

TESTS OF THE "CONVERGENCE HYPOTHESIS" :

A CRITICAL NOTE

by

Daniel COHEN¹

(Ecole Normale supérieure, CEPREMAP and CEPR)

N°9208

ABSTRACT

TESTS OF THE "CONVERGENCE HYPOTHESIS" :

A CRITICAL NOTE

I argue in this paper that the "speed of convergence" which has been estimated in recent works on "convergence" does not capture "actual" convergence towards a steady-state but rather conditional dynamics towards a moving target. Although this conditional convergence can be taken to imply that there are decreasing returns to such storable assets as physical and human capital, they cannot be taken to rule out unbounded growth (since human capital could well be indefinitely growing). I offer a test of the "convergence hypothesis" based on the analysis of a two dimensional system and find that it cannot be accepted.

RESUME

LES TESTS DE L'HYPOTHESE DE "CONVERGENCE" : UNE ANALYSE CRITIQUE

Cet article analyse les tests de l'hypothèse de "convergence" qui ont été récemment offerts dans la littérature économique et conclut que ces tests analysent uniquement une "convergence conditionnelle" et non pas une vraie convergence vers un état stationnaire (immuable). Même si ces tests permettent effectivement d'accepter l'hypothèse qu'il existe des rendements décroissants à l'accumulation du capital (physique et humain) ils ne permettent pas de rejeter l'hypothèse que la croissance puisse indéfiniment continuer (de façon endogène). Il est tout-à-fait possible, en effet, que le capital humain puisse croître indéfiniment. Je propose dans l'article un nouveau test de l'hypothèse deconvergence, basée sur l'analyse explicite d'un système dynamique bi-dimensionnel. Ce test conduit à rejeter l'hypothèse de convergence.

Key Words :Growth, Convergence.

Mots clés : Croissance, Convergence.

J.E.L. : 040

I - INTRODUCTION

In a series of recent papers, Barro and Sala-i-Martin (1990 and 1991) (henceforth BX) have thoroughly investigated the dependency of the growth rate of an economy upon the initial value of the economy's per capita income. When this correlation is unconditionnally tested across nations, only the OECD countries appear to exhibit a negative correlation between these two variables. When one controls for education, then a robust negative correlation is obtained. (Education is proxied by enrollment in primary and secondary school.) A fascinating aspect of BX's work is the "universality" of the coefficient that they obtain. Across European or U.S regions or across countries, this coefficient inexorably implies that an economy "converges" towards its "steady-state" at a rate of about 2% a year.

In order to analyze where this result comes from, let us first discuss how the steady-state (towards which the economy is converging "at 2% a year") is characterized. Such characterization is best offered by a paper by Mankiw, Romer and Weil (1990) -henceforth MRW- that shares many of the insights of BX's work. MRW explicitly derive the steady state towards which an economy is converging from an equation such as " $sQ_{\infty} = (d+n)K_{\infty}$ " which determines the steady-state to which a constant saving rate s takes the economy to. MRW take account of both the accumulation of physical and human capital. Investment in human capital is calculated in a way which is almost identical to BX's measure of education and is roughly equal to enrollment in secondary shool. (That the difference between the two approaches is inessential is shown in section 3 below). Taking the *observed* values of the rate of investment in human and physical capital to proxy their *steady-state* values allows MRW to determine explicitly the steady-state towards which, *ceteris paribus*, the economy is converging.

1. This paper was written while I was a consultant at the Debt and International Finance Division of the World Bank. I thank its members for their very warm hospitality and their useful comments, and J.F.Nivet for his invaluable research assistance. This paper does not necessarily represent the view of the World Bank.

It does not really matter that such a calculated steady-state actually proxies the true steady-state towards which the economy is converging, if one wants to capture the coefficient of the production function. As I show in section 2 below, differentiating the production function around the point where the economy initially stands delivers a law of motion that always "looks like" a convergence towards a "pseudo steady state" which would prevail if the saving rates were to stay indefinitely constant at their initial value. On this ground, one can consequently argue that BX and MRW get an indirect (and ingenious) way to calculate the parameters of the production function. They say less however on the *actual* steady-state (or on the speed of convergence towards it), nor even can they actually say if there is such a steady-state.

If enrollment in secondary school, rather than being an exogenous parameter reflecting perhaps the "taste" of a country toward education, is instead an endogenous function of the stage of development of the country, then the true steady-state, if it exists, will not be characterized by the "pseudo steady-state" (to prevail "all saving rates equal"). Furthermore, even though BX and MRW show that there are decreasing returns in human and physical capital, it need not be the case that there actually is a steady-state towards which the economy is going. If human capital can grow without bound (and if investment in human capital follow) as in the Lucas-Uzawa's analysis, then there is indeed no steady-state towards which the economy is converging even though controlling for the *current* saving rate would exhibit a negative correlation between growth and initial income. This negative correlation, which is the "universal" coefficient uncovered by BX, depends on the parameter of the production function only. It can only be interpreted as the speed of convergence towards the steady-state (rather than towards the "pseudo steady state") if savings (in both physical and human capital) have already reached their steady state values. Otherwise, one need to calculate a *two-dimensional* dynamic system (in both physical and human capital) and test whether it is stable. We perform such a test in section IV below : it leads us to reject the hypothesis that the system is convergent. Given the very imprecise

nature of the data, however, this result must be taken with great caution.

The paper proceeds as follows. In section II, we spell out the core of our argument by showing why controlling for the saving rates yields "pseudo-dynamics" rather than actual ones. In section III, we compare the results in BX to the results in MRW. In section IV we document the dependency of school enrollment on the level of development and we offer a direct test of the convergence hypothesis based on the analysis of a two dimensional system.

II - DYNAMICS AND "PSEUDO-DYNAMICS" IN A SOLOW MODEL

II.1. A simplified approach

In this sub-section, I first offer to give the essence of the analysis in a most simplified version of the Solow model.

Assume that production is :

$$(1) \quad Q_t = A K_t^\alpha L^{1-\alpha}$$

and take the law of motion of capital to be :

$$(2) \quad \dot{K}_t = -d K_t + I_t$$

in which Q is output, K is "capital", L is population and I_t is investment. The dot is the time derivative. Assume that population is constant.

In order to get the (transitional) dynamics of the economy, one usually writes (2) as :

$$(3) \quad \frac{\dot{K}_t}{K_t} = \frac{I_t}{K_t} - d, \quad \text{or :}$$

$$(4) \quad \frac{\dot{K}_t}{K_t} = d \left[\frac{I_t}{d K_t} - 1 \right]$$

i.e.

$$(5) \quad \frac{\dot{K}_t}{K_t} = d \left[\frac{I_t}{Q_t} \cdot \frac{A L^{1-\alpha}}{d} \cdot K_t^{\alpha-1} - 1 \right]$$

Let us now define $\hat{K}_t$ as the solution to :

$$(6) \quad d \hat{K}_t = \left(\frac{I_t}{Q_t} \right) \cdot A \hat{K}_t^\alpha L^{1-\alpha}$$

i.e.

$$(7) \quad \hat{K}_t = \left[\frac{I_t}{Q_t} \cdot \frac{A L^{1-\alpha}}{d} \right]^{\frac{1}{1-\alpha}}$$

$\hat{K}_t$ is simply the steady-state value towards which the stock of capital would converge to if the economy were to keep investing indefinitely at

the fixed rate $s = \frac{I_t}{Q_t}$. From equation (7) one can then write

$$(8) \quad \frac{\dot{K}_t}{K_t} = d \left[\left(\frac{K_t}{\hat{K}_t} \right)^{\alpha-1} - 1 \right]$$

and log-linearizing (8) :

$$(9) \quad \frac{d}{dt} k_t = d(1-\alpha) \left[\hat{k}_t - k_t \right]$$

in which $k_t = \text{Log } K_t$; $\hat{k}_t = \text{Log } \hat{K}_t$.

Let $y_t = \text{Log } Q_t$ and $\hat{y}_t \equiv \text{Log} \left[A \hat{K}_t^\alpha L^{1-\alpha} \right] = \frac{1}{1-\alpha} \text{Log } A + \text{Log } L$
 $+ \frac{\alpha}{1-\alpha} \text{Log } \frac{I_t}{Q_t}$, one can then write :

$$(10) \quad \frac{d}{dt} y_t = d(1-\alpha) \left[\hat{y}_t - y_t \right]$$

If one carefully avoids to interpret $\hat{y}_t$ as a predictor of the steady-state value of income, one sees that (10) offers nothing else but a compact form of (1) and (2). Controlling directly or indirectly for $\hat{y}_t$ and estimating equation (10) gives a "universal" coefficient for y_t which is $d(1-\alpha)$. This coefficient says little, however, about the actual dynamics of the economy since -in order to get those- one needs to know how $\frac{I_t}{Q_t}$ is actually set and -above all- how it comes to depend upon the income of the country.

It is this confusion between an estimate of equation (10) and an analysis of the actual dynamics of the economy which, we believe, lies at the heart of the results on convergence.

II.2. The "augmented" Solow model

Let us now briefly present how equation (10) is written in the more general case that is examined by Mankiw, Romer and Weil. Assume that production is given by

$$(11) \quad Q_t = K_t^\alpha H_t^\beta (A_t N_t)^{(1-\alpha-\beta)}$$

in which H_t is the stock of human capital and N_t is the total number of workers. Let us write the law of motion of K_t as in (2) and write the law of motion of H_t as :

$$(12) \quad \dot{H}_t = -d H_t + Z_t$$

in which Z_t is the total investment in human capital at time t . For simplicity, we follow MRW and assume the same rate of depreciation for physical and human capital. Call $Y_t = Q_t / (N_t A_t)$ the level of income per-effective capita. Let n be the rate of growth of the workers and μ the rate of growth of its "effectiveness".

One can then write

$$(13) \quad \frac{\dot{Y}_t}{Y_t} = \alpha \left[-d - \mu - n + \frac{I_t}{K_t} \right] + \beta \left[-d - \mu - n + \frac{Z_t}{H_t} \right]$$

Define, as in equation (7), $\hat{K}_t$ and $\hat{H}_t$ as the solutions to :

$$(14) \quad \begin{cases} (d + n + \mu) \hat{K}_t = \frac{I_t}{Q_t} \hat{K}_t^\alpha \hat{H}_t^\beta (A_t N_t)^{(1-\alpha-\beta)} \\ (d + n + \mu) \hat{H}_t = \frac{Z_t}{Q_t} \hat{K}_t^\alpha \hat{H}_t^\beta (A_t N_t)^{(1-\alpha-\beta)} \end{cases}$$

so that

$$(15) \begin{cases} \hat{K}_t = \left[\left(\frac{I_t}{Q_t} \right)^{1-\beta} \left(\frac{Z_t}{Q_t} \right)^{1-\alpha} / (d + n + \mu) \right]^{\frac{1}{1-\alpha-\beta}} (A_t N_t) \\ \hat{H}_t = \left[\left(\frac{I_t}{Q_t} \right)^{\alpha} \left(\frac{Z_t}{Q_t} \right)^{1-\alpha} / (d + n + \mu) \right]^{\frac{1}{1-\alpha-\beta}} (A_t N_t) \end{cases}$$

Again $\hat{K}_t$ and $\hat{H}_t$ would be the steady-state value of K_t and H_t if $\frac{I_t}{Q_t}$ and $\frac{Z_t}{Q_t}$ were to stay constant all along. Call $\hat{Y}_t = \hat{K}_t^{\alpha} \hat{H}_t^{\beta} / (N_t A_t)^{\alpha+\beta}$.

Call, as before, $y_t = \text{Log } Y_t$ and $\hat{y}_t = \text{Log } \hat{Y}_t$. One can log-linearize (13) as :

$$(16) \quad \frac{d}{dt} y_t = (1-\alpha-\beta) (d + n + \mu) [\hat{y}_t - y_t]$$

in which, one can write $\hat{y}_t$ as :

$$(17) \quad \hat{y}_t = \frac{\alpha}{1-\alpha-\beta} \text{Log } \frac{I_t}{Q_t} + \frac{\beta}{1-\alpha-\beta} \text{Log } \frac{Z_t}{Q_t} - \frac{\alpha + \beta}{1-\alpha-\beta} \text{Log } (d+n+\mu)$$

The same remarks as in the simplified version apply.

Equation (16) does *not* help depicting the dynamics of the economy. As long as one does not know the law of motion of $\hat{y}_t$, and in particular how $\hat{y}_t$ is correlated to y_t , one cannot extrapolate from (16) the law of motion of income. *However*, estimating (16) (when $\hat{y}_t$ is as in (17)) does help capture the coefficient of the production function (11). Indeed, to repeat, (16) is a compact form of (11), (2) and (12).

III - EMPIRICAL TESTS OF THE SOLOW MODEL AND OF THE "CONVERGENCE HYPOTHESIS"

We proceed, in this section, to compare the results obtained in Barro and Sala-i-Martin (BX) and Mankiw, Romer and Weil (MRW). (see also Levine and Revelt, 1990, for additional insights on the robustness of other factors).

Both authors estimate the growth rate over the period 1960-1985 as a function of the 1960 per-capita income (in Log terms) and other parameters.

In MRW, the other variables are $\text{Log } \frac{I_t}{Q_t}$ and $\text{Log } \frac{Z_t}{Q_t}$ (Log of the rate of investment in physical and human capital) over that period. In BX, the other variables are (essentially) the initial value of enrollment in primary and secondary school.

MRW proxy $\text{Log } \frac{Z_t}{Q_t}$ as follows. They take that saving rate to be proportional to the share of young people aged 15 to 19 (as a percentage of the working age population) that goes to secondary school. In order to get the enrollment in secondary school of the 15-19, they simply apply the enrollment rates obtained in the UNESCO data (which apply to the population aged 12-17) to the population aged 15-19. Clearly, their proxy for Z_t/Q_t is highly collinear to the secondary school enrollment data that are used by BX to proxy human capital. One difference, however, is that BX use *initial* enrollment data, while MRW use the average value over their period of estimation. Another important difference is that BX do not control for the investment rates but control for primary education.

In order to see which differences these discrepancies introduce we have re-estimated the following equations. Call Enrol2 the Unesco data on secondary enrollment, $\text{Log}(\text{Enrol2})_0$ the Log of its 1960 value and $\text{Log}(\text{Enrol2})_t$ its average value over the period 1960-1988. Call g the average growth rate of per capita income over the period 1960-1988

($g = \frac{1}{28} \text{Log} \frac{y_{88}}{y_{60}}$, in which y_t is per-capita income at t). Finally, let us follow MRW and take 0.05 to proxy $d + \mu$ in equation (16). We have run the following equations :

$$(18) \left\{ \begin{array}{ll} g = a+b \text{Log} (\text{Enrol2})_t + c \text{Log} \frac{I_t}{Q_t} + d y_0 + e \text{Log} (n+0.05) & (18.1) \\ g = a+b \text{Log} (\text{Enrol2})_0 + c \text{Log} \frac{I_t}{Q_t} + d y_0 + e \text{Log} (n+0.05) & (18.2) \\ g = a+b \text{Log} (\text{Enrol2})_t + c y_0 & (18.3) \\ g = a+b \text{Log} (\text{Enrol2})_0 + c y_0 & (18.4) \end{array} \right.$$

The results are shown in Tables A1 to A4 in appendix 1. (We use the latest Summers and Heston (1991) data).

Equation (18.1) essentially replicates the results in MRW. The value of the coefficient b , c , d and e are consistent with equations (16) and (17), (when one interprets Enrol2 as a proxy for the rate of investment in human capital), when $\alpha = 0.32$ and $\beta = 0.36$. One also sees from equation (18.2) that equation (18.1) is not much changed when the initial -rather than the average- value of secondary enrollment is taken as an explanatory variable. Interestingly, enrollment in primary school would be significant in equation (18.2) but not significant in equation (18.1). *Initial* enrollment in primary school (such as used in BX) may then be taken to help proxy *current* secondary enrollment.

One also sees from the comparison of equation (18.2) and (18.3) that the value of the partial correlation of growth with initial income is not much changed either when one does not control for investment and $\text{Log} (n + 0.05)$. Equation (3) is consistent with equation (13) and (17) when $\alpha = 0$ and $\beta = 0.66$. Compared to equation (18.1), this simply amounts to assume that K is collinear to H .

Finally the comparison between (3) and (4) shows that taking initial rather than current education does not alter the value of the partial correlation of growth with initial income. Equation (4) is essentially the form that is estimated in BX (The negative correlation between growth and initial income which is obtained when controlling for school enrollment was actually demonstrated earlier in Razin, 1977). On consequently sees through the chain that goes from equation (18.1) to (18.4) that the equations estimated by MRW and BX are essentially the same. In BX's interpretation, education was taken to proxy the steady-state. This is conform MRW's analysis, but from section II above, one sees that this is only correct if the "pseudo-steady state" obtained in equation (17) can be taken as a proxy of the steady-state. Otherwise equations (18) can only be taken to deliver an (indirect) estimate of the parameters of the production function.

IV - "CONVERGENCE" REVISITED

Let us take for granted, here, MRW's result that production can be written :

$$(19) Q = K^{1/3} H^{1/3} (AL)^{1/3}$$

This compact formulation leads to portray the economies as (at least) a two dimensional vector (K_t, H_t) . Using the latest Summers and Heston data set, we can get some capital stock data for a subsample of 24 countries. Extrapolating these results (see appendix 2) one can also expand these capital stock data to a larger sample of 65 countries. (In the sequel we shall rely on the original set of 24 countries, although we also report the results obtained for the larger data set).

Call $y_t = \text{Log } Q_t/L_t$.

One can write

$$h_t = 3 y_t - k_t$$

in which $h_t = \text{Log}(H_t A_t/L_t)$ and $k_t = \text{Log } K_t/L_t$

so that one can "construct" (augmented) human capital data based on the physical capital data and on the assumption that production can indeed be written as in (19) (in which h_t is a worldwide phenomenon) . We can then equally characterize the economies as a vector (h,k) , (h,y) or (k,y) .

IV.1. The "speed" of convergence

In view of MRW's interpretation of BX results, the "speed" of convergence which is estimated is the coefficient $(d+n+\mu)(1-\alpha-\beta)$ in equation (16) which is the now famous "2% a year" result. As I tried to make clear in my discussion in section II, this is only a valid description of the actual dynamics of the economies if the saving rates and the school enrollment ratio can be taken as exogenous. Let us focus here on school enrollment and take instead the opposite view that school enrollment is a function of the state of development. One can then characterize the dependency of school enrollment on the state of development -when the latter is proxied through the initial endowment (h_0, y_0) - through the following regression :

$$\text{Log (Enrol2)} = - 1.64 - 0.106 h_0 + 0.874 y_0 + \varepsilon_t ; R^2 = 0.75$$

(-1.79) (9.11)

$$\equiv z_0 + \varepsilon_t$$

Let us call z_0 the "structural" term and interpret the residual ε_t as an idiosyncratic noise which measures the specific "taste" of a country for schooling.

We have then run the following equation :

$$dy = \alpha + \beta z_0 + \gamma \varepsilon_t + \delta y_0$$

so as to separate the influence on growth of the "structural" determinants of education from their idiosyncratic terms.

The results (shown in appendix 2) come as follows :

$$dy = - 0.75.10^{-2} + 0.0235 z_0 + 0.0176 \varepsilon - 0.834.10^{-2} y_0$$

(2.43) (0.24) (-1.4)

$$R^2 = 0.38$$

One sees that the point estimate of the structural term z_0 is significant, while the idiosyncratic term is not. Albeit less precise (given the smaller sample) the point estimates of the correlation of growth with z_0 and y_0 appear to be quite similar to the results obtained in equation (18.3) (see appendix 2) in which we got:

$$dy = 2.80 + 0.017 \text{ Log(Enrol2)}_t - 0.86.10^{-2} y_0 \quad ; \quad R^2 = 0.40$$

(6.8) (-3.1)

(for the larger sample of 91 countries).

We can consequently argue that it is the endogenous (rather than the exogenous) part of the school enrollment data which appear to drive the results in BX. In order to go from "conditional dynamics" to "actual dynamics" one consequently need to give a full account of the dynamics of human and physical capital.

IV.2. Two-dimensional dynamics

Let us take the view that the economies can indeed be characterized as two-dimensional vectors (h_t, k_t) (disregarding persistent idiosyncradies) and let us attempt to account of their dynamics through the following two-dimensional system :

$$21) \quad \left\{ \begin{array}{l} \frac{d}{dt} h_t = a + b h_0 + c k_0 \\ \frac{d}{dt} k_t = a' + b' h_0 + c' k_0 \end{array} \right.$$

The system is then a stable one if and only if $b + c' < 0$ and $b c' - b' c > 0$.

We present in appendix 3 the results obtained (by the SUR estimator) for a sub-group of 24 countries for which capital stock data are available in Summers-Heston (1991) and for a larger group of 66 countries for which we reconstructed capital stock data. (see details in appendix 2). The results are (surprisingly) quite similar for the two sub-groups. We focus, here, on the regressions obtained for the 24 countries for which capital stock data are originally available. For this sub-sample, one gets the following results :

$$(22) \quad \begin{cases} dh = 0.0380 - 0.0172 h_0 + 0.0316 k_0 ; R^2 = 0.92 \\ \quad \quad \quad (-8.63) \quad \quad (+15.34) \\ dk = - 0.0737 + 0.0168 h_0 - 0.0202 k_0 ; R^2 = 0.84 \\ \quad \quad \quad (7.86) \quad \quad (-9.30) \end{cases}$$

(t statistics in parenthesis).

The accumulation of human capital, it appears, is negatively correlated to the initial stock of human capital and positively correlated to the initial wealth of the country.

One sees that the partial correlation of $\frac{d}{dt} h_t$ with h_0 and of $\frac{d}{dt} k_t$ with k_0 are (quite significantly) negative, implying a "partial" tendency for convergence. However the partial correlation of $\frac{d}{dt} h_t$ (resp $\frac{d}{dt} k_t$) with k_0 (resp h_0) is significantly positive, implying an opposite movement towards non-convergence. Interestingly, one sees that the positive influence of physical capital on human capital is quantitatively larger than the influence of human capital on the accumulation of physical capital. While the sum $b + c'$ is negative, the determinant of the system is $\Delta = - 1.771 \cdot 10^{-4}$ so that the (point-estimate of the) system is actually unstable. A Wald test leads us to reject (with 95% confidence) the hypothesis that it is positive (see appendix 4).

When explicit account is taken of the law of motion of human and of physical capital, we are then unable to get a speed of convergence toward a steady-state for the simple reason that the system appears to be exploding rather than converging. There are (at least) two potential explanations for this finding.

a) Endogenous growth cannot be ruled out

Even though MRW and BX find decreasing returns to scale with respect to the storable asset, it may still be the case that income can grow without bound if human capital can be accumulated indefinitely.

In MRW's formulation, however, human capital cannot -actually- grow without bound because investment in human capital is postulated to be a share (smaller than one) of GDP. There are obviously no compelling theoretical reasons to make such an hypothesis. Although in *value* terms, investment in human capital is likely to be a fraction (smaller than one) of GDP, in *volume* terms it can very well grow without bound as a *fraction of GDP* (with the price of a unit of human capital falling correlatively to zero).

b) Poverty-traps exist

Even if one accepts MRW's (implicit) view that the law of motion of human capital cannot grow unbounded, it may be that poor countries do not catch up the rich because school enrollment get stuck to low levels when initially low -not for exogenous reasons but rather for endogeneous (history-dependent) reason- as in Drazen and Azariadis (1991) or as in Durlauf (1991). In that case two-dimensional dynamics such as those which we reported above may fail to display convergence although -conditionally on some additional parameters- convergence may occur (see Durlauf and Johnson (1992) for such an analysis ; in a different spirit, see also Quah (1991)).

CONCLUSION AND SUMMARY

I have argued in this paper that the tests of the "convergence hypothesis" which have been performed in the work of Barro and Sala-i-Martin and of Mankiw, Romer and Weil amount to testing whether there are decreasing returns to the accumulation of the storable assets (human and physical capital). They do not, however, capture "actual" dynamics but rather conditional dynamics only. When postulating that the economies of the world behave as a two-dimensional system (in human and physical capital), I have shown that one is lead to reject the hypothesis that the system is a convergent one. More work is obviously needed before one can assess whether "poverty traps" or increasing returns to the accumulation of human capital lie behind this result.

REFERENCES

- Azariadis, C. and A. Drazen (1990) - "Threshold Externalities in Economic Development", *Quarterly Journal of Economics*, May.
- Barro, R. (1991) : "Economic Growth in a Cross-Section of Countries", *Quarterly Journal of Economics*, May.
- Barro, R. and X. Sala-i-Martin (1990) - "Economic Growth and Convergence Across the United States", NBER Working Paper n°3419.
- (1991) - "Convergence Across States and Regions", *Brookings Papers on Economic Activity*, n°1.
- Durlauf, S. (1991) - "Nonergodic Economic Growth", NBER WP n°3719.
- Durlauf, S. and P. Johnson (1992) - "Local Versus Global Convergence Across National Economies", LSE Financial Markets Group, D.P n°131.
- Levine, R. and D. Renelt (1990) - "A Sensitivity of Cross-Country Growth Regressions", World Bank.
- Mankiw, G., D. Romer and D. Weil (1990) - "A Contribution to the Empirics of Economic Growth", Harvard University.
- Lucas, R. (1988) - "On the Mechanics of Economic Development", *Journal of Monetary Economics*, XXII, 3-42.
- Quah, D. (1990) - "International Patterns of Growth : I - Persistence in Cross Countries Disparities", Working Paper, MIT.
- Razin, A. (1977) - "Economic Growth and Education : New Evidences", *Economic Development and Cultural Change*, January , 171.
- Summers, R. and A. Heston (1991) - "The Penn World Table (Mark 5) : An Expanded Set of International Comparisons 1950-1988", *Quarterly Journal of Economics*, May 91, Vol.56, pp. 327-368.

APPENDIX 1 **THE ESTIMATION OF EQUATIONS (18)**

Notations :

Y_t = Per-capita income at time t
 LYI = Log of per-capita income in 1960.
 G = average growth rate 1960-1988 = $\frac{1}{28} \text{Log} \frac{Y_{88}}{Y_{60}}$
 $LENR1I$ = Log of enrollment in primary school in 1960.
 $LENR2I$ = Log of enrollment in secondary school in 1960.
 $LENR2$ = Log average value of secondary enrollment over 1960-1988.
 $LINV$ = average value of Log of investment over 1960-88.
 n = average value of population growth, 1960-88.
 $LPOP$ = Log [0.05 + n].

In equations (18.1) and (18.2) we use the sample of 89 countries of non oil producers for which the data on investment and education are available. (see the list in appendix 5). In equations (18.3) and (18.4) we also include Chad and Nicaragua for which the data on education were available.

EQUATION 18.1

=====

METHOD OF ESTIMATION = ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: G

SUM OF SQUARED RESIDUALS = 0.147349E-01
 STANDARD ERROR OF THE REGRESSION = 0.132444E-01
 MEAN OF DEPENDENT VARIABLE = 0.178879E-01
 STANDARD DEVIATION = 0.177417E-01
 R-SQUARED = 0.468045
 ADJUSTED R-SQUARED = 0.442714
 F-STATISTIC(4, 84) = 18.4770
 LOG OF LIKELIHOOD FUNCTION = 261.139
 NUMBER OF OBSERVATIONS = 89

VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
C	-0.49980E-02	0.32266E-01	-0.15490
LENR2	0.12754E-01	0.26766E-02	4.7650
LINV	0.11541E-01	0.31217E-02	3.6972
LYI	-0.11500E-01	0.29687E-02	-3.8737
LPOP	-0.13702E-01	0.14717E-01	-0.93105

EQUATION 18.2

=====

METHOD OF ESTIMATION = ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: G

SUM OF SQUARED RESIDUALS = 0.148634E-01
 STANDARD ERROR OF THE REGRESSION = 0.133021E-01
 MEAN OF DEPENDENT VARIABLE = 0.178879E-01
 STANDARD DEVIATION = 0.177417E-01
 R-SQUARED = 0.463406
 ADJUSTED R-SQUARED = 0.437854
 F-STATISTIC(4, 84) = 18.1357
 LOG OF LIKELIHOOD FUNCTION = 260.753
 NUMBER OF OBSERVATIONS = 89

VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
C	0.20812E-01	0.35310E-01	0.58940
LENR2I	0.87624E-02	0.18775E-02	4.6672
LINV	0.12778E-01	0.30545E-02	4.1834
LYI	-0.11509E-01	0.29947E-02	-3.8432
LPOP	-0.10296E-01	0.14978E-01	-0.68740

EQUATION 18.3

=====

METHOD OF ESTIMATION = ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: G

SUM OF SQUARED RESIDUALS = 0.175030E-01
 STANDARD ERROR OF THE REGRESSION = 0.141031E-01
 MEAN OF DEPENDENT VARIABLE = 0.173136E-01
 STANDARD DEVIATION = 0.180041E-01
 R-SQUARED = 0.400031
 ADJUSTED R-SQUARED = 0.386395
 F-STATISTIC(2, 88) = 29.3371
 LOG OF LIKELIHOOD FUNCTION = 260.186
 NUMBER OF OBSERVATIONS = 91

VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
C	0.27972E-01	0.15365E-01	1.8205
LENR2	0.16764E-01	0.24639E-02	6.8041
LYI	-0.86227E-02	0.27884E-02	-3.0923

EQUATION 18.4

=====

METHOD OF ESTIMATION = ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: G

SUM OF SQUARED RESIDUALS = 0.183184E-01
 STANDARD ERROR OF THE REGRESSION = 0.144279E-01
 MEAN OF DEPENDENT VARIABLE = 0.173136E-01
 STANDARD DEVIATION = 0.180041E-01
 R-SQUARED = 0.372078
 ADJUSTED R-SQUARED = 0.357807
 F-STATISTIC(2, 88) = 26.0724
 LOG OF LIKELIHOOD FUNCTION = 258.114
 NUMBER OF OBSERVATIONS = 91

VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
C	0.52944E-01	0.18407E-01	2.8763
LENR2I	0.11681E-01	0.18397E-02	6.3496
LYI	-0.85051E-02	0.29208E-02	-2.9119

APPENDIX 2

Estimation of equation (25)

Notations :

Variable z_0 = LENROL2SS

ϵ = RESS2

y_0 = LYPC70

dy = average growth rate 1970-1988 = $\frac{1}{18} \text{Log} \frac{Y_{88}}{Y_{70}}$

METHOD OF ESTIMATION = ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: DY

SUM OF SQUARED RESIDUALS = 0.124050E-02
 STANDARD ERROR OF THE REGRESSION = 0.787559E-02
 MEAN OF DEPENDENT VARIABLE = 0.187550E-01
 STANDARD DEVIATION = 0.934813E-02
 R-SQUARED = 0.382811
 ADJUSTED R-SQUARED = 0.290233
 F-STATISTIC(3, 20) = 4.13500
 LOG OF LIKELIHOOD FUNCTION = 84.3890
 NUMBER OF OBSERVATIONS = 24

VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
C	-0.74853E-02	0.19006E-01	-0.39383
LENR2SS	0.23496E-01	0.96572E-02	2.4330
RESS2	0.17532E-02	0.74372E-02	0.23573
LYPC70	-0.83381E-02	0.59267E-02	-1.4069

APPENDIX 3

A - CAPITAL STOCK DATA

The Summers-Heston (1991) latest estimates include data on the capital stock for a sub-group of 29 countries for the years 1980-88 (in most cases) (see the list in appendix 4). We take an aggregate measure of capital (inclusive of residential capital, for lack of data on the decomposition of investment), and we reconstructed for the period 1970-1988 capital stock data by an inventory formula :

$$K_t = \sum_{s=0}^t I_s (1-\delta)^{(t-s)} + K_0 (1-\delta)^{t-s} .$$

We took $\delta = 12 \%$, which was the value that fitted best Summers-Heston's capital stock series after 1980. We had to drop, however, five countries (Zimbabwe, Korea, Phillipines, Thailand, Austria) for which the 1970 data made no sense.

In order to extrapolate these data to the larger sub-group of 75 countries analyzed in MRW (and consisting of non-oil countries with a population larger than one million), we have run the following regression :

$$\text{Log } K_{1970} = a + b \text{ Log } \left[\sum_{s=1960}^{1970} I_s (1-\delta)^{(1970-s)} \right] + c \text{ Log } Y_{60}$$

for the sub-group of 24 countries and use the prediction of the regression for the 51 other countries. For lack of data we had to drop 9 countries : Zimbabwe, Bangladesh, Burma, Indenesia, Korea, Phillipines, Singapore, Thailand, Austria. We are therefore left with two sub-samples: one of 24 countries and another one of 66 countries.

B - RESULTS

We ran equation (21) for both sub-groups, the results come as follows :

a) Equation (21) for the sub-group of 66 countries :

Notations : Y_t = Log of per-capita income at t (t= 1970,..., 1988)

k_t = Log of per-capita capital stock at t

$$h_t = 3y - k_t$$

$$dh = \frac{1}{18} [h_{88} - h_{70}]$$

$$dk = \frac{1}{18} [k_{88} - k_{70}]$$

$$\text{Model : } dh = EE + AA h_0 + BB k_0 \quad (\text{Equation CAPHU})$$

$$dk = FF + CC h_0 + DD k_0 \quad (\text{Equation CAPFI})$$

(sur estimator)

EQUATION 21 (for 66 countries)

=====

PARAMETER	ESTIMATE	STANDARD ERROR	T-STATISTIC
EE	0.84677E-01	0.43911E-01	1.9284
AA	-0.18751E-01	0.31397E-02	-5.9721
BB	0.28979E-01	0.27462E-02	10.552
FF	-0.94219E-01	0.31118E-01	-3.0278
CC	0.17630E-01	0.22250E-02	7.9236
DD	-0.20154E-01	0.19461E-02	-10.356

STANDARD ERRORS COMPUTED FROM QUADRATIC FORM OF ANALYTIC FIRST
DERIVATIVES (GAUSS)

EQUATION CAPHU

DEPENDENT VARIABLE: DH

SUM OF SQUARED RESIDUALS =	0.753035E-01
STANDARD ERROR OF THE REGRESSION =	0.337781E-01
MEAN OF DEPENDENT VARIABLE =	0.182435E-01
STANDARD DEVIATION =	0.559868E-01
R-SQUARED =	0.630402

EQUATION CAPFI

DEPENDENT VARIABLE: DK

SUM OF SQUARED RESIDUALS =	0.378172E-01
STANDARD ERROR OF THE REGRESSION =	0.239372E-01
MEAN OF DEPENDENT VARIABLE =	0.244786E-01
STANDARD DEVIATION =	0.403824E-01
R-SQUARED =	0.643228

b) Equation (21) for the sub-group of 24 countries

$$\text{Model : } dh = EEE + AAA h_0 + BBB k_0 \quad (\text{Equation CAPHU})$$

$$dk = FFF + CCC h_0 + DDD k_0 \quad (\text{Equation CAPFI})$$

(sur estimator)

EQUATION 21 (for 24 countries)
=====

PARAMETER	ESTIMATE	STANDARD ERROR	T-STATISTIC
EEE	0.37963E-01	0.34767E-01	1.0919
AAA	-0.17192E-01	0.19927E-02	-8.6272
BBB	0.31564E-01	0.20578E-02	15.339
FFF	-0.73740E-01	0.36677E-01	-2.0105
CCC	0.16532E-01	0.21022E-02	7.8641
DDD	-0.20192E-01	0.21708E-02	-9.3017

STANDARD ERRORS COMPUTED FROM QUADRATIC FORM OF ANALYTIC FIRST
DERIVATIVES (GAUSS)

EQUATION CAPHU

DEPENDENT VARIABLE: DH

SUM OF SQUARED RESIDUALS = 0.532974E-02
STANDARD ERROR OF THE REGRESSION = 0.149021E-01
MEAN OF DEPENDENT VARIABLE = 0.401059E-01
STANDARD DEVIATION = 0.532666E-01
R-SQUARED = 0.918329

EQUATION CAPFI

DEPENDENT VARIABLE: DK

SUM OF SQUARED RESIDUALS = 0.593148E-02
STANDARD ERROR OF THE REGRESSION = 0.157208E-01
MEAN OF DEPENDENT VARIABLE = 0.161590E-01
STANDARD DEVIATION = 0.400827E-01
R-SQUARED = 0.839483

APPENDIX 4

a) Eigenvalues and eigenvectors of the system (22)

The matrix $\begin{pmatrix} -0.0172 & 0.0316 \\ 0.0166 & -0.0202 \end{pmatrix}$ has two eigenvalues $\lambda_1 = 0.43 \cdot 10^{-2}$

and $-\lambda_2 = -4.2 \cdot 10^{-2}$, to which are associated the two eigenvectors

and $VP_2 = \begin{pmatrix} 1 \\ -0.78 \end{pmatrix}$. Call z_1 and z_2 the coordinates of h_t and k_t in the

base VP_1, VP_2 , one gets :

$$(3.1) \quad h = z_1 + z_2 \quad ; \quad k = 0.6831 z_1 - 0.07832 z_2$$

or, equivalently : $z_1 = 0.53 h + 0.67 k$; $z_2 = 0.47 h - 0.67 k$; z_1 and z_2 are solutions to the following equations :

$$\frac{dz_1}{dt} = \lambda_1 [z_1 - \hat{z}_1] \quad \text{in which } \hat{z}_1 = 6.71$$

$$\frac{dz_2}{dt} = -\lambda_2 [z_2 - \hat{z}_2] \quad \text{in which } \hat{z}_2 = 1.60$$

$$(3.2) \quad \begin{cases} z_1(t) = \hat{z}_1 + e^{\lambda_1 t} [z_1 - \hat{z}_1] \\ z_2(t) = \hat{z}_2 + e^{-\lambda_2 t} [z_2 - \hat{z}_2] \end{cases}$$

substituting these values into (3.1) delivers

$$y_t = 3.9 + e^{\lambda_1 t} \begin{bmatrix} 0.30 h_0 + 0.38 k_0 - 2.24 \end{bmatrix} + e^{-\lambda_2 t} \begin{bmatrix} 0.03 h_0 - 0.04 k_0 - 0.11 \end{bmatrix}$$

b) Test of non-negativity of the determinant

We want to know if the estimation of the system (21) leads us to reject the hypothesis H_0 that $\theta \equiv bc' - b'c > 0$. In order to do that, we perform the following Wald test.

Call :

$$u \equiv \left(\frac{\partial \theta}{\partial b}, \frac{\partial \theta}{\partial c}, \frac{\partial \theta}{\partial b'}, \frac{\partial \theta}{\partial c'} \right)$$
$$= (c' - b', -c, b)$$

and let $\hat{\Sigma}$ be the covariance matrix of $(\hat{b}, \hat{c}, \hat{b}', \hat{c}')$.

Call $\hat{V}(\hat{\theta}) = \hat{u} \hat{\Sigma} \hat{u}^T$. We calculate :

$$\xi = \frac{\hat{\theta}}{\hat{V}(\hat{\theta})^{1/2}} \text{ and find : } \xi = -2.07. \text{ We can then reject at the 5\% of}$$

confidence that θ is positive.

APPENDIX 5

Countries selected as those for which oil production is not the dominant industry	Countries for which education or investment data are missing	Countries with a population bigger than 1 million and with data rating better than D	Countries for which stock of capital in 1980 is available
Algeria		*	
Angola			
Benin			
Botswana		*	
Burkina Faso	*		
Burundi			
Cameroon		*	
Central Afr. Rep.			
Chad	*		
Congo			
Egypt			
Ethiopia		*	
Ghana			
Ivory Coast		*	
Kenya		*	*
Liberia			
Madagascar		*	
Malawi	*	*	
Mali		*	
Mauritania			
Mauritius			
Morocco		*	
Mozambique			
Niger			
Nigeria		*	
Rwanda	*		
Senegal		*	
Sierra Leone			
Somalia			
S. Africa		*	
Sudan			
Tanzania		*	
Togo			
Tunisia		*	
Uganda			
Zaire			
Zambia		*	
Zimbabwe		*	*
Canada		*	*
Costa Rica		*	
Dominican Rep.	*	*	*
El Salvador		*	
Guatemala		*	*
Haiti		*	
Honduras		*	
Jamaique		*	
Mexico		*	
Nicaragua	*	*	
Panama		*	

Countries selected as those for which oil production is not the dominant industry	Countries for which education or invest- ment data are missing	Countries with a population bigger than 1 million and with data rating better than D	Countries for which stock of capital in 1980 is available
Trinidad & Tobago		*	
United States		*	*
Argentina		*	*
Bolivia		*	
Brazil		*	
Chile		*	*
Colombia		*	*
Ecuador		*	
Paraguay		*	
Peru		*	
Uruguay		*	
Venezuela		*	
Bangladesh		*	
Burma	*	*	
Hong Kong		*	
India		*	*
Indonesia	*	*	
Israel		*	*
Japan		*	*
Jordan		*	*
Korea, Rep. of		*	
Malaysia		*	
Nepal			
Pakistan		*	
Philippines		*	*
Singapore		*	
Sri Lanka		*	
Syria		*	
Thailand		*	*
Austria		*	*
Belgium		*	*
Denmark		*	*
Finland		*	*
France		*	*
Germany, Fed Rep	*	*	*
Greece		*	*
Ireland		*	*
Italy		*	*
Netherlands		*	
Norway		*	*
Portugal		*	
Spain		*	*
Sweden		*	*
Switzerland		*	
Turkey		*	
United Kingdom		*	*
Australia		*	*
New Zealand		*	
Papua New Guinea			

Total number of countries:

98

9

75

24

;