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On Precommitment Effects Between Competing Agencies

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A B S T R A C T

We analyze a game situation where two principal-agent structures interact after having signed (adverse selection) contracts. Contracts can be publicly disclosed but they can also always be renegotiated secretly. Public disclosure of contract may induce precommitment effects and affect the opponent's behavior by granting an informational rent to the agent larger than if contracts were unobservable : this cannot be undone in renegotiations since the agent would refuse to forgo this rent. Such precommitment effects exist in particular in Cournot like situations. The general analysis hinges on whether actions are strategic substitutes or complements, and on the effect of one agent's action on the rival agency's aggregate payoff.

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Key words : Precommitment effects, Adverse selection, Principal-agent, Strategic substitutes or complements.

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Effets d'engagement et concurrence entre agences

R E S U M E

Nous analysons l'interaction stratégique de deux structures principal-agent qui signent des contrats incitatifs en situation de sélection adverse. Les contrats peuvent être rendus publics mais on ne peut exclure qu'ils soient aussi renégociés secrètement. La divulgation des contrats peut induire des effets d'engagement et affecter le comportement du concurrent dans la mesure où le contrat rendu public garantit à l'agent une rente informationnelle plus grande que si les contrats n'étaient pas observables ; lors de la phase de renégociation, les agents refuseraient en effet d'abandonner cette rente. On montre que des effets d'engagement existent notamment lorsque les agents jouent un jeu de concurrence à la Cournot. De manière générale, l'existence d'effets d'engagement dépend à la fois des modalités de l'interaction entre les agents (leurs actions sont-elles substituables ou complémentaires d'un point de vue stratégique) et de l'effet de l'action d'un agent sur le gain total de l'agence rivale.

Mots clefs : Effets d'engagement, Antisélection, Modèle principal-agent, subtituabilité ou complémentarité stratégique.

I. Introduction

In game-like situations, it is now well documented that hiring an agent to act on one's behalf may be a valuable opportunity, because it serves as a precommitment device to play some particular actions (see in particular Katz [1987]). By writing a contract which is observed by others, a player, called now a principal, can force her agent to some predetermined strategy before the game is actually played, thereby gaining a first-mover advantage when it is desirable.

When no asymmetry of information or risk-sharing problem arise, two assumptions have to be verified for contracts to be used as a precommitment device: contracts once signed should be observed by outsiders, and no secret renegotiation should be possible[1]. Indeed, in a perfect information setting, a principal can hire an agent under a very stringent contract to make the agent "tougher" in the ensuing game; but with possibly private (secret) renegotiations, the principal would actually cancel the "looking-tough" (or Stackelberg) contract so as to make the agent play the best response to the expected actions of outsiders given their beliefs that the agent will play tough.

This paper focuses on the incomplete information case. Agents have private information and contracts only bear on the sole verifiable variable for a principal-agent hierarchy, the action actually played by the agent. A hierarchy can publicly disclose a contract, but is always allowed to renegotiate secretly a mutually beneficial contract if it wants to. Precommitment effects occur when public disclosure of contracts modifies the outcome of the game.

Precommitment effects under incomplete information and renegotiation have previously been considered by Dewatripont [1988] in the framework of an entry deterrence example. As in Dewatripont's paper, we shall study these effects assuming that agents have private information at the renegotiation stage. There are however several major distinctions between Dewatripont's model and ours.

First, Dewatripont considers the case of one hierarchy competing with a third party. When signing an initial contract the hierarchy benefits from a first mover advantage and precommitment effects are maximal when no

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[1] See Katz [1987]

renegotiation can take place. We analyze the case of two hierarchies competing with each other. Initial contracts are signed simultaneously and no hierarchy has a first mover advantage. In this context, if renegotiation is impossible, no party can gain any precommitment advantage by the public disclosure of the contract signed, since at the time of the disclosure the competing hierarchy is already committed to its contract. Precommitment effects arise when renegotiation is allowed because this forces each party to react to its opponent's initial contract. Therefore the possibility of renegotiation is essential to our results.

Second, in Dewatripont's model initial contracts are signed ex-ante (before the agent receives his private information). The informational structure changes before renegotiation takes place although it is difficult to rule out the possibility that secret renegotiation takes place also ex-ante since this would be optimal for the parties. We will assume that both initial contracts and renegotiated contracts are signed at the interim stage so the information structure remains unchanged in the game.

As pointed above one hierarchy benefits from renegotiation because this forces the other hierarchy to react to its public announcement. This gain is limited to the extent that the hierarchy will itself renegotiate. For the initial announcement to be credible, the hierarchy must precommit not to renegotiate. How this precommitment is obtained is best seen on a simple example. Suppose a principal hires an agent who may be of two types, 0 or 1, with probability 1/2 each, and who may take two actions a_0 or a_1. Agent of type 0 has a monetary cost 0 for both actions while agent of type 1 has a cost 0 for a_0 and a cost $\theta >> 0$ for a_1. Let the principal propose an initial contract: transfer 0 to the agent if the action taken is a_0 and transfer t with $0<t<\theta$ if the action is a_1. Agent of type 0 chooses a_1 and agent of type 1 chooses a_0, while the expected monetary cost is t/2 for the principal. Now suppose that the principal wants to renegotiate so that agents of both types play a_0. She must then propose a contract that specifies a transfer t if a_0 is played. This means that both types will receive t so that the cost for the principal is t. The renegotiation costs t/2 to the principal. By signing publicly an initial contract with t large enough, the principal can increase its renegotiation cost and precommit not to renegotiate even when the optimal unobservable contract is: transfer 0 and action a_0 for both types.

The general idea is that an initial contract can modify the distribution of reservation utility across various types of agents at the renegotiation stage (it changes from (0,0) to (t,0) in the previous example) and therefore affect the outcome of renegotiation. With initial contracts signed at an interim stage only high rent differential across types can affect the renegotiation. It will follow that a hierarchy can only precommit to actions that are costlier to the agent than the actions prescribed by the optimal unobservable contract. Given that, the results of the paper follow naturally.

Whether precommitment effects actually occur will depend both on the relation between the contractual decision of an agent and the decision of his opponent, and on the effect of the opponent's action on each principal's welfare. Indeed we shall find that precommitment effects will exist if (as it is usually assumed, the higher an action the costlier it is):

-- either actions are strategic substitutes and a lower opponent's action increases one principal's welfare,

-- or actions are strategic complements and a higher opponent's action increases one principal's welfare.

There will be no precommitment effects in the two opposite cases (results are derived under some symmetry assumptions but can be easily extended to other cases).

For example, if principals are producers and agents are salesmen playing a Cournot game, paying a high rent to low cost salesmen allows to commit to higher production levels and to lower the opponent's production. Therefore public disclosure of retail contracts leads to precommitment effects.

One may wonder whether the informational structure is important for the results obtained. Clearly if initial contracts are signed and secretly renegotiated ex-ante, no precommitment effects can exist. We show that if initial contracts are signed ex-ante and renegotiated at an interim stage (as in Dewatripont), then precommitment effects always exist. The difference is that with contracts signed ex-ante a hierarchy can precommit to lower actions compared to those prescribed by the optimal unobservable contract by signing a contract with a low rent differential across types (the efficient ex-ante contract involves a higher rent differential than the efficient interim contract).

Several models have been developed to analyze the strategic aspect of contract disclosure in various settings. Fershtman-Judd [1987] and Sklivas [1987] study the implications of the separation of ownership and management in oligopoly models. Bonnano-Vickers [1988] analyze the strategic aspect of vertical separation in producer/retailer structures, while Rey-Stiglitz

[1987] study exclusive territory agreements (both in a similar model). Brander-Spencer [1983] and [1985] have developed models where governments use tax or subsidy policy as visible precommitment to help domestic firms in international competition. Most of these models asssume perfect information (except Ferschtman-Judd [1987]), very specific contractual possibilities and exclude the possibility of renegotiation. Allowing for renegotiation would then invalidate the results of these papers. Ferschtman-Judd [1987] assume imperfect information, ex-ante contracting and no renegotiation. They show that with public disclosure of managerial compensation schemes the new equilibrium appears more competitive with Cournot competition and less competitive with Bertrand competition. These findings are consistent with our conclusions and therefore robust to renegotiation.

The paper is organized as follows. Section 2 presents the general structure of the model and, in particular, the nature of contracts. Section 3 characterizes the outcome of the renegotiation stage. Section 4 develops the equilibrium analysis and gives sufficient conditions for precommitment effects to occur or not. Section 5 considers the example of producer/salesman relationships in the framework of Cournot competition. Finally Section 6 investigates the case where contracts are signed ex-ante.

II.The general model

The general situation we consider is summarized in the next figure :

$$\begin{array}{ccccccc} & P_A & & & & P_A & \\ \text{Contract} & \downarrow & & \text{market} & & \downarrow & \text{Contract} \\ & A & \rightarrow & \text{or} & \leftarrow & B & \\ & & & \text{general game} & & & \end{array}$$

Two principals P_A and P_B each hire an agent A and B to act on their behalf on some market or more generally in some game. Each pair of principal-agent, called a hierarchy, may be interpreted as a simple model of a firm, where shareholders (principal) hire a manager (agent) under some compensation scheme (contract). And the two firms interact on the market, competing in prices or in quantities for example. A hierarchy may also be composed of two firms within a vertical structure (producer/retailer, R&D/producer,....). Virtually any situation of strategic interaction in Industrial Economics involving two firms can be described by our model. We will focus on situations where the contractible variables are the actions of the agents.

We should also mention several alternative interpretations that might be of interest for future development. Principals could be different states or governmental agencies, and agents could be public firms or private regulated firms that compete on a common market with no trade barriers. The analysis will be therefore relevant for the problem of state monopoly in the European Market of 1991. Agents could also be domestic firms involved in a joint project, a research and development program for example. Given a surplus sharing rule describing such a program, a country would like to induce its home firm to invest the right amount in the project given the relative opportunity costs in each country. The study of such cooperative programs could be completed by an analysis of the performance of different program arrangements, or even of a common extra-national agency.

Coming back to our model, contracts are fully enforceable commitments by all parties that signed it. Moreover a contract can be published in newspapers or presented on TV, so that contracts may be publicly observable if desired by all contractual parties. However, we want to emphasize that secret renegotiations are always possible : public disclosure of a contract has no legal force in forbidding any change of contract thereafter, if all contracting parties agree to tear up the (publicly disclosed) contract. Therefore, we consider a global game that allows renegotiation.

<u>Stage 1 : Contracting Phase</u> : Contracts are signed simultaneously for all hierarchies, and may be publicly disclosed.
<u>Stage 2 : Renegotiation Phase</u> : Secret renegotiation may occur in every hierarchy.
<u>Stage 3 : Market Game</u> : Agents play a game in normal form, where agent A's pure strategy is denoted $a \in S_A$ compact interval of $\mathbb{R}$ ($b \in S_B$ compact interval of $\mathbb{R}$).

This timing allows any number of rounds of public renegotiations, in which case the last contract that is publicly disclosed constitutes the outcome of stage 1. The point we emphasize is that there is no credible commitment never to renegotiate secretly an agreement : stage 2 leaves this opportunity open. Stage 3 is mostly meant to model a one-shot simultaneous move game. We abstract from the possibility of renegotiation due to some sequentiality of moves in a general extensive form game, as considered in Dewatripont [1988] and in Caillaud-Jullien-Picard [1990].

We focus on situations of hidden knowledge between principals and agents : each agent will at some point get some private information about a parameter θ_A or θ_B , of his (dis)utility of playing the market game (θ may possibly affect the global aggregate payoffs accruing to a hierarchy). θ_A and θ_B are assumed to be independently drawn according to the distribution : $\theta_i \in \{\underline{\theta}_i, \bar{\theta}_i\}$, $\underline{\theta}_i < \bar{\theta}_i$ and $h_i \equiv \mathbb{P}\{\theta_i = \bar{\theta}_i\} \in (0,1)$.

We will study the game under alternative assumptions on the timing of information revelation (expressed only in terms of agent A) :

(i) interim initial contract : A observes θ_A before stage 1 .

(ii) ex-ante initial contract : A observes θ_A between stage 1 and stage 2.

Remark that the timing explicited above is fully general in the "interim contracts" case. However it is not, if contracts at stage 1 are signed at an ex-ante informational stage. Allowing renegotiations at any time would imply considering a stage 1', after stage 1 and before A observes θ, where the initial contract could be renegotiated ex-ante. We mention such a timing in the section on ex-ante contracts.

We assume all payoffs are linear/quadratic. As will become clear later, many results can be extended to a more general setting with some qualification corresponding to the possibility of multiple equilibria ; the linear-quadratic framework allows reaction functions to be linear (at least piecewise) in the opponent's expected action, so that the analysis becomes actually one-dimensional. Given a tranfer t_A from P_A to A, payoffs are :

for principal P_A : $$\pi_A = \phi^A(a,b,\theta_A) + \Psi^A(a,b,\theta_A) - t_A$$

for agent A : $$U_A = t_A - \Psi^A(a,b,\theta_A)$$

with
$$\Psi^A(a,b,\theta_A) = \theta_A a + m^A(b,\theta_A)$$
$$\phi^A(a,b,\theta_A) = [2A^*(b,\theta_A) - a]a + f^A(b,\theta_A)$$
$$A^*(b,\theta_A) = \alpha(\theta_A)b + \alpha_o(\theta_A) \in \mathrm{Int}(S_A) \text{ for all } b$$

and similarly for ϕ^B and ψ^B.

When agents A and B do not participate, they obtain zero utility : we then have $U_A = U_B = 0$.

Functions ϕ define the aggregate surplus of a firm/hierarchy, i.e. the global profits net of agent's costs. Hence $A^*(.)$ denotes the aggregate best response under full information of firm P_A-A if the opponent plays b : to avoid corner solutions in the game under full information $A^*(.)$ is assumed to be interior to S_A. All functions are assumed regular. We make one important assumption :

A1 : $\forall(a,b)\in S_A \times S_B$, $\quad \Psi^A(a,b,\bar{\theta}_A) \geq \Psi^A(a,b,\underline{\theta}_A)$

$\Delta^A \equiv \bar{\theta}_A - \underline{\theta}_A > 0$ and $A^*(b,\bar{\theta}_A) \leq A^*(b,\underline{\theta}_A)$.

Similarly for Ψ^B, Δ^B and B^*.

The monotonicity of Ψ^A in θ_A is a normalization : θ_A is a parameter of the cost of taking action a. Given this, the monotonicity of θ_A is the usual <u>Spence-Mirrlees</u> <u>condition</u> (sorting condition) and the monotonicity of A^* is the "<u>responsiveness</u>" <u>condition</u> corresponding to the monotonicity of θ_A. Responsiveness was characterized in Caillaud-Guesnerie-Rey-Tirole [1988] as requiring enough compatibility between the principal's and the agent's objectives so that the optimal contract under incomplete information be <u>separating</u> : here also, this condition ensures that any renegotiated contract will be separating.

The separability between a and b in the agent's utility allows to define the agent's "control utility" :

$$V_A \equiv t_A - \theta_A a$$

so that the total utility of agent A is the sum of his control utility and of a term of pure externality due to the opponent's action :

$$U_A = V_A - m^A(b,\theta_A).$$

Once a contract is signed, V_A is the only component of his utility on which the agent has some control and on which incentives depend.

In this paper, we focus on simple contracts that induce transfers t_A contingent only on the agent's action a and on messages sent privately by A to P_A, and symmetrically for P_B and B. Using the revelation principle (see e.g. Myerson [1979]) these contracts can be replaced by direct revelation mechanisms, denoted $[(\bar{a},\bar{t}_A), (\underline{a},\underline{t}_A)]$ for the contractual menu offered to A (similar notations for B). We will often use an equivalent representation in terms of control utilities : $\bar{V}_A = \bar{t}_A - \bar{\theta}_A\bar{a}$ and $\underline{V}_A = \underline{t}_A - \underline{\theta}_A\underline{a}$. A contract will then be denoted $[(\bar{a},\bar{V}_A),\ (\underline{a},\underline{V}_A)]$.

The remainder of this paper is devoted to the analysis of the equilibrium of our global game. Strategies are contract proposals, both at stage 1 and 2, by principals, and acceptance rules and announcements by agents. Given that stage 2 contracts specify stage 3 actions there is in fact no actual strategy choice in stage 3 (it is already solved by considering incentive compatible and individually rational mechanisms in stage 2). Our equilibrium concept is <u>strong perfect Bayesian equilibrium</u> (hereafter equilibrium) as defined in Fudenberg-Tirole [1989] : strategies must be sequentially optimal given beliefs, and beliefs are Bayesian consistent whenever possible. Moreover beliefs must respect the "no-signalling-what-you-don't-know" condition : for example after a deviation by P_B-B, P_A should not change her beliefs about agent A.

Note finally that we do not specify the game if no agent is hired. We simply assume principals must hire agents. Agents' individual rationality contraints have to be introduced at stage 1 as well as at stage 2. At stage 1, individual rationality requires that the agents' utility levels associated with any continuation equilibrium (including renegotiation) be nonnegative (whatever their type) and stage 2 contracts must specify utility levels which are greater than what the agent would get under stage 1 contract given B's anticipated action choice.

III. Renegotiation

This section is the central part of our analysis. It solves principal P_A's contracting problem at stage 2. At this stage P_A and A anticipate some opponent's actions for stage 2 and stage 3 that are summarized by a probability distribution on S_B, $\mu_B \in \Delta(S_B)$. Let us denote $U^1_A(\theta_A,\mu_B)$ the level of utility of an agent of type θ_A if the contract signed at stage 1 is enforced and agent B plays according to μ_B. At stage 2, principal P_A proposes a contract $[(\bar{a},\bar{V}_A),(\underline{a},\underline{V}_A)]$, or with obvious notations $[a(\theta),V_A(\theta)]$. This contract must first verify individual rationality constraints:

$$\forall\ \theta_A \in \{\underline{\theta}_A, \bar{\theta}_A\}, \qquad V_A(\theta_A) - \int m^A(b,\theta_A)d\mu_B(b) \geq U_A^i(\theta_A,\mu_B)$$

or
$$V_A(\theta_A) \geq V_A^i(\theta_A,\mu_B) \equiv U_A^i(\theta_A,\mu_B) + \int m^A(b,\theta_A)d\mu_B(b)$$

Now given that contracts contingent on the opponent's stage 3 action are not allowed, $V_A^i(\theta_A,\mu_B)$ will have to be is independent of μ_B : $V_A^i(\theta_A,\mu_B) = V_A^i(\theta_A)$ (although for this section such dependency can be allowed since B's strategy is fixed). As there will be no ambiguity we drop indexes for this section.

The problem is to find the best response for P_A in the renegotiation phase, in terms of contract proposed to A, if B is expected to play according to μ and P_A-A are already bound by an initial contract signed at stage 1 that promised control utilities $(\bar{V}^i,\underline{V}^i)$ to the agent. More generally the problem considered in this section is a standard principal-agent one, with type-dependent reservation utilites.

For what follows we will use $(\bar{V}^i,X)$ with $X \equiv \underline{V}^i - \bar{V}^i$ to characterize individual rationality/or renegotiation constraints (omitting the possible argument μ). Incentive constraints can then be writen as:

$$\forall\ (\theta,\theta') \in \{\underline{\theta},\bar{\theta}\}^2,$$
$$V(\theta) - \int m(b,\theta)d\mu(b) \geq V(\theta') + (\theta'-\theta)a(\theta') - \int m(b,\theta)d\mu(b)$$

Or
$$V(\theta) - V(\theta') \geq (\theta' - \theta)a(\theta')$$

So the final contract that we characterize is the solution of :

$$\underset{(\bar{a},\bar{V},\underline{a},\underline{V})}{\mathrm{Max}} \int \left\{ h\phi(\bar{a},b,\bar{\theta}) + (1-h)\phi(\underline{a},b,\underline{\theta}) \right\} d\mu(b) \quad -h\bar{V} \quad -(1-h)\underline{V}$$
$$\text{s.t.} \qquad \Delta\bar{a} \leq \underline{V} - \bar{V} \leq \Delta\underline{a}$$
$$\bar{V} \geq \bar{V}^i \ , \quad \underline{V} \geq \bar{V}^i + X$$

Where we use ψ - t = -V + m and omit m.

It appears that $\bar{V}^i$ acts only as a uniform constant affecting $(\bar{V},\underline{V})$, but does not affect the optimal actions. Moreover in our linear-quadratic setting, the optimal contract only depends on B's expected action denoted $b^e \equiv \int b.d\mu(b)$. In the next proposition we characterize completely this optimal contract, using the following notation :

$$\bar{A}(b^e) \equiv \underset{a\in S}{\mathrm{Argmax}} \left\{ h\int \phi(a,b,\bar{\theta})d\mu(b) - (1-h)\Delta a \right\}$$
$$= A^*(b^e,\bar{\theta}\ ) - \frac{1-h}{2h}\Delta \qquad \text{if interior to S;}$$

$$\underline{A}(b^e) \equiv \underset{a\in S}{\mathrm{Argmax}} \left\{ (1-h)\int \phi(a,b,\underline{\theta})d\mu(b) + h\Delta a \right\}$$
$$= A^*(b^e,\underline{\theta}) + \frac{h}{2(1-h)}\Delta \qquad \text{if interior to S.}$$

It is immediate that : $\forall b^e \in S_B$, $\bar{A}(b^e) < A^*(b^e,\bar{\theta}) < A^*(b^e,\underline{\theta}) < \underline{A}(b^e)$. $A^*(b^e,\theta)$ is clearly the full information best action in response to the expected action b^e, $\bar{A}(b^e)$ is the best action required from $\bar{\theta}$ that makes $\underline{\theta}$ just indifferent between truthful revelation and lying (between $A^*(b^e,\underline{\theta})$ and $\bar{A}(b^e)$), while $\underline{A}(b^e)$ is the best action required from $\underline{\theta}$ that makes $\bar{\theta}$ just indifferent between truthful revelation and lying.

<u>Proposition 1</u> : Under A1, after an initial contract $[(\bar{a}^i,\bar{V}^i),\ (\underline{a}^i,\underline{V}^i)]$, the (pure strategy) best response for P_A in stage 2 to a choice of actions μ by B, is a contract $[(\bar{a}(b^e,X),\bar{V}(\bar{V}^i,b^e,X));(\underline{a}(b^e,X),\underline{V}(\bar{V}^i,b^e,X))]$ defined by Table 1 below. Actions $\bar{a}(.)$ and $\underline{a}(.)$ are continuous piecewise linear in B's expected action b^e, weakly increasing in X.

<u>Table 1</u>

X	$\Delta\bar{A}(b^e)$ $\quad$ $\Delta A^*(b^e,\bar{\theta}\ )$ $\quad$ $\Delta A^*(b^e,\underline{\theta}\ )$ $\quad$ $\Delta\underline{A}(b^e)$				
$\bar{a}$	$\bar{A}(b^e)$	X/Δ	$A^*(b^e,\bar{\theta}\ )$	$A^*(b^e,\bar{\theta}\ )$	$A^*(b^e,\bar{\theta}\ )$
$\underline{a}$	$A^*(b^e,\underline{\theta})$	$A^*(b^e,\underline{\theta}\ )$	$A^*(b^e,\underline{\theta}\ )$	X/Δ	$\underline{A}(b^e)$
$\bar{V}$	$\bar{V}^i$	$\bar{V}^i$	$\bar{V}^i$	$\bar{V}^i$	$\underline{V}^i-\Delta\underline{A}(b^e)$
$\underline{V}$	$\bar{V}^i+\Delta\bar{A}(b^e)$	$\underline{V}^i$	$\underline{V}^i$	$\underline{V}^i$	$\underline{V}^i$

Table 1 should be read as follows. The horizontal axis illustrates variations of X, the level of precommitted rent differential in control utilities for which four critical levels are outlined. These critical levels determine 5 distincts regions, the 5 columns of Table 1, within which the optimal renegotiated contract is characterized. The derivation of this table is provided in Appendix 1 (in a more general setting).

Proposition 1 can also be considered as the solution of a standard contracting problem under incomplete information with type-contingent reservation utilities[2]. Presented this way, one clearly sees that what should be considered "the good type" (not distorted) and "the bad type" (IR binding) is actually related to the type for which the rationality constraint is binding at the optimum. We will still call $\underline{\theta}$ the good type (low cost) although if X is large this type of agent would be distorted and put at his individually rational level ! Let us now give some intuition for Proposition 1.

In our model, the full information actions $(A^*(b^e,\theta),\theta\in\{\bar{\theta},\underline{\theta}\})$ are implementable under incomplete information since $A^*(b^e,\bar{\theta}) \le A^*(b^e,\underline{\theta})$, which means that there exist transfer functions such that this action choice is incentive compatible. What matters actually for incentive compatibility is the rent differential X , so that for medium values of X, no type θ wants to deviate from $A^*(b^e,\theta)$ to $A^*(b^e,\theta')$. These actions maximize aggregate surplus, and if P_A has precommitted to leave $(\bar{V}^1,\bar{V}^1 + X)$ for $X\in[\Delta A^*(b^e,\bar{\theta}),\Delta A^*(b^e,\underline{\theta})]$, she cannot hope to make the agent accept lower rents.

If however P_A has precommitted to $(\bar{V}^1,\bar{V}^1+X)$ with X small $(X<\Delta A^*(b^e,\bar{\theta}))$ the inital rent levels do not implement the full information actions : $\underline{\theta}$ could pretend he is $\bar{\theta}$ and would earn $\bar{V}^1+ \Delta A^*(b^e,\bar{\theta}) > \underline{V}^1$. If an efficient agent is not rewarded enough, he will pretend he is inefficient so as to reduce the action he has to provide and therefore save on the disutility of playing the stage 3 game. To restore incentive compatibility, P_A can either distort the action required from $\bar{\theta}$, $\bar{a} < A^*(b^e,\bar{\theta})$, or increase the rent granted to $\underline{\theta}$, $\underline{V} > \bar{V}^1+ X$. Starting from $X = \Delta A^*(b^e,\bar{\theta})$ and full information actions, and lowering X slightly, it is clear that a distorsion of $\bar{a}$ has only second-order effects on aggregate surplus, whereas an increase in $\underline{V}$ has first-order effects on the principal's payoffs. So, for moderately small X, P_A chooses to distort $\bar{a}$ to X/Δ to restore incentive compatibility. For very

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2 The situation of Proposition 1 is economically different from Laffont-Tirole [1988] 2-period model with renegotiation, but results are similar. However Laffont and Tirole do not analyze the case of large X as it is irrelevant in their model. Here, at equilibrium X may be "large".

small X, X/Δ would imply too large a distortion in action, and P_A prefers to increase $\underline{V}$ beyond $\bar{V}^i+X$, while keeping $\bar{a} = \bar{A}(b^e)$.

The situation for X large is perfectly symmetric. Type $\underline{\theta}$'s action must be distorted for moderately large X, while P_A prefers to increase $\bar{\theta}$'s rent beyond $\bar{V}^i$ and to avoid distorting $\underline{a}$ beyond $\underline{A}(b^e)$, for very large X.

IV. Equilibrium Analysis and Precommitment Effects

IV.1. Continuation Equilibrium and Reduced Form

This subsection provides technical definitions about our equilibrium concept as well as existence and unicity results.

Consider an initial contract. Given the separability of ψ_A, the optimal choice of agent A, $a^i(\theta_A)$, and his control utility level $V^i_A(\theta_A)$ when the initial contract is enforced at stage 2 and 3 are independent of agent B's actions or contract. With regards to the continuation game starting at stage 2 and stage 3, we can summarize the initial contract by a pair $[(\bar{a}^i,\bar{V}^i_A),(\underline{a}^i,\underline{V}^i_A)]$ for P_A-A and $[(\bar{b}^i,\bar{V}^i_B),(\underline{b}^i,\underline{V}^i_B)]$ for P_B-B.

A continuation equilibrium following a pair of contracts $[(\bar{a}^i,\bar{V}^i_A),(\underline{a}^i,\underline{V}^i_A)]$ and $[(\bar{b}^i,\bar{V}^i_B),(\underline{b}^i,\underline{V}^i_B)]$ signed in stage 1, is a strong Perfect Bayesian equilibrium of the game starting in stage 2, with beliefs equal to priors. This reduces to a Bayesian equilibrium of the game where both principals choose simultaneously an incentive compatible contract that is preferred by each agent to the initial contract. Given the resulting expected equilibrium action by B, Proposition 1 gives P_A's optimal contract : it specifies optimal actions $(\bar{a}(b^e,X_A),\underline{a}(b^e,X_A))$ as functions of the opponent's expected action and of the initial rent differential. So we define:

$$a^e(b^e,X_A) \equiv h_A\bar{a}(b^e,X_A) + (1-h_A)\underline{a}(b^e,X_A)$$

the average best response in terms of average action required by P_A from A. One can then define a continuation equilibrium in terms of a pair of average actions $(a^*(X_A,X_B),b^*(X_A,X_B))$ such that :

$$\begin{cases} a^*(X_A,X_B) = a^e(b^*(X_A,X_B),X_A) \\ b^*(X_A,X_B) = b^e(a^*(X_A,X_B),X_B) \end{cases}$$

Since best responses are continuous functions and strategy sets are compact, a pure strategy continuation equilibrium exists. We will moreover make an assumption that guarantees unicity of the continuation equilibrium. This assumption is reminiscent of the stability/unicity condition for a Cournot equilibrium in a standard duopoly model : P_A-A's reaction curve is supposed to be steeper than P_B-B's reaction curve (with a in abscissa, b in ordinate) at the intersection point. In the linear-quadratic framework, this property is then true everywhere so that a unique perfect information equilibrium exists. Finally, it will turn out that this condition is also sufficient to guarantee unicity in our asymmetric information setting.

A2 : $\forall(\theta_A,\theta_B)\in\{\bar{\theta}_A,\underline{\theta}_A\}\times\{\bar{\theta}_B,\underline{\theta}_B\}, \qquad \alpha(\theta_A)\beta(\theta_B) < 1$

Lemma 1 : Under A1-A2, there exists a unique continuation equilibrium in stages 2-3 and it involves pure strategies in contract proposal.

Proof : Under A1, $a^e(.,X_A)$ and $b^e(.,X_B)$ are singletons (see Table 1) so that if a continuation equilibrium exists, it involves pure strategies : the reason is that from any pair of equilibrium expected actions, one recover using Table 1 only one pair of contracts that yield same expected action and that are not strongly dominated in terms of excessive rents left to agents.

Now, under A1, $a^e(.,X_A)$ has right and left derivatives everywhere that belong to $\left\{h_A\alpha(\bar{\theta}_A),\ (1-h_A)\alpha(\underline{\theta}_A),\ h_A\alpha(\bar{\theta}_A)+\ (1-h_A)\alpha(\underline{\theta}_A)\right\}$ (from Table 1). The same property is true for $b^e(.,X_B)$. So the mapping $b^e(a^e(.,X_A),X_B)$ from S_B to S_B is continuous, piecewise C^1 with left and right derivatives smaller than 1 by A2. The fixed point of this contraction mapping exists and is unique.

Q.E.D.

Let us now consider stage 1 contract proposals. If the contracts are $(\bar{V}_A,X_A)$ and $(\bar{V}_B,X_B)$, the continuation equilibrium will involve actions $\mu^*_A(X_A,X_B)$, $\mu^*_B(X_A,X_B)$ and rent differentials $X^*_A(X_A,X_B)$, $X^*_B(X_A,X_B)$. In particular it is not affected by $\bar{V}_A$ and $\bar{V}_B$. It follows that principals will always choose $\bar{V}_A$ and $\bar{V}_B$ at the minimum level compatible with agents' individual rationality constraints.

Lemma 2: Given initial rent differentials X_A and X_B, the optimal choice of $\bar{V}_A$ for P_A and $\bar{V}_B$ for P_B is such that the high type agent receive final control utility levels:

$$\bar{V}_A(X_A,X_B) = \int m^A(b,\bar{\theta}_A)d\mu_B^*(X_A,X_B)(b)$$
$$\bar{V}_B(X_A,X_B) = \int m^B(a,\bar{\theta}_B)d\mu_A^*(X_A,X_B)(a).$$

Proof: Fix X_A, X_B and the continuation equilibrium $\{ \bar{a}^*, \underline{a}^*, \bar{b}^*, \underline{b}^*, X_A^*, X_B^* \}$. Let $\bar{V}_A^*$ be the final control utility of type $\bar{\theta}_A$. Individual rationality implies that

$$\bar{V}_A^* - \int m^A(b,\bar{\theta}_A)d\mu^*(b) \geq 0 \quad \text{or} \quad \bar{V}_A^* \geq \bar{V}_A(X_A,X_B).$$

Suppose this is true. As $X_A^* \geq \Delta\bar{a}^*$, we can write

$$\bar{V}_A^* + X_A^* - \Delta\bar{a}^* - \int m^A(b,\bar{\theta}_A)d\mu^*(b) \geq 0$$

or
$$\bar{V}_A^* + X_A^* + \underline{\theta}_A\bar{a}^* - \int \psi^A(\bar{a}^*,b,\bar{\theta}_A)d\mu^*(b) \geq 0.$$

From A1,
$$\bar{V}_A^* + X_A^* + \underline{\theta}_A\bar{a}^* - \int \psi^A(\bar{a}^*,b,\underline{\theta}_A)d\mu^*(b) \geq 0$$

or
$$\bar{V}_A^* + X_A^* - \int m^A(b,\underline{\theta}_A)d\mu^*(b) \geq 0.$$

The individual rationality constraint for type $\underline{\theta}_A$ is verified. Now given X_A and X_B, $\bar{V}_A^*$ and $\underline{V}_A^*$ are increasing functions of $\bar{V}_A$. P_A will choose $\bar{V}_A$ so as to minimize the expected payment which is obtained at $\bar{V}_A^* = \bar{V}_A(X_A,X_B)$.

Q.E.D.

Now given this lemma we can consider the game as a one-shot game between principals with strategies X_A and X_B, and payoffs:

$$\pi_A(X_A,X_B) = \mathbb{E}\{ \phi^A(a,b,\theta_A)+m^A(b,\theta_A) \mid X_A,X_B \} - \bar{V}_A(X_A,X_B) - (1-h_A)X_A^*(X_A,X_B)$$

and similarly for $\pi_B(X_A,X_B)$.

IV.2. Equilibrium with unobserved contracts

We first analyze the equilibrium when stage 1 does not exist, that is when only unobserved contracts are signed. This situation is a benchmark in that it involves no precommitment effects of contracts. Since these are not observed and therefore opponents cannot react, there is actually one move in the game, the stage 2 revelation mechanism, so that a hierarchy cannot affect its rivals' future behavior.

The analysis of the previous subsection can be slightly adapted to yield existence and unicity. Functions $(\bar{a}(.),\underline{a}(.))$ and $(\bar{b}(.),\underline{b}(.)$ as defined in Proposition 1 again give the best response correspondence (in terms of actions) of the game with unobserved contracts. But now the reservation utility of A is $U^1_A=0$ so that his individual rationality constraint writes as $V_A(\theta_A) \geq \int m^A(b,\theta_A)d\mu_B(b)$ and X_A also depends on the anticipated equilibrium distribution of actions of his opponent, μ_B:

$$X_A(\mu_B) = \int \left(m^A(b,\underline{\theta}_A) - m^A(b,\bar{\theta}_A)\right)d\mu_B(b)$$

<u>Proposition</u> <u>2</u>: Under A1-A2, there exists a unique equilibrium when contracts are unobservable. It corresponds to

$\bar{a}^0 = \bar{A}(b^0)$, $\underline{a}^0 = A^*(b^0,\underline{\theta}_A)$, $a^0 = h_A\bar{a}^0 + (1-h_A)\underline{a}^0$

$\mu^0_A = \bar{a}^0$ with probability h_A, $\underline{a}^0$ with probability $(1-h_A)$

$\bar{V}^0_A = \int m^A(b,\bar{\theta}_A)d\mu^0_B(b)$, and $X^0_A = \Delta\bar{A}(b^0)$

and symmetrically for the P_B-B contract.

Profits are $\pi^0_A \equiv \pi_A(X^0_A,X^0_B)$ and $\pi^0_B \equiv \pi_B(X^0_A,X^0_B)$.

<u>Proof</u> : Fix B's equilibrium play at μ^0_B. $X_A(\mu^0_B)$ as defined above is continuous with respect to μ^0_B. Then best response actions $(\bar{a}(b^0,X_A(\mu^0_B)),\underline{a}(b^0,X_A(\mu^0_B)))$ and $(\bar{b}(a^0,X_B(\mu^0_A)),\underline{b}(a^0,X_B(\mu^0_A)))$ are <u>continuous functions</u> on compact strategy sets : so an equilibrium exists, and all equilibria involve pure strategies since best responses are always singletons.

The monotonicity of ψ^A in θ_A (see A1) implies :

$$\bar{\theta}_A\bar{A}(b^0) + \int m^A(b,\bar{\theta}_A)d\mu_B(b) \geq \underline{\theta}_A\bar{A}(b^0) + \int m^A(b,\underline{\theta}_A)d\mu_B(b)$$

i.e. : $X_A(\mu^0) \leq \Delta^A\bar{A}(b^0)$. Hence, from Table 1, best response actions must be

$$\begin{cases} \bar{a}(b^0,X_A(\mu^0_B)) = \bar{A}(b^0) \\ \underline{a}(b^0,X_A(\mu^0_B)) = A^*(b^0,\underline{\theta}_A) \end{cases}$$

and symmetrically for $\bar{b}$ and $\underline{b}$. Indeed, best responses do not depend on $X_A(\mu^0_B)$ and $X_B(\mu^0_A)$ in a neighborhood of an equilibrium, and then A2 guarantees unicity of the equilibrium as in lemma 1. Moreover, from Table 1:

$$\bar{V}^0_A = \int m^A(b,\bar{\theta}_A)d\mu^0_B(b)$$

$$X^0_A = \Delta^A\bar{A}(b^0)$$

Individual rationality constraints are thus satisfied. The expression of profits follows immediately.

Q.E.D.

It may be observed that this equilibrium with unobserved contract is also the continuation equilibrium (unique by lemma 1) following initial contracts with $X_A \le \Delta^A\bar{A}(b^0)$ and $X_B \le \Delta^B\bar{B}(a^0)$. The equilibrium with unobserved contracts thus corresponds in our global game to a path where initial contracts satisfy $X_A \le X^0_A$, $X_B \le X^0_B$. In particular $\pi_i(X_A,X_B)=\pi^0_i$ for $X_A \le X^0_A$, $X_B \le X^0_B$, i= A,B.

The analysis of Proposition 2 could be extended to the case where P_A does not sign any contract in stage 1 whereas P_B and B sign a contract specifying a given $(\bar{V}_B,X_B)$. X_B would be given, but $X_A(\mu_B)$ would depend on μ_B. Best responses are again continuous on compact strategy sets and are singletons. Moreover $X_A(\mu_B) \le \Delta^A\bar{a}^0(X_B)$ where

$$\bar{a}^0(X_B) = \bar{A}(b^0(X_B)) \quad \underline{a}^0(X_B) = A^*(b^0(X_B),\underline{\theta}_A) \quad \text{and}$$

$$b^0(X_B) = b^e(a^0(X_B),X_B)$$

are the equations determining the unique equilibrium after X_B.

In the next subsection, we analyze whether the equilibrium under unobservability of contracts remains an equilibrium (part of the equilibrium path, more precisely) when stage 1 contracts are publicly observed. For that, we will start from the equilibrium with unobserved contracts and we will evaluate the effects of a modification of stage 1 contracts on the principals' payoffs. Since the individual rationality constraints of $\bar{\theta}_A$ and $\bar{\theta}_B$ are tight at the equilibrium with unobserved contracts, we should worry that local modifications could be non feasible, i.e. non acceptable by agents. To avoid these problems, we will assume that principals and agents' objectives both increase or both decrease for a given variation of their opponent's action

A3 : $\forall(a,b,\theta_A,\theta_B) \in S_A \times S_B \times \{\bar{\theta}_A,\underline{\theta}_A\} \times \{\bar{\theta}_B,\underline{\theta}_B\}$,

$$\frac{\partial}{\partial b}(\phi^A(a,b,\theta_A) + \psi^A(a,b,\theta_A)).\frac{\partial}{\partial b}(\psi^A(a,b,\theta_A)) \le 0$$

$$\frac{\partial}{\partial a}(\phi^B(a,b,\theta_B) + \psi^B(a,b,\theta_B)).\frac{\partial}{\partial a}(\psi^B(a,b,\theta_B)) \le 0$$

A3 implies that if a given variation of B's action induces an increase in P_A's profits, it also induces a decrease in A's level of individual rationality and is therefore a feasible improvement of P_A's profits. The assumption thus allows us to assess without ambiguity whether an increase in the opponent's action is desirable from the point of view of the hierarchy or not.

Finally, it should be noted that in our model, the equilibrium with unobserved contracts is also the equilibrium with observable and not renegotiable contracts. The point is that in both cases, the global game is actually a one-shot game : both principals choose simultaneously their incentive compatible, individually rational contracts, which already incorporate the optimal acceptance and announcement rules by the agent and the optimal stage 3 actions. There is no second stage where a principal could modify her contract in response to a first move of her rival : there is no scope for precommitment effects in both cases. Renegotiation of observable contracts can add such a reaction stage as we will see. Sequential timing in contract proposals, or more complicated contingent contracts would also make precommitment effects appear with observable (and not renegotiable) contracts.

IV.3. Precommitment Effects.

As noted earlier, we will say that a global equilibrium of the game with observable and renegotiable contracts exhibits precommitment effects, if the unobserved equilibrium contracts are not stage 1 strategies of a global equilibrium. Then precommitment effects will arise by means of initial contracts such that $X_A > \Delta^A \bar{a}^{A-0}$ or $X_B > \Delta^B \bar{b}^{B-0}$. Such high levels of rent differentials will induce a continuation equilibrium characterized by $(a^*(X_A,X_B),b^*(X_A,X_B))$. Since $a^*(X_A,X_B) = a^e(b^e(a^*(X_A,X_B),X_B),X_A)$ by the fixed point property, equilibrium actions variations with respect to (X_A,X_B) will depend on whether market decisions (a and b) are strategic substitutes or complements. We make then the following assumptions :

Assumptions S:

SS : Actions are strategic substitutes : $\alpha(\bar{\theta}_A),\alpha(\underline{\theta}_A),\beta(\bar{\theta}_B),\beta(\underline{\theta}_B)$ are all strictly negative.

SC : Actions are strategic complements : $\alpha(\bar{\theta}_A),\alpha(\underline{\theta}_A),\beta(\bar{\theta}_B),\beta(\underline{\theta}_B)$ are all strictly positive.

That strategic complementarities are crucial for the nature of our results is indeed unsurprising from the industrial organization literature (Fudenberg-Tirole [1984], Bulow-Geanakoplos-Klemperer [1985], Tirole [1988]). SS or SC allow a simple characterization of the equilibrium variations with respect to initial rents (X_A,X_B).

Lemma 3 : Under SS, $b^*(X_A,X_B),\bar{b}^*(X_A,X_B)$ and $\underline{b}^*(X_A,X_B)$ (resp. $a^*,\bar{a}^*$ and $\underline{a}^*$) are weakly decreasing in X_A (resp. X_B) and weakly increasing in X_B (resp. X_A). Under SC, all equilibrium actions are weakly increasing in (X_A,X_B).

Proof : From Table 1, $a^e(b^e,X_A)$ is weakly increasing in X_A ($b^e(a^e,X_B)$ in X_B), and SS guarantees it is decreasing in b^e ($b^e(a^e,X_B)$ in a^e). From A_2 and the fixed point equation, $b^* = b^e(a^e(b^*,X_A),X_B)$, it follows that $b^*(X_A,X_B)$ is decreasing in X_A under SS, increasing under SC. Now, $a^*(X_A,X_B) = a^e(b^*(X_A,X_B),X_A)$ and therefore is weakly increasing in X_A. The results on type-contingent equilibrium actions then follow.

Q.E.D.

The higher the contractual rent differential guaranteed in stage 1 by P_A, the higher the induced equilibrium actions of agent A, even after renegotiation, in stage 3. This is the crux of the usual argument on precommitment effects : the design of stage 1 contracts can affect the opponent's behavior.

However, it should be emphasized that observability of contracts per se does not imply that precommitment effects exist. Here, as mentioned above, the equilibrium with observable, nonregenotiable contracts would not differ from the equilibrium under unobservability. To obtain precommitment effects in a model where contracts are publicly disclosed and cannot be renegotiated, one would typically need introducing some asymmetry in the game, for instance by assuming that at stage 1, P_A-A move before P_B-B. Our model considers a different case since both hierarchies are supposed to be in a symmetrical position. Nevertheless, we will show that precommitment effects may exist in such a case, if observed contracts can be secretly renegotiated. In our symmetrical setting, observable contracts will involve precommitment effects <u>when</u> they can be renegotiated, because they affect the conditions under which secret renegotiation will occur. The mere <u>disclosure</u> of a stage 1 contract affects the opponent's behavior, even though this contract can be renegotiated, because it limits the principal's ability to renegotiate to her simple optimal contract with no concern about the rival. Now the question is : can we characterize cases under which stage 1 contracts are effectively used as precommitment device in equilibrium ?

To answer this question, we start from the situation where both principals commit to $(\bar{V}^0_A, \underline{V}^0_A)$ and $(\bar{V}^0_B, \underline{V}^0_B)$, the unobserved contract equilibrium strategies. From this situation, P_A could try to propose instead a stage 1 contract with X_A larger in order to influence stage 3 outcome. This induces a decrease in the opponent's equilibrium play in the continuation game when decisions are strategic substitutes, an increase when they are strategic complements, and these are the only effects that can have stage 1 publicly disclosed contracts. Hence precommitment effects will actually exist if decisions are strategic substitutes and actions have adverse effects on opponent's payoffs or if decisions are strategic complements and have positive effects on opponent's payoffs. We will therefore focus attention of the role of SS/SC and of the following assumption, that determines whether a principal wants to increase or decrease her opponent's action :

<u>Assumptions E</u>[3]:

<u>PE</u>: $\dfrac{\partial(\phi^A + \psi^A)}{\partial b} > 0$ and $\dfrac{\partial(\phi^B + \psi^B)}{\partial a} > 0$

<u>NE</u>: $\dfrac{\partial(\phi^A + \psi^A)}{\partial b} < 0$ and $\dfrac{\partial(\phi^B + \psi^B)}{\partial a} < 0$

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3 In Fudenberg-Tirole [1984] terminology, assumption PE corresponds to a situation where a higher rent differential makes a principal "soft", NE to a situation where it makes her "tough".

Using the different configurations of S and E, one can then formalize the above discussion on the existence of precommitment effects.

Theorem 1 : Assume that the unobserved contract equilibrium is interior. Precommitment effects exist if SS and NE hold, or if SC and PE hold[4].

Remark : Note that the following proof shows that E could take a local, much weaker form.

Proof : It amounts to show that the right derivative of $\pi_A(X_A,X_B)$ with respect to X_A is positive at (X_A^0,X_B^0). Suppose stage 1 stragegies are to sign contracts characterized by X_A^0 and X_B^0, as in the equilibrium under unobserved contracts. Let us consider a deviation for P_A consisting in $X_A = X_A^0+\varepsilon$, where $\varepsilon > 0$ is small. With a slight abuse of notation, call $\bar{a}^*(\varepsilon)$, $\underline{a}^*(\varepsilon)$, $\bar{b}^*(\varepsilon)$ and $\underline{b}^*(\varepsilon)$ the continuation equilibrium actions.

For ε small enough, $\underline{a}^*(\varepsilon) = A^*(b^*(\varepsilon),\underline{\theta}_A)$ from Table 1. Now $\bar{a}^*(\varepsilon)$ could a priori be given for ε small enough, either by $\bar{A}(b^*(\varepsilon))$ or by $(X_A^0+ \varepsilon)/\Delta_A$. But in the first case, the equilibrium actions would actually not depend on ε, i.e. would be identical to the equilibrium under unobserved contracts. This, however, would imply renegotiated rent differential X_A^0 that would not satisfy renegotiation constraints, since $X_A^0 < X_A$ the precommitted rent differential. Hence, necessarily, for ε small positive, $\bar{a}^*(\varepsilon) = (X_A^0+ \varepsilon)/\Delta_A$.

From Lemma 3, one also knows that $\bar{b}^*(\varepsilon)$ and $\underline{b}^*(\varepsilon)$ decrease under SS when ε increases, increase under SC. Therefore, under SS and NE, using A3, it appears that individual rationality constraints relative to the forecast equilibrium are relaxed when ε increases. Similarly under SC and PE. The proposed deviation is thus feasible. P_A's expected payoff from the deviation X_A is :

$$\pi_A(X_A^0+\varepsilon,X_B^0) \equiv \pi_A(\varepsilon)$$

For ε small enough, all functions of ε are differentiable on a positive neighborhood $[0,\bar{\varepsilon}]$; differentiating and letting $\varepsilon\to 0^+$, it comes:

$$\frac{d\bar{a}^*}{d\varepsilon}(0^+) = \frac{1}{\Delta^A}$$

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[4] In Fudenberg-Tirole terminology, SS and NE correspond to a "top dog" situation, SC and PE to a "fat cat" situation.

Moreover, $\underline{a}^*(0^+) = A^*(b^0,\underline{\theta}_A)$ and $\bar{a}^*(0^+) = A^*(b^0,\bar{\theta}_A) - \frac{1-h_A}{h_A}\Delta_A$. So, by the interiority assumption, one gets :

$$\frac{d\pi_A}{d\varepsilon}(0^+) = \left\{h_A h_B\left[\frac{\partial(\phi^A+m^A)}{\partial\bar{b}}(\bar{a}^0,\bar{b}^0,\bar{\theta}_A)\right] + (1-h_A)h_B\left[\frac{\partial(\phi^A+m^A)}{\partial\bar{b}}(\underline{a}^0,\bar{b}^0,\underline{\theta}_A)\right]\right\}\frac{d\bar{b}^*}{d\varepsilon}(0^+)$$

$$+\left\{h_A(1-h_B)\left[\frac{\partial(\phi^A+m^A)}{\partial\underline{b}}(\bar{a}^0,\underline{b}^0,\bar{\theta}_A)\right] + (1-h_A)(1-h_B)\left[\frac{\partial(\phi^A+m^A)}{\partial\underline{b}}(\underline{a}^0,\underline{b}^0,\underline{\theta}_A)\right]\right\}\frac{d\underline{b}^*}{d\varepsilon}(0^+)$$

$$-h_B\frac{\partial m^A}{\partial b}(\bar{b}^0,\bar{\theta}_A)\frac{d\bar{b}^*}{d\varepsilon}(0^+) - (1-h_B)\frac{\partial m^A}{\partial b}(\underline{b}^0,\bar{\theta}_A)\frac{d\underline{b}^*}{d\varepsilon}(0^+)$$

Now, it is immediate that SS and $\frac{d\bar{a}^*}{d\varepsilon}(0^+) > 0$ imply $\frac{d\bar{b}^*}{d\varepsilon}(0^+) < 0$ and $\frac{d\underline{b}}{d\varepsilon}(0^+) < 0$. With NE, ϕ^A+m^A is decreasing with b while m^A is increasing with b (from A3), therefore one gets $\frac{d\pi_A}{d\varepsilon}(0^+) > 0$.

Similarly, SC and $\frac{d\bar{a}^*}{d\varepsilon}(0^+) > 0$ imply $\frac{d\bar{b}^*}{d\varepsilon}(0^+) > 0$ and $\frac{d\underline{b}}{d\varepsilon}(0^+) > 0$. With PE, one gets $\frac{d\pi_A}{d\varepsilon}(0^+) > 0$.

So in these two cases, the deviation, at least for ε small positive, is profitable and feasible ; therefore contracts that form the equilibrium when unobserved are not part of a global equilibrium : precommitment effects exist.

Q.E.D.

On the other hand, one can also characterize situations of our global game where precommitment effects do not exist, by which we mean that the equilibrium with unobserved contracts is also an equilibrium of our global game. Now the reasoning is very similar to the previous one. Contracts have two purposes : they have "internal" purposes to overcome asymmetric information problems between principals and agents, and "external" purposes to affect the opponent's renegotiation and action play. Internal purposes are maximized with unobserved equilibrium contracts, since these neglect any precommitment effects. Now if external purposes can only be worsen by using other contracts, principals will sign contracts that maximize internal concerns only. And this is the case if committing to a larger contractual rent differential can only affect the (equilibrium) opponent's action in the wrong direction, i.e. decrease it whereas PE holds or increase it whereas NE holds. This leads us to our second main result.

<u>Theorem 2</u> : Assume SS and PE, or SC and NE , then the equilibrium of the global game is unique in terms of payoffs and actions and it corresponds to the unobserved contracts equilibrium[5].

<u>Proof</u> : Suppose SC and NE hold ; the proof under SS and PE is similar. Consider an equilibrium path resulting in X_A and X_B as initial rent differentials , and μ_A^*, μ_B^* as equilibrium distribution of actions. W.l.o.g we can assume that $X_A = X_A^*(X_A,X_B)$ and $X_B = X_B^*(X_A,X_B)$ (X_A and X_B are renegotiation-proof) since (X_A,X_B) and (X_A^*,X_B^*) leads to the same equilibrium path. Profits are then $\pi_A(X_A,X_B)$ and $\pi_B(X_A,X_B)$. We write $\bar{V}_A$ for $\bar{V}_A(X_A,X_B)$.

Consider a deviation in stage 1 for P_A consisting of not proposing any contract. As explained in IV.2., this would result in a continuation equilibrium $(\mu_A^0(X_B),\mu_B^0(X_B))$ similar to the one obtained if P_A has signed a contract with $X_A \le X_A^0(X_B) \equiv \Delta^A\bar{a}^0(X_B)$. We will prove that such a strategy is dominant for P_A in stage 1.

Suppose that $X_A > X_A^0(X_B)$, then under SC, $\bar{b}^*(X_A,X_B) > \bar{b}^*(X_A^0(X_B),X_B)$, and $\underline{b}^*(X_A,X_B) > \underline{b}^*(X_A^0(X_B),X_B)$ by lemma 2 and the strict inequality comes from $\bar{a}^*(X_A,X_B) = X^A/\Delta_A$ strictly increasing immediately to the right of $X_A^0(X_B)$.

Let $\bar{\pi}_A$ be P_A's profits if the final contract is $\bar{V}_A$, X_A, μ_A^* and B plays according to $\mu_B^0(X_B)$. Then clearly, under NE:

$$\pi_A(X_A,X_B) < \bar{\pi}_A.$$

Note that $\bar{V}_A$, X_A is individually rational given $\mu_B^0(X_B)$ since

$$\bar{V}_A = \int m^A(b,\bar{\theta}_A)d\mu_B^*(b) > \int m^A(b,\bar{\theta}_A)d\mu_B^0(b)$$

$$\bar{V}_A + X_A \ge \int m^A(b,\underline{\theta}_A)d\mu_B^*(b) > \int m^A(b,\underline{\theta}_A)d\mu_B^0(b).$$

Clearly $\bar{V}_A$, X_A, μ_A^* is also incentive compatible.

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5 In Fudenberg-Tirole [1984] terminology, SS and PE correspond to a "lean and hungry look" situation, SC and NE to a "puppy dog" situation.

Now the contract $\bar{V}^0_A(X_B), X^0_A(X_B), \mu^0_A(X_B)$ is the best (unobserved) contract for P_A when B plays according to $\mu^0_B(X_B)$ and P_A has not signed any contract in stage 1, and it leads to profits $\pi_A(X^0_A(X_B), X_B)$. As $\bar{V}_A$, X_A, μ^*_A is feasible given $\mu^0_B(X_B)$ (when no initial contract has been signed) we immediately obtain:

$$\bar{\pi}_A < \pi_A(X^0_A(X_B), X_B).$$

Therefore $\pi_A(X_A, X_B) < \pi_A(X^0_A(X_B), X_B)$ when $X_A > X^0_A(X_B)$. It follows that in equilibrium $X_A \leq X^0_A(X_B)$ and $X_B \leq X^0_B(X_A)$, or $X_A = X^0_A$, $X_B = X^0_B$, $\mu^*_A = \mu^0_A$, $\mu^*_B = \mu^0_B$.

Q.E.D.

<u>Remark</u>: we should emphasize that this proof amounts to show that not signing an initial contract is a dominant strategy.

It is worth relating the results of Theorem 1 and 2 to the literature on strategic investment (see e.g. Tirole [1988], Chapter 8, pp 323-327). There, under the situation of entry accomodation, a given investment has a strategic effect that depends on : $\frac{\partial \pi_A}{\partial b} \frac{\partial b}{\partial a}$. The effect bears on the equilibrium of the continuation game so that the sign of reaction curves matters $\left(\frac{\partial b}{\partial a}\right)$; and $\frac{\partial \pi_A}{\partial b}$ measures the desirability of affecting the opponent's action in one direction or the other. Although in a more complex environment, a similar intuition applies in our case. Observed contracts, even though renegotiable, limit the ability of principals to renegotiate : they constitute a strategic investment. Distorting these contracts from their internal optimal value (when neglecting strategic effects) will be optimal when the sign of an expression similar to $\frac{\partial \pi_A}{\partial b} \frac{\partial b}{\partial a}$ is positive : this expression appears in the expression of $\frac{d\pi_A}{d\varepsilon}(0^+)$ in the proof of Theorem 1, but there $\frac{\partial b}{\partial a}$ is replaced by $\frac{db^*}{d\varepsilon}$ since $\frac{da^*}{d\varepsilon}$ is positive (Lemma 3)[6].

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6 There is an alternative set of assumptions under which Theorem 1 and 2 continue to hold. A3 ensures that whenever P_A's profits are increased via an effect on B's action, this modification is not forbidden by individual rationality. An alternative would be to allow deviations in both types'rents so that individual rationality constraints be verified ($\bar{V}^0_A + \nu$ in the proof of Theorem 1 for example). Then one can easily show that if A3-E are replaced by

V. Example : A Symmetric Model of Cournot competition

Consider the following example. Let P_A and P_B be two firms producing an homogeneous good with constant marginal cost c. Agents A and B are salesmen whose disutility of effort θ is privately known: $\theta \in \{\underline{\theta}, \bar{\theta}\}$ and $h = \mathbb{P}\{\theta = \bar{\theta}\}$. Firms compete in quantities and the industry inverse demand curve is $P(q_A + q_B) = d - q_A - q_B$, so that profits are $[P(q_A + q_B) - c]q_i$ for P_i. Firms propose contracts that specify payments to salesmen contingent on sales. In terms of previous notation :

$$\psi^A(q_A, q_B, \theta) = \theta q_A$$
$$\phi^A(q_A, q_B, \theta) = (d - q_A - q_B - c - \theta) q_A$$

We will focus on interior equilibria and we can forget compactness assumptions on the set of allowed quantities. Defining $a = q_A$, $b = q_B$, one gets $m^j \equiv 0$, $f^j \equiv 0$ and $A^*(b,\theta) = \frac{1}{2}(d-b-c-\theta)$ (symmetrically for $B^*(.)$). So $\alpha(\theta) = -\frac{1}{2}$, $\alpha_0(\theta) = \frac{1}{2}(d-c-\theta)$. Finally:

$$\frac{\partial^2 \phi^A}{\partial a \partial b} = -1 < 0$$

$$\frac{\partial \phi^A}{\partial b} + \frac{\partial \psi^A}{\partial b} = -a < 0 \text{ on the relevant range.}$$

So A1-A2-A3-SS and NE hold, and Theorem 1 applies. More precisely.

<u>Proposition 3</u> : In the model of Cournot competition (with interiority condition[7]), the equilibrium under unobserved contracts <u>is not</u> an equilibrium with observable but secretly renegotiable contracts.

<u>A3'</u> : $\frac{\partial \phi^A}{\partial b} \frac{\partial^2 m^A}{\partial b \partial \theta_A} \le 0$ and $\frac{\partial \phi^B}{\partial a} \frac{\partial^2 m^B}{\partial a \partial \theta_B} \le 0$

<u>PE'</u>: $\frac{\partial \phi^A}{\partial b} > 0$ and $\frac{\partial \phi^B}{\partial a} > 0$

<u>NE'</u>: $\frac{\partial \phi^A}{\partial b} < 0$ and $\frac{\partial \phi^B}{\partial a} < 0$

the proof of Theorem 1 still holds. Similarly a local argument would prove Theorem 2. The intuition behind A3' and E' is a bit less clear than for A3-E. A3' ensures that if a variation of b is beneficial for the aggregate structure (this depends on E') P_A can recover part of this benefit despite an eventual worsening of A's individual rationality constraints.

7 The interiority conditions are $\bar{a}^0 > 0$, $\bar{b}^0 > 0$ and $\underline{a}^0 + \underline{b}^0 < 0$ so that.

In a situation of Cournot competition, precommitment effects exist and firms have a strict incentive to publicly disclose the (non contingent) compensation mechanism under which their salesmen act. Hence in an equilibrium of (non-contingent) contracts, in stage 1 either $X_A > X_A^0 = \underline{V}_A^0$ or $X_B > X_B^0 = \underline{V}_B^0$. In order to go a bit further, we will focus on symmetric pure strategy equilibria in revelation mechanisms, with $X_A=X_B=X$.

Let us define the following benchmarks.

- *interim unobserved contract :*

$X^0=X_A^*=X_B^*$, $\bar{a}^0=\bar{b}^0, \underline{a}^0=\underline{b}^0$, corresponds to the equilibrium under unobserved contracts, given in proposition 2.

- *ex-ante unobserved contracts :*

This is the equilibrium contracts if contracts were signed at stage 1 under symmetric (but incomplete) information, i.e. ex-ante, and were not observed by opponents. So actions would be clearly efficient and the equilibrium would be (see section 7):

$\tilde{b} = \tilde{a} = A^*(a,\bar{\theta})$, $\underset{\sim}{b} = \underset{\sim}{a} = A^*(a,\underline{\theta})$, $a = h\tilde{a}+(1-h)\underset{\sim}{a}$ and $X_1^* \in [\bar{X},\underline{X}]$

where $\bar{X} \equiv \Delta\tilde{a}$ and $\underline{X} \equiv \Delta\underset{\sim}{a}$, $\underline{X} > \bar{X}$.

- *maximal degree of precommitment :*

X^1 is obtained as the highest rent differential X attainable in a continuation equilibrium. From Table 1:

$X^1 = \Delta\underline{a}^1$ where $\bar{a}^1= A^*(a^1,\bar{\theta})$, $\underline{a}^1 = \underline{A}(a^1)$ and $a^1 = h\bar{a}^1+(1-h)\underline{a}^1$

It is straightforward that $X^0 < \bar{X} < \underline{X} < X^1$ and we obtain:

<u>Proposition</u> <u>4</u> : In the symmetric model of Cournot competition , under interiority conditions, precommitment effects exist so that there is overproduction compared to the equilibrium under unobserved contracts (and incomplete information). Moreover :

. either $X^* \in]X^0,\bar{X}]$, and the <u>high-cost action is distorted downward</u> compared to the full information, unobserved contract situation: there is underproduction by inefficient managers $\bar{a}^* \leq A^*(b^*,\bar{\theta})$.

. or $X_A^* \in]\underline{X},X^1]$ and the <u>low cost action is distorted upward</u> : there is overproduction by efficient managers : $\underline{a}^* > A^*(b^*,\underline{\theta})$.

The proof of Proposition 4 is rather intuitive. Precommitment effects require $X^*\in]X^0,X^1]$, i.e. a larger informational rent to the efficient manager than under unobserved contracts (see Proposition 3). Now if $X^{*}\in]\bar{X},\underline{X}]$, actions required after renegotiation would be (full information) efficient so that the equilibrium would be similar to the one obtained with $X^{*}=\bar{X}$;however the rent left to efficient managers would be unnecessary higher than $\bar{X}$. So $X^{*}\in\ ]\bar{X},\underline{X}]$ is strictly dominated. When demand is high compared to the uncertainty about managerial production cost, a high degree of precommitment can be attained with a small distortion in the production and a marginal reduction in the rival's production becomes very profitable. Hence P_A takes much benefit from precommitment opportunities, and reverses the usual adverse selection distortion : she gives such a large informational rent to the good type that the efficient manager is induced to overproduce (compared to full information) so as to reduce the opponent's production. The need of aggressive production to reduce the rival's production overcome the tendency to reduced production (of high cost managers) due to incomplete information.

Principal P_A clearly gains from the opportunity to disclose publicly the compensation scheme of her salesman. It allows her to make her salesman look like a "top dog". Now it should be noted that firms are worse off when public disclosure of contract is possible, since they pay larger rents to their agents and the final outcome implies larger quantities for both firms and therefore smaller profits than under unobservable contracts. In equilibrium a firm is forced to fully use the precommitment opportunities, but both precommitment attempts cancel each others: the equilibrium is inefficient from the industry perspective, and a corporate association or a pressure group would presumably try to impose secrecy of compensation schemes or of quotas to salesmen. Consumers on the other hand would try to promote competition between firms by allowing them to disclose their internal aggressive incentives[8].

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[8] At the other extreme is the case where one hierarchy (say P_B-B) is under full information. The results concerning the rents received in equilibrium by A are similar, but welfare implications differ. P_B cannot benefit from precommitment effects since he would always renegotiate to the efficient action $B^{*}(a,\theta_B)$. Therefore public disclosure of contracts is beneficial to P_A-A and detrimental to P_B-B (see Caillaud-Jullien-Picard [1990]).

VI. Ex-ante Contracts.

In this section, we want to investigate how our game and solution should be modified if contracts in stage 1 were signed ex-ante, i.e. at a stage where the agents would still ignore their true characteristic θ_i. As noted in section 3, our timing implies that renegotiations occur at an interim stage, which is a restriction. We first derive the results under this restriction, and then relax it by allowing the possibility of renegotiation ex-ante also.

With ex-ante stage 1 contracts but interim stage 2 contracts, again only the levels of precommitted contractual rent differential, X_A and X_B, matter, and renegotiation and continuation equilibria are described by the same analysis as before (Proposition/Table 1, Lemma 2). A first modification concerns the equilibrium with unobserved ex-ante contracts, since individual rationality should now be satisfied ex-ante only :

$$h_A\bar{V}_A + (1-h_A)\underline{V}_A \geq \int \Big(h_A m^A(b,\bar{\theta}_A)+(1-h_A)m^A(b,\underline{\theta}_A)\Big)d\mu_B^0(b)$$

No separate constraints bear on $(\bar{V}_A,\underline{V}_A)$ (alternatively on $\bar{V}_A,X_A$), so the principal will, for a given X_A, optimally fix $\bar{V}_A$ so as to saturate this ex-ante individual rationality contraint. P_A's payoffs reduce then to the aggregate expected payoffs:

$$\pi_A = \mathbb{E}\ [\phi^A(a,b,\theta_A)]$$

So P_A will choose optimally X_A anywhere in $[\Delta^A A^*(b^0,\bar{\theta}_A),\Delta^A A^*(b^0,\underline{\theta}_A)]$ so as to induce full-information optimal actions. Hence, there is a continuum of equilibria under ex-ante unobserved contracts, all of them being (ex-ante) payoff-equivalent. They are defined as:

$\tilde{a}$, $\underset{\sim}{a}$, $X_A \in [\bar{X}_A,\underline{X}_A]$, $\bar{V}_A$ determined by individual rationality, with

$\tilde{a} = A^*(h_B\tilde{b} + (1-h_B)\underset{\sim}{b},\ \bar{\theta}_A)$

$\underset{\sim}{a} = A^*(h_B\tilde{b} + (1-h_B)\underset{\sim}{b},\ \underline{\theta}_A)$

$\bar{X}_A = \Delta^A\tilde{a}$, $\underline{X}_A = \Delta^A\underset{\sim}{a}$

and similarly for $\tilde{b},\underset{\sim}{b}$, $\bar{X}_B$ and $\underline{X}_B$.

This equilibrium corresponds to the absence of precommitment effects and will again serve as a benchmark. Starting from this equilibrium (from one pair $(X_A,X_B) \in [\bar{X}_A,\underline{X}_A]\times[\bar{X}_B,\underline{X}_B]$), P_A could try to commit in stage 1 to a larger rent differential $X'_A > X_A$. If X'_A still belongs to $[\bar{X}_A, \underline{X}_A]$ this cannot have any effect (from Proposition/Table 1) : but if X'_A is larger than $\underline{X}_A$, it will induce a decrease in the opponent's equilibrium play in the continuation game when actions are strategic substitutes. Indeed we get the analog of Theorem 1 :

Theorem 3 : Precommitment effects exist with ex-ante initial contracts if SS and NE, or if SC and PE hold : they involve high contractual rent differentials and equilibrium actions that are costlier than in the full information case.

Proof : It follows the proof of Theorem 1. One considers a deviation $X'_A = \underline{X}_A + \varepsilon$ and $\bar{V}'_A + (1-h_A)X'_A = \bar{V}_A + (1-h_A)\underline{X}_A$ to preserve exactly ex-ante individual rationality. Since one starts from full information optimal actions, ε has no first-order effects on P_A's payoffs for fixed actions played by B. Only the induced decrease (under SS) in B's actions has a first order effect on P_A's expected payoffs and it is profitable (under NE).

Q.E.D.

Now, a symmetric argument is true if P_A tries to commit in stage 1 to a smaller rent differential : $X'_A < \bar{X}_A$. If, for example, actions are strategic substitutes, a small rent differential will induce an increase of the opponent's actions in the continuation equilibrium. Under PE this will be profitable. The proof of Theorem 4 mimics the proof of Theorem 3 with $X'_A = \bar{X}_A - \varepsilon$ and $\bar{V}'_A$ adjusted to preserve ex-ante individual rationality.

Theorem 4 : Precommitment effects exists with ex-ante initial contracts if SS and PE or if SC and NE hold : they involve small contractual rent differentials and equilibrium actions that are less costly than in the full information case.

Ex-ante contracts are thus much different from interim (initial) contracts. Precommitment effects always exist, either because principals commit in stage 1 to large rent differentials, or because they commit to small rent differentials. In the Cournot example, principals commit to large rent differentials which are not renegotiated away and are intended to induce a decrease in the rival's quantities produced : it is a "top-dog" strategy. In the investment example with differentiated products, principals commit to small rent differentials, which are intended to induce an increase in the rival's prices : it is "puppy-dog" strategy.

The point is however very dependent on the fact that renegotiations occur in the interim stage, i.e. when agents have learnt their own types. Suppose instead that renegotiations could also occur before agents learn their types, at a stage 1' say. It is clear that any publicly disclosed ex-ante contract would be secretly renegotiated away to an efficient contract, because stage 1 commitment does not forbid such an efficient renegotiation as the agent is always indifferent ex-ante.

Theorem 5 : If renegotiations can occur ex-ante, ex-ante initial contracts cannot have precommitment effects.

As our initial goal was not to contrive artifially players in their renegotiation opportunities, it is clear that the framework of Theorem 5 is more natural than the one of Theorem 3 and 4 : if renegotiations are free to occur, they might well occur ex-ante also.

VII. Concluding Remarks.

This paper has characterized situations where competing vertical (Principal-Agent) structures have the opportunity to publicly disclose their internal adverse selection contracts, and where such public disclosure has precommitment effects, i.e. affects the equilibrium of the game, even though each internal contract may be privately and secretly renegotiated before competitive actions are actually undertaken.

Let us point out a final but important remark on our results. We have restricted the analysis to the case where in stage 1 a principal can only sign with an agent a contract contingent on this agent's future actions and announcements. We have excluded contracts contingent on the opponent's action (directly or indirectly through some aggregate variable as an equilibrium price in a Cournot game) or even on the opponent's contract (i.e. cross-contingent contracts).

The issue of "cross-contingent" contracts has been addressed by Katz [1987] who emphasized the fact that one should impose a priori restrictions on the class of admissible contracts to avoid a self-reference problem, as well as some kind of Folk theorem. We shall not attempt here to model such interactions. For one reason, our framework provides very useful insights and the problem of modelling competition through contracts is complex enough to justify a step by step approach. Secondly, the fact that competitors' contracts (or actions) are observable does not mean that they are verifiable. It is likely that such contingencies be ruled out by legal authorities as anticompetitive (with or without reason, this deserves formal treatment). Such complex contracts are hardly observed in practice and therefore competition through "simple" contracts is of particular interest for applications. One would presumably want to have good reasons to justify that such simple contracts emerge. Apart from their simplicity, one bit of answer and may be our best justification is their robustness to the choice of contract space. Indeed, it is easy to show that the equilibria in simple contracts that we exhibit remain equilibria for any extension of the space of allowed contracts. That is, complicated contingencies are useful to be considered by one principal only if the other principal signs a contract with such contingencies : this clearly leads to equilibria in the same spirit as "bootstraps equilibria" in supergames. In a forthcoming paper, we adopt a more formal approach which generalizes our results in this direction.

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APPENDIX 1.

General Proof of Proposition 1

We solve a general problem of optimal contracting with two types and type-dependent reservation utilities. To make the link with the text, one should interpret:

$\phi(a,\theta)$ as $\int \phi^A(a,b,\theta_A)d\mu_B(b)$

$H(a)$ as $\Delta^A a$

Then one obtains the solution of the renegotiation problem of the text. In the appendix, we use a general formalism to show the robustness of our results (see the discussion in Appendix 2).

We make the following assumptions :

$H(.)$ is strictly increasing in a on S_A and C^2

$\phi(.,.)$ is C^2, strictly concave in a with interior maximum at $A^*(\theta)$.

$A^*(.)$ is decreasing in θ.

The program to be solved is :

$$\max_{(\bar{a},\underline{a},\bar{V},\underline{V})} \quad h\phi(\bar{a},\bar{\theta}) + (1-h)\phi(\underline{a},\underline{\theta}) - h\bar{V} - (1-h)\underline{V}$$

$$\text{s.t.} \quad \begin{array}{ll} \underline{V} - \bar{V} \geq H(\bar{a}) & [\lambda] \\ \underline{V} - \bar{V} \leq H(\underline{a}) & [\mu] \\ \bar{V} \geq \bar{V}^i & [\bar{\alpha}] \\ \underline{V} \geq \underline{V}^i & [\underline{\alpha}] \\ (\bar{a},\underline{a}) \in S_A^2 & \end{array}$$

For later discussion (Appendix 2) we denote :

$$\bar{A}^0 \equiv \underset{a \in S_A}{\text{Argmax}}\ h\phi(a,\bar{\theta}) - (1-h)H(a)$$

which is assumed to be a singleton. And $X = \underline{V}^i - \bar{V}^i$.

Note that the program is not concave because of the two incentive constraints, except if $H(.)$ is linear in a as in the text. First order conditions can be written :

$$\bar{\alpha} + \underline{\alpha} = 1 \tag{A1}$$

$$\lambda - \mu = \bar{\alpha} - h \tag{A2}$$

$$\bar{a} \in \underset{a \in S_A}{\mathrm{Argmax}^{loc}}\{h\ \phi(a,\bar{\theta}) - \lambda\ H(a)\} \tag{A3}$$

$$\underline{a} \in \underset{a \in S_A}{\mathrm{Argmax}^{loc}}\{(1-h)\phi(a,\underline{\theta}) + \mu\ H(a)\} \tag{A4}$$

where Argmax^{loc} denotes the set of local maxima of a function.

Suppose at the optimum $\lambda.\mu > 0$, then incentive constraints imply $\bar{a} = \underline{a}$ since H(.) is strictly monotonic. But given the assumptions, (A3) implies $\bar{a} < A^*(\bar{\theta})$ and (A4) implies $\underline{a} > A^*(\underline{\theta})$, therefore $\underline{a} > \bar{a}$: a contradiction. So, $\lambda\mu = 0$.

Consider now the different cases

1) $\bar{\alpha} = 1$, $\underline{\alpha} = 0$, $\lambda = 1-h$, $\mu = 0$.

This implies $\begin{cases} \underline{a} = A^*(\underline{\theta}),\ \bar{V} = \bar{V}^1,\ \underline{V} = \bar{V}^1 + H(\bar{a}) \\ \bar{a} \in \underset{a \in S_A}{\mathrm{Argmax}^{loc}}\{h\ \phi(a,\bar{\theta}) - (1-h)H(a)\} \equiv \bar{A} \end{cases}$

Note that $\bar{a} < A^*(\bar{\theta})$ necessarily. In the linear-quadratic framework, $\bar{A}$ is singleton, equal to $\bar{A}^0$. A necessary condition for this case is $X \le H(\bar{a})$.

2) $\bar{\alpha} = 0$, $\underline{\alpha} = 1$, $\lambda = 0$, $\mu = h$

This implies $\begin{cases} \bar{a} = A^*(\bar{\theta}),\ \underline{V} = \underline{V}^1,\ \bar{V} = \underline{V}^1 - H(\underline{a}) \\ \underline{a} \in \underset{a \in S_A}{\mathrm{Argmax}^{loc}}\{(1-h)\phi(a,\underline{\theta}) + h\ H(a)\} \equiv \underline{A} \end{cases}$

Note that $\underline{a} > A^*(\underline{\theta})$. In the linear-quadratic framework, $\underline{a}$ is uniquely determined since the function is concave. A necessary condition for this case is $X \ge H(\underline{a})$.

3) $\bar{\alpha} \in (h,1)$, $\underline{\alpha} \in (0,(1-h))$, $\lambda = \bar{\alpha}-h$, $\mu = 0$

This implies $\begin{cases} \underline{a} = A^*(\underline{\theta}),\ \bar{V} = \bar{V}^1,\ \underline{V} = \underline{V}^1 \\ \bar{a} = H^{-1}(X) \equiv \mathcal{A}(X) \end{cases}$

A necessary condition is $X < H(A^*(\bar{\theta}))$ and $h\ \phi_a(\mathcal{A}(x),\bar{\theta}) - (1-h)\ H'(\mathcal{A}(x)) < 0$.

4) $\bar{\alpha} \in (0,h)$, $\underline{\alpha} \in ((1-h),1)$, $\lambda = 0$, $\mu = \underline{\alpha} - (1-h)$

$$\text{This implies} \begin{cases} \bar{a} = A^*(\bar{\theta}),\ \bar{V} = \bar{V}^1,\ \underline{V} = \underline{V}^1 \\ \underline{a} = \mathcal{A}(X) \end{cases}$$

A necessary condition is $X > H(A^*(\underline{\theta}))$ and $(1-h)\ \phi_a(\mathcal{A}(x),\underline{\theta}) + h\ H'(\mathcal{A}(x)) > 0$.

5) $\bar{\alpha} = h$, $\underline{\alpha} = 1-h$, $\lambda = 0$, $\mu = 0$.

$$\text{This implies} \begin{cases} \bar{a} = A^*(\bar{\theta}),\ \underline{a} = A^*(\underline{\theta}) \\ \bar{V} = \bar{V}^1,\ \underline{V} = \underline{V}^1 \end{cases}$$

A necessary condition is $H(A^*(\bar{\theta})) \le X \le H(A^*(\underline{\theta}))$.

Let us turn now to sufficiency. First remark that :

$$\sup_{a \in \bar{A}} \underline{H}(a) < H(A^*(\bar{\theta})) \le H(A^*(\underline{\theta})) < \inf_{a \in \underline{A}} H(a)$$

Now suppose $X \in [H(A^*(\bar{\theta})),\ H(A^*(\underline{\theta}))]$ (if non empty), then the program without the incentive constraints is concave, and its solution is precisely given by 5), which satisfies the incentive constraints. So the global solution of the (constrained) program is given by 5) in that case.

If $X \in [\sup_{a \in \bar{A}} H(a),\ H(A^*(\bar{\theta}))]$, then the solution must satisfy the <u>necessary</u> first order conditions, and it can only be as given by 3).

Similarly if $X \in [H(A^*(\underline{\theta})),\ \inf_{a \in \underline{A}} H(a)]$, then the solution is given by 4).

If $X < \sup_{a \in \bar{A}} H(a)$, then the solution could be given either by 1) or by 3). Figure A1 illustrates the situation. In this figure, $\bar{A} = \{\bar{a}_1, \bar{a}_2\}$. As explained earlier, for $X \ge H(\bar{a}_1)$ the solution is given by 3). Now if $X \in (H(\bar{a}_2), H(\bar{a}_1))$, e.g. $X = X_1$ on the figure, the principal maximizes

$$\begin{cases} h\phi(\bar{a},\bar{\theta}) - (1-h)(\bar{a}) & \text{for } \bar{a} \ge \mathcal{A}(X) \\ h\phi(\bar{a},\bar{\theta}) - (1-h)X & \text{for } \bar{a} \le \mathcal{A}(X) \end{cases}$$

So the maximum is now for $\bar{a}_1$ and the solution is of type 1). However if $X = X_2$ in the figure, the solution is now for $\mathcal{A}(X_2)$ and of type 3).

Note finally that when $X < \sup_{a \in \bar{A}} H(a)$, the solution cannot be such that

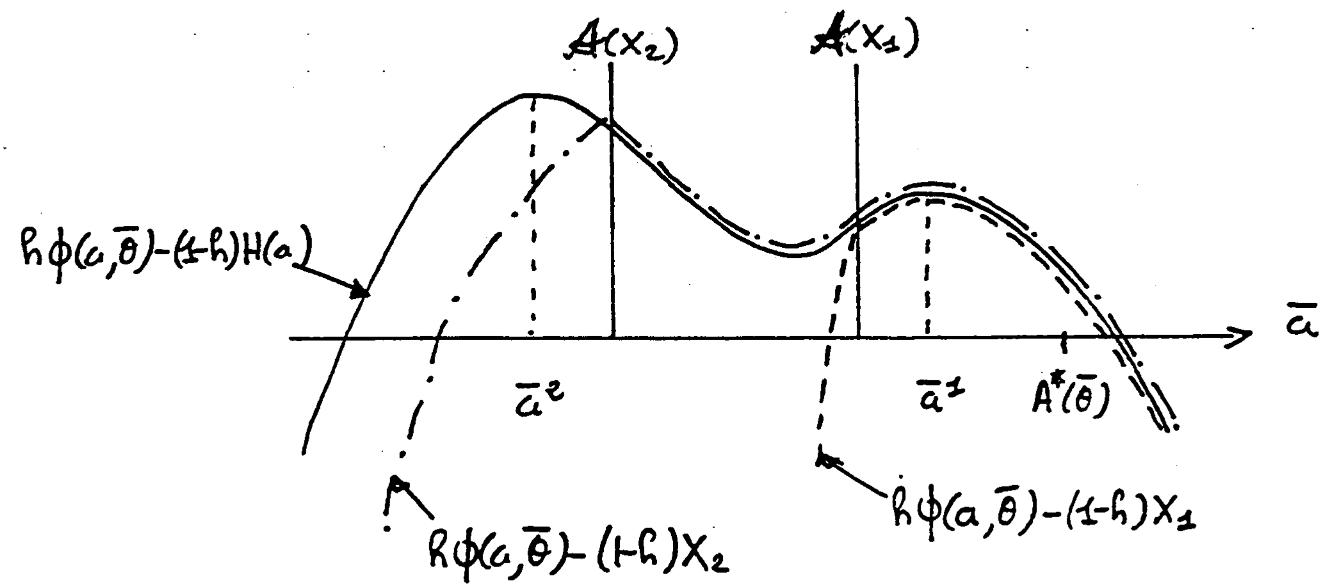
$A(X_2)$
$A(X_1)$
$h\phi(a,\bar{\theta})-(1-h)H(a)$
$\bar{a}$
$\bar{a}^2$
$\bar{a}^1$
$A^*(\bar{\theta})$
$h\phi(a,\bar{\theta})-(1-h)X_2$
$h\phi(a,\bar{\theta})-(1-h)X_1$

FIGURE 1

$\bar{a} < \bar{A}^0$; since $\bar{A}^0$ corresponds to the global maximum of $h\,\phi - (1-h)H$ and is always feasible for $X \le H(\bar{A}^0)$. So for extremely low values of X, $X \le H(\bar{A}^0)$, the solution is of type 1).

Similarly if $X < \inf_{a \in \underline{A}} H(a)$, the solution ight be of type 2) or 4).

To make the link with the text, note that because H(.) is linear in our model, the programs are all concave and $\bar{A}$ and $\underline{A}$ are singletons. The solution is then perfectly determined as in Table 1.

APPENDIX 2.

Extension of Theorem 1 and 2 : Informal discussion.

Consider the following assumptions :

$\phi^A(.,.,.) : S_A \times S_B \times \{\underline{\theta}_A, \bar{\theta}_A\} \longrightarrow \mathbb{R}$ is C^2, strictly concave in a with interior maximum at $A^*(b,\theta_A)$. $A^*(.,.)$ is decreasing in θ_A.

$\psi^A(.,.,.) : S_A \times S_B \times \{\underline{\theta}_A, \bar{\theta}_A\} \longrightarrow \mathbb{R}$ is C^2, increasing in θ_A.

$H^A(a,b) \equiv \psi^A(a,b,\bar{\theta}_A) - \psi^A(a,b,\underline{\theta}_A)$ is strictly increasing in a.

These assumptions are much weaker than in the text. θ is normalized as a cost parameter, and a single single property (H increasing in a) and the corresponding responsiveness condition (A^* decreasing in θ) are assumed to hold.

How, Appendix 1 applies and provides the solution to the renegotiation problem with incentive constraints and renegotiation constraints defined on indirect global utilities $U^A(\theta;\mu_B) = t^A(\theta) - \mathbb{E}\mu_B[\psi^A(a(\theta),b,\theta)]$:

$$H(\underline{a};\mu_B) \equiv \int H(\underline{a},b)d\mu_B(b) \ge \underline{U}(\mu_B) - \bar{U}(\mu_B) \ge H(\bar{a};\mu_B)$$

$$U^A(\theta;\mu_B) \ge U^{Ai}(\theta,\mu_B) \equiv t^{iA}(\theta) - \int \psi^A(a^i(\theta),b,\theta)d\mu_B(b)$$

where X now depends on μ_B.

Now Appendix 1 shows that the best response correspondence of P_A to $\mu_B \in \Delta(S_B)$ is a continuous correspondence $\mu_A(\mu_B, X_A(\mu_B))$. Indeed it is a continuous function except at points where, e.g. for $X_A(\mu_B)$, P_A is indifferent between $(\bar{A}(\mu_B), A^*(\mu_B,\underline{\theta}_A))$, and $(H^{-1}(X_A(\mu_B),\mu_B), A^*(\mu_B,\underline{\theta}_A))$. But at these points, any mixing between the two contracts will do, hence the continuity in terms of distribution of actions. Hence there exists a continuation equilibrium after any outcome of stage 1, but one looses unicity and the pure strategy property.

An equilibrium under unobserved contracts (i.e. with X = 0 here !) also exists. Now, suppose we make the following assumption : for any $\mu_B \in \Delta(S_B)$, $\bar{A}^0(\mu_B)$ is an <u>interior</u> singleton. That is to say, there exists one unique interior maximum of $\int [h\phi^A(a,b,\bar{\theta}_A) - (1-h)H^A(a,b)]d\mu_B(b)$. Then, mimicking the proof of Proposition 2, there exists pure strategy equilibrium under unobserved contracts, and no mixed strategy equilibria since $\bar{A}$ and $A^*(.,\underline{\theta})$ are continuous functions. Now there may exist several pure stragety equilibria, but we assume they are all isolated. Given an equilibrium under unobserved contract, it can be replicated by signing in stage 1 the equilibrium unobserved contracts and not renegotiating in stage 2. The question is now, is this play an equilibrium ?

Note first that one can start from the proposed play and make small enough perturbation in stage 1 contracts that induces a function $X_A(\mu_B)+\varepsilon(\mu_B)$ with $\varepsilon(\mu_B) > 0$ and $\varepsilon(.)$ small. Given a play that corresponds to an equilibrium with unobserved contracts, such a slight increase in initial rent induces a variation in the continuation equilibrium. <u>If the resulting continuation equilibrium is close to the initial one</u>, i.e. if a small deviation in stage 1 play does not induce a large difference in the selection of the continuation equilibrium, then it is easy to see (cf Figure A1) that P_A's induced actions are rent-constrained, i.e. given by $(H^{-1}(X_A(\mu_B),\mu_B),A^*(\mu_B,\underline{\theta}_A))$ and under A3-SS-NE or A3-SC-PE, theorem 1 applies. The continuity of the continuation equilibrium selection must be assumed, since it is related to the unmodelled coordination of principals on the same equilibrium. An alternative statement of Theorem 1 would be that the worst equilibrium under unobserved contracts for P_A cannot be an equilibrium of the global game.

Similarly Theorem 2 applies in a weaker form. Under A3-SS-PE or A3-SC-NE, any equilibrium under unobserved contract is also an equilibrium of the global game. Now the intuition is clearly the one provided in the text.