta [1994]). If model (1) is accompanied by an exponential function, one obtains a model in which the parameters change symmetrically about c with nh_{t-d}. Such a model is called an exponential STAR (ESTAR) model. It is a generalization of the exponential model of Ozaki [1978]. If $\gamma \to 0$ or if $\gamma \to \infty$, the ESTAR model becomes linear. The ESTAR model implies that contraction and expansion have rather similar dynamic structures, whereas the middle ground can have different dynamics.

Specification of STAR models is discussed in Teräsvirta [1994]. It consists of three stages : *step i)*: specification of a linear AR model with suitable model selection criterion, such as AIC, BIC, test of presence of autocorrelated residuals; *step ii)*: testing linearity for different values of the delay parameter d, and if linearity is rejected, determining d[3]; *step iii)*: choosing between

[3] In order to specify d, the test is carried out for the range of values $1 \leq d \leq p$. If linearity is rejected for more than one value of d, then d is determined as $\hat{d} = \arg\min p(d)$ where $p(d)$ is the p-value of the selected test. The underlying idea is that the test has maximum power if d is chosen correctly.

LSTAR and ESTAR models using a sequence of tests of nested hypotheses (see Teräsvirta [1994]).

The linearity hypothesis is given by $H_0 : \gamma = 0$ in equation (1). However it is worth noting that this test, assuming that nh_t is stationary and ergodic under H_0, is a nonstandard testing problem in the sense that the STAR model is only identified under the alternative $H_1 : \gamma \neq 0$. In this particular case the problem has a simple solution based on an approximation which is outlined in Teräsvirta [1994], and a Lagrange multiplier type test of linearity against STAR assuming d is known is equivalent to the test:

$$\begin{cases} H_0 & : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \quad (j = 1, ..., p) \\ H_1 & : H_0 \text{ is not valid} \end{cases}$$

in the auxiliary regression, obtained using a third-order Taylor's expansion of the LSTAR model:

$$\begin{aligned} nh_t &= \beta_0 + \sum_{j=1}^{p} \beta_{1j} nh_{t-j} + \sum_{j=1}^{p} \beta_{2j} nh_{t-j} nh_{t-d} + \sum_{j=1}^{p} \beta_{3j} nh_{t-j} nh_{t-d}^2 \\ &\quad + \sum_{j=1}^{p} \beta_{4j} nh_{t-j} nh_{t-d}^3 + v_t \end{aligned} \tag{2}$$

The linearity tests are built with a preset value for the number of lag of the

Table 1: Linearity tests results

nh_{t-d}	$p-STAR$	$p-val4$	$p-val3$	$p-val2$
nh_{t-1}	0.2208	0.6928	0.5293	0.0459
nh_{t-2}	0.2422	0.2986	0.6972	0.1155
nh_{t-3}	0.0504	0.2998	0.7957	0.0057
nh_{t-4}	0.0247	0.7102	0.5343	0.0009

AR model ($p = 4$). The linearity is rejected against STAR models for different values of d, and the p-value is minimized for $d = 4$: $p - STAR = 0.0247$. For $d = 4$, we look at LSTAR and ESTAR models, using the LM statistics for nested models. The identification of the Taylor expansion parameters with the parametric models, allows for the following binding function. For the LSTAR model, one has $\beta_{ij} = \Pi(\pi_j, \tilde{\pi}_j, \gamma, c)$ for $i = 1, \cdots, 4$ and $j = 1, \cdots, p$, and for the ESTAR model $\beta_{ij} = \Pi^*(\pi_j, \tilde{\pi}_j, \gamma^*, c^*)$ for $i = 1, \cdots, 3$ and $j = 1, \cdots, p$. If $\beta_{4j} \neq 0, j = 1, \cdots, p$ ($p - val4$) in table 1 is the associated p-value), then it can be interpreted as a rejection of the ESTAR model. If $\beta_{4,j} = 0$, then we test $\beta_{3,j} = 0/\beta_{4,j} = 0$ (see $p - val3$). If the null is not rejected, then this is taken as evidence for the LSTAR model, whereas a rejection is not very informative. If the hypothesis $\beta_{2,j} = 0/\beta_{3,j} = 0, \beta_{4,j} = 0$ is rejected (see $p - val2$), one can expected a LSTAR model. This sequential testing procedure allows to accept the null hypotheses $\{\beta_{4j} = 0, j = 1, \cdots, p\}$ and $\{\beta_{3j} = 0/\beta_{4j} = 0, j = 1, \cdots, p\}$ but we reject the null $\{\beta_{2j} = 0/\beta_{3j} = 0, \beta_{4j} = 0, j = 1, \cdots, p\}$. Then, the LSTAR model seems to be more adequate than the ESTAR model in order to explain the dynamics of hours worked.

We can conclude on the presence of significant nonlinearities in US hours worked dynamics. Nonlinear models are needed to describe the data-generating mechanisms of inherently asymmetric realizations. In order to provide a comprehensive explanation of such descriptive characterizations of the data, we now adopt a structural approach.

3 A structural approach to nonlinearity

We now propose a theoretical model in order to exhibit significant nonlinearities and persistence along the adjustment dynamics. Several theoretical specifications are natural candidates. We do not follow here usual approaches based on asymmetric adjustment costs. We prefer to specify firms labor demand behavior within a search model framework[4].

The allocation of resources is driven by a search process and the real wage is exogenous. Following Mortensen [1986] and Pissarides [1990], trade in the labor market is a costly and uncoordinated economic activity. In this model, labor demand is specified in terms of hours worked. There exists a constant return-to-scale *matching function* linking the number of hirings H_t in efficiency units to the number of vacancies V_t and hours worked Nh_t:

$$H_t = \left(\bar{H} - Nh_t\right)^{\alpha} V_t^{1-\alpha}$$

where $\bar{H}$ is the maximum amount of hours per employed worker. At each period, vacant jobs and unemployed hours worked are matched and move from trade to production. In equilibrium, some existing hours worked disappear. Since the size of the population is normalized to one, V_t corresponds to the vacancy rate and $\bar{H} - Nh_t$ denotes the unemployed hours rate, expressed in hours. The number of job vacancies and hours worked matched at time t are randomly selected from the sets V_t and Nh_t. The rate at which job vacancies are filled is given by $q_t = H_t/V_t$. This transition rate depends on the relative number of traders. In this model, firings are left unexplained. In each period t, the number of quits expressed in hours is ruled by an exogenous quit rate s applied to hours worked. Thus, hours worked evolve according to the process

$$Nh_{t+1} = (1-s)Nh_t + q_t V_t \tag{3}$$

We assume there is a large number of identical firms uniformally distributed on [0,1]. Each firm j has access to a constant returns-to-scale production function given by

$$Y_{j,t} = (1+\theta)(\bar{y} + a_t)Nh_{j,t} \text{ where } \bar{y}, \theta > 0$$

[4]This model is a partial equilibrium representation of previous labor-market search models (see Langot [1995], Fève and Langot [1996], Andolfato [1996]).

The aggregate productivity shocks a_t follows a stationary stochastic process. $\bar{y}$ denotes the average level of productivity. We assume that wages are indexed on total productivity:

$$w_{j,t} = \theta(\bar{y} + a_t)$$

Thus θ denotes the indexation rate. The number of hours worked is predetermined, but the firm can control its hiring policy through vacancies. However, the aggregate job availability only determines the rate at which its jobs vacancies are filled. This externality causes a stochastic rationing on the individual hiring policy. The dynamic problem of the firm j is to choose the number of vacancies $V_{j,t}$ that maximizes the expected discounted sum of its profit flows

$$\max_{V_{j,t}} E_t \left[\sum_{i=0}^{\infty} (1+r)^{-i} \Pi_{j,t+i} \right]$$

subject to

$$\begin{aligned} Nh_{j,t+1} &= (1-s)Nh_{j,t} + q_t V_{j,t} \\ V_{j,t} &\geq 0 \end{aligned}$$

where $\Pi_{j,t}$ is the profit flow of the firm j at time t, given w_t:

$$\Pi_{j,t} = (\bar{y} + a_t)Nh_{j,t} - \omega V_{j,t}$$

$\frac{\omega}{q_t}$ can be interpreted as a variable adjustment cost on hours worked. Indeed, the matching process involves an externality through the costs parameter. We impose an additional restriction, *i.e.* the number of vacancies is non negative, whatever the state of nature is. The first-order condition for a firm j says that firm j posts vacancies up to the point where the marginal value of a vacancy equals its marginal costs[5]:

$$\frac{1}{1+r}E_t\left[\frac{\partial \mathcal{V}_{j,t+1}}{\partial Nh_{j,t+1}}\right]=\frac{\omega}{q_t}-\frac{\lambda_t}{q_t} \tag{4}$$

where λ_t is the lagrangian multiplier associated to the constraint $V_{j,t}\geq 0$. This optimality condition is equivalent to the "free entry" condition in the labor market when firms can have only one filled hour worked or unfilled job. The dynamic adjustment of vacancies is given by the dynamics of the marginal value of hours worked implied by the time-consuming search process. Thus, the marginal real value of hours worked is given by:

$$\frac{\partial \mathcal{V}_{j,t}}{\partial Nh_{j,t}}=(\bar{y}+a_t)+\frac{(1-s)}{1+r}E_t\left[\frac{\partial \mathcal{V}_{j,t+1}}{\partial Nh_{j,t+1}}\right]$$

The symmetric partial equilibrium is defined by the set of functions $\{V(.), Nh(.)\}$ depending on the information set $\mathcal{I}_t$[6]. Thus, $V_t=V(\mathcal{I}_t)$ and $Nh_{t+1}=Nh(\mathcal{I}_t)$ solve the system of dynamic equations in q_t and Nh_t:

$$\begin{aligned}\frac{\omega}{q_t} &= \max\left\{0,\frac{1}{1+r}E_t\left[\bar{y}+a_{t+1}+(1-s)\frac{\omega}{q_{t+1}}\right]\right\} &(5)\\ Nh_{t+1} &= (1-s)Nh_t+q_tV_t &(6)\end{aligned}$$

for a given vector of prices.

In order to solve the model, we must specify the stochastic process governing the exogenous variable. We assume that a_t follows a first order autoregressive process

$$a_t=\rho a_{t-1}+\epsilon_t$$

where $|\rho|<1$ and ϵ_t is $iid\mathcal{N}(0,\sigma^2)$. Enriched specifications would obviously imply better dynamic properties in terms of fit. Nevertheless, this study aims

[5]We denote $\mathcal{V}_{j,t}$ the expected discounted sum of firm's profit flows, conditional on the information set available at time t.

[6]We denote $\mathcal{I}_t=\{Nh_t,a_t\}$. Note that in our case, the real wage correspond to a given arbitrary constant or may be indexed on aggregate productivity shocks.

at evaluating the internal propagation mechanism of a search model. Thus, we restrict the dynamic properties to a simple structure[7].

The non-negativity constraint on vacancies induces complicated decision rules. We thus adopt a minimum weighted residuals method to solve the model. We summarized the dynamics of hours worked by the following set of dynamic equations

$$\begin{aligned} Nh_{t+1} &= (1-s)Nh_t + \left(g(\bar{a}_t;\bar{\theta})/\omega\right)^{\frac{1-\alpha}{\alpha}} (\bar{H} - Nh_t) && (7) \\ \bar{a}_{t+1} &= \rho\bar{a}_t + \epsilon_t && (8) \end{aligned}$$

where $g(\bar{a}_t;\bar{\theta})$ is an approximation of the decision rule ω/q_t and its functional form is given by

$$g(\bar{a}_t;\bar{\theta}) = \exp\left(\sum_{i=1}^{p} \varphi_i v_i(\bar{a}_t)\right)$$

We guess an exponential form with a polynomial function (see the appendix 1 for more details on these computations). The non–linearities come from two possible sources. First, the approximation of the decision rule is non–linear in the exogenous variable because of the non–negativity constraint on vacancies. Second, equation (7) involves the product of the approximated decision rule with hours worked.

4 Combining the descriptive and the structural approach

Our structural model leads to the complicated reduced forms (7)-(8), which implies an intractable likelihood functions. In order to circumvent this difficulty, we employ an Indirect Inference method. An usual practice consists

[7]Note that persistence is necessary to obtain stochastic solutions. Indeed, setting $\rho = 0$ eliminates the effects of this shock and the solution is then purely deterministic. This is because employment is pre-determined and a pure transitory shocks has no effect on the marginal value of vacancies.

in replacing the initial model by an approximation (hereafter the auxiliary model), which is easier to handle and which replaces the unfeasible ML estimator of the model by the ML estimator computed from the auxiliary model[8]. Since the auxiliary model is misspecified under the structural model, such ML estimator is generally inconsistent. The idea of Indirect Inference is then to use simulations performed under the model in order to correct this bias (see Smith [1990], Gallant and Tauchen [1992], Gouriéroux et al. [1993] and Gouriéroux and Monfort [1994]).

The choice of an auxiliary model is an important feature of simulation based estimation methods. We have shown in the first section that the data display significant persistence and non–linearities. On the basis of the previous empirical findings, we use a simple, while still being non–linear, approximation of a STAR model[9]:

$$nh_t = \beta_0 + \sum_{j=1}^{4} \beta_{1j} nh_{t-j} + \sum_{j=1}^{4} \beta_{2j} nh_{t-j} nh_{t-4} + \sum_{j=1}^{4} \beta_{3j} nh_{t-j} nh_{t-4}^2 + v_t$$

The choice of this approximation bears on computational aspects of the estimation method. Indeed, the criterion function associated with the previous approximation is simple, because it "only" involves OLS. This criterion function is given by (See step 1 in appendix 2)

$$Q_T(\underline{nh}_T, \psi) = -\sum_{t=5}^{T} \left\{ nh_t - \beta_0 - \sum_{j=1}^{4} \sum_{i=1}^{3} \beta_{ij} nh_{t-j} nh_{t-4}^{i-1} \right\}^2$$

and similarly defined when simulated data, obtained from the structural model, are used (See step 3–4 in appendix 2). The vector of auxiliary pa-

[8]This method avoids the *ad-hoc* selection of unconditional moments to be matched, as required in the simulated method of moments suggested by Ingram and Lee [1991] and Duffie and Singleton [1993]. This is especially true if the auxiliary descriptive model is chosen only on statistical ground, without any theoretical *a priori*.

[9]This amounts to a second-order Taylor's expansion of the transition function.

rameters is given by

$$\psi = \{\beta_0, \beta_{11}, \cdots, \beta_{14}, \beta_{21}, \cdots, \beta_{24}, \beta_{31}, \cdots, \beta_{34}, \sigma_v\}$$

where dim$\psi = 14$. Hereafter, $\widehat{\beta}_T$ denotes the solution to the maximization of the preceding criterion function, while $\widetilde{\beta}^i_T(\theta, nh_0)$ is the corresponding solution based data generated by the ith simulation of the structural model. This last estimate is conditional on a given vector of structural parameters, θ, and an initial condition on hours worked, nh_0. The use of this flexible specification considerably reduces computational costs. Nonlinear least squares applied directly on parametric LSTAR or ESTAR models may fail to converge for particular realizations of the endogenous variable. Because we faced these difficulties, we follow the suggestions of Michaelides and Ng [1997] and keep a simple auxiliary model. As shown in the descriptive approach, an auxiliary model with $\beta_{31}, \cdots, \beta_{34} = 0$ should be preferred for statistical significance purposes. We prefer a more general flexible form because specification tests indicate the rejection of normality and significant ARCH effects when the restrictions $\beta_{31}, \cdots, \beta_{34} = 0$ are imposed.

The structural parameters are thus estimated in order to match the parameter estimates from the data of an auxiliary model (see the appendix 2 for more details on the estimation method). In order to satisfy identifying restrictions, the number of auxiliary parameters exceeds the number of structural parameters. Thus, one may conduct a global specification test (see the appendix 2 for the computation of the statistic, denoted $J-stat$). Further, we develop a simple diagnostic test, in the lines of Tauchen [1995] and Gallant, Hsieh and Tauchen [1994]. This test allows to locate some failures of the structural model. The main idea is that each element $g_{T,N} = \widehat{\psi}_T - \frac{1}{N}\sum_{i=1}^{N} \widetilde{\psi}^i_T(\theta, nh_0)$ of measures the discrepancy between the parameters of the auxiliary model computed from the data and from model simulations. Each element thus contains diagnostic information on how well the structural model matches the parameters of the auxiliary model. A small value for some elements indicates that the structural model is able to well explain some features of the auxiliary model, while large values may induce some trouble in the ability of the model to account for.

The first order condition associated to the minimization of the loss function (See step 5 in appendix 2) leads to:

$$\frac{\partial g'_{T,N}}{\partial \theta} \quad W_T \quad g_{T,N} = 0$$

Then, using the mean value approximation of $g_{T,N}$,

$$g_{T,N} \simeq (\widehat{\psi}_T - \psi_0) + \frac{\partial g_{T,N}}{\partial \theta} \left(\widehat{\theta}_T^N - \theta_0\right)$$

we get

$$\sqrt{T}\left(\widehat{\theta}_T^N - \theta_0\right) \simeq -\left(D'_\theta W_T D_\theta\right)^{-1} D'_\theta W_T \sqrt{T}\left(\widehat{\psi}_T - \psi_0\right)$$

where $D_\theta = \partial g_{T,N}/\partial \theta$. Plugging the latter expression into the mean value approximation and pre-multiplying both sides by $\sqrt{T}$, one gets:

$$\sqrt{T} g_{T,N} \simeq \left(I_q - D_\theta \left(D'_\theta W_T D_\theta\right)^{-1} D'_\theta W_T\right) \sqrt{T}(\widehat{\psi}_T - \psi_0)$$

Now, using the fact that the auxiliary parameters are normally distributed:

$$\sqrt{T}(\widehat{\psi}_T - \psi_0) \simeq \Omega^{1/2}\vartheta \qquad \vartheta \sim N\left(0, I_q\right)$$

and that the optimal weighting matrix corresponds to the inverse of the covariance matrix Ω , one obtains after some algebra

$$\sqrt{T} g_{T,N} \sim N\left(0, \Omega + D_\theta \left(D'_\theta W_T D_\theta\right)^{-1} D'_\theta\right)$$

Thus, each element of the following vector of *t-statistics:*

$$T_{T,N} = \left\{diag\left[\Omega + D_\theta\left(D'_\theta W_T D_\theta\right)^{-1} D'_\theta\right]\right\}^{-1/2} \sqrt{T} g_{T,N}$$

is asymptotically $\mathcal{N}(0,1)$. The test statistic may be then constructed after replacement of D_θ and Ω by consistent estimates.

5 Empirical results

We now apply the estimation method to the search model for the two approximations. The minimization of the simulated criterion function is carried out using a simplex method for minimization provided in the *Optim* numerical optimization package of MATLAB. We prefer this solution method to more traditional optimizers that use local gradient search methods, as the latter fails to converge in our experiments. The estimation is time consuming, despite the small number of parameters to be estimated. This is because the simplex method requires a large number of function evaluations, which implies for each function evaluation two steps : *i)* the computation of an approximated solution to the structural model and *ii)* the simulation of hours worked.

We use 20 simulations[10] for a sample size equals to 132. Simulation experiments of Michaelides and Ng [1997] indicates that efficiency gain becomes negligible when the number of simulations exceeds ten, especially with Indirect Inference. In order to reduce the effects of initial conditions, simulated samples includes 250 initial points which are subsequently discarded for estimation. The initial condition corresponds to the steady-state value of hours worked

[10]We use the same trial $e_1^n, \cdots, e_T^n$, $n = 1, \cdots, N$ for all the values of θ. These simulated values are redrawn from the same seed values.

We do not estimate all the structural parameters. Some of them are fixed because identification problems occurred during estimation. Our descriptive approach is univariate and we cannot introduce richer specification for the exogenous variables. For example, we cannot identify separately the contribution of the aggregate productivity shock and the real wage one. So we must impose the following restrictions on the structural parameters. The unconditional mean of the technological shock is $\bar{y} = 1$. Following Andolfato [1996], the rate at which job vacancies are filled is equal to 0.9 in steady state. The value for ω is then endogenously deduced from this value. Finally, we impose $\bar{H} = 1$. These pre–set values for some structural parameters imply the estimation of four free parameters: the parameter of the matching function α, the quit rate s, the autoregressive parameter of aggregate productivity shock ρ and the standard–error of the innovation σ. Thus, we have $\dim\theta = 4$ and we may conduct a global specification test for the structural model.

Table (2) gives the parameter estimates. The theoretical model contains 4 free parameters, so the asymptotic chi-square has 10 degrees of freedom. We are unable to estimate the structural parameters with large polynomial orders. This is because frequent convergence failures occurred during the minimization of the criterion function. The estimation results are thus obtained for a polynomial of order two. The estimation results are in favor of our theoretical model. More precisely, the global specification test indicates that the structural model with 4 free parameters is able to match the 14 auxiliary parameters. All the parameter estimates are significantly different from zero. The estimated value of α is quite large, if we compare it to estimation results of the matching function for the United States. Blanchard and Diamond [1989] find an elasticity of vacancies equal to 0.6, whereas our results imply a value of 0.11. This discrepancy could be partly explained by two main differences between our estimation and the precedings. First, previous studies provide estimates of the matching function without specifying the whole dynamic structure of the labor market. More particularly, our estimations are based on the specification of a structural search model on the labor market. Second, parameter estimates are obtained through the dynamics of hours worked. Conversely, previous studies use vacancies, unemployment and flow data to estimate the matching function. These elements are potentially responsible for this difference and illustrate the great sensitivity of the matching function parameters to the data set and the model specification. The quit rate is equal to 16% per quarter and appears to be surprisingly too large. Nevertheless, this might be explained by the lack of endogenous firing policy. Thus exogenous quits have to be large enough to match outflows due to firings. The estimated value of ρ induces significant persistence in the aggregate shock. The estimated standard error of this shocks is large, if we compare it to previous estimates. This result comes from the linear specification of the aggregate production shock. This contrasts with the usual log–linear specification. If we compute, using simulations, the implied dynamic properties of the productivity shock for a log-linear representation, we find a standard error of the innovation equal to 0.0547 with a confidence interval of $[0.014, 0.099]$. The lower admissible value for this standard–error is not that far from estimated value with US data.

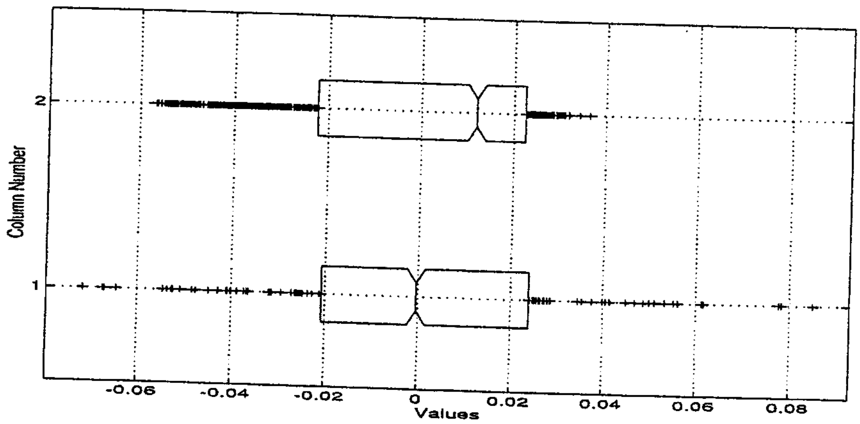
Figure 2 provides a "Boxplot" of the observed data and the simulated data from the theoretical model. The box has lines at the lower quartile, median and upper quartile values. Outliers are observed and simulated data with values beyond the end of the lines of lower and the upper quartiles. From figure 2, we can see that the theoretical model estimated from the auxiliary

Table 2: Estimation Results

α	0.8890
	(0.126)
s	0.1628
	(0.081)
ρ	0.8908
	(0.025)
σ	0.4890
	(0.181)
$J-stat$	11.61
	[31.2%]

Note: Standard-error in parentheses
P-value in brackets

Figure 2: Boxplot of observed and simulated hours worked



Note : Column number: 1, observed data ; 2, simulated data model reproduces the empirical distribution of hours worked. despite too much induced asymmetry.

First part of table (3) provides the parameter estimates of the auxiliary model from actual data and simulated data. The structural model reproduces well the auxiliary parameters. The comparison of estimated parameters does not provide any precise statistical inference on how the search model matches the data. It is more informative to look at our simple diagnostic test, to see on which dimensions the structural model works well or bad. The second part of the table 3 provides the diagnostic tests. Recall that a large value of the test statistic indicates that the structural model is not really able to reproduce the associated auxiliary parameters. From table 3, we see that the theoretical model matches the auxiliary model. All the linear components of the hours worked dynamics are reproduced. The fact that the structural model is not rejected essentially comes from its ability to match the linear dynamics[11].

We now look at the dynamic properties of the theoretical model. An illustration of the potential nonlinearities induced by our theoretical specification is based on the impulse response function (IRF) to a transitory random initial shock. It is worth noting that these IRF are based on "naive" forecasts, meaning that we put future innovations to zero. For an horizon equal to 12, we compute the IRF with a shock at period 0. This shock is normally distributed with zero mean and a standard-error corresponding to the estimated value (0.4890). If the model is linear or if the induced non-linearity is negligible, one may expect at period 1 a shape of the empirical

[11]This result is confirmed by the estimation of the structural model from a linear autoregressive model. The P-value associated to the chi-square distribution with two degrees of freedom is 55%. The diagnostic test associated to the auxiliary parameters never exceeds 1 in absolute value.

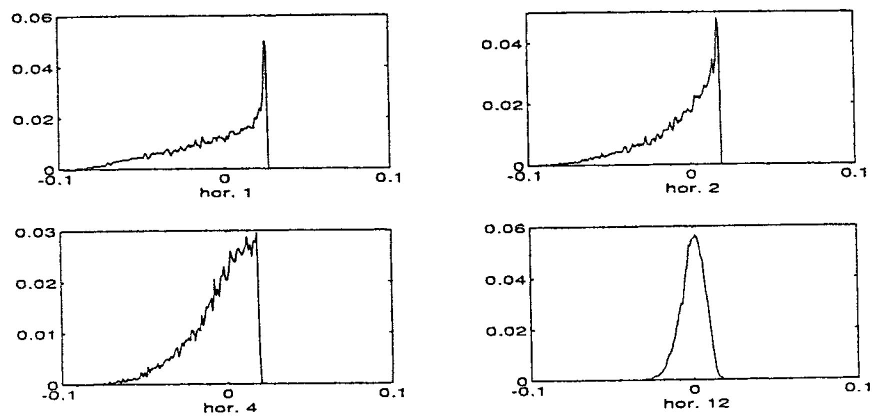
Table 3: Parameter estimates of the auxiliary models

	Actual data	Simulated data	Diagnostic test
β_0	-0.0029	-0.0006	1.38
	(0.002)	($3\ 10^{-4}$)	
β_{11}	0.7369	0.7957	0.43
	(0.111)	(0.096)	
β_{12}	0.0048	0.0001	-0.05
	(0.132)	(0.100)	
β_{13}	0.0838	-0.0005	-0.60
	(0.134)	(0.050)	
β_{14}	0.0544	0.0400	-0.11
	(0.131)	(0.010)	
β_{21}	-0.0515	-0.0224	0.80
	(0.032)	(0.015)	
β_{22}	0.0592	0.0448	-0.30
	(0.044)	(0.017)	
β_{23}	-0.0885	-0.0340	1.15
	(0.045)	(0.015)	
β_{24}	0.1110	0.0295	-2.82
	(0.028)	(0.005)	
β_{31}	0.0737	-0.0643	-1.25
	(0.091)	(0.065)	
β_{32}	-0.0740	0.0815	1.00
	(0.133)	(0.084)	
β_{33}	0.0151	-0.0325	-0.29
	(0.135)	(0.084)	
β_{34}	0.0362	0.0636	0.30
	(0.084)	(0.029)	
σ_v	0.0144	0.0139	-0.33
	(0.001)	(0.001)	

Note: Standard-error in parentheses.

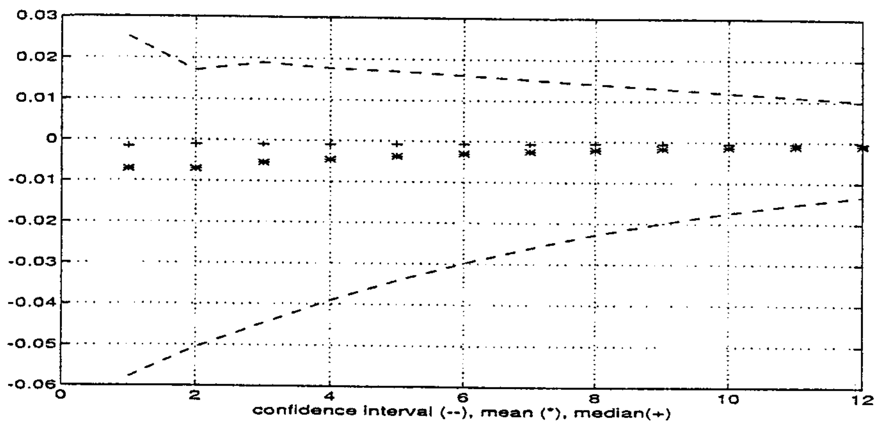
IRF close to a normal one. Because for any stationary model, as the horizon increases. the empirical probability density function (pdf) degenerates, we focus our attention on the IRF for a short horizon. In figure 3, the empir-

Figure 3: Empirical pdf of IRF



ical pdf are obtained from 5000 experiments. The shape at horizon one is essentially on the right, i.e. the distribution is concentrated on positive values. For an horizon between 1 and 4, the empirical pdf indicates significant asymmetries. This asymmetry disappears as the horizon increases and the distribution becomes more and more concentrated around zero (See horizon 12). For illustrative purposes, we report, in figure (4), the 95% confidence interval for impulse response functions, their mean and median. For the first horizons, asymmetry is relevant since the confidence interval displays asymmetry around zero. Moreover, mean and median are both negative. Further, they differ since the mean is lower than the median, indicating a leptokurtic distribution on the left. Indeed, one of the prediction of our estimated structural model is that hours worked diminishes more sharply after a negative shock than increase after a positive shock. This essentially comes from the fact that congestion externalities dominates the standard positive trade ex-

Figure 4: Impulse Response Functions



ternalities generated by this type of model. This illustrates the asymmetric of the empirical pdf of IRF. But, it also shed light on the difficulty one may have to propose an accurate measure of asymmetry in non–linear models.

6 Conclusion

This paper proposes to fully combine a descriptive and structural approach in order to look at the time series behavior of US hours worked. The empirical results indicate that our simple representation of the labor market is able to account for most of the observed dynamics, through both non–linearities and persistence. Moreover, this empirical investigation must be essentially considered as a first line of research in both descriptive and theoretical approaches to non–linear dynamics on the labor market. From an econometric point of view, the approach adopted here can be extended by considering various theoretical models for the labor market. For example, we may evaluate competitive models of hours worked using encompassing principles. From a theoretical point of view, the model is too simple. A natural extension which nests the actual model is to consider endogenous real wage rate, through the specification of a wage bargaining process between workers and firms. Wage bargaining induces additional non–linearities and this may improve the ability of the model to match various nonlinear descriptive models.

Appendix

1-The solution method

As soon as we consider that aggregate shocks may be large, the model does not admit any analytical solution. We thus have to turn to numerical methods. We use a method that relies on minimum weighted residuals (See Judd [1992] for a complete statement of this method, and Judd [1996]). The basic idea of this approach is to express equilibrium as the zero of an operator, Γ, which maps a function space into another : $\Gamma : \mathcal{A}_1 \longrightarrow \mathcal{A}_2$. Since this operator cannot be evaluated at all points of $\mathcal{A}_1$, the minimum weighted residuals method, or projection method, restricts its attention on a finite dimensional subspace of elements in $\mathcal{A}_1$, easily evaluable and likely to contain elements sufficiently close to the "true" solution. In other words, one searches to specify an *a priori* given function to approximate the operator. The Γ operator is then defined by Euler residuals :

$$\Gamma \equiv x_t - max\left\{\frac{1}{1+r}\int_A ((y+\bar{a}_{t+1}) + (1-s)x_{t+1})\, dG(a_{t+1}|a_t), 0\right\}$$

One then guesses the form of the approximate solution for x_t:

$$x_t = \mathcal{F}(a_t;\Phi) \equiv \exp\left(\sum_{i=1}^{p} \varphi_i \upsilon_i(a_t)\right)$$

where $\upsilon_i(a_t) \equiv T_{i-1}(\psi(a_t))$. Since we are using an orthogonal collocation method, $T_i(.)$ is a Chebychev polynomial function. Finally, it shall be noticed that Chebychev polynomials are defined on the interval $[-1,1]$, whereas a_t is not restricted to lie in this range. We transform it in order to lie in the interval $[-1,1]$ using $\psi(a_t) = \tanh\left((1-\rho^2)^{1/2}\sigma_\varepsilon^{-1} a_t\right)$.

Rational expectations involve the computation of an integral. This integral is approximated using a Gauss–Hermite quadrature, we use 20 nodes.

2-The Indirect Inference method

Given an auxiliary model and a good approximation of the policy function, the estimation method is conducted as follows:

Step 1: We define in a first step a criterion function, denoted $Q_T(\underline{nh}_T;\psi)$, where $\underline{nh}_T = (nh_1,\cdots,nh_T)$ is an observable variable and ψ a q-dimensional vector of auxiliary parameters, $\psi \in \Psi \subset \mathbb{R}^q$. $\underline{nh}_t$ includes current and lagged values of hours worked and ψ is the parameter of the approximated model. We denote $\widehat{\psi}_T$ the solution to the maximization of Q_T:

$$\widehat{\psi}_T = \operatorname{Arg}\max_{\psi\in\Psi} Q_T(\underline{nh}_T;\psi)$$

For $T \to \infty$, Q_T converges to deterministic limit $Q_\infty(G_0;\theta_0;\psi)$ where G_0 is the p.d.f. of ϵ_t and θ_0 the pseudo-true value of θ. We denote $\psi_0 = Arg\max_\psi Q_\infty(G_0;\theta_0;\psi)$ and $b(G,\theta) = Arg\max_\psi Q_\infty(G;\theta;\psi)$, where b is a binding function. When the binding function is known, θ can be estimated by solving the system $\widehat{\psi}_T = b(G,\theta)$. In our case, this function is unknown and we use simulations.

Step 2: For a given value of the structural parameters $\theta \in \Theta \subset \mathbb{R}^p$, we compute the policy functions using the two approximations described previously.
Step 3: From the approximated policy function, given the structural parameters θ and an initial condition on nh_t, we performs N simulated pathes, denoted $\underline{\tilde{nh}}^i_T(\theta, nh_0)$, $i = 1, \cdots, N$.
Step 4: From these simulations, we get:

$$\tilde{\psi}^i_T(\theta) = \operatorname{Arg}\max_{\psi \in \Psi} Q_T(\underline{\tilde{nh}}^i_T(\theta, nh_0); \psi) \quad i = 1, \cdots, N$$

Step 5: An Indirect estimator of θ is defined by choosing a value $\tilde{\theta}^N_T$ for which $\widehat{\psi}_t$ and $\tilde{\psi}^N_T$ are as close as possible:

$$\min_{\theta \in \Theta} \left[\widehat{\psi}_T - \frac{1}{N}\sum_{i=1}^{N} \tilde{\psi}^i_T(\theta, nh_0)\right]' \widehat{W}_T \left[\widehat{\psi}_T - \frac{1}{N}\sum_{i=1}^{N} \tilde{\psi}^i_T(\theta, nh_0)\right]$$

where $\widehat{W}_T$ is a symmetric nonnegative matrix defining the metric.

Steps 2 to 5 are conducted until convergence, *i.e.* until a value of θ which minimizes the objective function is obtained. Under standard regularity conditions, when N is fixed and T goes to infinity, $\sqrt{T}(\tilde{\theta}^N_T - \theta_0)$ is asymptotically normal, with a covariance matrix equal to $(1 + \frac{1}{N}) \left\{\frac{\partial^2 Q_\infty}{\partial\theta\partial\psi'} I_0^{-1} \frac{\partial^2 Q_\infty}{\partial\psi\partial\theta'}\right\}^{-1}$, where $I_0 = \lim_{T\to\infty} V_0 \left\{\sqrt{T}\frac{\partial Q_T(\tilde{nh}^N_T;\psi_0)}{\partial\psi}\right\}$. The computation of $\tilde{\theta}^N_T$ necessitates a preliminary consistent estimator of W_T. The optimal weighting matrix may be directly based on the observations (see Smith [1990]). It corresponds to the inverse of the covariance matrix of $\sqrt{T}(\widehat{\psi}_T - \psi_0)$, which is obtained from the first step of the implementation of the estimation method.

Identifying constraints impose that the dimension of ψ must be greater than or equal to the dimension of θ. A specification test for the structural model may be also based on the optimal value of the objective function. The statistic

$$J - stat = \frac{T\,N}{1+N} \min_{\theta\in\theta} \left(\widehat{\psi}_T - \frac{1}{N}\sum_{i=1}^{N} \tilde{\psi}^i_T(\theta, nh_0)\right)' \widehat{W}_T \left(\widehat{\psi}_T - \frac{1}{N}\sum_{i=1}^{N} \tilde{\psi}^i_T(\theta, nh_0)\right)$$

is asymptotically distributed as a chi-square with $\dim\psi - \dim\theta = q - p \geq 0$ degrees of freedom, under the null hypothesis than the model is well specified.

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