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Underemployment of Labour and Equipment in a Bargaining Model with Forward-looking Behavior*

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Abstract

This paper proposes a dynamic efficient bargaining model with a putty-clay technology and aggregation over heterogeneous micromarkets. We analyse the agents' optimal response to unexpected real changes in their economic environment. The model gives a useful description of the linkages between unemployment and the rate of utilization of productive equipment.

Key-words :
dynamic efficient bargaining model, aggregation , unemployment, rate of utilization of productive equipment.

Sous-emploi du travail et des équipements dans un modèle de négociations avec anticipations rationnelles

Résumé

L'objet de ce papier est de présenter un modèle dynamique de négociations efficaces avec une technologie de production putty-clay et agrégation de micro-marchés hétérogènes. On analyse les réponses optimales des agents à des changements réels non anticipés. Le modèle fournit une description utile des liens entre chômage et taux d'utilisation des capacités productives.

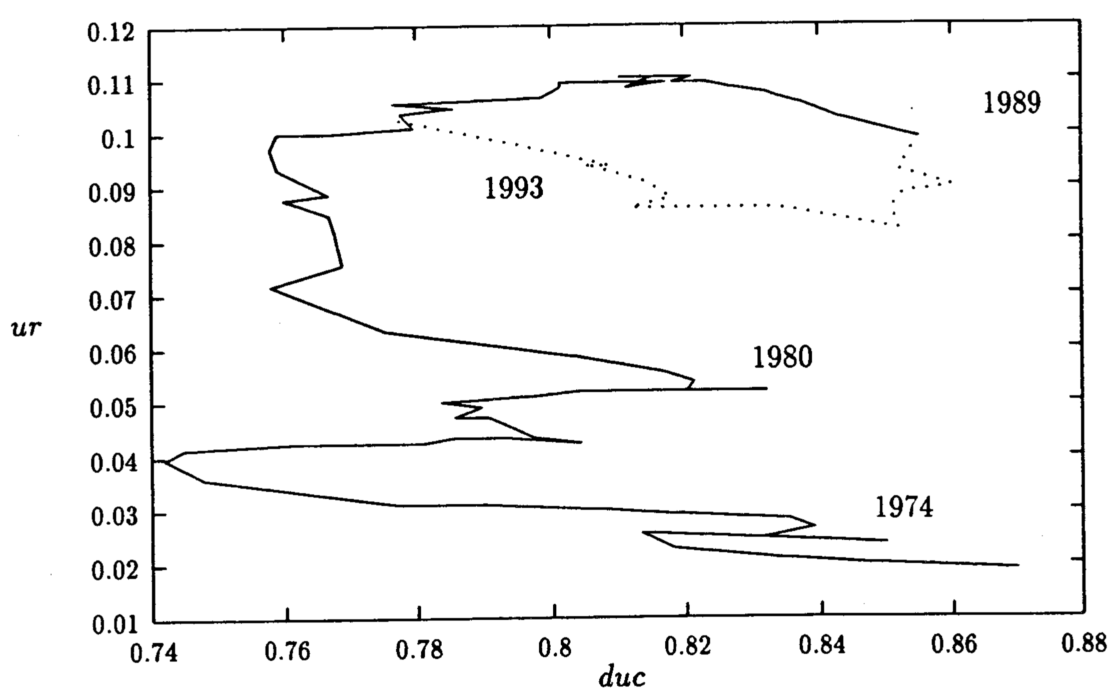
Mots clés :
modèle dynamique de négociations efficaces, agrégation, chômage, taux d'utilisation des capacités.

JEL classification number : D58, D92, E20.

Introduction

Beyond the persistent underemployment of existing capital and labour, Figure 1 outlines that the relation between the unemployment rate *ur* and the rate of utilization of productive capacities *duc* is non-monotonic: if a negative correlation between the two variables exists at the beginning of each business cycle, there seems to be no clear covariance in the long run. For instance, if in 1989 the rate of capital utilization is the same as in 1974, the unemployment is about six times higher. This may also suggest that there has been an increasing shortage of productive capacities with respect to full-employment output (see e.g. Bean (1989) and Burda (1986)). Accounting for the link between *ur* and *duc* is thus usefull to highlights the conditions under which an economy is able to create enough places of work.

Figure 1: Underutilization of Productive Factors in the EEC



The quantity rationing models (hereafter QRM) provide a natural framework for studying the behaviour of these two variables. Using an explicit aggregation over heterogeneous micromarkets, Sneessens (1987), Licandro (1992) and de la Croix (1993) show that imperfectly competitive wage and price settings combined with technological rigidities and market segmentation provide a convenient framework to study the underutilization of the productive factors. Following these previous works, our paper is interested in the joint dynamics of *ur* and *duc* in an intertemporal QRM model with imperfect competition. With respect to the dynamic bargaining models of Card (1986), Lockwood and Manning (1989) and Osano and Inoue (1991), the QRM framework stresses the role of productive capacities in determining employment.

As the optimal formation of prices and wages in the model is only responsible for real rigidities,[1] there is no Keynesian mechanism at work and the equilibrium unemployment can be assimilated to an equilibrium unemployment rate à la Layard et al. (1991) in an intertemporal context. Our explanations of the *ur* and *duc* dynamics do thus rely on real phenomena only.[2]

For this reason, our framework is not very far from the real business cycle methodology. Both try to explain macroeconomic stylized facts by the analysis of agents' optimal response to some external shocks. Our framework focusses on the explanation of the underutilisation of productive factors which cannot be analysed with current RBC models, even when they allow for non-competitive wage formation as in Danthine and Donaldson (1989). Another interesting part of the QRM model is its non linear structure. Contrary to what RBC models do, we solve a numerical version of our model without linearizing the trajectories towards the steady-state, using a technique proposed by Laffargue (1990) and Boucekkine (1993) for rational expectation non-linear models. However, our model has some weaknesses with respect to RBC. First we limit ourself to deterministic simulations of unexpected changes in exogenous variables instead of stochastic simulations. Secondly, the model is a partial equilibrium model of the union and of the firm with exogenous aggregate demand and interest rates. We therefore set aside any welfare analysis.

The paper is organised as follows: the model is presented in the first section which devotes an important part to describe the productive technology (putty-clay). After having derived the microeconomic first order conditions (section 2), we present the aggregate dynamic model (section 3) and analyse its steady state (section 4). In section 5, we propose numerical simulations describing the response of the model to unexpected changes in the environment analogous to those experienced by European economies during the last twenty years (oil shock, increased uncertainty, higher real interest rates). The induced dynamics of the endogenous variables gives a good qualitative insight on the way they could have react to these perturbations.

1 The model

• Technological choices and definition of productive capacities

Speaking of a "rate of capacity utilization" supposes to define an upper limit to the productive capacity of the firm. Of course, some short run technical rigidities are necessary to make this concept meaningfull. When installing such or such productive equipment, a firm invests simultaneously in capital goods and in some technology. If ex ante it can freely choose them among the available one, the decision of a couple capital stock - technology seems irreversible to a large extent: in most cases, modifying the technical coefficients of the productive factors will suppose new investments. In the litterature, two approaches deal with this idea of a putty-clay technology accounting for short-run tech-

[1] Introducing staggered contract in a dynamic wage bargaining framework as in Manning (1989) would open the door to nominal rigidities.

[2] This will alllow us to point out the ambiguity of some indicators given in business surveys. In particular, a higher proportion of firms reporting demand shortages do not reflect in itself a more Keynesian behavior of the economy. As far as this last point is concerned, a more detailed discussion on the respective role of nominal and real rigidities in a QRM model with aggregation can be found in Sneessens (1992).

nological rigidities. The most sophisticated one is undoutedly associated to the vintage models of capital. Unfortunately, this approach supposes to remember all the past of the investment process, which seems us prohibitive. The litterature on adjustment costs may provide another explanation for technical rigidities: if the firm bears some cost to modify its capital-labour ratio, it will only invest in technological change progressively[3] and the adjustment process will be spread optimally through time. As capital remains an homogeneous good, the overall investment process of the firm up to time t is perfectly summarized by the capital stock and the capital labour ratio at time t. However, a differentiable adjustment cost function is at best a reduced form of some unexplained microeconomic behavior.[4]

In this paper, we try to capture the basic idea of a capital vintage model without supporting its prohibitive cost. Fundamentally, a firm accumulates productive capacities instead of capital goods. At time t, when a firm h buys i_t^h units of capital goods, it uses them to increase its productive capacity by Δyc_t^h. Here we assume that these new installations are designed by combining i_t^h and potential labour $\bar{l}$ along a CES technology:

$$\Delta yc_t^h = \left[\, d\,(\bar{l}^h)^{(\sigma-1)/\sigma} \;+\; (1-d)\,(i_t^h)^{(\sigma-1)/\sigma}\,\right]^{\sigma/(\sigma-1)} \tag{1}$$

where σ is the elasticity of substitution between capital and labour and d is the parameter which weights the two factors.

On top of i_t^h, the firm h has thus to choose technical coefficients of i_t^h and $\bar{l}_t$:

$$\left\{\begin{array}{lll} b_t^{h*} & = \left[\dfrac{\Delta yc_t^h}{i_t^h}\right]^* & = \left[\, d\,(x_t^h)^{(\sigma-1)/\sigma} \;+\; (1-d)\,\right]^{\sigma/(\sigma-1)} \\[2ex] a_t^{h*} & = \left[\dfrac{\Delta yc_t^h}{\bar{l}_t^h}\right]^* & = \left[\, d \;+\; (1-d)\,(x_t^h)^{-(\sigma-1)/\sigma}\,\right]^{\sigma/(\sigma-1)} \end{array}\right. \tag{2}$$

where x_t^h is the labour-capital ratio $\bar{l}_t^h/i_t^h$. It is clear that $a_t^{h*} \;=\; b_t^{h*}\; x_t^h$.

These new production units are added to the old one yc_{t-1}^h net of depreciation

$$yc_t^h \;=\; yc_{t-1}^h\,(1-\delta) \;+\; \Delta yc_t^h \tag{3}$$

where δ is the average depreciation rate.

This accumulation equation is clearly an oversimplification with respect to what capital vintage models do. Here, we suppose that the firm will use all these productive capacities with the same intensity (i.e. at a mean rate of utilization) instead of using first the most profitable units.

Installing new capacities is supposed to take one period so that the contemporary investment Δyc_t^h becomes productive in $t+1$. The full capacity output at time t is then given by

$$yp_t^h \;= \tau\; yc_{t-1}^h\; \nu_{1t}^h \tag{4}$$

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[3]This supposes that the adjustment cost function is differentiable.

[4]Furthermore, it is not clear whether capital units could incorporate technical progresses that are posterior to their installation even if some adjustment cost is paid.

where τ is a productivity parameter which can be interpreted as the total factor productivity. ν_{1t}^h is a i.i.d random shock reflecting for instance the rate of technological breakdowns around a unitary mean. Hence

$$E_s(yp_t^h) = \tau \, yc_{t-1}^h$$

When acquiring Δyc_t^h production units, the firm creates a number of additional workplaces Δn_t^h given by

$$\Delta n_t^h = \frac{\Delta yc_t^h}{a_t^{h*}} = x_t^h \, i_t^h \tag{5}$$

The total number of workplaces in the firm is then

$$n_t^h = \frac{yc_t^h}{a_t^h}$$

This identity allows to compute the evolution of the average technical productivity of labour a_t^h:

$$\frac{yc_t^h}{a_t^h} = \frac{yc_{t-1}^h}{a_{t-1}^h}(1-\delta) + x_t^h \, i_t^h \tag{7}$$

If the labour market is segmented, the firm has an upper limit on its employment given by the number of workers in its segment of the labour market ls_t^h. It is then possible to define a level of output corresponding to the full employment of the available workforce:

$$ys_t^h = \tau \, a_{t-1}^h \, ls_t^h \, \nu_{2t}^h \tag{8}$$

where ν_{2t}^h is a i.i.d random shock of mean one representing the uncertainty on ls^h.[5] In expected terms,

$$E_s(ys_t^h) = a_{t-1}^h \, ls_t^h$$

- **Demand**

The utility function of the representative household is defined over a basket of imperfectly substitutable goods. The elasticity of substitution between the different goods, ϵ, is assumed constant and larger than one. The demand function for each good h at time t, yd_t^h, is a share of total real demand $\bar{C}_t/p_t$ depending upon the ratio of the good's price p_t^h to the general price index p_t and upon a good- specific random shock η_t^h, reflecting the fact that demand allocation among firms is not known with certainty.

$$yd_t^h = \left(\frac{p_t^h}{p_t}\right)^{-\epsilon} \frac{\bar{C}_t}{p_t} \eta_t^h, \qquad \epsilon > 1 \tag{9}$$

The random shock η_t^h is i.i.d. of mean 1 so that

$$E_s(yd_t^h) = \left(\frac{p_t^h}{p_t}\right)^{-\epsilon} \frac{\bar{C}_t}{p_t}$$

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[5]The relevant technical coefficient is a_{t-1}^h as the new workplaces in t will only be productive in $t+1$.

• Expected output

We suppose that each firm h announces its price p_t^h before knowing its demand and the technological shocks. The idea that prices are set are set before the complete revelation of demand is common to a lot of models, see e.g. Lucas and Woodford (1993). Given the technical rigidities (putty-clay production function, one period time to build, fixed labour supply and segmented labour market), the firm can be constrained after the realization of the shocks either by demand either by its productive capacity or by labour availability. Output is then the minimum of the three potential constraints:

$$y_t^h = \min(yd_t^h, ys_t^h, yp_t^h), \tag{10}$$

As there is no labour hoarding in the model, the effective employment level is decided after the realization of the shock and is given by

$$l_t^h = \frac{y_t^h}{\tau \ a_{t-1}^h} \tag{11}$$

If the shocks η_t^h, ν_{1t}^h and ν_{2t}^h are i.i.d. and lognormally distributed with a variance-covariance matrix with identical variances and identical covariances,[6] the expected output (before the realization of the shocks) can be written as a CES function of the three expected contraints (Lambert's (1988) theorem adapted by Sneessens (1983) to a three-constraint case):

$$E_s(y_t^h) = \left[\left(E_s(yd_t^h)\right)^{-\rho} + \left(E_s(ys_t^h)\right)^{-\rho} + \left(E_s(yp_t^h)\right)^{-\rho}\right]^{-1/\rho} \tag{12}$$

where ρ is a function of the variances and covariances of the shocks.

The interest of this formulation is twofold; first, the ex ante probability of each constraint yd, yp or ys is easily computed and equal to

$$\left\{\begin{array}{lcl} \phi_{Dt}^h & = & Prob(y_t^h = yd_t^h) = \left[\dfrac{E_s(y_t^h)}{E_s(yd_t^h)}\right]^{\rho+1} \\ \phi_{Pt}^h & = & Prob(y_t^h = yp_t^h) = \left[\dfrac{E_s(y_t^h)}{E_s(yp_t^h)}\right]^{\rho+1} \\ \phi_{Lt}^h & = & Prob(y_t^h = ys_t^h) = \left[\dfrac{E_s(y_t^h)}{E_s(ys_t^h)}\right]^{\rho+1} \end{array}\right. \tag{13}$$

The elasticities of expected output to each expected constraint (denoted π_{Dt}^h, π_{Pt}^h and

[6]This particular hypothesis on the variance-covariance matrix is made for analytical simplicity. More general cases are possibile but lead to less tractable functional forms for expected output. See e.g. Entorf et al. (1991).

π^h_{Lt}) are increasing function of these probabilities

$$\left\{\begin{array}{lcl} \pi^h_{Dt} & = & \eta_{E_s(y^h_t).E_t(yd^h_t)} = \left[\dfrac{E_s(y^h_t)}{E_s(yd^h_t)}\right]^\rho \\ \pi^h_{Pt} & = & \eta_{E_s(y^h_t).E_t(yp^h_t)} = \left[\dfrac{E_s(y^h_t)}{E_s(yp^h_t)}\right]^\rho \\ \pi^h_{Lt} & = & \eta_{E_s(y^h_t).E_t(ys^h_t)} = \left[\dfrac{E_s(y^h_t)}{E_s(ys^h_t)}\right]^\rho \end{array}\right. \tag{14}$$

with

$$\eta_{E_s(y^h_t).E_t(yd^h_t)} + \eta_{E_s(y^h_t).E_t(yp^h_t)} + \eta_{E_s(y^h_t).E_t(ys^h_t)} = 1$$

Secondly, if the number of firms is sufficiently large, Sneessens (1987) proposes an interesting way to link the microeconomic problem to the macroeconomic aggregates detailed in the following subsection (see also Arnsperger and de la Croix (1993)).

- **Aggregate output**

If the number of firms is large, aggregate output (resp. employment) is equal to each firm's expected output (resp. employment) times the number of firms. In that case, the ex post proportions of firms in each regime π_D, π_P and π_L are equal to the above ex ante individual elasticities and can be related to the two variables we are interested in, namely the unemployment rate, $ur_t = 1 - l_t/ls_t$ and the degree of capacity utilization $duc_t = y_t/yc_{t-1}$:

$$duc_t = (\pi_{Pt})^{1/\rho} \tag{15}$$

$$ur_t = 1 - (\pi_{Lt})^{1/\rho} = 1 - (1 - \pi_{D_t} - \pi_{P_t})^{1/\rho} \tag{16}$$

- **Unions**

Each union is firm-specific and composed of the ls^h_t households which supply their workforce to the firm h. As in Card (1986), the union's utility function is defined over employment l^h_t and over the difference between the real wage w^h_t/p_t and the disutility of work $\bar{u}_t$ (the alternative wage in Card's model):

$$U\left(l^h_t, \frac{w^h_t}{p_t}\right) = l^h_t \ \frac{1}{\nu} \ \left(\frac{w^h_t}{p_t} - \bar{u}_t\right)^\nu \tag{17}$$

The parameter ν can be interpreted as a relative preference for wages with respect to employment. It measures the concavity of the utility function with respect to the wage net of the disutility of work; in the presence of uncertainty, $1 - \nu$ is the coefficient of relative risk aversion of the workers, with $\nu = 1$ implying risk neutrality.

- **Efficient bargaining**

Efficient bargaining models in a static framework are often criticized as being time inconsistent. This time inconsistency is due to the fact that the firm has always an incentive to deviate from the contractual employment level and to return[7] on its labour demand curve on the wage has been fixed. When dynamic aspects are introduced, by considering for instance an infinitely repeated game, an opportunity exists for the agents to build a long-term relationship that may result in an efficient outcome. The conditions under which the efficient outcome emerges in a repeated bargaining game are derived in simple models by Espinoza and Rhee (1989) and Strand (1989). Basically, if the discount rates of the agents are high enough, the future consequences of any deviation from the contract (punishment or return to the non-cooperative outcome) have more weight than the instantaneous benefit from deviating from cooperation. Therefore, in that case, the cooperative outcome is time-consistent. Moreover, in the presence of a putty-clay technology, the rigidity of the technology restrains gains from deviating from cooperation, reinforcing the idea that efficient bargaining is the more appropriate framework for our set-up.

As prices and wages are set without knowing the realization of demand and productivity shocks, bargaining ex-ante efficiently imply that the firm and the union negotiate over wages and *expected* employment. At the firm level, expected employment is nothing else than the number of workplaces in the firm times their expected probability of utilization. In our setting, a negotiation over expected employment amounts to choose prices, investment and the technology. This decision is taken before the realization of the shocks, whereas output and effective employment are decided by the firm afterwards. This explains why output and employment are expressed in expected terms at the time of bargaining.

Given the imposed timing of the decisions, a cooperative agreement on p_t^h, w_t^h, x_t^h and i_t^h is the best solution for the agents. Note that, in the absence of unexpected changes in the exogenous variables, it is equivalent to negotiate at each point in time over p_t^h, w_t^h, x_t^h and i_t^h or to determine once for all the complete path of these variables. This is due to the fact that the idiosyncratic random shocks are i.i.d. and that the history of shocks does not affect future decisions.

The objective function is the weighted sum of all future utilities and profits:

$$\max_{w_t^h, p_t^h, i_t^h, x_t^h} \quad E_s \left[\beta \sum_{t=s}^{\infty} \theta_t \, U_t \left(l_t^h, \frac{w_t^h}{p_t} \right) + \sum_{t=s}^{\infty} \theta_t \, \frac{1}{p_t} \, \left(p_t^h y_t^h - w_t^h l_t^h - p_t^i i_t^h \right) \right]$$

such that for all t

$$yc_t^h = yc_{t-1}^h \, (1-\delta) + \Delta yc_t^h$$

$$\frac{yc_t^h}{a_t^h} = \frac{yc_{t-1}^h}{a_{t-1}^h} \, (1-\delta) + x_t^h \, i_t^h$$

$$i_t^h \geq 0$$

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[7] In the absence of a legal enforcement procedure.

and where l_t^h, i_t^h, y_t^h, yc_t^h and x_t^h have been defined before. The parameter β which weights the agents' objective function represents the union's relative power in the bargaining process.

θ_t is a discounting factor given by

$$\theta_t = \prod_{j=s}^{t} \left(\frac{1}{1+r_j} \right) \tag{18}$$

where r_j is the union's time-preference parameter and the firm's discount rate. Allowing r to differ across agents would not change the nature of our results.

Note finally that, from the terminological point of view, our bargaining framework is "efficient in expected terms" since there is no agreement on the effective employment level. In order to ensure non-negative profits, the firm has indeed to keep one degree of freedom in choosing optimally effective employment after the shocks.[8] However the ex ante decisions about prices and technology are more favourable to employment than what they would have been if the negociation had only born on wages.

Associating Lagrange's multipliers to the different constraints, one could recast the bargaining problem in the following way

$$\begin{aligned}
\max_{w_t^h, p_t^h, i_t^h, x_t^h} \quad \beta \sum_{t=s}^{\infty} \theta_t E_s \left(U_t \left(l_t^h, \frac{w_t^h}{p_t} \right) \right) \quad & + \quad \sum_{t=s}^{\infty} \theta_t \frac{1}{p_t} \left[\left(p_t^h - \frac{w_t^h}{\tau a_{t-1}^h} \right) E_s(y_t^h) - p_t^i i_t^h \right] \\
& + \quad \sum_{t=s}^{\infty} \theta_t \lambda_t \left[yc_t^h - yc_{t-1}^h (1-\delta) - b_t^{h*} i_t^h \right] \\
& + \quad \sum_{t=s}^{\infty} \theta_t \mu_t \left[\frac{yc_t^h}{a_t^h} - \frac{yc_{t-1}^h}{a_{t-1}^h} (1-\delta) - x_t^h i_t^h \right] \\
& + \quad \sum_{t=s}^{\infty} \theta_t \gamma_t i_t^h
\end{aligned}$$

2 First order conditions

- **Optimal pricing**

The first order condition for price is

$$p_t^h = \left[1 - \frac{1}{\epsilon \pi_{Dt}^h} \right]^{-1} \frac{\widetilde{w}_t}{\tau \, a_{t-1}^h} \tag{19}$$

where

$$\widetilde{w}_t = \left[w_t^h - \frac{\beta}{\nu} \left(\frac{w_t^h}{p_t} - \bar{u} \right)^{\nu} p_t \right] \tag{20}$$

The term $\widetilde{w}_t$ represents the marginal labour cost corrected for the effect of the efficient nature of the bargain: the price formation internalises the positive effect of employment on the union's utility level.

[8]That means that the effective rate of occupation of the existing workplaces is decided ex post.

According to this price equation, the price is a mark-up over the "social" marginal cost of labour. The mark-up rate depends positively on the firm market power and negatively on union power. Moreover, it decreases with the endogenous probability of facing a demand constraint.

Since, in (20), the gap between the social marginal cost of labour and the private cost w_t is a function of union power β, the mark-up rate of prices over money wages is a negative function of β.

- **Optimal real wage**

The first order equation for wages is:

$$\frac{w_t^h}{p_t} = (\beta)^{1/(1-\nu)} + \bar{u} \tag{21}$$

Real wages are equal to the disutility of work plus a positive term positively related to the union power. This equation is comparable to the usual result of bargaining models where the wage is a mark-up over the fall-back position of the union or over its "outside wage".

- **Investment and technological choices**

It is usefull to rewrite the real social marginal revenue per unit of output as:

$$MR_t^h = \frac{1}{p_t}\left(p_t^h - \frac{\widetilde{w}_t}{\tau\, a_{t-1}^h}\right)$$

The first order conditions for i_t^h is

$$\gamma_t = \frac{p_t^i}{p_t} + \mu_t^h\, b_t^{h*} + \lambda_t^h\, x_t \tag{22}$$

For x_t^h

$$\lambda_t^h + \frac{1}{d}\left[\frac{x_t^h}{b_t^{h*}}\right]^{\frac{1}{\sigma}} \mu_t^h = 0 \tag{23}$$

The Slatter's condition for the positivity of investment is

$$\gamma_t^h\, i_t^h = 0 \tag{24}$$

The optimality condition for yc_t^h is

$$(1+r_t)\left(\lambda_t^h + \frac{\mu_t^h}{a_t^h}\right) - (1-\delta)\left(\lambda_{t+1}^h + \frac{\mu_{t+1}^h}{a_t^h}\right) + \tau\, \phi_{Pt+1}^h\, MR_{t+1} = 0 \tag{25}$$

and for a_t^h

$$yc_{t-1}^h\, \left[(1+r_t)\,\mu_t^h - (1-\delta)\,\mu_{t+1}\right] = MR_{t+1}^h \phi_{Lt+1}^h\, \tau\, a_t^h\, ls_{t+1}^h + \frac{\widetilde{w}_{t+1}}{p_{t+1}} \frac{E_s(y_{t+1}^h)}{\tau\, a_t^h} \tag{26}$$

This set of optimality conditions on investment and technology is not really "reader-friendly". It is however possible to give it an economic interpretation if we rewrite these conditions as three equations on yc_t^h, x_t and μ_t^h (plus the Slatter's condition). The reader's understanding will not be altered substantially if he limits his reading to the intuitive comments proposed after each equation. These comments should give enough elements to understand the determinants of investment and technical choices. The more detailed analysis of the determination of the equilibrium at the steady state will give further insights on the working of the model.

The condition on yc_t^h can be rewritten as

$$\frac{(1+r_t)}{b_t^{h*}}\left(\frac{p_t^i}{p_t}-\gamma_t^h\right) = \frac{1-\delta}{b_{t+1}^{h*}}\left(\frac{p_{t+1}^i}{p_{t+1}}-\gamma_{t+1}^h\right) + (1+r_t)\mu_t^h\left(a_{t+1}^{h*}-a_t^{h*}\right)$$

$$+\,\tau MR_{t+1}^h\,\phi_{Pt+1}^h + \left\{MR_{t+1}^h\,\phi_{Lt+1}^h\,\tau\,a_t^h\,ls_{t+1}^h + \frac{\widetilde{w}_{t+1}}{p_{t+1}}\frac{E_s(y_{t+1}^h)}{\tau\,a_t^h}\right\}\frac{1}{yc_t^h}\left(1-\frac{a_t^h}{a_{t+1}^{h*}}\right)$$

This optimality condition on yc_t^h is to understand as follows. Let us suppose that investment is strictly positive. The terms in γ_t and γ_{t+1} vanish. As long as the firm doesn't change the nature of its technology, the technical coefficients remain constant and $a_{t+1}^* = a_t^* = a_t$. The condition on yc_t^h becomes simply :

$$(1+r_t)\frac{p_t^i}{p_t}\frac{1}{\tau\,b_t^{h*}} - \frac{p_{t+1}^i}{p_{t+1}}\frac{1-\delta}{\tau\,b_{t+1}^{h*}} = MR_{t+1}^h\,\phi_{Pt+1}^h$$

The left hand side represents the net cost in t of installing one unit of productive capacity (i.e. purchasing $1/(\tau\ b_t^*)$ capital goods at price p_t^i). Optimally, this net marginal cost must be equal to the expected marginal revenue generated by this capacity unit in $t+1$: this one corresponds to the extra social social marginal revenue MR times the probability of using this new capacity unit ϕ_P.

When the firm decides to modify the labour intensity of its production, investing in new capacity units is the only way to implement the required technical change. In this case, other terms come naturally in the optimality condition for yc_t^h.

First, when changing the technical productivity of labour, the firm modify the level of production corresponding to the full employment output. If a_t^h is increased (resp. decreased), this technical change will cause a gain (resp. a loss) in operating surplus if the employment constraint becomes binding. In expected terms, that means that this marginal variation in operating surplus has to be multiplied by the probability of a labour constraint ϕ_L. This justifies the presence of the following expression in the second line of the condition on yc_t^h :

$$MR_{t+1}^h\,\phi_{Lt+1}^h\,\frac{\tau\,ls_{t+1}^h\,a_t^h}{yc_t^h}$$

Secondly, at given output, the change in technical productivity of labour will affect the average wage cost of a workplace. If a_t is increased, the induced economy in wage bill is measured by

$$\frac{\widetilde{w}_{t+1}}{\tau\,a_t^h}\frac{E_s(y_{t+1}^h)}{yc_t^h}.$$

The optimality condition for the labour-capital ratio can be rewritten as:

$$\tau\,MR_{t+1}^h\left\{\phi_{Pt+1}^h - \phi_{Lt+1}^h\left(\frac{a_t^h}{da_{t+1}^{*1/\sigma}}-1\right)\frac{ls_{t+1}^h a_t^h}{yc_t^h}\right\}$$

$$-\frac{1}{p_{t+1}}\frac{\widetilde{w}_{t+1}}{\tau\ a_t^h}\frac{E_s(y_{t+1}^h)}{yc_t^h}\left(\frac{a_t^h}{da_{t+1}^{*1/\sigma}}-1\right)+\frac{(1+r_t)\mu_t^h}{d}\left(a_{t+1}^{*-1/\sigma}-a_t^{*-1/\sigma}\right)=0$$

The firm chooses its labour capital ratio in such a way that the expected marginal revenue of the increase in this ratio (line 1) is equal the extra wage bill at given output (line 2), due to the lower labour productivity. The expected marginal revenue is in fact a comparaison between a higher operating surplus due to a higher productivity of capital goods in the case ϕ_P of a capacity constraint and a lower one due to a lower productivity of labour in the case ϕ_L of a labour constraint.

Finally, using (22), (23) and (26), we have the law of motion of μ_t^h:[9]

$$\frac{yc_t^h}{a_t^h}\left((1+r_t)\ \mu_t^h\ -\ (1-\delta)\ \mu_{t+1}^h\right)=MR_{t+1}^h\ \phi_{Lt+1}^h\tau\ a_t^h ls_{t+1}^h+\frac{\widetilde{w}_{t+1}}{p_{t+1}}\ \frac{E_s(y_{t+1}^h)}{\tau\ a_t^h}$$

3 Aggregation

Using the properties of a symmetric Nash equilibrium between the different "union-firm" couples, the aggregation is particularly simple: as all agents are in the same situation at the time of bargaining, they choose the same wages, prices and the same technology. Consequently, the aggregate price level is

$$p_t=p_t^h=p_t^i$$

Furthermore, if the number of firms is large, the aggregate output is equal to the expected output of each firm times the number of firms. Normalizing this number to 1, we write aggregate output as :

$$y_t=\left[(yd_t)^{-\rho}+(\tau\ yc_{t-1})^{-\rho}+(\tau\ a_{t-1}\ ls_t)^{-\rho}\right]^{-1/\rho}$$

$$\text{with}\quad yd_t=\frac{\bar{C}_t}{p_t}$$

Removing the superscript h in marginal conditions given earlier, we may then rewrite the whole model in terms of aggregate variables. As there is no aggregate uncertainty, the expectation operator can be removed.

Note furthermore that the social operating surplus per unit of output can be simply recasted: indeed, using the equation of the optimal price, we have that

$$MR_t=1\ -\ \frac{\widetilde{w}_t/p_t}{\tau\ a_{t-1}}\ =\ \frac{1}{\epsilon\ \pi_{D_t}}.$$

[9]Denoting the number of workplaces yc_t^h/a_t^h at t by n_t^h and substituting recursively μ_{t+i}^h by its value in terms of variables of the subsequent period $t+i+1$, μ_t^h can be expressed as

$$\mu_t^h=\sum_{j=t+1}^{\infty}\frac{(1-\delta)^{j-t-1}}{\Pi_{s=t+1}^{\infty}(1+r_s)}\left[MR_j^h\ \pi_{Lj}^h\ +\ \frac{\widetilde{w}_j/p_j}{\tau\ a_{j-1}^h}\right]E_s(y_j^h)/n_j^h$$

μ_t is thus a measure of the discounted sum per workplace of the effect of an increase in the productivity of labour in t on expected future revenues.

The aggregate dynamic model is then given by:

A) Technical coefficients and accumulation equations

$$\left\{\begin{array}{rcl} b_t^* & = & \left[\, d(x_t)^{(\sigma-1)/\sigma} \;+\; (1-d)\,\right]^{\sigma/(\sigma-1)} \\ a_t^* & = & b_t^*\, x_t \\ yc_t & = & yc_{t-1}\,(1-\delta) \;+\; b_t^*\, i_t \\ yc_t/a_t & = & (yc_{t-1}/a_{t-1})\,(1-\delta) \;+\; x_t\, i_t \end{array}\right.$$

B) Aggregate demand and output and related business cycle indicators

$$\left\{\begin{array}{rcl} yd_t & = & \dfrac{\bar{C}_t}{p_t} \\ y_t & = & \left[(yd_t)^{-\rho} + (\tau\, yc_{t-1})^{-\rho} + (\tau\, a_{t-1}\, ls_t)^{-\rho}\right]^{-1/\rho} \\ duc_t & = & \dfrac{y_t}{\tau\, yc_{t-1}} \\ \pi_{Dt} & = & \left(\dfrac{y_t}{yd_t}\right)^{\rho} \end{array}\right.$$

C) Price and wage formation

$$\left\{\begin{array}{rcl} 1-\dfrac{1}{\epsilon\pi_{Dt}} & = & \dfrac{w_t/p_t}{\tau\, a_t} - \dfrac{\beta}{\nu}\,\dfrac{1}{\tau\, a_t}\left(\dfrac{w_t}{p_t} - \bar{u}\right)^{\nu} \\ \dfrac{w_t}{p_t} & = & \beta^{1/(1-\nu)} + \bar{u} \end{array}\right.$$

D) FOC's on capacities, investment and labour-capital ratio

$$\left\{\begin{array}{rcl} (1+r_{t+1})\dfrac{1-\gamma_t}{\tau\, b_t^*} & = & (1-\delta)\dfrac{1-\gamma_{t+1}}{\tau\, b_{t+1}^*} + (1+r_{t+1})\,\mu_t\,\dfrac{a_{t+1}^* - a_t^*}{\tau} + \dfrac{(duc_{t+1})^{\rho+1}}{\epsilon\pi_{Dt+1}} \\ & & +\left(1-\dfrac{1}{\epsilon}-\dfrac{duc_{t+1}^{\rho}}{\epsilon\pi_{Dt+1}}\right) duc_{t+1}\left(1-\dfrac{a_t}{a_{t+1}^*}\right) \\ \gamma_t\, i_t & = & 0 \\ \dfrac{duc_{t+1}^{\rho+1}}{\epsilon\,\pi_{Dt+1}} & = & (1+r_{t+1})\,\dfrac{\mu_t}{\tau\, d}\left({a^*_t}^{-1/\sigma} - {a^*_{t+1}}^{-1/\sigma}\right) \\ & & -\left(1-\dfrac{1}{\epsilon}-\dfrac{duc_{t+1}^{\rho}}{\epsilon\pi_{Dt+1}}\right) duc_{t+1}\left(1-\dfrac{a_t}{d\,{a^*_{t+1}}^{1/\sigma}}\right) \\ (1+r_{t+1})\,\mu_t\dfrac{yc_t}{a_t} & = & \mu_{t+1}\,(1-\delta)\,\dfrac{yc_t}{a_t} + \dfrac{\tau\, a_t\, ls_{t+1}}{\epsilon\,\pi_{Dt+1}}\left[\dfrac{y_{t+1}}{ys_{t+1}}\right]^{\rho+1} + \left(1-\dfrac{1}{\epsilon\,\pi_{Dt+1}}\right) y_{t+1} \end{array}\right.$$

4 The steady state

The steady state is characterized by the absence of both change in technical coefficients and net investment ($\gamma = 0$).

$$b^* = \left[\, d\, x^{(\sigma-1)/\sigma} \;+\; (1-d)\,\right]^{\sigma/(\sigma-1)} \text{ and } a^* = \; b^* \,/x$$

In particular the effective productivity of labour is equal to its optimal value

$$a_t \;=\; a_t^* \;=\; a^*$$

The long run level of productive capacities determines the stationary value of investment

$$i \;=\; \delta\, \frac{yc}{b^*}$$

Without the short run dynamics due to the accumulation and the technological change, the aggregate model becomes

$$\left\{\begin{array}{rcl}
b^* & = & \left[\, d\, x^{(\sigma-1)/\sigma} \;+\; (1-d)\,\right]^{\sigma/(\sigma-1)} \\
a^* & = & b^*\, x \\
yd & = & \dfrac{\bar{C}}{p} \\
y & = & \left[\, (yd)^{-\rho} \;+\; (\tau\, yc)^{-\rho} \;+\; (\tau\, ls\, a^*)^{-\rho}\,\right]^{-1/\rho} \\
\pi_D & = & \left(\dfrac{y}{yd}\right)^{\rho} \\
duc & = & \dfrac{y}{\tau\, yc} \\
1 - \dfrac{1}{\epsilon \pi_D} & = & \dfrac{w}{p\, \tau\, a^*} \;-\; \dfrac{\beta}{\nu}\, \dfrac{1}{\tau\, a^*} \left(\dfrac{w}{p} - \bar{u}\right)^{\nu} \\
\dfrac{w}{p} & = & \beta^{1/(1-\nu)} \;+\; \bar{u} \\
r \;+\; \delta & = & duc^{\rho+1}\, \dfrac{b^*\, \tau}{\epsilon\, \pi_D} \\
0 & = & \dfrac{duc^{\rho}}{\epsilon\, \pi_D} \;-\; \left[\dfrac{1}{\epsilon \pi_D}\, \left[\dfrac{y}{\tau\, a^*\, ls}\right]^{\rho} - \left(\dfrac{1}{\epsilon \pi_D} - 1\right)\right] \dfrac{1-d}{d}\, (x)^{-(\sigma-1)/\sigma} \\
i & = & \delta\, yc/b^*
\end{array}\right.$$

The first six equations are definitions. The following two describe the price-wage block of the model. The next equation concerns productive capacities and can be recasted as

$$\frac{r \;+\; \delta}{duc} \;=\; \frac{b^*\, \tau}{\epsilon\, \pi_D}\, duc^{\rho}$$

On the left hand side, $r + \delta$ is the real user cost of productive equipment. As each unit of capital is used at a rate *duc*, the true user cost of one unit of capital is $(r + \delta)/duc$. This marginal cost must be equal to the expected marginal revenue given by the physical productivity of equipment $\tau\, b^*$ times the product of the social operating surplus per unit of output $1/(\epsilon\pi_d)$ and the elasticity of output to capacities duc^ρ.

The following equation is the long run optimality condition on the labour capital ratio x. Let us rewrite it in the following way :

$$\frac{1}{\epsilon\, \pi_D}\left(duc^\rho - \left[\frac{y}{\tau\, a^*\, ls}\right]^\rho \frac{1-d}{d}\, x^{(1-\sigma)/\sigma}\right) = \left(1 - \frac{1}{\epsilon\, \pi_D}\right) \frac{1-d}{d}\, x^{(1-\sigma)/\sigma}$$

When the labour-capital ratio is increased, the productivity of capital (resp. labour) rises (resp. diminishes). The net effect on output depends on the size of the elasticity of output to capacities (duc^ρ) with respect to the elasticity of output to the labour constraint $((y/(\tau\, a\, ls))^\rho)$. The difference between the two which is measured for a unitary increase in capacities is multiplied by the social operating surplus $1/(\epsilon\pi_D)$. Furthermore, the lower labour productivity raises the unit labour cost $(1 - (\epsilon\pi_D)^{-1})$. At the margin, the sum of the two terms must be equal to zero.

The determination of the steady state equilibrium obeys to the following sequence: The equations of prices and wages determines the proportion π_D which is equal to

$$\pi_D = \frac{1}{\epsilon}\left[1 - \frac{1}{\tau\, a^*}\left(\bar{u} + \beta^{1/(1-\nu)}\left(1 - \frac{1}{\nu}\right)\right)\right]^{-1}$$

Note that the relative importance of this demand constraint is not linked to a Keynesian phenomenon. Here, firms and unions voluntary choose the possibility of a demand constraint: given the prevailing uncertainty, their pricing policy gives them an optimal probability ϕ_D to face insufficient demand. Thus, a proportion of firms π_D reporting such a shortage is simply the macroeconomic implication of these microeconomic choices. This does not mean at all that an anticipated boom in nominal demand would produce a rise in ouput. Rather, firms would change their prices in order to keep the same optimal π_D.

This value of π_D allows to write the long run first order conditions for the productive capacities and the technological choices as a system of two nonlinear equations with two unknowns *duc* and x (b^* and a^* are monotonic functions of x):

$$\begin{cases} duc = \left(\dfrac{\epsilon\, \pi_D\, (r+\delta)}{\tau\, b^*}\right)^{1/(\rho+1)} \\[2ex] x = \left(\dfrac{1-d}{d}\left[\dfrac{(\epsilon-1)\pi_D}{duc^\rho} - 1\right]\right)^{\sigma/(\sigma-1)} \end{cases}$$

This system determines the equilibrium values of *duc* and x and consequently π_D. Subsequent substitutions in the other equations allows to find the values of the remaining variables.

As mentionned in the first section of the paper, *ur* is fully determined by equilibrium π_D and *duc* (i.e. π_P as $\pi_P = duc^\rho$) since

$$ur = 1 - (\,1 - \pi_D - duc^\rho\,)^{1/\rho}$$

It is not possible to obtain closed form solutions for duc, x (and thus ur). The most likely sign of some partial derivatives of these variables with respect to parameters of the model can however be precised after further substitutions. The following table summarizes the sign of the derivatives of duc, x and ur with respect to the most important parameters:

	duc	x	π_D	ur
interest rate r	+	+	+	+
competition ϵ	0	0	+	−
productivity τ	−	0	−	−
micro uncertainty ρ	−	slightly +	$= 0$	+
wage claim $\bar{u}$	+	−	probably +	+
union power β	+	probably −	probably −	probably −

5 Dynamic Analysis

In spite of the smoothness of the equations of the model due to the aggregation over different "minimum" conditions, the above QRM model remains non linear enough to wish to avoid a linearization. For this reason we solve it numerically and verify the existence of a saddlepoint trajectory for the specific calibration that is presented in the next subsection. The resolution relies on a Newton-Raphson relaxation method which has been proposed by Laffargue (1990) for solving dynamic nonlinear models with rational expectations. The general problem is to solve a system of finite difference equations with initial and terminal conditions. Approximating the infinite horizon by a finite one,[10] the complete system has got as many equations as the number of equations at each period times the simulation horizon plus the initial and terminal conditions. This system is then solved using a Newton-Raphson algorithm including a specific triangulation[11] devoted to compute the Newton-Raphson improvements at each linearization.

Of course as any rational expectations model, our model may have an infinity of solutions. It is generally accepted that this aspect is not crucial if only one of these solutions is stable. The preliminary step of the resolution is then to determine whether the model admits either a saddlepoint path or an infinite number of stable solutions. Traditionally, this is done by computing the eigenvalues of the linearized model and checking whether they verify the Blanchard-Kahn (1980) conditions for the existence of a saddlepoint. As Boucekkine (1993) shows, this rigourous but cumbersome verification is unnecessary when Laffargue's method is used. This algorithm is indeed characterized by an explosivity property when the model has got an infinity of stable solutions (in fact, this property can be generalized to any convergent relaxation method, see Boucekkine-Le Van (1993)). The explosive behaviour can be revealed by a very simple numerical procedure

[10]That means that the transversality conditions on anticipated variables are replaced by the steady-state values of these variables at the end of the horizon of simulation.

[11]As the equations of the model at the different periods t are concatenated horizontally, the matrix of the complete system has a block recursive form: indeed, the first non zero elements in the rows representing the equations of the model at t are to the right of those in the rows describing the system at time $t-1$ (the gap between the first non zero columns at $t-1$ and the one at t is in fact given by the number of endogeneous variables in the model). This interesting block structure is of course taken into account for implementing the triangulation procedure.

relying on the initialization of the relaxation.[12] This procedure has allowed us to conclude unambiguously that a saddlepoint path exists.

Calibration

The calibration of the model has been done using information from the available econometric models in order to replicate the magnitude of Belgian aggregate data. The elasticity of substitution between capital and labour σ has been put to 0.4 which is what is find in general when one estimates a CES function for Belgium or France. The coefficient of labour in the CES function d has been put to 0.7 which should be equal to the labour share in added-value when $\sigma \to 1$. The depreciation rate δ linked to the capital stock series that we use is 0.04. The real long-term interest rate r on an annual basis is 0.08. Labour supply ls is 0.034 (3.4 millions). The global productivity of factors has been computed in order the retrieve the magnitude of Belgian GDP given the other parameters ($\tau = 7.75$).

The mismatch parameter ρ is estimated near 20 both by the Belgian Planning Bureau and by the estimates of presented in Drèze and Bean (1990). It is more arbitrary to choose the elasticity of substitution between goods ϵ since we do not have reliable econometric estimates of it. Moreover, different calibrations have shown us that the model responses are particularly sensitive to the value of ϵ. We have chosen 5, which means a mark-up rate of about 40% and a proportion of demand-constrained firms around 60%. This last proportion is consistent with the survey of the National Bank of Belgium.

Concerning the parameters related to the union we have still quite less information. A reasonable value for ν seems to be 0.5. This value is consistent with the estimation of the Euler equations of a dynamic contracting model for wages and employment by de la Croix, Palm and Pfann (1993). The value of the parameter related to the bargaining power of the union β is not interpretable since it includes also a normalisation factor of the utility function of the union. β has been set equal to 0.5. Finally, given ν and β the disutility of work $\bar{u}$ has been chosen to replicate the magnitude of real wage with respect to labour productivity. The induced long run values for endogeneous variables are: $\pi_d = 0.62$, $duc = 0.91$, $x = 0.285$, $y = 0.298$, $i = 0.0048$, $ur = 0.07$.

Simulation exercices

We propose hereafter 4 simulation exercices. 3 out of them concern unexpected changes in parameters of the model that are supposed to have occured in European economies over these last twenty years, i.e.

- a negative change in the global productivity parameter τ which can be interpreted as an oil shock.[13]

[12]That procedure consists in initializing the relaxation with values of the variables that slightly differ from the steady state one. If the model admits an infinity of stable solutions, the explosive behaviour appears at the first Newton-Raphson improvement.

[13]This comparaison between a negative schock on the productivity of the factors (in terms of added value) and an oil shock is meaningfull if for instance energy is a complementary factor to capital and labour in the production function. This sufficient (but not necessary) assumption seems realistic.

- A decrease in ρ which expresses a larger uncertainty at the microeconomic level and an increase in the mismatch between aggregate demand and supply. In empirical works, this lower ρ appears as an aftermath of the first oil shock (see Drèze and Bean (1990)).
- an increase in the real interest rate reflecting the tight monetary policies of the eighties.

The fourth exercice concerns the appearance of a new technology augmenting the productivity of labour. Given the putty-clay nature of the production function, it can only be incorporated in the productive equipment through new investments.

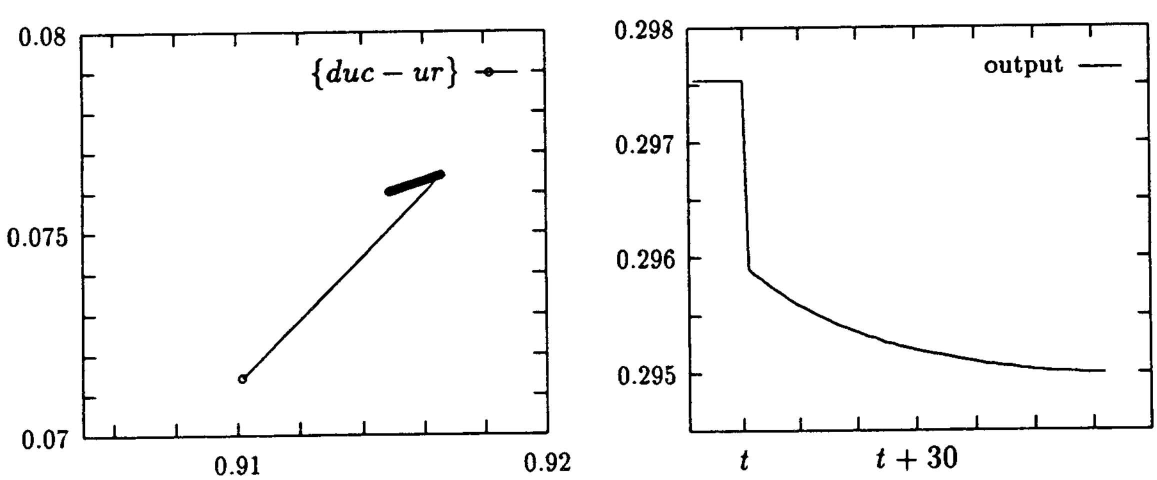
All these changes unanticipated but expected to be permanent once occured.

It is usefull to define the notion of capital gap cg. It is the unemployment rate that would prevail if all the existing productive capacities were fully utilized. Thus,

$$cg_t = 1 - \frac{yc_{t-1}}{a_{t-1}\, ls_t}$$

It measures the spread between the number of existing workplaces and the number necessary to reach full employment. With the above calibration, there is initially no positive capital gap.

Figure 2: Increase in Real Interest Rates

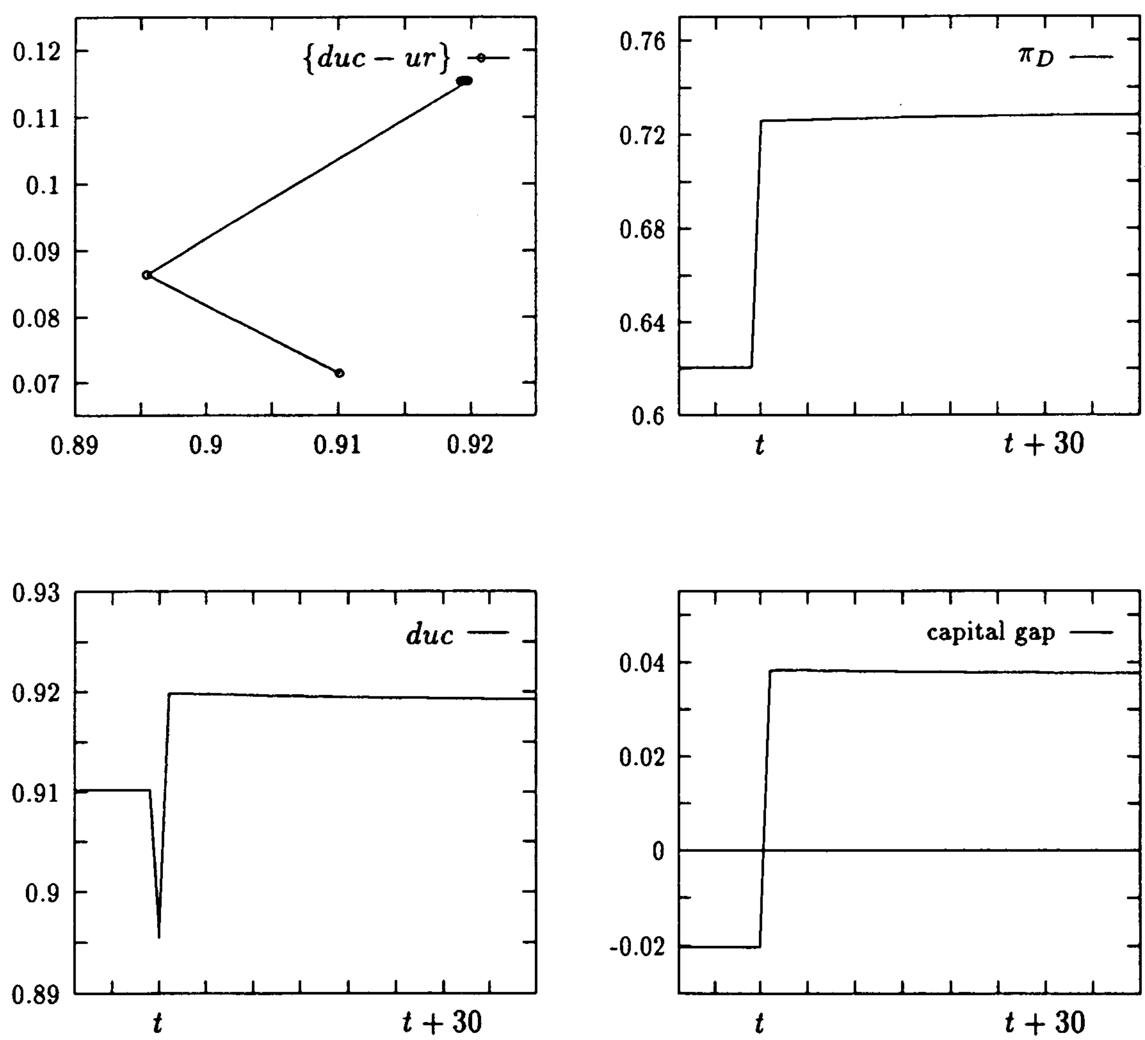


- **Increase in real interest rate**

The annual interest rate is raised from 0.08 to 0.10. Simulation results are presented in Figure 2. The main effect is linked to the consecutive loss of profitability for firms: the higher cost of capital reduces the optimal level of productive capacities (note that this leads to a higher price level as the optimal level of output for firms is lower at a given (exogenous) nominal demand). This effect on productive equipments is strengthened by the fact that the optimal rate of capacity utilization has risen too. Indeed, the firms revise their trade-off between their interest to accumulate excessive capacities because of the uncertainty on demand and the cost of this investment. All this means destructions of workplaces and thus a higher equilibrium unemployment rate. The positive effect of the capital-labour substitution on employment is much too weak to overcome the negative impact of the smaller profitability. Thus higher real interest rates mean higher unemployment and higher degree of capacity utilization.

Note that the magnitude of the rise in unemployment is small in this simulation. This is of course very dependent on the choosen calibration. For instance, a lower ϵ will lead to a much larger effect of interest rates on the endogenous variables. A larger σ increases the substitution effect, making the net effect on unemployment smaller but still positive.

Figure 3: Negative Productivity Change



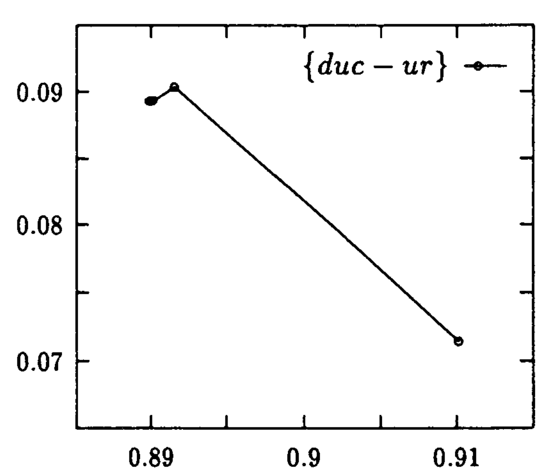
• Negative productivity change

We simulate here an unexpected change in the total factor productivity τ from 7.75 to 7.25.[14] Results are presented in Figure 3. This unexpected change causes a rise in marginal unit labour cost in response to which firms rise their prices. This leads to a fall in real demand explaining the drop in *duc* during the first period following the shock. Simultaneously, π_D rises explaining a lower mark-up over marginal costs. This rise in π_D implies also an increase in the labour share in added-value. During the following periods, the fall in productivity of factors and the loss in unitary operating surplus depress investment. This destruction of productive capacities explain the concomitant rise in *ur* and *duc* (and prices). The reason why the steady value of *duc* is higher than before the shock is of the same type as the one invoked for the previous simulation.

On top of these effects on *ur* and *duc*, it is worth noting that this productivity shock causes a set of stylized facts observed after the oil shocks :

1. A higher proportion of firms reporting to face demand shortages (without implying by itself a Keynesian problem);
2. Higher prices but lower mark-up;
3. The appearance of a capital gap.

Figure 4: Increase in Uncertainty



• Increased uncertainty and mismatch

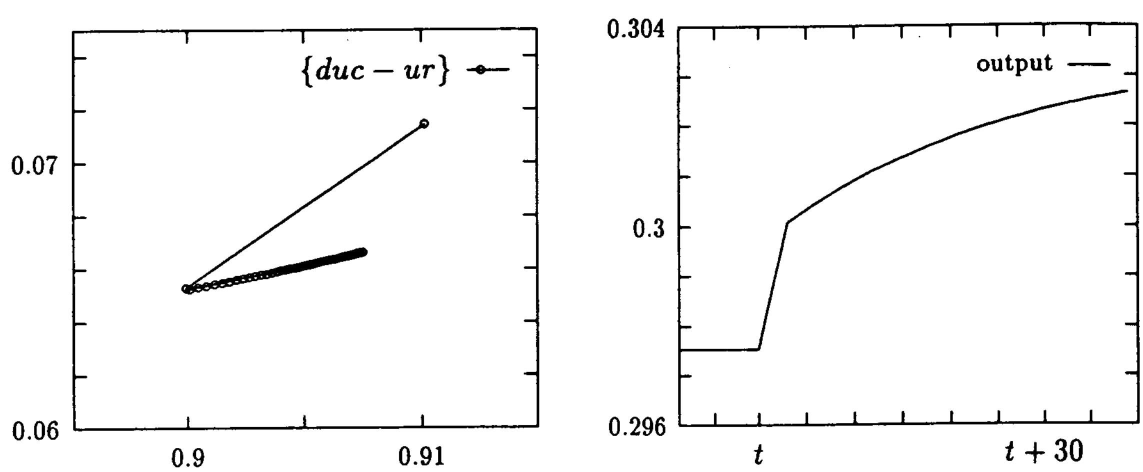
One of the main interest of the QRM framework is to link mismatch unemployment to microeconomic uncertainty and labour market segmentation. The main consequence of

[14]This change is close to a permanent change in the deterministic part of the Solow residual, this residual being the main driving force of a large class of real business cycle models.

a larger uncertainty is a lower expected output at given productive capacity. Firms are thus led to reduce the optimal level of their equipment. The macroeconomic impact of these micro-perturbations is a rise in ur. This increase in ur can be interpreted as a rise in structural unemployment. In the space unemployment-vacancies, it can be shown the corresponding Beverdige curve shifts outward. In our case, this shift is directly linked to an optimal response to increasing uncertainty.

The increased uncertainty (or mismatch) reduces the optimal duc for firms. The small effect of capital labour substitution which appears in Figure 4 is linked to the fall in the optimal rate of capacity utilisation as it raises the cost of equipment (one unit of capital good costs $p^i(r+\delta)$ per period. As this unit is used at a rate duc, its cost is multiplied by one over duc).

Figure 5: New Technology



• **New technology**

Finally, we simulate here a technological change augmenting the productivity of labour. Equation (1) should be rewritten as

$$\Delta yc_t^h = \left[\, d\,(\alpha\,\bar{l}^h)^{(\sigma-1)/\sigma} + (1-d)\,(i_t^h)^{(\sigma-1)/\sigma}\,\right]^{\sigma/(\sigma-1)} \qquad (1)$$

where α becomes larger than one (1.015) in the new technology.[15] Given the putty-clay nature of the production function, our model seems suitable to simulate such technical progress convincingly since it can only be incorporated in the firm's equipment through new investments.

The impact on the economic activity is immediately positive: investments in most productive equipments are indeed stimulated by the incentive to reduce rapidly the cost of labour. Prices decrease progressively (but the mark-up rate rises as π_D diminishes). As far

[15]The Euler equations have of course been modified accordingly but these modifications are not reported here.

as the evolution of the unemployment rate is concerned, the initial investment wave creates new workplaces (see Figure 5). When the investment rate comes back to the depreciation rate, the negative impact of the labour-capital substitution offsets progressively the initial gain in employment. The long term effect on unemployment remains however slightly negative.

Conclusion

Our analysis aims at understanding some aspects of the dynamics of unemployment and the rate of utilization of productive equipment by taking explicitly into account both the quantity constraints suggested by these two variables and the heterogeneity of microeconomic situations in a model with rational expectations. As the presence of unemployment in the short and the long run rely essentially on non-competitive wage and price formation, the model remains rather elementary. However, it seems able to characterize an important part of the evolution of *ur* and *duc* in the face of unexpected changes in the economic environment similar to those experienced by European economies. In particular, rises in interest rates and drops in total factor productivity increase both unemployment and the degree of capacity utilization in the long run. It is also worth noting that the productivity shock allows to understand a set of stylized facts observed after the oil shocks: A higher proportion of firms reporting to face demand shortages (without implying by itself a Keynesian problem); Higher prices but lower mark-up; The appearance of a capital gap. The model is also suited to study changes in firm-level uncertainty and their effect on structural unemployment. Of course, our framework provides no answer to the question of knowing why unemployment remained so high once these perturbations vanished (like after the counter oil shock). The introduction of mechanisms accounting for persistence would certainly be a priority for future works.

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