AGGREGATION, LEARNING

AND RATIONALITY

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ABSTRACT : The assumption of individual "rationality" is widely used in economics. The lecture reviews recent studies challenging two aspects of this assumption. The first issue concerns the well known fact that aggregation over optimizing households yields almost anything at the macroeconomic level. By contrast heterogeneity of individual characteristics alone (i.e. with only few individual rationality requirements) may generate strong macroeconomic regularities, with striking consequences for the prevalence in the aggregate of the weak axiom of revealed preference, of gross substitutability, and for uniqueness and stability of the Walrasian exchange equilibrium. In a partial equilibrium context, demand heterogeneity generates concave revenue functions and a unique Cournot oligopoly equilibrium. The second part of the lecture questions the "rational expectations" hypothesis that is widely used in dynamical economic models. Taking into account learning often makes "rational expectations" locally unstable, especially in markets where expectations matter significantly.

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Key words : Rationality, aggregation, heterogeneity, equivalence scales, competitive equilibrium, oligopoly equilibrium, expectations, learning, business cycles.

AGGREGATION, APPRENTISSAGE ET RATIONALITE

RESUME : L'hypothèse de "rationalité" individuelle joue un rôle important dans l'analyse économique. Cette étude passe en revue quelques travaux récents remettant en cause deux aspects de cette hypothèse. Le premier problème concerne le fait bien connu que l'hypothèse suivant laquelle les ménages maximisent une fonction d'utilité n'engendre, par agrégation, que peu de restrictions macroéconomiques. Par contraste, l'hétérogénéité des caractéristiques individuelles seule (i.e. avec très peu d'hypothèses sur la "rationalité" individuelle) peut engendrer de fortes régularités macroéconomiques, avec des implications remarquables concernant la validité, au niveau agrégé, de l'axiome faible de la préférence révélée, de la substituabilité brute, sur l'unicité et la stabilité de l'équilibre Walrasien des échanges. Dans un contexte d'équilibre partiel, l'hétérogénéité de la demande conduit à des fonctions de revenu concaves et à l'unicité de l'équilibre d'oligopole de Cournot. La seconde partie de la conférence est consacrée à l'examen de l'hypothèse d'anticipations "rationnelles". La prise en compte de l'apprentissage rend souvent les anticipations "rationnelles" localement instables, en particulier sur les marchés où l'influence des anticipations est significative.

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AGGREGATION, LEARNING AND RATIONALITY

Jean-Michel Grandmont *

The assumption that individual economic units behave "rationally" is widely used in many areas of economic theorizing, be it in microeconomics or macroeconomics. The purpose of this lecture is to review briefly a few recent studies that tend to challenge two aspects of this assumption.

The first issue concerns the pervasive idea that economic theory should start from "first principles" and portray individual economic units (in particular, households) as maximizing a well defined objective function. It has been known for a while, though, that the approach involves severe difficulties : if the distribution of individual characteristics is arbitrary, aggregation over optimizing households yields almost anything, hence nothing, at the macroeconomic level. In fact, postulates about individual rationality have to be used in practice jointly with additional, and often crucial, assumptions in order to get any meaningful result. For instance some macroeconomists make the extremely naive assumption that society as a whole behaves as a single optimizing individual. Imperfect competition or industrial organisation theorists often rely upon convenient but rather arbitrary assumptions about aggregate demand, such as concave revenue functions or even linear demand schedules. The problem with such auxiliary assumptions is that they do not appear to be grounded upon sound theorizing.

The first section of the paper reviews a few recent theoretical advances suggesting that aggregation over a large *heterogenous* population might provide a solution to these problems. The key point seems to be that heterogeneity tends to make aggregate expenditures more independent of prices. In a market equilibrium context, this fact has strong implications for the prevalence, in the aggregate, of the weak axiom of revealed preference, of gross substitutability, and for uniqueness and stability of a Walrasian exchange equilibrium. In (partial equilibrium) models of imperfect competition, demand heterogeneity leads to concave revenue functions and, for instance, to unicity of Cournot oligopoly equilibrium. One of the most intriguing features of the new approach is that, while it is compatible with the standard postulate that individual households are indeed optimizing, it relies essentially upon distributional assumptions about heterogeneity and imposes very few "rationality" requirements on individual behavior. As heterogeneity of individual characteristics appears to be a plausible hypothesis, the new approach suggests accordingly that, by contrast with currently accepted views, attention in economic theory might advantageously be shifted away from individual optimization and more focussed upon analyzing the consequences of heterogenous macroeconomic distributions.

The second part of the lecture questions the "rational expectations" hypothesis that is so widely used in dynamic economic models (and for that matter, in game theory), or specifically, the hypothesis that an individual's expectations about the future are correct at any moment, given current information. A frequently heard defense of the hypothesis is that it should be the asymptotic outcome of a dynamic process in which individuals learn about the laws of motion of their environment. The second section of this paper is devoted to a brief review of recent studies suggesting that taking into account learning often generates dynamic local instability. These studies suggest further that the dynamics with learning may be highly nonlinear and generate complex trajectories, and moreover that forecasting mistakes may never vanish, even in the "long run".

1. AGGREGATION AND RATIONALITY

It is well known that standard economic theory (i.e. individual optimizing behavior) does not generate many restrictions on aggregate market phenomena. The nagging result here is the Sonnenschein-Mantel-Debreu indeterminacy theorem (see Sonnenschein (1973, 1974) and the survey by Shafer and Sonnenschein (1982)) : if the distribution of microeconomic characteristics in the system is arbitrary, individual optimizing behavior does not place any restrictions on competitive market excess demand, on any given compact set of prices, other then homogeneity and Walras's law. Indeterminacy of this sort is clearly bound to be a pervasive phenomenon. In particular, it should not be confined to competitive models.

Underlying the result is the assumption that the distribution of microeconomic characteristics is arbitrary. The diagnosis hints accordingly at a possible way out that has been known for quite some time, namely to build our theories upon plausible restrictions on the distribution of individual characteristics. In particular, it has often been observed that there is apparently significant behavioral heterogeneity among economic agents, and it has been accordingly suggested that taking into account such heterogeneity might be useful in this area.

There has been some progress in recent times on this research front. Hildenbrand (1983) took indeed a decisive step by showing that heterogeneity of the income distribution may make macroeconomic income effects just right in multimarkets consumer demand analysis, to leave us with "nice" aggregate substitution effects only — specifically, the Jacobian matrix of the price derivatives of market demand becomes *negative quasi-definite* for every price system (see also Grandmont (1987)). Another important idea was due to Jerison (1982, 1984, 1992) who showed that, roughly speaking, *increasing dispersion of household Engel curves* as income goes up leads to the weak axiom of revealed preference in the aggregate when the distribution of income is fixed (see also Freixas and Mas-Colell (1987)). By marrying the two lines of argument, variants of the above increasing dispersion hypothesis were further shown to imply negative quasi-definiteness of the Jacobian matrix of price derivatives of market demand, and have been confronted to empirical data with rather convincing results (Härdle, Hildenbrand and Jerison (1991), Hildenbrand (1989, 1992)). While this approach is an outstanding methodological achievement, it applies essentially to the case in which incomes or expenditures are independent of prices, and it is apparently difficult to extend it beyond this restrictive situation. When specialized to a market for a single good or service, it yields only a downward sloping demand schedule, which still falls short of what is needed in the study of imperfect competition models.

Another approach, complementary to that of Hildenbrand and Jerison, has been recently implemented with fairly good success. It seeks to introduce plausible heterogeneities in other dimensions of individual characteristics, namely in consumers' demand schedules. This line of attack has it roots in, and generalizes the notion of, *household equivalence scales*, which has been much used in applied demand analysis (Prais and Houthakker (1955), Barten (1964), Jorgenson and Slesnick (1987)). Dispersion of demand schedules of this sort were introduced in general competitive equilibrium analysis some time ago by Mas-Colell and Neuefind (1977) and by E. Dierker, H. Dierker and W. Trockel (1984). A particular case was considered under the name of "replicas" in Jerison (1982, 1984). It has also been used by H. Dierker (1989) and E. Dierker (1991) to inquire whether demand heterogeneity might help in ensuring existence of equilibrium in imperfect competition models.

The recent discovery has been that demand heterogeneity of this sort makes aggregate expenditures more independent of prices. In multimarkets consumer demand analysis, with incomes or expenditures independent of prices, this yields an aggregate Jacobian matrix of price derivatives with a *negative dominant diagonal* on a large set of prices. In a competitive general equilibrium context, in which incomes *are* dependent on prices, this fact has strong consequences for the prevalence, in the aggregate, of the weak axiom of revealed preferences, of gross substitutability, and on uniqueness and stability of the Walrasian exchange equilibrium (Grandmont (1992a)). When specialized to a market for a single commodity, the approach implies that a relatively large heterogeneity in individual demand behaviors yields an aggregate demand that is not only downward sloping, but with an elasticity that is not too far from minus 1. Demand heterogeneity of this sort makes indeed easier the study of imperfect competition. In particular, it implies unicity of Cournot oligopoly equilibrium (Grandmont (1992b)).

An important feature of the approach is that, while it is compatible with the hypothesis that individual households do optimize, it is in fact much more general, for it relies upon very few "rationality" requirements on microeconomic behavior. The approach suggests accordingly that attention in economic theory might fruitfully be redirected away from the study of individual optimization toward the analysis of macroeconomic distributions of individual characteristics.

The whole analysis rests upon the estimation of bounds for the price elasticities of aggregate demand, that depend explicitly upon specific measures of heterogeneity. The derivation of these bounds is quite simple and we will present it in some detail here. We shall then explain heuristically how, on the basis of these bounds, heterogeneity generates strong macroeconomic regularities in general competitive exchange equilibrium and in models of imperfect competition.

<u>Equivalence scales</u>

We consider an economic system in which ℓ goods or services are exchanged. A commodity bundle is then described by a vector x of $\mathbb{R}^{\ell}$. To focus ideas, we concentrate on households' consumption behavior. A demand function will be a *continuous vector valued function* $x(p,b) \geqq 0$. It represents the commodity bundle demanded when the price system is p in $\mathrm{Int}\mathbb{R}^{\ell}_{+}$ and the household's income is $b > 0$. We shall assume throughout that Walras's law holds, i.e. $p \cdot x(p,b) \equiv b$. At this stage, we do *not* assume homogeneity of degree 0 of demand with respect to (p,w), as many intermediary results do not require this property.

The core of the approach is that whenever a household having the demand function x(p,b) is present in the population, there is a continuum of individuals who have the same demand function, up to a rescaling of the units of measurement of each commodity h by a factor $\beta_h > 0$. To see what should be the form of the "rescaled" demand function, consider a *fictitious* operation in which the unit of measurement of each commodity h is divided by β_h. If the price vector in the fictitious units is p, the price vector in the actual units is given by

$$(\beta_1 p_1, \ldots, \beta_\ell p_\ell) = \beta \otimes p.$$

Thus demand, expressed in the fictitious units, is $\beta \otimes x(\beta \otimes p, b) = [\beta_1 x_1(\beta \otimes p, b), \ldots, \beta_\ell x_\ell(\beta \otimes p, b)]$. By definition, the rescaled demand function corresponding to the vector β, *expressed in the actual unit system* (which is in fact fixed throughout !) is equal to the above expression, i.e. to $\beta \otimes x(\beta \otimes p, b)$. In the sequel, it will be much more convenient to work with the rescaling parameters $\alpha_h = \mathrm{Log}\beta_h$, which can take any positive or negative value. With the convention $e^{\alpha} = (e^{\alpha_1},\dots,e^{\alpha_\ell})$, *the rescaled demand function is thus given by* $e^{\alpha}\otimes x(e^{\alpha}\otimes p,b)$.

It should be noted that although we did not (and shall not) impose demand functions to be the outcome of utility maximization, rescaling operations are quite compatible with that requirement. If for instance the original demand function $x(p,b)$ is obtained by maximizing the utility function $u(x)$ under the budget constraint $p\cdot x = b$, then the rescaled demand maximizes the rescaled utility function $u(e^{-\alpha}\otimes x)$ under the budget constraint

$$p\cdot x = (e^{\alpha}\otimes p)\cdot(e^{-\alpha}\otimes x) = b.$$

Market demand

Whenever a household present in the population has the demand function $x(p,b)$, there is a continuum of individuals who have the rescaled demand function $e^{\alpha}\otimes x(e^{\alpha}\otimes p,b)$. These individuals are indexed by the vector α of rescaling parameters and they are assumed to be distributed according to the density function $f(\alpha)$. Aggregating demands over this subpopulation yields the *conditional market demand*

$$X(p,b) = \int e^{\alpha}\otimes x(e^{\alpha}\otimes p,b)\ f(\alpha)\ d\alpha$$

(the word "conditional" appears here because aggregation is carried out, for the moment, for a single initial demand function $x(p,b)$. This restriction will be relaxed shortly). We are now in a framework in which we can speak meaningfully of the "heterogeneity" of this subpopulation by looking at the dispersion of the density $f(\alpha)$. We shall see that a large dispersion implies a well behaved conditional market demand, even if individual demands are not.

Our strategy is to show that conditional market expenditures $p_hX_h(p,b)$ have continuous first derivatives and to estimate bounds for these derivatives that depend explicitly on the dispersion of the density $f(\alpha)$. To this effect, it is convenient to introduce the notation $w_h(p,b) = p_hx_h(p,b)$. Then it follows from the definition of conditional market demand that conditional market expenditures are given by

$$p_h X_h(p,b) = \int w_h(e^{\alpha} \otimes p, b)\ f(\alpha)\ d\alpha.$$

If we use the short hand notation $\mathrm{Logp} = (\mathrm{Logp}_1,\ldots,\mathrm{Logp}_\ell)$, and make the change of variable $r = \alpha + \mathrm{Logp}$, we obtain

$$(1.1) \qquad p_h X_h(p,b) = \int w_h(e^{r}, b)\ f(r - \mathrm{Logp})\ dr.$$

It is clear from this expression that if we assume that *the density* $f(\alpha)$ *is continuously differentiable and its partial derivatives are uniformy integrable, i.e.* $\int \left|\frac{\partial f}{\partial \alpha_k}(\alpha)\right|\ d\alpha \leqq m_k$, then conditional market expenditure is also continuously differentiable. Taking partial derivatives of (1.1) with respect to Logp_k and reverting to the original vector of variables $\alpha = r - \mathrm{Logp}$ yields then

$$(1.2) \qquad \frac{\partial[p_h X_h(p,b)]}{\partial \mathrm{Logp}_k} = -\int w_h(e^{\alpha} \otimes p, b)\ \frac{\partial f}{\partial \alpha_k}(\alpha)\ d\alpha.$$

It is now easy to find bounds for the price derivatives of market expenditure. Since the absolute value of the right hand side of (1.2) is bounded above by bm_k, we obtain indeed

$$(1.3) \qquad \left|\frac{\partial[p_h X_h(p,b)]}{\partial \mathrm{Logp}_k}\right| \leqq bm_k.$$

It should be noted that these bounds are valid for all densities $f(\alpha)$, even if they are rather concentrated. The coefficients m_k appearing in (1.3) measure in some sense the dispersion of the density. If they are small, the distribution of the rescaling parameters α should be spread out. In that case, the inequalities (1.3) tell us that conditional market expenditures do not vary much with prices. This is the simple but important fact around which the whole analysis is built.

A nice feature of the inequalities (1.3) is that they are *additive*, in the sense that if we put together two subpopulations satisfying these inequalities, then the mixture will also satisfy them. Specifically, consider a set of "types" a in some set A (we assume to fix ideas that A is a separable metric space). To each type a correspond a demand function $x_a(p,b)$ and an income level $b_a > 0$ (We assume for the moment that income is independent of prices, but this assumption will be relaxed later on). The population is distributed over types according to the probability distribution μ. For each type a present in the population (i.e. in the support of μ), there is a continuum of individuals who have the same demand up to a vector α of rescaling parameters, and these individuals are assumed to be distributed according to the conditional density $f(\alpha|a)$. Then for each type present in the population, *conditional market demand* is defined as before by

$$X_a(p,b) = \int e^{\alpha}\otimes x_a(e^{\alpha}\otimes p,b)\ f(\alpha|a)\ d\alpha, \tag{1.4}$$

while *total market demand* is obtained by aggregating over types

$$X(p) = \int_A X_a(p,b_a)\ \mu(da) \tag{1.5}$$

(We assume here enough regularity properties so that all these integrals are well defined). The important point is that if the conditional densities $f(\alpha|a)$ have uniformly integrable partial derivatives with $\int\left|\frac{\partial f}{\partial\alpha_k}(\alpha|a)\right|\ d\alpha \leqq m_k$, for every type a in the support of μ, then each *conditional* market demand satisfies the inequalities (1.3), with $b = b_a$. Then if per capita total income is finite, i.e. $\bar{b} = \int b_a\ \mu(d_a) < +\infty$, *total* market demand will also satisfy them, by integration over all types, that is

$$\left|\frac{\partial[p_h X_h(p)]}{\partial \mathrm{Log} p_k}\right| \leqq \bar{b}\ m_k. \tag{1.6}$$

We should therefore expect total market demand to be well behaved when there is significant heterogeneity in the system, i.e. when the conditional densities $f(\alpha|a)$ are spread out, since according to the inequalities (1.6), total market expenditures become relatively independent of prices when the coefficients m_k are small. In order to make the argument tight, however, we have to ensure that total market demand does not vanish when heterogeneity grows. To this end, we suppose that aggregate budget shares are uniformly bounded away from 0, or more precisely that *there exist* $\gamma_h > 0$ *such that*

$\gamma_h\ \bar{b} \leqq p_h X_h(p)$ *for all* p *and all* h. This assumption, together with (1.4), yields immediately

$$\left|\frac{\partial \mathrm{Log} X_h(p)}{\partial \mathrm{Log} p_k} + \delta_{hk}\right| \leqq \bar{b}\ m_k/p_h X_h(p) \leqq m_k/\gamma_h\ , \tag{1.7}$$

where δ_{hk} is the Kronecker symbol, i.e. δ_{hk} is equal to 1 when h = k and to 0 otherwise. These evaluations of market demand price elasticities show that when heterogeneity grows (if the coefficients m_k become small), other things being equal, the consumption sector behaves very well, since aggregate demand price elasticities become asymptotically close to those arising from the maximization of a Cobb Douglas utility function. One has of course to be careful about the *ceteris paribus* clause and make sure that the parameters γ_h appearing in (1.7) are actually independent of the coefficients of heterogeneity m_k. One simple way to guarantee this outcome is to assume that the conditional densities $f(\alpha|a)$ are actually independent of the type a, and that when aggregating the demands $x_a(p,b_a)$ over all types, one gets indeed

$$\gamma_h \bar{b} \leqq p_h \int_A x_{ah}(p,b_a)\ \mu(da),$$

for all prices and all commodities h. The results we review here are all derived under this simplifying independence assumption. But it should be emphasized that any other specification ensuring that aggregate budget shares are uniformly bounded below by $\gamma_h > 0$, independently of the coefficients of heterogeneity m_h, generates exactly the same picture.

Inequalities (1.7) imply that *the Jacobian matrix of total market demand price derivatives has a negative dominant diagonal on a set of prices that is large when, other things being equal, the coefficients of heterogeneity* m_k *are small*. To see this point, remark first that from (1.7), total market demand for commodity h is a decreasing function of its own price when $m_h < \gamma_h$. In that case, (1.7) implies

$$\frac{1}{X_h(p)}\ \left|\frac{\partial X_h}{\partial p_h}(p)\right| \geqq (\gamma_h - m_h)/(p_h\ \gamma_h)$$

while for $k \neq h$

$$\frac{1}{X_h(p)} \left|\frac{\partial X_h}{\partial p_k}(p)\right| \leqq m_k/(p_k\gamma_h).$$

Then it follows immediately that the Jacobian matrix $\frac{dX}{dp}(p)$ has a negative dominant diagonal, i.e. $\left|\frac{\partial X_h}{\partial p_h}(p)\right| > \sum_{k\neq h}\left|\frac{\partial X_h}{\partial p_k}(p)\right|$ on the set of prices defined by the inequalities $\sum_k(m_k/p_k) < \gamma_h/p_h$, for every commodity h. As announced above, this set of prices is large when, other things being equal, the densities $f(\alpha|a)$ are spread out, i.e. when the coefficients m_k are small.

Competitive exchange equilibrium

The methods we just presented yield sharp results when applied to a general exchange equilibrium. To be specific, let us assume now that a type *a* defines not only a demand function $x_a(p,b)$, but also an endowment of goods ω_a in $\mathrm{Int}\mathbb{R}^{\ell}_{+}$. For each type *a* present in the population, conditional market demand $X_a(p,b)$ is given by (1.4) as before. But income is now price dependent since it is equal to the value $p\cdot\omega_a$ of the endowment. *Conditional market excess demand* is thus equal to $Z_a(p) = X_a(p,p\cdot\omega_a) - \omega_a$, while *total market excess demand* is obtained by aggregation over all types

$$Z(p) = \int_A Z_a(p)\ \mu(da)$$

(we suppose here that per capita total endowment, i.e. $\bar{\omega} = \int_A \omega_a\ \mu(da)$, is finite). An *exchange equilibrium* is accordingly a price vector p^* such that total excess demand vanishes on each market, i.e. $Z(p^*) = 0$.

It turns out that demand heterogeneity, or more precisely conditional densities $f(\alpha|a)$ that are relatively spread out, has strong consequences for uniqueness and stability of equilibrium in a simple exchange economy of this sort. The method of analysis is, here as before, to evaluate bounds for price derivatives, that depend explicitly on the degree of heterogeneity in the system, i.e. on the coefficients m_k. The method is applied here to the *excess* demands $Z_a(p)$, so these price derivatives involve not only the price derivatives of conditional market demand

$X_a(p,b)$, but (since income is equal to $p\cdot\omega_a$) the income derivatives of X_a as well. One has therefore to suppose here the elementary demand functions $x_a(p,b)$ to be homogenous of degree 0 in prices and income, in order to keep agregate income effects under control. Then application of the same type of argument leading to the inequalitites (1.6) or (1.7) yields that total excess demand has the *gross substitutability* property, i.e. $\frac{\partial Z_h}{\partial p_k}(p) > 0$ for $h \neq k$, on a set of prices that is large when the degree of heterogeneity in the system is significant, i.e. when the coefficients m_k are relatively small, other things being equal. This implies *uniqueness* of the equilibrium price vector p^* (up to multiplication by a scalar). One gets in addition that the *weak axiom of revealed preferences* holds in the aggregate, i.e. $p^*\cdot Z(p) > 0$, as between the equilibrium price system p^* and any other price vector p that is not colinear to p^*. Finally, these results imply that the unique equilibrium price vector is *stable* in any standard tâtonnement process, its basin of attraction being large and filling eventually the entire price space when the degree of heterogeneity grows, i.e. when the coefficients m_k become small (for a precise analysis, see Grandmont (1992a)).

Cournot oligopoly equilibrium

Demand heterogeneity has strong consequences for the study of Cournot oligopoly equilibrium as well (Grandmont (1992b)). To see this, let us specialize our previous formulation of market demand to a partial equilibrium analysis of what happens for a single commodity h, the conditions prevailing on other markets being fixed. The evaluation (1.7) of market demand's own price elasticity reads then

$$(1.8)\qquad \left| \frac{\partial \mathrm{Log} X_h(p)}{\partial \mathrm{Log} p_h} + 1 \right| \leqq m_h/\gamma_h,$$

so we are sure that market demand is downward sloping whenever $m_h < \gamma_h$. This is not enough, however, if we wish to study Cournot competition among firms on that market. Indeed, the properties of market demand that are usually needed in configurations of this sort, such as concave revenue functions, require also that the second own price derivative of the market demand function be "well behaved". The important point about the approach presented here is that *heterogeneity allows us to keep under control not only the first price derivatives of market demand, but higher order price derivatives as well.* The fact is most immediate if we go back to the identity (1.1), which was the starting point of the whole analysis. This identity gives the expression of conditional market expenditure, when one aggregates over "rescaled" demand functions that are distributed according to the density $f(\alpha)$. Now, if we assume that the density function is twice continuously differentiable and that its second partial derivative with respect to α_h is uniformly integrable, with $\int \left|\frac{\partial^2 f}{\partial \alpha_h^2}(\alpha)\right| d\alpha \leqq m_h$, then the *same* argument that led to (1.3) shows, by differenciating (1.1) twice with respect to $\mathrm{Log}p_h$,

$$\left|\frac{\partial^2 [p_h X_h(p,b)]}{(\partial \mathrm{Log}p_h)^2}\right| \leqq b\ m_h$$

This inequality is also here additive, i.e. it is preserved when aggregating demand over different types. So the equivalent of (1.6) for the second derivative of total market demand becomes

$$\left|\frac{\partial^2 [p_h X_h(p)]}{(\partial \mathrm{Log}p_h)^2}\right| \leqq \bar{b}\ m_h. \tag{1.8}$$

It should be intuitively clear that the inequalities (1.6), (1.7) and (1.8) are bound to give us pretty good control of the first *and* second own price derivatives of market demand for commodity h when the degree of heterogeneity grows. As a matter of fact, when the coefficients m_h are small, other things being equal, market demand, as well as its first and second price derivatives, behave approximately like a unit elastic demand. It turns out that this is sufficient to guarantee concave revenue functions, and even a unique Cournot oligopoly equilibrium, when firms (with constant marginal costs) compete in quantities in the market under consideration (Grandmont (1992b)).

This brief review strongly suggests that equivalence scales give us very powerful tools to study aggregation over heterogenous economic agents. An important feature of the analysis is that *heterogeneity alone* is capable of generating striking macroeconomic regularities, even when individual economic units are not necessarily "rational" in the traditional sense, i.e. even when they do not maximize a well defined objective function. This outcome is to be contrasted with the fact that, in traditional economic theory, the assumption of individual optimization generated very few macroeconomic predictions, owing to aggregation problems, as examplified by the Sonnenschein-Mantel-Debreu indeterminacy theorem. The studies we surveyed suggest accordingly that attention might fruitfully be redirected away from individual optimization toward the analysis of the consequences of heterogenous macroeconomic distributions. It remains to be seen whether this approach can be as successful as it has been in the research reviewed here, to analyze aggregation over productive units, temporary equilibrium, other models of imperfect competition or even welfare issues.

2. LEARNING : ARE RATIONAL EXPECTATIONS UNSTABLE ?

An essential feature of economic models is that expectations matter. Individual expectations about the future influence current decisions, hence observed market outcomes. On the other hand, observations about market outcomes determine individual expectations. In the absence of firmly established empirical facts about actual individual forecasting behavior, quite a few economic theorists have chosen to impose in their models some kind of consistency between expectations and realized market outcomes, on the ground that arbitrary assumptions about expectations would allow to explain almost everything — and thus nothing. A common modelling strategy is indeed to postulate "rational" expectations, i.e. that an individual's expectations about the future are correct at any moment, given current information.

A frequently heard defense of the hypothesis is that it should be the asymptotic outcome of a dynamic process in which individuals learn about the laws of motion of their environment. It is by no means clear, however, that taking into account learning should lead to convergence to a dynamic state where the "rational" expectations hypothesis is satisfied. Economic "reality" is not independent of how individual players conceive it. Changing beliefs about the economic system modify the laws of motion of that system. The second part of this lecture is devoted to a brief review of recent studies suggesting that, indeed, taking into account learning often generates dynamic local instability (Champsaur (1983), Benassy and Blad (1989), Grandmont and Laroque (1991), Grandmont (1990)). Such results seem to agree with (admittedly casual) empirical observations. The economic time series that display most volatility are those for which it appears that expectations are important in shaping current decisions (investment in capital equipment, inventories, durable goods, financial and stock markets). As we shall see shortly, imposing "rational" expectations would lead to the exact opposite and counterfactual conclusion : under the "rational expectations" hypothesis the more expectations matter, the more stable (in the absence of exogenous shocks to the "fundamentals") the market should be. By contrast, the studies reviewed below suggest that *local dynamic instability induced by learning is most likely to occur in markets where expectations matter significantly*. They suggest further that in such markets, the dynamics with learning may be highly nonlinear and generate complex trajectories, and moreover that forecasting mistakes may never vanish, even in the "long run". [1]

Smooth foregasting rules

To illustrate the point most simply, we consider a deterministic formulation (no random shocks) in which the state of system in period t is described by a single real number x_t. The state at t is determined by the decisions made by the traders in the past, which we summarize by the immediately preceding state x_{t-1}, and by the traders' forecast about the future x^e_{t+1}, through the temporary equilibrium relation

$$(2.1) \qquad T(x_{t-1}, x_t, x^e_{t+1}) = 0.$$

We assume that there is a large number of traders, each of whom has a negligible influence on the market as a whole, so strategic considerations are also negligible. In (2.1), x^e_{t+1} should be interpreted as an *average* forecast (each individual's forecast being weighted by its relative influence on the dynamic evolution). The state variable can be viewed, say, as a price and (2.1) as a market clearing condition. The analysis will be local, i.e. near a steady state defined by $T(\bar{x},\bar{x},\bar{x}) = 0$. We shall assume throughout that T is smooth and denote by b_0, b_1 and a the partial derivatives of T with respect to x_t, x_{t-1} and x^e_{t+1}, evaluated at the stationary state. The parameter a, which measures the local influence of expectations in the market under consideration, is of course assumed to be different from 0, otherwise the issues we wish to analyze would disappear.

The other ingredient of the model is a specification of how forecasts are made. The traders mental processes may be quite sophisticated : they may have "models" of the world depending upon a number of unknown parameters, reestimate these parameters at each date by using past data and use these estimates to forecast the future. It turns out we do not have, for the purpose of the present analysis, to specify in great details the traders' mental processes. In *all* cases, forecasts with depend on, and only on, past data. To simplify matters, we assume that the average forecast is a time-independent function of a finite (but possibly very large) array of past states [2]

$$(2.2) \qquad x^e_{t+1} = \psi(x_t, x_{t-1}, \ldots, x_{t-L}).$$

We shall assume throughout that when presented with a long constant sequence of states equal to $\bar{x}$, then $x^e_{t+1} = \bar{x}$. We postulate (in this subsection) that learning is regular enough so that the forecasting rule ψ is smooth, and we shall denote by $c_0, \ldots, c_L$ its partial derivatives with respect to $x_t, \ldots, x_{t-L}$, evaluated again at the steady state.

The dynamics with learning that will be actually observed is defined by putting (2.1) and (2.2) together

$$(2.3) \qquad T(x_{t-1}, x_t, \psi(x_t, x_{t-1}, \ldots, x_{t-L})) = 0.$$

Clearly $x_t = \bar{x}$ is a stationary solution of (2.3). If we assume that the partial derivative of (2.3) with respect to x_t at the steady state differs from 0, i.e. $b_0 + ac_0 \neq 0$, the actual dynamics with learning is well defined near the stationary solution $x_t = \bar{x}$. The issue is to analyze its stability.

The usual procedure to evaluate local stability is to linearize the equation near the steady state, look at the corresponding characteristic polynomial and to see whether the resulting eigenvalues are stable (have modulus less than 1) or not. It is intuitively clear that all the information we need to proceed here is in fact embodied in the local behavior of (2.1) and (2.2). Indeed, (2.3) is obtained by "coupling" the dynamical systems (2.1) and (2.2) — in which the forecast x^e_{t+1} would be replaced by the actual state x_{t+1} — in such a way that the variable x_{t+1} actually disappears. Stability or instability of the actual dynamics with learning will accordingly be a consequence of the interaction of the local eigenvalues of (2.1) and those of (2.2).

The local eigenvalues of (2.1) are the two roots of the characteristic polynomial obtained by replacing x^e_{t+1} by x_{t+1} and linearizing near the steady state

(2.4) $$Q_T(z) \equiv b_0 + b_1 z + az^2 = 0.$$

The corresponding local eigenvalues λ_1, λ_2 summarize the local behavior of the economic system *under the assumption of perfect foresight*. One remarks that, as announced earlier, the hypothesis leads to the counterfactual conclusion that the more expectations matter (the larger the coefficient a is, given b_0 and b_1), the smaller the modulus of the two perfect foresight roots λ_1 , λ_2 and thus the more stable the local dynamics of the system should be.

The same procedure applied to the forecasting rule yields the polynomial

(2.5) $$Q_\psi(z) \equiv z^{L+1} - \Sigma_0^L c_j z^{L-j} = 0.$$

Since the characteristic polynomial is obtained by linearizing (2.2), the corresponding L+1 roots $\mu_1,\ldots,\mu_{L+1}$ (the local eigenvalues of the forecasting rule) describe the set of regularities, i.e. the trends and frequencies, that traders are on average able to filter out of current and past deviations Δx_t , $\Delta x_{t-1},\ldots,\Delta x_{t-L}$ from the stationary state. If people extrapolate constant sequences ($\psi(x,x,\ldots,x) \equiv x$ near $\bar{x}$), then $\mu = 1$ is solution of (2.5). If they extrapolate sequences that oscillate between two values ($\psi(x,y,x,y,\ldots) \equiv y$ near $\bar{x}$), then $\mu = 1$ and $\mu = -1$ are solutions of (2.5). If people are able to recognize and willing to extrapolate the specific trend r from past deviations, then $\mu = r$ is a solution of (2.5). More generally, the fact that $\mu = re^{i\theta}$ is a local eigenvalue of the forecasting rule means that people are able to recognize the trend r and the frequency associated to θ in past deviations from the stationary state. Of course, a smooth forecasting rule essentially acts locally as a linear filter and can extract only a finite set of regularities from a finite amount of data. When the memory L is large, and if the traders are relatively sophisticated, one should expect the set of local eigenvalues $\mu_1,\ldots,\mu_{L+1}$ of the forecasting rule to be somewhat spread out in the complex plane. It turns out that this configuration leads to local instability of the actual learning dynamics, especially when expectations matter significantly, i.e. when the coefficient a is large.

Specifically, let $\mu_1^* < \mu_2^*$ be the smallest and largest *real* local eigenvalues of the forecasting rule. Consider the situation where the two perfect foresight roots λ_1, λ_2 are either both complex or where, if they are real, they belong to the open interval (μ_1^*,μ_2^*). Then it is easy to show that the characteristic polynomial associated to the actual learning dynamics (2.3) has a real root ρ that lies outside the interval $[\mu_1^*,\mu_2^*]$. If we make the mild assumption that people are willing to extrapolate long sequences that oscillate between two arbitrary values x and y near the steady state, then as noted earlier, $\mu = 1$ and $\mu = -1$ are local eigenvalues of the forecasting rule. In that case $\mu_1^* \leqq -1$ and $\mu_2^* \geqq 1$, and the actual learning dynamics is bound to be locally unstable. This configuration, and thus local instability, is most likely to occur when expectations matter significantly, i.e. when the coefficient a measuring the local influence of forecasts on the evolution of the system is relatively large, for then the modulus of the two perfect foresight roots λ_1, λ_2 is small.

<u>Discontinuous forecasting rules</u>

A smooth forecasting rule (locally, essentially a linear filter) can only extract a finite set of trends and of frequencies from a finite sequence of past deviations from the steady state. It is not difficult to think of learning rules, e.g. through least squares regressions on such past deviations, that would allow to recognize and extrapolate, say, *any* real trend present in past data. Of course, one is then bound to lose smoothness and even continuity of the associated forecasting rule. Yet one would like to inquire whether the previous instability results carry over to such learning processes. We are going to show that this is indeed the case.

To simplify matters, we set $\bar{x} = 0$ (so x_t stands now for a *deviation* from the steady state) and linearize (2.1)

$$(2.6) \qquad b_1 x_{t-1} + b_0 x_t + a x^e_{t+1} = 0$$

with $a \neq 0$ (expectations matter) and $b_0 \neq 0$ so that we can actually solve (2.6) for the current state x_t. As for expectations, we assume that people believe, say, that the law of motion of the system is

$$(2.7) \qquad x_n = \beta x_{n-1} + \varepsilon_n \qquad \text{or} \qquad x_n = (\beta + \varepsilon_n)\, x_{n-1}$$

where β is an unknown coefficient and ε_n is white noise. Forecasts are generated as follows. At the outset of period t, traders form an estimate of the unknown coefficient β by looking at past states

$$(2.8) \qquad \beta_t = g(x_{t-1},\ldots,x_{t-L}),$$

and they formulate a forecast by iterating twice the relation (2.7)

$$(2.9) \qquad x^e_{t+1} = \beta^2_t\, x_{t-1}.$$

The relations (2.8) and (2.9) together define a forecasting rule $x^e_{t+1} = \psi(x_{t-1},\ldots,x_{t-L})$ exactly as before. One possible interpretation of this learning procedure is that people know where the steady state lies, but try to improve their performances by forecasting growth rates. We may, however, lose continuity if we wish, as here, that people be able to filter a continuum of trends out of past deviations from the steady state. For instance, if people estimate the models (2.7) through least squares, they will get

$$\beta_t = \frac{x_{t-1}\, x_{t-2} + \ldots + x_{t-L+1}\, x_{t-L}}{x^2_{t-2} + \ldots + x^2_{t-L}},$$

or

$$\beta_t = \frac{1}{L-1}\left[\frac{x_{t-1}}{x_{t-2}} + \ldots + \frac{x_{t-L+1}}{x_{t-L}}\right],$$

which are only defined out of the steady state and in fact highly discontinuous there. The nice feature of the above least squares learning schemes is that the estimates β_t are averages of past ratios x_{t-j+1}/x_{t-j}. As a result, the forecasting rule generated by (2.8) and (2.9) has the property that for every real number r,

$$\psi(r^{L-1}\, x,\ldots,rx,x) \equiv r^{L+1}\, x.$$

People can extract any real trend from past deviations from the stationary state, or in other words, any real number is a "local eigenvalue" of the forecasting rule. The price to pay for this nice feature is the loss of continuity.

The actual learning dynamics is obtained as before by putting together (2.6) with the forecasting rule defined by (2.8) and (2.9), which yields

$$(2.10) \qquad x_t = -b_0^{-1}[b_1 + a\,\beta_t^2]\, x_{t-1} \equiv \Omega(\beta_t)\, x_{t-1}\,,$$
$$\beta_t \text{ given by (2.8).}$$

The relation (2.8) defining the actual learning dynamics involves a map Ω (introduced in the literature on the subject by Marcet and Sargent (1989)) that has a remarkably simple interpretation. Indeed it describes the link that exists between the *beliefs* people have at the outset of period t about the dynamics of the growth rates x_n/x_{n-1}, as summarized by the estimate β_t, and the *actual* ratio x_t/x_{t-1} that will be observed in that period. It is easy to verify that the fixed points of Ω coincide with the perfect foresight roots λ_1, λ_2 when these are real, and that Ω has no fixed points when λ_1, λ_2 are complex (the equation $\Omega(\beta) = \beta$ is in fact identical to (2.5) with $z = \beta$).

The smallest and largest real "eigenvalues" of the forecasting rule are $-\infty$ and $+\infty$ whenever the estimate (2.8) is an average of past ratios x_{t-j+1}/x_{t-j} , $j = 2,\ldots,L$. By analogy with the smooth case discussed earlier, we should expect that the actual learning dynamics is locally unstable in the present case as well, and this for *all* configurations of the two perfect foresight roots λ_1 and λ_2. It can be shown that this conjecture is indeed true under quite general conditions (Grandmont and Laroque (1991)). When the perfect foresight roots λ_1 , λ_2 are complex, local instability occurs for all initial conditions. Suppose now that they are real, with $|\lambda_1| < |\lambda_2|$. Then local instability occurs for an open set of initial conditions, i.e. when the initial ratios x_{t-1}/x_{t-2} , etc ... have all the same sign as λ_2 and a modulus larger than $|\lambda_2|$. Were the forecasting rule smooth, getting local unstability for an open set of initial conditions would imply instability for almost every departure from the steady state. This may not be true here as the forecasting rule is discontinuous. If the map Ω is contracting at the perfect foresight root of smallest modulus, i.e. $|\Omega'(\lambda_1)| < 1$, and if that root is stable, i.e. $|\lambda_1| < 1$, then one will get local stability whenever all initial growth rates x_{t-1}/x_{t-2}, etc ... are close enough to λ_1. Of course if $|\Omega'(\lambda_1)| > 1$, the phenomenon disappears. Be it as it may, the size of the open set for which local instability occurs becomes larger as the coefficient a measuring the relative influence of expectations goes up (the modulus of the two perfect foresight roots goes down). Thus we reach the same qualitative conclusion as in the smooth case : *the more expectations matter, the more probable learning induced local instability becomes, and the more volatile market outcomes should be.*

To illustrate these points, we consider the particular situation where the estimate β_t is x_{t-1}/x_{t-2}. One gets then even sharper results that can be visualized through simple diagrams. In view of (2.10) the actual dynamics with learning is in that case described by the recurrence equation $x_t/x_{t-1} = \Omega(x_{t-1}/x_{t-2})$. The curve representing Ω (a parabola) is pictured in Fig. 1.a,b in the case where the sum of the two perfect foresight roots, i.e. $\lambda_1 + \lambda_2 = -b_0/a$, is positive (the reader will verify that one gets identical results in the opposite case, with the asymptotic branches of the parabola going down). The two roots λ_1, λ_2 are complex in Fig. 1.a, and one gets local instability for all initial conditions. The two perfect foresight roots are real in Fig. 1.b. There local instability occurs whenever the modulus of the initial ratio x_{t-1}/x_{t-2} exceeds $|\lambda_2|$. The ratios x_t/x_{t-1} converge to λ_1 whenever $|x_{t-1}/x_{t-2}| < \lambda_1$, and one gets local stability if $|\lambda_1| < 1$. The size of the region for which one gets instability grows as $|\lambda_2|$ goes down, i.e. when the coefficient a measuring the influence of expectations becomes large.

Fig. 1.a

Fig. 1.b

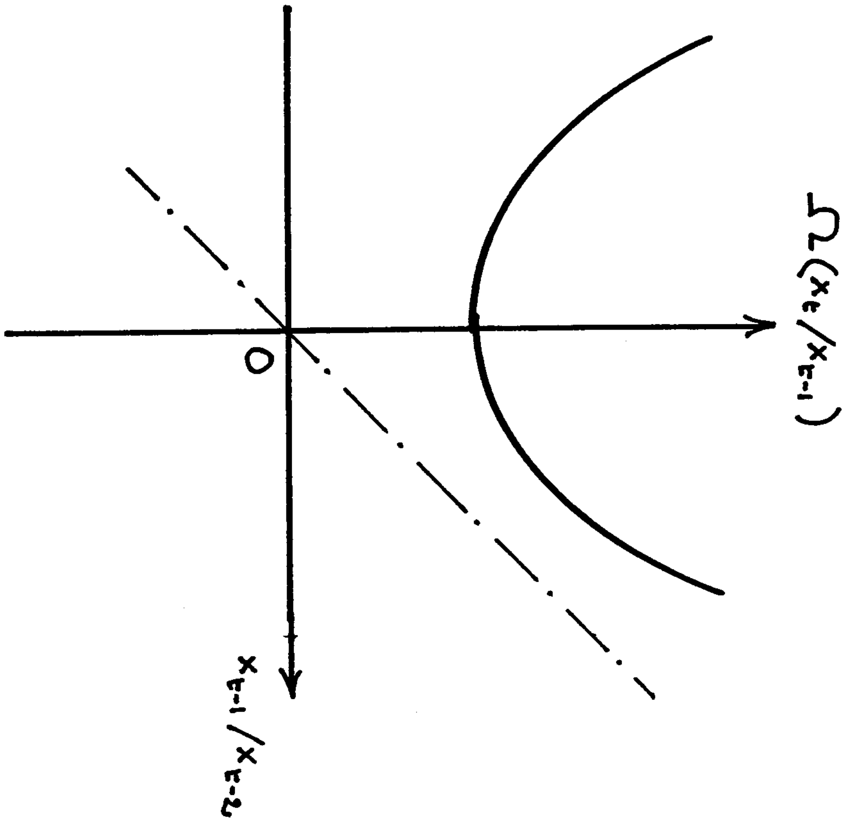
$\Omega(x_t/x_{t-1})$
0
x_{t-1}/x_{t-2}

Fig. 1.a

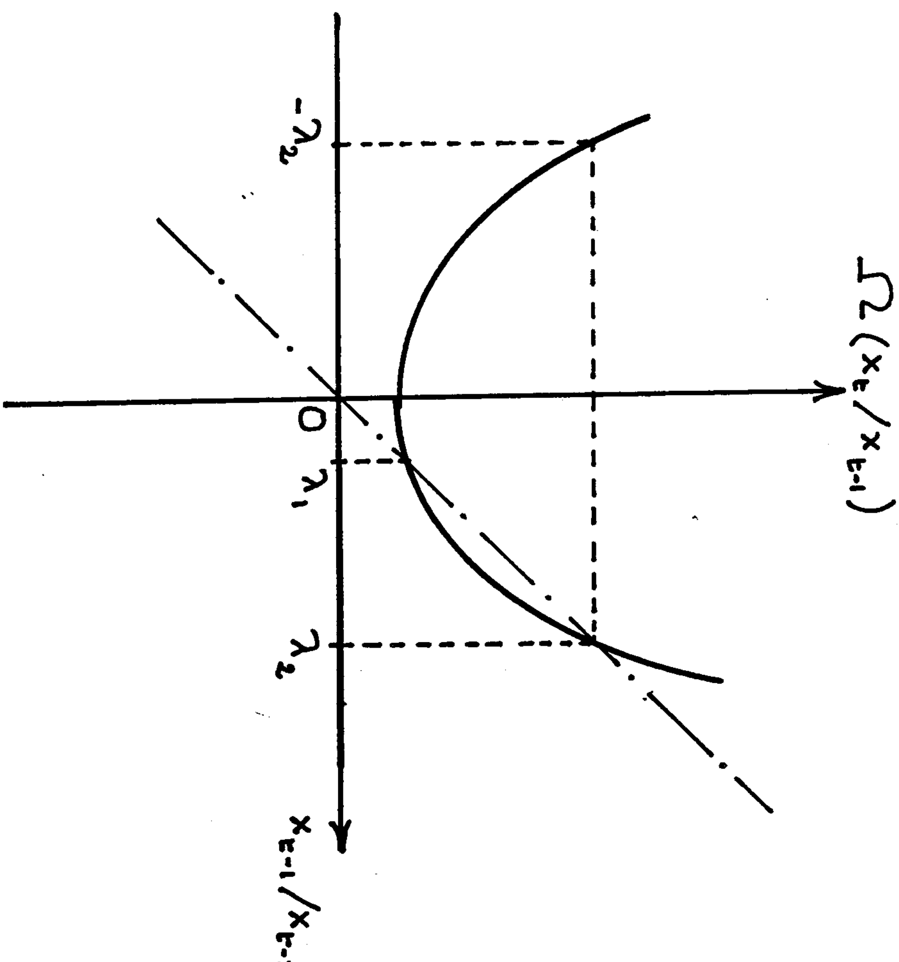
$\Omega(x_t/x_{t-1})$
$-\lambda_2$
0
λ_1
λ_2
x_{t-1}/x_{t-2}

Fig. 1.b

The foregoing analysis suggests strongly that learning is most likely to generate local instability in markets where expectations matter significantly. Another interesting feature is that although the world may be simple and close to linear (see (2.1) or (2.6)), the mental processes employed by economic agents when trying to learn the laws of motion of the system may be highly nonlinear. Then these nonlinearities will also show up in the actual observed dynamics : here, (2.10) involves the map Ω which describes a parabola. That feature suggests that the actual learning dynamics may generate non only local instability but also quite complex, even chaotic, nonexplosive expectations-driven fluctuations. The point is illustrated in Fig. 2, in the simple case where the estimate β_t is equal to x_{t-1}/x_{t-2}. There, the map Ω is expanding at both perfect foresight roots. The actual ratios x_t/x_{t-1} cannot converge to either of them but they may follow chaotic trajectories that are trapped in some invariant interval $[-c,c]$. [3]

<u>Fig. 2</u>

All this opens promising and largely unexplored avenues in business cycles theory. Although the "fundamentals" of the economic system may display only small nonlinearities and may not vary much over time, the traders' learning schemes are presumably highly nonlinear and this may lead to complicated expectations-driven nonexplosive fluctuations along which forecasting mistakes may never vanish, even in the "long run". This cannot, however, be the end of the story. For such a situation to be robust, of course, one should require some degree of consistency between the actual dynamics and private beliefs, so that traders have no incentive to change their views about how the world works. One might envision for instance a situation in which traders attribute their forecasting mistakes to "noise", although the observed dynamics are actually deterministic but chaotic, as in Fig. 2. As I said, this is largely unknown territory and should be the subject of further research.

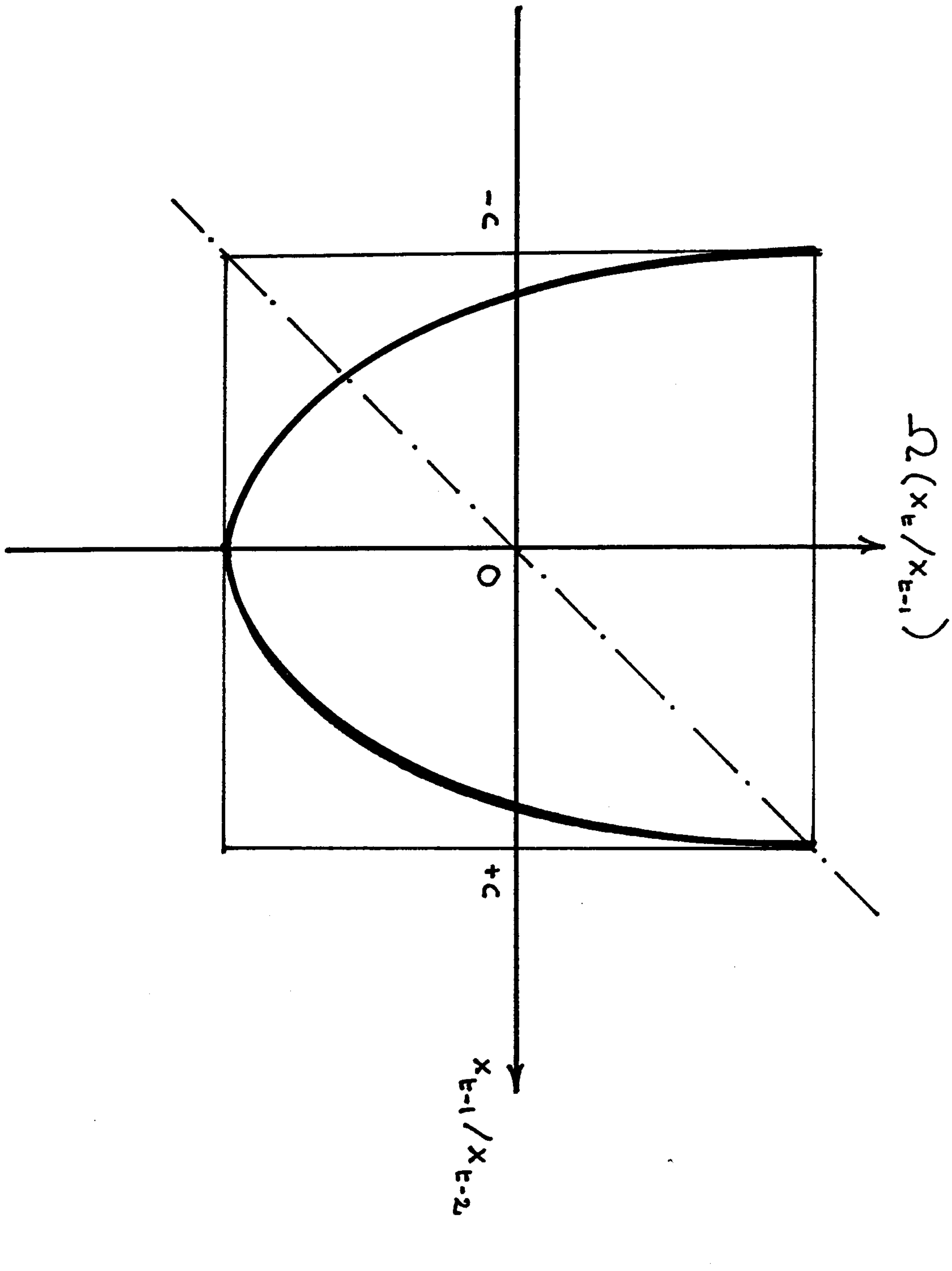

Fig. 2

R E F E R E N C E S

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F O O T N O T E S

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[1] In the sequel, we follow the presentation of Grandmont and Laroque (1991).

[2] Expectations may also depend on past forecasts, so as to allow people to learn from their past mistakes by comparing past forecasts with actual realizations. The results would be qualitatively the same (Grandmont (1990)).

[3] Examples of cycles and chaos generated by learning have been provided for instance by C. Hommes (1991, 1992), G. Negroni (1992).