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A B S T R A C T

This paper investigates whether rational expectations are actually rational, i.e. whether they emerge as the outcome of an individual maximizing process. For that purpose we construct a two-stage game. In the first stage an expectations scheme is chosen for each agent through utility maximization. In the second stage agents maximize subject to the chosen expectations scheme and a Walrasian equilibrium obtains. The traditional rational expectations literature simply assumes that rational expectations are given to all agents in the first stage, whereas we extend the framework by making expectations also an object of choice.

If rational expectations are individually rational, they should be an equilibrium in the "expectations game". Surprisingly it is found that they are not, and that "rational expectations" are usually not individually rational.

Keywords : Rational Expectations, Rationality.

Journal of Economic Literature Classification Numbers : 021, 022, 026.

LES ANTICIPATIONS RATIONNELLES SONT-ELLES VRAIMENT RATIONNELLES ?

R E S U M E

Nous nous demandons dans cet article si les anticipations rationnelles sont vraiment rationnelles, c'est-à-dire si elles peuvent résulter d'un processus de maximisation par des agents décentralisés. On construit pour cela un jeu à deux étapes. A la première étape du jeu un schéma d'anticipations est choisi pour chaque agent à travers un processus de maximisation de son utilité. Dans la seconde les agents maximisent en prenant en compte les anticipations choisies et un équilibre walrasien s'établit. La littérature traditionnelle utilisant l'hypothèse d'anticipations rationnelles fait l'hypothèse que les anticipations rationnelles sont automatiquement données à tous les agents dans la première étape ; nous étendons le cadre d'analyse en faisant des anticipations elles-mêmes l'objet d'un choix rationnel.

Si les anticipations rationnelles sont individuellement rationnelles, elles devraient être un équilibre dans le "jeu en anticipations" décrit ci-dessus. De façon assez surprenante on trouve qu'elles ne le sont pas, et que les "anticipations rationnelles" ne sont en général pas individuellement rationnelles.

Mots clefs : Anticipations rationnelles, rationalité.

Codes J.E.L. : 021, 022, 026.

1. INTRODUCTION (*)

The idea of "rational expectations" is clearly one which has enjoyed a steady success in the economics profession in the last twenty years or so. Though the general idea is always the same, its exact definition may differ a little according to authors : In the seminal contribution by Muth (1961), the (deterministic) anticipated price is taken as equal to the (mathematical) expected value of this price. In his influential paper, Lucas (1972) assumes that agents know the full distribution of future prices conditional on currently available information.

Generally in most rational expectations models the agents are assumed to know the model as well as the model-maker himself, and to make the best prediction (in the probabilistic sense) of the relevant variables conditional on currently available information, so that for example in deterministic models rational expectations are customarily identified with perfect foresight.

Quite strangely for all these years there was little or no questioning of whether "rational expectations" so defined are rational or not, i.e. whether they derive from some kind of utility maximizing behavior [(1)]. Of course there is always a presumption in the back of everybody's mind that rational expectations are "utility maximizing", simply because, other things equal, an agent will reach a higher utility level if he correctly forecasts the variables relevant to him rather than if he incorrectly

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(1) The word "utility" is used here as a generic term for whichever criterion the agents maximize (this could be for example profits for firms). We prefer to refer explicitly to utility maximization only, as other criteria have less firm choice-theoretic foundations.

forecasts them. However such an "other things equal" reasoning is at best a partial equilibrium reasoning, whereas such a question must clearly be posed in a general equilibrium framework, notably in view of the massive use of "rational expectations" models in macroeconomics.

Our purpose in this article is thus to study in a general equilibrium framework the issue of the individual rationality of rational expectations. For that, instead of simply imposing the rational expectations scheme as is usually done, we shall take the "expectations scheme" to be also subject to utility maximization. The model to be studied will be implicitly a "two-stage" game. In the first part of the game an expectations scheme" is chosen for each agent among a number of schemes, including of course the rational expectations one. In the second part of the game, agents maximize utility subject to the expectations scheme chosen. Note that the second part is completely standard, applying not only to rational expectations, but to any other type of expectations as well. The novelty of the paper thus lies in the first part, the choice of the expectations scheme, and we shall see that, contrary to common wisdom, rational expectations may fail to be rational in a general equilibrium framework, i.e. the equilibrium of our game is not the one with "rational expectations".

Given the complexity of many rational expectations models, studying in addition the choice of expectations schemes might yield overly complex models escaping the intuition. So, in order not to cloud the issue with irrelevant technicalities, we will purposely construct the simplest possible model allowing to study this problem. The model will be deterministic, and each agent will have only one parameter to forecast, "rational expectations" corresponding to the true future value of this parameter. We shall see that this "rational expectations" value will usually not emerge as an equilibrium of the game.

2. THE MODEL

The model comprises two agents (1 and 2), two goods (a and b) and two periods. Variables in the second period will be denoted by a superscript prime ' (for the true values) or e for the expected values.

Agent 1 has endowments of good a only, denoted (ω_a, ω'_a) , and a utility function :

$$U_1(x_{1a},\ x_{1b}, x'_{1a}) \tag{1}$$

where x_{1a} is his current consumption of good a , x_{1b} his current consumption of good b , x'_{1a} his future consumption of good a . Symmetrically agent 2 has endowments of good b (ω_b, ω'_b) and a utility function denoted :

$$U_2(x_{2a},\ x_{2b}, x'_{2b}) \tag{2}$$

There is only a single market held in period 1, where agents 1 and 2 can exchange good a against good b at numéraire prices p_a and p_b . In the second period they simply consume their endowment, i.e. :

$$x'_{1a} = \omega'_a \qquad\qquad x'_{2b} = \omega'_b \tag{3}$$

Expectations schemes come in the picture in the following way : In the first period the agents must form expectations on what their future endowments will be, and they forecast respectively ω^e_a for ω'_a and ω^e_b for ω'_b . The supplies and demands of agents 1 and 2 will thus be conditional on these expectations. For example the program P_1 giving the demand and supply of agent 1 is :

$$\text{Maximize } U_1(x_{1a},\ x_{1b}, x'_{1a}) \qquad \text{s.t.}$$

$$\begin{cases} p_a\ x_{1a} + p_b\ x_{1b} = p_a\ \omega_a \\ x'_{1a} = \omega^e_a \end{cases} \tag{P_1}$$

which yields x_{1a} and x_{1b} as functions of ω_a , ω^e_a and p_b/p_a . Calling $s_{1a} = \omega_a - x_{1a}$ and $d_{1b} = x_{1b}$ we find :

$$s_{1a} = \phi_a(\omega_a, \omega^e_a, p_b/p_a) \tag{4}$$

$$d_{1b} = \frac{p_a}{p_b}\ \phi_a(\omega_a, \omega^e_a, p_b/p_a) \tag{5}$$

Symmetrically the optimal trades of agent 2 are given by the following maximization program P_2 :

$$\text{Maximize } U_2(x_{2b},\ x_{2a},x'_{2b}) \qquad \text{s.t.}$$

$$\left\{ \begin{array}{l} p_a\ x_{2a} + p_b\ x_{2b} = p_b\ \omega_b \\ \\ x'_{2b} = \omega^e_b \end{array} \right. \tag{P_2}$$

yielding

$$s_{2b} = \phi_b(\omega_b,\omega^e_b,p_b/p_a) \tag{6}$$

$$d_{2a} = \frac{p_b}{p_a}\ \phi_b(\omega_b,\omega^e_b,p_b/p_a) \tag{7}$$

The condition of equilibrium in the market of a against b is given by the two equivalent equations :

$$s_{1a} = d_{2a} \qquad \text{or} \qquad s_{2b} = d_{1b} \tag{8}$$

which yields :

$$p_a\ \phi_a(\omega_a,\omega^e_a,p_b/p_a) = \quad p_b\ \phi_b(\omega_b,\omega^e_b,p_b/p_a) \tag{9}$$

and the final allocations in the first period are :

$$x_{1a} = \omega_a - \phi_a(\omega_a,\omega^e_a,p_b/p_a) \tag{10}$$

$$x_{1b} = \frac{p_a}{p_b}\ \phi_a(\omega_a,\omega^e_a,p_b/p_a) \tag{11}$$

$$x_{2b} = \omega_b - \phi_b(\omega_b,\omega^e_b,p_b/p_a) \tag{12}$$

$$x_{2a} = \frac{p_b}{p_a}\ \phi_b(\omega_b,\omega^e_b,p_b/p_a) \tag{13}$$

where p_b/p_a is a function of ω_a , ω^e_a , ω_b and ω^e_b , as given implicitly by equation (9).

3. THE QUESTION AND BASIC RESULT

The question asked initially can now be rephrased in the terms of our model. "Expectations schemes" for agents 1 and 2 correspond simply to a set of forecasted values ω^e_a and ω^e_b for the future endowments. "Rational expectations" thus corresponds to :

$$\omega^e_a = \omega'_a \qquad\qquad \omega^e_b = \omega'_b$$

Rational expectations are individually rational if the pair (ω'_a, ω'_b) is a Nash equilibrium of the "expectations game" (2). To find out whether this is true we shall compute first order variations dU_1 and dU_2 letting ω^e_a and ω^e_b vary in the neighborhood of (ω'_a, ω'_b) . At this point the allocations actually attained satisfy the first order conditions corresponding to programs P_1 and P_2 , so that we have :

$$\frac{1}{p_a}\frac{\partial U_1}{\partial x_{1a}} = \frac{1}{p_b}\frac{\partial U_1}{\partial x_{1b}} = \lambda_1 \tag{14}$$

$$\frac{1}{p_a}\frac{\partial U_2}{\partial x_{2a}} = \frac{1}{p_b}\frac{\partial U_2}{\partial x_{2b}} = \lambda_2 \tag{15}$$

where λ_1 and λ_2 are the "marginal utilities of numéraire income" for agents 1 and 2 respectively. Small variations in agent 1's utility are thus computed as (remember x'_{1a}, which is equal to ω'_a , does not move) :

$$dU_1 = \lambda_1 (p_a\, dx_{1a} + p_b\, dx_{1b}) \tag{16}$$

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(2) Note that this expectations game should be thought of as being played not by the agents i = 1,2 themselves, but rather by some "guardian angels" i = 1,2. The "guardian angel" i has the same utility function as agent i but, contrarily to him, knows the value of the future endowments so that he can compute the exact outcome corresponding to each expectations scheme. This framework generalizes the usual "rational expectations" framework, which corresponds implicitly to the case where each guardian angel would give rational expectations to "his" agent.

Differentiating equations (10) and (11) we find :

$$dx_{1a} = - [\phi_{a2} \, d\omega_a^e + \phi_{a3} \, d(p_b/p_a)] \tag{17}$$

$$dx_{1b} = \frac{p_a}{p_b} [\phi_{a2} \, d\omega_a^e + \phi_{a3} \, d(p_b/p_a)] + \phi_a \, d(p_a/p_b) \tag{18}$$

where ϕ_{a2} is the partial derivative of ϕ_a with respect to its second argument, and similarly for ϕ_{a3} . Combining (16), (17) and (18) a number of terms cancel out and we obtain the simple expression :

$$dU_1 = \lambda_1 \, p_b \, \phi_a \, d(p_a/p_b) \tag{19}$$

Similarly we find for agent 2 :

$$dU_2 = \lambda_2 \, p_a \, \phi_b \, d(p_b/p_a) \tag{20}$$

There remains only to compute $d(p_b/p_a)$ as a function of the variations in ω_a^e and ω_b^e . For that we differentiate logarithmically equation (9) and find :

$$(1 + \varepsilon_{b3} - \varepsilon_{a3}) \, d\mathrm{Log}(p_b/p_a) = \varepsilon_{a2} \, d\mathrm{Log} \, \omega_a^e - \varepsilon_{b2} \, d\mathrm{Log} \, \omega_b^e \tag{21}$$

where ε_{a2} is the elasticity of ϕ_a with respect to its second argument, and so on. Plugging this into equations (19) and (20) we obtain :

$$dU_1 = \frac{\lambda_1 \, p_a \, \phi_a [\varepsilon_{b2} \, d\mathrm{Log} \, \omega_b^e - \varepsilon_{a2} \, d\mathrm{Log} \, \omega_a^e]}{1 + \varepsilon_{b3} - \varepsilon_{a3}} \tag{22}$$

$$dU_2 = \frac{\lambda_2 \, p_b \, \phi_b [\varepsilon_{a2} \, d\mathrm{Log} \, \omega_a^e - \varepsilon_{b2} \, d\mathrm{Log} \, \omega_b^e]}{1 + \varepsilon_{b3} - \varepsilon_{a3}} \tag{23}$$

If we assume that goods a and b are gross substitutes, then $\varepsilon_{a3} \le 0$ and $\varepsilon_{b3} \ge 0$, and the denominator is always greater than 1.

Formulas (22) and (23) show us that as soon as either ε_{a2} or ε_{b2} is different from zero, it will be utility improving for at least one of the agents to deviate from the "true value" and the rational expectations point (ω'_a, ω'_b) will not be an equilibrium in the "expectations game". Note that in this model nonzero ε_{a2} and ε_{b2} simply means that expectations actually enter the demand and supply functions (equations 4-7), and, as equations 9-13 show, this is a necessary condition for expectations to matter at all in the final outcome. So whenever expectations matter, rational expectations are not an equilibrium in our model.

4. AN EXAMPLE

Let us take the following simple utility functions :

$$U_1 = \alpha \text{ Log}(x_{1a} + x'_{1a}) + (1-\alpha) \text{ Log } x_{1b} \tag{24}$$

$$U_2 = \beta \text{ Log}(x_{2b} + x'_{2b}) + (1-\beta) \text{ Log } x_{2a} \tag{25}$$

So each good a or b is perfectly substitutable with the same good tomorrow, but the utility function is Cobb-Douglas with respect to a and b. Simple calculations first give us the equilibrium relative prices :

$$\frac{p_b}{p_a} = \frac{(1-\alpha)\ (\omega_a + \omega_a^e)}{(1-\beta)\ (\omega_b + \omega_b^e)} \tag{26}$$

and the final allocations :

$$x_{1a} + x'_{1a} = \alpha\omega_a + \omega'_a - (1-\alpha)\ \omega_a^e \tag{27}$$

$$x_{1b} = (1-\beta)\ (\omega_b + \omega_b^e) \tag{28}$$

$$x_{2b} + x'_{2b} = \beta\omega_b + \omega'_b - (1-\beta)\ \omega_b^e \tag{29}$$

$$x_{2a} = (1-\alpha)\ (\omega_a + \omega_a^e) \tag{30}$$

which, plugged into (24) and (25), yield the agents' utilities :

$$U_1 = \alpha \text{ Log}[\alpha\omega_a + \omega'_a - (1-\alpha)\ \omega^e_a] + (1-\alpha) \text{ Log}[(1-\beta)\ (\omega_b + \omega^e_b)] \qquad (31)$$

$$U_2 = \beta \text{ Log}[\beta\omega_b + \omega'_b - (1-\beta)\ \omega^e_b] + (1-\beta) \text{ Log}[(1-\alpha)\ (\omega_a + \omega^e_a)] \qquad (32)$$

We see immediately that the optimal strategies of agent 1 and 2 are respectively $\omega^e_a = 0$ and $\omega^e_b = 0$ irrespective of the other's strategy, so that rational expectations are *never* individually rational. The Nash equilibrium corresponds to $(\omega^e_a, \omega^e_b) = (0,0)$ which is completely different from the perfect foresight equilibrium $(\omega^e_a, \omega^e_b) = (\omega'_a, \omega'_b)$.

5. CONCLUSIONS

The investigation pursued in this paper has lead us to a result which may be surprising to a number of people : In a general equilibrium context, rational expectations are not individually rational in the usual sense of the word, that is individual utility maximization does not lead to rational expectations schemes.

The mechanism at work here, as particularly evident in equations (19) and (20), is that the losses incurred in making computations with wrong expectations are outweighed by the benefits due to the changes in the "terms of trade". For the simplicity of exposition we chose the forecasted variable to be each agent's future endowments. Our results thus display some similarities with those of Hurwicz (1972) or Postlewaite (1979) who, in a different, atemporal, framework showed that it could be individually rational to misrepresent one's true preferences or endowments. But clearly the mechanism we displayed in this paper would be also at work in more general settings where not only individual variables, but also market ones, such as future prices, would have to be forecasted. The formalization would be quite heavier, involving in particular many more markets, but it is to be expected that there too rational expectations would not be individually rational.

Of course one may conjecture that if agents are negligible, in the sense that their individual expectations have negligible effects on the terms of trade, then rational expectations should be individually rational, at least in the Walrasian context considered here. One must be aware, however, that this would be anyway only a special limit case, and that in the general case "rational expectations" do not seem to have an actual claim on rationality.

The consequences should be at least twofold. First, though "rational expectations" are obviously a very important benchmark case, economists should work more on other expectations schemes (and notably everything concerned with learning). Secondly the terminology "rational expectations" should be abandoned in favor of a perhaps less glamorous, but more accurate terminology than the misleading "rational expectations" one.[3]

(3) For example Lindbeck (1989) proposes the quite accurate terminology of "Model-consistent expectations".

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