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THE RATIONALITY OF ADJUSTMENT BEHAVIOR IN A MODEL OF MONOPOLISTIC COMPETITION*

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RÉSUMÉ

LA RATIONALITÉ DES COMPORTEMENTS D'AJUSTEMENT DANS UN MODÈLE DE CONCURRENCE MONOPOLISTIQUE

La modélisation des comportements en terme d'ajustement, c'est-à-dire, la réaction au déséquilibre observés dans le passé par opposition à la maximisation d'une fonction objectif sur la base de l'anticipation des grandeurs futures, a été critiquée comme *ad hoc*. Dans cette étude, nous montrons que le comportement optimal des firmes, avec anticipations rationelles, peut être exprimé en termes d'ajustement aux déséquilibres passés entre l'offre et la demande. (c'est-à-dire à la quantité de stock involontaire). La perspective est celle de l'équilibre partiel. Deux cadres d'analyse sont considérés successivement : 1) Un monopole confronté à une fonction de demande soumise à des chocs aléatoires, et 2) La concurrence monopolistique sans incertitude.

ABSTRACT

THE RATIONALITY OF ADJUSTMENT BEHAVIOR IN A MODEL OF MONOPOLISTIC COMPETITION

The modeling of behavior in term of adjustment, *i.e.*, the reaction to disequilibria observed in the past, as opposed to the maximizing of an objective function on the basis of the expectation of future magnitudes, has been criticized as *ad hoc*. In this study, we show that the optimal behavior of firms, with rational expectations, can be expressed in terms of adjustment to past disequilibrium between supply and demand (*i.e.*, to the amount of involuntary inventories). The perspective is that of partial equilibrium. Two different frameworks are successively adopted : 1) A monopoly facing a demand function subject to random shocks, and 2) Monopolistic competition without uncertainty.

MOTS CLEFS : Rationalité, Ajustement, Concurrence monopolistique, Micro-économie

KEYWORDS : Rationality, Adjustment, Monopolistic competition, Microeconomics

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INTRODUCTION

Besides the dominant paradigm of the maximizing of an objective function on the basis of the expectation of the unknown values of magnitudes in the future, *adjustment* to disequilibria observed in the past defines an alternative approach to the modeling of decision making. We have made extensive use of adjustment in our work on the stability of capitalism (*cf.* for example, DUMÉNIL G., LÉVY D. 1987(c), 1987(a), 1987(b), 1989(a), and 1989(b)).

The description of adjustment behavior is as old as economic theory. The analysis of the competitive process and the formation of natural prices by Adam Smith, David Ricardo, and Karl Marx (SMITH A. 1776, Ch. 7, RICARDO D. 1817, Ch. 4, and MARX K. 1894, Vol. III, Ch. 10) was conducted in terms of adjustment. As they describe, capitalists seek the maximum return for their capital by moving fractions of this capital at each period, responding to profitability differentials. Our original adherence to adjustment is rooted in this analysis of competition by the classics.

The formal modeling of adjustment is, of course, more recent, and can be traced to works such as SAMUELSON P.A. 1939 and METZLER L.A. 1941. Since then adjustment survived in a few heterodox contributions such as LOVELL M. 1961 and 1962, KORNAI J., MARTOS B. 1973 and 1981, TOBIN J. 1975, DAY R., GROVES T. 1975 (ed.), KORNAI J. 1980, MALINVAUD E. 1982, and SIMONOVITS A. 1985.

Adjustment has been criticized under the banner of *rationality*. The description of behavior in terms of adjustment is rejected as *ad hoc*. In our opinion, however, the exact nature of the relationship between maximizing and adjustment has not been seriously addressed. Actually, the question of whether adjustment *is* or *is not* a rational behavior depends on the framework in which the analysis is conducted. More precisely, we believe that it is possible to define adjustment equations and values of reaction coefficients to disequilibrium, which are *rational* within given contexts. We take here *rational* in its full sense of the maximizing of an objective function with rational expectations.

The purpose of this paper is to illustrate this general proposition in a model of *partial equilibrium* in which we study the behavior of firms in their *price* and *output* decisions, when confronted with a demand function which varies over time. We will consider successively two distinct frameworks:

1. *Monopoly*. One firm faces a demand function which is subject to random shocks. This first case was studied as early as 1954, by Herbert Simon (*cf.* HOLT C.C., SIMON H.A. 1954 and also HOLT C.C., MODIGLIANI F., MUTH J.F., SIMON H.A. 1960) and is presently the object of an important literature. We will use in this study the work of Alan Blinder (*cf.* BLINDER A.S. 1982).
2. *Monopolistic Competition*. A number of firms are competing within a given

industry, and the vagaries of the demand function are *endogenous*: They result from the variation of the average level of prices in the industry (*i.e.*, they translate the changes in prices of competitors). To our knowledge, no result has been demonstrated in this framework, to date.

The study divides into two parts corresponding to the two above frameworks. In the first part, we study *rational adjustment* for the case of a monopoly. Then we make the assumption that demand is drifting slowly in comparison to the speed of adjustment of supply to demand by firms. Under this assumption, the exact form used in our work on stability for the modeling of the decision to produce is obtained. The second part is devoted to monopolistic competition. There we study the individual behavior of firms whose individual demand varies as a result of the changes in the prices of competitors in the industry. We derive the aggregate "behavior" at the industry level. This investigation requires the formation of price expectations. In conformity with the general line of argument in this paper, we assume rational expectations.

Two main conclusions follow:

1. For appropriate forms of the equations and values of the reaction coefficients, adjustment behavior is rational in frameworks in which demand is subject to variations.
2. This result does not necessarily follow from the existence of uncertainty, as in the case of the monopoly, but also from the mere variation of demand in a fully deterministic model, as in the case of monopolistic competition, from which we exclude uncertainty (which obviously could be added).

I - OPTIMAL MANAGEMENT: MONOPOLY

In the two first sections of this first part we borrow from BLINDER A.S. 1982 for the construction of the model, and from HOLT C.C., MODIGLIANI F., MUTH J.F., SIMON H.A. 1960, for its resolution. The model is presented in section A. Then in section B, the behavior of firms with respect to price and output decisions is derived. Last, section C is devoted to the assumption of the comparatively quick adjustment by firms of supply to demand. With this assumption the form of the equations for adjustment is modified.

A - THE FRAMEWORK OF ANALYSIS

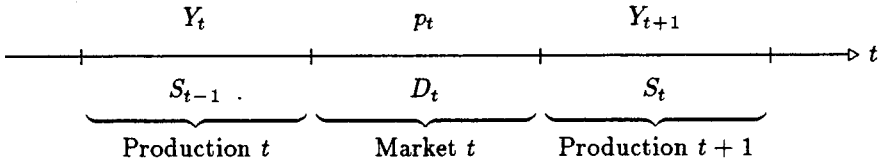
In this section, we introduce the model in three steps. The general assumptions are briefly sketched in subsection 1. Subsection 2 is devoted to the modeling of

demand. Last, production and inventory costs are discussed in subsection 3.

1. General Assumptions

The challenge is to derive the optimal behavior of a firm in the following environment:

1. The firm is a monopoly or, at least, behaves as such.
2. It is confronted with a given demand function which is known up to a stochastic additive disturbance.
3. The firm carries two types of costs: 1) Production costs, and 2) Disequilibrium costs such as inventory costs or the costs of varying production. However, in this study, we abstract from the costs of varying production for simplicity.
4. The firm must make its decisions concerning price and production levels ignoring the value of the demand shock for the period. However, it forms *rational* expectations of the demand shock.
5. Markets are held after each production period:



in which Y_t denotes the output of period t, D_t, demand or firm's sales, p_t, the price set by the firm on market t, S_t, inventories at the end of market t (held during production $t+1$). For accounting reasons, one obtains:

$$S_t = S_{t-1} + Y_t - D_t \tag{1}$$

6. To take advantage of certainty equivalence, *i.e.*, the fact that only the average value of random variables actually matters, the demand function is linear and cost functions are quadratic.

We consider the decision of the firm at the end of market 0. At this date S_0 is known, and the firm must decide on Y_1 and p_1.

2. The Demand Function

The demand function is:

$$D_t = d_0 - d_1 p_t + \eta_t \tag{2}$$

in which η_t is a random shock and d_0 and d_1 two constant parameters. With this model of demand, the revenue function is quadratic.

We assume that the stochastic variable η_t follows an $AR(1)$ process:

$$\eta_t = \rho \eta_{t-1} + \nu_t \quad \textit{with} \quad 0 \leq \rho \leq 1 \tag{3}$$

in which ν_t is an independently and identically distributed disturbance. For $\rho = 0$, equation 3 corresponds to the case in which η_t is itself such a disturbance term. The opposite case of $\rho = 1$ models a random walk. The assumption of an $AR(1)$ process is not required in the following computation. It is only necessary in subsection I.B.3 below in order to derive the value of coefficients φ and φ' in equations 19 and 20.

3. Production and Inventory Costs

Production costs and inventory costs are assumed to be quadratic:

$$\textit{Production costs:} \qquad \frac{c}{2}Y_t^2 \tag{4}$$

$$\textit{Inventory costs:} \qquad \frac{c_s}{2}(S_t - \overline{S})^2 + c'_s \tag{5}$$

Quadratic production costs correspond to the case of diminishing returns. The *inventory cost function* can be analyzed as the sum of two distinct costs:

1. The cost of storing and maintaining inventories which is an increasing quadratic function of the level of inventories.
2. The cost of loosing sales which is a decreasing function of the level of inventories (*cf.* BLINDER A.S. 1982, footnote 4, p. 337).

In equation 5, $\overline{S}$ is the value of S, for which the sum of these two costs is minimum. Since c'_s does not play any role in the determination of the optimal behavior, it can be set to zero.

B - THE DERIVATION OF PRICE AND OUTPUT DECISIONS

Subsection 1 presents the firm's profit maximizing behavior. Subsection 2 is devoted to the definition of the targets around which the firm will gravitate. Rational adjustment behaviors are defined in subsection 3.

1. Profit Maximizing

The firm *maximizes its expected profits*:

$$\Pi = E_0 \sum_{t=1}^{\infty} \delta^t \left(p_t(d_0 - d_1 p_t + \eta_t) - \frac{c}{2}Y_t^2 - \frac{c_s}{2}(S_t - \overline{S})^2 \right) \tag{6}$$

where δ is a discount factor and the expectation is taken as of time zero.

This maximizing is constrained by the accounting identity 1. With Lagrange multipliers, λ_t, the problem becomes that of the determination of a critical point of the Lagrangian:

$$\Pi + E_0 \sum_{t=1}^{\infty} \delta^t \lambda_t \left(-S_t + S_{t-1} + Y_t - d_0 + d_1 p_t - \eta_t \right)$$

The first order conditions in addition to equation 1 are:

$$cY_t = \lambda_t \tag{7}$$

$$p_t = \frac{1}{2}\left(\lambda_t + \frac{d_0 + \eta_t}{d_1}\right) \quad \text{or} \quad D_t = \frac{d_0 + \eta_t - \lambda_t d_1}{2} \tag{8 or 8'}$$

$$c_s(S_t - \overline{S}) = \delta\lambda_{t+1} - \lambda_t \tag{9}$$

The second order conditions obviously hold.

We use the same notation for variables S_t and η_t and for their expected values at time zero. (The firm decides on p_t and Y_t, and consequently knows these values with certainty.)

2. Targets

The equilibrium values can be obtained from equations 7, 8, and 9, for $\eta_t = 0$:

$$\tilde{Y} = \frac{d_0}{2 + d_1 c} \tag{10}$$

$$\tilde{p} = \frac{d_0}{d_1}\frac{1 + cd_1}{2 + cd_1} \tag{11}$$

$$\tilde{S} = \overline{S} - (1 - \delta)\frac{d_0 c}{c_s(2 + d_1 c)} \tag{12}$$

The two first equations correspond to the straightforward atemporal treatment of a monopoly. Only d_0, d_1, and c are considered in these equations. Conversely, inventory costs, c_s, and the discount rate, δ, are involved in the third equation which defines the target value for the stock of inventories.

If ρ is strictly smaller than 1, the equilibrium values determined above can be interpreted as the target around which the variables gravitate.

3. Rational Adjustment Behaviors

We now turn to the study of the dynamics of the above system. After substitution of 7 in 9, using 8, one obtains a second order inhomogenous difference equation in Y:

$$\delta Y_{t+1} - \alpha Y_t + Y_{t-1} = \beta_t \quad \text{for} \quad t = 2, 3, \ldots \tag{13}$$

$$\textit{with} \quad \alpha = 1 + \delta + \frac{c_s}{c}\left(1 + \frac{d_1 c}{2}\right) \quad \text{and} \quad \beta_t = -\frac{c_s}{2c}(d_0 + \eta_t)$$

Let μ be a real number. We first define:

$$Z = \sum_{t=1}^{\infty} \mu^{t-1} Y_t \quad \text{and} \quad B(\mu) = \sum_{t=2}^{\infty} \mu^{t-1} \beta_{t-1}$$

then multiply equation 13 by μ^{t-1}, and sum from $t = 2$ to $t = \infty$:

$$\frac{\delta}{\mu}(Z - Y_1 - \mu Y_2) - \alpha(Z - Y_1) + \mu Z = B(\mu) \tag{14}$$

We choose μ with a modulus smaller than 1, such that the coefficient of Z is equal to zero. Thus, μ is a zero of:

$$\mu^2 - \alpha\mu + \delta = 0 \tag{15}$$

It is easy to show that only one positive value of μ, μ_1, with a modulus smaller than 1, exists. Thus, equation 14 becomes:

$$\mu_1 Y_1 - \delta Y_2 = B(\mu_1) \tag{16}$$

A second equation is required in order to determine Y_1. We use equations 9 for $t = 1$, and 1, 7, and 8:

$$\delta Y_2 - (\alpha - \delta)Y_1 = \frac{c_s}{c}\left(S_0 - \overline{S} - \frac{d_0 + \eta_1}{2}\right) \tag{17}$$

Equations 16 and 17 allow for the determination of Y_1:

$$Y_1 = \tilde{Y} - \varepsilon\left(S_0 - \tilde{S} - \Delta\right) \tag{18}$$

$$\textit{with} \quad \varepsilon = \frac{c_s}{c(\alpha - \delta - \mu_1)} > 0 \quad \textit{and} \quad \Delta = \frac{1}{2}\sum_{t=1}^{\infty} \mu_1^{t-1}\eta_t$$

If η_t satisfies equation 3, then the rational expectation of η_t, after market 0, is $\eta_t = \rho^t\eta_0$, and $\Delta = \frac{1}{2}\frac{\rho\eta_0}{1 - \mu_1\rho}$, where η_0 measures the demand shock, *i.e.*, the difference between the actual demand D_0 and its model: $\eta_0 = D_0 - (d_0 - d_1 p_0)$.

Thus, equation 18, can be written:

$$Y_1 = \tilde{Y} - \varepsilon(S_0 - \tilde{S}) + \varphi\eta_0 \tag{19}$$

$$\textit{with} \quad \varphi = \frac{\varepsilon}{2}\frac{\rho}{1 - \mu_1\rho}$$

The price decision can be derived from equations 7, 8, and 19:

$$p_1 = \tilde{p} - \varepsilon'(S_0 - \tilde{S}) + \varphi'\eta_0 \tag{20}$$

$$\textit{with} \quad \varepsilon' = \frac{c}{2}\varepsilon \quad \textit{and} \quad \varphi' = \frac{c}{2}\varphi + \frac{\rho}{2d_1}$$

Both equations 19 and 20 describe typical adjustment behavior, and this brings to its conclusion the demonstration that adjustment can be a rational behavior. The following economic comments can be made:

1. A large value of inventories S_0 results in small values of Y_1 and p_1.
2. A large inventory cost, c_s, results in large values for the coefficients ε, φ, ε', and φ', which measure the reactions of firms to disequilibrium.
3. A large value of ρ, which means that demand is more sticky, has no effect on ε and ε', but increases φ and φ'.

The two last remarks correspond to Blinder's "main theorem" (*cf.* BLINDER A.S. 1982, Theorem 1, p. 337), for φ, and φ', and to Blinder's Proposition 4 (p. 341), for ε and ε'.

C - A QUICK RESPONSE OF SUPPLY TO DEMAND

In this section, we derive the specific forms of rational adjustment under the assumption that firms respond quickly in the short run to the disequilibrium between supply and demand, in comparison to the rather sticky drift of the demand function. The problem is introduced in subsection 1. Then, we analyze successively the quick and slow components of the process, in subsections 2 and 3 respectively.

1. The Problem

In sections A and B above, a relation of recursion with five variables Y, p, S, D, and η, has been defined (*cf.* equations 19, 20, 1, 2, and 3). These equations account for the transition from period 0 to period 1, but the same process can be repeated, and the same system accounts for any transition from $t-1$ to t.

In the overall process two different speeds of adjustment must be distinguished:

1. The exogenous speed of variation of demand shocks η. This speed is modeled by parameter ρ in equation 3. A large value of ρ is equivalent to a slow variation of the demand function.

2. The endogenous speed of adjustment of supply to demand which concerns the decision to produce, the price decision, and the level of inventory.

We believe that in real economies the first process is slow in comparison to the second. Firms react quickly to the disequilibrium between supply and demand, whereas the return of demand to its equilibrium value is slower. Output can be adjusted to demand far before the return of demand to equilibrium is realized. In extreme cases, such as, for example, in the steel industry during the recent years of hardship, the return to a normal capacity utilization rate can be very long, whereas inventories can be rescaled in a rather short span of time, as a result of the reduction of production.

We study the limit case, distinguishing two different dynamics:

1. For a given demand function after a shock, we analyze the dynamics of outputs, prices, and inventories.

2. Assuming that the above adjustment of outputs, prices, and inventories, constantly realized, we study the dynamics associated with the return of demand to equilibrium.

2. The Quick Component of Dynamics

Three variables are involved in this first component of the dynamical process,

since the shock has been "frozen" at its initial value η and D can be eliminated:

$$\begin{aligned} Y_t &= \tilde{Y} - \varepsilon(S_{t-1} - \tilde{S}) + \varphi\eta \\ p_t &= \tilde{p} - \varepsilon'(S_{t-1} - \tilde{S}) + \varphi'\eta \\ S_t &= S_{t-1} + Y_t - (d_0 - d_1 p_t + \eta) \end{aligned}$$

Since η is given, the fixed point in this recursion is not $(\tilde{Y}, \tilde{p}, \tilde{S})$ as in equations 10, 11, and 12, but is shifted. For example, one has:

$$Y^* = \tilde{Y} + \frac{2-\rho}{2+d_1 c}\eta \tag{21}$$

The study of the stability of this recursion is straightforward: Stability is always insured.

3. The Sticky Component of Dynamics

We now assume that η varies with time, and equation 21 must be written as:

$$Y_t = \tilde{Y} + \frac{2-\rho}{2+d_1 c}\eta_t \tag{22}$$

Variable η_t measures the difference between the actual demand observed, which is obviously known, and the model used by the firm for demand. The determination of this model is difficult, especially at a distance from equilibrium. We will derive another form of the decision to produce from which the demand shock has been eliminated.

Beginning with equation 19 at time t:

$$Y_t = \tilde{Y} - \varepsilon(S_{t-1} - \tilde{S}) + \varphi\eta_{t-1}$$

we substitute the value of η_{t-1} from equation 22 in the above equation:

$$Y_t = \tilde{Y} - \varepsilon(S_{t-1} - \tilde{S}) + \sigma(Y_{t-1} - \tilde{Y}) \tag{23}$$

$$\textit{with} \quad \sigma = \frac{2+d_1 c}{2-\rho}\varphi \tag{24}$$

In a similar manner for prices:

$$p_t = \tilde{p} - \varepsilon'(S_{t-1} - \tilde{S}) + \sigma'(Y_{t-1} - \tilde{Y}) \tag{25}$$

$$\textit{with} \quad \sigma' = \frac{2+d_1 c}{2-\rho}\varphi' \tag{26}$$

The strong point in equations 23 and 25 is that only observed values of the variables are involved, and no *computed* values (in which a model of the economy is implied as in the case of η). The problem the firm faces is only to determine the reaction coefficients as close as possible from their optimal values.

Parenthetically, one can notice that the same forms are obtained for equations 23 and 25, from the consideration of costs of varying production (*cf.* HOLT C.C., MODIGLIANI F., MUTH J.F., SIMON H.A. 1960).

II - OPTIMAL MANAGEMENT: MONOPOLISTIC COMPETITION

In the first part of this study, parameters d_0 and d_1 which define the demand function (*cf.* equation 2), are constant. The firm is a monopolist. The second part of this study is devoted to the case of monopolistic competition within an industry. Parameters d_0 and d_1 depend on the prices of competitors, or, rather, depend on the average price prevailing in the industry. Thus the individual demand schedule with which a firm, i, is confronted, is a function of p^i, its own price, and p, the average price.

Section A presents the general framework of analysis. Section B is devoted to the derivation of individual behavior. In section C, we analyze the overall dynamics at the industry level, under the assumption of rational expectations.

A - THE FRAMEWORK OF ANALYSIS

In this section we present the main assumptions made in the construction of the model.

We study the industry under the assumption that all firms remain in a vicinity of equilibrium. As a result of this assumption, it is possible to resort to linear developments of the equations. If X_t and Y_t are two variables whose equilibrium values are X^* and Y^*, we substitute $X^*Y^* + X^*(Y_t - Y^*) + Y^*(X_t - X^*)$ for X_tY_t, neglecting terms such as $(X_t - X^*)(Y_t - Y^*)$.

The perspective is that of *partial equilibrium*. We consider an industry composed of n identical firms. It is therefore necessary to make an assumption concerning the total demand which is addressed to the industry. We assume that the total purchasing power, M, destined for the purchase of the output of the industry is given (and independent from the prices in the industry).

Total purchasing power is shared among firms according to the *individual demand function*:

$$D^i = A\left(1 - \psi\left(\frac{p^i}{p} - 1\right)\right) \tag{27}$$

in which the average level of prices is defined by $p = \frac{1}{n}\sum_{i=1}^{n} p^i$, and A is a constant defined by $\sum_{i=1}^{n} D^i p^i = M$. If firms remain in a vicinity of equilibrium, one has $A = \frac{1}{n}\frac{M}{p}$. In equation 27, ψ is a parameter which accounts for the intensity of price competition among firms. If all prices are equal, each firm receives an equal share of total demand: M/np. Equation 27 can be written in a more traditional form as $D^i = d_0 - d_1 p^i$, with:

$$d_0 = \frac{1}{n}\frac{M}{p}(1 + \psi) \quad \text{and} \quad d_1 = \frac{1}{n}\frac{M}{p^2}\psi \tag{28}$$

Parenthetically, the demand function 27 can be derived from the maximizing of a C.E.S. utility function (*cf.* BLANCHARD O.J., KIYOTAKI N. 1987) limited to the consideration of the outputs of the various sellers in the industry:

$$U = \left(\sum_{i=1}^{n} (C^i)^{\frac{\psi}{\psi - 1}} \right)^{\frac{\psi - 1}{\psi}}$$

in which C^i is the consumption of good i and ψ a parameter.

Maximizing U under the budgetary constraint $\sum_{i=1}^{n} C^i p^i \leq M$, and for given prices p^i, one obtains the following demand function:

$$C^i = \frac{M}{n} \frac{1}{p'} \left(\frac{p^i}{p'} \right)^{-\psi}$$

in which:

$$p' = \left(\frac{1}{n} \sum_{i=1}^{n} (p^i)^{1-\psi} \right)^{\frac{1}{1-\psi}}$$

If all p^i are not too different, the above demand function and p' can be developed for p^i in the vicinity of p', *i.e.*, for $\frac{p^i}{p'} - 1$ small. The same function as in equation 27 is obtained.

In addition to the sharing of total demand as expressed in equation 27, one could add a random variable η^i, with the condition that the sum of all shocks at the industry level is equal to zero: $\sum_{i=1}^{n} \eta^i = 0$. From simplicity, we will abstract from this component which is not essential to the demonstration.

The firms must carry production costs and inventory costs. We assume that these costs do not depend on the average price p of the industry. This is equivalent to stating that inputs are purchased outside of the industry. These costs are considered given and constant.

When the firm maximizes its profits, it assumes that, p, the average price in the industry does not depend on its own price (the firm is small).

In this computation, the firm must make price expectations about p for the future. In conformity with the general purpose of this study—the derivation of rational adjustment behavior—we assume that the firm forms *rational expectations.*

B - INDIVIDUAL BEHAVIOR IN MONOPOLISTIC COMPETITION

Equilibrium is discussed in subsection 1. Subsection 2 introduces an assumption concerning ψ. Rational behavior is determined in subsection 3.

1. Equilibrium

A priori, the average price, p, in the industry can have any value. For a given p, the individual demand functions are determined (*cf.* equation 28). On the basis of these demand functions, and using equations 10 and 11, it is possible to define the price and output targets, $\tilde{p}(p)$ and $\tilde{Y}(p)$. Since d_0 and d_1 are independent from i, the same is true for the targets.

For a given p, the individual price targets, $\tilde{p}(p)$, can differ from p. We define the *equilibrium price* $p^\star$ *as the price for which firms choose the average price for individual targets, i.e.,* $\tilde{p}(p^\star) = p^\star$. One obtains:

$$\tilde{p}(p^\star) = p^\star = \left(\frac{\psi}{\psi - 1}\frac{cM}{n}\right)^{\frac{1}{2}} \tag{29}$$

$$\tilde{Y}(p^\star) = Y^\star = \frac{\psi - 1}{\psi c}p^\star \tag{30}$$

One can verify that, when equilibrium prevails, the sum of the output targets of the n firms is equal to total demand for $p^\star$, *i.e.*, $M/p^\star$.

When the industry is not at equilibrium in the sense that $p \neq p^\star$, we say that disequilibrium prevails. This obvious property must be stated unambiguously, since in sharp contrast with most models used in economics, our model still has a meaning out of equilibrium:

1. The behavior of agents in a Walrasian model is only rational in an equilibrium.
2. The behavior of agents in our model is rational in and out of equilibrium.

The ambiguity concerning equilibrium and disequilibrium is due to the two different meanings which can be given to the term equilibrium itself:

1. An equilibrium is a fixed point in a dynamic model. The term is used in this sense in this study.
2. At each period in the dynamic process described in this part, disequilibrium prevails in the above sense. However, each step can be characterized as a *Nash equilibrium*. Each behavior is optimal considering the behavior of other producers as given.

2. The Assumption that ψ is larger than 3

A priori, the output target $\tilde{Y}(p)$ can be an increasing or decreasing function of p, depending on the value of ψ. One can expect, under ordinary circumstances, $\tilde{Y}(p)$ to be an increasing function of its argument: If the prices, p, of all the competitors of firm i increase, whereas p^i remains constant, buyers will demand more from i, and the target $\tilde{Y}(p)$ should be raised. One can show that this is the case, in a vicinity of equilibrium, if $\psi > 3$.

In the remainder of this study, we will assume $\psi > 3$.

3. Rational Behavior

In order to derive the behavioral equations we resort to the same method as in part I (construction of the Lagrangian, derivation of the first order conditions). We first linearize the equations. For example, the substitution of 7 in 8', yields:

$$D_t^i = Y^* - \frac{\psi - 1}{2}\left(Y_t^i - Y^*\right) + \frac{\psi - 3}{2} Y^* x_t \quad \textit{with} \quad x_t = \frac{p_t - p^*}{p^*}$$

The derivation is identical to that in part I, up to equation 13, which becomes:

$$\delta Y_{t+1}^i - \alpha_t Y_t^i + Y_{t-1}^i = \beta_t \tag{31}$$

The important transformation is that α varies with time. We linearize $\alpha_t Y_t^i$, *i.e.*, substitute $\alpha^* Y_t^i + (\alpha_t - \alpha^*)Y^*$ for this term in equation 31. Using the notation:

$$\alpha^* = 1 + \delta + \frac{c_s}{c}\frac{1+\psi}{2} \quad \textit{and} \quad \beta_t' = \frac{c_s}{2c} Y^* \left(1 + \psi + (3 - \psi)x_t\right) \tag{32}$$

one obtains:

$$\delta Y_{t+1}^i - \alpha^* Y_t^i + Y_{t-1}^i = \beta_t'$$

Still following the demonstration in part I, the new form of equation 16 is:

$$\mu_1 \left(Y_1^i - Y^*\right) - \delta \left(Y_2^i - Y^*\right) = B \tag{33}$$

$$\textit{with} \quad B = \frac{c_s}{c}\frac{3 - \psi}{2} Y^* \mu_1 \Sigma_2 \quad \textit{and} \quad \Sigma_T = \sum_{t=T}^{\infty} \mu_1^{t-T} x_t$$

and μ_1 satisfying equation 15 with $\alpha = \alpha^*$. In a similar manner equation 17 can now be written as:

$$\delta \left(Y_2^i - Y^*\right) - \left(1 + \frac{c_s}{c}\frac{1+\psi}{2}\right)\left(Y_1^i - Y^*\right) = \frac{c_s}{c}\left(\frac{3 - \psi}{2} x_1 + S_0^i - S^*\right) \tag{34}$$

The equation for the decision to produce follows from equations 33 and 34:

$$Y_1^i = Y^* - \varepsilon(S_0^i - S^*) + \varepsilon \frac{\psi - 3}{2} Y^* \Sigma_1 \tag{35}$$

in which:

$$\varepsilon = \frac{c_s}{c}\frac{1}{\alpha - \delta - \mu_1} = \frac{\delta - \mu_1}{\delta}\frac{2}{\psi + 1} \tag{36}$$

For the price decision, one obtains:

$$p_1^i = p^* - \varepsilon'(S_0^i - S^*) + \varepsilon' \frac{\psi - 3}{2} Y^* \Sigma_1 + \frac{1+\psi}{2\psi} p^* x_1 \tag{37}$$

With $\varepsilon' = \frac{c}{2}\varepsilon$, the above forms for the output and price decisions are similar to those derived in part I (*cf.* equations 19 and 20), with two differences:

1. By construction no random shock appears in the equations.
2. A new term is present, the sum over time of $x_t = \frac{p_t - p^*}{p^*}$. This term corresponds to the specific effect of monopolistic competition. It measures the impact of

the variation of the expected average price in the future at the industry level, on individual behavior. Equations 35 and 37, do not define real adjustment behaviors, since expectations are still considered. These behaviors will be derived in the next section.

The behavior of firms is summarized in equations 35 and 37. In the determination of this behavior, they must, however, make a complete plan for future and also compute Y_t^i and p_t^i over time. The following equation for prices will be used in the next section:

$$p_t^i = p^\star - \varepsilon' \left(S_{t-1}^i - S^\star\right) + \varepsilon' \frac{\psi - 3}{2} Y^\star \Sigma_t + \frac{1+\psi}{2\psi}(p_t - p^\star) \tag{38}$$

C - ADJUSTMENT BEHAVIOR

In this section we derive the equations modeling rational adjustment at the level of the industry (subsection 1), then at the level of one individual firm (subsection 2).

1. Rational Adjustment at the Level of the Industry

Expectations are implied in the two behavioral equations above (*cf.* equations 35 and 37). We assume rational expectations: We sum equation 38 over i, for all firms, and identify $\frac{1}{n}\sum_{i=1}^n p_t^i$, the average price level derived from the price decisions of individual firms, and p_t, the average price expected by firms. One obtains:

$$\left(\frac{1}{\varepsilon} + \frac{3-\psi}{2}\right) x_t + s_{t-1} - \frac{\psi - 3}{2}\mu_1 \Sigma_{t+1} = 0 \tag{39}$$

$$\textit{with} \quad s_t = \frac{S_t - S^\star}{Y^\star} \quad \textit{and} \quad S_t = \frac{1}{n}\sum_{i=1}^n S_t^i$$

In this notation S_t is the average level of inventory in each industry.

In order to determine x_t, it is necessary to follow the dynamics of inventories. We define the notation Y_t for the average individual production level $Y_t = \frac{1}{n}\sum_{i=1}^n Y_t^i$. From: 1) The total value of inventories for the whole industry, $S_t = S_{t-1} + Y_t - \frac{1}{n}D_t$, 2) Total demand for the industry, $D_t = \frac{M}{p_t} = \frac{np^\star Y^\star}{p_t}$, and 3) The result of the aggregation of the first order condition as in equation 8, it follows:

$$s_t = s_{t-1} + 2x_t \tag{40}$$

The system formed by equations 39 and 40 allows for the determination of the unknowns. We seek a solution of the form $x_t = x_1 \rho^{t-1}$ and $s_t = s_0 \rho^t$. Such a solution exists if $x_1 = -\varepsilon_a s_0$ with $\varepsilon_a = \frac{1-\rho^\star}{2}$, and if $\rho^\star$ satisfies:

$$\left(\rho^\star - \frac{\mu_1}{\delta}\right)(1 - \mu_1 \rho^\star) - \varepsilon \frac{\psi - 3}{2}(1 - \mu_1)\rho^\star = 0 \tag{41}$$

It is easy to show that, for $\psi > 3$, equation 41 has one and only one zero ρ^* which verifies: $\frac{\mu_1}{\delta} < \rho^* < 1$ (the other zero is real and larger than $1/\mu_1$). Thus the two equations, which reflect the aggregate effects of individual behavior (equations 35 and 37) at the industry level, can be written as follows:

$$Y_1 = Y^* - \varepsilon_a(S_0 - S^*) \tag{42}$$

$$p_1 = p^* \left(1 - \varepsilon_a \frac{S_0 - S^*}{Y^*}\right) \tag{43}$$

A comparison with equations 35 and 37 leads to the following observations:

1. The terms corresponding to future prices are absent in the above equations, only past disequilibrium $(S_0 - S^*)$ is considered.
2. It can be shown that ε_a is larger than ε. This is due to the firms' consideration of the reactions of other producers, as will be shown below.

Conversely, equations 42 and 43 are very similar to those obtained in the first part of this study (equations 19 and 20) for the monopoly (without the effect of shocks, from which we abstracted in part II).

The aggregate behavior described above is absolutely distinct from that of a monopoly facing the aggregate demand function. With the assumptions made above, this monopoly would choose $Y = 0$ and $p = \infty$.

2. Individual Rational Adjustment Behaviors

It is now possible to rewrite the equations defining individual behavior, computing the sum over the expectations for the industry prices. From equations 35 and 37, one derives:

$$Y_1^i = Y^* - \varepsilon(S_0^i - S^*) - \varepsilon_s(S_0 - S^*) \tag{44}$$

$$p_1^i = p^* - \varepsilon'(S_0^i - S^*) - \varepsilon'_s(S_0 - S^*) \tag{45}$$

in which:

$$\varepsilon_s = \frac{1 - \rho^*}{2} - \varepsilon > 0, \quad \varepsilon' = \frac{c}{2}\varepsilon$$

$$\text{and} \quad \varepsilon'_s = \frac{c}{2}\left(\varepsilon_s + \frac{\psi + 1}{\psi - 1}\frac{1 - \rho^*}{2}\right) = \frac{c}{2}\left((1 - \rho^*)\frac{\psi}{\psi - 1} - \varepsilon\right)$$

The aggregation of equation 44, for the whole industry yields equation 42, since $\varepsilon_a = \varepsilon + \varepsilon_s$.

Equations 44 and 45 above describe a rational behavior expressed in terms of adjustment. Enterprises change their production levels and prices in response to the past differences between supply and demand:

1. Coefficients ε and ε' express the impact of the difference between supply and demand as it is felt at the level of individual firms, by the observation of their own inventories, on firms' output and price decisions respectively.

2. Coefficients ε_s and ε'_s expresss the impact of the difference between supply and demand at the industry level, on firms' price and output decisions respectively. Present excess supply in the industry leads to the expectation that competitors will diminish their prices and that individual demand will be reduced. This expectation induces firms to diminish both their outputs and prices, even more than they would do on the basis of their individual observations (their own inventories).

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