MONEY AND WAGE CONTRACTS IN AN OPTIMIZING MODEL OF THE BUSINESS CYCLE

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A B S T R A C T

This article presents a fully computable business cycle model with optimizing agents in an economy with money and wage contracts. We start from the wellknown Long-Plosser-Mc Callum real business cycle model and extend it in two directions : First money is introduced, still maintaining the market clearing assumption. Secondly this monetary model is studied under the assumption of predetermined wages. An explicit solution is given in both cases.It appears that the combination of money and nonclearing markets allows to give a synthetic view between usual "real business cycles" results and traditional Keynesian ones. We apply it in particular to specify the cyclical properties of real wages and prices.

Keywords : Monetary business cycles, real business cycles.

Journal of Economic Literature Classification Numbers : E3.

UN MODELE DE CYCLE AVEC MONNAIE ET CONTRATS SALARIAUX

R E S U M E

On présente dans cet article un modèle dans la tradition des "cycles réels", en introduisant à la fois monnaie et contrats salariaux. On part du modèle "réel" de Long-Plosser-Mc Callum, et on le généralise dans deux directions : Tout d'abord en introduisant la monnaie avec des marchés en équilibre, puis en étudiant ce modèle monétaire sous l'hypothèse de salaires prédéterminés. Une solution exacte est donnée dans les deux cas. Il apparaît que la combinaison d'une économie monétaire et de marchés en déséquilibre permet de synthétiser les résultats habituels de la littérature sur les "cycles réels" et ceux de la littérature Keynésienne traditionnelle. On donne une application particulière aux propriétés cycliques des salaires réels et des prix.

Mots Clefs : Cycles monétaires, cycles réels.

Codes J.E.L. : E3.

1. INTRODUCTION [1]

The purpose of this paper is to construct a simple and fully computable business cycle model with optimizing agents in an economy with money and wage contracts.

For a while research in the domain concentrated on the line laid down in the initial contributions by Kydland-Prescott (1982) and Long-Plosser (1983) and sought to reproduce actual business cycles features as the response of optimizing agents to random shocks in a purely real economy and under Walrasian market clearing at all times. Clearly these initial restrictions were bound not to last, and money and non-clearing markets have been quite naturally included in that line of research. King-Plosser (1984) and Cooley-Hansen (1989) introduced money in market clearing models, while non-clearing markets in real economies were introduced by Danthine-Donaldson (1990, 1991, 1992). Money and non-clearing markets were then successfully integrated, and a number of researchers (Cho, 1990, Cho-Cooley, 1990, King, 1990, Cho-Phaneuf, 1992, Hairault-Portier 1992) have convincingly argued that the consideration of price, and especially wage rigidities in a monetary economy subject to real and monetary shocks allowed to substantially improve the capacity of these business cycle models to match a number of stylized facts in actual economies.

All the above contributions have been made in "calibrated" models as in Kydland-Prescott (1982). We shall follow here the complementary line of research laid down by Long-Plosser (1983) and consider a fully computable "benchmark" model which will make most obvious the economic mechanisms at work. We shall actually consider here a one sector version as in Mac Callum (1989) [2] and extend it in the two directions we indicated : We shall first introduce money into the model, still maintaining the market clearing

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[1] I want to thank Pierre-Yves Hénin and Franck Portier for their useful comments on a first version of this paper. I am of course solely responsible for errors and opinions.

[2] Other fully computable "real" models with market clearing and endogenous growth are found in Basu (1990), Hercowitz-Sampson (1991).

assumption for all markets. We shall then study the same monetary model under the assumption of predetermined wages. We shall finally apply the results to a few stylized facts on prices, real wages and inflation.

2. THE MODEL

The economy studied is a monetary economy with two markets in each period t : Goods for money at the price P_t, labor for money at the wage W_t. There are two representative agents : A firm and a household.

As in Long-Plosser (1983), Mc Callum (1989), the firm has a Cobb-Douglas technology :

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \tag{1}$$

where K_t is capital, N_t labor input and Z_t a stochastic technological shock. All profits of the firm are distributed to the household. Capital fully depreciates in one period so that :

$$K_{t+1} = I_t \tag{2}$$

where I_t is investment in period t.

The representative household has an endowment of labor $\bar{N}$ and maximizes the expected value of discounted future utilities with the following utility :

$$U = \Sigma\, \beta^t \left[\text{Log } C_t + \theta \text{ Log } \frac{M_t}{P_t} + V(\bar{N} - N_t) \right] \tag{3}$$

where V is a concave function [3]. The household's budget constraint in period t is :

$$C_t + \frac{M_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \kappa_t I_{t-1} + \frac{\mu_t M_{t-1}}{P_t} \tag{4}$$

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[3] This class of utility functions (without money) has been shown to have good properties for growth models in King-Plosser-Rebelo (1988).

where μ_t is a stochastic multiplicative money shock and κ_t the real return in period t on capital invested in t-1.

3. THE WALRASIAN REGIME

We shall now, as in traditional Real Business Cycle theory, assume that the two markets clear in each period, and see how the economy reacts to the shocks on technology and money, assuming that agents have rational expectations.

Solving the model

In each period the firm demands labor competitively so that the real wage is equal to the marginal productivity of labor :

$$\frac{W_t}{P_t} = \frac{\partial Y_t}{\partial N_t} = (1-\alpha)\ Z_t\ K_t^{\alpha}\ N_t^{-\alpha} \tag{5}$$

Also the real return on capital is simply the marginal productivity of capital :

$$\kappa_t = \frac{\partial Y_t}{\partial K_t} = \alpha\ Z_t\ K_t^{\alpha-1}\ N_t^{1-\alpha} \tag{6}$$

The household maximizes the expected value of his discounted utility (3) subject to the sequence of budget constraints (4). Call λ_t the marginal utility of real wealth in period t (i.e. the Lagrange multiplier associated with the corresponding budget constraint). Then the optimality conditions for the consumer's program yield :

$$\frac{1}{C_t} = \lambda_t \tag{7}$$

$$V'(\bar{N} - N_t) = \lambda_t \frac{W_t}{P_t} \tag{8}$$

$$\lambda_t = \beta\ E_t\{\lambda_{t+1}\ \kappa_{t+1}\} \tag{9}$$

$$\lambda_t = \frac{\theta P_t}{M_t} + \beta\, E_t \left\{ \lambda_{t+1} \frac{\mu_{t+1} P_t}{P_{t+1}} \right\} \tag{10}$$

Combining (7) , (9), the definition of κ_t (6) and the condition $Y_t = C_t + I_t$, we obtain :

$$\frac{I_t}{C_t} = \alpha\beta + \alpha\beta\, E_t\left(\frac{I_{t+1}}{C_{t+1}} \right)$$

which, together with the transversality condition, yields :

$$C_t = (1 - \alpha\beta)\, Y_t$$

$$I_t = K_{t+1} = \alpha\beta\, Y_t \tag{11}$$

Now condition (10), using (7) and the definition of μ_{t+1}, is rewritten :

$$\frac{M_t}{P_t C_t} = \theta + \beta\, E_t\left\{ \frac{M_{t+1}}{P_{t+1}\, C_{t+1}} \right\}$$

which gives us similarly the level of real money balances :

$$\frac{M_t}{P_t} = \frac{\theta(1-\alpha\beta)}{1-\beta}\, Y_t = \nu\, Y_t \tag{12}$$

Finally combining condition (8) with the expression of the real wage (5) and the value of consumption just found, we obtain that N_t is constant and equal to N, where N is given by :

$$N.V'(\bar{N} - N) = \frac{1 - \alpha}{1 - \alpha\beta} \tag{13}$$

We further assume that V' becomes large enough near zero so that the solution, unique because of the concavity of V, is interior. As an example if we consider the usual specification $V(\bar{N} - N_t) = \gamma\, \mathrm{Log}(\bar{N} - N_t)$, then :

$$N = \frac{(1 - \alpha)\ \bar{N}}{1 - \alpha + \gamma(1 - \alpha\beta)}$$

which is the traditional solution (Long-Plosser, 1983, Mc Callum, 1989).

Walrasian dynamics

We shall now briefly describe the dynamics of this Walrasian economy. Let us start with the "real" variables, whose evolution is summarized by the following equations :

$$N_t = N \tag{14}$$

$$Y_t = Z_t\ K_t^{\alpha}\ N^{1-\alpha} \tag{15}$$

$$\frac{W_t}{P_t} = (1-\alpha)\ Z_t\ K_t^{\alpha}\ N^{-\alpha} \tag{16}$$

$$K_{t+1} = \alpha\beta\ Y_t \tag{17}$$

A first immediate and striking remark is that, although the economy is perpetually subjected to monetary shocks, fluctuations in real variables are driven by real shocks *only.* In fact the dynamics of the real variables are exactly the same as in the model without money. We thus see that the introduction of money per se does not give necessarily a role for monetary shocks [4].

Putting together equations (15) and (17), we obtain the expression for output in terms of current and past technology shocks, in logarithms [5] :

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[4] Of course the total lack of effect of monetary shocks is due to our specific utility for money and is thus not a robust result. But models with money usually yield relatively small effects for these shocks under market clearing (See for example Hairault-Portier 1993).

[5] Here and in what follows, lower case letters denote the logarithm of the corresponding upper case variables.

$$y_t = \frac{z_t}{1-\alpha L} + n + \frac{\alpha \text{ Log } \alpha\beta}{1-\alpha} \tag{18}$$

where L is the "lag operator". We see that the propagation mechanism is exactly the same as in the "pure" real models, going through the accumulation of capital.

Finally we can also compute the nominal wage and price

$$w_t = m_t + \text{Log}(1-\alpha) - \text{Log } \nu - n \tag{19}$$

$$p_t = m_t - (1-\alpha L)^{-1} z_t - \text{Log } \nu - n - \frac{\alpha \text{ Log } \alpha\beta}{1-\alpha} \tag{20}$$

At this stage, and though we will not embark in any actual "calibration" exercise, we may note a few correlations which have been a source of puzzle for researchers working in the "real business cycles" area.

The first puzzle is that real wages are too procyclical in this model. In fact equations (15) and (16) show a coefficient of correlation between Y_t and W_t/P_t equal to one. Although this correlation is a little weaker in calibrated models where N_t varies, it is usually much higher than what is observed in reality.

The second puzzle concerns prices : Comparison of equations (18) and (20) shows that prices in this model are always countercyclical, whatever the relative size of technological and monetary shocks (we assume they are independent). Although the "dogma" of the procyclicality of prices has now been seriously put into question, it is now admitted that there are periods where prices have been countercyclical, but also some where they have been procyclical (See for example Cooley-Ohanian, 1991, Todd-Smith, 1992). Clearly this model cannot reproduce this variety of experiences in the cyclical behavior of prices.

We shall now consider wage contracts, and see that this will seriously help alleviating the above problems.

4. WAGE CONTRACTS

We shall now assume that, instead of being determined by Walrasian market clearing, the level of wages is predetermined at the beginning of each period and that at this contract wage the household supplies all labor demanded by the firm.

As for the level at which the contract wage if fixed, we shall assume that parties to the contract aim at clearing the market ex-ante (in logarithmic terms). To that effect the contract wage will be set equal to the expected value of the Walrasian wage, which, using formula (19), yields :

$$w_t = Em_t + \mathrm{Log}(1-\alpha) - \mathrm{Log}\ \nu - n \tag{21}$$

where Em_t denotes the expectation of m_t formed at the beginning of period t before shocks have occurred (Em_t will thus denote $E_{t-1}\ m_t$ for short in what follows).

Solving the model

Since the goods market clears and the firm's demand for labor is always satisfied, equations (5) and (6) concerning the firm still hold.

As for the household, it maximizes the utility function (3) subject to the budget constraints (4), but this time taking N_t as *given* by the firm's demand instead of choosing it. As it turns out, except for the fact that N_t is not chosen by the household, and thus equations (8) and (13) are not valid anymore, the rest of the resolution of the model goes through unchanged and in particular equations (11) and (12) still hold. Putting equations (1), (5), (11) and (12) in logarithmic form, we thus obtain the system :

$$y_t = z_t + \alpha\ k_t + (1-\alpha)\ n_t \tag{22}$$

$$w_t - p_t = \mathrm{Log}(1-\alpha) + z_t + \alpha k_t - \alpha n_t \tag{23}$$

$$m_t = \text{Log } \nu + p_t + y_t \tag{24}$$

$$k_{t+1} = \text{Log } \alpha\beta + y_t \tag{25}$$

Dynamics

Putting together equations (21) - (24), we first obtain the equations for employment and production in period t :

$$n_t = n + (m_t - Em_t) \tag{26}$$

$$y_t = (1-\alpha)\ n + \alpha\ k_t + z_t + (1-\alpha)(m_t - Em_t) \tag{27}$$

Contrarily to what happened in the Walrasian model, unexpected money shocks now have an impact on the level of employment and production.

Now using equation (25) and lagging appropriately, we obtain :

$$y_t = \frac{z_t + (1-\alpha)(m_t - Em_t)}{1 - \alpha L} + n + \frac{\alpha \text{ Log } \alpha\beta}{1 - \alpha} \tag{28}$$

In which we see that unexpected money shocks get propagated in time via the same mechanism as technology shocks, i.e. capital accumulation. We should note, however that, eventhough persistent technology shocks can lead to persistent output, persistence in money shocks will not lead to a similar persistence in output, because it is only the unexpected part which matters [6].

Now the real wage and price are deduced from y_t through the simple formulas :

$$w_t - p_t = \text{Log}(1-\alpha) + y_t - n - (m_t - Em_t) \tag{29}$$

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[6] Of course this conclusion would be totally modified if we introduced elements of endogenous growth, as even temporary money shocks would have permanent effects. See the appendix for a very simple illustration.

$$p_t = m_t - \text{Log } \nu - y_t \tag{30}$$

5. THE CYCLICAL BEHAVIOR OF REAL WAGES, PRICES AND INFLATION

We shall now use the above results to throw some light on the cyclical properties of a few variables in response to technological and monetary shocks.

Real wages and prices

We shall start here with real wages and prices, for which we saw that the Walrasian model yielded too extreme correlations. In order to get these correlations clearer, let us rewrite output, real wage and prices under the following form (suppressing all irrelevant constant terms) :

$$y_t = (1-\alpha)(m_t - Em_t) + \frac{\alpha(1-\alpha)(m_{t-1} - Em_{t-1})}{1 - \alpha L} + \frac{z_t}{1 - \alpha L} \tag{31}$$

$$w_t - p_t = - \alpha(m_t - Em_t) + \frac{\alpha(1-\alpha)(m_{t-1} - Em_{t-1})}{1 - \alpha L} + \frac{z_t}{1 - \alpha L} \tag{32}$$

$$p_t = Em_t + \alpha(m_t - Em_t) - \frac{\alpha(1-\alpha)(m_{t-1} - Em_{t-1})}{1 - \alpha L} - \frac{z_t}{1 - \alpha L} \tag{33}$$

We see that, although all supply shocks and *lagged* money shocks induce a positive correlation between real wage and output and a negative one between output and prices, *contemporary* money shocks induce inversely a negative correlation between real wage and output, and a positive correlation between price and output. Our model thus allows to mix these last features, which are characteristic of traditional Keynesian models, with the more standard results of real business cycles models.

In order to have a most simple example of potential correlations, let us take the (unrealistic) case where m_t and z_t are both trend stationary with the following characteristics [7] :

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[7] Correlations are easy to compute for more complex processes, but the formulas become a little clumsy.

$$z_t = \mu_z \cdot t + \varepsilon_{zt} \qquad \mathrm{Var}(\varepsilon_{zt}) = \sigma_z^2 \tag{34}$$

$$m_t = \mu_m \cdot t + \varepsilon_{mt} \qquad \mathrm{Var}(\varepsilon_{mt}) = \sigma_m^2 \tag{35}$$

In that case we obtain [8] :

$$\mathrm{CORR}(w_t - p_t, y_t) = \frac{\sigma_z^2 - \alpha(1-\alpha)^2\sigma_m^2}{[\sigma_z^2 + (1-\alpha)^2\,\sigma_m^2]^{1/2}[\sigma_z^2 + 2\alpha^2(1-\alpha)\sigma_m^2]^{1/2}} \tag{36}$$

$$\mathrm{CORR}(p_t, y_t) = \frac{\alpha(1-\alpha)^2\,\sigma_m^2 - \sigma_z^2}{[\sigma_z^2 + (1-\alpha)^2\,\sigma_m^2]^{1/2}[\sigma_z^2 + 2\alpha^2(1-\alpha)\sigma_m^2]^{1/2}} \tag{37}$$

Formula (36) shows us that the correlation between real wages and output is still one if there are *only* supply shocks. But this correlation diminishes as soon as monetary shocks are present, and could even possibly become negative. The relatively low actual correlations can thus be reproduced by adequate combination of technological and monetary shocks.

Formula (37) shows that we can obtain procyclical prices if demand shocks are prevalent, countercyclical prices if technology shocks are prevalent. The different behavior of prices over different historical subperiods may thus simply be due to the nature of shocks faced by the economies during these periods.

The inflation-output correlation

Another wellknown relation in macroeconomics (though less emphasized in this literature), is that between inflation and output, which are usually thought to be positively correlated in the Keynesian tradition. We can compute this correlation for various processes for monetary and technological shocks.

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[8] One may note that the two correlation coefficients are exactly opposite. This is due to the particular process for money (35), which makes the wage w_t fully deterministic. This peculiar relation does not hold with a more general money process.

For example if we assume the two trend-stationary processes above (equations 34 and 35), we find :

$$\mathrm{CORR}(\Delta p_t, y_t) = \frac{(1-\alpha)^{1/2}\ [\alpha(1-\alpha)^2\ \sigma_m^2 - \sigma_z^2]}{[\sigma_z^2 + (1-\alpha)^2\ \sigma_m^2]^{1/2}[2\alpha^2(3-\alpha)\ \sigma_m^2 + 2\sigma_z^2]^{1/2}} \tag{38}$$

Formula (38), and similar ones for different processes on money or technology, shows us that the positive inflation-output correlation is very much related to the presence of monetary shocks, and that this correlation can actually be reversed if there are sufficiently strong technological shocks.

6. CONCLUSIONS

We constructed in this paper a fully computable "benchmark" model of a business cycle with optimizing agents, adding both money and nonclearing markets to the traditional "real business cycle" framework. A few observations can be made.

First we saw that introducing money per se does not necessarily give a role to money. In fact the dynamics of the monetary model under Walrasian market clearing are completely similar, as far as the real variables are concerned, to those of the pure real model.

Secondly, the combination of money *and* nonclearing markets does change things very substantially. In particular non-forecasted money shocks do have a clear impact on employment and production, and the effect on production is transmitted through time via the accumulation of capital. These effects allow to give a balanced view between usual "real business cycle" results and traditional Keynesian ones, since in particular money shocks induce countercyclical real wages and procyclical prices, whereas technological shocks induce procyclical real wages and countercyclical prices. This synthetic blend between "classical" and "Keynesian" outcomes is obtained via fully computable solutions to a rigorous intertemporal optimization problem under stochastic shocks. This should certainly encourage us to pursue this line of research further.

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A P P E N D I X

We shall briefly investigate here the introduction of elements of endogenous growth into the model, as this will be seen to alter somewhat drastically our results on the possible persistence of the effects of monetary shocks. To that purpose we shall modify the firm's technology, which becomes :

$$Y_t = Z_t \, K_t^{\alpha} (H_t \, N_t)^{1-\alpha}$$

where H_t, a labour augmenting technical progress, will be taken as in Basu (1990) and Hercowitz-Sampson (1991) to be directly related to the level of accumulated capital :

$$H_t = K_t^{\chi} \tag{39}$$

The important thing is that, although equation (39) will hold ex-post, the level of H_t is perceived as exogenous by firms and investors, and notably independent of their investment policy. As a result of this independence assumption, equations (21), (24), (25) and (26) hold as in the previous model, whereas equations (22) and (23) now become :

$$y_t = z_t + (1-\alpha)\, n_t + (\alpha + \chi - \alpha\chi)\, k_t \tag{40}$$

$$w_t - p_t = \mathrm{Log}(1-\alpha) + z_t + (\alpha + \chi - \alpha\chi)\, k_t - \alpha n_t$$

Combining equations (25), (26) and (40) and lagging appropriately, we obtain :

$$[1 - (\alpha + \chi - \alpha\chi)L]\; y_t = z_t + (1-\alpha)(m_t - Em_t) + (1-\alpha)\, n$$
$$+ (\alpha + \chi - \alpha\chi)\, \mathrm{Log}\, \alpha\beta$$

In which we see that if $\chi = 1$, which corresponds to endogenous growth, we obtain :

$$y_t - y_{t-1} = z_t + (1-\alpha)(m_t - Em_t) + (1-\alpha)\, n + \mathrm{Log}\, \alpha\beta$$

in which case unanticipated money shocks will have permanent effects as well as technological shocks.