October 1993
Revised March 1994

THE LONG RUN OF MACROECONOMETRIC MODELS: THE CASE OF MULTIMOD*

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N° 9413

* Paper written for presentation at a CCSO Workshop, University of Groningen, 15-17 December 1993. Forthcoming in S.K. Kuipers, L. Schoonbeek and E. Sterken Eds. *Methods and Applications of Economic Dynamics*, North Holland.

** We are grateful to Philippe Gudin de Vallerin and Steven Symansky for their useful help. We benefited greatly from numerous remarks and suggestions from Jan Jacobs, Bert Schoonbeek and an anonymous referee. Of course the responsibitity for any remaining errors is ours.

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ABSTRACT

The purpose of the paper is to implement a methodology to exhibit the long-run structure, behind a given model, to the case of the multinational model MULTIMOD. The first step is to look for the existence of such an underlying structure. We find that the way the model is written is indeed consistent with a well specified steady state growth path equilibrium. The simulation of the whole (long-run) model and the computation of basic multipliers give satisfactory results in line with the medium-run results of the dynamic model. However, in a few cases, the peculiar nature of the baseline solution implies the absence of any solution.

Keywords: Macroeconometric models, simulation, steady state growth path.

LE LONG TERME DES MODELES MACROECONOMETRIQUES : LE CAS DE MULTIMOD

RESUME

L'objet du papier est appliquer au cas du modèle multinational MULTIMOD une méthodologie destinée à mettre en évidence la structure de long terme sous-jacente à un modèle donné. La première étape consiste à vérifier les conditions d'existence d'une telle structure. Nous trouvons que la manière dont le modèle est construit le rend automatiquement cohérent avec avec une solution de croissance équilibrée correctement spécifiée. La simulation du modèle de long terme asocié et le calcul des multiplicateurs de base donne des résultats satisfaisants et cohérents avec les simulations à moyen terme du modèle dynamique de départ. Toutefois la nature particulière de la solution de référence implique dans certains cas une absence de solution.

Mots clefs : Modèles macroéconométriques, simulation, croissance équilibrée.

J.E.L: C53, C63

THE LONG RUN OF MACROECONOMETRIC MODELS: THE CASE OF MULTIMOD

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I - Introduction

Conventional dynamic macroeconometric models constitute a useful device, still widely used for shedding light on the preparation of quantitative economic policy. Their structure is generally based on the so-called "disequilibrium growth theory", an attempt at synthesis between short run keynesian theory and neo-classical growth theory. These models have been criticized on several grounds the most dramatic one being the numerous defects in their theoretical foundations.

The basic idea of this paper is to exhibit the long-run structure of a model taking opportunity of the fact that, most of the time, the derivation of the typical behavioral relations start from a long-run/equilibrium theory, on which dynamical mechanisms are plugged. We define the concept of long-run for macroeconometric models in the usual way of dynamic equilibrium of all adjustment mechanisms. Indeed, when the model is dynamically stable, this corresponds to its asymptotical behavior. Deriving long-run mechanisms associated with dynamical relationships is obviously not new. Most of the time, however, it has concerned single equation models and seldom an entire macro-model, except a small number of studies (see *e.g.* Bergstrom and Wymer (1976), Gandolfo (1981), Currie (1982) and more recently Wallis and Whitley (1987), or

Deleau *et al.* (1990)). The search for long-run properties, if any, of a model has been recently reinforced by the introduction of rational expectations. Indeed, the logic of rational expectations imposes, through the necessity of setting terminal conditions, to have a clear perception of the long-run solution of the model.

This paper is divided into five sections. In section 2, we present the methodology used for building the long-run of a given macroeconometric model. Section 3 gives a brief decription of the model analysed, MULTIMOD, and describes how its long-run is obtained. Section 4 shows the long-run properties of MULTIMOD. Section 5 concludes.

II - Methodology

In this section we recall briefly the methodology[1] we use for building the long-run version of a given macroeconomic model. A more detailed exposition can be found for instance in Deleau *et al.* (1990).

2.1 -Formalization

Let us consider a dynamic macroeconometric model written as follows:

$$(1) \qquad y(t) = f(\, y(t), y(t\text{-}1), y(t\text{+}1), x(t); t),$$

with y(t) [x(t)], vector nx1 [mx1] of the endogenous [exogenous] variables, y(t-1) and y(t+1), lagged and forward looking endogenous variables, t being the index of -discrete-time.

The search for a Steady State Growth Path (SSGP) is equivalent to finding particular solutions of (1) of the type:

$$(2) \qquad \begin{aligned} \overset{*}{x}_i(t) &= \overset{*}{x}_i(0)\, a_i^t \qquad && i = 1, \ldots, m \\ \overset{*}{y}_j(t) &= \overset{*}{y}_j(0)\, b_j^t \qquad && j = 1, \ldots, n \end{aligned}$$

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1 Another way to obtain the long-run structure of macro-models consists of running very long dynamic simulations. See e.g. Wallis and Whitley (1987).

In other words, we look for a set of n initial conditions $y^*(0)$ and of n growth rates $(b_1,\dots, b_n)$ for the endogenous variables, the corresponding exogenous elements $x^*(0)$ and $(a_1,\dots, a_m)$ being known, such that (2) satisfies the system (1) at each time period.

We immediately observe that, a solution of (1) of the type (2) will exist only if each equation of this system fulfills some homogeneity conditions. These conditions are imposed by an elementary mathematical property of exponentials: except for pathological cases, a sum of terms evolving through time at constant growth rates follows an exponential pattern only if all growth rates are identical.

2.2- Determination of Growth Rates

Almost all conventional macroeconometric models are based on the hypothesis that the long-term real growth of the economy results from the exogenously given growth rates of the population and of the technical progress. In the same way, long-term inflation of an open economy must be equal to foreign exogenous inflation, and in the case of a closed economy, from some -exogenous- money supply target growth. More generally, it is easy to see that all the rates (a_i) and (b_j) can be derived in a straightforward way from three basic exogenous growth rates which lie somewhere as parameters in the model:

ν : exogenous growth rate of population,

γ : exogenous growth rate of technical progress (necessarily Harrod neutral),

μ : exogenous growth rate of money supply,

or ρ : exogenous growth rate of foreign prices,

with the identity: $1+\mu = (1+\rho)\,(1+\gamma)\,(1+\nu)$

For instance, the -common- rate of growth of all quantities g is given by the identity:

$$1 + g = (1 + \nu)(1 + \gamma).$$

2.3- Determination of Initial Conditions

Once the growth rates, including those of the exogenous variables, being determined from the three basic growth rates (ν, γ, ρ), the principle of computation of the initial conditions $y(0)$, knowing $x(0)$, is straightforward. Rewriting the equations of model (1) at time t = 0, and substituting the lagged variables by their expression (2), yields:

$$(4) \qquad y^*(0) = f(\, y^*(0), (\overset{*}{y}_j(0)/\, b_j), (\overset{*}{y}_j(0)\, b_j),\; x^*(0), 0)_{\,j=1...n}$$

or, omitting the time index:

$$(5) \qquad y^* = f(\, y^*, x^*, \nu, \gamma, \rho)$$

The system (5), which we call the "long-run system" associated with the model (1), consists of a set of simultaneous equations, of which the solution y^*, given the exogenous vector x^*, defines a set of initial conditions for a SSGP.

One can notice that long-run (exogenous) growth rates intervene in this long-run system. The static multipliers associated with (5) correspond to the asymptotic effect of a sustained multiplicative shock on the exogenous variables, which could be obtained through long-run simulations of the initial model (1), provided it verifies the adapted stability conditions. Let us make a few preliminary remarks about this long-run system (5):

(i) An equation of model (1) written in terms of growth rates yields ultimately a long-run relationship between the three basic long-run growth rates only, without any implication for initial levels.

(ii) In a relation involving both rates and levels, only the last ones subsist in the long-run system. For instance an equation of the following type:

$\hat{y} = f(\, z)$, $\qquad z \in R$, $\qquad$ where $\hat{x}$ denotes the growth rate of x,

implies an autonomous determination of z^*:

$$z^* = f^{-1}(\, \hat{y}^{\,*}) = g(\, \nu, \gamma, \rho)$$

A typical case is the Phillips curve. It links the growth of wages and prices to the level of the unemployment rate, and thus leads to a long-run determination of the unemployment rate independently from the rest of the system.

(iii) The relations involved in the system (5) may receive the interpretation of being a set of cointegrating relations that the original model (1) postulates to hold.

III - Implementation of the Methodology to MULTIMOD

The version of the multinational dynamic econometric annual model MULTIMOD we study (Masson *et al.* (1990), slightly revised in Masson *et al.* (1991)) describes the behavior of the whole world decomposed into eight industrial countries/zones (Canada, -noted CA-, France -FR-, Germany -GR-, Italy -IT-, Japan -JA-, United-Kingdom -UK-, United-States -US-, and a zone, called "small industrial countries" -SI-, collecting the rest of OECD), and the rest of the world -RW- comprising two developing zones: high-income oil exporters -HO- and other developing countries -DC. Three goods are distinguished: oil, primary commodities, and other goods called manufactures.

3.1- Methodological Issues

It is important to note at this stage that MULTIMOD is not aimed at running baseline dynamic simulations, but has been designed mainly for evaluating effects of economic policy modulation or exogenous shocks, *around a reference path given from outside*. This reference path describes the transition of the world economy from the present state toward a SSGP in about ten years (the presently reference path available from IMF is increasing at a constant rate from the year 2004). It is simulated with MULTIMOD by introducing systematic residuals in each behavioral equation. In other words, it is important to note that *residuals are not given a reference value of zero but are computed so as the solution of the whole model coincides with the exogenously given baseline path.*

1/ The methodology described in the previous section has been applied to the model MULTIMOD by first simulating each country model in isolation, then connecting all the blocks together (let us mention that this is the first time a model of this size -more than 450 equations- is processed with such a methodology). We must keep in mind that, contrary to the dynamic original model, where the weight of the lagged and the forward looking variables makes the system rather sluggish (system (1)), the interdependences between variables in the long-run model are very strong because it is a purely simultaneous system (system (5)). Indeed it is our experience that this feature makes the solution much more sensitive to rather small errors than the solution of the corresponding original dynamic model. As a matter of fact, we found a few small inconsistencies between the reference path exogenously given and the version of MULTIMOD we used, which were easy to correct (slight modification of the normalization parameter in the production function of the industrial country models and addition of supplementary residuals in the two investment equations of the model of the rest of the world).

The simulation of the long-run whole system has been able to reproduce the reference solution with an excellent numerical precision. Indeed Table 1 gives the errors in per cent for the main variables between given and simulated data. One can see that the errors are of a very small order of magnitude (about 10^{-5}), as compared to the convergence criterion of the resolution algorithm[1], and are not worsened when considering the whole world model, relatively to the case of the country models in a non linked mode.

Table 1: Per Cent Errors Between the Given Reference Long-Run Path and the Computed Long-Run Solution

CA_GNP	0.0032	FR_GNP	0.0046
CA_PGNP	-0.0009	FR_PGNP	-0.0004
CA_R	-0.0019	FR_R	-0.0018
CA_E	0.0000	FR_E	0.0005
GR_GNP	0.0047	IT_GNP	0.0028
GR_PGNP	-0.0010	IT_PGNP	-0.0004
GR_R	-0.0018	IT_R	-0.0020
GR_E	0.0006	IT_E	0.0006
JA_GNP	0.0028	SI_GNP	0.0048
JA_PGNP	-0.0007	SI_PGNP	-0.0011
JA_R	-0.0018	SI_R	-0.0020
JA_E	0.0006	SI_E	0.0006
UK_GNP	0.0023	US_GNP	0.0053
UK_PGNP	0.0000	US_PGNP	-0.0004
UK_R	-0.0020	US_R	-0.0018
UK_E	0.0005	US_E	0.0000
DC_GDP	0.0004	POIL	-0.0002
DC_PGNP	0.0000	PCOM	0.0010
HO_GDP	0.0007	WTRADE	-0.0055

Legend: GNP = Gross National Product, PGNP = GNP deflator, R = nominal interest rate,

[1] The software used for simulating MULTIMOD, TROLL, is based on Newton's algorithm. It considers that a simultaneous system has reached its equilibrium level at iteration n when *the relative change of each variables* with respect to iteration n-1 is less than 0.001. Tightening this criterion left the solution unaltered.

E = exchange rate, GDP = Gross Domestic Product, POIL = price of oil, PCOM = commodity price index, WTRADE = World trade discrepancy

2/ As we have said above, contrary to usual macro-models, MULTIMOD does possess one steady state growth path solution *by construction,* through adequate values given to the residuals. However, in the present exercise, we are not looking for only one long-run path, but for a "long-run system" of the type (5), in order to be also able to compute directly long-run effects of permanent shocks. In order to check the existence of such a system, we have to examine whether the equations verify the relevant homogeneity properties. In fact, it is easy to show here that the homogeneity conditions needed are less stringent than in the general case without built-in time-varying residuals, because equations are generally specified in log-linear terms.

To explain that, let us consider first the following *linear* equation linking a non-trended variable R(t) to a trended one X(t):

$$R(t) = a\,X(t) + b(t) + \varepsilon(t),$$

with the reference stationary solution (exogenously given):

$$R^*(t) = R^*, \text{ and } X^*(t) = X^*(0)\,(1+g)^t.$$

The computation of the reference path for the residuals gives:

$$\varepsilon^*(t) = R^* - a\,X^*(0)\,(1+g)^t - b(t).$$

Hence we can rewrite the equation as:

$$R(t) = a\,(X(t) - X^*(0)\,(1+g)^t) + R^*.$$

We observe that we cannot get constant value for R(t) even if X(t) increases at the rate g, as soon as X(0) differs from $X^*(0)$.

If we consider now the following equation where the specification of the right hand side is expressed in *log-linear terms*:

$$R(t) = a\,\mathrm{Log}(X(t)) + b(t) + \varepsilon(t),$$

with the same reference solution, the computation of the reference $\varepsilon^*(t)$, consistent with the stationary path, leads to:

$$R(t) = a\,\mathrm{Log}(X(t)/(X^*(0)\,(1+g)^t)) + R^*.$$

This specification obviously allows now for a long-run structure of the type (5) even if $X(0) \neq X^*(0)$.

The homogeneity conditions that the model and the exogenous variables must fulfill do not need to be verified analytically but can be checked in a very straightforward way by simple simulation exercises. Indeed, this can be done by shocking some exogenous variables of the long run system (5) (without changing the residuals), and simulating it over several periods, in order to see if the shocked solution increases at a constant rate. This experiment has been carried out and led to the conclusion of the non existence of any heterogeneous behavior.

3/ At last, before getting into the presentation of the long-run structure, let us mention that the reference value for the long-run basic growth rates are (exogenously) given by the baseline path and amount to 3% for all the quantity variables and to 3.79% for all the price variables, respectively. The rate of growth of the labor force is not relevant in the model. In effect, demographic variables appear basically in the production function, through labor force. Employment as well as wages are substituted for in the supply function. Hence only the growth rate of labor in efficient units (i.e. multiplied by productivity growth) matters. This rate is of course identically equal to the rate of growth of quantities: 3%.

3.2-Overview of the Structure of MULTIMOD

The specification of the country models is the same across the eight industrial countries/zones, and includes about 50 equations. Their structure consists in the open economy Mundell-Fleming version of the standard IS-LM model. The short run behavior of agents is almost systematically described through dynamic error-correction mechanisms toward their long-run, theory-based, equilibrium. The estimation has been carried out with pooled data for the main country behavioral functions: consumption, capital, money demand, and price supply. The *world trade equilibrium* in the volumes (resp. values) is obtained, as is conventional when bilateral trade is not explicited, through a reallocation of discrepancies in export volumes (resp. in import prices).

The aggregate supply side of the model is given by an *inflation equation* which is a reduced form summarizing supply and demand on the labor market, the augmented Phillips curve, and a mark-up pricing on unit labor costs varying with the degree of capacity utilization. *Government* intervention is represented by public expenditures, money supply, bonds supply, and taxes. However, due to the intertemporal government budget constraint, taxes are not exogenous and there is a dynamic reaction function of the tax rate to adjust progressively the government debt toward a nominal target value.

The *rest of the world* is aggregated into two regions, capital exporting countries comprising mainly high-income oil exporters, and other developing countries. The former region does not face any constraint on its balance of payments and its structure is modeled in a simple way because of the high proportion of its oil exports in GNP. In contrast, the latter region, comprising dissimilar countries, is finance constrained and therefore its imports and its domestic investment depend on its ability to carry an additional debt. In addition, the manufactured goods are splitted in tradable goods, and nontradable good. The prices of these two goods are not perfectly flexible as opposed to to oil and primary commodities which are considered as homogeneous goods with a single world price.

3.3-Equations of MULTIMOD for a typical country

Table 2 lists the main equations of the dynamic original model for a typical country and their transcription into the long-run static version. They are slightly stylized with respect to the full version of MULTIMOD in order to make them more readable: oil and commodities are omitted; we do not consider the time structure of interest rates which is irrelevant in the steady state. The interessed reader will find a copmprehensive presentation of MULTIMOD in Masson *et al.* (1990, 1991).

The seven first equations describe the IS curve. *Consumption* C (eq. 1) is modelled through an adaptation of the Blanchard (1985) model, of finite but uncertain horizon agents, to an open and growing economy. In addition to wealth W and real interest rate RR, in the short run, disposable income influences consumption. The *demand for capital* K (eq. 2) is linked to real value of capital WK in conformity with the Tobin's q theory. *Foreign trade* in manufactured goods is formalized with conventional export and import country functions (eqs.4 and 5) with both demand and competitiveness effects. This ensures that the simulation of each country in isolation be economically meaningful. Equations 6 and 7 are accounting identities defining absorption (A), and GDP (Q).

The equation 8 gives the definition of total real wealth W as the sum of human wealth, WH, and non-human wealth, itself composed of real market value of domestic capital WK and of real values of money balances M, of domestic government bonds B and of net foreign assets converted in domestic currencies units NFA/E. The four following equations 9 to 12 describe the cumulation of these different stocks linked to the agents' intertemporal budget constraints. One can note that the rate at which WH and WK are discounted are different, in conformity with the implications of the Blanchard (1985) theory of finite horizon consumers, the main difference being the probability of death, here equal to 0.035. The model also includes a risk premium at a rate rpr paid by firms to the consumers.

The two following relations 13 and 14 describe the government behavior. The money supply is written as a reaction function of the interest rate toward a nominal money target MT (case u = 1), except for the non-German members of the European Monetary System which adjust their money supply through a reaction function toward a given exchange rate vis-à-vis the Deutsche mark (u = 0). The tax rate TR is endogenous and function of the gap between the public debt B and an exogenously given nominal target level BT, which is reached in the long run. Equation 15 is an accounting identity for taxes T.

The demand for real money balances M (eq. 16) is standard and is linked to absorption and interest rate. Import prices are expressed as a weighted average of exports prices of other countries expressed in dollars, these last being in turn functions of GDP prices and import prices (eq. 17). Prices on foreign markets PFM, used as intermediate variable to define the competitiveness, are a double weighted average of GDP and export prices of competitors (eq. 18). Equation 19 describes the open interest parity condition, in terms of an identity for the US, and, for other countries, of an adjustment of their interest rate to the US interest rate. Foreign absorption FA (eq.20) is defined as a weighted sum of the absorption of the other countries.

The supply side of the model is given by equation 21 by a CES production function with labor augmenting technical progress γ, and by an inflation equation which summarizes the working of the labor market. This price equation 22 ends up in the long run at an equilibrium between supply and demand in the goods market. The optimum capital share in production β is computed from the production technology (eq. 23). At last there is a straightforward relation between nominal and real interest rate.

Table 2: Main dynamic and long-run equations of a typical country model of MULTIMOD

(D1)	$\hat{C} = c_0 + c_1 (\widehat{Q-T}) + c_2\, RR + c_3\, Log(W_{-1} / C_{-1}) + \varepsilon\_C$	(S1)	$C = c\,(RR, g, \varepsilon\_C)\, W$
(D2)	$\hat{K} = k_0 + k_1\, Log(WK / K_{-1}) + k_2\, Log(WK_{-1} / K_{-2}) + \varepsilon\_K$	(S2)	$K = k\,(g, \varepsilon\_K)\, WK$
(D3)	$I = K - (1\text{-}\delta)\, K_{-1} + \varepsilon\_I$	(S3)	$I = (g + \delta)\, K + \varepsilon\_I$
(D4)	$\hat{X} = x_0 + x_1\, (\widehat{P/PFM}) + x_2\, \widehat{FA} + x_3\, Log\, (FA/X)_{-1} + x_4\, Log(P/PFM)_{-1} + \varepsilon\_X$	(S4)	$X = x\,(g, P / PFM, \varepsilon\_X)\, FA$
(D5)	$\widehat{IM} = i_0 + i_1\, (i'\, \hat{A} + (1\text{-}i')\, \hat{X}) + i_2\, (\widehat{PM/P}) + i_3\, Log(\, A^{i'}.X^{1-i'} / IM)_{-1} + i_4\, Log(PM/P)_{-1} + \varepsilon\_IM$	(S5)	$IM = i\,(g, PM / P, \varepsilon\_IM)\, A^{i'}\, X^{1-i'}$
(D6)	$A = C + I + G$	(S6)	$A = C + I + G$
(D7)	$Q = A + X - IM$	(S7)	$Q = A + X - IM$
(D8)	$W = WH + WK + (M + B + NFA / E) / P$	(S8)	$W = WH + WK + (M + B + NFA / E) / P$
(D9)	$WH = WH_{+1} / (1\text{+}rbar\text{+}.035) + (1\text{-}\beta)\,(Q - T) + rpr\, WK$	(S9)	$(rbar\text{+}.035\text{-}g)\, WH = (1 - \beta)(Q - T) + rprWK$
(D10)	$WK = WK_{+1} / (1\text{+}RR\text{+}g) + \beta\,(Q - T) - (rpr\text{+}\delta)\, WK$	(S10)	$(RR\text{-}g)\, WK = \beta\,(Q\text{-}\, T) - (rpr + \delta + g)WK$
(D11)	$\Delta B + \Delta M = R.B_{-1} + P\,(G - T + \varepsilon\_B)$	(S11)	$(R - gv)\, B = gv\, M - P\,(\, G - T + \varepsilon\_B)$
(D12)	$\Delta NFA = E\,(P.X - PM.IM) + US\_R.NFA_{-1} + \varepsilon\_NFA$	(S12)	$(R - gv)\, NFA = E.(PM.IM - P.X) + \varepsilon\_NFA$
(D13)	$\Delta R - r_3\, (R_{+1} - R_{-1}) = u\, r_1\, Log\, (MT/M) + (1\text{-}u)\, r_2\, (Log\, (GR\_E/E))^3$	(S13)	$M = MT\ \ (u = 1)$ or $E = e\, GR\_E\ \ (u = 0)$
(D14)	$\Delta TR = t_1\, (B - BT) / (P.Q) + t_2\, \Delta\, [(B - BT) / (P.Q)]$	(S14)	$B = BT$
(D15)	$T = TR\, (Q - \delta\, K_{-1} + R.B_{-1} / P)$	(S15)	$T = TR\, (Q - (\delta\, K - R.B / P) /(1\text{+}g))$
(D16)	$Log(M/P) = m_0 + m_1\, LogA + m_2\, R + m_3\, Log(M/P)_{-1} + \varepsilon\_M$	(S16)	$M / P = m\,(A, g, R, \varepsilon\_M)$
(D17)	$PM = (\Sigma\, \pi_i\, P_i.E_i) / E$	(S17)	$PM = (\Sigma\, \pi_i\, P_i.E_i) / E$
(D18)	$Log\, (PFM.E) = \Sigma\, \rho i\, Log\, (Pi.Ei)$	(S18)	$Log\, (PFM.E) = \Sigma\, \rho_i\, Log\, (P_i.E_i)$
(D19)	$(1 + US\_R) = (1 + R)\,(E_{+1} / E)$ or $US\_E = 1$	(S19)	$R = US\_R$ or $US\_E = 1$
(D20)	$FA = \Sigma\, f_i\ A_i$	(S20)	$FA = \Sigma\, f_i\ A_i$
(D21)	$CAP = (g_1\, K^{-\sigma} + g_2\, ((1\text{+}\gamma)^t\, L\, (1 + \varepsilon\_L))^{-\sigma})^{-1/\sigma}$	(S21)	$CAP = (g_1\, K^{-\sigma} + g_2\, ((1\text{+}\gamma)^t\, L\, (1 + \varepsilon\_L))^{-\sigma})^{-1/\sigma}$
(D22)	$\hat{P} - \hat{P}_{-1} = p_1\, (\hat{P}_{+1} - \hat{P}_{-1}) + p_2\, (Q/CAP)$	(S22)	$Q = CAP$
(D23)	$\beta = g_1\, (CAP / K)^{\sigma}$	(S23)	$\beta = g_1\, (CAP / K)^{\sigma}$
(D24)	$(1 + R) = (1 + RR)\, P_{+1} / P$	(S24)	$(1 + R) = (1 + RR)\,(1 + \rho)$

<u>Notations:</u> $\hat{x}$ means Log (x / x_{-1}), (approximately the rate of growth of x), and Δ x means x - x_{-1}. ε_x denotes the residual asociated to the equation defining x. g is the growth rate of quantities, gv is the growth rate of nominal values, γ the growth rate of productivity. The summations indexed by i run over all countries /zones except the home country.

Endogenous variables

A	Real domestic absorption
B	Government bonds
C	Real consumption expenditure
CAP	Capacity output
E	Exchange rate (dollar per local currency)
FA	Foreign real absorption
I	Real investment
IM	Real imports
K	Capital stock
M	Monetary base
NFA	Net foreign assets in US$
P	Absorption deflator
PFM	Prices in foreign markets
PM	Import prices
Q	Real GDP
R	Nominal interest rate
RR	Real interest rate
T	Government real tax receipts
TR	Tax rate
W	Real total net wealth
WH	Human wealth
WK	Real value of capital stocks
X	Real exports
β	Optimal capital share in production

Exogenous variables

BT	Target level of government debt
G	Real public expenditures
L	Labor force
MT	Target level of the monetary base
rpr	Risk premium
σ	Exponent in the CES production function ($1/(1+\sigma)$ = elasticity of substitution)
δ	Depreciation rate for capital stock

IV - The Long-run Properties of MULTIMOD

It is worth to present first the main multipliers associated with the country models taken in isolation, before analysing the properties of the long-run version of the whole multinational model MULTIMOD.

4.1 - Main Multipliers of the Country Models

The present section is devoted to the presentation of the multipliers obtained with some representative long-run country models, without taking into account the international feedbacks. These long-run multipliers may be interpreted as giving directly the asymptotic effects of sustained multiplicative shocks on the dynamic original MULTIMOD country models.

We said in section 3.3 that the foreign trade is specified in order that the simulation of each country in isolation could be economically meaningful. However, there is one important exception concerning the US model. Indeed, all the exchange rates are defined in MULTIMOD as the price of the national currency in US$, and the US exchange rate is by definition equal to one (see eq. 19). As a consequence, all the simulation of the US model alone, without taking into account the reaction of the foreign exchange rates, must be interpreted as if the US were in a situation of fixed exchange rates.

Table 3 gives the results of a *permanent increase of 1% of Government Spending* in the US, Germany and the Small Industrial Countries respectively, made on the relevant country models. As money supply and public debt are fixed to their target in the long-run, this increase must be exactly balanced by a rise in real taxes[1]. This rise in taxes implies a drop in consumption. We observe that there is more than 100% crowding out, resulting in a decline in absorption. This comes from the fact that the decrease in consumers wealth goes beyond the present value of the additional taxes at the market interest rate. This mechanism, which is a typical consequence of the hypothesis of finite horizons consumers, appears quantitatively fairly independent of the economy under consideration: the quasi-closed economy (US), the small open

[1] In the Small Industrial Countries there is no money target (see eq. 13), and the level of money supply results from the demand for money balances. The variation of this last is however rather small (0.05%) due to the opposite variation of quantities and prices.

economy with a flexible exchange rate (Germany), or the fixed exchange rate (Small Industrial Countries). The price level is greater, associated with LM re-equilibration, but in the US case, the interest rate rises, leading to an even greater price level.

The effects of *permanent augmentation of money supply of 1%* are given in Table 4. We first recall that in the Small Industrial Countries, as in other countries participating in the Exchange Rate Mechanism (ERM) of the European Monetary System, there is no concept of money target. If we consider the second column of the Table 4 giving the effects of an increase of 1% of the money target in the Germany, we observe that money is neutral: the shock induces a rise in prices of 1%, especially the DM price of the US$, which is equal to the reciprocal of E. The variation of nominal Net Foreign Assets NFA, expressed in US$, deserves specific comments because the behavior of this variable reveals an important feature of the model around the given long-run equilibrium. It must be first observed that in the equation (12), the relation between the stock of NFA and the flow of the trade balance degenerates if *the nominal interest rate R is equal to the nominal growth rate gv* as it is in the reference path -the variable NFA disappears from this equation[1]. The economic interpretation is that on a steady state growth path the interest received at each period on NFA is exactly compensated by the yearly purchase of foreign assets. Then it is easy to see that, in that case, NFA does not belong any more to the simulateous block of the model, but to the epilogue and is logically determined by the wealth equation (8). As a consequence, the variation of NFA can take any value contrary to other nominal variables of the simultaneous block which increase by 1%.

By contrast we see in the first column of Table 4 that money appears as being not neutral in the US long-run model. We recall that the way the exchange rates are defined in MULTIMOD implies that, when simulating the US economy in isolation, the system works as if exchange rates were fixed. Indeed US prices increase by less than 1% because of the stabilizing effect of import prices. Hence the interest rate must decrease because of the LM mechanism, leading to an upward shift of the equilibrium level of capital and of the output supply.

[1] This mathematical singularity of the reference path raised serious numerical problems for the simulation of the country models as well as of the whole model.

Table 3: Permanent increase in government spending by 1% of GNP - per cent deviation

US model		German model		Small Industrial countries model	
US_C	-1.67	GR_C	-1.65	SI_C	-1.72
US_I	-1.18	GR_I	-0.82	SI_I	-0.85
US_IM	0.03	GR_IM	0.07	SI_IM	0.04
US_X	-0.25	GR_X	-0.18	SI_X	-0.13
US_Q	-0.36	GR_Q	-0.24	SI_Q	-0.24
US_R	0.86	GR_R	0.	SI_R	0.
US_P	0.36	GR_P	0.17	SI_P	0.20
US_PM	0.03	GR_PM	-0.08	SI_PM	0.01
US_NFA	-22.94	GR_NFA	-22.93	SI_NFA	36.55
US_E	0.	GR_E	0.09	SI_E	0.

Table 4: Permanent increase in Money Target by 1% - per cent deviation

US model		German model	
US_C	0.47	GR_C	0.
US_I	0.75	GR_I	0.
US_IM	1.09	GR_IM	0.
US_X	-0.42	GR_X	0.
US_Q	0.21	GR_Q	0.
US_R	-1.45	GR_R	0.
US_P	0.64	GR_P	1.00
US_NFA	47.54	GR_NFA	1.11
US_E	0.	GR_E	-0.91

Table 5: Permanent increase in Public Debt Target by 1% - per cent deviation

US model		German model		Small Industrial countries model	
US_Q	0.	GR_Q	0.	SI_Q	0.
US_R	0.	GR_R	0.	SI_R	0.
US_P	0.	GR_P	0.	SI_P	0.
US_NFA	0.21	GR_NFA	-0.12	SI_NFA	0.25
US_E	0.	GR_E	0.	SI_E	0.

Table 5 gives the effects of a *permanent increase in Public Debt Target BT by 1%*. The special feature of the reference equilibrium path (R = gv), noted above implies also here that the variable B disappears from the government balance sheet identity, equation (11). Furthermore, in the wealth equation (8), the rise in B is exactly compensated by a symmetrical drop in NFA. Indeed, the change of this last variable produces no retroaction on the simultaneous block of the system.

4.3- Multinational Interdependencies in the Long-Run Multinational Model

We present in this section the multipliers obtained with the long-run version of the whole MULTIMOD.

Table 6 summarizes the incidence on the main variables of the model of an increase in Public Spending by 1% of GDP in the US, Germany and Small Industrial Countries, respectively. We first observe that the own country effects are indeed rather close to those given in Table 3 without taking into account international interdependences. More precisely, the shocks depress slightly less their own economy. In the case of the US, the international transmission of the shock creates an upward shift in the US (hence world) interest rate, an appreciation of the US Dollar and a worldwide economic contraction. A Public Demand shock in the Small Industrial Countries induces contracting results qualitatively quite similar to those generated by the US. By contrast, results are reversed in the case of a shock in Germany which initiates a worldwide expansion through a drop in interest rates.

Concerning the role of public debt, we can deduce from the results obtained in the previous section, that a shock in public debt Target BT will have here no more effect than in the country models in isolation.

Table 6: Permanent Increase in Government Spending by 1% of GNP - Per Cent Deviation

	G_US	G_GR	G_SI		G_US	G_GR	G_SI
CA_C	-0.25	0.04	0.01	SI_C	-0.02	-0.01	**-1.73**
CA_IM	-0.34	0.07	0.01	SI_IM	-0.05	-0.03	**0.03**
CA_X	0.03	0.02	-0.01	SI_X	-0.03	0.00	**-0.14**
CA_Q	-0.05	0.03	-0.01	SI_Q	-0.02	0.02	**-0.25**
CA_R	0.15	-0.21	0.02	SI_R	0.15	-0.21	**0.07**
CA_P	-0.01	-0.05	-0.01	SI_P	0.03	-0.10	**0.20**
CA_E	-0.25	0.02	0.02	SI_E	-0.05	0.02	**-0.02**
FR_C	-0.02	-0.01	-0.05	UK_C	-0.03	-0.06	-0.09
FR_IM	-0.04	-0.03	-0.09	UK_IM	-0.03	-0.08	-0.10
FR_X	-0.02	0.03	-0.01	UK_X	-0.07	0.19	0.04
FR_Q	-0.02	0.02	-0.02	UK_Q	-0.04	0.03	-0.04
FR_R	0.15	-0.21	0.07	UK_R	0.15	-0.21	0.07
FR_P	0.02	-0.12	-0.02	UK_P	0.09	-0.34	-0.07
FR_E	-0.05	0.02	-0.02	UK_E	-0.05	0.02	-0.02
GR_C	-0.01	**-1.66**	-0.05	US_C	**-1.63**	0.03	-0.01
GR_IM	-0.03	**0.07**	-0.10	US_IM	**-0.04**	0.06	-0.03
GR_X	-0.04	**-0.15**	-0.04	US_X	**-0.24**	0.02	-0.01
GR_Q	-0.02	**-0.21**	-0.02	US_Q	**-0.28**	0.03	-0.01
GR_R	0.15	**-0.21**	0.07	US_R	**0.15**	-0.21	0.07
GR_P	0.03	**0.12**	0.01	US_P	**0.18**	-0.05	0.02
GR_E	-0.05	**0.02**	-0.02	US_E	**0.00**	0.00	0.00
IT_C	-0.05	-0.01	-0.07	DC_CTOT	-0.08	0.00	-0.05
IT_IM	-0.07	-0.02	-0.08	DC_IM	-0.14	0.04	-0.04
IT_X	-0.01	0.06	0.01	DC_X	-0.10	0.02	-0.02
IT_Q	-0.03	0.03	-0.02	DC_Q	-0.05	0.00	-0.02
IT_R	0.15	-0.21	0.07	DC_P	-0.02	-0.06	0.02
IT_P	-0.01	-0.14	-0.04				
IT_E	-0.05	0.02	-0.02				
JA_C	-0.05	-0.01	0.00	HO_X	-0.22	0.04	-0.06
JA_IM	-0.12	-0.02	0.00	HO_IM	-0.13	0.01	-0.04
JA_X	-0.03	0.08	-0.03	HO_Q	-0.08	0.01	-0.02
JA_Q	-0.03	0.03	-0.01				
JA_R	0.15	-0.21	0.07				
JA_P	0.03	-0.06	0.02	POIL	0.03	-0.07	0.03
JA_E	-0.13	-0.07	0.02	PCOM	-0.12	-0.08	-0.05

The last and most puzzling experiment concerns the role of money supply target. Indeed, as in the country models, there is no money supply target in the countries participating in the ERM. An increase in the money supply target in Germany has again no real effects, and produces a rise in the same proportion in prices and nominal variables in Germany, but implies now the same nominal evolutions in France, Italy, United Kingdom and the Small Industrial Countries whose exchange rates are supposed to be pegged to the German one. By contrast, it is impossible to find a solution to the system in the case of an increase of the money supply target in the US. However, a *decrease* of the same amount leads to the results displayed in Table 7. This result can be explained by the special feature of the long-run structure of MULTIMOD, not easy to exhibit: there is another equilibrium corresponding to the same exogenous data generating the baseline reference path given by the IMF, which is very close to the first. This equilibrium can be generated by making exogenous the US interest rate and by recomputing it through the US money demand equation (eq. 16). The following Chart 1 plots, for values of the nominal interest rate between 6.82% and 6.96%, the difference between the interest rate, computed from equation 16, and the exogenously "supplied" one. The bold line, which describes the baseline case, is almost tangential to the horizontal axis. Actually, it intersects this last twice for the very close values of interest rate of 6.88% and 6.90%. The money shock curve of the chart corresponds to a 1% permanent decrease of the US money target. The case of an increase is symmetrical and, indeed, does not allow for equilibrium.

Table 7: Permanent Decrease of Money Supply Target in the US by 1% - Per Cent Deviation

	CA	FR	GR	IT	JA	SI	UK	US
IM	-0.23	-0.14	-0.11	-0.16	0.02	-0.17	-0.02	-0.23
X	-0.06	-0.11	-0.15	-0.12	-0.30	-0.11	-0.41	-0.09
Q	-0.10	-0.10	-0.10	-0.13	-0.10	-0.09	-0.17	-0.09
P	0.15	0.12	0.16	0.13	0.19	0.15	0.55	-0.83
R	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71
E	-1.09	-0.92	-0.92	-0.92	-0.75	-0.92	-0.92	0.00

V - Conclusion

In the present paper, we have shown that the determination of the long-run of models is a workable framework for large scale macroeconometric models. The problem is not the complexity of this task but the fact that most macroeconometric models do not have well-specified long-run properties, even models with rational expectations which should possess correctly defined terminal conditions. Our results confirm that MULTIMOD is one of the rare large scale models to include a well-specified long-run. However the complexity of the long-run comes from two main peculiar features. On the one hand, the general interdependence nature of the (static) system gives no weight to the past and to the future as in the dynamic version. The other source of complexity lies in stock-flow relations which lead to highly sensitive equations in the long-run.

The most noticeable property of the steady-state structure of MULTIMOD is the existence of two close solutions. We do not understantd well the reasons for this peculiar feature. They need careful investigations which are left for further research.

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CHART I: LONG-RUN EQUILIBRIA OF MULTIMOD