AUDITING CLAIMS IN INSURANCE MARKET WITH FRAUD : THE CREDIBILITY ISSUE

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THE CREDIBILITY ISSUE

A B S T R A C T

This paper characterizes the equilibrium of an insurance market where opportunist policyholders may submit fraudulent claims. We assume that insurance policies are traded in a competitive market where insurers cannot distinguish honest policyholders from opportunists. The insurer-policyholder relationship is modelled as an incomplete information game, in which the insurer decides to audit or not. The market equilibrium depends on whether insurers can credibly commit or not to their audit strategies. We show that a no-commitment equilibrium results in a welfare loss for honest individuals that may even be so large that the insurance market completly shuts down. Finally, we show that transferring monitoring costs to a budget-balanced common agency would mitigate the commitment problem.

Keywords : Insurance, fraud, audit, credibility, commitment.

JEL Classification Number : D8, K4.

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LE CONTROLE DES DEMANDES D'INDEMNITES DANS LES MARCHES

D'ASSURANCE AVEC FRAUDE : LE PROBLEME DE LA CREDIBILITE

R E S U M E

Cet article caractérise l'équilibre d'un marché d'assurance où des individus opportunistes peuvent réclamer des indemnités de manière frauduleuse. Nous supposons que les contrats d'assurance s'échangent sur un marché concurrentiel où les assureurs ne peuvent pas distinguer les assurés honnêtes de ceux qui sont oportunistes. La relation assureur-assuré est modélisée comme un jeu à information incomplète dans lequel l'assureur peut décider de procéder à un audit. L'équilibre de marché dépend de la capacité des assureurs à s'engager de manière crédible sur leur stratégie d'audit. Nous montrons qu'un équilibre sans engagement conduit à une perte de bien-être pour les individus honnêtes qui peut même conduire à une fermeture du marché d'assurance. Enfin, nous montrons que le problème de l'engagement est atténué si on transfère les coûts de contrôle à une agence commune dont le budget est équilibré.

Mots clefs : Assurance, fraude, audit, crédibilité, engagement.

I. INTRODUCTION

Nowadays combatting fraudulent claims on insurance policies is a major concern of most insurance companies. From minor overestimation of losses to arson or other criminal activities, there are many different degrees of severity of insurance fraud. *Build-up* is a soft fraud in which policyholders misrepresent the damages in an attempt to obtain a larger insurance payment. *Opportunistic fraud* occurs when the insured seizes the opportunity of an accident or theft to fake the damage for personal profit. Finally *planned fraud*, is the result of deliberate criminal behaviour [1].

Although it is impossible to measure exactly what proportion of the cost of claims is attributable to fraud, many insurers say that fraud occurs in all sectors of insurance and presents, at the lowest estimates, about 5 % of the total annual claims bill for property-casualty insurance, excluding build-up. Much higher rates of suspicious claims are frequently put forward, in particular for fire insurance, automobile theft claims and bodily injury liability insurance [2]. On the whole, as stressed by the head of public affairs of the Association of British Insurers : "There is a realization that if even a tiny percentage of policyholders are willing to tell lies, or not disclose relevant information when they take out their policy, it does present the industry with a multi-million pound problem" [3].

More often than not, insurers seem to be unable to deter pervasive fraudulent claims. Collective actions at the industry level are helpful in providing information about how to deal with suspected fraudulent claims and establishing common databases from the claims experiences of each policyholder. More severe sentences for fraudsters also help to deter insurance fraud [4]. However, the main obstacle to strong action against fraud remains that monitoring suspicious claims is costly. Monitoring costs include the cost of claim adjusters, investigators and lawyers, and this cost is all the more important given that hard evidence on fraudulent claims can be difficult to obtain. Furthermore, withholding payment on claims may be perceived as unfair by honest policyholders, which may result in a costly reputation effect, or even in bad-faith penalties imposed by courts.

Nevertheless, in a situation where fraud is pervasive, it is essential for companies to credibly announce that a tough monitoring policy will be enforced, with a high level of claim verification and scrutiny for suspected fraud. However, since monitoring is costly to the insurer, what is desirable *ex ante* may be sub-optimal *ex post*, and a commitment to subject claims to close scrutiny may not be credible.

The commitment problem is exacerbated in so far as it is not easy for an insurer to attain the reputation of being tough. First, optimal claim handling usually involves random auditing, which makes it difficult for policyholders to monitor deviations of the company from its pre-announced strategy. Second, the probability of suffering a loss may be too low for a policyholder to experiment with the credibility of the insurer's auditing strategy. Third, policyholders may have only aggregate information on the average probability among insurers for a claim to be audited. Such global information could be volunteered by insurance regulators, but then we are faced with the problem of the credibility of this public announcement, since any insurer has an incentive to deviate.

This paper characterizes the equilibrium of an insurance market where policyholders may file fraudulent claims. Its purpose is to analyze the social inefficiency that results from insurance fraud by focusing on the consequences of the commitment problem. We will also show that a common agency that contracts with insurance companies to subsidize claims monitoring may help to solve the commitment problem.

Our starting point is the relationship between an insurance company and its policyholders. Some policyholders may file fraudulent claims (we call them "opportunists"), while honest policyholders always tell the truth, regardless of their pecuniary interest. This insurer-policyholder relationship is modelled as a game in which the insurer has incomplete information about his customer's type. Policyholders may experience a loss of a given size or not, and opportunists may put in fraudulent claims. When a claim is filed, the insurance company decides whether to audit it or not. A policyholder who is discovered to have submitted a fraudulent claim is prosecuted and fined.

Of course, this is a very crude way to model insurance fraud, since it reduces the opportunists'strategy to a simple fraud honesty choice, where build-up does not occur. Its main advantage is to allow an explicit characterization of equilibrium strategies in monitoring games, which will prove essential in characterizing the competitive market equilibrium.

The equilibrium of an audit game depends on whether the insurance company can commit or not to its auditing strategy. Commitment gives a Stackelberg advantage to the insurer, as in the literature on income taxation where the tax authority acts as a Stackelberg leader in its choice of tax-audit strategy (see Reinganum and Wilde (1985), Border and Sobel (1987), and Mookherjee and P'ng (1989)). In the absence of commitment, the auditing strategy of the insurance company should be a best response to opportunists' fraud strategy, as in the papers by Graetz, Reinganum and Wilde (1986) and Melumad and Mookherjee (1989).

We will assume that insurance policies are traded in a competitive market with free entry, where trades are affected by adverse selection because insurance companies cannot distinguish honest policyholders from opportunists. As in the literature initiated by Rothschild and Stiglitz (1976) and Wilson (1977) in which insurers have imperfect information on policyholders' risk type, several equilibrium concepts can be contemplated. Rothschild and Stiglitz (RS) assume that companies play Cournot-Nash strategies in that they take the offer of their competitors as given. Then a RS-equilibrium is characterized by a set of profitable policies such that there is no other policy which, when offered in addition, earns positive profits. As in the Rothschild-Stiglitz model, we will show that a RS-equilibrium may not exist and we will give a necessary and sufficient condition for existence. In Wilson's definition (W), firms anticipate that any policy that becomes unprofitable will be withdrawn. Then a W-equilibrium is reached with a set of profitable policies such that there does not exist any other policy that would remain profitable after all non-profitable policies are withdrawn in reaction to the offer. We will show that, a RS-equilibrium is a W-equilibrium and that a W-equilibrium always exists.[5]

In our economy, RS-or W-equilibrium has a simple interpretation. Equilibrium is characterized by a pooling contract that maximizes the expected utility of honest policyholders, under the constraint that it should also break even when it is taken out by opportunists. In other words, insurance companies compete to attract honest policyholders under the additional constraint that opportunists cannot be set aside.

It will be shown that the equilibrium expected utility of honest policyholders is higher under a commitment to auditing policy than under no such commitment. The efficiency loss due to a no-commitment constraint may even be so large that the insurance market completely shuts down at equilibrium. Honest policyholders would then rather abstain from taking out any insurance policy than pay too high premiums because of the load of fraudulent claims.

Finally, we show that a common agency for insurance companies may help to solve the commitment problem. In this setting, the common agency takes charge of a part of the monitoring expenditures decided by insurers and is financed by lump-sum participation fees paid by companies. It is shown that such a mechanism mitigates the commitment problem and may even settle it completely if monitoring costs can be fully transferred to the agency. This latter case is unlikely however, since insurance companies may have private information about monitoring costs.

The organization of the paper is as follows. Section 2 describes the general framework of our study and characterizes the equilibrium of auditing games both under commitment and no-commitment to auditing policy. Section 3 is devoted to the analysis of market equilibrium. We prove the existence of an RS – or W-equilibrium. We also emphasize the social inefficiency entailed by a no-commitment constraint. Section 4 shows how a common agency that subsidizes claims monitoring may mitigate market inefficiency due to the absence of commitment. Section 5 concludes.

II. GENERAL FRAMEWORK AND AUDIT GAME

The general framework of our study may be described as follows. Risk-averse insurance buyers own an initial wealth W. They face the possibility of loss L with probability δ, $0 < \delta < 1$. Risk-neutral companies offer insurance contracts described by a premium P and a level of coverage q. The insurance market is supposed to be competitive with free entry.

Individuals' preferences over state contingent net wealth R is represented by a von Neumann-Morgenstern utility function U(R). U is twice continuously differentiable with $U' > 0$, $U'' < 0$. Individuals maximize expected utility EU(R). Their reservation utility is $\bar{U} = \delta U(W-L) + (1-\delta) U(W)$. They may be either honest with probability $1-\theta$ or opportunist with probability θ, $0 < \theta < 1$. Honest policyholders truthfully report losses to their insurance company : they never cheat. Opportunists may choose to fraudulently report a loss. We let α denote the (endogeneously determined) probability for an opportunist to file a fraudulent claim when no loss has been incurred.

Companies cannot distinguish honest policyholders from opportunists. They audit claims with probability p at cost k. When a fraudulent claim is audited, the company discovers the truth and, following a law suit, the cheater has to pay a fine M, a part of which denoted m is paid as award to the company. M and m may depend on the contract (q,P) with $m(q,P) \leq M(q,P)$. The functions m(q,P) and M(q,P) are exogeneously determined by law. They are supposed to be twice continuously differentiable.[6]

Let $\tilde{p}$ denote the audit probability that makes an opportunist (who has not experienced any loss) indifferent between honesty and fraud. Honesty gives $R = W - P$ with certainty. Fraud gives $R = W - P - M(q,P)$ if the claim is audited and $R = W - P + q$ otherwise. $\tilde{p}$ is then given by

$$U(W-P) = \tilde{p}\, U(W-P-M(q,P)) + (1-\tilde{p})\, U(W-P+q)$$

which yields

$$\tilde{p} = \frac{U(W-P+q) - U(W-P)}{U(W-P+q) - U(W-P-M(q,P))} \equiv \tilde{p}(q,P) \in (0,1)$$

Once an insurance policy (q,P) has been taken out, the relationship between a policyholder and his insurance company may be described as a three-stage audit game with three players : nature, the insurance company and the policyholder.

Assume that the proportion of opportunists among the policyholders of the insurance company is σ. Note that σ may conceivably differ from θ when honest and opportunist customers are not evenly distributed among companies, which may occur when various contracts are offered. As we will show below, all companies offer the same contract when the insurance market is at equilibrium and, under a uniform distribution, we will then have $\sigma = \theta$ for all companies. However, out of equilibrium contract proposals may attract a proportion of opportunists σ that differs from θ. An important assumption is that the distribution of honest individuals and opportunists among offered contracts is common knowledge : the proportion σ is thus publicly known at the beginning of an audit game. Given (q,P,σ), the audit game may be described as follows :

- At stage 1, Nature determines whether the policyholder is honest or opportunist, with probabilities $1-\sigma$ and σ respectively. *Nature* also determines whether the policyholder experiences a loss with probability δ.

- At stage 2, the *policyholder* decides to file a claim or not. Honest customers always tell the truth. When no loss has been incurred, opportunists defraud with probability α.

- At stage 3, when a loss has been reported at stage 2, the *insurance company* audits with probability p.

Opportunists who do not experience any loss choose α to maximize expected utility

$$EU = \alpha[p\ U(W-P-M(q,P)) + (1-p)\ U(W-P+q)] + (1-\alpha)\ U(W-P)$$

The insurance company chooses p to minimize its expected profit or, equivalently, to minimize the expected cost C defined by

$$C = IR + AC - AP$$

where IR = expected insurance reimbursement

AC = expected audit cost

AP = expected award paid to the company.

Expected profit is then equal to P-C.

Reimbursements are paid to policyholders who actually experience a loss and to opportunists who fraudulently report a loss and are not audited. This gives

$$IR = q[\delta + \alpha\sigma(1-\delta)(1-p)]$$

Expected audit cost is given by

$$AC = pk[\delta + \alpha\sigma(1-\delta)]$$

Awards are paid by opportunists who are caught cheating :

$$AP = \alpha\sigma p(1-\delta)\ m(q,P)$$

Two cases will be considered, according to whether the insurance company can commit to an audit policy or not. In the commitment case, the company has a Stackelberg advantage in the audit game : the verification probability p is chosen to minimize expected cost, given the reaction function of opportunist policyholders. In the no-commitment case, the equilibrium audit policy is constrained to be a best response to opportunists' behaviour. More explicitly, under no-commitment the outcome of the audit game is associated with a perfect Bayesian equilibrium where :

(a) Opportunists' strategy is optimal given audit policy,

(b) Audit policy is optimal given insurer's beliefs about the probability of a claim to be fraudulent,

(c) Insurer's beliefs are obtained from the probability of loss and opportunists' strategy using Bayes' rule.

Let $C^c(q,P,\sigma)$ be the expected cost associated with a commitment equilibrium of an audit game induced by contract (P,q) and probability σ. Likewise, we denote $C^n(q,P,\sigma)$ as the expected cost at a no-commitment equilibrium. $EU_h(q,P)$ and $EU_0^i(q,P,\sigma)$ denote respectively the expected utility of an honest policyholder and of an opportunist at the equilibrium of an audit game (q,P,σ) under commitment $(i = c)$ or no-commitment $(i = n)$. Note that the proportion σ may affect the expected utility of opportunists through its effect on the equilibrium audit strategy . Let also $\alpha^i(q,P,\sigma)$ and $p^i(q,P,\sigma)$, be the equilibrium strategy of opportunists and insurance companies respectively in an audit game (q,P,σ), with $i = c$ or n.

Multiple equilibria may occur in the commitment game since any probability α in the interval $[0,1]$ is optimal for the customer when $p = \tilde{p}(q,P)$. However a strategy $p = \tilde{p}(q,P) + \varepsilon$, with $\varepsilon > 0$ arbitrary small, leads to a unique best response $\alpha = 0$ which is optimal for the company. When $p = \tilde{p}(q,P)$, we will thus consider $\alpha = 0$ as the equilibrium strategy. Multiple equilibria may also occur in the (most unlikely) case where cost and benefit from auditing are equal for the company. In such a case, it is assumed that companies choose their audit strategy so as to dissuade opportunists from cheating (i.e. they choose $p = \tilde{p}(q,P)$ although other strategies $p' < \tilde{p}$ may be also optimal). In other cases equilibrium strategies are unambiguously defined.

Propositions 1 and 2 characterize the outcome of an audit game under commitment and in the absence of commitment respectively.

Proposition 1 : Under commitment to audit policy, the equilibrium of an audit game (q,P,σ) is characterized by :

$$p^c(q,P,\sigma) = 0 \text{ and } \alpha^c(q,P,\sigma) = 1 \qquad \text{if } k > \frac{(1-\delta)\sigma q}{\delta\, \tilde{p}(q,P)}$$

$$p^c(q,P,\sigma) = \tilde{p}(q,P) \text{ and } \alpha^c(q,P,\sigma) = 0 \qquad \text{if } k \leq \frac{(1-\delta)\sigma q}{\delta\, \tilde{p}(q,P)}$$

and

$$C^c(q,P,\sigma) = \mathrm{Min}\{(\delta+\sigma(1-\delta))q \ , \ \delta q + \delta\tilde{p}(q,P)k\}$$

Proof :

Given an audit policy p, the optimal report of opportunists when no loss occurs is given by

$$\left.\begin{array}{lll} \alpha = 0 & \text{if} & p > \tilde{p}(q,P) \\ \alpha \in [0,1] & \text{if} & p = \tilde{p}(q,P) \\ \alpha = 1 & \text{if} & p < \tilde{p}(q,P) \end{array}\right\} \qquad (1)$$

Consider first an equilibrium of the audit game where $\alpha = 0$. This may be obtained with an audit strategy $p \geq \tilde{p}(q,P)$. Then expected cost is

$$C = \delta(q + pk)$$

which is minimized at $p = \tilde{p}(q,P)$. This gives

$$C = \delta[q + \tilde{p}(q,P)k] \equiv C_1$$

Let us now consider an equilibrium where $\alpha = 1$, which may be obtained with $0 \leq p \leq \tilde{p}(q,P)$. We then have

$$C = \delta(q + pk) + \sigma(1-\delta)[p(k-m(q,P)) + (1-p)q]$$

C is linear in p and we have

$$C = [\delta + (1-\delta)\sigma]q \equiv C_2 \text{ at } p = 0$$

$$C = \delta[q + \tilde{p}(q,P)k] + \sigma(1-\delta)[\tilde{p}(q,P)(k - m(q,P)) + (1 - \tilde{p}(q,P))q] \equiv C_3 \text{ at } p = \tilde{p}(q,P)$$

Note that concavity of U(.) implies

$$[1 - \tilde{p}(q,P)]q - \tilde{p}(q,P)\, M(q,P) > 0 \quad \text{for all } q,P \qquad (2)$$

Using (2) and $k - m(q,P) > - m(q,P) \geq - M(q,P)$ gives

$$\tilde{p}(q,P)\ [k - m(q,P)] + [1 - \tilde{p}(q,P)]q > 0 \qquad (3)$$

and thus $C_3 > C_1$.

Lastly, assume $\alpha \in (0,1)$ at equilibrium which is obtained for $p = \tilde{p}(q,P)$. We then have

$$C = \delta[q + \tilde{p}(q,P)k] + (1-\delta)\ \alpha\sigma[\tilde{p}(q,P)(k - m(q,P)) + (1-\tilde{p}(q,P))q] \equiv C_4$$

and (3) then gives $C_4 > C_1$.

As above indicated, we select $\alpha = 0$ among opportunists' best responses when $p = \tilde{p}(q,P)$. Then the optimal strategy choice by the insurance company is $p = \tilde{p}(q,P)$ when $C_1 \leq C_2$ and $p = 0$ when $C_1 > C_2$. Expected cost is obtained at the minimum of C_1 and C_2. ■

The interpretation of proposition 1 is straightforward. Only two strategies may be optimal for the company : either fully preventing fraudulent claims by auditing claims with probability $p = \tilde{p}(q,P)$ or to abstain from any audit policy ($p = 0$). $\alpha = 0$ is an optimal strategy for opportunists when $p = \tilde{p}(q,P)$ (it is the only optimal strategy if $p = \tilde{p}(q,P) + \varepsilon$, $\varepsilon > 0$) and then expected cost is the sum of expected coverage δq and expected audit cost pk. When $p = 0$, fraudulent claims occur with probability $(1-\delta)\sigma$, which gives expected cost $(\delta + (1-\delta)\sigma)q$. The optimal audit strategy is $p = \tilde{p}(q,P)$ if $\delta[q + \tilde{p}(q,P)k] \leq [\delta + (1-\delta)\sigma]q$ and $p = 0$ otherwise. In particular, given the contract (q,P), preventing fraud through audit policy is optimal if audit cost k is low enough and the proportion of opportunists σ is large enough.

Let us now consider the no-commitment case. Let π be the probability for a claim to be fraudulent. π is deduced from the opportunists' strategy α using Bayes' rule, which gives

$$\pi = \frac{\alpha\sigma(1-\delta)}{\alpha\sigma(1-\delta) + \delta} \tag{4}$$

Once a policyholder puts in a claim, the expected cost for the insurer is $k + (1-\pi)q - \pi\ m(q,P)$ if the claim is audited and equal to q otherwise. In the absence of a commitment to audit policy, the equilibrium audit probability minimizes the (conditional) expected cost

$$\begin{aligned}\bar{C} &= p[k + (1-\pi)q - \pi\ m(q,P)] + (1-p)q \\ &= q + p[k - \pi(q + m(q,P))]\end{aligned}$$

Note that expected cost is equal to $\bar{C}$ times the claim probability $\delta + \alpha\sigma(1-\delta)$.

This yields

$$\left.\begin{array}{lll} p = 0 & \text{if} & \pi[q + m(q,P)] < k \\ p \in [0,1] & \text{if} & \pi[q + m(q,P)] = k \\ p = 1 & \text{if} & \pi[q + m(q,P)] > k \end{array}\right\} \quad (5)$$

The optimal strategy of opportunists is given by (1). The equilibrium of the no-commitment audit game is a solution (α,p,π) to (1)-(4)-(5). Proposition 2 characterizes such an equilibrium and the corresponding expected cost.

Proposition 2 : Without commitment to an audit policy, the equilibrium of an audit game (q,P,σ) is characterized by :

$$p^n(q,P,\sigma) = 0 \text{ and } \alpha^n(q,P,\sigma) = 1 \qquad \text{if} \quad k > \frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta}$$

$$p^n(q,P,\sigma) = \tilde{p}(q,P) \text{ and } \alpha^n(q,P,\sigma) = 1 \qquad \text{if} \quad k = \frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta}$$

$$p^n(q,P,\sigma)=\tilde{p}(q,P) \text{ and } \alpha^n(q,P,\sigma) = \frac{\delta k}{\sigma(1-\delta)[q+m(q,P)]-k]}$$

$$\text{if } k < \frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta}$$

and

$$C^n(q,P,\sigma) = \mathrm{Min}\left\{[\delta + (1-\delta)\sigma]q \ , \ \frac{\delta q[q + m(q,P]}{q + m(q,P)-k}\right\} \text{ if } k < q + m(q,P)$$

$$C^n(q,P,\sigma) = [\delta + (1-\delta)\sigma]q \quad \text{otherwise.}$$

Proof :

Let us consider in turn the three possible cases $p = 1$, $p = 0$ and $p \in (0,1)$.

Assume first that $p = 1$. Then (1) and (4) give $\alpha = 0$ and $\pi = 0$. Using (5), we deduce that $p = 0$, which is thus a contradiction. The equilibrium of the no-commitment audit game thus involves either random auditing $p \in (0,1)$ or no audit at all $p = 0$.

Assume $p = 0$. Then (1) and (4) give $\alpha = 1$ and

$$\pi = \frac{\sigma(1-\delta)}{\sigma(1-\delta) + \delta}$$

Using (5), we obtain an equilibrium of this type if

$$\frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta} < k \tag{6}$$

Assume now that $p \in (0,1)$. Then (5) gives

$$\pi = \frac{k}{q + m(q,P)}$$

and using (4) we deduce

$$\alpha = \frac{\delta k}{\sigma(1-\delta)[q + m(q,P) - k]} \tag{7}$$

This is an equilibrium if $0 \le \alpha \le 1$, $0 \le \pi \le 1$. Condition $0 \le \pi \le 1$ gives $k \le q + m(q,P)$. Then, condition $0 \le \alpha \le 1$ gives

$$k \le \frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta} \tag{8}$$

which is just the opposite of (6).

If (8) is not binding, we have $0 < \alpha < 1$ and (1) gives $p = \tilde{p}(q,P)$. If (8) is binding, we get $\alpha = 1$ and any audit probability p in the interval $[0,\tilde{p}(q,P)]$ is an equilibrium strategy. For above mentioned reasons, we select $p = \tilde{p}(q,P)$ as the equilibrium strategy in that case.

When (6) holds, we have $\alpha = 1$, $p = 0$ and expected cost is

$$C = [\delta + (1-\delta)\sigma]q = C_2$$

When (8) holds, α is given by (7) and we obtain :

$$C = [\delta + \alpha\sigma(1-\delta)]q$$

$$= \frac{\delta q[q + m(q,P)]}{q + m(q,P) - k} \equiv C_5$$

When $k < q + m(q)$, we have $C_2 < C_5$ iff (6) holds ; hence $C^n = \mathrm{Min}\{C_2,C_5\}$ in this case. When $k \ge q + m(q,P)$, then (6) holds and $C^n = C_2$ ∎

Proposition 2 deserves some comment. First, by using (2) and $M(q,P) \geq m(q,P)$, we deduce

$$\frac{\sigma(1-\delta)[q + m(q,P)]}{\sigma(1-\delta) + \delta} < \frac{\sigma(1-\delta)q}{\delta\tilde{p}(q,P)}$$

Consequently, the audit strategy $p = \tilde{p}(q,P)$ that discourages fraud is optimal for a larger set of contracts and parameters in the commitment game than in the no-commitment game.

Second, (3) implies

$$\frac{\delta q[q + m(q,P)]}{q + m(q,P) - k} > \delta[q + \tilde{p}(q,P)k] \tag{9}$$

when $k < q + m(q,P)$ and consequently $C^n(q,P,\sigma) \geq C^c(q,P,\sigma)$, with a strict inequality when the no-commitment game involves $p > 0$ at equilibrium : inability to commit to an audit policy ultimately leads to higher expected insurance costs. This may be explained as follows : $\alpha = 0$ cannot be an equilibrium strategy in the no-commitment game, since any policy that totally prevents fraud is not credible. Indeed, fully preventing fraud implies auditing at least with probability $\tilde{p}(q,P)$ but, when no claim is fraudulent, auditing with positive probability is not an optimal strategy for the insurer. Consequently, at the equilibrium of the no-commitment audit game, there must be some degree of fraud for audit policy to be credible and these unavoidable fraudulent claims increase insurance cost.

The equilibria of audit games under commitment or no-commitment share several common properties that will be useful for what follows. They are summarized in the following corollary.

Corollary 1 : Under commitment ($i = c$) or no-commitment ($i = n$), the outcome of an audit game (q,P,σ) is such that :

P1 : $C^i(q,P,\sigma)$ is non-decreasing with respect to σ,

P2 : If $\alpha = 1$ at the equilibrium of the audit game induced by (q,P,σ) and $q > 0$, then $C^i(P,q,\sigma)$ is locally increasing with respect to σ and $C^i(q,P,\sigma) = q$ if $\sigma = 1$,

P3 : $C^i(0,0,\sigma) = 0$ for all σ,

P4 : $\alpha^i(q,P,\sigma)$ is non-increasing with respect to σ,

P5 : $EU_h(q,P) \leq EU_0^i(q,P,\sigma)$ for all q,P,σ,

P6 : If $EU_h(q,P) < EU_0^i(q,P,\sigma)$, then $\alpha^i(q,P,\sigma) = 1$ and $p^i(q,P,\sigma) = 0$.

P7 : If $EU_0^i(q,P,\sigma) > \bar{U}$ then $q > 0$.

Proof : immediate.

P1, **P2** and **P3** characterize the equilibrium expected cost. A larger proportion of opportunists among policyholders cannot decrease expected cost (**P1**). When opportunists systematically defraud, expected cost increases with their weight (**P2**). Of course, when the set of customers only includes opportunists who systematically defraud, then expected cost coincides with coverage (**P2**) and a zero premium – zero coverage contract entails no cost (**P3**). **P4** means that, at equilibrium, a larger proportion of opportunists leads the company to more frequent audits and, finally, the equilibrium probability for an opportunist to report a fraudulent claim is lower. **P5** - **P6** describe the outcome of the audit game for policyholders. Since opportunists can mimic honest policyholders (they can refrain from defrauding), their expected utility cannot be lower (**P5**). At an equilibrium where expected utility is larger for opportunists than for honest policyholders, opportunists systematically defraud (**P6**), since a mixed strategy $\alpha < 1$ would mean that fraud does not dominate honesty. In that case, the company does not audit at equilibrium. Lastly, the expected utility of opportunists may be larger than their reservation utility only for positive coverage (**P7**).

Finally, note that any contract (q,P) such that $q = P > 0$ which only attracts opportunists who systematically defraud ($\sigma = 1$, $\alpha = 1$) is equivalent to a no-insurance situation $q = P = 0$: in both cases, profit is nil and (opportunist) policyholders only reach their reservation utility level. To avoid a trivial indeterminacy of equilibrium, we restrict attention to contracts (q,P) such that $0 \leq P \leq q$ with $P < q$ if $q > 0$.

III. EQUILIBRIUM

We will consider a competitive insurance market with free entry, where insurance companies compete by offering policies. An adverse selection feature is brought in because of the impossibility for companies of distinguishing opportunists from honest policyholders.

It is well-known since contributions by Rothschild and Stiglitz (1976) and Wilson (1977) that the nature of equilibrium that may emerge in such a competitive market with adverse selection depends on how firms anticipate the reactions of competitors. In the simplest case, firms have static expectations with respect to the policy offers of their competitors : insurers then follow a Cournot-Nash strategy in which each insurer takes the action of other firms as given. In such a case, an equilibrium (under commitment or no-commitment to audit policy) is characterized by a set of profitable insurance contracts such that there is no other contract which, when offered in addition to this set, earns positive profits. We will refer to such an equilibrium with static expectations as to an RS-equilibrium (or, more explicitly, a type-i RS-equilibrium, with i = c under commitment and i = n under no-commitment).

An alternative equilibrium concept has been provided by Wilson (1977). Here it is assumed that firms anticipate that any policy which becomes unprofitable will be withdrawn. An equilibrium is then defined as a set of profitable contracts such that no company can offer another contract which remains profitable after other insurers have withdrawn all non profitable contracts in reaction to the offer. Such an equilibrium will be called a W-equilibrium (or, more explicitly, a type-i W-equilibrium, with i=c or n).

A formal definition of RS - and W-equilibria is provided in the appendix.[7] We show that, in our economy, a RS-equilibrium is a W-equilibrium, that an RS-equilibrium may not exist when audit costs are very high, and that a unique W-equilibrium always exists. The profitability of an insurance contract is measured at the equilibrium of the subsequent audit game. In particular, the profitability of a contract (q,P) depends on the proportion of opportunists σ among individuals who choose it. It should be noted at this stage that the distribution of honest individuals only depends on the characteristics of the contracts that are offered but that the distribution of opportunists also depends on the equilibrium audit strategies.

The existence and characterization of equilibrium arises from two additional assumptions :

A1 : Honest individuals are uniformly distributed among best contracts ; likewise for opportunists.

A2 : $$(q^i, P^i) = \underset{q,P}{\text{Arg Max}} \left\{ \delta\, U(W-L+q-P) + (1-\delta)\, U(W-P) \quad \text{s.t.} \quad P \geq C^i(q,P,\theta) \right\}$$

is a singleton, for i = c,n.

Assumptions **A1** and **A2** are made for the sake of simplicity : they guarantee the uniqueness of the equilibrium contract. Of course, in **A1** the set of best contracts may differ between honest policyholders and opportunists. At the cost of additional technicalities, these results could be extended to the case where we only assume that the distribution of agents among best contracts has full support. In **A2**, (q^i,P^i) is the best contract that can be offered to honest individuals, under the profitability constraint, when all individuals, be they honest or opportunist, choose the same policies. We have $P^i = C^i(q^i,P^i,\theta)$ since function C^i is continuous with respect to (q,P).

Proposition 3 : Under **A1-A2**, (q^i,P^i) is the unique type-i W-equilibrium for i = c or n. It is a type-i RS-equilibrium if $\alpha^i(q^i,P^i, \frac{2\theta}{1+\theta}) < 1$ or if $q^i=P^i=0$

Proof : See the appendix.

According to proposition 3 an RS- or W-equilibrium is defined by a unique contract (q^i,P^i) that maximizes the expected utility of honest policyholders, under the constraint that opportunists cannot be set aside.

The arguments at work in the proof of proposition 3 can be summarized as follows. Consider a type-i equilibrium, without specifying at the moment whether it is an RS – or W-equilibrium. All contracts which are offered at equilibrium are equivalent for honest customers, otherwise from **A1** some equilibrium contracts would only attract opportunists. From **P6**, defrauding with probability 1 is the equilibrium strategy of opportunists for such contracts and, from **P2**, the contract could not be profitable. Equilibrium contracts are also equivalent for opportunists, otherwise opportunists would concentrate on a subset of equilibrium contracts for which the proportion of opportunists would be greater than θ. Then **P1** implies that honest individuals prefer (q^i,P^i) to these contracts and consequently a contract $(q^i-\varepsilon,P^i)$, $\varepsilon > 0$ would attract all honest individuals for ε small and would remain profitable even if opportunists finally also opt for the new contract.

Hence, for any contract (P,q) offered at equilibrium, the profitability constraint is $P \geq C^i(q,P,\theta)$ and (q^i,P^i) is the best choice for honest consumers among these profitable contracts. If (q^i,P^i) were not offered, another contract could be proposed that would be strictly preferred by honest individuals and that would remain profitable whatever the reaction of opportunists. Hence (q^i,P^i) is the only possible type-i equilibrium.

To prove the existence of a type-i RS-equilibrium, it remains to show that there does not exist any profitable contract that is preferred to (q^i,P^i) by some individuals. Let us first assume that $\alpha^i(q^i,P^i,\theta) < 1$. Then, from **P5-P6**, opportunists and honest individuals reach the same expected utility level when (q^i,P^i) is the only contract to be offered. Consequently, any additional contract that attracts honest individuals also attract opportunists since the latter cannot be better off by staying alone or with a smaller proportion of honest policyholders. Hence this new contract cannot be profitable, since (q^i,P^i) is the best pooling contract that breaks even. Furthermore, if only opportunists were attracted by a new offer, this would mean that $\alpha = 1$ in the corresponding audit game (from **P6**) and this new offer would not be profitable (from **P2**). In that case (q^i,P^i) is an RS-equilibrium (and also a W-equilibrium).

Let us now assume that $\alpha^i(q^i,P^i,\theta) = 1$. In this case, we have $C^i(q,P,\theta) = [\delta + \theta(1-\delta)]q$ in the neighbourhood of $q = q^i$, $p = P^i$. A new offer (q,P) will be profitable if it attracts honest individuals only (once again opportunists cannot benefit from separating and (q^i,P^i) is the best pooling contract) and $P > \delta q$. Equivalently, (q^i,P^i) is <u>not</u> a RS-equilibrium if there exists (q,P) such that :

(i) honest individuals prefer (q,P) to (q^i,P^i) or they think both contracts are equivalent and $P > \delta q$

(ii) opportunists go on defrauding with probability 1 at (q^i,P^i).

Let us show that such a policy (q,P) does not exist under the assumptions given in proposition 3. Assume first that $q_i > 0$. If $EU_0^i(q^i,P^i,1) > EU_h(q^i,P^i)$, then $\alpha^i(q^i,P^i,1) = 1$ and $p^i(q^i,P^i,1) = 0$ from **P6**. Conditions (i) - (ii) are satisfied for contracts (q,P) located in the interior of the dashed area in figure 1. [8] Furthermore, any contract located on arc AE^i is equivalent to (q^i,P^i) for honest individuals. If such a contract were offered (in a neighbourhood of q^i,P^i), then the proportion of opportunists among individuals who choose (q^i,P^i) becomes $2\theta/(1+\theta)$ and (ii) is verified if $EU_0^i(q^i,P^i,\frac{2\theta}{1+\theta}) > EU_h(q^i,P^i)$, which implies $\alpha(q^i,P^i,\frac{2\theta}{1+\theta}) = 1$. Hence **P4** and $\theta < 2\theta/(1+\theta) < 1$ imply that $\alpha(q^i,P^i,\frac{2\theta}{1+\theta}) < 1$ is a necessary and sufficient condition for (q^i,P^i) to be a type-i RS-equilibrium, when $q_i > 0$. In any case, if such a new contract (q,P) were offered, companies that go on offering (q^i,P^i) lose money. If (q^i,P^i) is withdrawn, all individuals will turn toward the new contract (q,P). Finally this new contract will show a deficit and it will not be offered, which establishes (q^i,P^i) as a type-i W-equilibrium.

Finally note that the dashed area vanishes when $q^i = 0$: $q^i = P^i = 0$ is then a type-i RS-(and W-) equilibrium where no insurance policy is taken out.

In what follows we will focus attention on the case where fraudulent behaviour is widespread enough for actual insurance $q > 0$ to imply auditing with positive probability. This occurs when θ is sufficiently large and honest customers would prefer to take out no insurance policy at all than to pay high premiums that cover the cost of systematic fraud by opportunists. Indeed, when opportunists systematically defraud ($\alpha = 1$) and insurance companies never audit ($p = 0$), the zero-profit condition for a pooling contract may be written as $P = [\delta + \theta(1-\delta)]q$. Then the expected utility of an honest policyholder is

$$EU_h = \delta\, U[W-L+(1-\delta)(1-\theta)q] + (1-\delta)\, U[W-(\delta+\theta(1-\delta))q]$$

Maximizing EU_h with respect to $q \geq 0$ gives $q = 0$ if $\theta \geq \theta^*$ with

$$\theta^* = \frac{\delta[U'(W-L) - U'(W)]}{\delta U'(W-L) + (1-\delta)U'(W)} \in (0,1)$$

When $\theta \geq \theta^*$, then either the equilibrium contract (q^i, P^i) entails positive coverage and companies audit with positive probability and opportunists do not systematically defraud ($q^i > 0$, $P^i > 0$, $p^i(q^i,P^i,\theta) > 0$ and $\alpha^i(q^i,P^i,\theta) < 1$), or consumers prefer not to insure against risk ($q^i = P^i = 0$). In fact, individuals will actually take out an insurance policy when the audit cost k is not too large and total or partial deterrence of opportunists from cheating does not result in too high premiums. However, the threshold audit cost is higher when insurance companies can commit to their audit policies than under no-commitment. This is more explicitely stated in propositions 4 and 5. Note that we do not specify the equilibrium concept (RS or W) since q^i, P^i is at the same time an RS- and W-equilibrium when $\theta \geq \theta^*$.

Proposition 4. Let $\theta \geq \theta^*$. Then, under commitment to audit policy, there exists $k^c > 0$ such that :

- when $k \leq k^c$, the equilibrium insurance policy involves positive coverage $q = q^c > 0$, a positive premium $P = P^c > 0$ and random auditing that deters fraud : $p = \tilde{p}(q^c,P^c)$, $\alpha = 0$
- when $k > k^c$, no insurance policy is taken out.

Proof : Propositions 1 and 3 imply

$$\begin{aligned} EU_h(q^c,P^c) = \text{Max}\ \{EU_h(q,P) \quad & \text{s.t.} \\ & P \geq \text{Min}\{[\delta + (1-\delta)\theta]q,\ \delta q + \delta\tilde{p}(q,P)k\} \\ \equiv \phi^c(k) & \end{aligned}$$

We have

$$\phi^c(k) = \text{Max}\ \{\phi_1\ ,\ \phi_2^c(k)\}$$

with

$$\phi_1 = \text{Max}\ \{EU_h(q,P) \text{ s.t. } P \geq [\delta + (1-\delta)\theta]q\}$$

$$\phi_2^c(k) = \text{Max}\ \{EU_h(q,P) \text{ s.t. } P \geq \delta q + \delta\tilde{p}(q,P)k\}$$

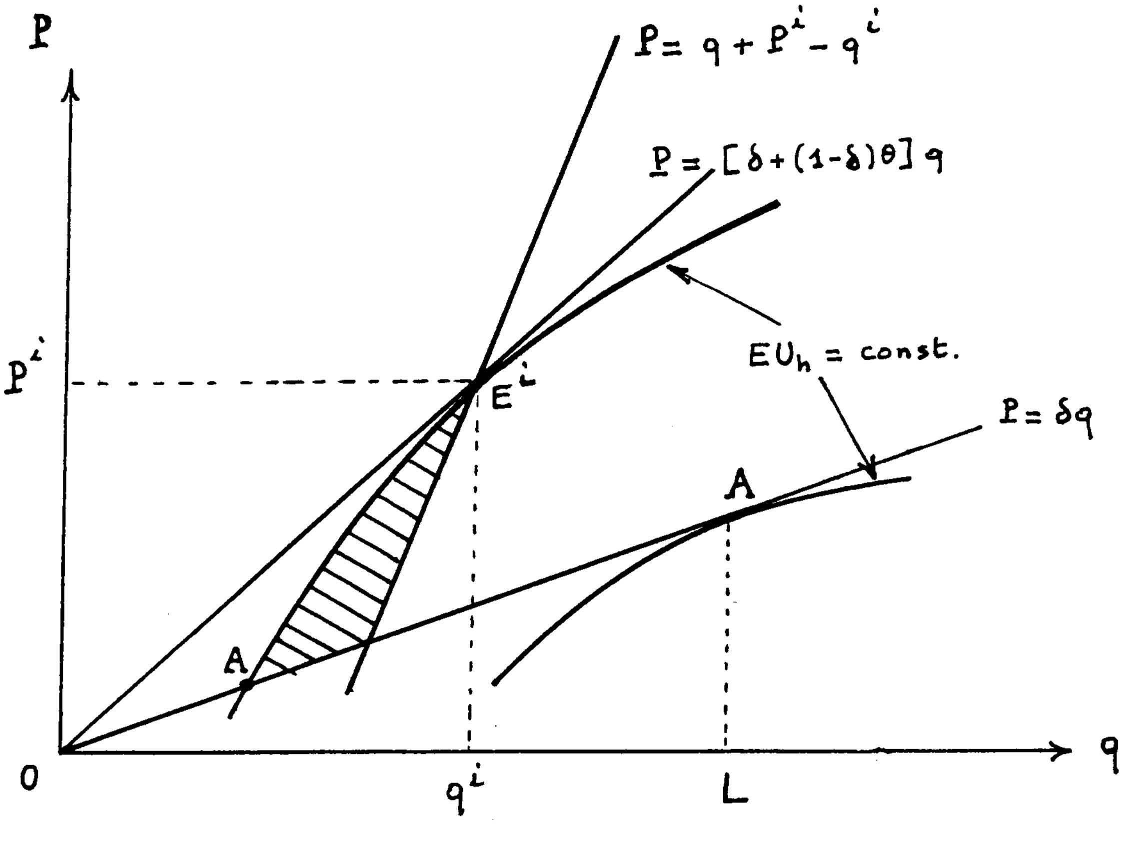

Figure 1

$\theta \geq \theta^*$ gives $\phi_1 = EU_h(0,0) = \bar{U}$. Furthermore $\phi_2^c(k)$ is strictly decreasing and concavity of U gives

$$\phi_2^c(0) = U(W - \delta L) > \bar{U}$$

Consequently $\phi_2^c(k)$ is larger than $\bar{U}$ on an interval $[0,k^c]$ with $k^c > 0$. When $k < k^c$, we have $\phi^c(k) = \phi_2^c(k) \geq \bar{U} = \phi_1$ which gives $P^c = \delta q^c + \delta\tilde{p}(q^c,P^c)\ k \leq [\delta + (1-\delta)\theta]q^c$. Proposition 1 then gives $p^c = \tilde{p}(q^c,P^c)$ and $\alpha = 0$. When $k > k^c$, we have $\phi^c(k) = \phi_1 = \bar{U} \geq \phi_2^c(k)$ which gives $q^c = P^c = 0$.

■

Proposition 5. Let $\theta \geq \theta^*$. Then, under no-commitment to audit policy there exists $k^n > 0$ such that $k^n < k^c$ and

- when $k \leq k^n$, the equilibrium insurance policy involves positive coverage $q = q^n > 0$ and a positive premium $P = P^n > 0$, insurance companies audit with positive probability $p = \tilde{p}(q^n,P^n)$ and opportunists cheat with positive probability $0 < \alpha \leq 1$. Furthermore $EU_h(q^n,P^n) < EU_h(q^c,P^c)$.
- When $k > k^n$, no insurance policy is taken out.

Proof. Propositions 2 and 3 imply

$$\begin{aligned} EU_h(q^n,P^n) = \text{Max } \{EU_h(q,P) \quad & \text{s.t.} \\ & P \geq \text{Min}\{[\delta + (1-\delta)\theta]q,\ \frac{\delta q[q + m(q,P)]}{q + m(q,P) - k}\} \text{ if } k < q + m(q,P) \\ & P \geq [\delta + (1-\delta)\theta]q \text{ if } k \geq q + m(q,P)\} \\ & \equiv \phi^n(k) \end{aligned}$$

we have

$$\phi^n(k) = \text{Max } \{\phi_1,\phi_2^n(k)\}$$

with

$$\phi_2^n(k) = \text{Max } \{EU_h(q,P) \text{ s.t. } P \geq \frac{\delta q[q + m(q,P)]}{q + m(q,P) - k},\ k < q + m(q,P)\}$$

$\phi_2^n(k)$ is strictly decreasing and

$$\phi_2^n(0) = U(W - \delta L) > \bar{U} = \phi_1$$

Furthermore (9) gives $\phi_2^n(k) < \phi_2^c(k)$ for all $k > 0$. Hence $\phi_2^n(k)$ is larger

than $\bar{U}$ on an interval $[0,k^n]$ with $0 < k^n < k^c$.
When $k \leq k^n$, we have $\phi^n(k) = \phi_2^n(k) \geq \bar{U} = \phi_1$ which gives

$$P^n = \frac{\delta q^n[q^n + m(q^n,P^n)]}{q^n + m(q^n,P^n) - k} \leq [\delta + (1-\delta)\theta]q^n$$

Proposition 2 then gives $p = \tilde{p}(q^n,P^n)$ and $\alpha = \alpha^n(q^n,P^n,\theta) \in (0,1]$. We also have $EU_h(q^n,P^n) = \phi_2^n(k) < \phi_2^c(k) = EU_h(q^c,P^c)$.

When $k > k^n$, we have $\phi^n(k) = \phi_1 = \bar{U} \geq \phi_2^n(k)$ which gives $q^n = P^n = 0$.

■

Propositions 4 and 5 deserve some comment. First of all, when $k \leq k^c$, the expected utility of honest policyholders is lower in the no-commitment setting than under commitment. Second, when $k^n \leq k \leq k^c$, the insurance market collapses in the no-commitment case, although commitment would allow trade to take place. Thus, even if the commitment equilibrium only involves some kind of second-best optimality (the first-best optimum would correspond to $k = 0$ and full insurance $q = L$ with actuarial premium $P = \delta q$), the inability to commit entails an additional inefficiency. As mentioned above, the absence of commitment to a monitoring policy leads to some fraud at equilibrium, which ultimately increases insurance costs and penalizes honest individuals. This increase in insurance cost may even be so high that individuals prefer not to take out any insurance policy at equilibrium. Figure 2 illustrates this case. We here assume constant risk aversion : $U(R) = -e^{-\lambda R}$, $\lambda > 0$ and we assume that M and m do not depend on P. In this case $\tilde{p}$, C^c and C^n do not depend on P and we have

$$\tilde{p}(q) = \frac{1 - e^{-\lambda q}}{e^{\lambda M(q)} - e^{-\lambda q}}$$

We also assume

$$\theta > \theta^* = \frac{\delta[e^{\lambda L} - 1]}{\delta e^{\lambda L} + (1-\delta)}$$

which means that maximizing $EU_h(q,P)$ with respect to (q,P), subject to $P = [\delta + (1-\delta)\ \theta]\ q$, $q \geq 0$ leads to a corner solution at $q = 0$.

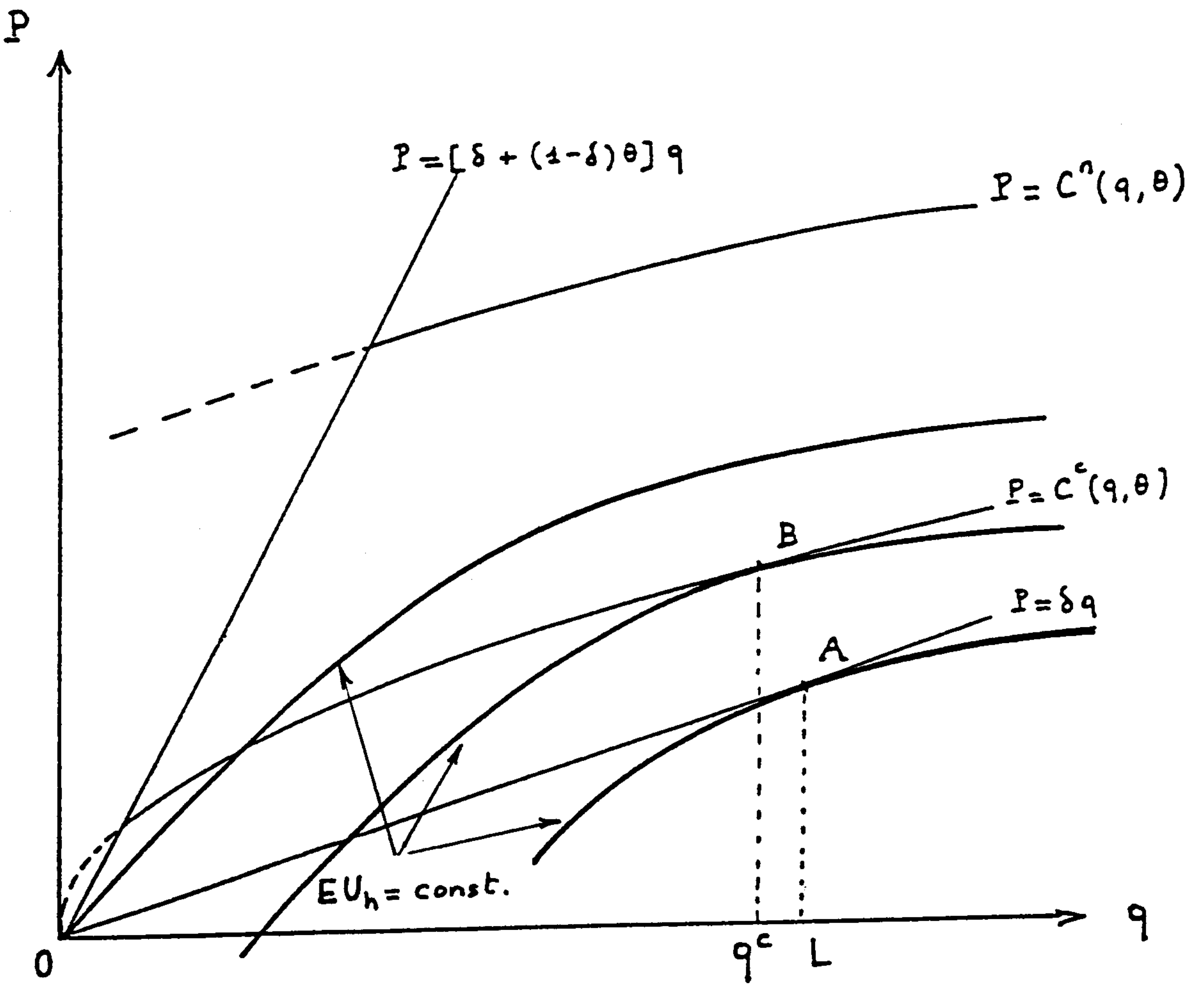
P
$P=[\delta+(1-\delta)\theta]\,q$
$P=C^n(q,\theta)$
$P=C^c(q,\theta)$
B
$P=\delta q$
A
$EU_h=$ const.
0
q^c
L
q

Figure 2

Indifference curves are in the position indicated in figure 2 and the first-best optimum (obtained when $k = 0$) is at point A with full coverage. When monitoring is costly but firms can commit to their monitoring strategy, the zero profit line is $P = C^c(q,\theta)$ and the equilibrium contract is at point B with $q = q^c$. Under co-commitment, the zero-profit line is $P = C^n(q,\theta)$ and the market shuts down completely at $q = q^n = 0$.

IV. MITIGATING MARKET INEFFICIENCY THROUGH A COMMON AGENCY

Inevitably, in an institutional setting where punishments are limited by law, insurance fraud involves some degree of market inefficiency, even if companies act as Stackelberg leaders in audit games. This inefficiency will be all the more important as the penalty paid by individuals caught cheating is low : the higher is the penalty $M(q,P)$, the lower is the audit probability $\tilde{p}(q,P)$ that dissuades opportunist policyholders from cheating and the lower is the insurance premium. In figure 2, this corresponds to the fact that the locus $P = C^c(q,\theta)$ shifts downwards as $M(q)$ increases.

However the previous analysis also emphasizes the specific social cost due to the inability to commit to a monitoring policy. This social cost is mitigated when the awards paid to the insurance company by fraudsters is high : the locus $P = C^n(q,\theta)$ shifts downwards as $m(q)$ increases.

The question naturally arises whether this particular inefficiency could be overcome. A conceivable solution to the commitment problem is to delegate authority over audit policy to an independent agent who is in charge of investigating claims. Then, an incentive contract signed by the insurance company and the investigator could induce a tough monitoring strategy by the latter. For instance the investigator's salary could be an increasing function of the number of claims that are actually audited. Pre-commitment effects would be obtained by publicly announcing that such an incentive contract has been offered to the investigator [9].

In this section we explore another way to solve the commitment problem, from a more market regulation point of view. Assume that a common agency is created for all insurance companies. Its function is to take charge wholly or partly of monitoring costs : auditing is still companies' own work, but for each claim which is actually audited, the agency pays h to the company with $h \le k$. In other words, there is cost sharing but no delegation of audit decision making. The agency is financed through participation fees F paid by companies so that its budget is balanced. The agency may conceivably propose a menu $h = h(q,P)$, $F = F(q,P)$ in which the subsidy rate h and the participation fee F are expressed as functions of the policy offered by companies.

Now companies bear a monitoring cost $k - h(q,P)$ and (for each contract) they also bear a fixed cost $F(q,P)$. In the absence of commitment to monitoring policy, expected cost is now written as

$$C^r(q,P,\sigma) = \mathrm{Min}\{[\delta + (1-\delta)\sigma]q + F(q,P)\ ,\ \frac{\delta q[q + m(q,P)]}{q + m(q,P) - k + h(q,P)} + F(q,P)\}$$

and equilibrium strategies $\alpha^n(q,P,\sigma)$ and $p^n(q,P,\sigma)$ are obtained by replacing k by $k - h(q,P)$ in proposition 2.

We may then define a regulated RS-equilibrium as a set of profitable contracts such that : *(a)* There is no other contract which, when offered in addition, earns positive profits, and *(b)* The budget constraint of the common agency is satisfied, i.e. participation fees are equal to expected subsidies. Likewise, a regulated W-equilibrium is defined by stating instead of (a) that no out of equilibrium contract would be profitable after non-profitable contracts are withdrawn in reaction to the offer. As before, the expression "equilibrium" is used when both definitions apply.

We let

$$(q^r,P^r) = \underset{q,P}{\mathrm{Arg\ Max}}\{\delta\ U(W - L + q - P) + (1-\delta)\ U(W-P) \qquad \text{s.t.}\quad P \ge C^r(q,P,\theta)\}$$

The following proposition states that a common agency may help to solve the commitment problem.

Proposition 6. Assume that $\theta \ge \theta^*$, $k < k^c$ and that insurance companies cannot commit to audit policy. Let

$$h(q,P) = \bar{h} \qquad \text{for all } q,P$$

$$F(q,P) = \frac{\delta\,\bar{h}\,\tilde{p}(q,P)[q + m(q,P)]}{q + m(q,P) - k + \bar{h}}$$

Assume also **A1** and that (q^r,P^r) is a singleton. Then there exists h_0 in $[0,k)$, with $h_0 = 0$ iff $k \le k^n$, such that an unique regulated equilibrium exists with positive coverage $q = q^r > 0$ and premium $P = P^r$ when $\bar{h} > h_0$, and the expected utility of honest policyholders increases with $\bar{h}$ over $[h_0,k]$. At $\bar{h} = k$ the equilibrium coincides with the commitment equilibrium in terms of coverage, premium and monitoring probability, and opportunist policyholders do not cheat.

Proof : Properties **P1-P6** are satisfied under audit subsidization, replacing $C^i(q,P,\sigma)$ by $C^r(q,P,\sigma)$, except the last statement in **P2** and **P3**. We now have

$$C^r(q,P,1) = q + F(q,P) \ge q \quad \text{if} \quad \alpha = 1 \tag{10}$$

and

$$C^r(0,0,\sigma) = F(q,P) \tag{11}$$

(10) does not modify the proof of proposition 3. **P3** was used only to ensure that $EU^h(q^i,P^i) \ge \bar{U}$. We hereafter check that $EU^h(q^r,P^r) > \bar{U}$ for $\bar{h}$ large enough. In that case, the proof of proposition 3 applies and (q^r,P^r) is the unique regulated equilibrium if the agency's budget constraint is satisfied.

Let $\theta \ge \theta^*$. If $EU^h(q^r,P^r) > \bar{U}$ then $\alpha < 1$ and $p > 0$ at the (no-commitment) equilibrium of the audit game $q = q^r$, $P = P^r$, $\sigma = \theta$. In such a case Proposition 2 gives

$$p = \tilde{p}(q^r,P^r)$$

$$\alpha = \frac{\delta(k - \bar{h})}{\theta(1-\delta)[q^r + m(q^r,P^r) - k + \bar{h}]}$$

The expected cost of subsidies (per contract) paid by the agency to the company when only (q^r,P^r) is offered is

$$[\delta + \theta(1-\delta)\alpha]p\ \bar{h} = \frac{\delta\ \bar{h}\ \tilde{p}(q^r,P^r)[q^r + m(q^r,P^r)]}{q^r + m(q^r,P^r) - k + \bar{h}} = F(q^r,P^r)$$

which establishes (q^r,P^r) as the regulated equilibrium if $EU_h(q^r,P^r) > \bar{U}$.

If $EU_h(q^r,P^r) > \bar{U}$, then $\theta \geq \theta^*$ implies $P^r < [\delta + (1-\delta)\theta]q^r + F(q^r,P^r)$ Hence (q^r,P^r) maximizes $EU_h(q,P)$ subject to

$$P - \frac{\delta[q + m(q,P)][q + \bar{h}\ \tilde{p}(q,P)]}{q + m(q,P) - k + \bar{h}} \geq 0 \qquad (12)$$

Let ψ denote the left-hand side in (12). We have

$$\frac{\partial\psi}{\partial\bar{h}} = \frac{\delta[q + m(q,P)][q(1 - \tilde{p}(q,P)) + (k - m(q,P))\ \tilde{p}(q,P)]}{q + m(q,P) - k + \bar{h}}$$

and (3) gives $\dfrac{\partial\psi}{\partial\bar{h}} > 0$.

Let $h_1 < h_2 < k$ and let $(q^r,P^r) = (q_k,P_k)$ when $\bar{h} = h_k$ for $k = 1,2$. We thus have

$$P_1 - \frac{\delta[q_1 + m(q_1,P_1)][q_1 + h_2\ \tilde{p}(q_1,P_1)]}{q_1 + m(q_1,P_1) - k + h_2} > 0$$

Consequently there exists a policy (q_1,P'_1) with $P'_1 < P_1$ that satisfies (12) when $\bar{h} = h_2$. We deduce $EU^h(q_2,P_2) \geq EU_h(q_1,P'_1) > EU_h(q_1,P_1)$ which proves that $EU_h(q^r,P^r)$ increases with $\bar{h}$ when $EU_h(q^r,P^r) > \bar{U}$.

At $\bar{h} = k$, we have $\alpha = 0$ and (12) is written as

$$P - \delta[q + k\ \tilde{p}(q,P)] \geq 0$$

We then have $(q^r,P^r) = (q^c,P^c)$ and $k < k^c$ gives $EU^h(q^c,P^c) > \bar{U}$.

Let $h_0 = \text{Inf}\{\bar{h}\ \text{s.t.}\ EU_h(q^r,P^r) > \bar{U},\ \bar{h} \geq 0\}$. At $\bar{h} = 0$, we have $(q^r,P^r) = (q^n,P^n)$. This gives $h_0 = 0$ if $k \leq k^n$ and $0 < h_0 < k$ if $k > k^n$.

■

Figure 3 illustrates proposition 6. Here we make the same assumptions as in figure 2, particularly $\theta > \theta^*$ and $k^n < k < k^c$. Under no-commitment, the insurance market shuts down at the no-trade point 0. When $0 < \bar{h} < k$, the locus $P = C^r(q,\theta)$ is located between $P = C^c(q,\theta)$ and $P = C^n(q,\theta)$. It shifts downwards when $\bar{h}$ increases. At $\bar{h} = h_0$, the equilibrium jumps from 0 to C and it converges to B when $\bar{h}$ goes to k.

Our results on insurance market regulation should not be interpreted in an exaggeratedly optimistic way, since administrative costs or imperfect information issues may perturbate the action of the common agency. In particular, there may be asymmetric information between the agency and insurance companies about audit costs. However, proposition 6 shows that, to some extent, audit subsidization can lead to greater efficiency of insurance markets. Assume for instance that the agency only knows that audit costs are *ex-ante* distributed in an interval $[k_0,k_1]$. Assume also that the audit cost of a particular claim is known to the insurer. Then, any claim whose cost is lower than $\bar{h}$ will be verified with probability 1 so as to pocket the subsidy and policyholders with low monitoring costs will be over-audited. If policyholders know their own monitoring cost, over-auditing low cost claims cannot be compensated by under-auditing high cost claims, otherwise cheating will be systematic when audit cost is high. Ultimately, excessive monitoring will be reflected in larger participation fees. However, monitoring subsidization at rate $\bar{h} = k_0$ improves the honest policyholders' welfare, without inciting insurers to always monitor low cost claims.

Finally, it is worth mentioning that market regulation through a common agency may be a volunteer-based policy, since non-participating insurance companies would have larger expected cost and could not break even at equilibrium.

V. CONCLUSION

According to many insurers, fraud is a pervasive plague that penalizes honest policyholders by bringing about unduly high premiums. Fraudulent claims may even threaten the durable equilibrium of some insurance markets. Collective action at the industry level may help to facilitate a more

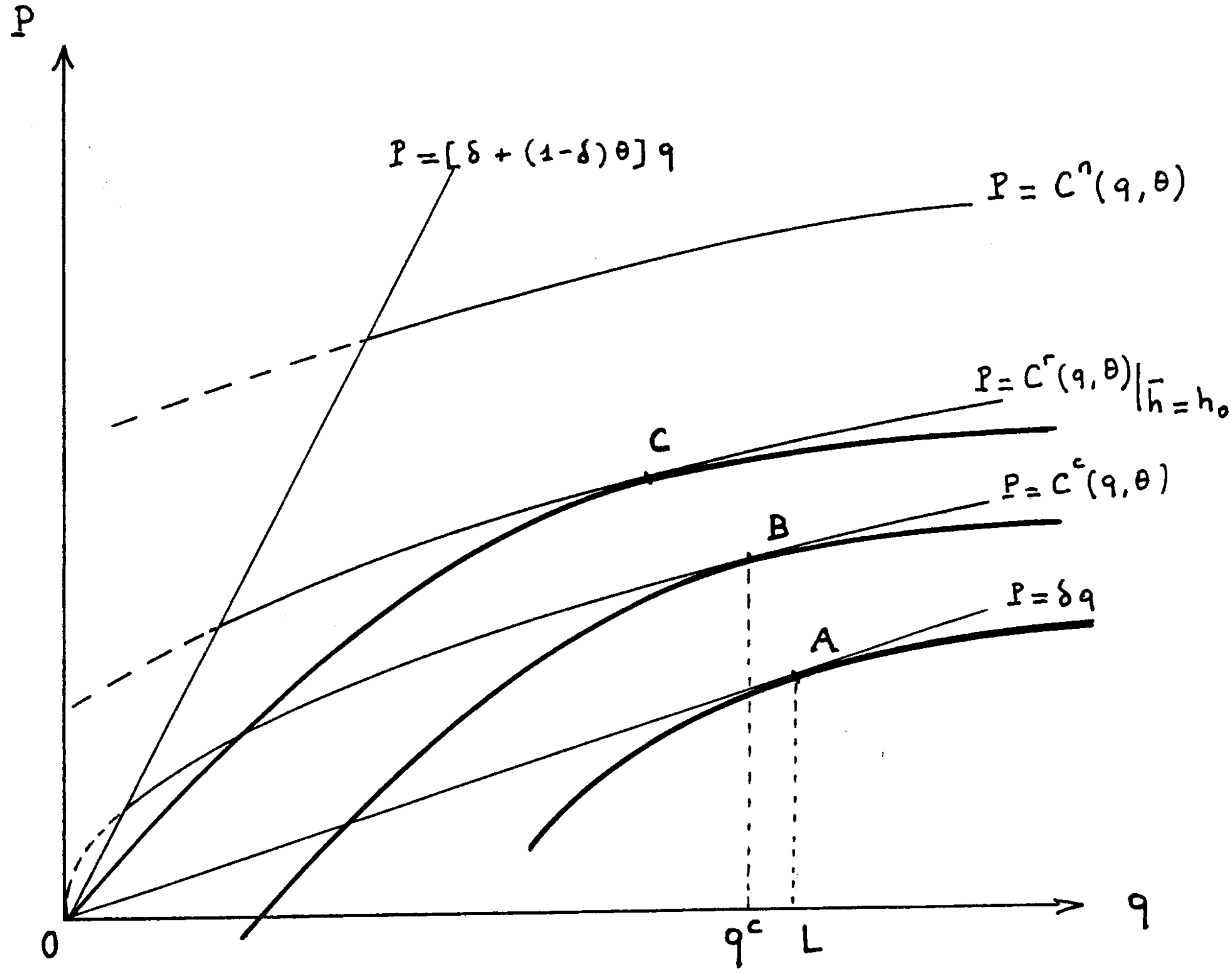

Figure 3

efficient attack on fraud, by providing information to insurers and intermediaries on how to better identify suspicious claims and on how to prosecute defrauders. However, the fact remains that detecting and establishing evidence on fraudulent claims is costly and the insurers' monitoring policy often suffers from a severe credibility problem.

This paper has characterized the equilibrium of an insurance market where claims may be fraudulent, and we have focused on this credibility problem. Our model includes adverse selection features since we assume that insurers cannot distinguish honest policyholders from opportunists. The absence of commitment to a monitoring policy is responsible for a welfare loss that may be even so large that the insurance market shuts down at equilibrium. Finally, we have shown that the commitment problem is alleviated if monitoring is subsidized by a common agency, that is financed by insurers through volunteer-based participation fees.

Our study could be extended in several ways. In particular, we have described insurance fraud in a very rudimentary way that excluded build-up, and where the opportunists' strategy comes down to a simple fraud/honesty alternative. It would be more satisfactory (but also technically much more difficult) to assume that policyholders may suffer a loss whose magnitude is random.[10] Furthermore, fraud may also result from lies which are told when the policy is taken out, as for instance when an individual does not truthfully reveal their wealth in the case of property insurance. We may also consider more complex audit games, in which policyholders engage in costly actions that increase monitoring costs [11] or collude with insurance intermediaries so as to obtain a larger indemnification. These issues certainly deserve further research.

APPENDIX

Given the integer N, we let $\mathbf{N} = \{1,\dots,N\}$.

Definition 1 : A type-i RS-equilibrium (for i = c or n) is characterized by N different contracts (q_j,P_j) in R^2_+ and proportions σ_j in [0,1], j = 1,...,N such that:

[1] For all j :

either $$EU_h(q_j,P_j) \geq \mathrm{Max}\,\{\bar{U}\ ,\ \underset{k\in\mathbf{N}-\{j\}}{\mathrm{Max}}\ EU_h(q_k,P_k)\}$$

or $$EU^i_0(q_j,P_j,\sigma_j) \geq \mathrm{Max}\,\{\bar{U}\ ,\ \underset{k\in\mathbf{N}-\{j\}}{\mathrm{Max}}\ EU^i_0(q_k,P_k,\sigma_k)\}$$

[2] Given $EU_h(q_j,P_j)$ and $EU^i_0(q_j,P_j,\sigma_j)$ for j = 1,...,N, proportions $\sigma_1,\dots,\sigma_N$ verify assumption **A1**.

[3] $P_j \geq C^i(q_j,P_j,\sigma_j)$ for all j in **N**.

[4] There does not exist (q_{N+1},P_{N+1}), different from (q_j,P_j) for all j in **N**, such that :

[4.1] $P_{N+1} > C^i(q_{N+1},P_{N+1},\sigma'_{N+1})$

[4.2] either $$EU_h(q_{N+1},P_{N+1}) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{k\in\mathbf{N}}{\mathrm{Max}}\ EU_h(q_k,P_k)\}$$

or $$EU^i_0(q_{N+1},P_{N+1},\sigma'_{N+1}) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{k\in\mathbf{N}}{\mathrm{Max}}\ EU^i_0(q_k,P_k,\sigma'_k)\}$$

where given $EU_h(q_k,P_k)$ and $EU^i_0(q_k,P_k,\sigma'_k)$, the proportions σ'_k verify **A1** when contracts (q_k,P_k), $k \in \mathbf{N} \cup \{N+1\}$ are offered.

Definition 2 : A type-i W-equilibrium (for i = c or n) is characterized by N different contracts (q_j,P_j) and proportions σ_j , j = 1,...,N such that [1], [2], [3] hold and such that :

[5] There does not exist (q_{N+1},P_{N+1}) different from (q_j,P_j) for all j in **K** and $\mathbf{K} \subset \mathbf{N}$ such that :

[5.1] $P_k \geq C^i(q_k,P_k,\sigma'_k)$ for all k in **K**

[5.2] $P_{N+1} > C^i(q_{N+1},P_{N+1},\sigma'_{N+1})$

[5.3] either $EU_h(q_{N+1},P_{N+1}) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{k\in\mathbf{K}}{\mathrm{Max}}\ EU_h(q_k,P_k)\}$

or $EU^i_0(q_{N+1},P_{N+1},\sigma'_{N+1} \geq \mathrm{Max}\{\bar{U},\underset{k\in\mathbf{K}}{\mathrm{Max}}\ EU^i_0(q_k,P_k,\sigma'_k)\}$

[5.4] $P_j < C^i(q_j,P_j,\hat{\sigma}_j)$ for all j in $\mathbf{N} - \mathbf{K}$ such that

either $EU_h(q_j,P_j) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{k\in\mathbf{K}\cup\{N+1\}}{\mathrm{Max}}\ EU_h(q_k,P_k)\}$

or $EU^i_0(q_j,P_j,\hat{\sigma}_j) \geq \mathrm{Max}\{\bar{U},\underset{k\in\mathbf{K}\cup\{N+1\}}{\mathrm{Max}}\ EU^i_0(q_k,P_k,\hat{\sigma}^j_k)\}$

where, given $EU_h(q_k,P_k)$ and $EU^i_0(q_k,P_k,\sigma'_k)$, the proportions σ'_k correspond to **A1** when contracts (q_k,P_k), $k \in \mathbf{K} \cup \{N+1\}$ are offered, and, given $EU_h(q_k,P_k)$ and $EU^i_0(q_k,P_k,\hat{\sigma}^j_k)$, proportions $\hat{\sigma}^j_k$ correspond to **A1** when contracts (q_k,P_k), $k \in \mathbf{K} \cup\{j\} \cup \{N+1\}$ are offered, with $\hat{\sigma}^j_j \equiv \hat{\sigma}_j$.

In definitions 1 and 2, the interpretation of [1], [2] and [3] is straightforward. From [1], any equilibrium contract is chosen either by opportunists or by honest individuals or by both ; [2] recalls that agents are uniformly distributed among their best contracts ; [3] requires that, in equilibrium, each policy earns non-negative profits.

In definition 1, [4] states that at an RS-equilibrium there is no other policy which earns a positive profit when offered in addition to the existing market offer of policies : [4.1] requires that the premium is greater than the expected cost and [4.2] states that the new policy is actually chosen by some individuals.

In definition 2, [5] states that at a W-equilibrium there is no other policy which earns a positive profit, when insurance companies withdraw any policy that becomes unprofitable. In other words, companies anticipate that only a subset in the existing market offer will remain after a new policy is offered. Note that there may exist several subsets of policies which preserve the profitability of the remaining policies. The definition of a W-equilibrium requires that, once a new contract is offered, there does not exist any subset of remaining policies **K** such that : (i) remaining policies are still profitable while any other (previously offered) policy would earn negative profits if it were offered in addition, and (ii) the new contract earns positive profits [12]. From a game theory perspective, this may be interpreted as the outcome of a two stage game in which at stage one a new contract may be offered by a potential entrant, and at stage two incumbents simultaneously decide whether to remain or leave [13]. [5] means that for any subgame-perfect equilibrium of this two-stage game, the incumbent earns non-positive profits.

Proof of Proposition 1

The proof is in three steps.

Step 1 : $\{(q^i,P^i)\}$ is a type-i W-equilibrium.

Proof of step 1

Let $N = 1$, $(q_1,P_1) = (q^i,P^i)$ and $\sigma_1 = \theta$. Then [1], [2] and [3] are obviously satisfied. To show that [5] also holds, we consider another contract (q_2,P_2), with $\mathbf{K} = \emptyset$ or $\{1\}$, and we prove that [5.1] - [5.4] cannot hold simultaneously. In steps 1 and 2, σ_j, σ'_j, $\hat{\sigma}_j$ and σ^k_j are defined as in definition 2.

Case 1 : $EU_h(q_2,P_2) > EU_h(q_1,P_1)$

Subcase 1.1 : $\mathbf{K} = \{1\}$ and $EU^i_0(q_2,P_2,\sigma'_2) > EU^i_0(q_1,P_1,\sigma'_1)$

We then have $\sigma'_2 = \theta$ since all individuals prefer (q_2,P_2). We have $C^i(q_2,P_2,\sigma'_2) = C^i(q_2,P_2,\theta) > P_2$ since $EU_h(q_2,P_2) > EU_h(q_1,P_1)$, which contradicts [5.2].

Subcase 1.2 : **K** = {1} and $EU_0^i(q_2,P_2,\sigma'_2) \le EU_0^i(q_1,P_1,\sigma'_1)$

Then, only opportunists choose (q_1,P_1) which gives [14] $\sigma'_1 = 1$, $\sigma'_2 < \theta$. Assume $\alpha_1 \equiv \alpha^i(q^i,P^i,\sigma'_1) < 1$. Then **P5-P6** give:

$$EU_0^i(q_2,P_2,\sigma'_2) \ge EU_h(q_2,P_2) > EU_h(q_1,P_1) = EU_0^i(q_1,P_1,\sigma'_1)$$

hence the impossibility. We then have $\alpha_1 = 1$. Furthermore, from **P5** we have $EU_0^i(q_1,P_1,\sigma'_1) \ge EU_0^i(q_2,P_2,\sigma'_2) \ge EU_h(q_2,P_2) > EU_h(q_1,P_1) \ge \bar{U}$. **P7** then gives $q_1 > 0$. Using **P1** and **P2** and $\sigma'_1 > \theta$, we then deduce

$$P_1 = C^i(q_1,P_1,\theta) < = C^i(q_1,P_1,\sigma'_1)$$

which contradicts [5.1].

Subcase 1.3 : **K** = ∅

We then have $\sigma'_2 = \theta$. Since $EU_h(q_2,P_2) > EU_h(q_1,P_1)$, we deduce $P_2 < C^i(q_2,P_2,\theta) = C^i(q_2,P_2,\sigma'_2)$, which contradicts [5.2].

Case 2 : $EU_h(q_2,P_2) < EU_h(q_1,P_1)$

Subcase 2.1 : **K** = {1} and $EU_0^i(q_2,P_2,\sigma'_2) < EU_0^i(q_1,P_1,\sigma'_1)$

[5.3] does not hold in that case.

Subcase 2.2 : **K** = {1} and $EU_0^i(q_2,P_2,\sigma'_2) \ge EU_0^i(q_1,P_1,\sigma'_1)$

Then, only opportunists choose (q_2,P_2) which gives $\sigma'_1 < \theta$, $\sigma'_2 = 1$.

Assume $\alpha_2 \equiv \alpha^i(q_2,P_2,\sigma'_2) < 1$. Then **P5-P6** give

$$EU_h(q_2,P_2) = EU_0^i(q_2,P_2,\sigma'_2) \ge EU_0^i(q_1,P_1,\sigma'_1) \ge EU_h(q_1,P_1)$$

hence the impossibility. We thus have $\alpha_2 = 1$. Using **P1** and **P2**, we deduce

$$C^i(q_2,P_2,\sigma'_2) = C^i(q_2,P_2,1) = q_2$$

For [5.2] to be fulfilled, it is necessary that $P_2 > q_2$. This implies

$$EU_0^i(q_2,P_2,\sigma'_2) < \bar{U}$$

which contradicts [5.3].

Subcase 2.3 : $\mathbf{K} = \emptyset$

We then have $\hat{\sigma}_1 \le \theta$ (using notation of [5.4]) since honest individuals prefer (q_1,P_1) to (q_2,P_2). Using **P1** gives

$$C^i(q_1,P_1,\hat{\sigma}_1) \le C^i(q_1,P_1,\theta) = P_1$$

which contradicts [5.4].

Case 3 : $EU_h(q_2,P_2) = EU_h(q_1,P_1)$

Subcase 3.1 : $\mathbf{K} = \{1\}$ and $EU_0^i(q_2,P_2,\sigma'_2) \ge EU_0^i(q_1,P_1,\sigma'_1)$.

In that case, we have $\sigma'_1 \le \theta$, $\sigma'_2 \ge \theta$. This gives (using **P1**) :

$$C^i(q_2,P_2,\theta) \le C^i(q_2,P_2,\sigma'_2)$$

Then, for [5.2] to hold, we should have

$$C^i(q_2,P_2,\theta) < P_2$$

Consider the contract $q = q_2$, $P = P_2 - \varepsilon$, $\varepsilon > 0$. For ε small enough, we have

$$C^i(q,P,\theta) \le P$$

which gives (from the definition of q^1,P^1 and the fact that $EU_h(q,P)$ is decreasing with respect to P) :

$$EU_h(q_2,P_2) < EU_h(q,P) \le EU_h(q_1,P_1)$$

hence a contradiction.

Subcase 3.2 : $\mathbf{K} = \{1\}$ and $EU_0^i(q_2,P_2,\sigma'_2) < EU_0^i(q_1,P_1,\sigma'_1)$.

Then, we have $\sigma'_1 = \frac{2\,\theta}{1+\theta} < 1$ and $\sigma'_2 = 0$.

Assume $\alpha_1 \equiv \alpha^i(q_1,P_1,\sigma'_1) < 1$. Then, **P5-P6** give

$$EU_0^i(q_2,P_2,\sigma'_2) \geq EU_h(q_2,P_2) = EU_h(q_1,P_1) = EU_0^i(q_1,P_1,\sigma'_1)$$

hence a contradiction. We thus have $\alpha_1 = 1$. As in subcase 1.2, **P7** gives $q_1 > 0$. **P1** and **P2** yield $C^i(q_1,P_1,\sigma'_1) > C^i(q_1,P_1,\theta) = P_1$ which contradicts [5.1].

Subcase 3.3 : $\mathbf{K} = \emptyset$

We then have $\sigma'_2 = \theta$, and using **A2** a contradiction is obtained in the same way as in subcase 3.1. ■

Step 2 : $\{(q^i,P^i)\}$ is the unique type-i W-equilibrium.

Proof of step 2

Let $\{(q_1,P_1),\ldots,(q_N,P_N)\}$ be a type-i W-equilibrium.

Claim 1 : For all j in N, if $EU_0^i(q_j,P_j,\sigma_j) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{\substack{k\neq j\\ k\in\mathbf{N}}}{\mathrm{Max}}\ EU_0^i(q_k,P_k,\sigma_k)\}$ (i)

then $EU_h(q_j,P_j) \geq \mathrm{Max}\{\bar{U}\ ,\ \underset{\substack{k\neq j\\ k\in\mathbf{N}}}{\mathrm{Max}}\ EU_h(q_k,P_k)\}$ (ii)

Proof

Assume that (i) holds, but not (ii). Then **P5-P6** give

$$EU_h(q_j,P_j) < EU_0^i(q_j,P_j,\sigma_j)$$

and

$$\alpha_j = \alpha^i(q_j,P_j,\sigma_j) = 1.$$

We have $\sigma_j = 1$, since contract (q_j,P_j) is chosen only by opportunists. Then, from **P2** we would have

$$C^i(q_j,P_j,\sigma_j) = C^i(q_j,P_j,1) = q_j$$

and [3] would give $P_j \geq q_j$. This implies $P_j = q_j = 0$ (see the assumption at the end of section 2) and (ii) holds with $EU_h(q_j,P_j) = \bar{U}$, hence a contradiction. ■

From claim 1, the set of equilibrium contracts can be split into two subsets : contracts (q_k,P_k) are chosen by honest individuals and opportunists if $k = 1,\ldots,m$, with $m \leq N$, and they are chosen only by honest individuals if $k = m+1,\ldots,N$

Claim 2 : $m = 1$ and $(q_1,P_1) = (q^i,P^i)$.

Proof

For all k in $\{1,\ldots,m\}$, we have $\sigma_k = \dfrac{\theta N}{\theta N + (1-\theta)\ m} > \theta$ and (using [3] and **P1**), we may write $P_k \geq C^i(q_k,P_k,\theta)$. **A1** then gives $EU_h(q_k,P_k) < EU_h(q^i,P^i)$ if $(q_k,P_k) \neq (q^i,P^i)$.

Let us consider the case where $m > 1$ and $(q_j,P_j) = (q^i,P^i)$ for some index j in $\{1,\ldots,m\}$. Then $EU_h(q_k,P_k) < EU_h(q_j,P_j)$, for $k \in \{1,\ldots,m\}$, $k \neq j$. This contradicts the fact that both contracts (q_j,P_j) and (q_k,P_k) are simultaneously chosen by honest individuals.

Let us now contemplate the case where $m \geq 1$ and $(q_k,P_k) \neq (q^i,P^i)$ for all k in $\{1,\ldots,m\}$. Let $(q_{N+1},P_{N+1}) = (q^i-\varepsilon,P^i)$, with $\varepsilon > 0$. For ε small enough, we have $EU_h(q_{N+1},P_{N+1}) > EU_h(q_k,P_k)$ for all k in $\{1,\ldots,m\}$. Let $\mathbf{K} = \emptyset$, we then have $\sigma'_{N+1} = \theta$. $C^i(q,P^i,\theta)$ is locally increasing [15] with respect to q, $q \leq q^i$, in the neighbourhood of $q = q^i$. Hence $P_{N+1} = P^i = C^i(q^i,P^i,\theta) > C^i(q_{N+1},P_{N+1},\theta)$, hence [5.2].

Using [1], we may also write for ε small :

$$EU_h(q_{N+1},P_{N+1}) > EU_h(q_k,P_k) \geq \bar{U} \qquad \text{for all k in } \mathbf{N}$$

hence [5.3].

If a contract (q_j,P_j), $j \in \mathbf{N}$ and (q_{N+1},P_{N+1}) are simultaneously offered and some individuals choose (q_j,P_j), we have $\hat{\sigma}_j = 1$ and $q_j > P_j$ for ε small since all honest individuals choose (q_{N+1},P_{N+1}). If $\alpha_j \equiv \alpha^i(q_j,P_j,\hat{\sigma}_j) < 1$, then **P5-P6** give

$$EU^i_0(q_j,P_j,\hat{\sigma}_j) = EU_h(q_j,P_j) < EU_h(q_{N+1},P_{N+1})$$

$$\leq EU^i_0(q_{N+1},P_{N+1}, \hat{\sigma}^j_{N+1})$$

and *a fortiori*

$$EU^i_0(q_j,P_j,\hat{\sigma}_j) < \text{Max}\{\bar{U} , EU^i_0(q_{N+1},P_{N+1},\hat{\sigma}^j_{N+1})\}$$

Likewise

$$EU_h(q_j,P_j) < \text{Max}\{\bar{U} , EU_h(q_{N+1},P_{N+1})\}$$

If $\alpha_j = 1$, **P2** gives :

$$C^i(q_j,P_j,\hat{\sigma}_j) = q_j > P_j$$

hence [5.4].

(q_{N+1},P_{N+1}) and $\mathbf{K} = \varnothing$ satisfy [5.1], [5.2], [5.3] and [5.4] which contradicts the equilibrium definition. We thus have $m = 1$ and $(q_1,P_1) = (q^i,P^i)$. ■

Claim 3 : $N = 1$.

Proof

Assume $N > 1$. Then from claim 2, all opportunists choose (q^i,P^i) and honest individuals are uniformly distributed among N contracts. Consequently, we have $\sigma_1 = \dfrac{\theta N}{\theta N + (1-\theta)} > \theta$. Assume $\alpha_1 \equiv \alpha^i(q_1,P_1,\sigma_1) < 1$. Then, using **P6**, for $k \in \{2,\dots,N\}$, we would have $EU_h(q_k,P_k) = EU_h(q_1,P_1) = EU^i_0(q_1,P_1,\sigma_1) > EU^i_0(q_k,P_k,0)$ which contradicts **P5**. We thus have $\alpha_1 = 1$. Furthermore $EU^i_0(q_1,P_1,\sigma_1) > EU^i_0(q_k,P_k,0) \geq EU_h(q_k,P_k) = EU_h(q_1,P_1) \geq \bar{U}$. Using **P7** then gives $q_1 > 0$. Using **P2**, we deduce

$$C^i(q_1,P_1,\sigma_1) > C^i(q_1,P_1,\theta) \equiv C^i(q^i,P^i,\theta) = P^i = P_1$$

which contradicts [3]. ■

This achieves the proof of step 2.

Step 3 : If $q^i = 0$ or $\alpha^i(q^i, P^i, \frac{2\theta}{1+\theta}) < 1$, then $\{(q^i, P^i)\}$ is a type-i RS-equilibrium.

Proof of step 3

Let $N = 1$, $(q_1, P_1) = (q^i, P^i)$, $\sigma_1 = \theta$. It remains to show that [4] holds when $q^i = 0$ or $\alpha^i(q^i, P^i, \frac{2\theta}{1+\theta}) < 1$. Note that **P4** gives $\alpha^i(q^i, P^i, 1) < 1$ if $\alpha^i(q^i, P^i, \frac{2\theta}{1+\theta}) < 1$.

Equivalently, [4] holds if there does not exist (q_{N+1}, P_{N+1}) that verifies [5.2] and [5.3] with $\mathbf{K} = \mathbf{N} = \{1\}$. We consider all possible cases in turn. As shown in the proof of step 2 :

Subcase 1.1 contradicts [5.2]
Subcase 1.2 implies $q^i > 0$ and contradicts $\alpha^i(q^i, P^i, 1) < 1$
Subcase 2.1 contradicts [5.3]
Subcase 2.2 contradicts [5.2] - [5.3]
Subcase 3.1 contradicts [5.2]
Subcase 3.2 implies $q^i > 0$ and contradicts $\alpha^i(q^i, P^i, \frac{2\theta}{1+\theta}) < 1$.

The proof is now complete. ■

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FOOTNOTES

1 This terminology follows Weisberg and Derrig (1993).

2 For instance, in studies of automobile bodily injury claims in Massachusetts, Weisberg and Derrig (1991, 1992) have found that the level of suspected fraud was about 10 %. The percentage of suspicious claims, including build-up, reached 48.2 % in 1989 after the Auto Insurance Reform Law of 1988 replaced the previous $ 500 medical damage threshold with a $ 2.000 one. Likewise, according to Mooney and Salvatore (1990), about 13 % of automobile insurance claims in Florida are fraudulent. Econometric studies by Dionne and St. Michel (1991) and Dionne, St. Michel and Vanasse (1992) of workers' compensation suggest that build-up is particularly important for injuries like spinal disorder or back pain that are difficult to diagnose.

3 *Lloyd's List*, Oct. 16, 1993.

4 In the U.S., the Insurance Fraud Protection Act of 1993 makes it a federal crime to defraud an insurance company. Conviction for an offense carries fines and/or a prison term of up to 10 years.

5 As in Rothshild-Stiglitz' (1976) and Wilson's definitions, we restric attention to situations in which each policy breaks even at equilibrium. Miyazaki (1977) and Spence (1978) extended Wilson's approach and considered cross-subsidization between contracts.

6 The case $m(q,P) \equiv M(q,P)$ covers the situations where, after a fraud has been detected, companies and fraudsters settle differences out of a court by negotiating on a compensatory transfer to the company, so as to economize on a cost $M(q,P) - m(q,P)$ that would go to lawyers in case of prosecution in court.

[7] In the definition given in appendix, we assume that companies compete by offering policies (q_j,P_j). Any policy actually offered at equilibrium is associated with a (rationally expected) proportion of opportunists σ_j and an audit probability p_j such that opportunists and honest individuals are distributed among best contracts and p_j is an equilibrium strategy in the audit game (q_j,P_j,σ_j). Under commitment to policy, an alternative definition is to assume that companies compete by simultaneously offering policies (q_j,P_j) and associated audit probabilities p_j. Proportions σ_j are then be rationally expected given opportunists's reaction function. At such a market equilibrium we have $p_j = p^c(q_j,P_j,\sigma_j)$, otherwise an out-of-equilibrium profitable offer would exist and consequently policies (q_j,P_j) satisfy the definition of a type-c equilibrium. Hence, under commitment to audit policy, allowing firms to compete by simultaneously offering policies and audit probabilities would not enlarge the set of market equilibria.

[8] Contracts that are preferred to (q^i,P^i) by honest individuals are located in the area under the curve EU_h = constant. Furthermore, from $\alpha^i(q^i,P^i,1) = 1$ and $p^i(q^i,P^i,1) = 0$, we have

$$EU_i^O(q^i,P^i,1) = \delta\ U(w-L-P^i+q^i) + (1-\delta)\ U(w-P^i+q^i)$$

We also have for any (q,P,σ) :

$$EU_i^O(q,P,\sigma) \leq \delta\ U(w-L-P+q) + (1-\delta)\ U(w-P+q)$$

which gives

$$EU_i^O(q,P,\sigma) \leq EU_i^O(q^i,P^i,1)$$

for all (q,P,σ) such that $P \geq q + P^i - q^i$

[9] On delegation as a commitment device in the case of income tax audits, see Melumad and Mookherjee (1989). It should be observed that delegation is a commitment device only in so far as investigators' incentive contracts cannot be secretly renegotiated. If secret renegotiation cannot be prevented, once a tough monitoring policy has been publicly announced insurance companies and investigators may benefit from renegotiating a weaker policy. This renegotiation will be anticipated by opportunists and, ultimately, pre-commitment effects will not exist. This argument assumes that incentive contracts are signed and implemented under perfect information about the investigator's type. When the investigator has private information, pre-commitment effects may exist (see Dewatripont (1988) and Caillaud, Jullien and Picard (1993)).

[10] Note however that our model could be adapted to deal with the case where occurence of accidents is verifiable but policyholders may file fraudent claims to receive compensation of a large loss, although they only suffer a small damage. See Cummins and Tennyson (1994) for such an analysis about automobile liability insurance.

[11] Bond and Crocker (1993) characterize the optimal insurance contract under endogenous monitoring costs.

[12] In this interpretation, it is assumed that each firm offers only one policy.

[13] Note that this condition slightly differs from Wilson's (1977) definition – see Assumption 6 in Wilson's paper. Wilson postulates that there exists a deterministic relation between the set of policies that remain from an initial offer and the new policy which is offered. However, several definitions for this relation are possible. Our definition escapes from this difficulty. Both definitions coincides in the Rothschild-Stiglitz (1976) insurance market model as well as in the present model.

[14] In that case, we have

$$\sigma'_2 = 0 \quad \text{if } EU^i_0(q_2,P_2,\sigma'_2) < EU^i_0(q_1,P_1,\sigma'_1)$$

$$\sigma'_2 = \frac{\theta}{2-\theta} \quad \text{if } EU^i_0(q_2,P_2,\sigma'_2) < EU^i_0(q_1,P_1,\sigma'_1)$$

[15] If $C^i(q,P^i,\theta)$ is differentiable with respect to q at $q = q^i$, the definition of (q^i,P^i) given in **A2** implies that $C^i(q,P^i,\theta)$ is increasing with respect to q at $q = q^i$. Furthermore, we know from propositions 1 and 2 that $C^i(q,P^i,\theta) \leq [\delta+(1-\delta)\theta]q$ with equality at $q = q^i$ in case of non-differentiability at $q = q^i$, hence the result.