FROM A HICKS-GRANDMONT TEMPORARY EQUILIBRIUM
TO A RATIONAL EXPECTATIONS
EQUILIBRIUM AND CONVERSELY*

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ABSTRACT

A condition is given under which a Hicks-Grandmont equilibrium is a rational expectations equilibrium. A generic converse holds: the condition is recovered when a rational expectations equilibrium is a Hicks-Grandmont equilibrium. Finally, the condition can be used to decompose the price-effect and scrutinize the "interior" of the REH. A "law of demand" supposes a further axiomatization of the Roy-consistency type.

D'UN EQUILIBRE TEMPORAIRE A LA HICKS-GRANDMONT A UN EQUILIBRE AVEC ANTICIPATIONS RATIONNELLES ET RECIPROQUEMENT

RESUME

Les fonctions d'anticipation peuvent se spécifier de manière qu'un équilibre temporaire à la Hicks-Grandmont soit un équilibre à anticipations rationnelles. La réciproque est générique: on retrouve cette spécification quand un équilibre avec anticipations rationnelles peut se représenter comme un équilibre de Hicks-Grandmont. La décomposition de l'effet-prix qui s'ensuit permet de caractériser la structure des anticipations rationnelles et de la compléter, éventuellement. La Roy-compatibilité forte conduit à une "loi de demande" plausible.

Noms-clés : Equilibre temporaire, fonctions d'anticipation, anticipations rationnelles, Roy-compatibilité rationnement quantitatif, effet de substitution intertemporel, Economie de distribution.

Key words : Temporary equilibrium, expectation function, rational expectation hypothesis, Roy-consistency, quantity rationing, intertemporal substitution effect, distribution economy.

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INTRODUCTION

In their presentation of a temporary equilibrium, Hicks (1946) and Grandmont (1983) used expectation functions in order to close their model on the present. This was mainly interpreted as an assumption of bounded rationality and seen as a device hardly reconcilable with the Rational Expectations Hypothesis (REH). (See for instance, Radner (1982) and Gale (1985)). Even if not every economist accepts such an opinion, it is clear, at least to our knowledge, that the machinery allowing a complete reconcilement is not built. The first purpose of our paper is to elaborate such a machinery.

Suppose we succeed in doing so. Then, this machinery can be used to characterize the REH itself in terms of intratemporal substitution effects, intertemporal substitution effects and real balance effects. Its links with respect to the Sonnenschein-Mantel-Debreu arbitrariness (the problem of the "law of demand") can then considered. Such is our second -and last- purpose.

The paper falls into four sections. In the first one, a HG-equilibrium (that is, a temporary equilibrium à la Hicks-Grandmont) and a RE-equilibrium (a temporary equilibrium with Rational Expectations) are presented and discussed in the case of a distribution economy.

Their reconcilement is the object of the second section. A condition is given under which a HG-equilibrium is a RE-equilibrium. A somewhat converse theorem holds: from a generic viewpoint, a RE-equilibrium is a HG-equilibrium displaying the previous given condition.

In the third section, we study the implications of this reconcilement. After decomposing the price effect into a pure substitution effect, a current wealth effect (or real balance effect) and an expected wealth effect, it is remarked that this decomposition a)
follows a logical structure equivalent to the one found in a Mixed Model (which is a special case of a Quantity Rationing Model and will be discussed below); b) perfectly transforms intertemporal substitution effects into intratemporal substitution effects; c) amplifies the Sonnenschein-Mantel-Debreu arbitrariness, at least a priori; d) can be used to reduce this arbitrariness with the help of a mixed axiomatisation of the Roy-consistency type; e) induces a temporary demand system involving no more observability problems than the standard one in spite of its more complex structure.

Proof are gathered in section 4.

1. THE MODELS

1.1. Basic concepts

The preferences of a consumer $i$, $i=1,2,...,m$, are defined on an open and convex subset of the commodity space and representable by a utility function $u_i$ of class $C^k$, $k \geq 2$, on this subset. $u_i$ is strongly monotonic and strongly quasi-concave. $(x_{0i}, x_{1i})$ is a point in the commodity space representing the consumption plan of consumer $i$. $x_{0i}$ has $n_0$ components, $n_0 \geq 1$, and $x_{1i}$ has $n_1$ components, $n_1 \geq n_0$. The subscripts 0 and 1 refer to current and future consumptions respectively.

The problem of consumer $i$ is to maximize $u_i(x_{0i}, x_{1i})$ subject to

$$P_0 x_{0i} + \gamma A_1 = R_{0i} \quad (1)$$

$$p_1^e x_{1i} - A_1 = R_1^e \quad (2)$$

In these relations, $P_0$ and $\gamma$ are strictly positive and represent respectively the vector of current prices and the discount factor. $\gamma A_1$ is a bank deposit and $A_1$ is the balance the consumer will have in the beginning of the next period. This balance is negative for a borrower. $R_{0i}$ is the consumer's current wealth, that is, his current income plus the balance of his previous financial operations. $p_1^e$ is the expected vector of prices and $R_1^e$ the expected wealth. The future budget
constraint can be interpreted as one representing trade over several future periods. \( p_1^e \) and \( R_1^e \) are then discounted back to the first of these future periods. Bankruptcy is excluded.

This problem leads to the existence of demand functions \( \xi_{01} \), \( \xi_{11} \) such that

\[
x_{01} = \xi_{01}(p_0, \gamma p_1^e, R_{01} + \gamma R_1^e)
\]

\[
A_1 = p_1^e \xi_{11}(p_0, \gamma p_1^e, R_{01} + \gamma R_1^e) - R_1^e
\]

\[
x_{11} = \xi_{11}(p_0, \gamma p_1^e, R_{01} + \gamma R_1^e)
\]

Relations (3) and (4) represent a temporary complete demand system. Relation (5) represents expected or planned demands.

Total demands are defined by \( x_0 = \sum \xi_{01}(.) \), \( A_1 = \sum A_1 \) and \( x_i = \sum \xi_{11}(.) \).

1.2. HG-equilibrium and RE-equilibrium

Let \( \omega_0 \) be a vector of initial current endowments. In a distribution economy, that is, an economy where the \( R_{01} \)'s are exogenous, the conditions of a temporary equilibrium are written

\[
\sum \xi_{01}(.) = \omega_0
\]

\[
\sum A_1 = 0
\]

In a distribution economy, there is no linear combination among market-clearing conditions; consequently, in principle, they determine \( p_0 \) and \( \gamma \) in the usual way when expectations are exogenous. But, in general, expectations are not exogenous; so we have to represent their determination. To do so, two devices are currently used. The first one is the introduction of expectation functions representing learning processes. It can be attributed to Hicks and Grandmont (HG). Let \( (\psi_i, \rho_i) \) be some functions such that
\[ p^e_i = \psi_i(p_0, \gamma, R_0, \sigma) \]  
(8)

\[ R^e_i = \rho_i(p_0, \gamma, R_0, \sigma) \]  
(9)

where \( R_0 = (R_{01} \ldots R_{0m}) \) and where \( \sigma \) represents other signals. We shall say that when (8) and (9) complete (6) and (7), this defines the conditions of a HG-equilibrium.

The second one is the introduction of "perceived" future markets. This is the Rational Expectation Hypothesis (REH). It is assumed that agents have a mental representation (perhaps after concertation) of what markets will be in the future and can solve them. Let \( \omega_1 \) be a vector of perceived future endowments. The conditions of an expected equilibrium are written

\[ \Sigma \xi_{i1}(.) = \omega_1 \]  
(10)

Relation (10) represents a model of price formation. Prices are thus the object of rational expectations (RE). Wealth expectations can be represented as in the HG-equilibrium:

\[ R^e_i = \rho_i(p_0, \gamma, R_0, \sigma) \]  
(11)

We shall say that when (10) and (11) complete (6) and (7) this defines the conditions of a RE-equilibrium.

1.3. The reconcilement problem

The HG-equilibrium model and the RE-equilibrium model have been seen as a new dichotomy. For Gale (1985) the HG-equilibrium model is irrational and assumed to be irreconcilable with the RE-equilibrium model. For Radner (1982), it is designed to represent "bounded rationality" and is presented as a particular specification of the temporary equilibrium.
2. SOLVING THE RECONCILEMENT PROBLEM

Roughly speaking, we shall prove that there exists a set of conditions under which a HG-equilibrium is a RE-equilibrium and vice versa. We first assume that information is symmetric among agents and that the price expectation function is the same for each agent, that function is, $\Psi_i = \Psi$, $i = 1, 2, \ldots, m$. Consequently, the expectation functions in the economy can be represented by the function $(\Psi, (\rho_1))$. Of course $p_e^i = p_e$ for $i = 1, 2, \ldots, m$ and we can write

$$p_e = \Psi(p_0, \gamma, R_0, \sigma)$$

$$R_e = \rho(p_0, \gamma, R_0, \sigma)$$

where $\rho = (\rho_1, \ldots, \rho_m) = (\rho_1)$.

Considering $\sigma$ as a parameter and given that $(\Psi, \rho)$ is of class $C^{k-1}$, the Jacobian matrix of these relations is written

$$M = \begin{pmatrix}
\frac{\partial \Psi}{\partial p_0} & \frac{\partial \Psi}{\partial \gamma} & \frac{\partial \Psi}{\partial R_0} \\
\frac{\partial \rho}{\partial p_0} & \frac{\partial \rho}{\partial \gamma} & \frac{\partial \rho}{\partial R_0}
\end{pmatrix}$$

(14)

Now, let us consider the total planned demand function. Its Jacobian matrix is written

$$\left[ \begin{array}{c}
J_{10} : J_{11}
\end{array} \right] = \left[ \begin{array}{ccc}
\frac{\partial \xi_1}{\partial p_0} & \frac{\partial \xi_1}{\partial p_1} & p_e + \sum_i \frac{\partial \xi_1}{\partial w_i} R_e \\
\frac{\partial \xi_1}{\partial \gamma} & \frac{\partial \xi_1}{\partial p_1} & \frac{\partial \xi_1}{\partial \gamma} \frac{\partial \xi_1}{\partial \gamma}
\end{array} \right]$$

(15)

where $\xi_1 = \sum_{i=1}^{n} \xi_{1i}$, $p_1 = \gamma \ p_e$ and $W = R_0 + \gamma \ R_e$ and where the partition separates the effects of current variables from the effects of future ones.
Proposition 1:

If $J_{11} M = - J_{10}$ for any $(p_0, \gamma, R_0) \in V_0$ open and connected, a
HG-equilibrium is a RE-equilibrium. The induced $\Sigma \xi_{11}(.) = \omega_1$ has a
unique solution $p^e$ for any given $(p_0, \gamma, R_0)$.

We are now looking for some converse to Proposition 1. Two lemmas
will be useful.

Lemma 1:

The matrix $J_{11}$ has rank $n_1$ (the perceived future equilibrium is
regular).

Lemma 2:

Except for a subset of measure zero of the future wealth
distributions, the matrix has rank $n_1$ if $\Sigma \xi_{11}$ is sufficiently
differentiable (that is for some $k$).

Remark also that the second part of Proposition 1 is congenial to
the REH (if $p^e$ is not unique, agents may choose different ones,
contradicting the assumption of a future equilibrium relative to a price
system). Consequently, it is natural to start our second proposition
with such an assumption.

Proposition 2:

If $\Sigma \xi_{11}(.) = \omega_1$ is sufficiently differentiable and has a unique
solution $p^e$ for any given $(p_0, \gamma, R_0) \in V_0$, a RE-equilibrium is a
HG-equilibrium. The induced $\psi$ is such that $J_{11} M = - J_{10}$ almost
everywhere on $V_0$. 6
Remark 1

From Proposition 1, $\Psi$ implies a unique $w_1$ if $\rho$ is given. From Proposition 2, $w_1$ implies a unique $\Psi$ if $\rho$ is given. This bijection between $\Psi$ and $w_1$ defines a bijection between HG-equilibria (having the property $J_{11} M = -J_{10}$) and RE-equilibria (having $\Sigma \xi_{11}$ sufficiently differentiable). The reconcilement is generic and point to point.

3. THE IMPLICATIONS

At this point, the remarkable fact is that we are now in a position to scrutinize the REH via a "rational expectations function", that is, any function $(\Psi, \rho)$ displaying the property $J_{11} M = -J_{10}$.

Let $K = \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix}$ be the sum of the intertemporal substitution matrices and let us set

$$S_{00} = K_{00} - K_{01} K_{11}^{-1} K_{10} \text{ and } \beta_{01} = \frac{\partial \xi_0}{\partial w_1} - K_{01} K_{11}^{-1} \frac{\partial \xi_1}{\partial w_1}.$$ 

Proposition 3:

If $J_{11} M = -J_{10}$, the current total price effects admit the following decomposition:

a) $\frac{\partial x_0}{\partial p_0} = S_{00} - \Sigma_{i} \beta_{01} x_{0i} + \gamma \Sigma_{i} \beta_{01} \left[ \frac{\partial \rho_1}{\partial p_0} - x_{1i} \frac{\partial \Psi}{\partial p_0} \right]$

b) $\frac{\partial x_0}{\partial \gamma} = [0] - \Sigma_{i} \beta_{01} A_i + \gamma \Sigma_{i} \beta_{01} \left[ \frac{\partial \rho_1}{\partial \gamma} - x_{1i} \frac{\partial \Psi}{\partial \gamma} \right]$

where $S_{00} = S_0$, $S_{00} p_0 = 0$, $\xi_0 S_{00} \xi_0 < 0$ for any $\xi_0 \neq \theta p_0$, $\theta \in \mathbb{R}$, $p_0$ $\beta_{0i} = 1$ for any $i$.  

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Thus, a current price effect decomposes into a pure substitution effect, a current wealth (or real balance) effect and an expected real wealth effect. The interest of this decomposition lies in the following remarks.

Remark 2:

From Proposition 3, current demands display the structure of a Quantity Rationing model. In equation a), $S_{00}$ and the $\beta_{01}'s$ are the constrained substitution effects and the constrained income effects of a complete demand system under Quantity Rationing (or, more sharply, of a Mixed Model, that is, a demand model in which some prices are endogenous, the corresponding quantities being exogenous). This is not really surprising as the perception of a given $w$ and future equilibrium is kind of a rationing (défines an exogenous $x_1^e$).

Remark 3:

In equation b), the intertemporal substitution effects associated to a variation in the discount factor vanish. They are "killed" by their perfect integration into the present via $S_{00}$ which includes the spill-over effects $K_{01}K_{11}^{-1}$.

Remark 4:

A priori, there is no reason for a temporary equilibrium with Rational Expectations to be more immune from arbitrariness than the standard atemporal general equilibrium. In other words, the price effects are, in principle, arbitrary according to the Sonnenschein-Mantel-Debreu theorem. In fact, in Proposition 3, the expressions

$$\frac{\partial p_1}{\partial p_0} - x_1 \frac{\partial \psi}{\partial p_0}$$

$$\frac{\partial p_1}{\partial \gamma} - x_1 \frac{\partial \psi}{\partial \gamma}$$
can be interpreted as additional arbitrary factors and can be thought of as adding to the initial arbitrariness. This complication can remain even if a model of income formation is added to the model of price formation and is embodied in the definition of the REH. For example, it can be argued that, in the case of a distribution economy, the REH implies that $\rho$ is a constant mapping. As suggested by Proposition 3 and as it could be carefully checked, the price effects remain arbitrary under such an assumption.

Remark 5:

However, expectation functions represent the potentialities of a new structure and can be used to go further. We shall illustrate this point by considering the problem of the "law of demand".

Let us consider the relation a). It is clear that the diagonal of $\frac{\partial x_0}{\partial p_0}$ will be negative (a necessary condition for the matrix to be negative quasi-definite) if

$$\sum_1^b \beta_{01} \left[ x'_{01} - \gamma \left( \frac{\partial p_1}{\partial p_0} - x'_{11} \frac{\partial \psi}{\partial p_0} \right) \right] \geq 0.$$  

Because $p_0$ is positive and taking into account the relation $p_0 \beta_{01} = 1$, the previous inequality implies

$$\sum_1^b x'_{01} \geq \gamma \sum_1^b \left[ \frac{\partial p_1}{\partial p_0} - x'_{11} \frac{\partial \psi}{\partial p_0} \right].$$  

A sufficient condition for this inequality is, of course, its validity for each agent:

$$x'_{01} \geq \gamma \left[ \frac{\partial p_1}{\partial p_0} - x'_{11} \frac{\partial \psi}{\partial p_0} \right] \quad i = 1, 2, \ldots, m.$$  

Such individual inequalities are natural: they just imply that the signs of the indirect utility functions are preserved when $x'_{01} \geq 0$ (a
buyer dislikes any increase in his cost of living). In fact, the previous inequalities define an assumption of weak Roy-consistency. Thus an assumption on individual preferences and expectations is consistent with a "macroeconomic result" (namely the negativity of the demand slope).

Conversely our individual inequalities can be used to imply the result. Indeed, suppose first they are satisfied as equalities or nearly equalities. Then, it is clear from Proposition 3 that the price effects can be signed. Such is the case near a compensated or quasi compensated equilibrium. Second, suppose the inequalities are fulfilled because expectations are strongly Roy-consistent. Then the sum of the income effects reduces to \( \Sigma \alpha_i \beta_{01} x_{0i}' \) where \( \alpha_i \) is, for each consumer \( i \), the variation of expected real wealth consecutive to a variation of current income and is positive. In practice, \( \alpha_i \) is large for a young and poor agent while, in the same time and for the same person, \( \beta_{01} \) is strictly positive. \( \alpha_i \) is near zero for a well-off person, due in particular, to taxation. In short, positive income effects are positively weighted and negative income effects are quasi eliminated. This sketch is sufficient to yield a temporary "law of demand" and is similar to the intuition behind Hildenbrand's results (1983) except that our \( \alpha_i \) combine with Hildenbrand's density function to yield a more empirical content (a more precise formulation of this last point supposes the introduction of a continuum of consumers. The negative quasi definiteness of the matrix \( \frac{\partial x_0}{\partial p_0} \) can then be proved).

Remark 6:

From Proposition 3, the differential of the current total demands can be written

\[
dx_0 = S_{00} \, dp_0 + \Sigma \beta_{01} \frac{du_i}{\lambda_i}
\]

where \( \lambda_i \) is the marginal utility of wealth. In this relation, \( S_{00} \) is intertemporally compensated. Consequently this form is not generally
observable. But, even if \( \frac{du_1}{\lambda_1} \) contains future variables, observability can be recovered as usual. Indeed, suppose for instance that the \( \beta_{01} \)'s are identical. Then, the additivity properties of Proposition 3 imply
\[
\sum_{1} \frac{du_1}{\lambda_1} = p_0 \, d \, x_0
\]
and, consequently, the observability of the differential form (16).

4. PROOFS

4.1. In order to prove Proposition 1, one considers the expected demands
\[
x_1 = \sum_{1} \xi_{11} (. )
\]
under the assumption \( J_{11} M = - J_{10} \). This assumption implies \( dx_1 = 0 \) everywhere on \( V_0 \) which is a connected set. Consequently, \( x_1 \) is equal to a vector of constants. This constant vector is not arbitrary since consistent with a given value \( (p_0, \gamma) \) of the temporary equilibrium. We denote it \( \omega_1. \sum_{1} \xi_{11} (.) = \omega_1 \) has a unique solution since \( p^e \, \sum_{1} \xi_{11} \) is defined on \( V_0 \times \Psi(V_0) \times \rho(V_0) \).

The simplicity of this proof should not obscure the fact that each consumer is endowed with the capacity of reconstructing future markets.

4.2. In order to prove Lemma 1, we start from the intertemporal Slutsky matrix \( K = \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix} \). The assumptions on preferences and the
convention $n_1 \geq n_0 \geq 1$ imply that $K_{11}$ has rank $n_1$. $K_{11}$ is the product of $J_{11}$ with the matrix $[I_{n1}]$ and a matrix cannot exceed the rank of its factors. Then, $J_{11}$ has rank $n_1$.

4.3. Lemma 2 is a consequence of Lemma 1. By transversality, the rank of $J_{11}$ is the rank of $\frac{\partial \xi_1}{\partial p_1}$ (see Mas-Colell (1985), Proposition 8.3.1). This may suppose some small perturbations in the $\sigma$'s and consequently is true for almost every future wealth distribution (if $p$ is a constant mapping in $p_0$, $\gamma$, $R_0$, we can set $R^e = \sigma$).

4.4. Proposition 2 follows from Lemma 2. The implicit function theorem can be applied almost everywhere and its local functions are pieced together by the unicity assumption on $p^e$.

4.5. In order to prove Proposition 3, we first solve the equations $J_{11} M = -J_{10}$. From Lemma 2, $J_{11}$ generically admits as a right inverse, the matrix $\left[ \frac{1}{2} \left( \frac{\partial \xi_1}{\partial p_1} \right)^{-1} \right]$. Consequently, one has

\[ \left[ \begin{array}{c} 1 \\ \frac{\partial \xi_1}{\partial p_1} \\ 0 \end{array} \right] \]
\[ \frac{\partial \psi}{\partial p_0} = \frac{1}{\gamma} \left( \frac{\partial \xi_1}{\partial p} \right)^{-1} \frac{\partial \xi_1}{\partial p_0} \]

\[ \frac{\partial \psi}{\partial \gamma} = \frac{1}{\gamma} \left( \frac{\partial \xi_1}{\partial p} \right)^{-1} \left[ \frac{\partial \xi_1}{\partial p} p e + \sum \frac{\partial \xi_1}{\partial \omega_1} R_1 \right]. \]

Since
\[ \frac{\partial \xi_1}{\partial p_1} = K_{11} - \sum \frac{\partial \xi_1}{\partial \omega_1} x_{1i} \]
\[ \frac{\partial \xi_1}{\partial p_0} = K_{10} - \sum \frac{\partial \xi_1}{\partial \omega_1} x_{0i} \]

the previous relations can be written
\[ \frac{\partial \psi}{\partial p_0} = \frac{1}{\gamma} K_{11}^{-1} \left\{ K_{10} - \sum \frac{\partial \xi_1}{\partial \omega_1} \left[ x_{0i} - \gamma \frac{\partial \xi_1}{\partial p_0} - x_{1i} \frac{\partial \psi}{\partial p_0} \right] \right\} \]
\[ \frac{\partial \psi}{\partial \gamma} = \frac{1}{\gamma} K_{11}^{-1} \left\{ K_{11} p e - \sum \frac{\partial \xi_1}{\partial \omega_1} \left[ A' - \gamma \frac{\partial \xi_1}{\partial \gamma} - x_{1i} \frac{\partial \psi}{\partial \gamma} \right] \right\} \]

(This transformation amounts to using \( \left[ I_{n1} \right] \frac{1}{\gamma} K_{11}^{-1} \) as a right inverse of \( J_{11} \)).

Now, let us consider the current demand functions for commodities \( x_0 = \sum \xi_{0i} p_i \), \( \gamma \psi (.), R_{01} + \gamma \rho_1(.) \).
a) Differentiating with respect to $p_0$, one has

\[
\frac{\partial x_0}{\partial p_0} = \frac{\partial \xi_0}{\partial p_0} + \frac{\partial \xi_0}{\partial p_1} \frac{\partial \psi}{\partial p_0} + \sum_1 \frac{\partial \xi_0}{\partial w_1} \frac{\partial \psi}{\partial p_0}
\]

and

\[
\frac{\partial x_0}{\partial p_0} = K_{00} + \gamma K_{01} \frac{\partial \psi}{\partial p_0} - \sum_1 \frac{\partial \xi_0}{\partial w_1} \left[ x_0' - \gamma \left( \frac{\partial \rho_1}{\partial p_0} - x_1' \frac{\partial \psi}{\partial p_0} \right) \right].
\]

Using $J_{11} \, M = -J_{10}$, this can be written

\[
\frac{\partial x_0}{\partial p_0} = K_{00} - K_{01} K_{11}^{-1} K_{10} - \sum_1 \beta_{01} \left[ x_0' - \gamma \left( \frac{\partial \rho_1}{\partial p_0} - x_1' \frac{\partial \psi}{\partial p_0} \right) \right],
\]

where $\beta_{01} = \frac{\partial \xi_0}{\partial w_1} - K_{01} K_{11}^{-1} \frac{\partial \xi_1}{\partial w_1}$.

b) Differentiating with respect to $\gamma$, one has:

\[
\frac{\partial x_0}{\partial \gamma} = \frac{\partial \xi_0}{\partial p_1} p + \sum_1 \frac{\partial \xi_0}{\partial w_1} R_{1} e + \frac{\partial \xi_0}{\partial p_1} \frac{\partial \psi}{\partial \gamma} + \sum_1 \frac{\partial \xi_0}{\partial w_1} \frac{\partial \psi}{\partial \gamma}
\]

\[
= K_{01} \left( p + \gamma \frac{\partial \psi}{\partial \gamma} \right) - \sum_1 \frac{\partial \xi_0}{\partial w_1} \left[ A_{1} - \gamma \left( \frac{\partial \rho_1}{\partial \gamma} - x_1' \frac{\partial \psi}{\partial \gamma} \right) \right].
\]

Using $J_{11} \, M = -J_{10}^t$, this can be written

\[
\frac{\partial x_0}{\partial \gamma} = [0] - \sum_1 \beta_{01} \left[ A_{1} - \gamma \left( \frac{\partial \rho_1}{\partial \gamma} - x_1' \frac{\partial \psi}{\partial \gamma} \right) \right].
\]

The properties of $S_{00}$ and $\beta_{01}$ stem from the properties of $K$, $\frac{\partial \xi_0}{\partial w_1}$ and $\frac{\partial \xi_1}{\partial w_1}$.  

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NOTES

1. The concept of a distribution economy was first studied by Malinvaud ((1969), English translation (1972)). It was applied to a temporary equilibrium by Malinvaud ((1982), English translation (1985)). This is done in harmony with his initial works on the intertemporal equilibrium (Malinvaud (1953)). The atemporal version is currently used in the discussion of the "law of demand" (Hildenbrand (1983), Freixas and Mas-Colell (1987), Grandmont (1987)).

2. As only point expectations are considered here, we are assuming correspondingly that future total endowments are forecasted by each agent with a subjective probability equal to one. This convention is sufficient to establish a sharp distinction between perfect foresight and rational expectations for two reasons. First, historically, the perfect foresight hypothesis was an hypothesis of exogenous expectations. Here, the agents will have to solve equation (10). This rationalization of the perfect foresight hypothesis is already the REH even if currently identified with the perfect foresight hypothesis itself. Second, and more basically, we are not assuming that $\omega_1$ corresponds to the true future state of the world and this makes our model consistent with an Arrow-Radner equilibrium (in the terminology of Laffont (1985)). Indeed, suppose an Arrow-Radner model where each agent agrees, with a subjective probability equal to one, that a given state of the world will occur. Such a model reduces to ours. On the other hand a HG-equilibrium can be extended to cover the case of an Arrow-Radner equilibrium if we introduce a complete set of financial markets and expectation functions matching with every state of the world. This complication would not change the nature of the reconcilement problem.

3. This point will be discussed farther. At this moment, it can be said that this convention is general enough to cover the case where the $R^e_1$'s are exogenous and the case where a model of income formation is used. After seeing that the reconcilement is "parameterized" on the $\rho_1$'s we shall come back to that point. The corresponding difficulty in a private ownership economy would be the expectations of the $\omega_{11}$'s.

5. For the importance of this point see Grandmont (1983).

6. For an introduction to this literature, see Debreu (1986) and Schafer and Sonnenschein (1982).

7. This problem is discussed in an atemporal framework by Hildenbrand (1983), Freixas and Mas-Colell (1987) and Grandmont (1987) among others.

8. The weak Roy-consistency hypothesis is presented with strict inequalities in Allard, Bronsard and Richelle (1990). It is complemented by a study of the "boundary" in Bronsard and Salvas-Bronsard (1988). The strong Roy-consistency hypothesis is presented in Allard, Bronsard and Richelle (1990) and amounts to assume that temporary Roy identities do exist.
REFERENCES


Malinvaud, E. (1972) - Lecture on Microeconomic Theory, Amsterdam: North Holland (2nd édition (1985)).


