ENDOGENOUS GROWTH AND CYCLES THROUGH RADICAL AND INCREMENTAL INNOVATION

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Abstract

This paper develops a model of endogenous growth where innovation can take two forms, namely radical and incremental. The latter type of innovation means that a new intermediate good is introduced in the production of the final good, thus raising productivity in a Ethier-Romer-type way. A radical innovation means first that the level of knowledge is multiplied by a constant factor, which raises productivity in an Aghion-Howitt way, and second that the previous innovations are made obsolete. The economy is populated by a constant number of researchers, who may either engage in radical or incremental innovation activity. Incremental innovation is deterministic and continuous, radical innovation is discrete and stochastic. The market equilibrium is an allocation of the researchers between radical and incremental innovation. Different types of equilibria with perfect foresight are possible: fixed as well as periodic or aperiodic allocations.

Keywords: endogenous growth, radical and incremental innovation, cycles

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CROISSANCE ET CYCLES ENDOGENES INDUITS PAR L'INNOVATION RADICALE ET INCREMENTALE

Résumé

Un modèle de croissance endogène avec deux types d'innovation, radicale et incrmentale, est développé dans ce papier. Une innovation incrmentale se manifeste par l'apparition d'un nouveau bien intermédiaire dans la production du bien final. Une innovation radicale signifie premièrement que le degré de connaissance est multipliépar un facteur constant, ce qui augmente la productivité de l'économie, et deuxièmement que toutes les innovations précédentes deviennent dépassées. L'économie est constituée d'un nombre fixe de chercheurs, qui peuvent s'engager dans les activités d'innovation radicale ou incrémentale. L'innovation incrémentale se déroule selon un processus déterministe et continu, l'innovation radicale est discrète et stochastique. L'équilibre décentralisé est une allocation des chercheurs entre l'innovation radicale et incrémentale. Différents types d'équilibres avec prévision parfaite sont possibles : allocation fixe, cycles et allocations apériodiques.

Mots-clés : croissance endogène, innovation radicale et incrémentale, cycles.

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1 Introduction

Some endogenous growth models rely on the accumulation of a particular factor to which is associated some form of positive technological externality: knowledge, human capital or any factor with a public good aspect, as in Romer (1986a) or Lucas (1988). Other models are based on the discovery of new products and/or processes, i.e. innovation, which is then perceived as a regular economic activity, in opposition to the conception of the traditional neo-classical growth theory where technology stayed outside of the economic realm. Endogenous growth is made possible by the introduction of new products or processes that contribute to increasing productivity. A further distinction among this latter category of models is possible.

In Romer (1990), a particular type of production function is assumed. Following Ethier (1982), it is supposed that the introduction of a new intermediate product raises productivity in the production of the final good. The interpretation of this effect follows the line of argument put forward in Young (1928). Division of labour at the macroeconomic level means an increasing specialisation of industries, accompanied by an increase in production efficiency.

On the other hand, the model of Aghion and Howitt (1992) incorporates an important aspect of innovation: obsolescence and creative destruction. New products do not just add-up to old ones, they may alternatively replace them. Each innovation then takes the place of the previous one and increases the level of productivity. Such innovations have different technological as well as economic consequences from the type of innovation found in Romer (1990), e.g. the business stealing effect, well-known in the industrial organisation literature[1]. Besides, an individual innovation shocks the entire economy.

This paper considers an model of endogenous growth where two types of innovations, namely radical and incremental, are possible. A distinction is made between the innovations that "outstretch the boundaries of known science" and those that exploit the already existing knowledge and put it into use for production purposes. Radical innovations have a destructive aspect for the existing technical system. They replace the preceding dominant technology with a new one, similarly to what happens in Aghion and Howitt (1992). In parallel, there also exists a possibility of incremental innovation, whereby a new intermediate good is introduced in the production of the final good. Incremental innovation is always made within the framework of a given technical system, the latter being defined by a radical innovation.

Researchers may engage into either type of innovation. In the long run, growth depends both on a continuous stream of incremental innovations and on radical breakthroughs in technology. The discovery of radically new products is uncertain so that

[1]The distinction between the two types of innovation-based endogenous growth models can also be found in Grossman and Helpman (1991). Segerstrom, Anant and Dinopoulos also take into account the vertical dimension of innovation.

the time period between two radical innovations is random. On the other hand, incremental innovation is guaranteed as long as some research personnel is devoted to it. At each moment, an investor will have to choose between investing in research for the next radical innovation and investing in research for another incremental innovation within the existing technical system. The consideration of two types of innovations considerably enriches the dynamics of innovation and growth. Instead of a regular improvement in productivity, the model displays growth and innovation cycles with may have any periodicity, as well as irregular trajectories.

The paper is organised as follows. Section 2 discusses the two forms of innovation, and their relation to division of labour. Section 3 presents the basic model, section 4 discusses perfect foresight equilibria and Section 5 presents cycles. A brief conclusion is found in section 6.

2 Two types of innovation

The opposition between the two forms of innovation mentioned above and considered respectively in the endogenous growth models of Romer (1990) and Aghion and Howitt (1992) corresponds to an important difference in the treatment of technical change in economic theories. Some economists see technical change as relatively smooth and continuous, involving a cumulative addition of small improvements, as for instance in a continuous process of learning by doing (Arrow, 1962). These changes may be conceived as the result of a deliberate research and development activity or be the outcome of inventions conceived in the production process. Empirical studies have shown that incremental improvements may indeed account for a substantial share of technical progress (Hollander, 1965). This conception of technical progress was that of Gilfillan (1970), whose first social principle of invention stated that " *What is called an important invention is a perpetual accretion of little details, probably having neither beginning, completion nor definable limits (..) An invention is an evolution, rather than a series of creations...*"(p. 5).

A series of creations (and destructions) is precisely what characterises the "Schumpeterian" conception of technical change. It emphasizes the irregular aspect of technical change and represents it as a series of shocks rather than a continuous trend. Indeed, technical change is not necessarily limited to the consequences of a multitude of small improvements that end up in a widening of the technology set. Technological change need not be smooth and continuous. Breakdowns in production methods that deeply alter the structure of the economy do exist. According to a broad "Schumpeterian" view, technical change is an irregular, routine- breaking process, generating 'instability' as well as growth. Drastic changes in technology cannot be reduced to small incremental changes. As Schumpeter put it in a now well-known citation, "*Add as many mail coaches as you please, you will never get a railroad by doing so*".

Nevertheless, the analysis of technical change cannot be limited to the study of major innovations either. A related "Schumpeterian" aspect is that swarms of innovations usher structural change and the birth of new sectors, a characteristics of the diffusion of important innovations (Freeman, 1989). Rosenberg (1982) pointed out that some technologies exhibited a strong complementarity aspect in their simultaneous development. The success of an innovation depends on the success of complementary techniques developed elsewhere in the economy. Schumpeterian growth cycles, or long waves, are related to a major innovation, i.e. an innovation that has such a large influence on the economy that it starts a wave of investment in other sectors. Technical change appears under the guise of 'swarms of innovations'. A major technical breakthrough gives birth to many innovations of lesser importance. Kondratiev cycles are based on the occurence of a radical innovation every fifty years. There is a large literature on the theoretical and empirical relevance of such an hypothesis (Freeman, 1987).

In a related perspective, Gille (1978) developed the concept of "technical system" as a group of complementary techniques around a few major innovations. More recently, Mokyr (1990) distinguished between "micro-" and "macro-" inventions, emphasizing that technical change involves both regular improvements and radical changes of technological perspective. The list of references could easily be extended since such classifications of innovations is very often found in the literature on the economics and history of technical change.

Central to such a view of innovation is a hierarchy of innovations as regards their technical and economic impacts. A new growth phase must be initiated by a major innovation. Some discoveries are radically different from the products previously in use in the economy (the steam machine, electrical engines,...) and their introduction revolutionizes technique. It may then be useful to distinguish between 'incremental' innovation, i.e. innovation on products and processes within an already existing technological framework, and 'radical' innovation, i.e. innovation that changes the very framework. Both continuous improvements and major innovative leaps are part of technical change. Incremental changes possess the limitations inherent to the initial major improvement upon which they are built. On the other hand, a radical innovation could not alone achieve the improvements that numerous incremental changes will bring about after it.

In the model considered in this paper, an incremental innovation means that a new intermediate good is added to an already existing range, as in Romer (1990). A radical innovation means two things:

- the preceding radical innovation is discarded, as well as all the intermediate goods that had been developed on it. The new radical innovation replaces the previous one. Therefore, a radical innovation introduces new goods that are substitutable for older ones[2].

- A new level of knowledge is reached and it allows to produce goods that perform functions similar to those developed on the preceding radical innovations. These

[2]This is a somewhat extreme but not unrealistic assumption. For instance, the introduction of a radical innovation such as the transistor needed some skills that were very different from those for advancing vacuum tube technology. The companies that had assembled the skills for making vacuum tubes found them totally obsolete when the transistor appeared and very few of them successfully made the transition to the new technique.

goods are produced under perfect competition since the competence for producing the goods that replace the former intermediate products is not appropriable.

The creative destruction aspect of a radical innovation is present. The radical innovation allows the implementation a continuum of fixed size of radically new products. Growth is also the result of an increase in the specialization of inputs and an enlargement of the range of intermediate industries. Ethier (1982) has proposed a production function where specialization of inputs led to a form of increasing returns. Romer (1986b, 1990) proposed a version of this production function which takes the following form:

$$Y = \int_0^G x(i)^\alpha di \tag{1}$$

$\alpha \leq 1$. Y is the production of the final good and $x(i)$ is the amount of intermediate good i used in its production. New intermediate goods are invented, increasing the roundaboutness in the production of the final good, measured by G. However, contrary to Romer (1990), it is not assumed that the same intermediate goods are perpetually used in the production of the final good[3].

An important hypothesis of the model presented here is that the increase in knowledge brought about by a new radical innovation enables to produce goods that performed the tasks of the former intermediate goods in the production function. This knowledge is public so that the production of such goods can be made under perfect competition. A radically new intermediate good increases the level of knowledge. This good is produced by a monopoly since its inventor has been granted a life-long patent. In parallel, new intermediate products, compatible with the new radical invention are discovered, and their inventors are granted life-long patents too, so that the incremental goods are produced by monopolies.

3 The basic model

3.1 The technology of production

The model considered here is simplified so as to emphasize the key aspects. The economy under consideration is populated with a continuum of length H of identical researchers. In addition, as in Aghion and Howitt (1992), N specialized workers assist the researchers in radical innovation. There is no traditional factor accumulation nor population growth. Preferences are identical across individuals, intertemporally additive and with a constant marginal utility of consumption at each date. Thus, the subjective discount rate, assumed to be constant, equals the economy-wide interest rate r. Contrary to Romer (1990), Aghion and Howitt (1992), Grossman and

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Helpman (1991) or many other endogenous growth models, the allocation of skilled personnel will not result from a choice between research and production, but between different types of research. The researchers may either engage in radical or incremental innovation activity.

The model can be exposed more precisely. The unique consumption good is produced according to the following production function:

$$y(t) = \omega^n \left\{ \int_0^{G_n} x_c(j)^\alpha dj + \int_0^G x_m(i)^\alpha di + \int_0^1 x_R(u)^\alpha du \right\} \tag{2}$$

At time t, n radical innovations have occurred, each one having improved the level of knowledge by a factor ω $(\omega > 1)$. $y(t)$ is the production of the final good, G is the length of the range of intermediate products specifically developed on the $n-th$ radical innovation at time t, G_n is the length of the total range of intermediate products developed on the $n-1$ preceding radical innovations, $x_k(i)$ is the quantity of intermediate good i used in the production of the final good, $k = c, m$ according to whether the good is produced under perfect or monopolistic competition. $x_R(u)$ is the quantity of the radically new product u corresponding to the latest radical innovation. Each radical innovation allows for the production of a continuum of length 1 of radically new products. All these products appear each time a radical innovation happens. All intermediate goods producers are atomistic.

The inventions corresponding to either a radical or an incremental innovation are protected by patents. These patents are sold or licenced to intermediate good producers, so that the goods corresponding to these innovations are produced under monopoly conditions, whereas the goods corresponding to the preceding radical innovations are produced under perfect competition.

A radical innovation means a departure from the previous technological framework and the discovery and implementation of new technological principles. The discovery of a new intermediate good does not challenge the existence of those already in use in the production of the final good. Incremental innovation means that new intermediate products may increase the range of goods used in the production of the final output. Such innovation is not subject to uncertainty. A radical innovation will be described by a discrete stochastic process whereas incremental innovation will be a deterministic and continuous process.

If c is the flow of researchers employed in radical innovation, the appearance of a radical innovation is characterized by a Poisson process with parameter $\lambda \cdot \varphi(c, N)$, φ is a concave constant return production function. may be thought of as a non-increasing return function in c alone. The presence of an additional factor, specialized workers, makes it a constant returns function when it has partial decreasing returns in c. To simplify notations, the factor N will be dropped from the notations of φ .It is further assumed that $\varphi'(c)$ remains bounded when c goes to 0. λ is a constant parameter. The Poisson process being memory-less, the arrival rate of the radically new technology depends only on the current research-input flow. Past research exerts no influence on the instantaneous probability of the arrival of an innovation.

Time is continuous, indexed by t, and it is spanned by a sequence of radical innovations. Time between two radical innovations will be indexed by s. A new level of knowledge corresponds to each new radical innovation, equal to times the preceding level. Setting the initial level of knowledge to 1, the level of knowledge corresponding to the $n-th$ radical innovation is ω^n. Thus, there is an intertemporal externality of research attached to radical innovations. Incremental innovations exert a positive externality insofar as they increase the economy-wide division of labour.

An intermediate good must be invented before being produced with η units of the final good. Contrary to Romer (1990), the intermediate goods are not durable. To discover new products, one must hire a certain amount of researchers. If l is the number of researchers engaged in incremental innovation, the range of intermediate goods increases to the tune of:

$$\dot{G} = \psi(l) \tag{3}$$

where ψ is a concave decreasing returns function. One notices that there is no spillover associated with knowledge creation in this production function, contrary to Romer (1990), where the innovation production function is of the type: $\dot{G} = \psi(l) \cdot G$, so that an increase in the range of available goods increases the productivity of research. In the specification adopted here, a re- searcher will have a constant productivity in incremental innovation.,

Considering the distinction made between incremental and radical innovations, there is no reason to believe that the incremental innovation production function should exhibit the type of increasing returns found in the technical progress function of Romer (1990). The discovery of an additional intermediate good is no substantial contribution to knowledge, merely the result of the exploitation of existing knowledge.

3.2 The determination of prices and quantities

The demand for an intermediate good i follows from the maximization of profits in the perfectly competitive final goods sector (we note $x_j = x(j)$): $x_j \in ArgMax\,[y - p_j x_j]$, so that the demand curve for good j:

$$p_j = \alpha \cdot \omega^n \cdot x_j^{\alpha-1} \tag{4}$$

Each producer of an intermediate good developed upon a new radical innovation is a monopolist. Profit maximization for an intermediate good producer is such that: $x_i \in ArgMax\,[p_i \cdot x_i - \eta \cdot x_i]$. The price pmi of an intermediate good is thus:

$$p_i = \frac{\eta}{\alpha} \tag{5}$$

The price is the same for all such goods since they are produced in the same conditions and have the same demand curve. The profit for a producer of an intermediate good belonging to the "technical system" created by a new radical innovation is therefore:

$$\pi_i = \eta^{\frac{\alpha}{\alpha-1}} \cdot \alpha^{\frac{2}{1-\alpha}} \cdot \left[\frac{1-\alpha}{\alpha}\right] \cdot \omega^{\frac{n}{1-\alpha}} \tag{6}$$

the profit flow stays constant during the lifetime of each radical innovation, and increases from one radical innovation to the other. The production size of each intermediate good x_i stays constant for each time interval corresponding to a radical innovation, and increases from one radical innovation to the other:

$$x_i = \left[\frac{\eta}{\alpha^2}\right]^{\frac{1}{\alpha-1}} \cdot \omega^{\frac{n}{1-\alpha}} \tag{7}$$

The price of the intermediate goods that correspond to previous radical innovations is equal to its marginal cost since it is produced under perfect competition:

$$p_j = \eta \tag{8}$$

and these goods are produced in the following quantity:

$$x_j = \left[\frac{\eta}{\alpha}\right]^{\frac{1}{\alpha-1}} \cdot \omega^{\frac{n}{1-\alpha}} \tag{9}$$

Research is undertaken by research firms. They hire researchers to discover innovations and obtain a patent that they will license or sell to intermediate goods producers. Free entry ensures that monopoly profits in the intermediate goods production business can be reaped by the research firms in order to cover their development costs. Therefore, patents are priced so that the research firms are rewarded with the monopoly profits stemming from the implementation of the new products corresponding to the innovation. At each moment, investors have a choice between hiring researchers for incremental innovation or hiring them for radical innovation. l researchers engaged in incremental innovation will produce $\psi(l)$ new intermediate goods and c researchers engaged in radical innovation will find a radical innovation with an instantaneous probability of $\lambda \cdot \varphi(c)$. If V_I is the current value of the profits generated by an incremental innovation, the employment of l researchers in the incremental innovation activity will induce $\dot{G} \cdot V_I$, with:

$$V_I = \int_t^{\infty} e^{-r(s-t)} \cdot e^{-\lambda \cdot \varphi(c_n) \cdot (s-t)} \pi_i(s) \cdot ds \tag{10}$$

One notices that the probability of the discovery of a new radical innovation is accounted for in the computation of expected profits. If incremental innovation involves no uncertainty, investing in the production of a new incremental good does, because the appearance of a radical innovation will terminate the lifetime of all incremental goods belonging to the current technical system.

It is assumed here that all research firms take the wage rate of researchers as given. If w is the wage rate of the researchers, profit maximization leads to the following condition:

$$w = \psi(l) \cdot V_I \tag{11}$$

for $l \neq 0$.

Concerning the $n+1-st$ radical innovation, researchers will be employed in radical innovation up to the level where their marginal productivity equals their marginal cost of employment. If c_n researchers are employed in radical innovation, the expected profit for the innovator is $\lambda \cdot \varphi(c_n) \cdot V_{n+1}$, with:

$$V_{n+1} = \int_t^{\infty} e^{-r\cdot(s-t)} \cdot e^{-\lambda\cdot\varphi(c^e_{n+1})\cdot(s-t)} \cdot \pi_{n+1}\, ds \tag{12}$$

$\pi_{n+1} = \eta^{\frac{\alpha}{\alpha-1}} \cdot \alpha^{\frac{2}{1-\alpha}} \cdot \left[\frac{1-\alpha}{\alpha}\right] \cdot \omega^{\frac{n+1}{1-\alpha}}$. V_{n+1} is the present value of the profits π_{n+1} generated by the $n+1-st$ innovation and c^e_{n+1} is the expected number of researchers engaged in radical innovation during the period between the $n+1$-st and the $n+2$-nd innovations. Profit maximization leads to the following condition:

$$w = \lambda \cdot \varphi'(c_n) \cdot V_{n+1} \tag{13}$$

for $c_n \neq 0$.

Therefore, the behaviour of each type of investor is described by the market equilibrium equations. In what follows, a particular type of equilibrium allocation of research will be considered, the case where agents have perfect foresight.

4 Perfect foresight market equilibria

This section considers the case of perfect foresight equilibria, i.e. it is assumed that $c_{n+1} = c^e_{n+1}$. The equilibrium on the perfectly competitive research-labour market gives the equilibrium allocation of research personnel that one is looking for. c_n is the number of researchers devoted to the discovery of the $n+1$-st radical innovation during the time interval corresponding to the life of the n-th radical innovation, and c_{n+1} the number of researchers devoted to the discovery of the $n+2$-nd radical innovation during the time interval corresponding to the life of the $n+1$-st radical innovation.

The following notation is adopted: $\psi(l) = \psi(H-c) \equiv \theta(c)$, $\theta' < 0$, $\theta'' < 0$. The equilibrium condition reads:

$$\frac{-\theta'(c_n)}{[r + \lambda \cdot \varphi(c_n)] \cdot \varphi'(c_n)} = \frac{\lambda \cdot \omega^{\frac{1}{1-\alpha}}}{r + \varphi(c_{n+1})} \tag{14}$$

This formula can be expressed as:

$$f(c_n) = g(c_{n+1}) \tag{15}$$

g is, without ambiguity, a decreasing function of c_{n+1}. But the monotonicity of f is not assured. The sign of $f'(c_n)$ depends on the sign of a difference of positive terms: and is a-priori indeterminate.

EXAMPLE. If the innovation functions are specified as follows: $\psi(l) = \ln(1+\beta \cdot l)$ and $\varphi(c) = \sigma \cdot c$, $\sigma, \beta > 0$, f is first decreasing and then increasing on $[0, H]$ when $\mid \frac{r}{\lambda \cdot \sigma} - \frac{1}{\beta} \mid < H$ (Figure 1). Indeed, $f\prime(0) < 0$ when $\beta \cdot r - \lambda \cdot \sigma < \lambda \cdot \sigma \cdot \beta \cdot H$. and f' goes to zero and changes signs strictly before H when $\frac{1}{\beta} - \frac{r}{\lambda \cdot \sigma} < H$.

g is invertible, so that the forward dynamics can be written as:

$$c_{n+1} = g^{-1}\left[f(c_n)\right] \equiv \xi(c_n) \tag{16}$$

and:

$$\xi'(c) = \frac{f'(c)}{g\left[\xi(c)\right]} \tag{17}$$

In what follows, it is supposed that ξ maps $[0, H]$ into itself. The behaviour of ξ rests on that of f. If f is first decreasing and then increasing, as in the example above, ξ is first increasing and then decreasing, its graph exhibiting a hump, as in Figure 3. This means that there exists c^* in $[0, H]$ such that $\xi'(c) = 0$, with $\xi' > 0$ for $c < c^*$ and $\xi' < 0$ for $c > c^*$. This condition ensures that there exists a unique fixed point $\hat{c}$.

A fixed point of ξ represents a perfect foresight equilibrium with a constant allocation of research personnel. The properties of this constant allocation equilibrium are summarized in the following proposition:

Proposition 1. At a constant allocation equilibrium, (i) the rate of time interest exerts no influence on the number of researchers engaged in either type of innovation activity and (ii) the amount of research personnel devoted to the discovery of the next radical innovation increases with the size of each radical innovation, the arrival parameter λ and the production parameter α.

Proof. A constant allocation of research personnel $\hat{c}$ corresponds to a fixed point of ξ such that:

$$\frac{\psi'(H - \hat{c})}{\varphi(\hat{c})} = \lambda \cdot \omega^{\frac{1}{1-\alpha}} \tag{18}$$

One can immediately see in the expression above that the value of the interest rate does not matter for the allocation of research. Indeed, the uncertainty-adjusted rate of discount $r + \lambda \cdot (c)$ is the same for both types of innovation when the allocation of research personnel is fixed. Therefore, the value of the interest rate is of no consequence on the equilibrium allocation that depends only on the relative expected marginal productivities of the research production functions. At the equilibrium allocation, the marginal productivity of research labour in the incremental innovation sector must be equal to the marginal productivity of labour in the incremental innovation activity multiplied by the expected factor by which profits will be multiplied when the radical innovation occurs. The proof of the second part of the proposition follows lies in the fact that $\frac{\partial}{\partial c}\left[\frac{\psi'(H-c)}{\varphi'(c)}\right] \geq 0$ □

The equilibrium $\hat{c}$ is asymptotically stable if $\mid \xi'(\hat{c}) \mid < 1$, that is if:

$$\left| \frac{[r + \lambda \cdot \varphi(\hat{c})] \cdot [\theta''(\hat{c}) \cdot \varphi'(\hat{c}) - \varphi''(\hat{c}) \cdot \theta'(\hat{c})] - \lambda \cdot \theta'(\hat{c}) \cdot \varphi'(\hat{c})^2}{\lambda^2 \cdot \omega^{\frac{1}{1-\alpha}} \cdot \varphi'(\hat{c})^3} \right| < 1 \qquad (19)$$

For the example given above where $\psi(l) = \ln(1 + \beta \cdot l)$ and $\varphi(c) = \sigma \cdot c$, one obtains: $\xi(c) = a_0 + a_1 \cdot c - a_2 \cdot c^2$, with: $a_0 = r \cdot \left[\frac{\omega^{\frac{1}{1-\alpha}} \cdot (1+\beta \cdot H)}{\beta} - \frac{1}{\lambda \cdot \sigma} \right]$, $a_1 = \omega^{\frac{1}{1-\alpha}} \cdot \left[\frac{\lambda \cdot (1+\beta \cdot H) \cdot \sigma}{\beta} - r \right]$, $a_2 = \sigma \cdot \lambda \cdot \omega^{\frac{1}{1-\alpha}}$, with a_0, a_1 and a_2 assumed positive. The fixed point is asymptotically stable if: $\left| 1 - \sqrt{(1 - a_1)^2 + 4 \cdot a_0 \cdot a_1} \right| < 1$, that is if: $\left| 2 - \omega^{\frac{1}{1-\alpha}} \cdot (r + \lambda \cdot \sigma \cdot \nu \right| < 1$ with $\nu = \frac{1+\beta \cdot H}{\beta}$.

One may thus obtain a unique stable allocation $(\hat{l}, \hat{c})$ for the population of researchers. At each moment, $\hat{c}$ researchers are busy trying to discover what will be the next radical innovation, while $H - \hat{c}$ are implementing the current technological system through the discovery of additional intermediate goods. Growth results both from the discrete jumps in the level of productivity and the continuous improvements due to incremental innovations. Between two radical innovations, productivity grows arithmetically with respect to time. The time intervals between two radical innovations are random.

5 Cycles

A single fixed point is not the only possibility for the allocation of research personnel. First, if there exists a c_{ng} such that $\xi(c_{ng}) = 0$, then there is the equivalent of the "no-growth trap" equilibrium present in the models of Romer (1990) and Aghion and Howitt (1992). In these models, a no-growth trap means that all skilled personnel is affected to production, so that productivity improvements stop. Here, the equivalent of a no growth trap means that radical innovation stops for all research personnel is allocated to the incremental innovation activity. The economy grows at a decreasing rate, which asymptotically reaches zero, so that growth ceases in the long run. Second, besides the possible existence of several fixed points, i.e. multiple fixed allocation equilibria, dynamical systems such as equation (19) may generate very complex dynamics. Other equilibria than a fixed point may exist, for instance periodic points or cycles. Of course, 0 must not belong to a periodic orbit since radical innovation activity would stop altogether. Therefore, only orbits with non-zero periodic points make economic sense.

The most simple cycle is a cycle of period two, where periods where few researchers are allocated to the discovery of the next radical innovation alternate with periods where a high proportion of research personnel is engaged in the pursuit of the next radical innovation. The interpretation of such cycles is straightforward. A low c_{n+1} means an increase in the expected life- time of the $n + 1$-st radical innovation. The higher expected profits make the search for the next radical innovation all the more attractive, inducing a high c_n. Cycles of period 2 were already a possibility in

Aghion and Howitt (1992). However, more complex trajectories were not possible, whereas they are in this model, due to the fact that "creative destruction" affects both incremental and radical innovations.

Of particular interest are the maps ξ that admit a cycle of period three. The existence of a period-3 cycle may be proved with the help of theorem II-9 in Block and Coppel (1992), which states that if for some $c \in [0, H]$ and for some odd $n > 1$, $\xi^n(c) \leq c \leq \xi(c)$, where ξ^j is the j-th iterate of ,ξ then ξ has a periodic point of period q, for some odd q satisfying $1 < q < n$ (see Appendix 1)[4].

Going back to the example ($\psi(l) = \ln(1 + \beta \cdot l)$ and $\varphi(c) = \sigma \cdot c$), one obtains a single- peaked map for ξ, so that if there exists c_0 in $[0, \hat{c}]$ such that $\xi^3(c_0) \leq c_0$, then ξ has a cycle of period 3[5]. For some values of the parameters, one can readily obtain an example of the existence of such a cycle (Figure 4).

Proposition 2. If $\psi(l) = \ln(1 + \beta \cdot l)$ and $\varphi(c) = \sigma \cdot c$, $\sigma, \beta > 0$, ξ admits a cycle of period 3 if :

(i) $r + \lambda \cdot \sigma \cdot \nu \cdot \left(1 - 2 \cdot r \cdot \omega^{\frac{1}{1-\alpha}}\right) \geq 0$,

(ii) $\omega^{\frac{2}{1-\alpha}} \cdot (r + \lambda \cdot \sigma \cdot \nu)^3 \cdot \left[4 - \omega^{\frac{1}{1-\alpha}} \cdot (r + \lambda \cdot \sigma \cdot \nu)\right] - 16 \cdot r > 0,$

(iii) $\omega^{\frac{1}{1-\alpha}} \cdot (r + \lambda \cdot \sigma \cdot \nu) \geq \Lambda$

with $\Lambda \approx 3.83$.

Proof. Let c_2 be such that $c^* = \xi(c_2)$ and c^* such that $\xi'(c^*) = 0$, with $c_2 < c^*$. (i) ensures that c_2 is positive since $\xi(c)$ is increasing for $c \leq c^*$. (ii) ensures that $\xi^2(c^*) > 0$. According to the theorem mentioned above, there exists a period-3 cycle if $\xi^2(c^*) \leq c_2$. Since: $c_2 = \frac{a_1 - \sqrt{a_1^2 - 2 \cdot a_1 + 4 \cdot a_0 \cdot a_2}}{2 \cdot a_2}$, further calculations give that the sign of $c_2 - \xi^2(c^*)$ is the sign of $(z - 4) \cdot z - 8 \cdot z$, with $z = \left[\omega^{\frac{1}{1-\alpha}} \cdot (r + \lambda \cdot \sigma \cdot \nu) - 1\right]^2 - 1$, from which (iii) follows◇

Of course, the conditions stated in proposition 2 are sufficient but not necessary to the existence of a period-3 cycle. Furthermore, in more general cases, ξ may very well not be unimodal.

The example above gives a single-peaked ξ with a unique fixed point. Restraining ξ to $[0, \xi(c^*)]$, one supposes that $\xi(c) > c$ for all c in $[0, x^*]$, $\xi'(0) > 1$ if $\xi(0) = 0$, ξ has a unique fixed point in $[x^*, H]$. One has: $\xi(c) = a_0 + a_1 \cdot c - a_2 \cdot c^2$, $\xi'(c) = a_1 - 2 \cdot a_2 \cdot c$, $\xi''(c) = -2 \cdot a_2$ and $\xi'''(c) = 0$. The stability of at most one cycle of any period is ensured if the Schwarzian derivative of ξ is negative for $c \neq c^*$. It is obvious that for every c in $[0, H]$ different from c^*, $S\xi$ is negative. Therefore, at most one weakly stable cycle exists. This weakly stable periodic orbit, *when it exists*, attracts the critical point c^* and all points in $[0, c^*]$ except for a set of measure 0 if in addition $\xi''(c^*) < 0$, which is obviously the case in the example.

The stability of periodic orbit(s) cannot be guaranteed no matter what ξ is. If ξ has no stable cycle, the behaviour of the c_i's is aperiodic for any initial condition that does not belong to a periodic orbit. The trajectory of the number of researchers

[4]The applicability of this theorem is not limited to one-hump-shaped maps.

[5]Economic models with complex dynamics can be found in Grandmont (1985) or Benhabib (1992).

engaged in radical innovation will then become very complex, looking "random" or unpredictable.

To sum up, long term growth in this model is an irregular process. When the equilibrium allocation of research personnel is fixed, radical innovations occur at random, the expected time period between each innovation being fixed. Between radical innovations, the economy's growth rate is not constant, but decreasing, asymptotically tending to zero. However, the occurrence of radical innovations prevents the extinction of growth. At certain time intervals, the economy's productivity makes a discrete jump, so that high and low growth rates will alternate. Since the expected life-time of each technological system (defined by a given radical innovation and the other intermediate goods compatible with it) is fixed, it is possible to find some regularity in the growth process.

When the equilibrium allocation of research is not fixed, i.e. when it follows a cycle or when it is aperiodic, the expected time period between two radical innovations is no longer fixed, so that the pattern of growth may be considered as even more irregular than in the fixed alloca- tion case. The allocation of research personnel varies from one period to the other, and the frequency of occurrence of a radical innovation varies. A very long period without a radical innovation may follow periods where radical breakthroughs have been frequent, making technological stagnation succeed to periods of rapid increase in the level of knowledge. Consequently, some radical innovations will have a much larger range of incremental intermediate products developed on them than others.

6 Conclusion

The model presented here accounts for two types of innovations. Incremental innovation is a continuous stream of improvements of a given technological system. Such a pattern of technical change has its limitations. There are limits to the achievements possible within a given technological framework. Radical innovation is an important breakthrough in the accumulation of knowledge that alters the technological system and allows for further incremental advances in technology. Researchers may engage in either type of innovation and are allocated to each type of innovating activity through market mechanisms, so that innovation is considered as a normal economic activity.

Growth in the present model is endogenously determined, resulting from the productivity improvements that different innovations enable, either through discrete jumps in the level of knowledge, or through a continuous increase in the degree of economy-wide division of labour. Most of the productivity gains are not necessarily associated with a single radical innovation but may alternatively stem from the discovery of new 'incremental' intermediate goods, which are follow-up inventions. Contrary to most models of endogenous growth, growth is an irregular process, where phases of growth at a decreasing rate alternate with sudden jumps in the productivity level. The growth process is all the more irregular that the allocation of research personnel between radical and incremental innovation may follow a cyclical pattern, or even be aperiodic. Thus, periods of arithmetic growth are disrupted by innovative leaps that are unevenly distributed over time.

Several important features are not captured by the model. The study of the equilibria has been limited to the case of perfect foresight. This is a limit case. Expectations could be taken account of, which would result in a different dynamics than the one presented here. An extreme assumption has been made, that each radical innovation dissipated all the monopoly rents in the economy. A more gradual process could be devised, taking account of the pattern of diffusion of an innovation in the economy. In fact, the diffusion of the new radical technology has been assumed instantaneous for the sake of simplification. Actually, the problem of the diffusion of innovations is seldom treated in endogenous growth models. A better account of diffusion problems could be a subject for further research.

The conception of technological change adopted in this paper is at odds with most approaches of innovation in endogenous growth models, where technological progress is regular and no distinction is made between the different types of innovations. Jovanovic and Rob (1990) have made a distinction between intensive and extensive search, leading to one type of extension of knowledge. Young (1991, 1993a) combined an expansion in the range of intermediate inputs with an increase in the quality of the new inputs, so that both the horizontal and the vertical dimensions of technological change were taken into account. Nevertheless, there was no radical innovations since change occurred as a smooth process. Young (1993b) considers innovations that complement earlier ones and innovations that substitute for earlier ones. However, there is no choice concerning the type of innovation for the agents. Since this paper was written, some models of endogenous growth have taken better account of the diversity of innovation. A paper by Aghion and Howitt (1994) takes into account a distinction between research and development, research producing fundamental knowledge, development generating secondary knowledge.

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APPENDIX 1. Complex dynamics

The reader may check Grandmont (1988) and Block and Coppel(1992) for a detailed exposition of discrete dynamics in one dimension. In the context of this article, the existence of a period 3 cycle for ξ matters because of Sarkovskii's theorem which states that if ξ has a periodic orbit of period n and if $n \prec m$ in Sarkovskii's ordering, then has also a periodic orbit of period m. To recall, Sarkovskii's ordering is the following:

$$3 \prec 5 \prec 7 \prec 9 \prec ... \prec 2 \cdot 3 \prec 2^2 \cdot 3 \prec 2^2 \cdot 5 \prec ... \prec 2^3 \prec 2^2 \prec 2 \prec 1$$

Therefore, if a cycle of period 3 exists, there also exist cycles of every period. The following results will help to characterize ξ.

f, an arbitrary continuous map of an interval I into itself, is said to be turbulent if there exists compact subintervalsJ, K with at most one common point such that: $J \cup K \subseteq f(J) \cap f(K)$. It will be said to be strictly turbulent if the subintervals J, K can be chosen disjoint. Turbulent maps have periodic points of all periods. By contrast, the trajectories of non-turbulent maps are subject to considerable restrictions.

The three following conditions are equivalent:

(i) f has a periodic point whose period is not a power of 2, (ii) f^m is strictly turbulent for some positive integer m, (iii) f^n is turbulent for some positive integer n.

The map f is said to be chaotic when one, and hence all three, of these conditions is satisfied. Therefore the existence of a period-3 cycle for ξ makes it a chaotic map (Block and Coppel, 1992).

In such a case, there exist cycles of every period, but they may be unstable. If $(c_0, c_1, ..., c_{k-1})$ is a periodic orbit of a cycle of period k for ξ, the cycle is weakly stable if $\left|\left(\xi^k\right)'\right| = |\xi'(c_0)...\xi'(c_{k-1})| \leq 1$, stable if the inequality is strict and superstable if c^* belongs to the periodic orbit.

The stability of at most one cycle of any period is ensured if the Schwarzian derivative of ξ is negative for $c \neq c^*$ (Grandmont, 1988), i.e.: $S\xi = \frac{\xi'''}{\xi'} - \frac{3}{2} \cdot \left(\frac{\xi''}{\xi'}\right)^2 < 0$. It is obvious that for every c in $[0, H]$ different from c^*, $S\xi$ is negative. Therefore, at most one weakly stable cycle exists. This weakly stable periodic orbit, *when it exists*, attracts the critical point c^* and all points in $[0, c^*]$ except for a set of measure 0 if in addition $\xi''(c^*) < 0$, which is obviously the case in the example given in the text.

APPENDIX2. Strategic monopsony effect

It had been assumed so far that research firms took the wage rate of researchers as given. If only one firm hires all research personnel employed in radical innovation, this firm controls a sizeable part of the research labour force. In this section, the possibility fo··r the radical innovator to determine its research labour demand taking into account the effect it will have on the wage rate is considered. Adopting the same notations as before, the radical innovator has the program: $Max \lambda \cdot \varphi(c_n) \cdot V_{n+1} - w \cdot c_n$, subject to: $w = \frac{-\theta'(c_n) \cdot \pi_n}{r + \lambda \cdot \varphi(c_n)}$. The first order condition is:

$$\frac{\lambda \cdot \varphi'(c_n) \cdot \pi_{n+1}}{r + \lambda \cdot \varphi(c_{n+1}^e)} = w + \frac{\partial w}{\partial c} \cdot c$$

The second term in the above expression is the effect of the labour demand of the innovator on the wage rate of the researchers.

Since there is free entry in research, the monopsony effect is only valid when it leads to a higher wage rate for researchers than in the perfectly competitive case, because the monopsony would not be sustainable otherwise. In order to simplify the treatment, only the case of example 2 will be considered. In this case, the graph of $w(c)$ may be represented as in Figure 5. $w(c)$ is decreasing for c between 0 and c_m, and increasing for c between c_m and H. Therefore, the curve representing $w + c \cdot w'$ lies below that of $w(c)$ for c between 0 and c_m and above it for c between c_m and H. The market structure that prevails is the one that guarantees a lower number of researchers in radical innovation on the interval $[0, c_m]$ and a higher number of researchers on the interval $[c_m, H]$. This way, the prevailing market structure can offer a higher wage rate to researchers.

In a similar way to what has been derived for the competitive solution in section V, one can define a difference equation corresponding to the monopsony case. In this case:

$$c_{n+1} = \xi_M(c_n)$$

with:

$$\xi_M(c) = g^{-1}\left[f_M(c)\right]$$

and:

$$\begin{aligned} f_M(c) &= \frac{w(c) + c \cdot w'(c)}{\varphi'(c) \cdot \pi_n} \\ &= \frac{\lambda \cdot c \cdot [\theta'(c) \cdot \varphi'(c) - \varphi(c) \cdot \theta''(c)] - [r + \lambda \cdot \varphi(c)] \cdot \theta'(c) - r \cdot c \cdot \theta'(c)}{\varphi'(c) \cdot [r + \lambda \cdot \varphi(c)]^2} \end{aligned}$$

ξ_M possesses the same shape as ξ (Figure 6), except that attains a maximum for $c = c_m$ and ξ_M attains a maximum for $c = c_M$, with $c_M \leq c_m$. The two curves intersect at c_m. As was mentioned before, the monopsony only holds when it can offer a higher wage rate. To the left of c_m, the competitive solution gives a higher level of employment and thus a higher wage rate. To the right of c_m, the monopsony leads to a lower level of employment of researchers in radical innovation and thus to a higher wage rate. The monopsony is not sustainable for high levels of employment in the radical research activity.

Therefore, the dynamics of the number of researchers employed in radical innovation is given by:

$$c_{n+1} = \begin{cases} \xi_M(c_n) & for \quad c \in [0, c_m] \\ \xi(c_n) & for \quad c \in [c_m, H] \end{cases}$$

The dynamics for the cns is defined by a map composed with ξ_M and ξ (Figure 7) which is not unimodal and is not differentiable at $c = c_m$. The same type of complex behaviour as with the purely competitive case may arise. Cycles concerning the competitive structure of the labour market may exist too. The trajectory of the c_ns will see periods of competition alternate with periods of monopsony. As in the competitive case, growth will be characterized by periods of smooth technological change disrupted by radical innovations.

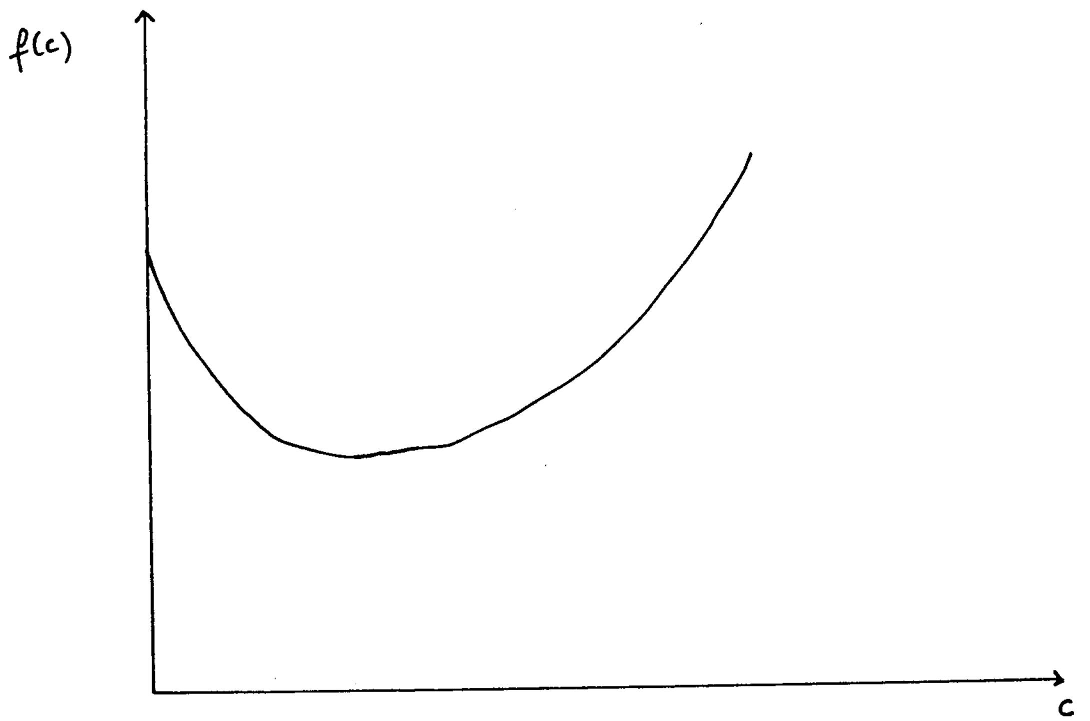

Figure 1

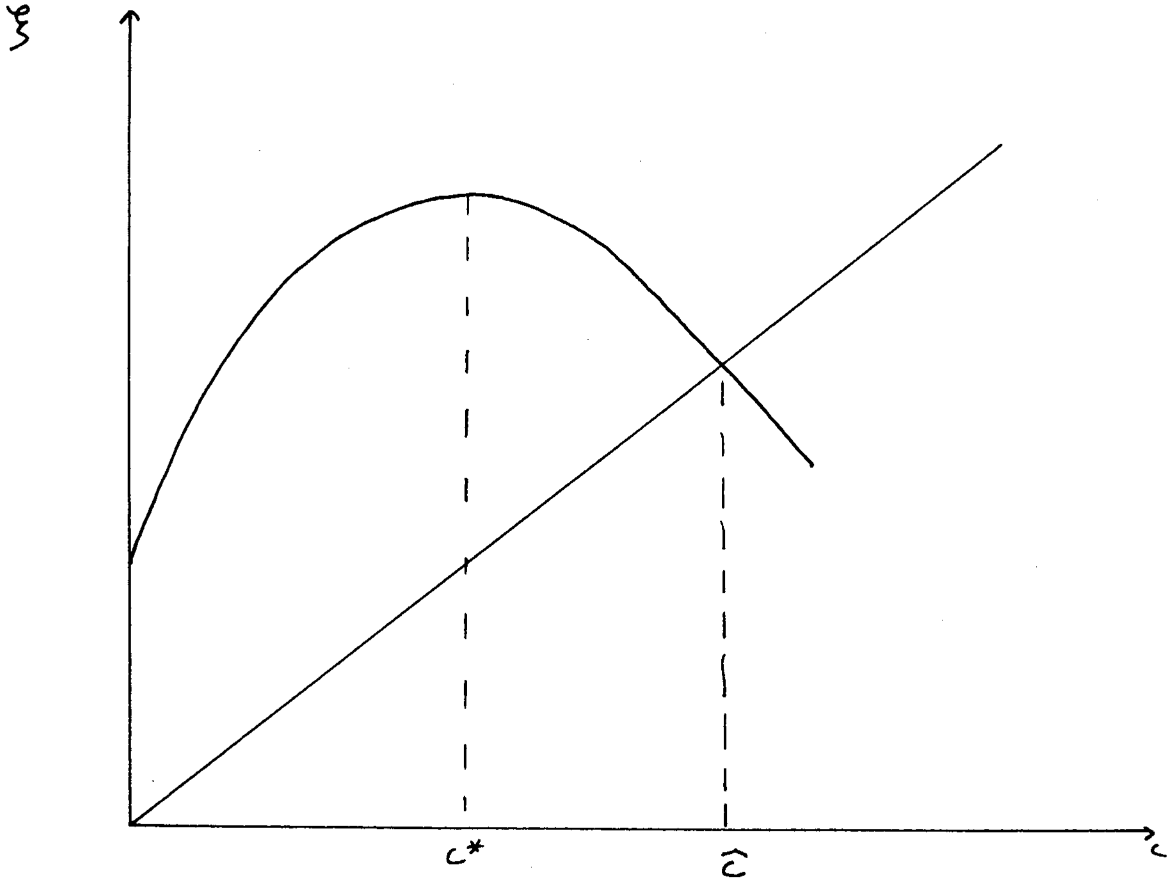

Figure 2

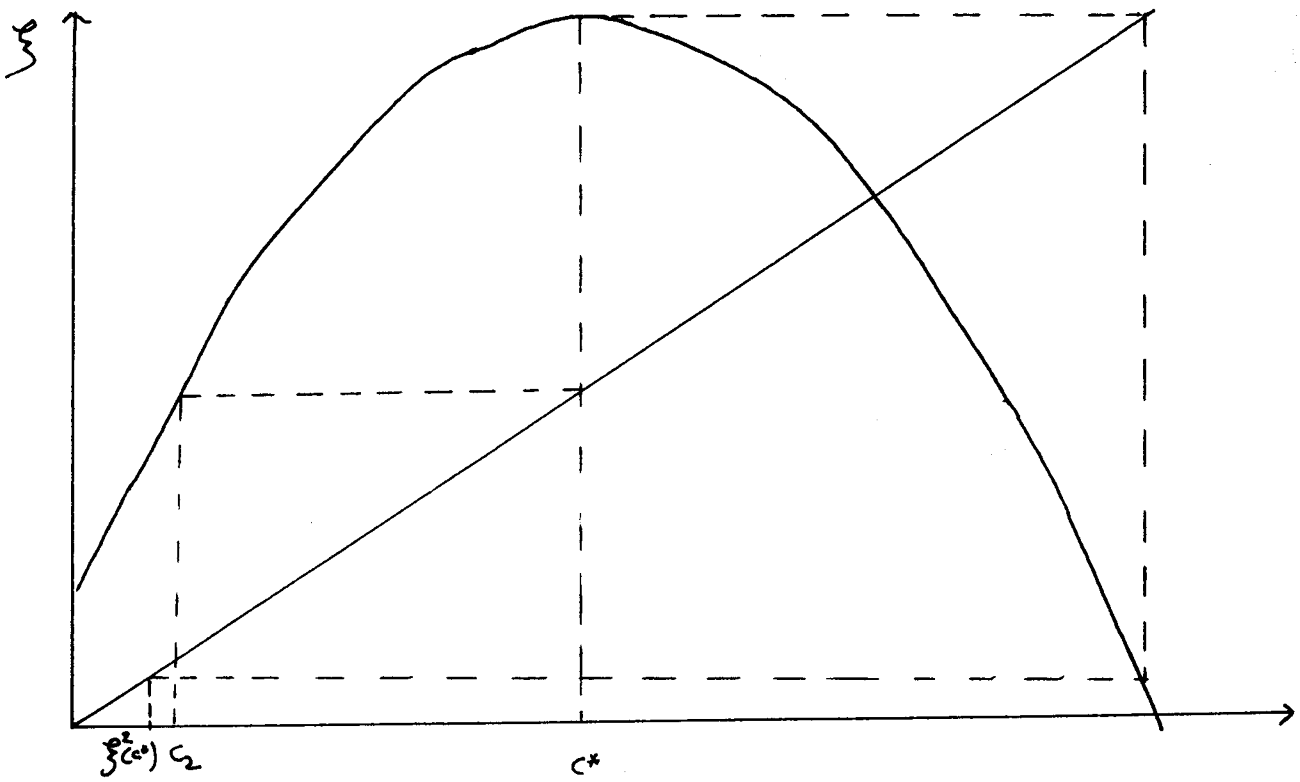

Figure 3

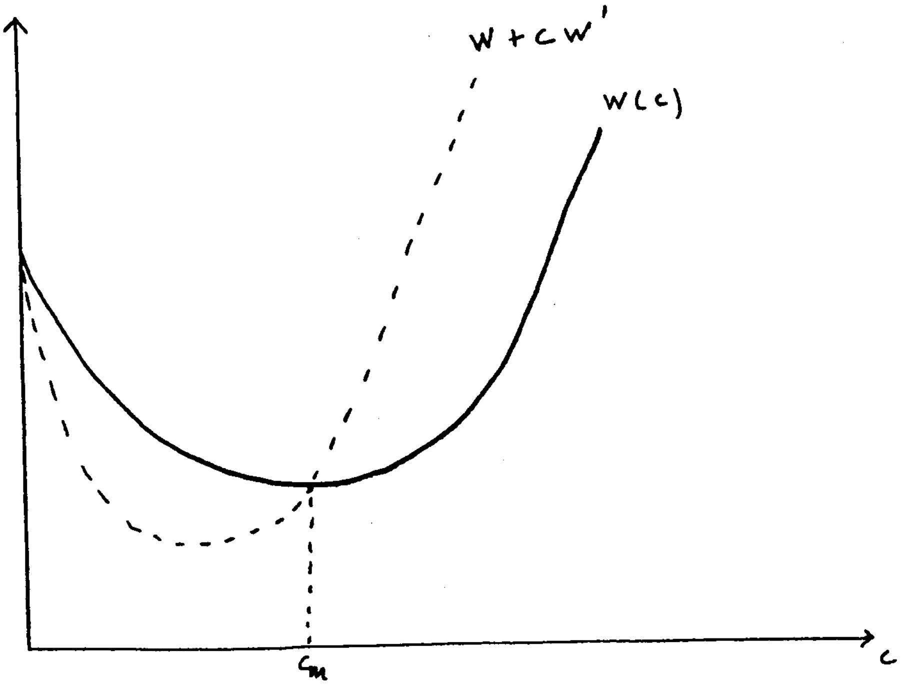

Figure 4

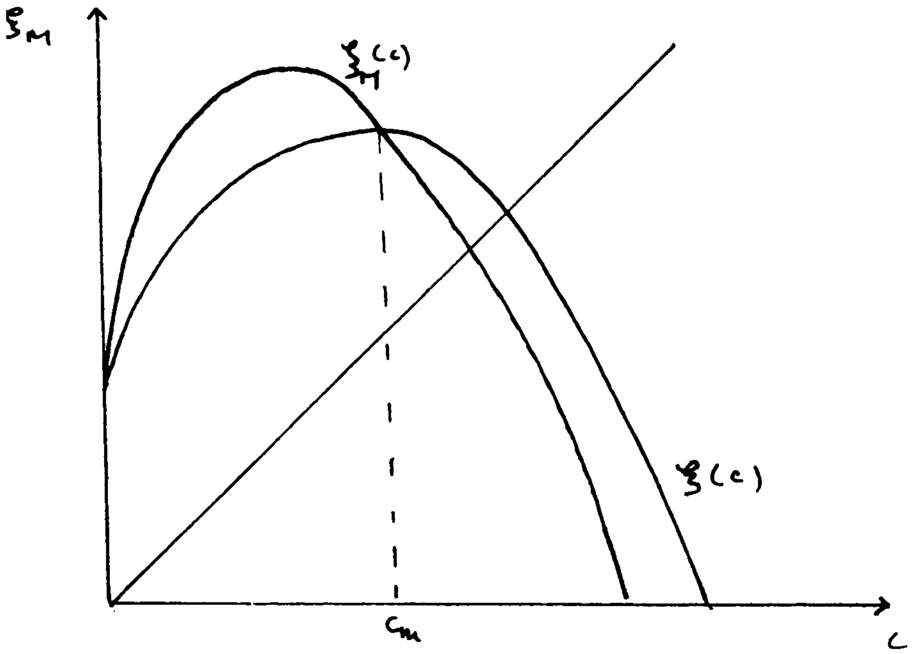

Figure 5

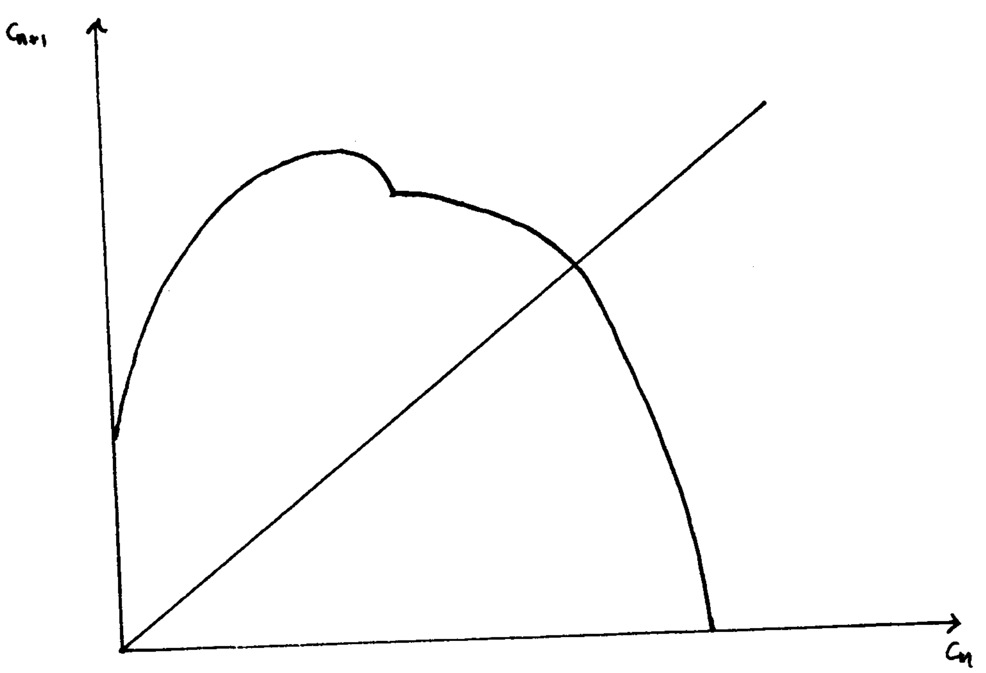

Figure 6