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A VALUATION FORMULA FOR LDC DEBT

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ABSTRACT

A VALUATION FORMULA FOR LDC DEBT

The paper gives a valuation formula for LDC debt which is used for assessing: 1) What is the price at which a buy-back of the debt is advantageous to the country (we shall see that it is likely to be half the observed market price); 2) What is the value to the creditors of having the flows of payment guaranteed against the extrinsic stochastic disturbance faced by the country (we shall see that it may not exceed 25%); 3) What is the trade-off between growth of payments and levels of payments (we show that a 1% additionnal growth rate is worth a 15% increase of the flows).

RESUME

UNE EVALUATION THEORIQUE DE LA VALEUR DE LA DETTE DES PAYS EN VOIE DE DEVELOPPEMENT

Ce papier propose une évaluation théorique de la valeur de la dette des pays en voie de développement. Nous montrons que, sous certaines hypothèses raisonnables: 1) le prix auquel un rachat de la dette est avantageux pourrait ne représenter que la moitié du prix qui est observé sur les marchés; 2) une garantie offerte aux créditeurs représenterait 25% de la valeur faciale de la dette; une augmentation de 1% de la croissance des paiements est équivalente à une augmentation de 15% du niveau des paiements réalisés par le marché.

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I - INTRODUCTION

The Brady plan has triggered a number of proposals aimed at reducing the face value of LDC debt. As now well understood in the literature on this topic (see e.g Bulow and Rogoff (1988), Dooley (1988)), there is a gap which may be very large between the debt relief which is nominally granted to a debtor and the relief which is actually given up by the creditors (when measured in terms of net transfers which the debtor is expected to pay). This paper aims at giving an exact valuation formula for LDC debt which will help, it is hoped, to assess empirically this question. In particular, we shall assess: 1) What is the price at which a buy-back of the debt is advantageous to the country (we shall see that it is likely to be less than half the observed market price) 2) what is the value (to the creditors) of having the flows of payment guaranteed against the extrinsic stochastic disturbances which plague the debtor's economy. (we shall see that it may not exceed 25%); 3) what is the trade-off between growth of payments and current payments that the creditors are willing to consider (we shall see that an additional 1% growth rate may be traded against a 15% reduction of the levels of payments).

The model presented here is essentially identical to the one in Genotte, Kharas and Sadeq (1986). While they offer a simulation study, this paper relies on a closed form solution which will allow to calculate directly the marginal value of the debt.

Perhaps the best way to motivate this paper will come from the following observation. In 1987, the ratio of net transfers paid by the Highly Indebted Countries (HICs) to their private creditors (such as calculated in the World Debt Tables) on their outstanding long term debt (due to private creditors) was approximately equal to 5 %. In theory, non-withstanding risk, this ratio should be equal to the difference between the interest rate on the debt and the growth rate of the country's transfers, presumably equal to the country's growth rate. From that perspective, 5% seems about right. Yet, an average discount of about 50 cents on the dollar was observed in December 1987. In this paper, we will investigate how far risk can go in explaining this discrepancy and relate to it various debt relief proposal.

Section II gives the valuation formula and gives the discount associated to various parameters of the country. Section III shows the difference between average and marginal prices and estimate the value for the country to making a "take it or leave it" offer to its creditors; section IV gives an estimate of the cost of guaranteeing the debt. A conclusion gives some perspectives on the Mexican deal which has been negotiated in July 1989. On one interpretation of the deal we shall see that it is not a bad one for Mexico; on another one, that it is a bad deal for the multilateral agencies (World Bank and IMF) which helped financing it.

II - A VALUATION FORMULA FOR LDC DEBT

Assume that the debtor country is expected to make a net transfers P_t to its creditors, with P_t a Brownian process with drift :

$$(1) \qquad \frac{dP_t}{P_t} = \mu\, dt + \sigma\, dz_t \qquad ; z_t \text{ a Wiener process.}$$

For instance, think of P_t as a fixed fraction of the country's GDP (see Cohen (1990) for such a framework) with GDP following a stochastic process as in (1).

Call r the world rate of interest and assume that the unserviced part of the debt is rescheduled at the rate r. The law of motion of the country's debt then follows the law of motion :

$$(2) \qquad d\,D_t = [r\,D_t - P_t]\,dt \qquad \text{as long as} \qquad D_t \geq 0$$

Call V_t the market value of the debt. Assuming risk-neutral lenders, it is a solution to

$$(3) \qquad E_t \frac{\dot{V}_t}{V_t} + \frac{P_t}{V_t} = r$$

whenever $D_t > 0$.

Call q_t the market price of the debt so that $V_t = q_t\ D_t$ and call $x_t = \frac{P_t}{D_t}$ the apparent yield on the country's debt. We shall seek a function q(x) which is a solution to (3). Making use of Ito's lemma, one can show that q(x) is a solution to the following differential equation:

$$(4) \qquad \frac{1}{2}\, q''(x)\, x^2\, \sigma^2 + x\, q'(x)\, [\mu - r + x] - q(x)\, x + x = 0$$

One can then check that the price $q(x)$ of the debt can then be written :

$$(5)\quad \begin{cases} \text{(a) } q(x) = \frac{x}{x_0} - C(1-\frac{x}{x_0}) \int_0^{x/x_0} \varphi(t)dt & \text{when } x < x_0 \\ \text{(b) } q(x) = 1 - C(\frac{x}{x_0} - 1) \int_{x/x_0}^{\infty} \varphi(t)dt & \text{when } x > x_0 \\ \text{(c) } q(x_0) = 1 - C\, e^{-\beta} \end{cases}$$

in which

$$(6)\quad \begin{cases} x_0 = r - \mu \\ \varphi(t) = \frac{1}{(t-1)^2} e^{-\beta t}\, t^{\beta} \quad ; \quad \beta = 2\frac{r-\mu}{\sigma^2} \\ C = \frac{1}{\beta \int_0^{\infty} e^{-\beta x}\, x^{\beta}\, dx} \end{cases}$$

C is obtained by "smooth pasting" equations (5a) and (5b). $q(x)$ is a twice differentiable function which can be depicted as follows

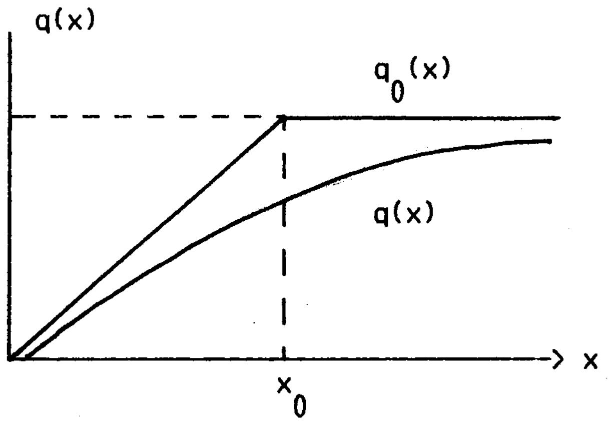

DIAGRAM 1

On Diagram 1, $q_0(x)$ represents the market price of the debt when there is no risk ($\sigma=0$). It is worth $q_0(x) = \frac{x}{x_0}$ when $x < x_0$ and 1 otherwise. In the zero-risk case, the country's debt is quoted below par when the present discounted value of the transfers paid by the debtor is below the face value of the debt.

When the present discounted value of these transfers exceeds the face value of the debt, then there will necessarily come a point when the country will have repaid all its debt, so that the price is necessarily one all the time.

When the transfers are risky ($\sigma > 0$), the expected present discounted value of all P_t is again $\frac{P_0}{r-\mu}$ and it does not depend on σ. The difference between the risky and the non risky case is due to the fact that a good fortune may help the country repay all its debt so that the value of the claim held by the debtor is less than the present discounted value of all future transfers P_t.

In the case when $x = x_0$, for instance, present discounted value calculation (based on the assumption that creditors would receive P_t for ever) would lead to the misleading implication that the debt should be quoted at par. In table 1, we have shown the discount which instead appears when the risk component is taken into account, in that case when $x = x_0$ and in the case when $x = \frac{x_0}{2}$.

The market price of the debt

β	Case $x = x_0$	Case $x = x_0/2$
1	0.63	0.390
2	0.73	0.444
5	0.825	0.487
10	0.875	0.498

When $x = x_0$ (respectively $x = \frac{x_0}{2}$), the country can generate in "average" enough cash (respectively twive less) cash than necessary to repay its debt over the entire future).

TABLE 1

$\beta = 2\frac{r-\mu}{\sigma^2}$ so that a standard deviation $\sigma = 0.2$ (which is that which is observed on Wall Street) and a difference between interest rate and growth of 4 percentage point lead to $\beta = 2$ which we stall take as our banckmark case in the sequel.

As a simple application of Table 1, one may ask : what is the equivalence, from the creditors' view point, between an increase in the growth rate of payment and a reduction of current payment?

When the value of the debt is infinite, the equivalence is straightforwart to calculate. The value of the debt is simply $V_0 = \frac{P_0}{r-\mu}$ so that $\frac{dP}{P} = -\frac{d\mu}{r-\mu}$. If growth is increased by 1 pourcentage point then, say when $r-\mu = 4$ %, lenders can accept a 25% reduction of the transfers made by the country. This 25% reduction is obviously the maximum amount of debt service reduction that lenders can accept when the debt is finite. When the debt is very small, for example, the trade-off becomes a negligible one as the lenders care less about the future growth of the country's ability to pay in the future (since they do not expect to cash it in full).

As an intermediate case, it is possible to use formula (1) to see how this trade-off operates when $x = x_0$, $\beta = 2$ and $r-\mu = 0.04$. One can calculate in that case that an additional 1 percentage point of the country's growth can "buy" a 15% reduction of the current payment that the country makes to its creditors.

III - THE VALUE OF A DEBT WRITE-OFF

1 - Let us first assume that the country wants to repurchase its debt (say out of the Industrialized Countries' money). It if makes an offer to the banks, and if we assume that the banks are a group of competitive investors, the only price at which the transaction can take place is the ex-post market value of the debt which will prevail after the transaction has taken place. As an application of the numbers displayed in table 1, consider the case when the country's debt is initially such that $x = \frac{1}{2} x_0$ and assume that the country

wants to repurchase half of its debt. Consider for instance the case when $\beta = 2$; we see from table 1 that the debt must be repurchased at 73 cents on the dollar, despite an initial price of 45 cents on the dollar before the transaction was announced. This indicates that repurchasing half the face value of the debt costs 62% of its initial market value.

2 - Assume now instead that banks can rationnally coordinate their collective behavior and that the country can make the banks a credible "it or leave it" offer such as: "repurchase the debt at such price or I use the money in an alternative way which yields no benefits to you (say I consume it)". What is now the cost —under this hypothesis— of repurchasing half the debt. When x goes from $\frac{x_0}{2}$ to x_0, the value of the banks' claims is reduced by a number which is

$$\Delta V = q(\frac{x_0}{2})\ D - q(x_0)\ D/2$$

This implies that the banks must be compensated for reducing the debt by half by a fraction θ of their initial claim which is equal to :

$$\theta \equiv \frac{\Delta V}{q(x/2).D} = [1 - \frac{1}{2}\frac{q(x)}{q(x/2)}]$$

When $\beta = 2$, we find that $\theta = 18\%$. This number can therefore be advantageously compared to the 62% which was found in the previous paragraph. Under the hypotheses which have adopted, the capability of making a credible offer to the banks therefore represents as much as 44% of the market value of the debt.

3 - When small transactions are involved, the relevant statistic to analyze is the marginal value of the debt. (This point was usefully emphasized by Bulow and Rogoff (1988)). Define the marginal price of the debt as :

$$\rho \equiv \frac{dV}{dD}$$

ϱ measures the market value which the banks as a whole are actually giving up when they reduce the face value of the debt by one dollar. Mathematically one finds $\varrho = q(x) - xq'(x)$. When the debt is infinite (x=0), it is easy to check that the ratio of the marginal to the average price of the debt is zero (the lenders -as a whole- do not care at all about one more or one less dollar). Conversely, as the debt goes to zero, the marginal and the average price do converge one towards the other. In table 2, I have calculated the marginal price of the debt in the two corresponding cases which were desplayed in table 1. We see from tables 1 and 2 that the difference between the marginal and the average price remains very substantial, even in the case when $x=x_0$. Indeed, in such cases, the marginal price always appear be less than half the value of the average price. For instance, a relatively solvent country such as one obtained when $x=x_0$ and $\beta = 10$, whose debt only show a 12.5% discount, exhibits a marginal price of its debt of only 42 cents on the dollar. For a country with the same characteristic ($\beta=10$) and twice bigger a debt, the marginal price is virtually zero, while the discount which is observed the secondary market is about 50%.

These results confirm the econometric evidences which are shown in Cohen (1989) where it was indeed found -out of a direct analysis of the data on the secondary market- that the hypothesis that the marginal price was zero could not be statistically rejected when the debt was quoted at a 50% discount.

(The evidence brought by Bulow and Rogoff (1988) for the Bolivian debt also strongly pointed to the fact that its marginal price was zero).

TABLE 2 - MARGINAL PRICE OF THE DEBT

	(1) $x=x_0$	(2) $x=x_0/2$
$\beta=1$	0.266	0.148
$\beta=2$	0.336	0.110
$\beta=5$	0.388	0.017
$\beta=10$	0.420	0.007

IV - THE VALUE OF GUARANTEING THE DEBT

a - The value of a guarantee

Offering a guarantee on the payment of the debt usually involves two distinct mechanisms. One is to guarantee that, say, the interest will always be paid, the other one amounts to protect the lenders from the stochastic disturbances which afflict the ability of the country to service its debt. The first mechanism is generally not a pure guarantee and may actually enhance the average ability of the country to service its debt. To that extent, it involves a partial bail out of the banks as well as a pure insurance mechanism. In this section we limit our analysis to a second mechanism and ask: what is the value of protecting the banks against the fluctuations of the countries' ability to pay. Mathematically, this simply amount to substitute to the stochastic streams of repayment P_t a deterministic pattern $\hat{P}_t = P_0\, e^{\mu t}$ which has the same expected mean and offer the banks Min $[D_0, \frac{P_0}{r-\mu}]$.

When the face value of the debt is infinite, the market value of such an insurance scheme is simply zero. Indeed, the banks are assumed to be risk-neutral and they already expect –in present value terms– to receive $\frac{P_0}{r-\mu}$. When the debt is not infinite, the value of the guarantee is simply given by the difference between the market price of the debt and the $q_0(x)$ line displayed in diagram 1. From this diagram, it is apparent that the maximum value of such a guarantee is obtained at the point when $x=x_0$. At this point the country would be solvent if the banks could make sure to get $P_0\, e^{\mu t}$ rather than P_t. One also sees that the maximum value of the guarantee is nothing else but the market discount which the debt would exhibit at this point. Table 3 also shows the value of guaranteeing the debt when $x = \frac{x_0}{2}$.

TABLE 3 - THE VALUE OF GUARANTEING THE DEBT
AGAINST DEBTORS' RISK
(IN % OF THE MARKET VALUE OF THE DEBT)

	Maximum value of such guarantee ($x=x_0$)	Value of the guarantee when $x=x_0/2$
$\beta=1$	37 %	22 %
$\beta=2$	27 %	10.7 %
$\beta=5$	17.5 %	2.6 %
$\beta=10$	12.5 %	1.4 %

We therefore see that the value (to the banks) of offsetting the stochastic disturbances of the debtor's transfers represents (only) approximately 10% of the market value of the debt in the cae when $\beta=2$ and when the debtor is "half-solvent". If the Mexican deal signed in August 1989 was to he interpreted as a pure "guarantee" scheme, we see that it would fit the first line shown when β is between 1 and 2.

b - <u>The timing of a guarantee</u>

Assume that the offer to guarantee the banks agains the debtor's income fluctuations is a standing one which the banks can accept at any point in time they wish. <u>When</u> will the swap take place ?

A priori there is a trade-off between accepting the swap early on (and enjoying the guarantee immediately) and postponing the time when the swap is done so as to —perhaps— enjoy a guarantee based upon a larger income base. We shall now see that this is <u>not</u> the case and that the banks will always immediately accept to swap their claims against the riskless asset they are offered.

Assume, instead, that they choose to swap their claim at only a specific point, when $x = x^*$. First assume that $x < x^*$. In that case, the price of their asset is still a solution of the differential equation (4) and it can be written as :

$$q(x) = \frac{x}{x_0} + \hat{C}(1 - \frac{x}{x_0}) \int_0^{x/x_0} \varphi(s)\, ds \qquad \text{(when } x* < x_0\text{)}.$$

in which $\hat{C}$ is a new parameter (presumably larger than C_β such as calculated in (6)). The swap will take place at a point x* when the new value of the banks' asset is equal to the guarantee they are offered x*. It must therefore be such that:

$$V* = q(x*)\ D* = \frac{P_{t*}}{r-\mu}$$

or $$q(x*) = \frac{x*}{x_0}$$

But from equation (8) one sees that this is possible if and only if : C=0, or x*=0. Both conditions have the same economic content: the swap is immediately carried on, at the time the offered is made. The banks cannot again by trying to postpone the deal until a time when it has more value (such as for instance, $x = x_0$). The same result would hold true when one tries $x* > x_0$.

The intuition behind this result comes as follows. When $x < x_0$, the debt-to-payment ratio is expected to raise, so there is no point in postponing the swap. When $x > x_0$, the debt-to-payment ratio goes away from the preferred point $x = x_0$, hence there is no point either to postpone the deal. (Mathematically, one can check that the "smooth pasting conditions" à la Dixit (1988) and Krugman (1988) holds when x = 0).

V - SOME PERSPECTIVES ON THE MEXICAN DEAL

In July 1989, Mexico and its creditors agreed on a debt relief plan offering the banks three options: 1) Reduce the face value of the debt by 35 % 2) Reduce the interest rate down to 6.25% ; 3) capitalize about 2/3 of the interest due. Options (1) and (2) will take the form of a swap of the banks loans against a 30 years bonds which will be offeed guarantees of two kinds : the principal will be collaterized through a zero-coupon bond issued by the US treasury and financed by the IMF; a fund worth 5.3 bls (financed by the IMF, the World Bank and the Japanese Government) will be used to guarantee (part of) the interest due by Mexico.

In total 7 bls (at a minimum) will be lent by the Industrialized Countries and Multilateral Agencies to finance the deal. What is the market value of this money (that is: by how much is really enhanced to the market value of the banks claims ?).

The simplest way to come up with a market value figure for these 7 billions may come as follows. Out of these 7 billions only 4 bls can be interpreted as new money "earmarked" for debt relief (2 from the World Bank and the IMF, 2 from the Japanese Government). Let us interpret those 4 bls as a loan which is understood by the parties to be junior to the newly issued bonds. The market price of a junior claim is simply measured by the marginal price reported in Table 4. Depending upon which interpretation we have of the Mexican situation we can come up with the two following numbers (taking the case $\beta=2$ as a benchmark).

$x = x_0$	$x = x_0 /2$
2.7	3.6

TABLE 4 - ENHANCEMENT OF THE MARKET VALUE OF COMMERCIAL CLAIMS OUT OF 4 bls OF "NEW" JUNIOR MONEY

Now, how much money was given up by the banks in exchange for this deal ? It appears (in early December 1989) that only 10% of the banks will take option (3) while the rest will be evenly split between options (1) and (2).

Option (2) can be estimated to represent a 28% nominal write-off. Out of the 54 bls $ at stake this represents a 6.8 bls nominal write-off. Option (1), on the other hand, represents a write-off amounting to 8.5 bls. In total, one sees that a nominal write-off of 15.3 bls will be granted. The market value of this nominal write-off must be estimated at the marginal price shown in table 2. for $\beta = 2$, we get the following :

$x = x_0$	$x = x_0/2$
5.1	1.7

TABLE 5 - MARKET VALUE OF THE DEBT WRITE-OFF

Compared to table 4, one sees that it is a bad or a good deal for the banks depending upon whether $x = x_0$ or $x = x_0/2$. If one takes the observed market price (of about 45 cents) to be an accurate measure of Mexico's solvency, one should conclude (from Table 1) that the hypothesis that $x = x_0/2$ is the correct one.

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