

TEMPORARY EQUILIBRIUM :

MONEY, EXPECTATIONS AND DYNAMICS

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A B S T R A C T

It is attempted in this review to present in accessible terms modern developments in the theory of temporary equilibrium and of economic dynamics initiated by the publication of Value and Capital. Attention is focussed on 1) the integration of money and financial assets in general equilibrium theory, and the clarification of the choice-theoretic structure of Keynesian macroeconomic models and of their links with the theory of imperfect competition ; 2) the occurrence and multiplicity of long run steady states, with self-fulfilling expectations, of the economic or social system (stationary states, deterministic cycles, stationary stochastic processes), and 3) the stability, or unstability, of these long run steady states when traders employ pre-specified learning processes.

Keywords : Temporary equilibrium, economic dynamics, money, expectations, business cycles, learning.

JEL Classification : 021, 031, 131, 311.

EQUILIBRE TEMPORAIRE : MONNAIE, ANTICIPATIONS ET DYNAMIQUE

R E S U M E

On essaie dans cette revue de présenter en termes accessibles les développements modernes de la théorie de l'équilibre temporaire et de la dynamique économique initiés par la publication de Valeur et Capital. L'accent est porté sur 1) l'intégration de la monnaie et des actifs financiers dans la théorie de l'équilibre général, et la clarification de la structure décisionnelle des modèles macroéconomiques Keynésiens et de leurs liens avec la théorie de la concurrence imparfaite ; 2) l'existence et multiplicité des états de long terme, avec anticipations auto-réalisatrices, du système économique ou social (états stationnaires, cycles déterministes, processus stochastiques stationnaires), et 3) la stabilité, ou instabilité, des états de long terme quand les agents utilisent des processus d'apprentissage donnés.

Mots clés : Equilibre temporaire, dynamique économique, monnaie, anticipations, fluctuations économiques, apprentissage

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TEMPORARY EQUILIBRIUM :
MONEY, EXPECTATIONS AND DYNAMICS^{*}

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My personal debt toward Value and Capital is important. As a graduate student in math and physics in Paris in the early sixties, I had the fortune indeed to come across this great book. By contrast with the few other pieces in economics I had tried to read before, Value and Capital struck me as proposing a unified and consistent theoretical framework, I should say a research program, for the study of static as well as dynamic economic systems. It is fair to say that my work is still, some 25 years later, very much influenced by this early intellectual encounter.

The emphasis throughout the book was to model economic units as maximizing their objective functions under well defined constraints, and interacting through markets. The first part of the book (Chapters I to VII) dealt with Static Walrasian equilibrium. The later chapters on Temporary Equilibrium, which laid down broad foundations for the study of "the economic system as a process in time," were most fascinating. By elaborating upon Marshall's period analysis, Hicks introduced there the notion of a Spot Economy, in which economic units traded goods and services for immediate delivery, money and a short list of financial assets, at the outset of each period – on "Monday" of each "week". At each date every individual would draw a plan for the current and future weeks in view of current prices and interest rates, and of her/his expectations about the future values of these variables. On each Monday, however, only current demands and supplies were matched by

movements of current prices and interest rates, whereas plans for the future and expectations, being uncoordinated, might, and in general would, be inconsistent. Hicks then sketched (Chaps. XXIII and XXIV) how the trade cycle could be studied by looking at the evolution in time of the sequence of temporary equilibria. Most of the analysis in Value and Capital dealt with a competitive economic system ; but Hicks was careful enough to note that the whole approach could be applied to models with imperfect competition and wage or price rigidities. He remarked in fact, in 1939, that Keynes' views on unemployment could be reinterpreted as a temporary equilibrium model with sluggish wages (Chap. XXI), thus anticipating somewhat what was to become known among macroeconomists in the fifties as the "neoclassical synthesis".

Hicks contrasted his Spot Economy with what he called a Futures Economy, in which economic units would exchange goods and services at some initial date, not only for immediate delivery, but also for delivery in the future for a considerable period ahead. He remarked that in such a set-up, "everything was fixed up in advance" through forward trading. Then "plans would be co-ordinated; and, for practical purposes, expectations would be co-ordinated too" (p. 136) ¹. Hicks argued that such a Futures Economy "can have no claims to be a good approximation to reality" (p. 140). "Uncertainty about the future, and the desire to keep one's hands free to meet that uncertainty [are bound to] limit the extent of forward trading" (p. 139). Hicks worked in Value and Capital with "definite", i.e. subjectively certain, expectations, and did not consider contracts that would be contingent upon uncertain events. Yet one can view the modern theory of competitive equilibrium, as elaborated later so beautifully by Kenneth Arrow, Gerard Debreu and Lionel McKenzie (see Debreu,

1959), as giving rigorous foundations to the concept of a Futures Economy in a broader context involving risky prospects and a complete set of markets for contingent contracts. Hicks' arguments, when transposed to a such a modern framework, however, make clear that one should expect markets for contingent contracts to be incomplete in actual economies, because it would be too difficult and costly to describe exhaustively in advance all possible contingencies of a complex, uncertain environment. Considering an incomplete market structure such as Hicks' Spot Economy, leads to study "the economic system as a process in time" : trade takes place sequentially, expectations and plans for the future are no longer co-ordinated through forward trading, forecasting mistakes and thus learning have to be explicitly taken into account.

1. The Temporary Equilibrium approach

The central features of the Temporary Equilibrium approach can be described with the help of a simple example. Let the state of the economy in any period be represented by an n -dimensional vector x , in some feasible set X , and assume, for the sake of the exposition, that a temporary equilibrium at date t depends only on the (subjectively certain) forecasts made at that date about the future by the economic units $i = 1, \dots, m$, that is $x_{t+1}^e, \dots, x_{m,t+1}^e$, through the Temporary Equilibrium map (a function from the $(m+1)$ -fold product of X into R^n).

$$(1.1) \quad T(x_t, x_{t+1}^e, \dots, x_{m,t+1}^e) = 0.$$

This relation summarizes how individual current decisions are made, and co-ordinated at any date, for a given state of expectations about the future. It is determined solely by the fundamentals of the economic system (which need

not, I should like to emphasize, be competitive) or the game under consideration. To describe the evolution of the system "as a process in time", one needs to specify in addition how individuals forecast the future at each moment, on the basis of their a priori knowledge of the structure of the system, and of their information about current and past history. For the sake of concreteness, let information at any date t be completely described by the sequence of states (x_t, \dots, x_{t-L}) , where L is the largest memory of the individuals; the way in which economic units process that information may then be summarized by the forecasting rules, or expectations functions

$$(1.2) \quad x_{i,t+1}^e = \psi_i(x_t, \dots, x_{t-L}), \quad i = 1, \dots, m,$$

which are taken as given characteristics of the individuals, on the same level as tastes, endowments or technologies.² A temporary equilibrium is then determined, given past history, by replacing forecasts in (1.1) by their expressions (1.2), and solving for x_t the resulting relation

$$(1.3) \quad W(x_t, x_{t-1}, \dots, x_{t-L}) = 0.$$

Studying the system "as a process in time", amounts to analyzing the properties of the trajectories generated recursively through (1.3).

Anticipating again modern debates, Hicks discussed in Value and Capital the wider notion of Equilibrium over Time, in which the system is not only in temporary equilibrium, but expectations are also realized, at all dates (p. 132). He dismissed it as a poor approximation to reality, however, out of special circumstances, e.g. when prices are stable. In the simple model described here, an Equilibrium over Time may be (and usually is) defined as an infinite sequence of states x_t , for $t = 1, \dots$, and of forecasts such that at

all times, the temporary equilibrium relation (1.1) is met and expectations are realized, $x_{i,t+1}^e = x_{t+1}$ for all i and t . The corresponding sequence of equilibrium states satisfies then the recurrence equation

$$(1.4) \quad T(x_t, x_{t+1}, \dots, x_{t+1}) = 0.$$

While logically consistent, defining in this way an Equilibrium over Time is a little awkward, for it makes forecasts formulated in period t depend upon the state that will actually prevail in the future. We want clearly to describe "the economic system as a process in time", that is the past to determine the present and not the other way around, whether or not expectations are realized. To meet this requirement, one must define an Equilibrium over Time as a sequence of temporary equilibria generated recursively by (1.1), (1.2) as before, with the traders' expectations functions being given but appropriately chosen so as to lead indeed to perfect foresight. There are usually many forecasting rules that achieve this purpose. For instance, if there is a stationary state defined by $T(\bar{x}, \dots, \bar{x}) = 0$, specifying $\psi_i = \bar{x}$ in (1.2) yields $x_t = \bar{x}$ as a sequence of temporary equilibria along which expectations are realized. More generally, if (1.4) defines implicitly x_{t+1} as a function of x_t , i.e. $x_{t+1} = F(x_t)$, choosing $\psi_i = F(F(x_{t-1}))$ in (1.2) implies that the corresponding sequences of temporary equilibria obey $x_t = F(x_{t-1})$, and there is indeed perfect foresight.³ The Temporary Equilibrium approach encompasses therefore, in principle, the case where expectations are realized, through an appropriate choice of the traders' forecasting rules (self-fulfilling expectations functions).

The approach is of course more general. Requiring a priori perfect

foresight is very demanding, since it means that traders know beforehand the true structure of the system, i.e. the temporary equilibrium equation T , and co-ordinate their expectations (forecasting rules) to achieve intertemporal consistency. The temporary equilibrium approach allows us more realistically, to portray the economic units as learning only progressively, in a decentralized fashion, the dynamic laws of their environment. Expectational mistakes then occur in the short run, but one can ask whether the resulting sequences of temporary equilibria do or do not converge in the long run to particular trajectories along which forecasting errors vanish. In this way, self-fulfilling expectations would be the asymptotic outcome, as they should, of a well defined learning procedure.

Clearly, the approach sketched here is not confined to the simple example presented above. The state vector may include exogenous as well as endogenous variables, and parameters that are characteristic of the Government's policies (or policy rules). The current temporary equilibrium state may depend upon decisions made in the far distant past ; traders may make plans for many periods ahead. The state variable may be random, and the traders' expectations functions take the form of conditional probability measures. The dynamic evolution of the economic system is then described by stochastic processes of temporary equilibria.

There are at least three sets of issues on the general research agenda of the temporary equilibrium approach, as already spelled out sometime ago (see Grandmont, 1977). There is first the analysis of the existence and the properties of a temporary competitive or non-competitive equilibrium for given general expectations functions. I summarize in Section 2 how consideration of

this matter by general equilibrium theorists led in the early seventies to a significant clarification of quite a few central questions in the theory of money or of capital asset markets, and to a better understanding of the choice theoretic structure of Keynesian macroeconomic models of unemployment. The second set of issues to be studied concerns the existence, multiplicity and properties of long run "steady states" of the system (stationary states, deterministic cycles, stationary stochastic processes) along which expectations are realized. This topic will be briefly reviewed in Section 3. The third set of issues is to evaluate the stability of long run steady states when traders employ given learning procedures (expectations functions), and to assess accordingly when learning may, or may not, lead asymptotically to self-fulfilling expectations. I shall have a brief look at this matter in Section 4.

2. Short run monetary equilibrium, and inelastic expectations

Hicks was very much concerned in Value and Capital by the potential instability of a temporary competitive equilibrium during the prices and interest rates adjustments that take place on a given "Monday". He considered in Parts III and IV of the book a competitive economy in which traders exchange sequentially on spots markets, goods and services, money and bonds. He asked then whether there were automatic stabilizers on a given "Monday" that would, after a perturbation away from a temporary equilibrium, make the system go back to it. As in the first, static part of the book, Hicks tended to focus attention on substitution effects. He argued that a temporary equilibrium should be stable when elasticities of expectations are less than 1, i.e. when a change in current prices changes expected prices in less than the same proportion. "So long as all changes in current prices are regarded as being

temporary changes, any change in current prices will induce substitution effects.... This substitution over time will be ... stabilizing" (pp. 250-51). By contrast, if elasticities of expectations were above unity, substitution effects should be destabilizing. Hicks laid great stress on the borderline case in which expectations of future prices are unit elastic, a condition that "has been simply taken for granted by the majority of economists" through "the habit of working in real terms" (p. 251), and that he believed "to have some relevance to actual situations" (p. 258). Hicks concluded that in such a case, the lack of intertemporal substitution effects would lead to an indeterminate price level. He argued that unit elastic expectations were at the root of Wicksell's famous disequilibrium "cumulative process", and of Keynes' claim in the General Theory that nominal wage and price flexibility would be unable to restore equilibrium in the short run market adjustment of a competitive monetary economy.

Hicks' analysis applied, strictly speaking, to what we call now an inside money economy – when private debts to and claims on the banking sector or the Government are equal. He assumed in fact that income effects due to variations in the purchasing power of initial debts and claims when current prices change, would vanish in the aggregate or might even be destabilizing, as the marginal propensity to spend of the debtor class was likely to be predominant (p. 264) – an argument spelled out again recently by Tobin (1980, Ch. 1) who traced it back to Irving Fisher. Hicks acknowledged in the 1946 edition of Value and Capital, after the contributions of Lange, Mosak, Pigou, that income effects would provide in theory the missing stabilizers when outside money was positive. But he doubted that these effects were strong enough to

alter significantly his previous analysis (Additional Note B, pp. 333-35).

Strangely enough, theoretical studies of monetary economics focussed after the war exclusively on such income effects. One of the most authoritative works in the area was Patinkin's remarkable attempt, in his Money, Interest and Prices (1956), at integrating money in the theory of value by using Hicks' Temporary Equilibrium method. Patinkin assumed static expectations – expected prices were equal to current prices – and concluded that income (real balance) effects were indeed effective stabilizers of the short run competitive mechanism when outside money is positive. The apparent implicit consensus among "neoclassical" macroeconomists in the sixties was that the picture would not be fundamentally affected if elasticities of expectations below or mildly above 1 were allowed.

This optimistic consensus was proved soon to be wrong. Neither Hicks nor Patinkin considered the question of the existence of a temporary competitive equilibrium with money – an issue raised forcefully by Frank Hahn (1965). But it was discovered in the early seventies through a class of simple examples (see Grandmont 1970, 1974) that income effects could in fact be too weak to guarantee the existence of a temporary competitive equilibrium, even with positive outside money and unit elastic expectations. Strong substitution over time, induced by elasticities of expectations significantly below 1, had to be brought back into the picture, not so much to stabilize short run adjustments of prices at a given date as Hicks thought, but to ensure merely the existence of a temporary competitive equilibrium with money. This was achieved in the early seventies by general equilibrium theorists in a temporary equilibrium setting through a series of independent contributions : by Arrow and Hahn

(1971) for the case of subjectively certain expectations as in Value and Capital, by Grandmont (1970, 1974) and Stigum (1969, 1972) for the general case in which forecasts take the form of probability distributions.

In the context of a pure exchange economy with outside money considered in Grandmont (1974), individuals trade at each date given but varying over time endowments of perishable goods and services, and nonnegative money holdings, the total supply of money being constant and positive. For simplicity, traders make plans for the current period and the immediate future only. A temporary competitive equilibrium is then found by equating to 0 aggregate excess demands for current goods and money,

$$(2.1) \quad Z(p, \mu_1^e, \dots, \mu_m^e) = 0 .$$

Here the vector Z of excess demands depends on the vector p of current positive money prices of the goods, on each trader's forecast μ_i^e – a probability distribution over positive future money prices of goods – and implicitly on the individuals' initial (non-negative) money balances. The model is completed by a specification of each trader's expectation function : a conditional probability distribution that links the forecast μ_i^e to current prices and past history, as in (1.2). In the simple case of only one good, there is typically a positive aggregate excess demand for the current good (excess supply of money) at all prices $p > 0$ if all traders' expected probabilities about the gross rates of inflation, i.e. μ_i^e/p , are bounded below by a large enough number, uniformly in p . The phenomenon may arise with unit elastic expectations, i.e. when the expected probabilities μ_i^e move proportionally to current prices. Real balance effects are then present indeed, but they are too weak to ensure the existence

of a temporary monetary equilibrium. One can also generate an excess supply of the current good at all prices p if all traders expect a significant deflation with elasticities of expectations well above 1 (see e.g. Grandmont, 1983, Ch. 1). By contrast, in the general case with many goods and services, one can show existence if there is a single "insensitive" trader i with positive endowments of goods and money who has "bounded" (uniformly tight) expectations ; or in other words if her/his forecast μ_i^e , considered as a function of current prices alone, given past history, lies in a compact set of probability distributions over future positive money prices. For in that case, when some or all current prices go to 0 or to infinity, the insensitive trader believes that with increasing probability, prices will go back to normal values. Substitution over time at the level of the insensitive trader is then enough to generate eventually an aggregate excess demand for some current good or money, and existence of a temporary monetary equilibrium follows from a standard fixed point argument.

These theoretical advances have been pursued in the early seventies, in general temporary equilibrium frameworks involving overlapping generations or infinitely long lived traders, to clarify basic issues in monetary theory such as the control of nominal interest rates or of the money supply by monetary authorities, when there are inside and outside money, open market operations or liquidity constraints on transactions of the Clower cash-in-advance type. Inexistence of a temporary competitive monetary equilibrium was found to be a pervasive phenomenon when elasticities of expectations are equal to or above 1. The phenomenon is particularly acute when credit (inside money) is taken into account. For then, in a competitive set-up with no credit rationing, even a

small group of traders who expect real interest rates to be very low may destabilize the whole market : they may wish to borrow a lot from banking institutions, and by attempting to spend the borrowed money immediately, generate an aggregate excess demand for current goods that persists at all current prices and nominal rates of interest. With credit, the conditions needed to guarantee the mere existence of a temporary competitive monetary equilibrium, subject to the policies of the monetary authorities, are accordingly quite stringent : all traders must then have "insensitive" (uniformly tight) expectations (Grandmont and Laroque, 1975)⁴. Important studies by Jerry Green (1973), Oliver Hart (1974) have shown moreover that in order to ensure existence of a temporary competitive equilibrium in markets for futures contracts or financial securities with unlimited short sales, the traders' beliefs about future spot prices should not only be "uniformly tight" as above but also mutually consistent, i.e. assign positive probability to a common open set of future spot prices (overlapping expectations).

While succeeding at integrating rigorously the theory of money and of asset markets into modern general equilibrium theory, these contributions confirmed in some broad sense the intuition of Hicks – and of Keynes for that matter – by giving it a precise meaning. In periods of fairly stable conditions, one may expect economic units to develop a notion of "normal" prices and interest rates. Deviations of current prices and interest rates from their "normal" values will then be regarded as temporary changes, and the conditions above for the existence of a temporary competitive equilibrium (uniformly tight and overlapping expectations), will be presumably satisfied. In periods of rapid change, elasticities of expectations are likely to be

fairly large, beliefs of various individuals are likely to differ sharply, and the competitive mechanism may go astray in the sense that a temporary competitive monetary equilibrium may not even exist. The necessity to close down actually financial markets in troubled times may be viewed as real life instances of the phenomenon.

I should mention briefly here that the temporary equilibrium method was also used successfully in the seventies to clarify the choice theoretic structure of Keynesian macroeconomic models. Following the early leads of Hansen (1951), Patinkin (1956) and the reappraisal of Keynes made by Clower (1965), Leijonhufvud (1968), simple examples designed by Barro and Grossman (1971, 1974, 1976) showed that postwar Keynesian macroeconomic models could be interpreted as short run equilibria with money at given prices and wages, in which workers and firms face binding, mutually reinforcing quantitative constraints on their supplies of labour and of output (multiplier, spillover effects). Other types of equilibria were also possible, in particular unemployment equilibria in which the activity level was limited not so much by aggregate demand but rather by the profitability of production (stagflation, classical unemployment)⁵. Models with quantity rationing of this sort were imbedded explicitly into general temporary equilibrium frameworks with endogenous price setting by optimizing traders, by Benassy (1973, 1976), Grandmont and Laroque (1976), Negishi (1977, 1979) along the lines laid down by Negishi (1961) to analyze monopolistic competition. Traders are endowed with beliefs about the relation between perceived quantity rations and quoted prices (perceived demand or supply schedules). Traders choose prices optimally at the outset of each date – for instance by equating marginal cost and expected

marginal revenue ; the ensuing temporary equilibrium is achieved by quantity rationing at the quoted prices. A number of studies, e.g. by O. Hart (1982), Benassy (1987a), have made more precise the circumstances under which the traders may be endowed consistently with beliefs about demand or supply schedules that are in some sense self-fulfilling – an issue raised by Hahn (1978)⁶. These theoretical investigations have made clear, at a formal level, that Keynesian macroeconomic models are in fact temporary equilibrium models with optimizing traders operating under conditions of imperfect (monopolistic, oligopolistic) competition, thereby confirming the intuitions of Hicks and of other early writers on that issue (see e.g. Harcourt (1987), Tobin (1987)). Modern research on the "microeconomic foundations" of Keynesian economics has followed this approach (see Blanchard 1987, L.E.O. Svensson 1986).

3. Steady states with self-fulfilling expectations

Once the evolution of the economic or social system is viewed as "a process in time", a natural question is whether this process has long run steady state equilibria along which expectations are realized.

Research on this question focussed initially in the seventies on deterministic stationary states, or stationary stochastic equilibria when there are random shocks. Analysis of deterministic stationary states aimed essentially at clarifying when money has positive value in the long run, in various set-ups. It was found early on that this property requires the presence of "frictions" in the trading process through the introduction of an overlapping generations structure, or of constraints on transactions of the Clower cash-in-advance type when traders are infinitely long lived. In the latter case, however, a long run monetary equilibrium is typically (constrained) ineffi-

cient ; but efficiency can be re-established by implementing an optimum rate of deflation through fiscal policy (lump sum monetary transfers). – an observation due to Friedman (1969). In all cases, monetary policy was found to be superneutral in the long run (Grandmont and Younès, 1973, Grandmont and Laroque, 1975).

Studying the existence of a stationary stochastic equilibrium with self-fulfilling expectations when there are random shocks required the introduction of new techniques which I review now⁷. Let $x = (s, y)$ be the state of the system in any period, where s describes all exogenous variables (shocks) and y all variables that are endogenously determined. Both s and y may involve lagged variables, i.e. variables that were determined prior to the period under consideration. The set X of feasible states is taken to a complete separable metric space.

Let the exogenous variables s be random and follow a time homogenous Markov process. In this abstract stochastic framework, the analogue of the Temporary Equilibrium map (1.1) is a relation that describes how the endogenous variables y are determined in any period, given the current exogenous variables s and the traders' beliefs about the future, i.e. μ_1^e, \dots, μ_m^e , where each μ_i^e is a probability distribution over X . An Equilibrium over Time is in this context a stochastic process of states and of probabilistic expectations such that, at all times, the Temporary Equilibrium relation is satisfied, and expectations are equal to the true (conditional) probability of the future state. These conditions lead to a recurrence relation of the form

$$(3.1) \quad x \longrightarrow G(x) ,$$

which says that the (true) probability distribution μ_{t+1} of the future state variable x_{t+1} must lie in a set $G(x)$, given that $x_t = x$ in the current period.⁸ I assume, as is often done in economic or game theoretic models, that the relation (3.1) has a closed graph: if the sequence x_n of states tends to x and the probabilities $\mu_n \in G(x_n)$ converge weakly to μ , then μ lies in $G(x)$. I assume also that we have found a compact invariant set for (3.1), or equivalently that X is compact, with $G(x)$ being non empty for every x .

If (3.1) defines actually a function G , stochastic Equilibria over Time follow a standard time homogenous Markov process on X : $G(x, A)$ is the probability that the random variable x_{t+1} lies in the (Borel) subset A , given that $x_t = x$. One can then compute recursively the probability distributions of the state variables by noting that when the probability distribution of x_t is μ_t , the distribution μ_{t+1} of x_{t+1} is

$$(3.2) \quad \mu_{t+1}(A) = \int_X G(x, A) d\mu_t.$$

The analogue of a stationary state in this context is the notion of an invariant probability measure, i.e. a fixed point of the application

$\mu_{t+1} = \Gamma(\mu_t)$ defined by (3.2). The existence of an invariant probability follows easily from a standard fixed point argument when the function G is continuous. For then the mapping Γ is also continuous and takes the space of probability distributions μ , which is compact and convex, into itself (Grandmont and Hildenbrand (1974, p. 264), Grandmont (1977, p. 566)).

Matters are more complicated when the application G defined in the recurrence equation (3.1) is multivalued. Since G has a closed graph, there is indeed a (measurable) selection g such that the probability distribution $g(x)$

lies in $G(x)$ for all x . For every such selection, a stochastic Equilibrium over Time follows a time homogenous Markov process with transition probability $x \rightarrow g(x)$ on the compact invariant set X , but there may not exist an invariant probability, since g is not necessarily continuous.⁹ In fact, the multivalued application (correspondence) $\mu_t \rightarrow r(\mu_{t+1})$ defined as in (3.2) when G is replaced by all of its possible measurable selections g , has a closed graph but may have no fixed point. The existence of such a fixed point, and thus of a measurable selection with an invariant probability, will be guaranteed through Kakutani's fixed point theorem if $G(x)$ is convex for every x , for then the correspondence r is also convex-valued (Hellwig (1980), Blume (1982)). The convexity property may arise in some contexts when there is enough "dispersion" of the underlying characteristics of the system. In other contexts, one can convexify the correspondence G by adding noise, i.e. by choosing randomly at each date equilibria among which traders are indifferent¹⁰.

The analysis outlined above yields a set of fairly general conditions under which a stationary stochastic equilibrium with self-fulfilling expectations does exist. These techniques do not give any information, however, about the multiplicity of these equilibria, nor about the possible existence of other types of steady states, such as cyclic stochastic or deterministic equilibria. A lot of work has been undertaken on these issues in the seventies and the eighties, and led to the growing realization that there may be an amazing multiplicity of such steady states in economic or social settings. Multiplicity of equilibria has been of course a fact of life for a long time in game theory (the folk theorem in repeated games, correlated equilibria). Work in economics has shown that there may be a large number of cycles in simple

competitive contexts when there are no shocks to the system (Benhabib and Day (1982), Grandmont (1985)). Following suggestions by Cass and Shell (1983), Azariadis (1981) also proved that there may be a large multiplicity of stationary stochastic equilibria driven by random factors that influence the traders' expectations but leave unaltered the system's fundamentals (sunspots). Further research has shown that such random expectations-driven fluctuations arise in a wide variety of contexts ¹¹. These results have obvious consequences for the theory of business cycles. They confirm in particular the views of earlier students of the trade cycle (Keynes (1939), Hicks (1950)), who were very much concerned by the possibility that recurrent, non-explosive business fluctuations may be caused significantly by volatile expectations (market psychology, animal spirits). I sketch now how the phenomenon can arise in a simple example.

Consider an overlapping generations model with a single representative individual in each generation who lives two periods, works and saves in the form of money when she/he is young, and consumes when she/he is old. On the production side, one unit of labour yields one unit of output (perishable consumption good) in the same period. The supply of money $M > 0$ is constant. With competitive markets, the equilibrium real wage is 1 at all times, and a temporary equilibrium state may be described by a single real number : equilibrium output y_t , which is equal to the labour supply ℓ_t of the young, or to the consumption c_t of the old, or to the equilibrium real balance M/p_t . The traders' characteristics are assumed to be the same in each generation. The "fundamentals" of the system are thus constant over time.

I look first at deterministic equilibria when expectations are "definite", i.e. subjectively certain. A young trader born in period t will then maximize her/his utility for current leisure and future consumption $V_1(\ell_t^* - \ell_t) + V_2(c_{t+1}^e)$ subject to the budget constraints $p_t \ell_t = m_t$ and $p_t \ell_t = p_{t+1}^e c_{t+1}^e$, where the superscript "e" stands for "expected". Under standard assumptions, an interior solution is then characterized by the budget constraints and the first order condition

$$(3.3) \quad \ell_t V_1'(\ell_t^* - \ell_t) = c_{t+1}^e V_2'(c_{t+1}^e).$$

The Temporary Equilibrium relation between the current state $y_t = M/p_t$ and the trader's expectation about the future state takes thus the form

$$(3.4) \quad v_1(y_t) = v_2(y_{t+1}^e),$$

where v_1 and v_2 stand for the functions on the left and right members of (3.3), respectively. In fact, since v_1 is increasing and can be inverted, the

Temporary Equilibrium relation can be put in the equivalent form $y_t = x(y_{t+1}^e)$,

where x is equal to the composition of v_1^{-1} and v_2 . The graph of the function

x , i.e. of the relation $\ell_t = x(c_{t+1}^e)$, represents the traditional offer curve

of the representative trader, that is the locus of all optimum combinations

(ℓ_t, c_{t+1}^e) when the ratio of the current price to the expected price varies.

A deterministic Equilibrium over Time is described by a sequence of states y_t that satisfies the recurrence equation $y_t = x(y_{t+1})$. Thus whether or not this simple system displays multiple steady state equilibria with self-fulfilling expectations depends ultimately on the shape of the offer curve.

Increasing the ratio p_t^e/p_{t+1}^e induces as usual substitution and income effects. These effects lead both to an increase of the demand for future consumption, but work in opposite directions for current leisure. If the substitution effect dominates everywhere, the offer curve is monotone (the function x is increasing), and there can be no cycle. By contrast, Fig. 1.a describes the case of an extreme conflict between substitution and income effects, which leads to the existence of cycle of period 3. This fact then implies, from a theorem by Sarkovskii, that the system has infinitely many cycles with perfect foresight : at least one cycle of period k , for every k (Benhabib and Day (1982), Grandmont (1985)). The existence of a cycle of period 3 is fairly easy to verify in this simple case, since it can be identified to a fixed point of the third iterate x^3 of the function x , i.e. of the composition of x with itself three times, that differs from a stationary state. In the case of Fig. 1.a, one has $x^3(y) > y$ for y close to 0 , while $x^3(y^*) < y^*$; the application x^3 has thus a fixed point between 0 and y^* .

Fig. 1.a

Fig. 1.b

The above cycles are driven by cyclical, self-fulfilling expectations. One can also show that there are many stationary stochastic equilibria that are driven by random but self-fulfilling expectations, when the conflict between substitution and income effects is significant.

Let a young trader born at t believe that the price of the good p_{t+1}^e , and thus $y_{t+1}^e = M/p_{t+1}^e$, will be random in the future. If the trader is an expected utility maximizer, working through the first order conditions as in (3.3) leads to a Temporary Equilibrium relation that looks like (3.4), or

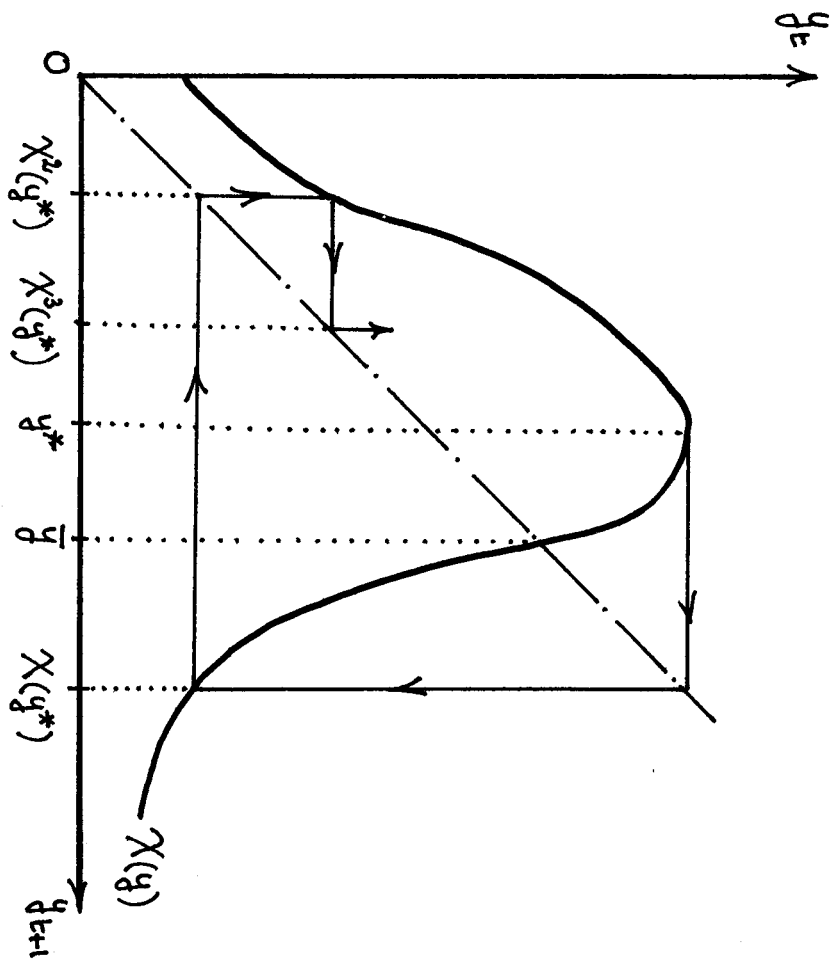


Fig.1.a

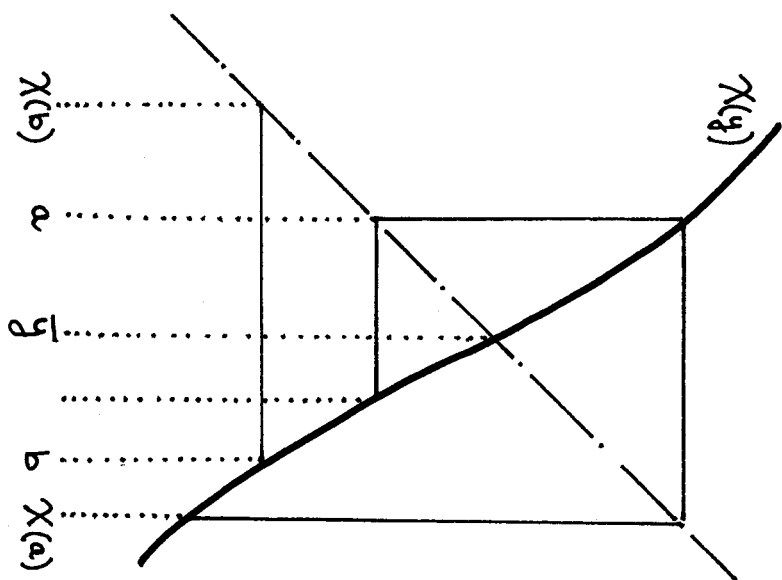


Fig.1.b

$$(3.5) \quad v_1(y_t) = E_t v_2(y_{t+1}^e),$$

where the mathematical expectation is taken relatively to the subjective conditional probability distribution of the random variable y_{t+1}^e . A stochastic equilibrium with self-fulfilling expectations will be thus a sequence of random state variables y_t satisfying (3.5) for $t = 1, \dots$, with the expected super-script removed. There are many such equilibria : they are all generated through the stochastic difference equation

$$(3.6) \quad v_2(y_{t+1}) = v_1(y_t) \eta_{t+1},$$

where the positive random variables η_{t+1} satisfy $E_t \eta_{t+1} = 1$, but are otherwise arbitrary.¹²

We are interested in stochastic equilibria that remain bounded away from 0 and from infinity at all times. By looking at (3.5) or (3.6), one sees that, given a stochastic equilibrium, the smallest closed interval $[a, b]$ containing all y_t with probability one, is such that the image of $[a, b]$ by v_1 is contained in its image by v_2 . Or equivalently, since v_1 is increasing, the interval must be contained in its image by the application χ : the interval is invariant in the deterministic recurrence from y_t to y_{t+1} induced by $y_t = \chi(y_{t+1})$. Conversely, once an invariant interval has been found, one can construct recursively stochastic equilibria that remain in it at all times – in fact an infinity of them – by choosing appropriately the random variables η_{t+1} in (3.6). This argument shows that there cannot exist stochastic equilibria, other than the monetary stationary state itself, in an invariant interval $[a, b]$ satisfying $a > 0$ when the substitution effect dominates everywhere, since then the offer curve is monotone : the only closed invariant intervals reduce in that case to

the monetary stationary state \bar{y} , to the origin $y = 0$, and to $[0, \bar{y}]$.

By contrast, there are many invariant intervals arbitrarily near the stationary state \bar{y} when the income effect is significant enough to ensure that $\chi'(\bar{y}) < -1$ (Fig. 1.b). For any such interval, one can solve (3.6) for y_{t+1} (the function v_2 is invertible near \bar{y}) and choose – again in infinitely many ways – particular random variables η_{t+1} so that the process y_t is time homogenous Markov with a continuous transition probability. The existence of an invariant probability distribution follows then by the arguments presented earlier in this section¹³.

Ongoing research along these lines shows that the existence of multiple expectations-driven cycles or stochastic equilibria is a pervasive phenomenon in a variety of settings. Equilibria of this sort do occur in higher dimensions, for instance in overlapping generations models with productive capital and a monotone offer curve. They do also occur in optimum growth models for high discount factors, or in models with infinitely long lived traders facing binding liquidity constraints on their transactions. These studies confirm indeed, under increasingly plausible assumptions, that the old Keynesian view according to which a significant part of actual business cycles may be attributed to volatile expectations, has to be taken seriously, even if one requires that expectations be realized¹⁴.

4. Learning and the saddle point property

An important feature of the Temporary Equilibrium approach is that it allows to describe the individual traders as learning progressively the dynamics of their environment. A central issue is then to know whether

the corresponding sequences of temporary equilibria do or do not converge to steady states with self-fulfilling expectations, and to assess to which extent the models the traders use eventually coincide with the correct one(s). The issue is all the more acute when there are many possible long run steady states, as was shown to be often the case in the preceding section.

There has been a significant amount of work in the seventies on the stability of a stationary state in deterministic overlapping generations models with pre-specified learning rules, see Fuchs and Laroque (1976), Fuchs (1976, 1977, 1979). The issue has been taken up again more recently in various settings : stability of a stationary state, of deterministic cycles or stochastic sunspot equilibria with overlapping generations (Benassy and Blad (1985), Grandmont (1985), Grandmont and Laroque (1986), Woodford (1986b)), convergence to rational expectations in stochastic linear macroeconomic models (see, e.g. Bray (1982), Bray and Savin (1986), Evans (1985, 1986), Fourgeaud, Gourieroux and Pradel (1986), Marcet and Sargent (1986)), stability of specific equilibrium solutions in game theory (Fudenberg and Kreps (1988)). Although the diversity of the specifications does not seem to lead yet to a unified theory, two elements appear to play often a significant role in the analysis. The first one is the saddle point property : given past history in a small neighborhood of a steady state, there is a unique Equilibrium over Time in that neighborhood that converges to it. In a deterministic set up, this means that the number of stable characteristic roots of the steady state in the dynamics with perfect foresight, is equal to the number of variables that are pre-determined at each date. The second element involves the spectral (extrapolative) properties of the traders' learning processes. When the traders are able

to extrapolate from past time series a rich enough set of frequencies, convergence to a steady state along which expectations are realized seems to be closely related to the steady state having the saddle point property. Ongoing research suggests that as a consequence, all traders cannot succeed in learning the dynamics with self-fulfilling expectations completely, i.e. in a whole neighborhood of the steady state. There is however no general result yet available along these lines, and I shall only sketch here a simple deterministic example that is drawn from some current joint work with Laroque ¹⁵.

Let a temporary equilibrium state x_t at date t , a real number, be determined by the previous state x_{t-1} and the forecasts made at date t about the immediate future by the traders $i = 1, \dots, m$, through the equation

$$(4.1) \quad T(x_{t-1}, x_t, x_{1,t+1}^e, \dots, x_{m,t+1}^e) = 0,$$

where the Temporary Equilibrium map T is real-valued and continuously differentiable. An Equilibrium over Time is then a sequence of states satisfying

$$(4.2) \quad T(x_{t-1}, x_t, x_{t+1}, \dots, x_{t+1}) = 0.$$

We focus attention on what happens near a stationary state defined by

$T(\bar{x}, \dots, \bar{x}) = 0$. If a_{-1} , a_0 , a_{i1} denote the partial derivatives of T with respect to x_{t-1} , x_t and $x_{i,t+1}^e$, evaluated at \bar{x} , (4.2) can be solved uniquely for x_{t+1} near the stationary state when $a_1 = \sum_i a_{i1} \neq 0$. This determines locally a perfect foresight dynamics of the form

$$(4.3) \quad x_{t+1} = F_{\text{loc}}(x_t, x_{t-1}).$$

Stability of the stationary state in these dynamics means that all roots of the corresponding characteristic polynomial

$$(4.4) \quad Q_F(\lambda) = a_1 \lambda^2 + a_0 \lambda + a_{-1} = 0$$

are stable, i.e. satisfy $|\lambda| < 1$ (I assume away from now on the exceptional cases where some roots have modulus 1). As there is here a single pre-determined variable, the stationary state will be a saddle point if and only if (4.4) has a stable real root λ_1 in the interval $(-1,1)$, while the other root λ_2 lies outside that interval. Since the characteristic polynomial can be written $a_1(\lambda - \lambda_1)(\lambda - \lambda_2)$, this means that $Q_F(\lambda)$ has opposite signs for $\lambda = 1$ and $\lambda = -1$

$$(4.5) \quad Q_F(1) Q_F(-1) < 0, \text{ or } |a_0| > |a_{-1} + a_1|.$$

Consider now the dynamics of temporary equilibria when the traders do not know beforehand the structure of the system, and are endowed with given learning behaviours that are summarized by the forecasting rules

$$(4.6) \quad x_{t,i,t+1}^e = \psi_i(x_t, \dots, x_{t-L}).$$

I shall make precise later how this formulation embodies the particular case where the traders try to discover some unknown parameters that characterize their environment. As in section 1, replacing forecasts in (4.1) by their expressions (4.6) yields a recurrence equation

$$(4.7) \quad W(x_t, x_{t-1}, \dots, x_{t-L}) = 0,$$

which determines implicitly x_t given past history. The assumptions below will imply that $\psi_i(\bar{x}, \dots, \bar{x}) = \bar{x}$; in that case $x_t = \bar{x}$ is a stationary solution of (4.7). I shall suppose, although the assumption is quite strong, that expectations functions are continuously differentiable, with c_{ij} being the partial derivative of ψ_i with respect to x_{t-j} , evaluated at \bar{x} . Then (4.7) can be

solved uniquely for x_t near the stationary state when $a_0^* = a_0 + \sum_i a_{i1} c_{i0} \neq 0$.

Stability of \bar{x} in the resulting local dynamics with learning means that the roots of the corresponding characteristic polynomial

$$(4.8) \quad Q_W(\lambda) = a_0 \lambda^L + a_{-1} \lambda^{L-1} + \sum_{i,j} a_{i1} c_{ij} \lambda^{L-j} = 0$$

are all stable, i.e. satisfy $|\lambda| < 1$. This implies that $a_0^* Q_W(1) > 0$,

otherwise (4.8) would have a real root exceeding 1; similarly, one must have

$a_0^* (-1)^L Q_W(-1) > 0$, otherwise there would be a real root less than -1. Thus

if \bar{x} is stable in the local dynamics with learning induced by (4.7), one has

$$(4.9) \quad (-1)^L Q_W(1) Q_W(-1) > 0.$$

It is now fairly easy to verify that the stability of the stationary state in the dynamics with learning implies that it is a saddle point in the perfect foresight dynamics, under mild conditions on the spectral

(extrapolative) properties of the expectations functions. Suppose that if the traders observe a time series that oscillates between two values, they believe that the pattern will also obtain in the future, i.e. $\psi_i(x,y,x,y,\dots) = y$ for x,y near \bar{x} . Differentiating and rearranging yields

$$(4.10) \quad \sum_j c_{ij} = 1 \quad \text{and} \quad \sum_j c_{ij} (-1)^{-j} = -1.$$

Computing the values of $Q_W(\lambda)$ and $Q_F(\lambda)$ for $\lambda = \pm 1$ shows that $Q_W(1) = Q_F(1)$

and $(-1)^L Q_W(-1) = -Q_F(-1)$ in such a case. Then stability of the stationary

state in the dynamics with learning implies (4.9), which is equivalent to

(4.5): the stationary state is a saddle point under perfect foresight.

When the stationary state is stable in the dynamics with learning, forecasting errors do vanish in the long run since both states x_{t+1} and forecasts x_{t+1}^e converge to \bar{x} . Each trader learns eventually the value \bar{x} of the stationary state. Yet it is in the interest of each trader to make more accurate predictions on the transition path and to try to learn more about the dynamics of her/his environment. What follows suggests however that they cannot all succeed, i.e. learn the perfect foresight dynamics (4.3) completely.

To address this question, we have to specify how the foregoing formulation embodies the particular case where the traders try to learn progressively some unknown parameters that characterize their environment. Let the views of a trader i about the dynamics of the system be represented by a class of models φ_i indexed by a vector of unknown parameters β_i . Forecasts at date t are then made according to the rules

$$(4.11) \quad x_{t+1}^e = \varphi_i(\beta_{it}, x_t, \dots, x_{t-K}) .$$

Each trader is supposed furthermore to revise at each date, in view of past data, her/his estimate of the unknown parameters through some more or less sophisticated statistical procedure that is summarized by a given estimation function

$$(4.12) \quad \beta_{it} = g_i(x_t, \dots, x_{t-L}) .$$

Replacing β_{it} in (4.11) by the estimate (4.12) yields an expectations function as in (4.6). Thus the dynamics with learning of the temporary equilibrium states x_t is determined by the recurrence (4.7) as before ; the corresponding sequences of estimates β_{it} are given in turn by (4.12).

The following argument shows that under our maintained assumption that traders extrapolate correctly time series that have period 2, one cannot generally expect the stationary sequences $x_t = \bar{x}$ and $\beta_{it} = \bar{\beta}_i = g_i(\bar{x}, \dots, \bar{x})$ to be stable in the dynamics with learning, and at the same time the model φ_i corresponding to $\bar{\beta}_i$ to coincide with the perfect foresight model (4.3) in some neighborhood of the stationary state \bar{x} .

Suppose indeed that for every i , and every sequence of states generated recursively through (4.3) from two arbitrary initial values x, y near \bar{x} , the subjective model (4.11) corresponding to $\bar{\beta}_i$ and the perfect foresight recurrence (4.3) lead to identical predictions. Assume furthermore that the estimation procedure (4.2) yields the estimate $\bar{\beta}_i$ for any such sequence, i.e. that the model corresponding to $\bar{\beta}_i$ is "identifiable". When the models φ_i and the estimation functions g_i are continuously differentiable, these conditions imply that every root λ of the perfect foresight polynomial (4.4) satisfies $\sum_j c_{ij} \lambda^{L-j} = \lambda^{L+1}$, and is thus also a root of characteristic polynomial (4.8), i.e. $Q_W(\lambda) = 0$. It should be now clear that $x_t = \bar{x}$, $\beta_{it} = \bar{\beta}_i$ is then unstable in the dynamics with learning. For if the stationary state were stable, it should have the saddle point property in the perfect foresight dynamics when the traders extrapolate time series displaying period 2; but that would mean that the characteristic polynomial (4.8) associated to the dynamics with learning has an unstable real root, and one gets a contradiction.

The foregoing example is rudimentary, but is nevertheless suggestive of the sort of results one might expect in the area. If the spectral (extrapolative) properties of the traders' learning processes are rich enough, a steady state that is stable in the dynamics with learning should have the

saddle point property in the dynamics with self-fulfilling expectations. In that case, the traders may succeed eventually in learning part – specifically, the stable part – of the dynamics with self-fulfilling expectations (as in Marcet and Sargent (1986)). But if the traders try to be more clever and learn more about their environment, these attempts will be self-defeating as they will generate unstability of the system. The intuition is that if the traders were successful, the dynamics with learning should eventually mimic the dynamics with self-fulfilling expectations, and therefore be unstable. There are still no satisfactory general results available along these lines, but the recent spur of activity on the topic makes one hope that decisive progress may not lie too far ahead.

F O O T N O T E S

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1. All references to Value and Capital are to the 2nd edition (1946).

2. One may envision more complex formulations in which the traders' information includes past forecasts as well, or an increasing time series of past states as the system evolves in time. It is shown in section 4 how this formulation embodies the particular case where the traders try to discover some unknown parameters that characterize the motion of their environment, as in Grandmont (1977, Section 2.4).

3. It may happen that (1.4) defines x_{t+1} as a multivalued function (correspondence) of x_t , in which case the function F of the text must be viewed as a (usually measurable) selection of this correspondence. I am voluntarily vague about the domain of definition of these functions, although the issue is sometimes tricky. Note that it is important not to make the expectations functions depend on the current state alone : specifying $\psi_i = F(x_t)$ in (1.2) would make the temporary equilibrium solution x_t of (1.3) indeterminate. Remark finally that when one defines an Equilibrium over Time as a sequence of states satisfying (1.4) at all dates $t = 1, 2, \dots$, one gets a multiplicity of equilibria that is due in part to the fact that this definition is essentially "static" (does not give any weight to past history). By contrast, viewing such equilibria as sequences of temporary equilibria with prespecified self-fulfilling expectations functions such as $\psi_i = F(F(x_{t-1}))$ gives back to history a major role as a determinant of the equilibrium path : the sequence of temporary equilibria generated recursively by (1.3) for $t = 1, 2, \dots$ is then determinate, given the initial condition x_0 .

4. For a relatively non-technical account of these developments, see Grandmont (1983).

5. Elements of these examples can be found in Solow and Stiglitz (1968), Younes (1970). The structure of equilibria with quantity rationing at given prices was made precise in a general microeconomic framework by Benassy (1973, 1975), Dreze (1975), Younes (1975). The terminology Keynesian-Classical unemployment was coined by Malinvaud (1977).
6. For a good account of these developments, see Benassy (1982, 1986, 1987b).
7. The abstract framework used below arose in the study by Grandmont and Hildenbrand (1974) of a general version of the overlapping generations model with money and stochastic endowments – see also Radner (1974).
8. The analogue of (3.1) in the deterministic case is obtained by solving (1.4) for x_{t+1} . The formulation of the text requires at first sight that traders have complete information about the state. It is unclear to me at this stage how the incomplete information case can fit in this abstract set up. One gets also a formulation like (3.1) with arbitrary expectations functions and exogenous shocks, see Grandmont and Hildenbrand (1974, Section 5), Grandmont (1977, Section 5), Hellwig (1980), Blume (1982, Section 4).
9. See Grandmont and Hildenbrand (1974, p. 259). There may be stochastic equilibria generated by (3.1) that are not time homogenous or even that are not Markov, since the selection g may depend on time, or may be randomly chosen at each date in a non-Markovian fashion.
10. See Hellwig (1980), Blume (1982), Duffie, Geanakoplos, Mas-Colell and Mc Lennan (1988). It may even happen that with enough dispersion of the characteristics of the system, every measurable selection induces an invariant probability, see Hellwig (1980).
11. See e.g. Azariadis and Guesnerie (1986), Grandmont (1986), Guesnerie (1986), Woodford (1984, 1986a).
12. The technique of transforming stochastic difference equations involving conditional expectations as in (3.5), into the more manageable form (3.6), can be found in studies of linear rational expectations models, see Broze, Gourieroux and Szafarz (1985). See also Farmer and Woodford (1984).
13. More generally, if there is a deterministic cycle, the interval $[a,b]$ determined by the smallest and the largest elements of the corresponding

periodic orbit is also a compact invariant interval that can support stochastic equilibria.

14. The general picture which seems to emerge when the fundamental characteristics of the system are constant, is that one can construct infinitely many stochastic equilibria driven by random but self-fulfilling expectations, arbitrarily near a deterministic stationary state, if (and only if when traders are expected utility maximizers) it is locally indeterminate. This property, which is a generalization to higher dimensions of the situation represented in Fig. 1.b, says essentially that the dimension of the stable manifold of the stationary state, in the deterministic dynamics with perfect foresight, exceeds the number of variables that are predetermined at each date. There is no general proof of this conjecture as yet, however. For more information on these questions, see the symposium issue of the Journal of Economic Theory (Oct. 1986), as well as Benhabib and Nishimura (1979, 1985), Benhabib and Laroque (1988), Woodford (1986c).

15. See Grandmont (1985, Section 3), Grandmont and Laroque (1986 ; 1987, Section 2).

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