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Competing Vertical Structures: Precommitment and Renegotiation[†]

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COMPETING VERTICAL STRUCTURES: PRECOMMITMENT AND RENEGOTIATION
REVISED VERSION

ABSTRACT

We consider a model where two agents, privately informed on their own characteristics, play a (normal form) game on behalf of two uninformed principals. We analyze the existence of *precommitment effects* through public announcements of contracts, in a model where agency contracts, designed ex-ante, can always be *secretly renegotiated*, at ex-ante and interim stages. We show that the existence of precommitment effects depends both on the strategic complementarity of the agents' actions and on the direct effect of the opponents' actions on each principal's welfare. In our model, the possibility of renegotiation is crucial for the existence of precommitment effects. Our results are robust to other specifications of the timing, in particular when all contracts are designed at an interim stage. The results are introduced through an example of Cournot and Bertrand competition between firms, viewed as vertical structures.

Key Words: Principal-agent theory, agency contracts, renegotiation, precommitment effects, strategic complementarity, imperfect competition.

JEL Classification number: C72, D43, D82, L13, L14.

CONCURRENCE ENTRE STRUCTURES VERTICALES: ENGAGEMENT ET RENEGOCIATIONS
VERSION REVISEE

RESUME

On considère un modèle où deux agents, dotés d'une information privée sur leur propre caractéristique, jouent un jeu (sous forme normale) pour le compte de deux principaux. On étudie l'existence d'*effets d'engagement* à travers la divulgation publique des contrats, dans un modèle où les contrats sont établis ex-ante, mais peuvent être *secrètement renégociés* ex-ante et à une étape interim. On montre que l'existence d'effets d'engagement dépend, d'une part, de la complémentarité stratégique entre les actions des agents, et, d'autre part, de l'effet direct de l'action de l'adversaire sur les objectifs de chaque principal. Dans notre modèle, la possibilité de renégocier est centrale pour l'existence d'effets d'engagement. On montre que les résultats sont préservés si tous les contrats sont signés et renégociés à l'étape interim. L'article illustre les résultats généraux dans un modèle de concurrence imparfaite à la Cournot et à la Bertrand.

Mots Clés: Théorie principal-agent, contrats, renégociation, effets d'engagement ou effets d'annonce, complémentarités stratégiques.

Classification CEPREMAP: I, V.

1. INTRODUCTION

The idea that a contract between two parties may be used as a precommitment device vis-à-vis third parties dates back to Schelling [1960].[1] By publicly disclosing the contract that determines her agent's incentives, a principal can force her opponents to take her agent's actions as given, thereby gaining a first-mover advantage when desirable. We will say that *precommitment effects* exist when allowing public disclosure of agency contracts leads to a different outcome than if all contracts were kept secret.[2]

Following Vickers [1985], the theoretical literature on rivalrous agencies has developed these ideas, usually focusing on perfect information situations (with specific forms of contracts) in industrial organization, corporate finance or international trade. Among others, economic applications include the strategic aspect of vertical separation or of vertical restraints (Bonnano-Vickers [1988], Rey-Stiglitz [1987])[3], the effect of the separation of ownership and management in oligopoly models (Fershtman-Judd [1987a], Sklivas [1987]), the use of tax or subsidy policy to help domestic firms in international competition (Brander-Spencer [1983]

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1 More game-theoretical analyses can be found in Fershtman-Judd-Kalai [1986], Katz [1991] and Fershtman-Judd [1987b]. The last two articles investigate situations of, respectively, incomplete and imperfect information within agencies.

2 The mere fact of hiring an agent, as opposed to playing the game oneself, may also have precommitment effects, even though the content of the contract is not public: see Katz [1991] and Caillaud-Hermalin [forthcoming]. Here we concentrate on situations where principals must hire agents to play on their behalf.

3 See Slade [1992] for an empirical test of the theory in the retail - gasoline sector.

& [1985]).[4] A well documented case of strategic commitment is the public announcement of Employees Stock Ownership Plans by firms. ESOPs affect employee behavior and the distribution of voting power. In particular, they have significant strategic effects on the market for corporate control.[5]

The literature raises two questions. First, in a perfect information setting, given a public contract, a principal could always propose her agent a new secret contract that implements the best response to the opponents' actions, for the same expected compensation.[6] Therefore, as emphasized by Dewatripont [1988] and Katz [1991], precommitment effects seem to rest on the crucial assumption that contracts, once publicly disclosed, cannot be secretly renegotiated. But this is at odds with reality: whether legally enforceable or of a more implicit nature, actual contracts may almost always be renegotiated if both parties agree.

The second question is more empirical: why is public disclosure of contracts not universal ? Although business newspapers abound in announcements of contractual agreements, not all contracts are disclosed. Katz [1991] argues that it is costly to publicly disclose a contract, but it is hard to see why it would be more costly in one branch than in another.

This paper provides a partial and theoretical answer to these two questions. It analyzes precommitment effects under incomplete information

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4 Along these lines, let us mention a still vivid example. As a defense for publicly renegotiating the Common Agricultural Policy before the summer of 1992, European governments commonly invoked the fact that it had sent the Bush Administration a credible signal about the maximal reduction in subsidies that european countries were committed to accept in the GATT talks that took place by the end of 1992.

5 See e.g. Gordon-Pound [1990] and Chang-Mayers [1992].

6 We consider models with transferable utilities. With non-transferable utilities, precommitment effects may exist under perfect information (Bensaid - Gary-Bobo [1992]).

when public contracts can always be *secretly renegotiated* and it characterizes economic situations where precommitment effects exist or not.

More precisely, we consider two competing agencies where agents have private information. Contracts specify transfers from principals to agents as a function of the actions chosen by both agents. In most of the paper, contracts are signed and renegotiated before information is learnt by the agents. We also assume that agents are free to quit at any point in time. The two agencies sign their public contracts simultaneously. Precommitment effects arise when renegotiation is allowed because each hierarchy has the opportunity to react to its opponent's public contract disclosure. A hierarchy benefits from the inability of the other to preclude renegotiation, but this gain is limited by the fact that the hierarchy can itself renegotiate.

We show that credible commitment is possible but only on actions that are costlier to the agent than the actions that would be implemented under secret contracting. The consequence is that the existence of precommitment effects depends both on the strategic complementarity of the agents' actions and on the direct effect of the opponents' actions on each principal's welfare.[7] These results are shown to extend to the case where contracts are signed at an interim stage, after information is learnt by the agents.

The analysis relies on a general property of the Principal-Agent model. It is well known that this model predicts underproduction (e.g. the agent's private cost of action is suboptimal), and we show that this level of underproduction is <u>maximal</u> among the set of (constrained) efficient contracts. Therefore efficient renegotiation implies that the final

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[7] The fact that the results depend on these two features is not surprising since contracts constitute a form of strategic investment: see Bulow-Geanakoplos-Klemperer [1985], Fudenberg-Tirole [1984] and Tirole [1988] on the taxonomy of business strategies.

contract induces a lower level of underproduction. Public contracting can only modify the rent-efficiency trade-off by promising a high level of utility to the agent, which affects the choice within the set of efficient contracts.

Our results can be interpreted in the I.O. examples mentioned before. If agents compete in a Cournot way, our results are in line with previous contributions provided there are asymmetries of information within agencies: precommitment effects exist, yielding, under mild assumptions, a more competitive outcome than if no contract could ever be disclosed. If, however, agents compete in a Bertrand way, say in differentiated products, no precommitment effects arise: therefore, in contrast with previous results, collusion is not facilitated by the possibility of disclosing agency contracts, if secret renegotiation cannot be prevented. Under secret renegotiation, firms can only precommit to be tough competitors, a clear result contrasting with the previous literature.

Precommitment effects under incomplete information and renegotiation have been previously considered by Dewatripont [1988]. There are however several major differences between Dewatripont's model and ours. First, Dewatripont [1988] considers the case of one hierarchy competing with a third party. When signing an initial contract, the hierarchy therefore benefits from a first-mover advantage due to the specification of the timing of the model. Precommitment effects are then maximal when no renegotiation can take place. Second, in Dewatripont [1988], initial contracts are signed ex-ante (before the agent receives his private information) while renegotiation occurs at the interim stage. In contrast, we will consider more symmetric situations between two competing vertical structure and we will investigate timings in which both initial and renegotiated contracts are signed either at the ex-ante stage or at the interim stage. So renegotiation matters here despite the facts that the information structure remains unchanged and that vertical structures play simultaneously.

The paper is organized as follows. Section 2 presents two introductory examples based on a Cournot competition game and on a Bertrand competition game. Section 3 presents the general model where contracts are signed at an ex-ante stage. Section 4 characterizes the outcome of the renegotiation stage, i.e. the set of ex-ante incentive efficient outcomes; it is therefore of general theoretical interest. Section 5 (1 and 2) develops the equilibrium analysis and contains our main results about existence of precommitment effects. Section 6 discusses various modifications of the timing. In particular, Subsection 6.2 considers the case where contracts are signed and renegotiated at the interim stage. We characterize the set of interim incentive efficient outcomes and generalize our results to this case.

2. ILLUSTRATIVE EXAMPLES

2.1/ Precommitment effects in a Cournot setting

Consider two firms, A and B, that compete on some market. Firm B is integrated: B produces and sells at unit cost equal to 1. Firm A is a Principal - Agent structure: principal P^A is the owner of the firm, while agent A is a seller, who bears a unit cost of θ for each unit sold. θ is unknown and can take 2 values $\theta_1 = 1-\delta$ and $\theta_2 = 1+\delta$, with probability 1/2 each, and $\delta < 3/10$.

If quantities a and b are produced respectively by firms A and B, let the inverse demand function be: $p = \frac{1}{2}(5 - a - b)$. With monetary transfers t between P^A and A, the agent's and principal's payoffs are respectively:

$$U(a,t,\theta_i) = t - \theta_i a$$

$$\pi(a,b,t,\theta_i) = \frac{1}{2}(5 - a - b)a - t.$$

Consider the following game:

stage 1: P^A offers A a contract $t = t(a,b)$. If A refuses, firm A-P^A produces $a=0$, but P^A pays A an ex-ante reservation utility R. If A accepts, stage 2 begins. B knows R but does not observe stage 1.

stage 2: A privately learns θ, at which point he may quit or stay. If A quits, firm A-P^A produces $a=0$, but no transfers are necessary (A's ex-post reservation utility is 0). If A stays, A and B simultaneously choose quantities a and b, and the contract is implemented.

We focus on equilibrium outcomes where A accepts the contract and always stays; this corresponds to δ not too large, as appears in the following. Any (pure strategy) equilibrium outcome can be characterized by equilibrium sales, $\mathbf{a}=(a_1,a_2)$ and b, and equilibrium utilities for A, $\mathbf{U}=(U_1,U_2)$; equilibrium transfers are then given by: $t_i = U_i + \theta_i a_i$. The profile $(\mathbf{a},\mathbf{U})$ must necessarily be incentive compatible, ex-ante individually rational and ex-post individually rational for agent A. Moreover, given B's action b, there cannot exist an incentive compatible, ex-ante and ex-post individually rational profile $(\mathbf{a}',\mathbf{U}')$ that P^A would prefer, since then P^A should have proposed $t(a) = U'_i+\theta_i a'_i$ if $a=a'_i$, $t(a)=0$ otherwise. It follows that $(\mathbf{a},\mathbf{U})$ must be solution of:

$$\max_{\mathbf{a},\mathbf{U}} \sum_{i=1}^{2} \frac{1}{2}\left\{\frac{1}{2}(5 - a_i - b - 2\theta_i)a_i - U_i\right\} \qquad \text{(P1)}$$

$$\text{s.t.} \quad 2\delta a_1 \geq U_1 - U_2 \geq 2\delta a_2 \qquad \text{(IC)}$$

$$U_i \geq 0 \quad \text{for } i = 1,2 \qquad \text{(EPIR)}$$

$$U_1 + U_2 \geq 2R \qquad \text{(EAIR)}$$

For different values of R, the solution, denoted $\mathbf{A}(b,R)$ and $\mathbf{U}(b,R)$, is:

Table 1

R	$R^0(b)$		$R^*(b)$
a_1	$A_1^*(b)$	$A_1^*(b)$	$A_1^*(b)$
a_2	$A_2(b,0)$	R/δ	$A_2^*(b)$
U_1	$2R^0(b)$	$2R$	*indeterminate*
U_2	0	0	*indeterminate*

$A_i^*(b) \equiv \frac{1}{2}(5 - b) - \theta_i$ is the full information best response to production b; $A_2(b,0) \equiv \frac{1}{2}(3 - b) - 3\delta$, $R^0(b) \equiv \delta A_2(b,0)$ and $R^*(b) \equiv \delta A_2^*(b)$.

Table 1 characterizes the best responses for firm P^A-A to a level of production b by firm B. It shows that the larger R, the higher the actions implemented, the minimum being obtained for R=0.

When R=0, and for a given b, it is well known that action a_1 must be efficient and action a_2 must be distorted downward so as to reduce type-1 agent's ex-post utility, equal to $2R^0(b)$. As long as $R < R^0(b)$, the previous solution is interior to the set of constraints and therefore is optimal. Now when R becomes larger then $R^0(b)$, the previous solution violates EAIR. The action a_2 that allows to maintain EAIR as an equality increases. As increasing this action improves on efficiency, P^A increases a_2 while keeping a_1 at its efficient level. For large R, the full information productions (given b) can be implemented while giving the agent exactly his ex-ante reservation utility.

For fixed R, any equilibrium of the game must then be characterized by:

$$\mathbf{a} = \mathbf{A}(b,R) \text{ and } \mathbf{U} = \mathbf{U}(b,R)$$

$$b = \frac{3 - a}{2} \quad \text{where} \quad a = \frac{1}{2}(a_1+a_2).$$

The characterization of equilibria reveals that:[8]

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8 An Appendix, containing the complete treatment of this example, is

* for $R \leq R^0 \equiv \delta\,(1 - \frac{10}{3}\delta)$, the equilibrium (expected) quantities are $a^0 = 1 - \frac{4}{3}\delta$ and $b^0 = 1 + \frac{2}{3}\delta$; we denote P^A's expected payoff by Π^0;

* for $R^0 \leq R \leq R^1 \equiv \delta\,(1 - \delta)$, the equilibrium (expected) quantities are $a = a^0 + \frac{4}{7}\frac{(R - R^0)}{\delta}$ and $b = b^0 - \frac{2}{7}\frac{(R - R^0)}{\delta}$ and the principal's payoffs:

$$\Pi(R) = \Pi^0 + \frac{1}{7}\,(1 - \frac{4}{3}\delta)\,\frac{(R - R^0)}{\delta} - \frac{17}{98}\left(\frac{(R - R^0)}{\delta}\right)^2$$

If $R \leq R^0$, the equilibrium outcome is independent of R, in particular, it is the same as if R=0. As soon as $R^0 < R < R^1$, however, firm B expects A's production to increase with R and therefore B reduces its own production, as usual in Cournot models (strategic substitutability). This improves P^A's profit since the principal is interested in reducing her opponent's output. Of course, A's actions increase beyond a^0, but this has only a second order effect on P^A's profits, since a^0 corresponds to the unconstrained optimum, i.e. when the EAIR constraint is not binding. As a result, $\Pi(R)$ increases in R on $[R^0,\ R^0 + \frac{7}{17}\,(1-\frac{4}{3}\delta)[$. This means that P^A prefers to face an agent characterized by an ex-ante reservation utility R^0+dR rather than by any other level smaller or equal to R^0, <u>provided firm B knows this ex-ante reservation utility</u> R^0+dR.

Consider now an agent with $R=0$ and suppose that, prior to stage 1, P^A has the opportunity to propose a contract $t_0(a,b)$ to the agent. If the agent refuses, the game ends and everyone gets 0. If he accepts, the contract is publicly disclosed, and the game goes on to stage 1 and 2. Stage 1 is then a secret renegotiation stage with an ex-ante reservation level R that corresponds to the agent's expected utility, given an anticipated action for B, in a continuation path where $t_0(.)$ is enforced.

Now, is it possible that in equilibrium, P^A does not commit through a

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available upon request from the authors.

public contract in stage 0 ? If so, stage-1 negotiation would be driven by the true ex-ante reservation utility of 0, and would lead to $\mathbf{U}^0 \equiv (2R^0, 0)$ and Π^0. Suppose instead that P^A publicly commits to: $t_0 = R^0 + dR$, for all a and b. Whatever B decides to produce, if this contract were enforced, agent A would get an expected utility of R^0+dR; therefore, in the renegotiation stage, the relevant ex-ante reservation utility is publicly known to be R^0+dR. P^A would then receive a payoff $\Pi(R^0+dR)$ that she prefers to Π^0. Hence, <u>not signing a public contract is not an equilibrium</u>.

To summarize, there exist <u>precommitment effects of public contracting</u> in a Cournot situation, despite the fact that any public contract can be secretly renegotiated. This illustrates the main results that appear in Theorems 1, 3i and 4i.

<u>2.2/ The absence of precommitment effects in a Bertrand setting</u>

We will now consider a similar model in a situation of Bertrand competition between differentiated products. Let the demand functions be $q^A = 5 - 2p^A + \sigma p^B$ and $q^B = 5 - 2p^B + \sigma p^A$. The agent now chooses the price, but bears a cost of θq^A, and prices are verifiable.

<u>For a given price decision p^B</u>, let us define $a = q^A$ and $b = -\sigma p^B$. It is immediate to see that P^A and A's payoffs have the same expression as in the previous example. Consider the game consisting of stages 1 and 2, as in subsection 2.1. Note that, given that p^B is taken as given, it is equivalent at stage 1 for the agent to choose p^A or a: a choice of a_i must be interpreted as the choice of a price $p_i^A = \frac{1}{2}(5 - b - a_i)$. The previous reasoning still holds and the final equilibrium productions must verify:

$$\mathbf{a} = \mathbf{A}(-\sigma p^B, R) \text{ or equivalently, } p_i^A = \frac{1}{2}\left(5 - A_i(-\sigma p^B, R) + \sigma p^B\right)$$

$$\mathbf{U} = \mathbf{U}(-\sigma p^B, R).$$

Moreover, for all $R \geq 0$, $A_i(-\sigma p^B, R) \geq A_i(-\sigma p^B, 0)$: the price p_i^A is smaller than the price that P^A would have implemented in response to p^B, if the agent's ex-ante reservation utility were 0.

Taking into account the equilibrium condition $p^B = \frac{7}{4} + \frac{\sigma}{8}(p_1^A + p_2^A)$, the previous property translates into an equilibrium property: any possible equilibrium outcome following $R \geq 0$ is such that $p^B \leq p_0^B$, since a decrease in A's price induces a decrease in B's prices in the Bertrand logic (strategic complementarity). The effect of an increase of R beyond 0 is thus, on one side, to decrease the opponent's equilibrium price which reduces P^A's payoff, and on the other side, to increase the expected rent paid to the seller which also reduces P^A's payoff. Consequently, at stage 1, P^A prefers to face an agent with ex-ante reservation utility of 0 rather than of $R \geq 0$.

If, as in subsection 2.1, public contracting is possible at a stage 0, it is clearly an equilibrium strategy for P^A <u>not to commit through a public contract in stage 0</u>, since it leads, after stages 1 and 2, to the most preferred outcome for P^A, corresponding to $R=0$ (note that signing $t^0 \equiv 0$ would achieve the same outcome, and is therefore also an equilibrium strategy). In this case, we conclude that public contracting has no precommitment effects, when secret renegotiation is possible.[9] This result illustrates Theorem 2, 3ii and 4ii.

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[9] The analysis in both examples would be the same if the contract between P^A and A was a franchise contract with non-linear wholesale price. In this case, a contract would be a transfer function $t(a,p^A) = p^A a - q(a)$, where $q(a)$ is the wholesale price. The equilibrium will exhibit precommitment effects in the Cournot case (by decreasing the wholesale price), but not in the Bertrand case (increasing the wholesale price beyond its optimal level is not renegotiation proof).

3. THE GENERAL MODEL

The general situation we consider is summarized in the next diagram:

$$\begin{tikzcd} P^A \arrow[d, "\text{Contract}"'] & & P^B \arrow[d, "\text{Contract}"] \\ A \arrow[r, "\text{action } a"] & \boxed{\begin{array}{c}\text{market}\\ \text{or}\\ \text{general game } \Gamma\end{array}} & B \arrow[l, "\text{action } b"'] \end{tikzcd}$$

Two principals P^A and P^B each hire an agent A and B, via contracts, to act on their behalf on some market or more generally in some game. Contracts are fully enforceable, unilateral commitments by principals. They are offered by principals on a take-it-or-leave-it basis. A contract between P^A and A is a function of the agents' actions, a and b in game Γ, that determines monetary transfers from P^A to A: $t^A = t^A(a,b)$.

A contract may be *publicly observable*. However, we want to emphasize that secret renegotiations are always possible; therefore, we consider a global game that allows *renegotiation*.

We focus on situations of hidden knowledge between principals and agents. All contracts are signed under symmetric but imperfect information about a parameter θ^A or θ^B of the (dis)utility of playing the market game, i.e. at an ex-ante stage. After he has been hired, an agent privately learns the relevant parameter; at this interim stage, the agent can freely decide to quit the relationship. The possibility of costless breach of contract seems natural in employment relationships. In a situation of subcontracting between firms, similar interim participation constraints would arise with an assumption of limited liability.[10]

Formally, the game is the following.

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[10] See Sappington [1983] on limited liability contracts.

Stage 0: Public contracting: Each principal can offer a public contract to the agent or decide to wait for stage 1. If no offer is made, or if an offer is made and accepted, the game goes to stage 1. If an offer is made and rejected, the game between the two parties ends, the agent gets a payoff of 0, the principal gets $\underline{\Pi}^A$ or $\underline{\Pi}^B$. This stage is public.

Stage 1: Secret (re)negotiation: Each principal offers a secret contract to her agent. If the agent accepts, the new contract signed is enforced. If he rejects and a public contract has been signed, this public contract is enforced. If he rejects and no public contract has been signed, the game between the two parties ends with reservation payoffs of 0 and $\underline{\Pi}^A$ or $\underline{\Pi}^B$. Secret (re)negotiations are not observed by outside parties.

Stage 2: Agents' market game : An agent who has been hired, privately learns the parameter θ^A or θ^B. He may decide to quit and get his reservation payoff 0, his principal getting $\underline{\Pi}^A$ or $\underline{\Pi}^B$, or to stay and play a game in normal form Γ with pure strategy $a \in S^A=[\underline{a},\bar{a}]$ (or $b \in S^B=[\underline{b},\bar{b}]$). The final outcome (a,b) is public and verifiable, and contracts are enforced.[11]

This three stage game will be called game G. Stage 0 allows for the possibility of precommitment through public disclosure of contracts. Stage 1 leaves the opportunity of secret renegotiation open. Stage 2 is mostly meant to model a one-shot simultaneous move game. We abstract from the possibility of renegotiation due to some sequentiality of moves in a general extensive form game, as considered in Dewatripont [1988] and in Caillaud-Jullien-Picard [1990a]. We shall assume that $\underline{\Pi}^I$, the reservation

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11 We omit the description of the degenerate subgame with only one player, since only the payoffs of the non-contracting principal and agent are relevant for our concern. We could be perfectly rigorous by using the notation a or $b=\varnothing$ in the payoff functions to describe the situation where no agent plays on behalf of a principal in game Γ, be it because no contract was ever accepted or because the agent quit.

outcome for a principal, is sufficiently small (see Appendix A), compared to the payoff generated by any type of agent under incomplete information; that is, the principal loses much from not participating in the market or general game Γ. This assumption implies that, in any equilibrium, no principal will ever propose a contract that could be rejected or that could induce some type of agent to quit at stage 2.

In what follows, we shall also consider a game G_0, in which no public contracting is allowed. This game coincides with the subgame of G starting at stage 1 when no contract has been signed at stage 0. The basic issue we want to address is whether "no public contracting" is an equilibrium of G or not. We shall say that *precommitment effects exist* when no equilibrium outcome can be such that no public contract is signed in stage 0. In the opposite case, the equilibrium outcome of G_0 may be obtained as an equilibrium outcome of G. In this case, we shall also look for conditions that guarantee that any equilibrium outcome of G can be obtained as an equilibrium outcome of G_0.

We can now present the detailed description of the model. All assumptions are presented with respect to hierarchy P^A-A; but symmetric assumptions hold for hierarchy P^B-B. Parameters θ^A and θ^B are assumed to be independently determined by nature; θ^A is drawn from a C^2 cumulative distribution function $F^A(.)$ on $[\underline{\theta}^A,\bar{\theta}^A]$, with density $f^A(.)$.

Given transfers t^A from P^A to A and actions a and b chosen by the agents, payoffs are denoted Π^A for principal P^A, and U^A for agent A, with:

$$\Pi^A = \Omega^A(a,b,\theta^A) - t^A$$

$$U^A = t^A - v^A(a,\theta^A)$$

Let us define the aggregate payoff of hierarchy P^A-A by:

$$\phi^A(a,b,\theta^A) = \Omega^A(a,b,\theta^A) - v^A(a,\theta^A).$$

We assume that $\phi^A(.,.,.)$ is C^2, linear in b, strictly concave in a, and

$v^A(.,.)$ is C^3. From now on, we make the following specific assumptions:

A1: $v^A_\theta(.,.)$ is positive, increasing and weakly convex in a.

A2: **i)** $\dfrac{F^A(\theta^A)}{f^A(\theta^A)}$ is non-decreasing in θ^A,

ii) $\phi^A_{a\theta} \le 0$, $v^A_{a\theta\theta} \ge 0$.

iii) For all b, θ^A, $\dfrac{\phi^A_a}{v^A_{a\theta}}(\bar{a},b,\theta^A) < \dfrac{F^A(\theta^A)}{f^A(\theta^A)} < \dfrac{\phi^A_a}{v^A_{a\theta}}(\underline{a},b,\theta^A)$.

A3: $\forall\ (a,a',b,b',\theta^A,\theta^B),\quad 0 < \dfrac{\phi^A_{ab}}{\phi^A_{aa}}(a,b,\theta^A)\ .\ \dfrac{\phi^B_{ba}}{\phi^B_{bb}}(a',b',\theta^B) < 1.$

A1 and A2 are standard from contract theory (see e.g. Caillaud et al [1988]). $v^A_{a\theta}>0$ is the Spence-Mirrlees condition. The monotone likelihood ratio property (A2i) and the responsiveness conditions (A2ii) ensure that in an equilibrium of G_0, P^A and A negotiate a *separating* contract, with interior actions (A2iii). A3 is a global stability/unicity condition, familiar from the I.O. literature: it will ensure that the game G_0 has a unique equilibrium. The linearity in the opponent's action allows to restrict attention to expected actions instead of the complete distribution.

In the game G, strategies are contract proposals, both at stage 0 and 1, by principals, acceptance rules at stage 0 and 1 and actions (including stay or quit) at stage 2 by agents. Our equilibrium concept is *strong perfect Bayesian equilibrium* (hereafter equilibrium) as defined in Fudenberg-Tirole [1991]. As usual, we solve the game backward.

4. SECRET (RE)NEGOTIATION: THE SET OF EX-ANTE INCENTIVE EFFICIENT ACTIONS

In this section, we consider the subgame of G consisting of stages 1 and 2. At stage 1, the (public) history of the game is taken as given; it consists of $\mathcal{H} = (t^A_0(.), t^B_0(.))$, the pair of public contracts offered in stage

0, with $t_0(.) = \emptyset$ if no contract has been signed. Moreover, all players share the same prior beliefs $(F^A(.), F^B(.))$ on (θ^A, θ^B). This section studies the properties of the pair of A's type-contingent action and utility profiles, in any continuation equilibrium following $\mathcal{H}$. We shall omit superscripts A since the subgame is viewed from P^A and A's perspective.

Consider a (pure strategy) continuation equilibrium following $\mathcal{H}$, characterized by contracts $t^A(.)$ and $t^B(.)$ and profiles of type-contingent actions and type-contingent interim utility levels, $a(.)$ and $U(.)$ for agent A, $b(.)$ and $U^B(.)$ for agent B.[12] Let $d\mu^B(b)$ denote the image probability distribution of $dF^B(.)$ by $b(.)$ and let b^e denote agent B's average action. As noted in the previous section, $\underline{\Pi}$ is small enough so that in any equilibrium, a contract is eventually signed in every vertical structure and no type of agent ever quits at stage 2; therefore,

$$U(\theta) = \int_{S^B} \Big(t(a(\theta), b) - v(a(\theta), \theta) \Big) \, d\mu^B(b)$$

Let $V(\theta)$ denote the interim utility that a type-θ agent can get by refusing any new contract at stage 1, given the distribution of B's action μ^B. If $t_0(.) \neq \emptyset$ has been signed at stage 0, the agent may quit and get his reservation payoff, or he may stay under the terms of the public contract:

$$V(\theta) = \operatorname{Sup} \left\{ \operatorname*{Max}_{\underline{a} \le a \le \bar{a}} \int_{S^B} \Big(t_0(a, b) - v(a, \theta) \Big) \, d\mu^B(b) \; ; \; 0 \right\} \tag{1}$$

and if no public contract has been signed: $V(\theta) = 0$.

Let us define $R \equiv \int_{\underline{\theta}}^{\bar{\theta}} V(\theta) dF(\theta)$. R is A's ex-ante reservation utility at stage 1.

Given $t^B(.)$ and μ^B, equilibrium conditions imply: (a) that A's

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[12] Interim utilities are evaluated at stage 2, after the agent has learnt his type, decided to stay and played an action in Γ, but before the actual observation of the opponent's action.

decisions (acceptance of $t^A(.)$, stay and subsequent action in Γ) must be best response to $t^A(.)$ given $\mathcal{H}$, and (b) that there must not exist a contract that, if accepted by A, improves P^A's expected payoff. (a) above implies that, in equilibrium, for all $(\theta,\tilde{\theta}) \in [\underline{\theta},\bar{\theta}]^2$,

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)dF(\theta) \geq R \tag{2}$$

$$U(\theta) \geq 0 \tag{3}$$

$$U(\theta) - U(\tilde{\theta}) \geq v(a(\tilde{\theta}),\tilde{\theta}) - v(a(\tilde{\theta}),\theta) \tag{4}$$

(2) is an ex-ante individual rationality constraint. (3) is an interim individual rationality constraint (no quit). (4) is an incentive compatibility constraint; if it did not hold, some type would mimic the behavior of some other type.

For a fixed $\mu^B(.)$ and any pair of action and utility profiles, $\tilde{a}(.)$ and $\tilde{U}(.)$, there exists a contract that implements $\tilde{a}(.)$, $\tilde{U}(.)$ at stage 2. Then, using the linearity in b, equilibrium condition b) above implies the following lemma:

Lemma 1: Given history $\mathcal{H}$, any continuation equilibrium $\left(t^A(.),t^B(.),a(.),U^A(.),b(.),U^B(.)\right)$ must necessarily be such that:

$$a(.),U^A(.) \in \underset{\tilde{a}(.),\tilde{U}(.)}{\text{Argmax}} \int_{\underline{\theta}^A}^{\bar{\theta}^A} \left(\phi^A(a(\theta),b^e,\theta) - U^A(\theta)\right)dF^A(\theta) \tag{P}$$

s.t. (2), (3) and (4) hold for $\tilde{a}(.),\tilde{U}(.)$

where R^A (or V^A) in (2) is defined by (1); similarly for $(b(.),U^B(.))$.

(P) has a unique solution (see Appendix A) characterized by the pair of action and interim utility profiles, $\left(A(b^e,R,.),U(b^e,R,.)\right)$ and the value function $\Pi(b^e,R)$. For any (R,b^e), defining the ex-ante equilibrium utility:

$$R(b^e,R) = \int_{\underline{\theta}^A}^{\bar{\theta}^A} U(b^e,R,\theta)dF^A(\theta),$$

it is immediate to prove that, for any R and b^e,

$$\Big(A(b^e,R(b^e,R),.),U(b^e,R(b^e,R),.)\Big) = \Big(A(b^e,R,.),U(b^e,R,.)\Big)$$

The interpretation of Lemma 1 is therefore that, in equilibrium, the final outcome $(a(.),U(.))$ must be feasible, ex-ante incentive efficient for P^A-A, given $\mathcal{H}$ and P^B-B behavior, where feasibility accounts for the possibility that the agent quits at the interim stage. That is, given P^B-B behavior, the interim feasibility constraints (stay or quit) and the (interim) incentive constraints, there is no final outcome that increases the principal's (ex-ante) expected payoffs while preserving the agent's (ex-ante) expected utility. The remaining part of the section analyzes the solution of program (P), i.e. it characterizes the set of ex-ante incentive efficient outcomes in a problem of contracting under incomplete information. This part is therefore quite general and of independent interest.

At this point, let us remark that introducing an additional round of secret contract renegotiation at the interim stage would not change the set of continuation equilibrium outcomes of the game following $\mathcal{H}$. The fundamental reason is that feasible, ex-ante incentive efficiency implies feasible, interim incentive efficiency.[13] Therefore, a stage-1 contract that satisfies Lemma 1 is necessarily immune to interim renegotiation. Now, any final outcome of the game with ex-ante and interim renegotiation must satisfy Lemma 1, otherwise the solution to (P) would be proposed, accepted and not renegotiated at the interim stage, and would then be a profitable deviation for the principal. Since (P) has a unique solution for given $V(.)$ and μ^B under our assumptions (see Appendix A), this solution is also the best response to μ^B after $\mathcal{H}$, when interim negotiations are allowed, and

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[13] See Holmstrom-Myerson [1983] and Maskin-Tirole [1992]. Interim incentive efficiency should be understood w.r.t. the prior beliefs, since ex-ante contract proposals and acceptance/rejection decisions cannot affect the principal's beliefs in a strong perfect Bayesian equilibrium.

then, the set of continuation equilibria after $\mathcal{H}$ is preserved.

It will be important to compare $A(b^e,R,.)$ with some benchmark profiles that we define now. The full information optimal action profile for a given b^e, denoted $A^*(b^e,.)$, satisfies: for any $\theta \in [\underline{\theta},\bar{\theta}]$,

$$\underline{a} < A^*(b^e,\theta) \equiv \underset{\underline{a}\le a\le \bar{a}}{\mathrm{Argmax}}\ \phi(a,b^e,\theta) < \bar{a} \tag{5}$$

The solution of program (P) when $V=0$, i.e. when no stage 0 contract has been signed, will also play a central role in the following. It is the standard adverse selection solution of a principal - agent problem with only incentive constraints and interim, type-independent, individual rationality constraints. Under our assumptions, it is well known that, $A(b^e,0,\theta)$ is non-increasing in θ, and for any $\theta>\underline{\theta}$,

$$\underline{a} < A(b^e,0,\theta) < A^*(b^e,\theta) < \bar{a}$$

$$U(b^e,0,\theta) = \int_{\theta}^{\bar{\theta}} v_{\theta}\Big(A(b^e,0,x),x\Big)dx$$

$$R(b^e,0) = \int_{\underline{\theta}}^{\bar{\theta}} v_{\theta}\Big(A(b^e,0,\theta),\theta\Big)\ F(\theta)d\theta$$

We are now in a position to prove the following proposition.[14]

Proposition 1: For any profile of reservation utilities $V(.)$ corresponding to a reservation expected utility R, and any opponent's expected action $b^e \in S^B$, $A(b^e,R,\theta)$ is continuous in (b^e,R,θ) and $\forall\theta\in]\underline{\theta},\bar{\theta}]$,

if $R \le R(b^e,0)$, then $A(b^e,R,\theta) = A(b^e,0,\theta)$

if $R > R(b^e,0)$, then $A^*(b^e,\theta) \ge A(b^e,R,\theta) > A(b^e,0,\theta)$

This proposition contains the basic intuition for the rest of the paper. It provides a lower bound on the set of actions on which P^A is able to precommit through public contracting: $A(b^e,0,.)$. This limit profile is

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[14] All results in sections 4 and 5 are proved in Appendix A.

obtained precisely when P^A does not precommit at all through public contracting, i.e. under standard adverse selection conditions, when the agent's acceptance / rejection decision is only based on his interim, type-independent, reservation utilities, here normalized at zero. $A(b^e,0,.)$ is chosen so as to trade-off a marginal increase of type-θ's action, which induces a marginal increase of (full information) aggregate payoffs ϕ^A, with a marginal increase of all informational rents paid to types $\tilde{\theta}$, $\tilde{\theta}<\theta$, in excess of their reservation utilities. This trade-off leads to suboptimal actions and positive informational rents.

When considering a positive ex-ante reservation utility R, the previous solution may be altered only if R is larger than the expected utility associated with the optimal solution for $R=0$, i.e. when R is larger than $R(b^e,0)$. In this case, P^A must leave larger interim utilities to some types of agents to satisfy the ex-ante individual rationality constraint. In the trade-off mentioned above, an increase in interim utilities of types $\tilde{\theta}$ for ex-ante individual rationality reasons allows to increase type-θ's action without violating the incentive compatibility constraints, compared to $A(b^e,0,\theta)$. Therefore, the previous trade-off tips more in favor of increasing type-θ's action, and the final profile is less (ex-post) inefficient than $A(b^e,0,.)$.

The conclusion of this section is that P^A can gain a precommitment advantage by signing a public contract that modifies the ex-ante reservation utility of the agent at stage 1. This advantage necessarily takes the form of a precommitment to larger actions than without previous public contracting, given some beliefs about the opponent's actions. In more game-theoretical terms, it is a precommitment to a larger best response action profile in the continuation game, to the opponent's action profile.

5. EQUILIBRIUM ANALYSIS AND PRECOMMITMENT EFFECTS.

5.1/ Equilibrium without Public Contracts and Classification of Games

We first analyze the equilibrium when no contract has been signed at stage 0 (game G_0). This situation serves as a benchmark: since contracts are not observed, a hierarchy cannot affect its rival's future behavior and the game is in fact simultaneous as far as the interaction between hierarchies is concerned.

Define:

$$A^0(b^e) = \int_{\underline{\theta}^A}^{\bar{\theta}^A} A(b^e,0,\theta^A)dF^A(\theta^A) \tag{6}$$

$$B^0(a^e) = \int_{\underline{\theta}^B}^{\bar{\theta}^B} B(a^e,0,\theta^B)dF^B(\theta^B) \tag{7}$$

Lemma 1 implies that if an equilibrium outcome of G_0 is obtained at expected actions a^0 and b^0, these must satisfy:

$$a^0 = A^0(b^0) \quad \text{and} \quad b^0 = B^0(a^0) \tag{8}$$

Equilibrium actions and payoffs for hierarchy P^A-A are then given by $A(b^0,0,\theta^A)$, $U^A(b^0,0,\theta^A)$ and $\Pi^A(b^0,0)$. One equilibrium strategy is to offer the contract:

$$t^{A0}(a) = U^A(b^0,0,\theta^A) + v(a,\theta^A) \qquad \text{if } a = A(b^0,0,\theta^A),$$

$$t^{A0}(a) = -\infty \qquad \text{otherwise,}$$

which A accepts and which induces A to stay whatever his type.

Proposition 2: G_0 has a unique equilibrium outcome, (a^0,b^0) defined in (8).

Proof: $A^0(b)$ and $B^0(a)$ are continuous and a.e. differentiable:

$$\frac{\partial A^0(b)}{\partial b} = \int_{\underline{\theta}^A}^{\bar{\theta}^A} \frac{\partial A}{\partial b}(b,0,\theta)dF^A(\theta) = - \int_{\underline{\theta}^A}^{\bar{\theta}^A} \frac{\phi^A_{ab}}{\phi^A_{aa} - \frac{F^A}{f^A} v_{aa\theta}} (A(b,0,\theta),b,\theta)\ dF^A(\theta)$$

Since $v^A_{aa\theta}$ is positive, the global stability assumption A3 implies:

$$0 \le \frac{\partial A^0(b)}{\partial b} \cdot \frac{\partial B^0(a)}{\partial a} < 1 ,$$

Therefore a^0 and b^0 that solve (8) are uniquely defined. ∎

Starting from an initial situation with no public contract, P^A may consider whether to offer a public contract or not. If she offers a public contract, she can only induce her agent to increase his action. This will in turn induce P^B to increase or decrease B's expected actions depending on whether expected actions are strategic complements or strategic substitutes in the game G_0. This leads naturally to a first classification of games (see e.g. Fudenberg-Tirole [1984]):

<u>Classification S</u>

SS (Strategic Substitute): $\phi^A_{ab} < 0$ and $\phi^B_{ba} < 0$

SC (Strategic Complement): $\phi^A_{ab} > 0$ and $\phi^B_{ba} > 0$

We can cross this first criterion with a second one related to how each principal would like her opponent to move her action:

<u>Classification E</u>

PE (Positive Effect): $\phi^A_b > 0$ and $\phi^B_a > 0$

NE (Negative Effect): $\phi^A_b < 0$ and $\phi^B_a < 0$

For example, the model of Cournot and Bertrand competition described in Section 2.1 and 2.2 lead respectively to SS-NE and SC-NE.[15] In the following, we will omit the asymmetric cases where both hierarchies do not satisfy the same criteria, since these cases do not raise new difficulties.

5.2/ <u>Precommitment Effects, General Results.</u>

Let us first provide the intuition for the existence of precommitment

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[15] In the case of Bertrand, action *a* should be defined as the opposite of the price; moreover, one has to consider an extended framework where *v* can depend on *b*, since B's price affects A's production. This extension is presented in section 6.1.

effects. Suppose that P^B-B have not signed a public contract at stage 0. In this case, B will choose the action $B^0(a^e)$. Let $\mathcal{C}^0$ be an optimal contract at the equilibrium of G_0. Suppose that we start from the equilibrium of G_0 and that P^A proposes a public contract $\mathcal{C}$ close to $\mathcal{C}^0$, but leading to actions slightly above $A^0(b^0)$. Then the direct effect of $\mathcal{C}$ on P^A's payoff is second order since $\mathcal{C}^0$ is optimal given b^0. $\mathcal{C}$ has an indirect effect through the variation of B's expected action. The sign of this variation is the sign of the slope of the average reaction curve of the opponent hierarchy, which, in turn, is the sign of the slope of full information reaction curves, i.e. the sign of $-\phi^B_{ab}/\phi^B_{bb}$ or of ϕ^B_{ab}. The sign of the effect on P^A's payoff is the sign of $\phi^B_{ab}\phi^A_b$, obtained by crossing the classifications S and E. Therefore we find two cases where precommitment effects exist:

Theorem 1 : If SS and NE, or SC and PE, hold, then precommitment effects exist: not signing a public contract is not an equilibrium of G.

The proof proceeds along the lines suggested above, but it requires some care because there may well exist a multiplicity of continuation equilibria when at least one public contract has been signed at stage 0. The proof also shows that S and E could take a local, much weaker form.

We view Theorem 1 as providing a possible explanation of the practice of public disclosure of contracts between firms when the agreement is about pre-competitive investments, such as R&D investments or strategic alliances.[16] One can also argue that Cournot-style competition (and negative

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16 For example, one of the goals of the highly mediatized agreement between IBM and French state-owned Bull was the development of the RISK technology; see also the public announcement of preliminary (and obviously renegotiable) agreements between Boeing and DASA about the marketability of a 600-seat plane.

direct effects) may reflect interactions in markets with highly regulated prices such as in the agricultural and food industry.

There also exist situations where precommitment effects may not exist, i.e. such that the equilibrium outcome of G_0 (with unobserved contracts) is also an equilibrium outcome of G. The reasoning is very similar to the previous one. Contracts have two purposes: they have "internal" purposes to overcome asymmetric information problems between principals and agents, and "external" purposes to affect the opponent's renegotiation and action play. Internal purposes are maximized when no constraints are imposed by public contracting, since, then, no precommitment effects arise. Now if external purposes can only be worsen by resorting to public contracting, principals will sign contracts that maximize internal concerns only. And this is the case if committing to larger actions, on average, can only affect the average (equilibrium) opponent's action in the wrong direction, i.e. decrease it whereas PE holds or increase it whereas NE holds.

Theorem 2 : If SS and PE, or SC and NE, hold, then, not signing a public contract is an equilibrium of G. Under SC and NE, it is a Pareto dominant equilibrium for the pair of principals.

The theorems are stated in a weak form. Theorem 1 does not exclude the possibility that the actions and transfers of the equilibrium of G_0 be obtained as the outcome of some equilibrium of G. It just implies that, if this is the case, such an equilibrium cannot be sustained by secret contracts only: at least one firm must sign a *public contract*. The possibility of such an outcome stems from the fact that a contract, say $t_0^A(.)$, may depend upon the opponent's action b; such a dependency, if publicly disclosed, can serve as a credible precommitment for hierarchy P^A-A to renegotiate very high actions if, following the disclosure of a public contract $t_0^B(.)$, P^A believes that B's action will differ from b^0.

Similarly Theorem 2 does not say that the equilibrium allocation of G_0 is the unique equilibrium allocation of G. This is due to the same reason. Notice however that, from the principals' point of view, the Pareto optimality of the "no public contract" equilibrium outcome, along with its simplicity in terms of strategies, provides a good case for this equilibrium in the case SC-NE.

5.3/ Specific Results

In this subsection, we point to some economically relevant situations where stronger results can be obtained.

It is clear that a hierarchy which is not subject to asymmetric information cannot use public contracts as a precommitment device. Indeed, whatever the initial public contract, the hierarchy will always renegotiate so as to choose the action that maximizes the full information aggregate payoff. This leaves the opponent free to use all the precommitment possibilities. Such a situation might be relevant to describe competition between an independent seller and an integrated seller. We then obtain:

Theorem 3: Assume $\underline{\theta}^B=\bar{\theta}^B$, then:

i) under SS-NE and SC-PE, in any equilibrium of G, P^A signs a public contract at stage 0 and A's equilibrium expected action differs from a^0

ii) under SS-PE and SC-NE, all equilibrium outcomes of G coincide with the equilibrium outcome of G_0 in terms of final actions and transfers.

A second possibility to strengthen the results is to restrict attention to games where only non-contingent contracts can be signed, i.e. a contract is a transfer function $t^A(a)$. These games may be of some practical relevance when the opponents' behavior may be known but not verifiable, or costly to verify. Admittedly, in other situations, contracts may very well be indexed on industry wide variables, in which case, the strong form of the results, as it appears in Theorem 4, may not hold.

To refine the results, one has to guarantee uniqueness of the continuation equilibrium after a public contract has been signed. Unfortunately, it turns out that, in the general case, the slope of the expected reaction curves, when a hierarchy is committed to a public contract at the renegotiation stage, may vary in a non trivial way (it may even change sign). This leads us to the following definition.

Definition: The game G is R-monotonic (for Renegotiationproof-monotonic) if $a(b,R^A)$ has the same monotonicity in b for every R^A, where

$$a(b,R^A) \equiv \int_{\underline{\theta}^A}^{\bar{\theta}^A} A(b,R^A,\theta^A)dF^A(\theta^A)$$

and similarly for $b(a,R^B)$.

R-monotonicity extends the properties of the average reaction curves in G_0 to any possible continuation equilibrium. In particular, R-monotonicity guarantees that the classification according to the criteria S and E extends to any continuation game after public contracting, and it moreover implies that the corresponding stability condition holds.

Theorem 4: Assume G is R-monotonic and that only non-contingent contracts are allowed. Let a^* and b^* be expected actions obtained at some equilibrium of G, then:

i) under SS-NE or SC-PE: $a^* \neq A^0(b^*)$ and $b^* \neq B^0(a^*)$;

ii) under SS-PE and SC-NE: $a^* = a^0$ and $b^* = b^0$.

As an example, one can easily show, using the formulas provided in Appendix A, that if $\phi^A$and ϕ^B are quadratic respectively in a and b, and v^A

and v^B are linear, then the game G is R-monotonic.

6. EXTENSIONS

This section first mentions an extension of Theorems 1 and 2 to situations where the cost of action of agent A may depend on B's action directly. Then we investigate the robustness of our results to the specification of other extensive forms for game G. There are obvious modifications that would not change the set of continuation equilibria after any history $\mathcal{H}$ at the beginning of stage 1. We could allow the game to go to stage 1 even after a rejection at stage 0; at an ex-ante stage, no information can be conveyed by a rejection so that this modification does not affect the set of equilibria of the game. Equivalently, we could assume that only stage 0 contracts are public, not all stage 0 decisions. We could consider several rounds of secret renegotiation without affecting the analysis. Finally, the analysis would not be changed if contracts were also renegotiated at an interim stage, i.e. after the agent learns his type and before he plays in Γ (or quits). Our four theorems are then robust to these modifications.

6.1 Direct externalities between agents

As already suggested, the Bertrand example of section 2.2 does not fit into the general framework presented so far, since B's action has a direct effect in terms of A's cost of undertaking his own action. To encompass such cases, we allow v^A to depend on b. We assume separability and linearity:

$$v^A(a,b,\theta^A) = w^A(a,\theta^A) + m^A(b,\theta^A) \tag{9}$$

where m^A is linear in b. Nothing is changed in A1-A2-A3, and Proposition 1 is still valid. The new issue is, however, that a modification of the opponent's actions induced by public disclosure also affects the agent's cost of action. To respect individual rationality, the principal must compensate the agent for this effect, which could be prohibitive. Then, let us add the following assumption:

A4: $\Omega^A_b \cdot m^A_b \leq 0$, and similarly for B.

A4 guarantees that, *ceteris paribus*, both contracting parties P^A and A agree on the desirability of increasing or decreasing the opponent's action. Appendix A shows that this is sufficient to preserve Theorems 1 and 2; indeed, the proofs are all presented with the externality terms.

Theorem 5: Under (9), A1-A2-A3-A4, Theorems 1 and 2 are still valid.

We continue to consider the general framework with (9) and **A4** in the other extensions.

6.2 Privately informed agents at stage 0

Consider a situation where the agents learn their types before stage 0 begins. Then the game is played as described in section 3.[17] This situation is close to common practices in franchising or in labor contractual relationships that are covered by collective agreements where firms hire a large number of agents according to typical public contracts, which does not preclude from privately adding specific provisions later on.

As in section 4, we first consider a continuation equilibrium following a public history $\mathcal{H}$ after stage 0, from P^A-A's perspective (superscripts A omitted). It is characterized by contracts $t(.)$ and $t^B(.)$, action and interim utility profiles $a(.)$, $b(.)$, $U(.)$ and $U^B(.)$, and assuming $\underline{\Pi}$ low

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17 In this case, it is important to assume that the game ends after a rejection at stage 0, as discussed below.

enough, principals still have prior beliefs F^A, F^B (see also previous footnote). $V(.)$, defined by (1), is agent A's interim utility profile if no new contract is signed at stage 1. Individual rationality at stage 1 now imposes:

$$\forall\theta\in[\underline{\theta},\bar{\theta}],\ U(\theta) \geq V(\theta) \tag{10}$$

instead of (2). Given $\mathcal{H}$ and μ^B, equilibrium profiles must solve program (P') instead of (P), where (2) is replaced by (10). Let $\mathbf{A}(b^e,V(.),\theta)$ and $\mathbf{U}(b^e,V(.),\theta)$ for $\theta\in[\underline{\theta},\bar{\theta}]$ respectively denote the profiles of actions and interim utilities that solve (P'). Again, note that this solution also solves (P') where $V(.)$ in (10) is replaced by the function that maps θ to $\mathbf{U}(b^e,V(.),\theta)$. Hence, in equilibrium, implemented profiles are feasible, interim[18] incentive efficient for P^A-A, given $\mathcal{H}$ and the opponent's behavior.

If $V\equiv 0$, it is immediate that (P') is equivalent to (P) with $R = 0$. Therefore, in this case, the solution to (P') is $A(b^e,0,\theta)$ and $U(b^e,0,\theta)$.

Proposition 3 below characterizes the set of feasible, interim incentive efficient action profiles; as such, it is of general and independent interest.[19]

Proposition 3: For any profile $V(.)$ and any expected action b^e by the opponent, $\mathbf{A}(b^e,V(.),\theta)$ is continuous in b^e and θ, and for all $\theta\in[\underline{\theta},\bar{\theta}]$,

$$\mathbf{A}(b^e,V(.),\theta) \geq A(b^e,0,\theta)$$

As in the ex-ante case developed in Proposition 1, $A(b^e,0,\theta)$ is a lower bound on the set of actions on which P^A is able to precommit through public contracting. This limit profile is obtained precisely when P^A does not

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18 for prior beliefs

19 All results of this subsection are proved in Appendix B.

precommit at all through public contracting; it leads to suboptimal actions, to solve the trade-off explained in section 4.

In (P'), the previous solution may be altered only if, for some type θ, $V(\theta) > U(b^e,0,\theta)$. If, in this case, P^A decides to implement the profile $A(b^e,0,\theta)$, she has to increase interim utilities, for all $\theta\in[\underline{\theta},\bar{\theta}]$, up to:

$$U_1(\theta) = U(b^e,0,\theta) + \left\{V(\theta_1) - U(b^e,0,\theta_1)\right\}$$

where θ_1 is the largest element of $\underset{\theta}{\text{Argmax}} \left\{V(\theta) - U(b^e,0,\theta)\right\}$. Consider another possible pair of action and interim utility profiles, $a(.)$ and $U(.)$, under the condition $U(\theta_1) \geq V(\theta_1)$. Since $A(b^e,0,.)$ is the optimal action profile over $[\underline{\theta},\theta_1]$ under the condition at θ_1, P^A cannot do better than $a(.) = A(b^e,0,.)$ on $[\underline{\theta},\theta_1]$. On $[\theta_1,\bar{\theta}]$ however, all interim individual rationality constraints are slack for the profile $A(b^e,0,.)$; it is then feasible and profitable for P^A to increase actions of all types above θ_1, $a(\theta) > A(b^e,0,\theta)$ on $[\theta_1,\bar{\theta}]$ and therefore decrease the agent's interim utility for those types, $U(\theta) < U_1(\theta)$ on $[\theta_1,\bar{\theta}]$, while preserving incentive compatibility. This clearly suggests that type-dependent reservation utility profiles may lead to larger actions being implemented.

Proposition 3 shows that, by signing a public contract, P^A can gain an advantage that necessarily takes the form of a precommitment to larger actions than without previous public contracting. Consequently, the logic of general results on precommitment effects with interim public and secret negotiations is the same as with ex-ante public and secret negotiations.

Theorem 6: In the game with interim public contracting, Theorems 1 and 2 still hold.

Considering an extensive form where the game proceeds to stage 1 even after a rejection of a public contract at stage 0 may admittedly constitute a more attractive timing to describe business practices related to exceptional public disclosure of top managers' compensation schemes, or to exceptional announcement of suppliers' or distribution contracts. But such a timing raises technical difficulties. If agents may be hired after rejecting a public offer at stage 0, the principal can use the acceptance / rejection decision at stage 0 as a screening device that would lead to updated beliefs when negotiating at stage 1. Another difficulty arises if the acceptance / rejection decision is public; through this decision, the principal can make the agent partly signal his type to the opponent.[20] The global equilibrium would then mix phenomena due to precommitment effects as defined earlier and phenomena related to the choice of an information structure at stage 1 and to signaling issues.

6.3 Ex-ante public contracting and interim renegotiations

The complicated issues mentioned at the end of the previous sub-section do not arise if public contracting occurs at an ex-ante stage, i.e. when types are not known by agents. Consider then a new timing for game G where agents learn their types between stage 0 and stage 1; public contracting occurs ex-ante while renegotiation is interim.[21]

In this case, public contracting still affects the renegotiation stage through interim individual rationality constraints (10). Equilibria of the game and characterization of precommitment effects are thus similar to the case described in sub-section 6.2, when public contracting occurs at an interim stage.

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20 See Caillaud-Hermalin [forthcoming] for a simple model along these lines.

21 Dewatripont [1988] and Beaudry-Poitevin [1992] consider similar timings.

Note, however, that this version of the game is somewhat artificial since we allow hierarchies to sign public contracts at an ex-ante stage but forbid them from renegotiating these contracts at an ex-ante stage. Allowing such a possibility would bring us back to our basic model of sections 3-5.

6.4 Binding contracts

In the games considered so far, agents were allowed to quit after having learnt their types and after all (re)negotiations were over. This assumption introduces a lower bound on the interim utility profile at the renegotiation stage, summarized by (3), as would be the case with a limited liability assumption or an assumption of infinite risk aversion. In this sub-section, we show that this property plays a crucial role in our basic model, although not in the model of subsection 6.1.

In our basic model, with ex-ante public contracting and ex-ante secret renegotiation, fully binding contracts, i.e. with no quit or limited liability condition, will lead the principals to renegotiate to full information best response action profiles $A^{*}(b^{e},.)$; the reason is that, at stage 1, for a given R^{A} (defined without the quit opportunity), renegotiation will solve program (P) where (3) is omitted. It follows that no precommitment effects will ever arise in this situation, independently of the assumptions about strategic complementarity or substitutability and about the direct effect of the opponent's action.

In the framework of sub-section 6.2, with interim public contracts and secret renegotiation and with binding contracts, renegotiations lead to a solution of program (P') where (3) is omitted and where $V(.)$ may possibly be negative, depending on the history $\mathcal{H}$. The final outcome still satisfies Proposition 3 in terms of action profiles. In any equilibrium of the global game in this case, a type-θ agent cannot end up with negative interim utility at the beginning of stage 2, since he should have rejected the initial public offer in the first place. Moreover, with $\underline{\Pi}$ low enough, there cannot be equilibria of the game where some type of agent rejects a public offer at stage 0. This means that any equilibrium with binding contracts, would also be an equilibrium with non-binding contracts, i.e. with a quit opportunity. This implies that Theorem 1 would still hold and precommitment effects may exist with fully binding contracts. Finally, note that Theorem 2 would still hold, as the proof does not rely at all on this aspect of the model.

7. CONCLUSION

Although in most contracting situations, legal rules may impose restrictions on the form of the renegotiation process, we see renegotiation as a widespread phenomenon. Basically one cannot effectively prevent renegotiation between agreeing parties. (In particular renegotiation may be implicit in the context of long term relationship.) Therefore a sensible theory of precommitment through contracts has to take into account renegotiation stages.

The paper shows that precommitment effects can obtain when allowing for renegotiation at all stages of the game under asymmetric information. But mostly it provides a clear set of conditions for these effects to occur effectively or not. Contracts can only be altered so as to increase the cost incurred by the agent and the informational rent transferred. Moreover the results are shown to be robust to changes in the informational structure of the game.

The analysis has considered only vertical structures including a single agent. For some situations, it would be of interest to analyze the multiple-agent framework, in particular with strong interactions or side-contracting and delegation between agents.[22] In particular, side-contracting between agents may limit renegotiation possibilities.[23] The conclusions of the present paper may then not extend to this case.

To conclude, if one thinks in terms of imperfect competition on product markets, it appears that when renegotiation is possible, firms can only precommit to be tougher competitors, at the benefit of consumers. This provides a good case for laissez-faire on this type of practice. On the other end, firms would benefit from preventing any disclosure of contracts. There may then be a tendency to include secrecy as an implicit rule of corporate culture, calling for an activist competition policy.

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[22] Rey-Stiglitz [1987] is precisely along this line: exclusive territories are used to restrict intrabrand competition among retailers, which allows precommitment on local markets.

[23] If the principal proposes to renegotiate an initial contract, agents may operate some arbitrage between contracts by having only part of the agents accepting the new contract and subcontracting with the others, which limits the advantage of renegotiation for the principal.

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APPENDIX A: EX-ANTE CONTRACTS

The results are proved in the more general context of the model presented in 6.1, with externality terms $m(.)$ satisfying (9) and **A4**. A reference in optimal control theory is Seierstad-Sydsæter [1987].

Proposition 1 and related results

For this part, we fix b^e. We analyze the following program:

$$\underset{a(.),U(.)}{\text{Max}} \int_{\underline{\theta}}^{\bar{\theta}} \left(\phi(a(\theta),b^e,\theta) - U(\theta)\right) f(\theta)d\theta \tag{PA}$$

$$\text{s.t. } \forall(\theta,\tilde{\theta})\in[\underline{\theta},\bar{\theta}]^2, \quad U(\theta) \geq 0 \tag{1A}$$

$$U(\theta) \geq U(\tilde{\theta}) + \int_{\theta}^{\tilde{\theta}} v_{\theta}(a(\tilde{\theta}),b^e,x)dx \tag{2A}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)f(\theta)d\theta \geq R \tag{3A}$$

$$a(\theta) \in \{\varnothing\} \cup [\underline{a},\bar{a}]$$

$$v(\varnothing,b^e,\theta) = 0 \text{ and } \phi(\varnothing,b^e,\theta) = \underline{\Pi}$$

(2A) is a straightforward transformation of (4). $a(\theta)=\varnothing$ corresponds to the situation where type-θ agent quits or where no contract has been signed.

General analysis for arbitrary values of $\underline{\Pi}$: existence of a cut-off point

If there exist θ_1,θ_2 such that $\theta_2>\theta_1$, $a(\theta_1)=\varnothing$ and $a(\theta_2)\neq\varnothing$, incentive compatibility imposes:

$$U(\theta_2) \geq U(\theta_1) \geq U(\theta_2) + \int_{\theta_1}^{\theta_2} v_{\theta}(a(\theta_2),b^e,x)dx > U(\theta_2)$$

This is clearly impossible, and therefore there exists $\theta^*\in[\underline{\theta},\bar{\theta}]$ such that for any $\theta\in[\underline{\theta},\theta^*[$, $a(\theta)\neq\varnothing$ and for any $\theta\in]\theta^*,\bar{\theta}]$, $a(\theta)=\varnothing$ at the optimum. Incentive compatibility implies that for any $\theta\in]\theta^*,\bar{\theta}]$, $U(\theta)=U^*$.

Within $[\underline{\theta},\theta^*[$, it is well known that incentive compatibility constraints (2A) are equivalent to:

$$\begin{cases} U(\theta) = U(\theta^*) + \int_{\theta}^{\theta^*} v_{\theta}(a(x),b^e,x)dx & \text{(4A)} \\ a(.) \text{ non-increasing} & \text{(5A)} \end{cases}$$

Moreover, incentive compatibility around θ^* implies that $U(\theta^*)=U^*$. After integrating (4A) by part in the maximand, Program (PA) reduces to:

$$\underset{a(.),U^*,\theta^*}{\text{Max}} \int_{\underline{\theta}}^{\theta^*}\left(\phi(a(\theta),b^e,\theta) - \frac{F(\theta)}{f(\theta)} v_\theta(a(\theta),b^e,\theta)\right)f(\theta)d\theta + (1-F(\theta^*))\underline{\Pi} - U^*$$

s.t. $U^* \geq 0$ (6A)

$a(.)$ non-increasing (7A)

$$\int_{\underline{\theta}}^{\theta^*} v_\theta(a(\theta),b^e,\theta)F(\theta)d\theta + U^* \geq R \qquad (8A)$$

$a(\theta) \in [\underline{a},\bar{a}]$ for any $\theta \in [\underline{\theta},\theta^*[$

Necessary and sufficient conditions of optimization

As usual, we start by neglecting the monotonicity constraint (7A), as well as the boundary conditions on a. With multipliers τ and γ associated respectively to (6A) and (8A), the Lagrangean of this relaxed problem is:

$$\mathcal{L}\left(a(.),U^*,\theta^*,\gamma,\tau;b^e,R\right) = \int_{\underline{\theta}}^{\theta^*}\left(\phi(a(\theta),b^e,\theta) - (1-\gamma)\frac{F(\theta)}{f(\theta)} v_\theta(a(\theta),b^e,\theta)\right)f(\theta)d\theta$$
$$+ (1-F(\theta^*))\underline{\Pi} - (1-\tau-\gamma)U^* - \gamma R \qquad (9A)$$

Given the concavity of ϕ and v_θ in a, the first order conditions with respect to $a(.)$ and U^* are necessary and sufficient:

$$\phi_a(a(\theta),b^e,\theta) = (1-\gamma)\frac{F(\theta)}{f(\theta)} v_{a\theta}(a(\theta),b^e,\theta) \qquad (10A)$$

$$1 - \gamma - \tau = 0 \qquad (11A)$$

plus the complementary slackness conditions.

A sufficient condition for all types of agents to stay in equilibrium

Let us note that if:

$$\underline{\Pi} < \underset{(a,b,\theta)}{\text{Min}}\left\{\phi(a,b,\theta) - \frac{\phi_a(\underline{a},b,\theta)}{v_{a\theta}(\underline{a},b,\theta)} v_\theta(a,b,\theta)\right\} \qquad (12A)$$

then, whatever the solution in terms of $a(.)$, the derivative of the Lagrangean in θ^* is positive, and therefore $\theta^*=\bar{\theta}$. As suggested in section 4 of the text, we assume (12A) holds all along the paper and in this appendix. But the presentation above clearly shows that the model could easily be adapted to a more general setting.

Characterization of the solution and proof of Proposition 1

$\tau=0$ would yield the full information action profile, $A^*(b^e,.)$ and then,

$$U^* = R - \int_{\underline{\theta}}^{\bar{\theta}} v_\theta(A^*(b^e,\theta),b^e,\theta)F(\theta)d\theta \equiv R - R^*(b^e)$$

(6A) then implies that $R \geq R^*(b^e)$. Conversely, it is immediate that if $R \geq R^*(b^e)$, the solution is to implement $A^*(b^e,.)$, which is interior from A2iii.

$\gamma=0$ corresponds to $A(b^e,0,.)$, which is interior from A2iii, and an expected rent for the agent of $R(b^e,0)$. (8A) implies that $R \leq R(b^e,0)$. Conversely, if $R \leq R(b^e,0)$, the solution that ignores (8A) is precisely $A(b^e,0,.)$ and therefore satisfies (8A).

Then for $R \in]R(b^e,0),R^*(b^e)[$, both γ and τ are positive, and $U^*=0$. (10A), A1 and A2 ensures that the solution obtained is interior and non-increasing in θ; so it is the solution $A(b^e,R,.)$ of the original program. The concavity of ϕ and v_θ in a immediately implies: $\forall\theta\in]\underline{\theta},\bar{\theta}]$,

$$A(b^e,0,\theta) < A(b^e,R,\theta) < A^*(b^e,\theta) \tag{13A}$$

and finally, $A(b^e,R,.)$ is clearly continuous in every argument. This completes the proof of Proposition 1.

<u>Further useful results</u>

On $R\in[R(b^e,0),R^*(b^e)]$, $\left(A(b^e,R,.),\gamma\right)$ is the solution to (10A) and:

$$\int_{\underline{\theta}}^{\bar{\theta}} v_\theta(A(b^e,R,\theta),b^e,\theta)F(\theta)d\theta \equiv R \tag{14A}$$

By the implicit function theorem, the restriction of $A(b^e,R,\theta)$ on this domain is C^1 in every argument. Differentiating (10A) yields:

$$\frac{\partial A}{\partial R}(b^e,R,\theta) = \frac{-v_{a\theta}(\%)\dfrac{F(\theta)}{f(\theta)}}{\phi_{aa}(\%) - (1-\gamma)\dfrac{F(\theta)}{f(\theta)}v_{aa\theta}(\%)}\frac{\partial\gamma}{\partial R} \equiv \lambda(\%)\frac{\partial\gamma}{\partial R}$$

$$\frac{\partial A}{\partial b}(b^e,R,\theta) = \frac{-\phi_{ab}(\%) - v_{a\theta}(\%)\dfrac{F(\theta)}{f(\theta)}\dfrac{\partial\gamma}{\partial b}}{\phi_{aa}(\%) - (1-\gamma)\dfrac{F(\theta)}{f(\theta)}v_{aa\theta}(\%)} \equiv \nu(\%) + \lambda(\%)\frac{\partial\gamma}{\partial b}$$

where (%) denotes $\left(A(b^e,R,\theta),b^e,\theta\right)$. Differentiating (14A) then gives:

$$\frac{\partial \gamma}{\partial R}\int_{\underline{\theta}}^{\bar{\theta}} v_{a\theta}(\%)\lambda(\%)F(\theta)d\theta = 1$$

$$\frac{\partial \gamma}{\partial b}\int_{\underline{\theta}}^{\bar{\theta}} v_{a\theta}(\%)\lambda(\%)F(\theta)d\theta + \int_{\underline{\theta}}^{\bar{\theta}} v_{a\theta}(\%)\nu(\%)F(\theta)d\theta = 0$$

As $\lambda(\%)$ and $v_{a\theta}(\%)$ are positive, one obtains $\frac{\partial \gamma}{\partial R} > 0$ for $R \in [R^0(b^e), R^*(b^e)]$, and therefore, on this domain,

$$\frac{\partial A}{\partial R}(b^e, R, \theta) > 0 \tag{15A}$$

And since $\nu(\%)$ has the same sign as ϕ_{ab}, $\frac{\partial \gamma}{\partial b}$ has the same sign as $(-\phi_{ab})$.

■

Proof of Theorem 1

We focus on the case SS-NE, the other case being similar, and, when no confusion is possible, we will omit the superscripts A.

We start from the equilibrium outcome of G_0 and consider a deviation for P^A consisting in proposing a public contract close to $\mathscr{C}^0$. More precisely, for a given (small) δ, define $\hat{a}(x,\theta)$, $\hat{a}(x)$, $\hat{R}(x)$ and $\hat{\gamma}(x)$, for any $x \in [0,\delta[$, by:

$$\hat{a}(x,\theta) = A(b^0 - x, \hat{R}(x), \theta) \tag{16A}$$

$$\hat{a}(x) = \int_{\underline{\theta}}^{\bar{\theta}} \hat{a}(x,\theta)f(\theta)d\theta \tag{17A}$$

$$b^0 - x = B^0(\hat{a}(x)) \tag{18A}$$

$\hat{\gamma}(x)$ is the value of the Lagrange multiplier associated to the ex-ante individual rationality constraint in P^A's problem corresponding to $R=\hat{R}(x)$ and $b^e = b^0 - x$. For $x=0$, the situation corresponds to the equilibrium outcome of G_0. For small values of x, it is then possible to derive some properties of the close-by situation described by (16A)-(17A)-(18A), which are recorded in the following claim.

Claim 1: There exists δ small enough so that $\hat{R}(.)$, $\hat{a}(.)$, $\hat{a}(.,.)$ and $\hat{\gamma}(.)$ are uniquely defined and C^1 on $[0,\delta[$; moreover $\frac{\partial\hat{a}}{\partial x}(x,\theta) > 0$.

Proof of Claim 1: For $x=0$,

$$b^0 = B^0(A^0(b^0))$$

$B^0(A^0(.))$ is a contraction, so that there exists δ_1 such that, $\forall x\in]0,\delta_1[$,

$$B^0(A^0(b^0-x)) > b^0-x \tag{19A}$$

From the proof of Proposition 1 ("further results"), for $R\in[R(b^0-x,0),R^*(b^0-x)]$, $B^0\left(\int_{\underline{\theta}}^{\bar{\theta}}A(b^0-x,R,\theta)dF(\theta)\right)$ is C^1 in x and R, and from (15A) and SS, it is strictly decreasing in R, starting from $B^0(A^0(b^0-x))$ when $R = R(b^0-x,0)$. Then (19A) implies that, on $[0,\delta_1[$, there exists a unique $\hat{R}(x)$ solution of:

$$B^0\left(\int_{\underline{\theta}}^{\bar{\theta}}A(b^0-x,R,\theta)dF(\theta)\right) = b^0-x \tag{20A}$$

and it is C^1. Uniqueness and smoothness of $\hat{a}(x,\theta)$ and $\hat{a}(x)$ follow. (10A) can be rewritten as:

$$\phi_a(\hat{a}(x,\theta),b^0-x,\theta) = (1-\hat{\gamma}(x))\,\frac{F(\theta)}{f(\theta)}\,v_{a\theta}(\hat{a}(x,\theta),b^0-x,\theta) \tag{21A}$$

So $\hat{\gamma}(x)$ is also well defined and C^1. Since $\hat{R}(x) > R(b^0-x,0)$, Proposition 1 implies that for any $x\in]0,\delta_1[$, $\hat{\gamma}(x)>0$, whereas $\hat{\gamma}(0)=0$. Consequently, there exists $\delta\le\delta_1$ such that $\hat{\gamma}'(x)\ge 0$ on $[0,\delta[$. This implies, by differentiating (21A), that $\frac{\partial\hat{a}}{\partial x}(x,\theta) > 0$, on $[0,\delta[$.

□

Let us consider the contract $\hat{\mathcal{C}}(x)$, defined as follows:

$$\begin{aligned}
\hat{t}(a,b) &= \int_{\underline{\theta}}^{\bar{\theta}}v_\theta(\hat{a}(x,s),b,s)ds + v(\hat{a}(x,\theta),b,\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}}w_\theta(\hat{a}(x,s),s)ds + w(\hat{a}(x,\theta),\theta) + m(b,\bar{\theta}) && \text{if } a = \hat{a}(x,\theta) \qquad (22A)\\
&= -\infty && \text{if } \forall\theta,\ a \neq \hat{a}(x,\theta).
\end{aligned}$$

$\hat{\mathcal{C}}(x)$ is such that, whatever b^e, an agent A of type θ is induced to stay and choose action $\hat{a}(x,\theta)$, and the worst possible type of agent A earns an interim (and even ex-post) utility of 0. More precisely, if agent A expects B to play on average b^e and if contract $\hat{\mathscr{C}}(x)$ is not renegotiated, it yields interim utility:

$$\int_{\theta}^{\bar{\theta}} v_{\theta}(\hat{a}(x,s),b^e,s)ds = \int_{\theta}^{\bar{\theta}} w_{\theta}(\hat{a}(x,s),s)ds + m(b^e,\bar{\theta}) - m(b^e,\theta)$$

which is positive by A2, and ex-ante utility at stage 1:

$$\int_{\underline{\theta}}^{\bar{\theta}} v_{\theta}(\hat{a}(x,\theta),b^e,\theta)F(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} w_{\theta}(\hat{a}(x,\theta),\theta)F(\theta)d\theta + m(b^e,\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} m(b^e,\theta)f(\theta)d\theta$$

By construction, $\hat{\mathscr{C}}(x)$ is ex-ante incentive efficient <u>if B is expected to play on average</u> $b^e = b^0-x$. This implies that:

$$\hat{R}(x) = \int_{\underline{\theta}}^{\bar{\theta}} w_{\theta}(\hat{a}(x,\theta),\theta)F(\theta)d\theta + m(b^0-x,\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} m(b^0-x,\theta)f(\theta)d\theta \tag{23A}$$

We will prove that this contract $\hat{\mathscr{C}}(x)$, if publicly signed at stage 0, constitutes a profitable deviation for P^A *in any continuation equilibrium*, compared to not signing any public contract, if the opponent hierarchy does not sign any public contract. If P^A publicly commits to $\hat{\mathscr{C}}(x)$ at stage 0, and P^A-A expects B to play on average b^0-x, agent A's expected utility at stage 1 will be $\hat{R}(x)$ and $\hat{\mathscr{C}}(x)$ will not be renegotiated and will not generate any quit decision. Then, there exists a continuation equilibrium with expected action $\hat{a}(x)$ and b^0-x. There may exist other continuation equilibria, but they will satisfy the following claim:

<u>Claim 2:</u> If P^A and A publicly sign $\hat{\mathscr{C}}(x)$ at stage 0, then any continuation equilibrium is such that $b^e \le b^0-x$.

<u>Proof of Claim 2:</u> Let us denote a^e and b^e the equilibrium expected action for some continuation equilibrium. Proposition 1 implies that $a^e \ge A^0(b^e) = A^0(B^0(a^e))$.

If b^e were strictly larger than b^0, then a^e would be strictly smaller than a^0. Since $A^0(B^0(.))$ is a contraction by A4, $A^0(B^0(a^e))$ would be strictly larger than a^e, a contradiction. So $b^e \leq b^0$.

Suppose that $b^e = b^0 - y$ with $0 \leq y < x$. If agent A expects B to play on average b^0-y and if $\hat{\mathcal{C}}(x)$ is not renegotiated, A's expected utility would be:

$$S(x,y) \equiv \int_{\underline{\theta}}^{\bar{\theta}} v_\theta(\hat{a}(x,\theta), b^0-y, \theta) F(\theta) d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} w_\theta(\hat{a}(x,\theta),\theta) F(\theta) d\theta + m(b^0-y,\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} m(b^0-y,\theta) f(\theta) d\theta$$

Claim 1 showed that $\frac{\partial \hat{a}}{\partial x}(x,\theta) > 0$ on $[0,\delta[$. With $y<x$, $w_{a\theta}>0$ then implies:

$$S(x,y) > \int_{\underline{\theta}}^{\bar{\theta}} w_\theta(\hat{a}(y,\theta),\theta) F(\theta) d\theta + m(b^0-y,\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} m(b^0-y,\theta) f(\theta) d\theta = \hat{R}(y).$$

Using (15A), it comes:

$$A(b^0-y, S(x,y), \theta) > A(b^0-y, \hat{R}(y), \theta)$$

and therefore:

$$B^0\left(\int_{\underline{\theta}}^{\bar{\theta}} A(b^0-y, S(x,y), \theta) dF(\theta) \right) < b^0-y$$

Hence, b^0-y cannot be part of a continuation equilibrium outcome after P^A has committed to $\hat{\mathcal{C}}(x)$ at stage 0.

□

Let us consider now principal P^A's expected profits when she commits to $\hat{\mathcal{C}}(x)$ at stage 0. If the continuation equilibriumm is characterized by $\hat{a}(x)$ and b^0-x, these profits are $\Pi(b^0-x, \hat{R}(x))$. If another continuation equilibrium arises, Claim 2 ensures that $b^e \leq b^0-x$. Given b^e, if P^A did not renegotiate $\hat{\mathcal{C}}(x)$ at stage 1, she would earn expected payoffs:

$$\tilde{\Pi}(b^e) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(\hat{a}(x,\theta), b^e, \theta) f(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} v_\theta(\hat{a}(x,\theta), b^e, \theta) F(\theta) d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \Omega(\hat{a}(x,\theta), b^e, \theta) - w(\hat{a}(x,\theta),\theta) - w_\theta(\hat{a}(x,\theta),\theta)\frac{F(\theta)}{f(\theta)} \right\} f(\theta) d\theta - m(b^e, \bar{\theta})$$

NE and A4 imply that $\Omega_b < 0 \leq m_b$; therefore, $\tilde{\Pi}(b^e)$ decreases when b^e increases, and then, for any continuation equilibrium $b^e \leq b^0-x$,

$$\tilde{\Pi}(b^e) \geq \Pi^0(b^0-x, \hat{R}(x))$$

Now, in the continuation equilibrium characterized by b^e, it is possible that P^A and A renegotiate, but then, P^A will end up with profits Π^A that are larger than $\tilde{\Pi}(b^e)$ and thus, larger than $\Pi^0(b^0-x,\hat{R}(x))$. It remains to show that $\Pi^0(b^0-x,\hat{R}(x))$ is larger than $\Pi^0(b^0,\hat{R}(0))$. From (9A), it comes:

$$\frac{d\Pi^0}{dx}(b^0-x,\hat{R}(x)) = -\int_{\underline{\theta}}^{\bar{\theta}}\Omega_b(A(b^0-x,0,\theta),b^0-x,\theta)f(\theta)d\theta + \gamma(x)\int_{\underline{\theta}}^{\bar{\theta}}m_b(b^0-x,\theta)f(\theta)d\theta$$
$$+ (1-\gamma(x))m_b(b^0-x,\bar{\theta}) - \gamma(x)\frac{d\hat{R}}{dx}(x)$$

Taking the limit when x goes to zero, and using $\gamma(0)=0$,

$$\frac{d\Pi^0}{dx}(b^0-x,\hat{R}(x))\Big|_{x=0} = -\int_{\underline{\theta}}^{\bar{\theta}}\Omega_b(A(b^0,0,\theta),b^0,\theta)f(\theta)d\theta + m_b(b^0,\bar{\theta})$$

which is strictly positive from NE and A4; the derivative is therefore positive and publicly proposing $\hat{\mathcal{C}}(x)$ for x small enough would be profitable for P^A, if no public contract is signed between P^B and B.

∎

Proof of Theorem 2

Consider the case SS-PE first. Suppose that P^B does not offer a public contract. If P^A does not offer a public contract, the (unique) continuation equilibrium coincide with the equilibrium of G_0 and P^A obtains Π^{A0}.

If, however, P^A signs a public contract at stage 0, let a^e and b^e denote A's and B's expected actions in a continuation equilibrium and R^A be the final expected utility for the agent. Then, $b^e = B^0(a^e)$ and $a^e \geq A^0(b^e)$ by Proposition 1. This implies that $b^e \leq b^0$. Then P^A obtains expected equilibrium payoffs $\Pi^A(b^e,R^A)$. Now,

$$\Pi^A(b^e,R^A) \leq \Pi^A(b^e,0)$$

since setting $R^A = 0$ in (P') relaxes a constraint and, consequently, can only increase the value of the program. Moreover, using (9A),

$$\frac{d\Pi^A}{db}(b,0) = \int_{\underline{\theta}}^{\bar{\theta}} \Omega_b(A(b,0,\theta),b,\theta)f(\theta)d\theta - \gamma(b)\int_{\underline{\theta}}^{\bar{\theta}} m_b(b,\theta)f(\theta)d\theta - (1-\gamma(b))m_b(b,\bar{\theta})$$

A4 and PE imply that $m_b \le 0 < \Omega_b$, so that $\Pi^A(b,0)$ increases in b; with $b^e \le b^0$, one gets:

$$\Pi^A(b^e,R^A) \le \Pi^A(b^e,0) \le \Pi^A(b^0,0)$$

Hence, not making any offer at stage 0 is, indeed, an equilibrium strategy.

Consider now the case SC-NE and assume again that P^B does not offer a public contract at stage 0. The same reasoning holds with now $b^e \ge b^0$. Suppose, moreover, that there exists another equilibrium of G leading to expected actions (a^e,b^e) and expected agent's utility R^A and R^B. Proposition 1 implies that $a^e \ge A^0(b^e)$ and $b^e \ge B^0(a^e)$, so that $a^e \ge a^0$ and $b^e \ge b^0$. Therefore, by the same reasoning as before,

$$\Pi^A(b^e,R^A) \le \Pi^A(b^0,0) \text{ and } \Pi^B(a^e,R^B) \le \Pi^B(a^0,0),$$

with equality only if $a^e=a^0$, $b^e=b^0$, $R^A=R^A(b^0,0)$ and $R^B=R^B(a^0,0)$. This proves that, among all possible equilibria of G, not signing a public contract at stage 0 is a Pareto dominant outcome for the pair of principals.

■

Proof of Theorem 3:

If $\underline{\theta}^B = \bar{\theta}^B = \theta^B$, then for all R^B,

$$B(a,R^B) = B^0(a) = B^*(a)$$

P^B will always induce the full information optimal choice of actions $B^*(a)$. Under **SS-NE** and **SC-PE**, the proof of Theorem 1 can be replicated with $B^*(.)$ in stead of $B^0(.)$. Part i) follows. Under **SS-PE** and **SC-NE**, the proof of Theorem 2 clearly shows that P^A can obtain the maximum payoff by not signing any public contract. If P^A signs a public contract that leads to $a^e \ne a^0$, the inequalities in the proof of Theorem 2 are strict (whether **SS-PE** or **SC-NE** hold) and this would be a dominated move. Part ii) follows.

■

Proof of Theorem 4:

Assume **SS**, and define the expected reaction functions:

$$a(b,R^A) = \int_{\underline{\theta}^A}^{\bar{\theta}^A} A(b,R^A,\theta^A)dF^A(\theta^A)$$

and $b(a,R^B)$ similarly. From the proof of Proposition 1 ("further results")

$$\frac{\partial A}{\partial b}(b^e,R^A,\theta) = \frac{-\phi_{ab}(\%) - v_{a\theta}(\%)\dfrac{F(\theta)}{f(\theta)}\dfrac{\partial\gamma}{\partial b}}{\phi_{aa}(\%) - (1-\gamma)\dfrac{F(\theta)}{f(\theta)}v_{aa\theta}(\%)} > -\frac{\phi_{ab}(\%)}{\phi_{aa}(\%)}$$

almost everywhere, where $(\%)$ denotes $\left(A(b^e,R^A,\theta),b^e,\theta\right)$, since $\frac{\partial\gamma}{\partial b} \geq 0$, $v_{a\theta}\geq 0$, $v_{aa\theta}\geq 0$, $\phi_{aa}<0$ and $\gamma\leq 1$. R-monotonicity implies first that $\frac{\partial a}{\partial b}(b^e,R^A)$ has the same sign as ϕ_{ab}, and second, that:

$$\left| \frac{\partial a}{\partial b}(b^e,R^A) \right| \leq \left| \int_{\underline{\theta}^A}^{\bar{\theta}^A} -\frac{\phi_{ab}(\%)}{\phi_{aa}(\%)} dF^A(\theta^A) \right|$$

It follows that $a(b^e,R^A)$ and $b(a^e,R^B)$ have the same monotonicity than $A^0(b^e)$ and $B^0(a^e)$ and satisfy the same stability condition:

$$0 < \frac{\partial a}{\partial b}(b^e,R^A) \cdot \frac{\partial b}{\partial a}(a^e,R^B) < 1 \qquad (24A)$$

The same reasoning holds in the case **SC**.

Let us now turn to the proof of Theorem 4. Consider cases **SS-NE** and **SC-PE**. Suppose that $a^* = A^0(b^*)$, and that P^B-B have signed a public contract leading to agents' expected equilibrium utilities R^{B*}. With simple contracts and no externality terms, R^{B*} corresponds exactly to the ex-ante rent promised to B by public contracting, whatever A's expected action. Therefore, if P^A-A deviate in stage 0, any continuation equilibrium will still be on B's average reaction curve $b(a^e,R^{B*})$.

Now, the proof of Theorem 1 can be applied to show that P^A would sign a public contract so as to increase A's action. One just has to replace $B^0(a)$

by $b(a,R^{B*})$, a^0 by a^* and b^0 by b^* in (16A)-(18A) and subsequent expressions. R-monotonicity preserves the monotonicity properties as well as the contraction mapping property used in Claim 1; and since the externality terms are zero, (22A) defines a non-contingent contract.

Consider now the case **SS-PE**. Suppose that an equilibrium is characterized by a^*, b^*, R^{A*} and R^{B*}, with $a^* \neq A^0(b^*)$. Proposition 1 implies that:

$$a^* = a(b^*,R^{A*}) \geq a(b^*,0) = A^0(b^*)$$

So, $a^* > A^0(b^*)$ and necessarily $R^{A*} > R^{A0}(b^*)$. Let P^A decide not to sign a public contract. The new equilibrium is obtained at:

$$a = A^0(b) = a(b,0) \quad \text{and} \quad b = b(a,R^{B*}).$$

All we have to show is that $b > b^*$. This is where R-monotonicity is required. If $b \leq b^*$ and $a < a^*$, then R-monotonicity should be violated on B's average reaction curve for R^{B*}. If $b \leq b^*$ and $a > a^*$, R-monotonicity implies the stability condition (24A) and therefore:

$$b(a^*,R^{B*}) < b(A^0(b^*),R^{B*}) \leq b^* = b(a^*,R^{B*})$$

a contradiction. Therefore $b > b^*$. P^A has thus reduced its incentive costs and increased B's expected action, which altogether increases P^A's payoff.

The proof is similar for the case **SS-PE**

■

APPENDIX B: INTERIM CONTRACTS

Proof of Proposition 3

Following the proof of Proposition 1, we fix b^e, assume (12A) so that all types of agents stay at the optimum, and transform the incentive compatibility constraints to end up with the following program:

$$\max_{a(.),U^*} \int_{\underline{\theta}}^{\bar{\theta}} \left(\phi(a(\theta),b^e,\theta) - \frac{F(\theta)}{f(\theta)} v_\theta(a(\theta),b^e,\theta)\right) f(\theta)d\theta - U^* \tag{PB}$$

$$\text{s.t. } \forall\theta\in[\underline{\theta},\bar{\theta}],\ a(.) \text{ non-increasing} \tag{1B}$$

$$\int_{\theta}^{\bar{\theta}} v_\theta(a(s),b^e,s)ds + U^* \geq V(\theta) \tag{2B}$$

$$\underline{a} \leq a(\theta) \leq \bar{a}$$

We allow for a finite number of jumps in the state variable $a(\theta)$, and choose $a(.)$ to be right-continuous.

Let $\gamma(\theta) = \frac{d\Gamma}{d\theta}(\theta)$ be the multiplier of the participation constraint (2B), with $\Gamma(\underline{\theta}) = 0$ and $\Gamma(.)$ non-decreasing, $\mu(\theta)$ the multiplier associated with (1B) and $\bar{\alpha}(\theta)$ and $\underline{\alpha}(\theta)$ the multipliers associated with the interiority conditions. The Lagrangean of this problem is:

$$\mathcal{L}\left(a(.),U^*,\Gamma(.),\mu(.),\bar{\alpha}(.),\underline{\alpha}(.);b^e,V(.)\right) = -\int_{\underline{\theta}}^{\bar{\theta}} \mu(\theta)da(\theta) - (1-\Gamma(\bar{\theta}))U^*$$

$$+\int_{\underline{\theta}}^{\bar{\theta}}\left(\phi(a(\theta),b^e,\theta) - \left(\frac{F(\theta)}{f(\theta)} - \frac{\Gamma(\theta)}{f(\theta)}\right)v_\theta(a(\theta),b^e,\theta) + a(\theta)\frac{\underline{\alpha}(\theta)-\bar{\alpha}(\theta)}{f(\theta)}\right)f(\theta)d\theta$$

The first order necessary conditions are: for a.e. $\theta\in[\underline{\theta},\bar{\theta}]$,

$$\phi_a(a(\theta),b^e,\theta) = \left(\frac{F(\theta)}{f(\theta)} - \frac{\Gamma(\theta)}{f(\theta)}\right)v_{a\theta}(a(\theta),b^e,\theta) - \frac{1}{f(\theta)}\frac{d\mu}{d\theta}(\theta) + \frac{\bar{\alpha}(\theta)-\underline{\alpha}(\theta)}{f(\theta)} \tag{3B}$$

$$\Gamma(\bar{\theta}) = 1 \tag{4B}$$

plus the complementary slackness conditions. Recall that $\mu(.)$ must be continuous, non-negative, and $\mu(\theta) = 0$ if $a(.)$ is decreasing around θ or jumps at θ (see Theorem 7, page 196, in Seierstad-Sydsæter [1987]).

Suppose that for some θ^0 and $\varepsilon>0$: $\forall\theta\in[\theta^0,\theta^0+\varepsilon[$, $\mathbf{A}(b^e,V(.),\theta)<A(b^e,0,\theta)$. Let $\theta_1 \equiv \sup\left\{\theta<\theta^0 \text{ s.t. } \mathbf{A}(b^e,V(.),\theta)\geq A(b^e,0,\theta)\right\}$ if it exists, $\theta_1\equiv\underline{\theta}$ otherwise.

P^A could keep unchanged the action on $[\theta^0+\varepsilon,\bar{\theta}]$ and on $[\underline{\theta},\theta_1[$, and implement $A(b^e,0,\theta)$ on $[\theta_1,\theta^0+\varepsilon[$, adjusting upward the utilities of types $\theta<\theta^0+\varepsilon$ to maintain incentive compatibility. The new profile of actions would still be non-increasing and this profile of utility preserves individual rationality. The change in the principal's profits would be:

$$\int_{\theta_1}^{\theta^0+\varepsilon}\left(\left(\phi(A(b^e,0,\theta),b^e,\theta) - \frac{F(\theta)}{f(\theta)}\, v_\theta(A(b^e,0,\theta),b^e,\theta)\right)\right. \tag{5B}$$

$$\left. - \left(\phi(\mathbf{A}(b^e,V(.),\underline{\theta}),b^e,\theta) - \frac{F(\theta)}{f(\theta)}\, v_\theta(\mathbf{A}(b^e,V(.),\underline{\theta}),b^e,\theta)\right)\right)f(\theta)d\theta$$

Since $\left(\phi(a,b^e,\theta) - \dfrac{F(\theta)}{f(\theta)}\, v_\theta(a,b^e,\theta)\right)$ is concave, maximum for $a = A(b^e,0,\theta)$, the change of profits would be strictly positive, a contradiction. Therefore,

$$\mathbf{A}(b^e,V(.),\theta^0) \geq A(b^e,0,\theta^0) \text{ for all } \theta^0 \tag{6B}$$

■

Proof of Theorem 6:

It is immediate that, given Proposition 3, the proof of Theorem 2 can be replicated with R replaced by $V(.)$.

Let us consider Theorem 1 and focus on the case SS-NE. Consider again $\hat{a}(x,\theta)$, $\hat{a}(x)$ and $\hat{R}(x)$ and contract $\hat{\mathcal{C}}(x)$; let $\hat{V}(.;b,x)$ denote the profile of interim utilities obtained with $\hat{\mathcal{C}}(x)$, if the opponent's average action is b:

$$\hat{V}(\theta;b,x) = \int_\theta^{\bar{\theta}} w_\theta(\hat{a}(x,s),s)ds + m(b,\bar{\theta}) - m(b,\theta).$$

If $b = b^0-x$, $\hat{\mathcal{C}}(x)$, being ex-ante incentive efficient, is also interim incentive efficient and therefore would not be renegotiated. The proof of

Theorem 1 then remains valid if we can show that, following $\hat{\mathcal{C}}(x)$ signed at stage 0, there cannot exist a continuation equilibrium with $b = b^0-y$, $0<y<x$ (as in Theorem 1, it is obvious that $y > 0$). For this purpose, using the notation $\mathbf{a}(\theta) \equiv \mathbf{A}(b^0-y,\hat{V}(.;b^0-y,x),\theta)$ for given x and y, we will show that $\mathbf{a}(\theta) > \hat{a}(y,\theta)$ for (almost) all θ.

Suppose instead that there exists θ^0 such that on a neighborhood of θ^0:

$$\mathbf{a}(\theta) \le \hat{a}(y,\theta) \tag{7B}$$

i) Suppose that $\frac{d\mu}{d\theta}(\theta^0) = 0$.

If $F(\theta^0) < \Gamma(\theta^0)$, then (3B) would imply that $\phi_a(\mathbf{a}(\theta^0),b^0-y,\theta^0) < 0$, i.e. that $\mathbf{a}(\theta^0) > A^*(b^0-y,\theta^0)$ which is not compatible with (7B). Hence $F(\theta^0) \ge \Gamma(\theta^0)$. Now (3B), (7B) and (21A) imply that: $F(\theta^0) \ge \hat{\gamma}(y)F(\theta^0) \ge \Gamma(\theta^0)$. Then, consider on $[\theta^0,\bar{\theta}]$ a solution to the first order condition: $\Gamma(\theta) = \Gamma(\theta^0)$, $\mu(\theta) = \mu(\theta^0)$ and (3B)-(4B). This defines a non-increasing profile of actions, and since for these values of the multipliers, the Lagrangean is indeed concave in a for $\theta\in[\theta^0,\bar{\theta}]$, this must be <u>the</u> solution $\mathbf{a}(.)$. Since $\Gamma(\theta) = \Gamma(\theta^0) \le \hat{\gamma}(y)F(\theta^0) < \hat{\gamma}(y)F(\theta)$ for $\theta>\theta^0$, it comes:

$$\mathbf{a}(.) < \hat{a}(y,\theta) < \hat{a}(x,\theta) \text{ for } \theta>\theta^0.$$

Now, (4B) implies that $\mathbf{U}(\bar{\theta}) = 0$, and $\forall\theta>\theta^0$,

$$\mathbf{U}(\theta) = \int_\theta^{\bar{\theta}} v_\theta(\mathbf{a}(s),b^0-y,s)ds < \int_\theta^{\bar{\theta}} v_\theta(\hat{a}(x,s),b^0-y,s)ds = \hat{V}(\theta;b^0-y,x)$$

a contradiction.

ii) Suppose then that $\frac{d\mu}{d\theta}(\theta^0) \neq 0$.

θ^0 must then belong to a pooling region. Define:

$$\theta_1 \equiv \text{Inf}\left\{\theta \text{ s.t. } \mathbf{a}(\theta) = \mathbf{a}(\theta^0)\right\}.$$

Two cases can arise.

ii-1) If $\theta_1=\underline{\theta}$, then, using Proposition 3, $\mathbf{a}(\theta^0)=\mathbf{a}(\theta_1)$ must be larger or equal to $A^*(b^0-y,\theta_1) = A(b^0-y,0,\theta_1) = \hat{a}(y,\theta_1)$, and therefore, with $\theta^0 > \underline{\theta}$, (7B) would be violated.

ii-2) If $\theta_1 > \underline{\theta}$ then $\mu(\theta_1)=0$, as $a(.)$ must either be decreasing on the left of θ_1 or jump downward. Moreover on $]\theta_1, \theta_1+\varepsilon[$, $\bar{\alpha}(\theta)=\underline{\alpha}(\theta)=0$ and $\frac{d\mu}{d\theta}(\theta)\geq 0$; as $\mathbf{a}(\theta) < \hat{a}(y,\theta) \leq A^*(b^0-y,\theta)$ we obtain $F(\theta) > \Gamma(\theta)$.

If at θ_1, $\frac{d\mu}{d\theta}(.)$ jumps upward, then, by (3B) and $F>\Gamma$ in a right neighborhood of θ_1, either $\mathbf{a}(.)$ jumps upward which violates (1B), or $\Gamma(.)$ jumps downward, which is also impossible. Thus $\frac{d\mu}{d\theta}(.)$ can only jump downward, which, given the non-negativity of $\mu(.)$ implies that $\mu(\theta_1)>0$, a contradiction. Therefore, $\frac{d\mu}{d\theta}(.)$ must be continuous at θ_1, and $\frac{d\mu}{d\theta}(\theta_1)=0$.

Then, the same reasoning as in i) applies at θ_1, yielding a contradiction.

Therefore, $\mathbf{A}(b^0-y, \hat{V}(.;b^0-y,x),\theta) > \hat{a}(y,\theta)$ for (almost) all θ, and one can fully replicate the proof of theorem 1.

■