BEHAVIORAL HETEROGENEITY AND

COURNOT OLIGOPOLY EQUILIBRIUM

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ABSTRACT : It is not unfrequent to see studies of imperfect competition or of industrial organization rest upon questionable foundations such as the hypothesis that inverse market demand is, whenever it is positive, concave or even linear. Assumptions of this sort are not robust (i.e. "additive") in the sense that they are not usually preserved through aggregation of different sectors that would satisfy them individually. The present paper investigates an alternative specification that is based upon the plausible existence of significant heterogeneities among demanders. It is demonstrated that specific forms of demand heterogeneity tend to stabilize market expenditures. In a partial equilibrium context, sufficient demand heterogeneity is shown to imply existence and unicity of a Cournot oligopoly equilibrium.

Jel Code : D00, D43, L13.

Key words : Aggregation, heterogeneity, equivalence scales, oligopoly equilibrium.

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HETEROGENEITE ET OLIGOPOLE DE COURNOT

RESUME : Il n'est pas rare de voir des études portant sur la concurrence imparfaite ou l'organisation industrielle partir d'hypothèses douteuses, comme une demande inverse globale concave ou même linéaire. De telles hypothèses ne sont pas robustes (i.e. additives), car elles ne sont pas habituellement préservées par agrégation de secteurs qui les vérifieraient individuellement. L'article examine une spécification alternative, fondée sur la présence plausible d'hétérogénéités significatives dans les comportements des acheteurs, qui est préservée par agrégation. On montre que des formes d'hétérogénéité spécifiques tendent à stabiliser la dépense globale. Ceci implique, dans un cadre d'équilibre partiel, l'existence et l'unicité de l'équilibre d'oligopole de Cournot.

Code JEL : D00, D43, L13.

Mots clés : Agrégation, hétérogénéité, échelles d'équivalence, oligopole.

BEHAVIORAL HETEROGENEITY AND COURNOT OLIGOPOLY EQUILIBRIUM

Jean-Michel Grandmont *

Partial equilibrium models of imperfect competition typically rely upon convenient assumptions on, say, market demand curves, that are tailored so as to ensure concave revenue functions, hence the existence or uniqueness of the corresponding Nash equilibrium (in socalled pure strategies). It is even not unfrequent to see specifications in which the inverse market demand curve is supposed to be, whenever it is positive, linear or concave in price (see e.g. Tirole (1988)). The advantage of such specifications is of course tractability but the trouble is that they appear to lack robust economic foundations. It is well known that traditional economic theory, or more specifically individual optimizing behavior, does not typically generate demand curves displaying the kind of nice properties imperfect competition theorists like to assume. A related point is that these properties are not usually additive : if one takes two demand sectors that are "nice" when functionning separately, adding them together often results in a demand sector that behaves quite unpleasantly. The point is illustrated in Fig. 1 in the case of linear or concave inverse demand schedules. These difficulties have led quite a few theorists (see, e.g., Mc Manus (1962, 1964), Roberts and Sonnenschein (1976, 1977), Bamon and Frayssé (1985), Frayssé (1986), Novshek (1985)) to try to dispense with concave revenue functions and other niceties in models of imperfect competition. While there has been significant progress in the area, the outcome is still frustrating as it has not yet generated models that are both general and tractable.

<u>Fig. 1</u>

The difficulties we just alluded to are not novel, nor are they limited to theories of partial equilibrium or imperfect competition. It is well known that traditional economic theory does not place many restrictions, to say the least, on aggregate market behavior in general competitive equilibrium models. The devastating result is here Sonnenschein-Mantel-Debreu's indeterminacy theorem (see Sonnenschein (1973,

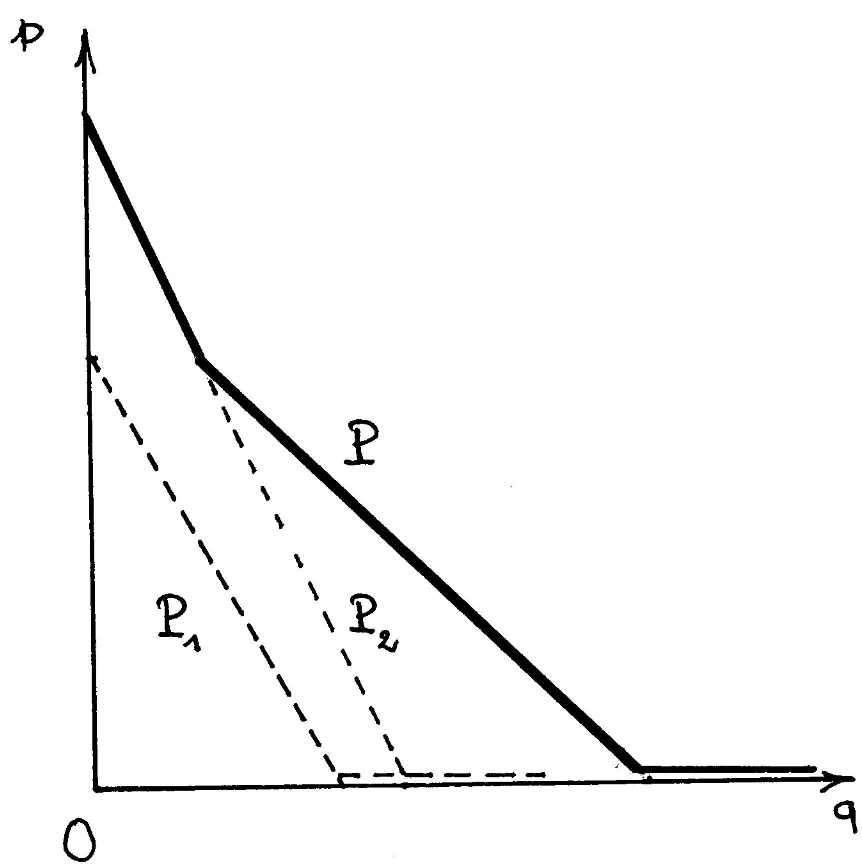

Fig. 1.a

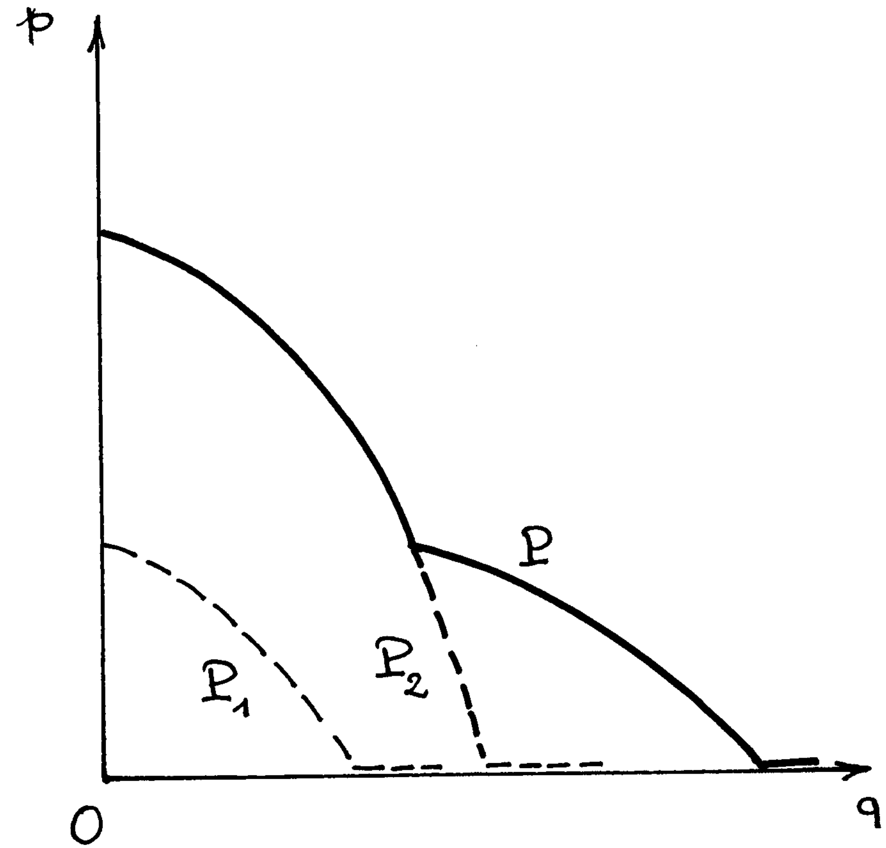

Fig. 1.b

1974) and the survey by Shafer and Sonnenschein (1982)) : if the distribution of microeconomic characteristics in the system is arbitrary, individual optimizing behavior does not place any restrictions on competitive aggregate excess demand, on any given compact set of prices, other than homogeneity and Walras's law. That is not much, and of course one should not expect concave revenue functions.

The main source of the problem, whether in general competitive analysis or in partial equilibrium models of imperfect competition, lies in the single word : *aggregation*. One can get almost anything, hence nothing, at the macroeconomic level, when the distribution of individual characteristics is arbitrary. The diagnosis suggests a possible way out that has been known for quite some time, namely to start with plausible restrictions on the distribution of individual characteristics to get testable (and hopefully pleasant) macroeconomic regularities. In particular, it has been often asserted that *heterogeneity* should help in this respect.

There has been some progress in recent times on this research front. Hildenbrand (1983) took indeed a decisive step by showing that heterogeneity in the income distribution may make macroeconomic income effects just right or weak enough in multimarkets consumer demand analysis, to leave us more or less, in the aggregate, with "nice" substitution effects only (see also Grandmont (1987)). Another important idea originated in the work of Jerison (1982, 1984, 1992), who showed that, roughly speaking, increasing dispersion of household Engel curves as income goes up leads to the weak axiom of revealed preference in the aggregate when the distribution of income is fixed. This approach has been extended quite successfully by Hildenbrand, who showed that it implied in fact the "Law of Demand" at the macroeconomic level, and confronted to empirical data with fairly convincing results (Härdle, Hildenbrand and Jerison (1991), Hildenbrand (1989, 1992)). While this approach has been an outstanding methodological achievement, it seems however difficult to apply it beyond the restrictive situation in which incomes are independent of prices. When specialized to a partial equilibrium analysis, it apparently yields only a downward sloping demand schedule. That is already a lot, but we are still rather far from concave revenue functions.

Another approach, complementary to that of Hildenbrand and Jerison, has been recently implemented with fairly good success. It seeks to introduce plausible heterogeneities in other dimensions of individual characteristics, namely in consumers' demand schedules. This line of attack has it roots in, and generalizes the notion of, *household equivalence scales*, which has been much used in applied demand analysis (Prais and Houthakker (1955), Barten (1964), Jorgenson and Slesnick (1987)). Dispersion of demand schedules of this sort have been introduced in general competitive equilibrium analysis some time ago by Mas-Colell and Neuefind (1977) and by E. Dierker, H. Dierker and Trockel (1984). It has also been used by H. Dierker (1989) and E. Dierker (1991) to inquire whether demand heterogeneity might help in ensuring existence of equilibrium in imperfect competition models.

The recent discovery has been that increasing demand heterogeneity tends to make aggregate expenditures more independent of prices. In a competitive general equilibrium context, this fact has strong consequences for the prevalence, in the aggregate, of the weak axiom of revealed preferences, of gross substitutability, and on uniqueness and stability of the Walrasian exchange equilibrium (Grandmont (1992)). When specialized to a market for a single commodity, the approach implies that a relatively large heterogeneity in individual demand behaviors yields an aggregate demand that is not only downward sloping, but with an elasticity that is not too far from minus 1. We *are* now within reach of a concave revenue function ... Moreover, by contrast to the case of linear or concave inverse market demands depicted in Fig. 1, this sort of configuration is robust, i.e. additive. If one puts together two heterogenous demand sectors, one still gets heterogeneity (alternatively, adding two market demand schedules having an elasticity that is not too far from -1 yields a market demand with the same property).

A reasonable conjecture is accordingly that demand heterogeneity, along the line of Grandmont (1992), may generate concave revenue functions and help in imperfect competition analysis. The purpose of the present paper is to demonstrate that this conjecture is correct, in the context of a partial equilibrium model in which firms compete in quantities, *à la Cournot*. We give in Section 1 approximate bounds for market demand elasticities that depend explicitly on specific measures of demand heterogeneity. When the degree of demand heterogeneity grows, the elasticity of market demand is not too far from minus -1. We establish in Section 2 that demand heterogeneity leads essentially to strictly concave revenue functions and thus, for a given firm, to a unique optimum supply level. We show in Section 3 that enough demand heterogeneity, in a sense that is made quantitatively precise, implies not only existence, but also unicity of a Cournot oligopoly equilibrium.

1. HETEROGENEITY AND AGGREGATE DEMAND

We consider a market for a particular good or service, with a large number – in fact a continuum – of individual buyers. All buyers behave "competitively", i.e. formulate their demands while taking the price $p > 0$ of the good as given. It is known that individual demands may not be well behaved even if they come say, out of utility maximization (in the case of a consumer good). The purpose of this section is to show that if there is sufficient heterogeneity among individual buyers, aggregate market demand displays indeed enough nice properties to be a potentially successful building block of partial equilibrium models of imperfect competition.

The population of buyers is described as follows. There is first a set A of "types". Each type *a* in A defines a demand function $q_a(p) \geqq 0$. Second, we assume that for each type that is present in the population, there is a continuum of individuals who have the same demand function as $q_a(p)$, *up to a rescaling of the unit of measurement of the good by a factor* $\beta > 0$. To see what should be the form of this "rescaled" demand function, consider a *fictitious* operation in which the unit of measurement of the good is divided by β. If the price of the good in the fictitious unit is $p > 0$, the price in the actual unit is βp. Thus the original demand, expressed in the fictitious unit, is $\beta q_a(\beta p)$. By definition, the rescaled demand function corresponding to the parameter β, *expressed in the actual unit system* (which is in fact fixed throughout !) is equal to the above expression, i.e. to $\beta q_a(\beta p)$. [1] In the sequel, it will be in fact much more convenient to work with the parameter $\alpha = \mathrm{Log}\beta$, which can be a positive or a negative number. With this convention, the rescaled demand function is $e^{\alpha} q_a(e^{\alpha}p)$.

The distribution of characteristics in the demand sector can accordingly be represented by a probability distribution over types, and for each type *a* present in the population, by a conditional distribution over the rescaling parameter α, which we shall suppose to have a density $f(\alpha|a)$. Market demand $Q(p)$ is then obtained by aggregating (i.e. integrating) individual demands over the whole population, first conditionally for each type and second, over types. We are then in a framework in which we can speak in a meaningful way of the "heterogeneity" of this population by looking at the dispersion of the conditional densities $f(\alpha|a)$. Our program is indeed to show that a large dispersion implies a well behaved market demand, even if individual demands are not.

To simplify the exposition, we shall assume that the conditional densities are actually independent of the type a (see the Appendix for the consequences of relaxing this assumption). The independence hypothesis allows us to compute market demand by changing the order of integration. We can first aggregate the demands $q_a(p)$ over types ; this yields the demand function $q(p)$. Market demand is then obtained by integration over the rescaling factor α

$$Q(p) = \int e^{\alpha}q(e^{\alpha}p)\ f(\alpha)\ d\alpha.$$

We shall make throughout the following assumptions. The first one, on the demand function $q(p)$, states essentially that expenditure on the good, aggregated over all types, is uniformly bounded above.

(1.*a*) *The demand function* $q(p)$ *is continuous and satisfies* $0 \leqq pq(p) \leqq b$ *for all* $p > 0$.

The immediate consequence is that market expenditure $pQ(p)$ is also bounded above by b. The second assumption states that the density function $f(\alpha)$ is regular enough to ensure that market demand $Q(p)$ has continuous first and second derivatives.

(1.*b*) *The density function* $f(\alpha)$ *is twice continuously differentiable for every* α *in the real line. Its first and second derivatives are uniformly integrable, i.e.* $\int|f'(\alpha)|d\alpha \leqq m$ *and* $\int|f''(\alpha)|d\alpha \leqq m$.

The coefficient m appearing in (1.*b*) is an indicator of the dispersion of the density $f(\alpha)$. If it is small, the distribution over the rescaling factor α should be widely spread out. We are going to show that in such a case, market demand $Q(p)$ is "well behaved" even though the individual demands may not be so.

Market demand

Our strategy is to show that market expenditure $pQ(p)$ has continuous first and second derivatives and to give bounds for these derivatives that depend explicitly on the dispersion of the density $f(\alpha)$, i.e. on the coefficient m. To this effect, it is convenient to introduce the notation $w(p) = pq(p)$. Then it follows from the definition of market demand that market expenditure is given by

$$pQ(p) = \int w(e^{\alpha}p)\, f(\alpha)\, d\alpha.$$

By making the change of variable $r = \alpha + \mathrm{Log}p$, we obtain

$$pQ(p) = \int w(e^{r})\, f(r - \mathrm{Log}p)\, dr.$$

This expression shows immediately that since the density f was assumed to have continuous first and second derivatives that are uniformly integrable, market expenditure has also continuous first and second derivatives. Taking derivatives with respect to Logp and reverting to the original variable $\alpha = r - \mathrm{Log}p$ yields

$$(1.1) \qquad \frac{d[pQ(p)]}{d\mathrm{Log}p} = - \int w(e^{\alpha}p)\, f'(\alpha)\, d\alpha,$$

$$(1.2) \qquad \frac{d^2[pQ(p)]}{(d\mathrm{Log}p)^2} = \int w(e^{\alpha}p)\, f''(\alpha)\, d\alpha.$$

The last step is to use again assumption (1.*b*) to bound the right hand sides of (1.1) and (1.2). We get then

$$(1.3) \qquad \left| \frac{d[pQ(p)]}{d\mathrm{Log}p} \right| \leqq bm \quad , \quad \left| \frac{d^2[pQ(p)]}{(d\mathrm{Log}p)^2} \right| \leqq bm.$$

It should be noted that these bounds are valid for *all* densities $f(\alpha)$, even if they are rather concentrated. On the other hand, when the coefficient m becomes very small, the inequalities (1.3) tell us that market expenditure is eventually approximately independent of the price. We shall make throughout the following assumption

(1.c) *Expenditure on the good, aggregated over all types, is uniformly bounded away from 0, i.e.* $pq(p) \geqq \delta b > 0$ *for every* $p > 0$.

The immediate consequence is that market expenditure $pQ(p)$ is also uniformly bounded below by δb. Thus if the coefficient m becomes very small, other things (in particular the lower bound δb) remaining fixed, market demand should be asymptotically close to a unit elastic demand of the form A/p. One should expect accordingly that when m is not too large (the density $f(\alpha)$ is relatively dispersed), market demand should be rather well behaved. This is exactly the property we shall exploit in the sequel [2].

We transpose now, for later use, the inequalities (1.3) in terms of the elasticities of market demand and of its first derivative. Here we should expect these elasticities to be close to -1 and -2, respectively, when the coefficient m becomes small, other things being equal.

LEMMA 1.1. *Let* $\varepsilon_Q(p) = pQ'(p)/Q(p)$ *and* $\varepsilon_{Q'}(p) = pQ''(p)/Q'(p)$ *be the elasticities of market demand and of its first derivative. Then*

$$|\varepsilon_Q(p) + 1| \leqq m/\delta. \tag{1.4}$$

Therefore, market demand is downward sloping, i.e. $\varepsilon_Q(p) < 0$, *when* $m < \delta$. *In that case*

$$|\varepsilon_{Q'}(p) + 2| \leqq 2m/(\delta - m). \tag{1.5}$$

In particular, $Q'(p) < 0$ *and* $Q''(p) > 0$ *for every* p *whenever* $m < \delta/2$.

Proof : It follows from the definition that

$$\frac{d[pQ(p)]}{dLogp} = pQ(p)\ [1 + \varepsilon_Q(p)]\ .$$

Hence from the first inequality of (1.3) and assumption (1.*c*)

$$|\varepsilon_Q(p) + 1| \leqq mb/pQ(p) \leqq m/\delta,$$

which is (1.4). Then it is clear that $\varepsilon_Q(p) < 0$ when $m < \delta$ since

(1.6) $$- (\delta+m)/\delta \leqq \varepsilon_Q(p) \leqq - (\delta-m)/\delta.$$

To prove (1.5), remark first that, using simple calculations

$$\frac{d^2[pQ(p)]}{(dLogp)^2} = pQ(p)\ [1 + \varepsilon_Q(p)\ (\varepsilon_{Q'}(p) + 3)].$$

This implies, in view of the second inequality of (1.3) and assumption (1.*c*)

$$|\varepsilon_Q(p) + 1 + \varepsilon_Q(p)\ (\varepsilon_{Q'}(p) + 2)| \leqq mb/pQ(p) \leqq m/\delta.$$

Hence on account of (1.4)

$$|\varepsilon_Q(p)|\ |\varepsilon_{Q'}(p) + 2| \leqq 2m/\delta.$$

If $m < \delta$, we have $\varepsilon_Q(p) < 0$ and therefore from (1.6),

$$|\varepsilon_{Q'}(p) + 2| \leqq 2m/(\delta|\varepsilon_Q(p)|) \leqq 2m/(\delta-m),$$

which is (1.5). Since $m/(\delta-m)$ is increasing with m and is equal to 2 when $m = \delta/2$, one has $\varepsilon_{Q'}(p) < 0$ hence $Q''(p) > 0$ if $m < \delta/2$. Q.E.D.

<u>Inverse market demand</u>

The inverse market demand is often the object of prime interest in models of imperfect competition. We assume from now on

(1.*d*) $$m < \delta.$$

We know from Lemma 1.1 that market demand is then downward sloping. Moreover $\delta b \leqq pQ(p) \leqq b$ and the range of $Q(p)$ is the whole open interval $(0,+\infty)$. Thus market demand has an inverse $P = Q^{-1}$, which satisfies also $\delta b \leqq qP(q) \leqq b$ and is twice continuously differentiable for every $q > 0$. The next result gives useful bounds on the elasticities of the inverse market demand $P(q)$ and of its first derivative. Here again the inverse market demand is well behaved when the density $f(\alpha)$ of the rescaling factor α is "flat", since these elasticities are close to -1 and -2, respectively, when the coefficient m is small, other things being equal.

LEMMA 1.2. *For every* $q > 0$ *let* $\varepsilon_P(q) = qP'(q)/P(q)$ *and* $\varepsilon_{P'}(q) = qP''(q)/P'(q)$ *be the elasticities of the inverse market demand and of its first derivative. Then*

$$(1.7) \qquad |\varepsilon_P(q) + 1| \leqq \eta \qquad \textit{and} \qquad |\varepsilon_{P'}(q) + 2| \leqq \eta',$$

with $\eta = m/(\delta-m)$ *and* $\eta' = \eta[1 + \delta/(\delta-m)]$.

Proof : By definition $P(q) = Q^{-1}(q)$, hence by differentiation, $P'(q) = -1/Q'(P(q))$ and $\varepsilon_P(q) = 1/\varepsilon_Q(P(q)) < 0$. Thus in view of (1.6)

$$(1.8) \qquad -\delta/(\delta-m) \leqq \varepsilon_P(q) \leqq -\delta/(\delta+m)$$

and therefore

$$-m/(\delta-m) \leqq \varepsilon_P(q) + 1 \leqq m/(\delta+m)$$

which implies $|\varepsilon_P(q) + 1| \leqq m/(\delta-m)$.

By differentiation of $P'(q) = -1/Q'(P(q))$, we get

$$\varepsilon_{P'}(q) = -\varepsilon_{Q'}(P(q))\, \varepsilon_P(q),$$

$$\varepsilon_{P'}(q) + 2 = 2[\varepsilon_P(q) + 1] - \varepsilon_P(q)[\varepsilon_{Q'}(P(q)) + 2].$$

Then using the bound on $|\varepsilon_P(q) + 1|$ we just got, together with (1.8) and (1.5), we obtain

$$|\varepsilon_{p'}(q) + 2| \leqq \frac{2m}{\delta-m}\left[1 + \frac{\delta}{\delta-m}\right]. \qquad \text{Q.E.D.}$$

We shall occasionally work directly with the coefficients η and η' appearing in (1.7), with understanding that, in view of their expressions, they can be made small by decreasing the coefficient m, given δ.

Remark 1.3. The *normal distribution* may play a significant role in applications of the theory presented here. It might be accordingly useful to see how the coefficient m involved in assumption (1.*b*) is related to the variance of the distribution in such a case. The coefficient m being invariant through translations of the density $f(\alpha)$, we may assume with loss of generality the mean of the distribution to be 0. The density is then

$$f(\alpha) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\alpha^2/2\sigma^2} ,$$

where σ^2 is the variance. By differentiating we get

$$f'(\alpha) = -\alpha f(\alpha)/\sigma^2 \text{ and } f''(\alpha) = -f(\alpha)[1 - \alpha^2/\sigma^2)]/\sigma^2 ,$$

and this implies

$$\int |f'(\alpha)| \, d\alpha = 2f(0) = \frac{2}{\sigma}(2\pi)^{-1/2} ,$$

$$\int |f''(\alpha)| \, d\alpha = 2[f'(-\sigma) - f'(\sigma)] = \frac{4}{\sigma^2} (2\pi e)^{-1/2}.$$

These two expressions go to 0 when the variance σ^2 becomes large.

2. FIRMS' REVENUE AND REACTION FUNCTIONS

We wish to study Cournot oligopoly competition in the present framework by using our previous findings on the demand sector. We analyze in this section the behavior of a single firm. To simplify matters we assume a constant marginal cost of production $c > 0$, and no fixed cost. The firm's total cost of producing $y \geqq 0$ is thus cy. The firm takes as given the total output z of the other competing firms (which we will assume to be strictly positive [3]) and chooses its own output supply $y \geqq 0$ so as to maximize its profit

$$\pi(y,z) = R(y,z) - cy,$$

in which $R(y,z) = yP(y+z)$ is the firm's revenue function.

Under our assumptions revenue is bounded above by b, so the firm's choice of output cannot exceed b/c, otherwise profit would be negative. Thus an optimum output level always exists, as it maximizes the continuous profit over the interval [0,b/c]. The optimum output level needs not be unique however, unless the revenue function is for instance strictly concave in y. This creates usually great difficulties in Cournot equilibrium or other imperfect competition models, as economic theory does not provide any reason for such a concavity property to be satisfied. In view of the results obtained so far, we may expect to be able to give a solution to the problem in our framework : with sufficient heterogeneity in the demand sector (in the sense we have been talking about it in the preceding section), the firm's revenue function should be well behaved and the optimum supply unique. The purpose of this section is to make this intuition precise.

Differentiating twice the revenue function with respect to y gives

$$\frac{\partial R}{\partial y}(y,z) = P(y+z) + yP'(y+z),$$

$$\frac{\partial^2 R}{\partial y^2}(y,z) = 2P'(y+z) + yP''(y+z).$$

The last expression can be put in the equivalent form

$$\text{(2.1)} \qquad \frac{\partial^2 R}{\partial y^2}(y,z) = \frac{P'(y+z)}{y+z}\,[2z + y(2 + \varepsilon_{P'}(y+z))].$$

Were market demand of the form $Q(p) = A/p$, the term $\varepsilon_{P'} + 2$ would vanish and the revenue function would be strictly concave for every nonnegative output y. In our case we can hope, by exploiting the bounds established in the previous section, to show the same property for a range of supply levels that would be large as the degree of heterogeneity in the demand sector grows. In view of Lemma 1.2, we know that $\varepsilon_{P'}(y+z) + 2$ is bounded below by $-\eta'$, which implies

$$\frac{\partial^2 R}{\partial y^2}(y,z) \leqq \frac{P'(y+z)}{y+z}\,(2z - y\eta').$$

Therefore the revenue function is strictly concave in y whenever $0 \leqq y < 2z/\eta'$. Given the other firms' total output z, the interval $[0, 2z/\eta')$ becomes indeed large when, other things being equal, the coefficient m, hence η', becomes small.

The study of the unicity of the firm's optimum supply is now rather simple. If $P(z) \leqq c$, it is too costly for the firm to produce anything, for marginal revenue $\frac{\partial R}{\partial y}(y,z)$ is strictly bounded above by P(z) and thus less than marginal cost c, for all output levels $y > 0$. If $c < P(z)$, a necessary condition for profit maximization is obtained by equating marginal revenue and marginal cost

$$P(y+z) + yP'(y+z) = c,$$

which is equivalent to, after rearranging terms

$$\frac{y}{y+z} = -\frac{1}{\varepsilon_P(y+z)}\,[1 - \frac{c}{P(y+z)}]\;.$$

Thus in both cases $P(z) \leqq c$ and $c < P(z)$, profit maximization requires

$$\text{(2.2)} \qquad \frac{y}{y+z} = \text{Max}\left[0,\; -\frac{1 - c/P(y+z)}{\varepsilon_P(y+z)}\right]$$

If $P(z) \leqq c$, the unique profit maximizing output is $y = 0$ and it is the only solution of the first order condition (2.2). When $c < P(z)$, we expect the firm's optimum supply to be unique if there is sufficient demand heterogeneity. Specifically, we know that the firm's revenue function is strictly concave for $0 \leqq y \leqq 2z/\eta'$ and thus on the whole interval $[0, b/c]$ if $b/c \leqq 2z/\eta'$ or equivalently $\eta' \leqq 2zc/b$. Since any profit maximizing output must belong to that interval, the optimum supply level is unique if

$\eta' \leqq 2zc/b$. It is in fact the sole solution of the first order condition (2.2) (see Fig. 2).

Fig. 2

The following lemma summarizes our findings

LEMMA 2.1. *Let* $z > 0$ *be the other firms' total output. Then*

1. *A necessary condition for profit maximization is*

$$\text{(2.3)} \qquad \frac{y}{y+z} = \text{Max}\left[0,\ -\frac{1 - c/P(y+z)}{\varepsilon_P(y+z)}\right] \equiv G(y+z)$$

2. *The firm's revenue function* $R(y,z) = yP(y+z)$ *is strictly concave in its own output, i.e.* $\frac{\partial^2 R}{\partial y^2}(y,z) < 0$, *when* $0 \leqq y < 2z/\eta'$. *If either* $P(z) \leqq c$ *or* $\eta' \leqq 2zc/b$, *the profit mamximizing supply* y *is unique. In both cases, it is the unique solution of the first order condition* (2.3).

The analysis confirms our intuition. As the degree of demand heterogeneity grows, i.e. when the coefficient m, hence η', becomes small, a firm's revenue function becomes strictly concave for a large range of supply levels and its optimum output is unique, other things (including the other firms' total supply) being equal.

3. COURNOT OLIGOPOLY EQUILIBRIUM

We study now existence and unicity of a Cournot oligopoly equilibrium, in relation with the degree of heterogeneity in the demand sector. We assume that there are $n \geqq 2$ firms competing in quantities. Firm j produces its output $y_j \geqq 0$ at constant marginal cost $c_j > 0$ (and no fixed cost). Without loss of generality, we assume that firms are ordered by increasing marginal costs, i.e. $c_1 \leqq c_2 \leqq \dots \leqq c_n$. A *Cournot equilibrium* is an array of outputs $(y_1^*,\dots,y_n^*)$, with $\Sigma_j\, y_j^* > 0$, such that the j-th firm's supply y_j^* maximizes its profit $y_j\, P(y_j + z_j^*) - cy_j$ with respect to y_j, given the other firms' total output $z_j^* = \Sigma_{k \neq j}\, y_k^*$, and this for every j.

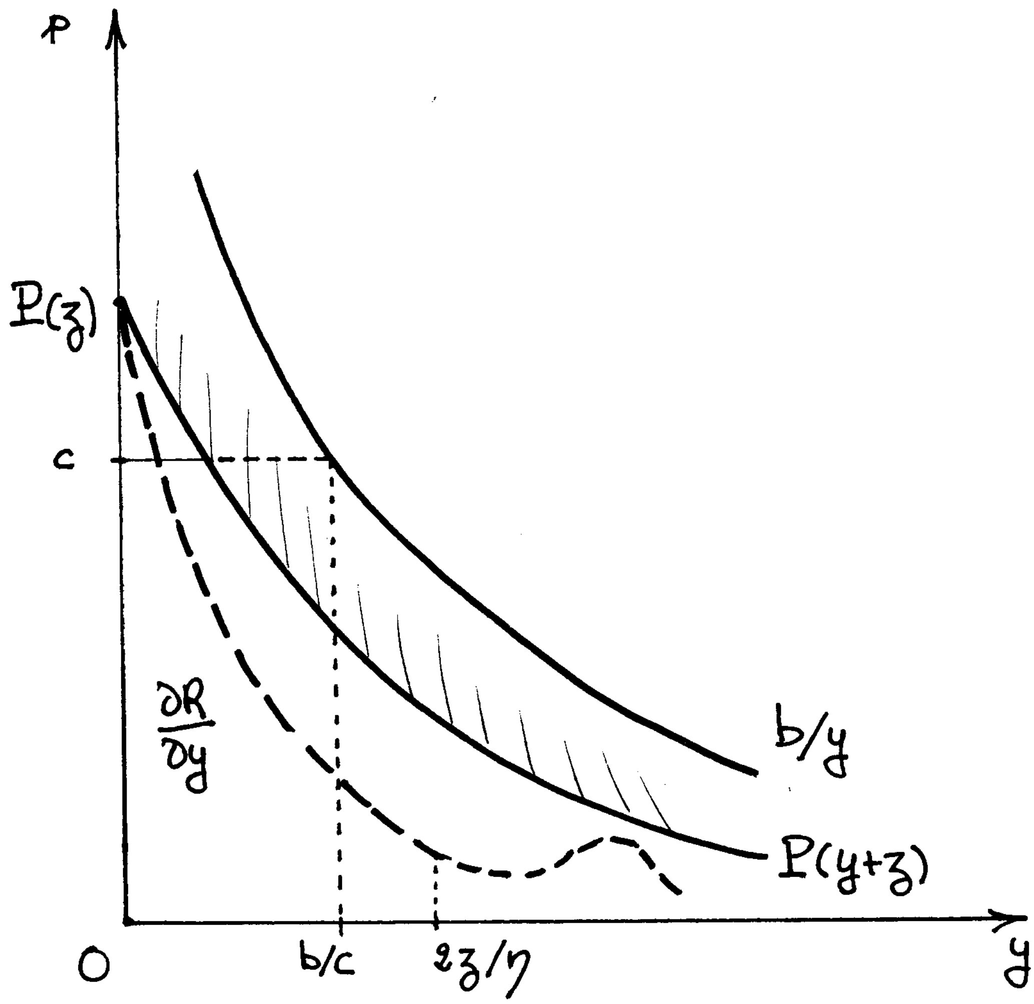

Fig. 2

Since the case of a unit elastic market demand is the asymptotic situation that arises when the degree of demand heterogeneity grows, we begin by reviewing what happens in that case.

Unit elastic demand

If market demand is of the form $Q(p) = A/p$, $A > 0$, the j-th firm's profit function is $Ay_j/(y_j + z_j) - c_j y_j$, where $z_j \geqq 0$ stands for the other firms' total output. Profit is strictly concave in y_j when $z_j > 0$ and linearly decreasing when $z_j = 0$. The optimum supply is thus unique and is the unique solution of the following first order condition, which is the exact analogue of (2.3)

$$\frac{y_j}{y_j+z_j} = \text{Max}\left(0, 1 - \frac{c_j(y_j + z_j)}{A}\right) \equiv G_j(y_j + z_j).$$

This formulation shows that a Cournot equilibrium can be characterized by a total output level $Y^* > 0$ that is solution of the equation $G(Y) = 1$, with $G(Y) = \Sigma_j G_j(Y)$. Each firm's equilibrium output is then determined by $y_j^*/Y^* = G_j(Y^*)$.

The function $G(Y)$ is piecewise linear (Fig. 3). Its value for $Y = 0$ is equal to the number of firms $n \geqq 2$. It is strictly decreasing when $0 \leqq Y \leqq A/c_1$ and vanishes for $Y \geqq A/c_1$. Thus there is unique $Y^* > 0$ that satisfies $G(Y) = 1$. The first n^* firms are active in equilibrium and the number n^* is determined by looking at the nondecreasing sequence

$$r_1 = G(A/c_1) = 0,$$
$$r_2 = G(A/c_2) = 2 - (c_1 + c_2)/c_2,$$
$$\cdot$$
$$\cdot$$
$$\cdot$$
$$r_n = G(A/c_n) = n - (c_1 +\ldots+ c_n)/c_n.$$

Then n^* is the largest integer such that $r_{n^*} < 1$. Since $r_2 < 1$, there are at least two active firms in equilibrium. Total equilibrium output is then $Y^* = A(n^*-1)/\sum_{j\leqq n^*} c_j$.

Fig. 3

Unicity of Cournot equilibrium

We wish to demonstrate that when there is sufficient heterogeneity in the demand sector, there exists a unique Cournot equilibrium. In view of the first order condition (2.3) and by analogy with the case of a unit elastic demand, we define for every total output $Y > 0$ and for every firm j

$$G_j(Y) = \mathrm{Max}\left[0 \ , \ -\frac{1 - c_j/P(Y)}{\varepsilon_P(Y)}\right] .$$

Each G_j is strictly positive for $Y < Q(C_j)$ and vanishes when $Q(c_j) \leqq Y$. If a Cournot equilibrium exists, with total output Y^*, then $G_j(Y^*)$ should give the equilibrium market share of firm j. The reader will note that, here as usual in Cournot equiibrium models (with constant marginal costs), the firms separate in two disjoint sets. The firms that are active in equilibrium are those for whom $c_j < P(Y^*)$, while all firms with $P(Y^*) \leqq c_j$ do not produce anything.

Since market shares have to add up to 1, equilibrium total output must satisfy the equation $G(Y^*) = 1$, with $G(Y) = \Sigma_j G_j(Y)$. Our strategy will be therefore to show that with enough heterogeneity, the equation $G(Y) = 1$ has a unique solution $Y^* > 0$, and that the array of individual outputs defined by $y_j^* = Y^* G_j(Y^*)$ corresponds indeed to a Cournot equilibrium.

PROPOSITION 3.1. *Let* $\bar{Y} = \delta b/2(c_1 + c_2)$. *Consider the set* J *of firms such that* $\bar{Y} < b/c_j$ *(which is nonempty since it contains firms* 1 *and* 2*). Assume that*

$$4\,\frac{m}{\delta} \leqq \frac{c_j\,\bar{Y}}{b} \quad \textit{and} \quad \frac{4m}{\delta} \leqq \frac{3}{2}\left(\frac{c_j\,\bar{Y}}{b}\right)^2$$

for every firm in J.

Then there is a unique Cournot equilibrium. Moreover each function $G_j(Y)$, *hence* $G(Y)$, *is non increasing, and strictly decreasing when positive, for all* $Y \geqq \bar{Y}$, *while* $G(Y) > 1$ *for all* $Y \leqq \bar{Y}$. *In equilibrium, at least firms* 1 *and* 2 *are active, i.e.* $y_1^* > 0$ *and* $y_2^* > 0$.

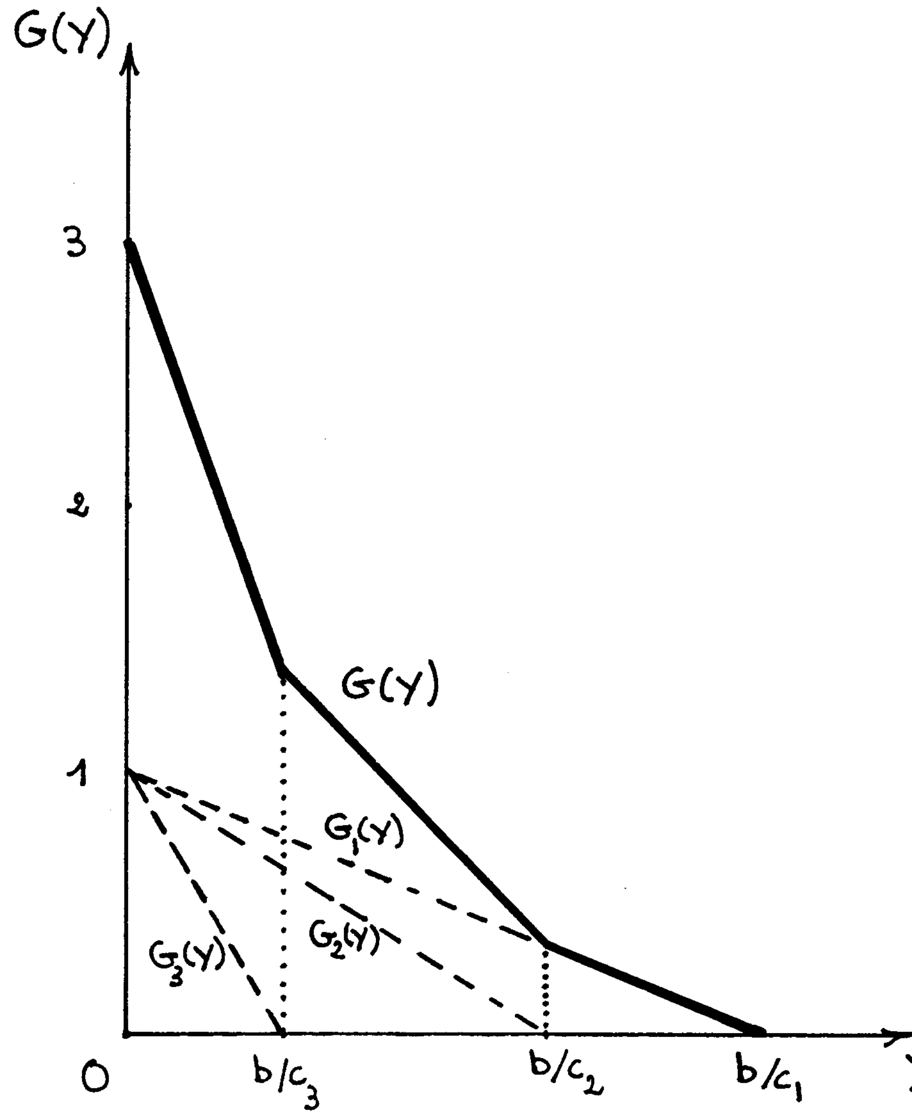

Fig. 3

Proof. The first item to show is that when the coefficient m is small, the equation $G(Y) = 1$ has a unique solution $Y^* > 0$ that is bounded away from 0 when m decreases.

Remark that one can rewrite G_j as

(3.1) $$G_j(Y) = \mathrm{Max}[0, -\varepsilon_Q(P(Y))\ (1 - c_j/P(Y))].$$

Each G_j is positive for $0 < Y < Q(c_j)$ and vanishes when $Y \geqq Q(c_j)$. Next, in view of the bounds of ε_Q given in Lemma 1.1 (see (1.4)), and since $\delta b \leqq Y\,P(Y) \leqq b$, we have $G_j^-(Y) \leqq G_j(Y) \leqq G_j^+(Y)$ for all $Y > 0$, with

$$G_j^+(Y) = \mathrm{Max}[0,\ (1 + \tfrac{m}{\delta})\ (1 - \tfrac{c_j Y}{b})],$$

$$G_j^-(Y) = \mathrm{Max}[0,\ (1 - \tfrac{m}{\delta})\ (1 - \tfrac{c_j Y}{\delta b})],$$

and therefore, $G^-(Y) \leqq G(Y) \leqq G^+(Y)$, where $G^-(Y) = \Sigma_j\, G_j^-(Y)$ and $G^+(Y) = \Sigma_j\, G_j^+(Y)$. Here G is positive for $Y < Q(c_1)$ and vanishes when $Y \geqq Q(c_1)$. In particular $G(Y)$ is always 0 when $Y \geqq b/c_1$, independently of m.

We wish to show now that $G(Y) = 1$ has all its solutions bounded away from 0 when m become small, other things being equal. Specifically,

(3.2) *Let* $\bar{Y} = \delta b/2(c_1 + c_2)$. *Then* $G(Y) > 1$ *for every* $0 < Y \leqq \bar{Y}$ *whenever* $m < \delta/3$.

To prove this, notice that $G(Y) \geqq G_1^-(Y) + G_2^-(Y)$. Now the map $G_1^- + G_2^-$ is piecewise linear, vanishes for $Y \geqq \delta b/c_1$ and its values for $Y = 0$ is $2(1 - m/\delta)$, which exceeds 1 as soon as $m < \delta/2$ and *a fortiori* when $m < \delta/3$ (Fig. 4). The equation $G_1^-(Y) + G_2^-(Y) = 1$ is equivalent to

$$\left(1 - \frac{m}{\delta}\right) \left[2 - \frac{(c_1 + c_2)\ Y}{\delta b}\right] = 1,$$

which yields

$$Y_0 = \frac{\delta b}{c_1 + c_2}\left[2 - \frac{\delta}{\delta - m}\right].$$

If $m < \delta/3$, then

$$Y_0 > \frac{\delta b}{2(c_1 + c_2)} = \bar{Y}.$$

In that case, we get for all $0 < Y \leqq \bar{Y}$

$$G(Y) \geqq G_1^-(Y) + G_2^-(Y) > G_1^-(Y_0) + G_2^-(Y_0) = 1$$

This proves (3.2).

Fig. 4

Our next step is to demonstrate that when m is small, each function G_j is non increasing (in fact strictly decreasing when positive) for all output levels that exceed some prespecified value γ. Of course such properties will show up in $G = \Sigma_j G_j$.

(3.3) *For fixed* $0 < \gamma < Q(c_j)$, *the function* G_j *is strictly decreasing on* $[\gamma, Q(c_j)]$ *provided that* $4m/\delta \leqq c_j\gamma/b$.

Fix $0 < \gamma < Q(c_j)$. For any $\gamma \leqq Y \leqq Q(c_j)$, one has

$$G_j(Y) = -\varepsilon_Q(P(Y))\,(1 - c_j/P(Y)).$$

The elasticity $\varphi(Y)$ of this function is the elasticity $\varphi_1(Y)$ of $\varepsilon_Q(P(Y))$ plus the elasticity $\varphi_2(Y)$ of $(1 - c_j/P(Y))$. Simple calculations show that

$$\varphi_1(Y) = \varepsilon_P(Y)\,[\varepsilon_{Q'}(P(Y)) + 1 - \varepsilon_Q(P(Y))]$$

$$= \varepsilon_P(Y)\,[\varepsilon_{Q'}(P(Y)) + 2 - (\varepsilon_Q(P(Y)) + 1)],$$

$$\varphi_2(Y) = \varepsilon_P(Y)\, c_j/[P(Y) - c_j].$$

Since $\varepsilon_P(Y) < 0$, one will have $\varphi(Y) < 0$ if and only if

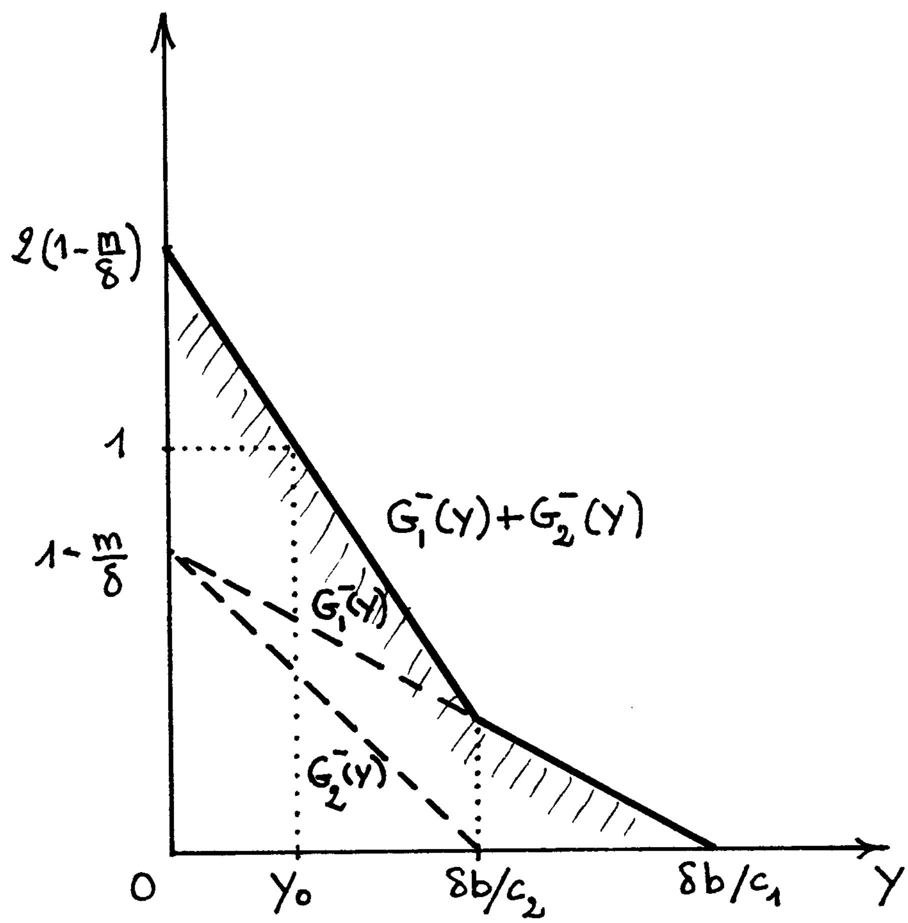

Fig. 4

$$(\varepsilon_Q(P(Y)) + 1) - (\varepsilon_{Q'}(P(Y)) + 2) < c_j/[P(Y) - c_j].$$

The leftside of the inequality is, from Lemma 1.1, bounded above by $\frac{m}{\delta} + \frac{2m}{\delta - m}$. The right side is greater than $c_j/P(Y) \geqq c_jY/b \geqq c_j\gamma/b$. Thus we are sure that $\varphi(Y) < 0$ if

(3.4) $$\frac{m}{\delta} + \frac{2m}{\delta - m} \leqq \frac{c_j\gamma}{b}.$$

Now assume $4m/\delta \leqq c_j\gamma/b$. Since $\gamma < Q(c_j) \leqq b/c_j$, one has $c_j\gamma/b < 1$, hence $m < \delta/4$. This implies that the left side of (3.4) is less than

$$\frac{m}{\delta} + \frac{2m}{\delta - (\delta/4)} = \frac{11}{3}\frac{m}{\delta} < \frac{4m}{\delta}.$$

Therefore the inequality $4m/\delta \leqq c_j\gamma/b$ implies (3.4), hence $\varphi(Y) < 0$, or equivalently that G_j is strictly decreasing on the interval $[\gamma, Q(c_j)]$. This proves (3.3).

We are now ready to prove that when m is low, the equation $G(Y) = 1$ has a unique solution. This will be done by putting together (3.2) and (3.3), with $\gamma = \bar{Y}$.

(3.5) *Let* $\bar{Y} = \delta b/2(c_1 + c_2)$. *Let* J *be the set of firms for which* $\bar{Y} < b/c_j$. *Assume that* $4m/\delta \leqq c_j\bar{Y}/b$ *for all* j *in* J. *Then* $G(Y) > 1$ *for all* $Y \leqq \bar{Y}$. *Moreover each* G_j, *hence* G, *is nonincreasing, and strictly decreasing when positive, for all* $Y \geqq \bar{Y}$. *The equation* $G(Y) = 1$ *has then a unique solution* $Y^* > 0$.

Remark first that $\bar{Y} < \delta b/c_j \leqq b/c_j$ for $j = 1, 2$, so the set J is nonempty as it contains firms 1 and 2. The assumption $4m/\delta \leqq c_j\bar{Y}/b$ for each firm j in J implies that $4m/\delta < 1$, so surely $m < \delta/3$ and (3.2) applies. The statements in (3.5) then follow by putting together (3.2) and (3.3) with $\gamma = \bar{Y}$.

Under the assumptions of (3.5), there is a unique solution Y^* of $G(Y) = 1$. Hence there is at most one Cournot equilibrium, and it must involve the total output Y^*. To complete the proof of the Proposition, it remains to show that the array of individual outputs defined by $y_j^* = Y^* G_j(Y^*)$ indeed corresponds, with enough heterogeneity, to a Cournot equilibrium. To this end, we have to use Lemma 2.1. Let

$$z_j^* = Y^* - y_j^* = Y^*[1 - G_j(Y^*)] \geqq 0$$

be the other firms' total output from the j-th firm's point of view. What we have to show is that y_j^* is an optimal response to z_j^*, for every j. If firm j is inactive, i.e. $y_j^* = 0$, this is clear, since then $z_j^* = Y^*$ and from the definition of G_j, $P(Y^*) \leqq c_j$.

Consider now a firm j that is active, i.e. $y_j^* > 0$. For such a firm

$$\bar{Y} < Y^* < Q(c_j) \leqq b/c_j,$$

so that an active firm belongs to the set J.

(3.6) *Assume, as in* (3.5), *that* $4m/\delta \leqq c_j\bar{Y}/b$ *for all firms in the set* J. *Let* $y_j^* = Y^* G_j(Y^*)$ *and* $z_j^* = Y^* - y_j^*$. *Then* $z_j^* > 3c_j\bar{Y}^2/4b$ *for all* j *in* J. *If in addition* $4m/\delta \leqq 3(c_j\bar{Y}/b)^2/2$, *then* y_j^* *is an optimal response to* z_j^*, *for every firm in* J.

To prove this statement, notice first that

$$z_j^* = Y^*(1 - G_j(Y^*)) \geqq \bar{Y}(1 - G_j(\bar{Y})) \geqq \bar{Y}(1 - G_j^+(\bar{Y})).$$

If j belongs to J, then $c_j\bar{Y} < b$ and one has

$$G_j^+(\bar{Y}) = (1 + \frac{m}{\delta})\ (1 - \frac{c_j\bar{Y}}{b}) < 1 - \frac{c_j\bar{Y}}{b} + \frac{m}{\delta}\ ,$$

which implies

$$z_j^* > \bar{Y}\left(\frac{c_j\bar{Y}}{b} - \frac{m}{\delta} \right) \geqq \frac{3c_j\bar{Y}^2}{4b}\ ,$$

under the assumption that $4m/\delta < c_j\bar{Y}/b$ for all j in J.

According to Lemma 2.1, for y_j^* to be an optimal response to $z_j^* > 0$, it is sufficient to have $\eta' \leqq 2z_j^*c_j/b$. In view of the expression of η' that is given in Lemma 1.2, the sufficient condition reads

$$(3.7) \qquad \frac{m}{\delta-m}\left[1+\frac{\delta}{\delta-m}\right] \leqq 2z_j^* \, c_j/b.$$

The assumption $4m/\delta \leqq c_j\bar{Y}/b$ for j in J implies $m < \delta/4$. An upper bound for the left side of (3.7) is thus

$$\frac{m}{\delta-(\delta/4)}\left[1+\frac{\delta}{\delta-(\delta/4)}\right] = \frac{28}{9}\,\frac{m}{\delta} < \frac{4m}{\delta}\ .$$

On the other hand a lower bound for the right side of (3.7) is obtained by replacing z_j^* by $3c_j\bar{Y}^2/4b$. Therefore (3.7) is satisfied if

$$\frac{4m}{\delta} \leq \frac{3}{2}\left(\frac{c_j\bar{Y}}{b}\right)^2.$$

In that case, y_j^* is indeed an optimal response to z_j^*, for every firm in J. This completes the proof of (3.6).

The final remark is that since firm 1 is necessarily active and $z_1^* > 0$, then firm 2 must also be active. The statement of Proposition 3.1 is simply a reformulation of (3.5) and (3.6). Its proof is complete.

Q.E.D.

4. CONCLUSION

We have shown that enough heterogeneity in demand behavior may lead to a well behaved aggregate market demand and to revenue functions for firms that display attractive concavity properties. We have then applied these findings to a market in which firms compete in quantities *à la Cournot* and showed uniqueness of equilibrium when there is enough demand heterogeneity. It remains to be seen whether the theory presented here can be a successful building block in the study of other models of imperfect competition.

APPENDIX

We assumed in the text that conditional densities over the rescaling parameter α were independent of the type *a*, in order to simplify the exposition. We sketch briefly here how the analysis must be amended when the independence axiom is relaxed.

Let μ be the probability distribution over all types *a* (we assume for instance that the set of types A is a separable metric space and shall make all necessary regularity assumptions, i.e. continuity with respect to types, to ensure that the integrals presented below are well defined). For each type *a* present in the population (in the support of μ), the conditional density over the rescaling parameter α is $f(\alpha|a)$. Market demand is defined in two stages. Integration over α, for a given type in the support of μ, yields *conditional market demand*

$$Q_a(p) = \int e^{\alpha} q_a(e^{\alpha} p)\ f(\alpha|a)\ d\alpha.$$

Total market demand is then obtained by aggregating over all types

$$Q(p) = \int_A Q_a(p)\ \mu(da).$$

The analysis leading to the bounds (1.3) applies without any change to conditional market demand $Q_a(p)$, provided that $pq_a(p) \leqq b_a$ and that the derivatives of the conditional density $f(\alpha|a)$ satisfies (1.*b*) with $m = m_a$. One gets then (1.3) for conditional market demand Q_a, with $b = b_a$ and $m = m_a$. If the distribution of all b_a in the population has a finite mean b and if m_a is bounded above by m for all types present in the population, one gets (1.3) for total market demand by integration over a.

To sum up, total market demand Q(p) satisfies (1.3) if assumptions (1.*a*) and (1.*b*) are replaced by

(1.*a'*) *For each type* a *in the support of* μ *and every price* p, *the individual demand function* $q_a(p)$ *is continuous in* (a,p), *with* $0 \leqq pq_a(p) \leqq b_a$ *for all* $p > 0$ *and* $\int_A b_a \, \mu(da) = b < +\infty$.

(1.*b'*) *For every type* a *in the support of* μ :

- *the conditional density* $f(\alpha|a)$ *is continuous in* (α,a), *and so are its first and second derivatives* $f'(\alpha|a)$ *and* $f''(\alpha|a)$.
- $\int |f'(\alpha|a)| \, d\alpha \leqq m$ *and* $\int |f''(\alpha|a)| \, d\alpha \leqq m$.

Then it follows that all formal results of the paper are valid if assumption (1.*c*) is made directly on total market demand

(1.*c'*) *Total market expenditure* pQ(p) *is bounded below by* $\delta b > 0$.

One has, however, to be a little more careful in the *interpretation* of the results when working with (1.*a'*), (1.*b'*), (1.*c'*) instead of the assumptions of the text. The independence assumption, together with (1.*c*), allowed us to treat the coefficient of heterogeneity m as independent of the lower bound δb on aggregate expenditure, and to argue that increasing heterogeneity should make things nicer, other things, *including the lower bound* δb, being equal. When the independence axiom is relaxed as here, one has to make sure that the lower bound on total market expenditure appearing in assumption (1.*c'*), is actually independent of the coefficient of heterogeneity m in the class of density functions $f(\alpha|a)$ considered, in order to apply consistently this interpretative argument.

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FOOTNOTES

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[1] Since only changes of measurement units are involved in transformations of this type, it should be clear that if the original demand $q_a(p)$ was derived from utility maximization, then the rescaled demand function could be also viewed as being derived from utility maximization – the new utility function being obtained from the original one by the same change of unit of measurement. The approach, however, is much more general and is valid *even if people don't maximize*. For more on this, in a multicommodity context, see Grandmont (1992).

[2] All formal results of this paper are still valid when (1.*c*) is postulated directly on market expenditure, i.e. $pQ(p) \geqq \delta b$, but one has to be more careful when interpreting them. For more on this, see the Appendix.

[3] The reason is that we wish to look at a truly oligopolistic situation. In the case of a monopoly ($z = 0$), we would run into trouble. Indeed when demand heterogeneity increases, the asymptotic situation is a unit elastic demand, and a monopolist's profit maximizing output would then be 0. That could not be a market equilibrium as the price would be infinite.