INCOMPLETE MARKETS AND THE SUBOPTIMALITY OF RATIONAL EXPECTATIONS

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A B S T R A C T

We study in this article a model of incomplete markets which can function under both rational and non rational expectations. It is shown that, because of the incompleteness of markets, rational expectations equilibria can be Pareto dominated by equilibria with non rational expectations.

Keywords : Incomplete Markets, Rational Expectations, Optimality.

Journal of Economic Literature Classification Numbers : 021, 022, 024.

MARCHES INCOMPLETS ET SOUS-OPTIMALITE

DES ANTICIPATIONS RATIONNELLES

R E S U M E

On étudie dans cet article un modèle à marchés incomplets où les anticipations peuvent être rationnelles ou "irrationnelles". Il est montré que, du fait de l'existence de marchés incomplets, des équilibres à anticipations rationnelles peuvent être dominés au sens de Pareto par des équilibres avec anticipations non rationnelles.

Mots clefs : Marchés incomplets, anticipations rationnelles, optimalité.

Codes J.E.L. : 021, 022, 024.

1. INTRODUCTION

The study of incomplete market economies is a subject that has gained an increasing amount of attention in the late years (see for example the special 1990 issue of the Journal of Mathematical Economics on this topic). Walrasian equilibria with incomplete markets have a number of inefficiency properties. Typically they are not Pareto-optimal, and can even be Pareto dominated by other incomplete markets equilibria (Hart, 1975). It was also shown that adequate interventions on some asset markets could lead to Pareto improvements (Geanakoplos and Polemarchakis, 1976).

Quite curiously, and though a number of markets are assumed to be held at some future dates, incomplete markets equilibria have mostly been studied in the framework of "rational expectations" or perfect foresight. What we want to do in this article is to compare incomplete markets equilibria under various expectations schemes from a welfare point of view. Our main result is that under incomplete markets, equilibria with irrational expectations can Pareto dominate rational expectations equilibria.

For the convenience of the exposition, we shall carry our investigation in the framework of an overlapping generations model *without* fiat money. The market incompleteness will come from the fact that agents cannot trade current wealth against future wealth. Rational expectations and irrational ones have already been compared in an O.L.G. model without our market incompleteness problem by Teit-Nielsen (1988). He showed that in an O.L.G. model *with* fiat money (i.e. where wealth can be transferred from period to period), some rational expectations equilibria are dominated by some irrational expectations ones. The dominated equilibria, however, are nonstationary ones which, due to the particularity of the O.L.G. model, do not satisfy the first theorem of welfare. The stationary equilibrium is not Pareto dominated (and in fact is a full Pareto optimum).

Here we do not want to focus on the peculiar nonstationary behavior of the OLG model, but rather on the market incompleteness issue, and we shall thus study only stationary equilibria. We shall then show that stationary equilibria with irrational expectations can Pareto dominate stationary equilibria with rational expectations because of the market incompleteness.

2. THE MODEL

We shall thus study an overlapping generations model with two goods, denoted a and b, and a single representative agent living two periods (there are thus in every period one young and one old agent). This agent has endowments ω_{1a} and ω_{1b} of goods a and b respectively when young and ω_{2a} and ω_{2b} when old. His utility function over consumptions x_{1a} , x_{1b} , x_{2a} , x_{2b} is denoted :

$$U(x_{1a},x_{1b},x_{2a},x_{2b})$$

We shall assume that in each period only the market of good a against good b is open. The market structure is thus incomplete because there is no market on which the agents can trade goods when young against goods when old. Let us take good a as the numeraire and call p the price of good b. In a stationary state equilibrium the consumptions of the representative consumer are solutions of the program :

$$\text{Maximize } U(x_{1a},x_{1b},x_{2a},x_{2b}) \qquad \text{s.t.}$$

$$x_{1a} + px_{1b} = \omega_{1a} + p\omega_{1b}$$

$$x_{2a} + px_{2b} = \omega_{2a} + p\omega_{2b}$$

Calling λ_1 and λ_2 the Kuhn-Tucker multipliers of the first and second period budget constraints respectively, we obtain the first order conditions :

$$\frac{\partial U}{\partial x_{1a}} = \frac{1}{p}\,\frac{\partial U}{\partial x_{1b}} = \lambda_1 \tag{1}$$

$$\frac{\partial U}{\partial x_{2a}} = \frac{1}{p}\,\frac{\partial U}{\partial x_{2b}} = \lambda_2 \tag{2}$$

In general, because of the market incompleteness $\lambda_1 \neq \lambda_2$ and the stationary equilibrium is not Pareto efficient.

3. IRRATIONAL EXPECTATIONS AND MARKET EQUILIBRIUM

Let us now assume that the young household, who is faced with the price p in the current period, expects a price π in the future period. His trade on the current market is thus given by the following program :

$$\text{Maximize } U(x_{1a},x_{1b},x_{2a},x_{2b}) \qquad \text{s.t.}$$

$$x_{1a} + px_{1b} = \omega_{1a} + p\omega_{1b}$$

$$x_{2a} + \pi x_{2b} = \omega_{2a} + \pi\omega_{2b}$$

The solutions in x_{1a} and x_{1b} of this program depend upon p and π . We shall denote by $\phi(p,\pi)$ the net purchase of good b by the young agent. We shall thus have :

$$x_{1a} = \omega_{1a} - p\phi(p,\pi) \tag{3}$$

$$x_{1b} = \omega_{1b} + \phi(p,\pi) \tag{4}$$

Consider now the old consumer. He has consumed x_{1a} and x_{1b} in the previous period. The program giving his optimal trade on the current market is the following :

$$\text{Maximize } U(x_{1a},x_{1b},x_{2a},x_{2b}) \qquad \text{s.t.}$$

$$x_{2a} + px_{2b} = \omega_{2a} + p\omega_{2b}$$

The solutions in x_{2a} and x_{2b} of this program depend upon x_{1a} , x_{1b} and p. We shall denote by $\psi(x_{1a},x_{1b},p)$ the net sale of good b by the old agent. We shall thus have:

$$x_{2a} = \omega_{2a} + p\psi(x_{1a},x_{1b},p) \tag{5}$$

$$x_{2b} = \omega_{2b} - \psi(x_{1a},x_{1b},p) \tag{6}$$

The equilibrium price p is determined by equations (3) to (6) and the equilibrium condition on the current market, i.e. :

$$x_{1b} + x_{2b} = \omega_{1b} + \omega_{2b} \tag{7}$$

which using (4) and (6) becomes :

$$\phi(p,\pi) = \psi(x_{1a},x_{1b},p) \tag{8}$$

If the equilibrium is stationary, x_{1a} and x_{1b} are those given by equations (3) and (4), so that p will be determined by the following equation :

$$\phi(p,\pi) = \psi[\omega_{1a} - p\phi(p,\pi) \ , \ \omega_{1b} + \phi(p,\pi) \ , \ p] \tag{9}$$

Equation (9) gives us p as a function of π . Expectations will matter if $\partial p/\partial\pi$ is nonzero, which requires in particular that $\partial\phi/\partial\pi$ be different from zero. Once p is computed through equation (9), all quantities at the stationary equilibrium can be computed via equations (3) - (6), and will also depend on π .

A particularly interesting equilibrium is of course the stationary rational expectations equilibrium. This is characterized by a price p* given by equation (9), where π has been replaced by p , i.e. :

$$\phi(p,p) = \psi[\omega_{1a} - p\phi(p,p) \ , \ \omega_{1b} + \phi(p,p) \ , \ p] \tag{10}$$

4. THE RESULT

What we want to find out is whether a deviation from rational expectations can lead to a Pareto improvement. For that we shall consider a small variation $d\pi$ around the rational expectations equilibrium p^* , and compute the utility increment dU of the representative consumer.

$$dU = \frac{\partial U}{\partial x_{1a}} dx_{1a} + \frac{\partial U}{\partial x_{1b}} dx_{1b} + \frac{\partial U}{\partial x_{2a}} dx_{2a} + \frac{\partial U}{\partial x_{2b}} dx_{2b} \tag{11}$$

Using formulas (1) and (2) which are valid at the rational expectations equilibrium, we find :

$$dU = \lambda_1 [dx_{1a} + pdx_{1b}] + \lambda_2 [dx_{2a} + pdx_{2b}] \tag{12}$$

Using formulas (3) and (4), we find

$$dx_{1a} + pdx_{1b} = - \phi(p,\pi)\ dp \tag{13}$$

Similarly using formulas (5) and (6) we find :

$$dx_{2a} + pdx_{2b} = \psi(x_{1a}, x_{1b}, p)\ dp \tag{14}$$

Further using the equilibrium equation (8), and plugging into (12) we finally obtain :

$$dU = (\lambda_2 - \lambda_1)\ \phi\ \frac{\partial p}{\partial \pi}\ d\pi \tag{15}$$

where $\partial p/\partial \pi$ is that coming from equation (9). We see that as soon as expectations matter $(\partial p/\partial \pi \neq 0)$, the incompleteness of markets is actually binding $(\lambda_1 \neq \lambda_2)$ and there is exchange on the market $(\phi \neq 0)$, a deviation from rational expectations will lead to a Pareto improvement.

More specifically one will choose a deviation $d\pi$ such that the resulting variation in p , i.e. $dp = (\partial p/\partial \pi)\ d\pi$, has the same sign as $(\lambda_2 - \lambda_1)\ \phi$. The intuition behind this result is quite straightforward :

Formulas (12), (13) and (14) show that utility variations in each period depend on the variation in the "terms of trade" p . The idea is thus to improve these terms of trade in the period where the representative consumer is "poorest" (i.e. where his marginal utility for income is highest).

Take for example the case where $\lambda_2 > \lambda_1$ (the consumer is "poorer" when old) and $\phi > 0$ (he purchases good b when young and sells it when old). Then one will want to engineer, via a variation $d\pi$, a positive dp so as to increase the consumer's terms of trade when old (i.e. poorest). All other cases would work in a similar way.

5. CONCLUSIONS

We have shown in this paper that in a world of incomplete markets irrational expectations may Pareto dominate rational ones. The point has been made for stationary equilibria in an O.L.G. model, but clearly could be made in other incomplete markets structures, though evidently at the price of a much more clumsy and intricate exposition. The incomplete markets hypothesis is certainly a realistic one, and a very worthy subject for study. This paper shows that, from a normative point of view, it would be interesting not to restrict research on incomplete markets to rational expectations models, but to investigate other expectations schemes as well.

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