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*THE STRUCTURE OF DYNAMIC MACROECONOMETRIC MODELS**

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ABSTRACT

THE STRUCTURE OF DYNAMIC MACROECONOMETRIC MODELS

The paper presents a method for analysing a given macroeconometric model aimed at enlightening its theoretical foundations hidden behind the apparently numerous and complex equations. The basic idea is to operate on the model as if it was a theoretical disequilibrium growth model, but with numerical parameters. A thorough application to the French Quarterly Model METRIC is given.

RESUME

LA STRUCTURE DES MODELES MACROENOMIQUES DYNAMIQUES

Le papier présente une méthode d'analyse d'un modèle macroéconomique quantitatif destinée à mettre pleinement en lumière ses fondements théoriques cachés dans ses équations en général nombreuses et complexes. La procédure consiste à traiter le modèle comme un modèle théorique de croissance en déséquilibre, mais avec des coefficients numériques. Une application approfondie au modèle METRIC illustre la méthode.

Mots clés : Modèles macroéconométriques - Sentiers stationnaires - Analyse structurelle des modèles - Systèmes dynamiques.

Key words : Macroeconometric Model - Steady State Growth Paths - Structural Analysis of Models - Dynamical Systems.

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Introduction

The building of macroeconometric models has generally two goals: produce explanatory structures able to test a theory about the working of a whole economy, and deliver forecasts with the best possible degree of accuracy. The production of reliable forecasts generates, in a quite natural way, a rather large number of equations in the model. As a matter of fact, a "medium size" model is understood as including two or three hundreds of equations, and a "large size" one, up to several thousends. Comparatively, a model of fifty equations or less is called a "core model" or a "maquette", i.e. a demonstration model unable to deliver "serious" forecasts. This large scale feature has the very unfortunate consequence of making the working of these representations rather opaque to the user. In other words, they are treated as "black boxes", thus contradicting their other ambition of being explanatory structures, stylized explanations of the working of the economy. This has been a strong argument against macroeconometric models because very small size "black boxes" are able to produce much cheaper competing forecasts (Sims (1980)).

So there has been a need, in defense of structural models, for methods especially designed to achieve a good control on their diferent mechanisms, and there exists now a substancial set of methods aimed at enlightening various aspects of a given model. (See, for instance,as general references : Hickman Ed. (1972), Deleau, Malgrange (1978), Kmenta, Ramsey Eds.(1981), Chow, Corsi Eds.(1984), Wallis Ed. (1984), Kuh et Al. (1985)).

We present here a method aimed at exhibiting the theory behind the specification of the model, exploiting the peculiar structure of standard macroeconomic models, which combine standard Keynesian disequilibrium in the short run with dynamic capital accumulation. Thus the basic idea is to operate on them as if they were purely deterministic disequilibrium growth models.

So, starting from a given macroeconomic model,the strategy consists in first looking for a steady state growth path solution, and in trying to interpret this path as equilibrium path. Then, by a conventional change of coordinate, the steady growth path is transformed into a steady state, constant through time, and so the disequilibrium dynamics can be studied around a well defined equilibrium state.

This procedure has the extra advantage of allowing a study of the model without being limited by any horizon of simulation, and the rigorous use of linear methods of investigation, which are from far the most powerful ones to highlight the true important features of the model.

We first present details of the methodology, and then give a thorough illustration on the basis of a compacted version of the French quarterly model METRIC. The developments stem essentially on our previous work (see e.g. Deleau, Malgrange (1978)), and form a synthesis of two unpublished studies of this model (Malgrange (1983), Kuh,Le Van, Malgrange (1984))

I - METHODOLOGY

1. Long run

1. Without loss of generality, macroeconometric models may be written under the form :

(1) $y(t) = f(y(t), y(t-1), x(t) ; \beta, t)$

with

$y(t) = (y_1(t), \ldots y_n(t))$, a nx1 vector of endogenous variables,

$x(t) = ((x_1(t), \ldots, x_m(t))$, a mx1 vector of exogenous variables,

β, a vector of parameters and t the time variable.

The variables x(t) and y(t) are supposed to represent levels. We recall that we are concerned here only by the deterministic part of the model. As the capital accumulation is endogenized in macroeconometric modelling, the consistent concept of "long run", or equilibrium, is that of steady state growth path –SSGP–, a moving equilibrium where the rates of growth of the levels, are constant through time.

Formally we are looking for the following solution to (1) :

$$(2) \quad \begin{cases} \bar{x}_i(t) = \bar{x}_i(0)\ a_i^t & i=1, \ldots, m \\ \bar{y}_j(t) = \bar{y}_j(0)\ b_j^t & j=1, \ldots, n \end{cases}$$

Thus a SSGP will consist of the determination of the vector of the n initial conditions $\bar{y}(0)$ and of the n trends $(b_1, \ldots, b_n)$, given $\bar{x}(0)$ and $(a_1, \ldots, a_m)$, verifying (1) at each date.

. 2. Some necessary specification constraints on the system (1) are required for a SSGP to exist. These constraints are not always satisfied in most macro-econometric models because they are not generally long run oriented. The most typical example is the presence of additive constants in equations the evolution of which is trended. Indeed the relation :

(3) $y(t) = \alpha\, x(t) + \beta,$

has no solution of the type (2) except if a=b=0 (no trend) or if $\beta=0$. It is however possible to reach homogeneity without altering substancially the structure of the system, for instance by reestimating (3) with the constraint $\beta=0$, or by introducing a time varying constant $\beta(t)$ consistent with a SSGP.

Our past experience suggests that a careful examination of the violations of these long run constraints may lead to question the model specification.

3. The equilibrium growth factors (b_i) can be determined in general before the level y(0), because they all derive necessarily from only three basic exogenously given rates :

γ = rate of growth of (Harrod neutral) technical progress

ϱ = rate of growth of foreign prices in domestic units

n = rate of growth of population

All the endogenous <u>and exogenous</u> variables of the system must actually grow at a rate determined by γ, ϱ and n: $g = (1+n)(1+\gamma)-1$ for <u>all</u> real variables, ϱ for <u>all</u> prices, n for <u>all</u> variables relative to population, $(1+g)(1+\varrho)-1$ for <u>all</u> nominal variables, and $(1+\gamma)(1+\varrho)-1$ for nominal wage rate.

This may readily be seen by considering the relations of a typical macro-model, especially the various accounting identities.

The only exception to this strong rule would occur in a closed economy (or an open economy with perfectly flexible exchange rates) in which no monetary variable were exogenously fixed. In that case it is easy to see that ϱ

would be determined simultaneously with initial levels, but then price level indetermination would occur.

Note that, not only the b_i's, but also the a_i's must be fixed consistently with γ, n and ϱ which form actually the only 3 degrees of freedom as far as steady state rates of growth are concerned.

4. We are now able to derive the n initial conditions $\bar{y}_j(0)$. Putting (2) into (1), for t=0, we get :

$$(4) \qquad \bar{y}_j(0) = f_j(\bar{y}_1(0) \ldots, \bar{y}_n(0) \;;\; \bar{y}_1(0)/b_1, \ldots, \; \bar{y}_n(0)/b_n, \; x(0), \beta, 0), j=1, \ldots, n$$

or equivalently, including rates of growth restrictions and dropping time index t :

$$(5) \qquad \varphi(\bar{y}, \; \bar{x}, \; \beta, \; \gamma, \; n, \; \varrho) = 0$$

Then, <u>any solution in y of (5) is the initial condition for a SSGP of (1)</u> (modified to eliminate long run heterogeneities).

<u>We observe that the (static) multipliers associated with (5) correspond to the "long run" effect of a sustained unitary multiplicative shock on $\bar{x}$.</u>

5. The system (5) is actually easier to analyse than a dynamic model, and building exact reduced size representation is straightforward by substitutions because of its static nature. So, we can make its underlying structure and theory more apparent, as we will see later on.

At this stage, we must note that the static model (5) may display a rather different behaviour than its dynamic counterpart (1).

This point can readily be seen from the example of a Phillips relation (6).

$$(6) \qquad \frac{W(t) - W(t-1)}{W(t-1)} = \beta_1 + \beta_2 \quad \frac{P(t) - P(t-1)}{P(t-1)} + \beta_3 \; U_N(t)$$

with $W(t)$, nominal wages, $P(t)$, price level and $U_n(t)$, unemployment rate.

On a SSGP, (6) becomes :

$$(7) \qquad (1+\gamma)(1+\varrho) - 1 = \beta_1 + \beta_2\,\varrho - \beta_3\,U_N$$

There are two implications of (7) : U_N becomes predetermined as a function of the only exogenously given rates of growth and the coefficients of the Phillips curve ; nominal wages W must be implicitly determined elsewhere in the model.

2. Dynamics

A straightforward but important step to tackle the dynamics of the model (1) is to put it under a detrended form, by the transformation :

$$(8) \quad \begin{cases} \hat{x}_i(t) = a_i^{-t}\, x_i(t) \\ \\ \hat{y}_j(t) = b_i^{-t}\, y_j(t), \end{cases}$$

so as to get :

$$(9) \qquad \hat{y}\,(t) = g(\hat{y}(t),\, \hat{y}(t-1),\, \hat{x}(t)\ ;\ \gamma,\, n,\, \varrho,\, \beta,\, t)$$

The usual structure of macroeconometric models is such that, when we correct from long run heterogeneities, time disappears in the system (9), as an explicit variable.

Let us give a short illustration of this point on the case of the Cobb-Douglas model :

$$q_t = A\, k_{t-1}^{\alpha}\, n_t^{\beta}\, \gamma^t,$$

with q_t production, k_t capital n_t employment and γ technical progress.

Looking for a SSGP of the form :

$$\begin{cases} \bar{q}_t = \bar{q}_0 \, a^t \\ \bar{k}_t = \bar{k}_0 \, b^t \\ \bar{n}_t = \bar{n}_0 \, c^t, \end{cases}$$

generates the two long run conditions :

$$a = b^{\alpha} \, c^{\beta} \, \gamma \qquad \text{- rates of growth}$$

$$\bar{q}_0 = A\left(\frac{\bar{k}_0}{b}\right)^{\alpha} n_0^{\beta} \qquad \text{- initial levels}$$

The transformation (8) of the initial model, taking into account the long run constraints, leads then to the stationnary following form :

$$\hat{q}_t = \frac{A}{b^{\alpha}} \, \hat{k}_{t-1}^{\alpha} \, \hat{n}_t^{\beta}.$$

So, in general, (9) may be rewritten as :

$$\text{(10)} \qquad \hat{y}(t) = g(\hat{y}(t), \hat{y}(t-1), \hat{x}(t) \; ; \gamma, n, \varrho, \beta)$$

2. The form (10) is especially suited for a careful study of the model of the type we want to work out, because it describes a stationnary dynamical system fluctuating around a well defined and time invariant equilibrium (formula (5)).

Simulation based methods can be undertaken on (very) long time span without risk of large bias in interpreting results due to non stationarity (and sometimes no well defined reference path).

The linear methods which are in general highly powerful tools to illuminate the "most active" part of a given model, can be carried out with rigor. Indeed, the approximation in the neighborhood of the equilibrium of (10) by the following linear <u>stationary</u> form :

$$(11) \quad \Delta\hat{y}(t) = D\Delta\hat{y}(t) + E\Delta\hat{y}(t-1) + F\Delta\hat{x}(t)$$

with D, E, F constant matrices of suitable dimension and Δ, deviations from the equilibrium, works in general very well, because of the usually "weak" non-linearity of macroeconometric models.

At last, the knowledge of the long run allows for a study of the model from now —time t=0— up to infinity and for a precise definition of the concepts of delays and more generally of the dynamical hierarchy between variables in reaching their asymptotic equilibrium value.

II - AN ILLUSTRATION ON A CONDENSED VERSION OF THE METRIC MODEL

We now give an illustration of the above methodology on the case of a French quarterly forecasting model. We first give a brief overview of the way the model has been "prepared" (section 2.1), before presenting the model itself (section 2.2) and studying successively its long run and dynamic features (section 2.3 and 2.4)

2.1 - Derivation of the model

1. METRIC is a dynamic quarterly model of about 900 equations built and used by the French government as a support for short term macroeconomic forecasting and for economic policy studies —see Artus et al.(1981)— Several abridged versions of this model have been built to better understand its properties, especially one in 1980 for pedagogical purpose by P.Artus.

This version which imbeds the same major behavioural equations as the large scale version, is unfortunately not published.

As we said in the previous section, the present methodology aims at shedding some light on the specification and on theoretical foundations of a given model. So, at the same time as we put it in a stationnary form, according to formula (10) it is sensible to eliminate mechanisms whose contributions to the overall behaviour of the model are modest.

Let us give here briefly the different steps of these preliminaries.

2. The first step has been to make the model consistent with SSGP constraints. For METRIC this is not a big issue. The corrections introduce a "bias" growing with time, but reaching hardly 8% after 20 quarters, of which more than 4/5 come from a quicker growth of foreign trade than production in the original model –EEC integration effect–, which contradicts the requirement of unique growth rate for all quantities.

We nead from the very beginning numerical values for the three basic long run growth rate, γ, n and ρ. In the present case they have been chosen from the average growth of the relevant variables within the sample period (1962.1 to 1978.4), leading to :

$\gamma = 0.8\ \%$, $n = 0.2\ \%$, $\rho = 2\ \%$,

on a quarterly basis[1] .

3. We then have transformed the model into a stationary dynamic system, on which a first sensitivity analysis has been run to eliminate the numerically unimportant mechanisms, and to compact its dynamic structure.

The criterium to evaluate the "importance" of a mechanism has been the proximity between the initial and the modified model of selected dynamic multipliers.

The most noticeable simplification in the case of METRIC has been the deletion of the financial sector whose contribution, in this version of the model, reveals to be very modest, as it can be seen on the two following figures 1 and 2, giving the dynamic effects of government spending on output Q and price level p with and without financial sector[2].

FIGURES 1 and 2 about here

This simplification had the consequence of reducing the dynamic order (i.e the number of lagged variables weighted by the lag length) of METRIC by roughly one third. The equations of investment (putty-clay structure), imports and exports (relative price effects) coagulating more than one half of the remaining lags, have been simplified too on the same basis.

2.2 - Presentation of the model

1. The "representative maquette" of METRIC model (3) is presented on table 1, and the numerical values of coefficients on table 2.

The left hand side of the table 1 (equations D1 to D25), gives the stationary dynamic model, and the right hand side (equations S1 to S25) its long run static correspondant.

The set of equations below represents in a conventional keynesian, demand oriented model, a small open economy with fixed exchange rate, sluggish prices, and wages determination by a Phillips curve.

We recall again the fact that some interesting features of the original METRIC are not present here because of their revealed minor influence, as the financial sector, or the presence of survey variables in expectations. Furthermore one must keep in mind that most fiscal and other policy instruments have been fixed and are imbedded in parameters.

TABLES 1 and 2 about here

2. The three first equation D1 to D3 describe the working of the employment market, leading to the imbalance in the job market, defined as the

logarithm of the ratio between unfilled demands and supplys for employment. The employment vacancies equation, D2, is written in the original version of METRIC in first differences to take into account the vintage structure of the technology, but the writing in levels does not change the working of the model as long as we do not perturbe directly the equation.

The four following equations, D4 to D7, derive in a rather standard way from the theory of labor and capital demand by firms facing a putty-clay technology with adjustment costs on both factors.

Consumption –eq.D8– depends from the real disposable income R_t in line with the traditionnal permanent income theory.

The inventory investment equation –eq.D10– formalizes a dynamic buffer stock mechanism.

Foreign trade –eq.D11 and D12– includes the three classic explaining variables, corresponding to the demand or "pull effect", the relative prices or "competitivity effect" and the capacity utilization or "push effect" (4).

The (optimal) production capacity –eq.D16– is determined consistently with the vintage technology, and the ratio between this supply and effective demand Q defines the degree of capacity utilization –eq.D15 –.

The production prive level is taken by firms, supposed in situation of monopolistic competition, as a sluggish adjustment toward the classical mark up pricing –eq.D17 and D18–.

Equation D19 to D22 describe the price structure of the different components of aggregate demand with a common specification of partial adjustment to a composite index of production price and import price.

The cost of capital is defined in a conventional way, D23, remembering that tax rates have been merged with the coefficients.

Equation D24 is the standard Phillips curve (5) with lagged indexation of wages to inflation, and the last equations summarizes the traditionnal "cascade" of interest rates by a relation between the long run real interest rate and the very short run money market rate.

The model includes only five explicit exogenous variables :

$\underline{N}$: Labor force

$\underline{D}_e$: Foreign demand

$\underline{P}_e$: Foreign price index

$\underline{G}$: Government spending

$\underline{r}_c$: Money market interest rate (nominal)

2.3 - Analysis of the structure of the long run

The present section is devoted to the analysis of the long run static version of the maquette. The equations S1-S25 of table 1 describe the set of conditions that must satisfy initial conditions of endogenous variables to generate, through D1-D25, a dynamic path constant over time, i.e a steady state. We will label this version SSM.

Thus SSM has the technical caracteristics of an equilibrium. Furthermore one can observe that the theory (or theories) behind the equations is (are) more transparent than in the dynamic version. At last, as a static system, it allows very easily size reduction and analytical study.

1. SSM has the form of a simultaneous equations model which must be solved by a fix point algorithm, knowing the values of the five exogenous variables. Table 3 gives thus the "long run" levels of the endogenous variables associated with the historical values of exogenous in 1979-1 and the percentage deviations between actual and long run levels.

TABLE 3 about here

So the actual level of production would exceed its long run level by almost 6%, or, put other words, with the chosen steady growth rates for quantities, would lead by about 6 quarters its long run value.

The price level would be too low by 4% –lag of six months– and the employment too high by 4% –advance of 20 quarters–.

These deviations look reasonnably modest, and validate the SSGP as a valuable reference path for the type of studies of the dynamic model we want to work out.

Although, at first sight, the "reasonable" gaps obtained between short run and long run, may seem essentially casual luck, this result is not totally surprising. Indeed, one can think that, the dynamic model goes through the mean values of the sample data set, if it has been estimated by least squares methods, and so does the long run model –because it is the same model–. As a direct consequence, if we fix exogenous variables at their mean value over the sample date set, the SSM should give too the mean values for the endogenous variables.

This property is however far from exactly verified in a model as the maquette :

(i) This model has not been reestimated after the numerous simplifications and alterations.

(ii) The "mean value" argument is valid only in the case of stationnary data with a sufficiently long sample set. Taking growth into account may lead to important biases, more especially as the results may be rather sensitive to the value of the three basic long run growth rates.

There is no space to discuss more deeply this topic here,[6] except just to conclude that one must be very careful in interpreting the numerical values of the levels given by SSM as a benchmark for an evaluation of the amplitude of actual disequilibrium.

Let us rewrite now SSM in a compact form, familiar to macro-theorists. We first observe that we can express all relative prices in terms of $\pi = p/p_e$. Secondly we need only the real wages $\omega = W/p$. At last we observe that U_c becomes a constant by combination of S6 and S16.

We are now able to get a compressed form for SSM.

The demand for goods results from equations S6 to S14 :

(C1) $\quad Q = C + I + IS + \underline{G} + X - M$

with :

$$C = \varepsilon_1 + (\varepsilon_2\, \omega\, L + \varepsilon_3\, Q)\, \pi^{\varepsilon_4}$$

$$I = \varepsilon_5 \left[(\omega/r)\, \pi^{\varepsilon_6}\right]^{1-b} Q$$

$$IS = \varepsilon_7 + \varepsilon_8\, (C+I+\underline{G}+X)$$

$$X = \varepsilon_9\, \underline{D}_e^{\varepsilon_{10}}\, \pi^{-\varepsilon_{11}}$$

$$M = (\varepsilon_{12} + \varepsilon_{13} \operatorname{Log} \pi)\, (Q+M),$$

or, formally :

(C1) $\quad Q = f_1(\pi, \omega L, \underline{D}_e, \underline{G}) + f_2(\pi, \omega, r)\, Q.$

The labor demand by firms is derived from S4 and S5 :

(C2) $\quad L = \varepsilon_{14} + \varepsilon_{15} \left[(\omega/r)\, \pi^{\varepsilon_6}\right]^{-b} Q = \varepsilon_{14} + f_3(\pi, \omega, r)\, Q$

Three more equations are necessary in order to determine π, ω and r.

The combination of S17 and S18 gives the outcome of the general price level equation :

(C3) $\quad Q = \omega\ (\varepsilon_{16} + \varepsilon_{17}L)$

We saw in the methodological section that a Phillips curve of the type of D24 resulted in the long run in a predetermined "unemployment rate" (eq.S24). Using S1 to S4 leads then to :

(C4) $\quad L = \varepsilon_{18} + \varepsilon_{19}\ \underline{N}$

At last the interest rate r is given by the simple transcription of S25 :

(C5) $\quad r = \varepsilon_{20} + \varepsilon_{21}\ \underline{rc}$

The following table 4 gives the numerical values associated with this system (C1-C5).

TABLE 4 about here

The system (C1-C5) may be interpreted as describing a monetary economy with three markets : output, labor and bonds.

The two demands for goods and labor are given by (C1) and (C2)) : The supply for goods, deriving from a monopoly mark up behaviour by firms is given by (C3) and amounts, as is well known, to an income distribution relation. The relation (C4) is rather ambiguous because it is not a supply relation, but the long run outcome on employment of wage begaining between firms and unions, the NAIRU. At last, the bonds market is described by (C5), with the implicit assumption that the supply by government clears the market at the given interest rate r[7]. The formalization of the demand for bonds does not matter as long as we are interessed mainly in goods and labor market, because of the

–keynesian– hierarchical structure of the behaviour of consumers : saving rate independant of portfolio decisions on the use of this saving.

We will consider r as directly exogenous and focus on the output and labor markets only.

We first observe that (C3) and (C4) correspond to excess supply on both markets : indeed firms, as monopolies, produce less than they would wish if prices were given[8], and the NAIRU implies obviously a certain amount of unvolontary unemployment.

So the demands (C1) and (C2) must be considered as "effective" demands, with the standard spillover mechanism between the two markets. We can compute the multipliers of public expenditure on the demand block (C1,C2) at given prices and wages. In the precise case of SSM, it comes to the value of 1.94, and would be approximately twice as much without imports –closed economy multiplier–.

To study the incidence of prices,we follow the IS-LM tradition, deriving "aggregate demand" and "aggregate supply" in terms of two relations between output and prices. Here, however, we find the non usual presence of the real wages ω coming from income repartition effects in consumption C and in the substitution mechanism between the two factors L and I.

Basically the relation (C1) gives the demand schedule and (C4), the supply schedule, (C2) and (C3) helping to eliminate L and ω. Let us just give the numerical outcome for SSM, in terms of log-linearized variations around the equilibrium :

$$\begin{cases} Q^d = - .027\ p + 2.54\ \underline{G} \\ Q^s = \ \ .22\ p + 2.41\ \underline{N} \end{cases}$$

(i) The reader will easily verify that the slope of the supply function is proportionnal to the elasticity of investment price index to foreign price index, $\varepsilon_6 = 1 - \beta_{33}$. In other words, the absence of import content of capital goods would imply "classical", horizontal, supply curve. Indeed $\varepsilon_6 = 0$ implies that the labor demand does not depend on π, and, as L is fixed by (C4), the relations (C2) and (C3) jointly determine Q and ω. Here the flexibility is generated by the Laursen-Metzler effect in relative costs –lesser variation of p_i than of p–.

The surprising high effect of $\underline{N}$ on Q at given prices comes from the fact that L is much lower than N, because L covers only the wage earners, amplified by casual numerical specifications details –constant terms in employment equations–. In a theoretical model we would find an elasticity of 1 instead of 2.41.

(ii) The effect of government spending on demand at given prices, is greater than the multiplier with given prices and wages –2.54 against 1.94–. It is easy to see that the presence of the positive constant terms ε_{14} and ε_{16} in (C2) and (C3) implies an elasticity of L with respect to Q of .7 and a co-variation of ω of .45 with a result of an elasticity of ωL amounting to 1.15. Hence wages increase contribute to increment consumption and investment.

Furthermore the demand function is very weakly negatively sloped. The explanation lies once again in the Laursen-Metzler effect on both consumption (ε_4) and investment (ε_6) playing against the "normal" negative effect of prices on foreign trade (ε_{11} and ε_{12}), with an ambiguous resulting sign.

(iii) The complete resolution of the system gives :

$$\begin{cases} Q = 2.26\ \underline{G} + .26\ \underline{N} \\ p = 10.3\ \underline{G} - 9.76\ \underline{N} \end{cases}$$

One finds a substancial multiplier of public spending, greater than in the case of fixed wages and prices (2.26 against 1.94). In other words the crowding out of prices is not enough to even compensate the crowding in of wages.

We also observe a very large sensitivity of prices, contrasting sharply with short and medium range rather small effects, as we will see below.

3. As a conclusion to this study of long run properties of METRIC, let us comment some properties of SSM so far exhibited from the point of view of their theoretical consistency.

We have seen that the long run is characterized by generalized excess supply, as in the short run. It is then excessive to say that macroeconometric models are keynesian in the short run and walrasian in the long run.

Actually the long run model shares common features with the model of monopolistic competition proposed by Benassy (1976,1987). But, is it rational to combine, in the long run, a constant return to scale technology and monopolistic behaviour ? Furthermore the Benassy equilibrium describes a state of the economy in which perceived demand curves coincide with the realized ones, while in SSM, demand is found almost insensitive to prices, and supply is built from a numerically quite low mark-up –eq.S18, β_{29} = 1.084–[9].

Another aspect of interest, when analyzing the theoretical specification behind SSM, is to study its consistency in terms of agents. Indeed model builders think in generally terms of "macroeconomic functions" rather than agents behaviour to be formalized, in the IS-LM tradition, and this contributes greatly to make their construction obscure for an economist trained in microeconomic theory.

In SSM, firms choose the levels of their factor demands, I and L, of their inventory investment IS, and of their output price, but these are not derived from a unified optimization set up. The theoretical model does exist –see e.g. Maccini (1981)–, but it is never tested as such. However we do not believe this to be a crucial issue. The usual way of doing has the effect of mainly generating "not enough" interdependence in the system.

On the consumers side, it is hard to understand why they would have a marginal propensity to spend their disposable income not exactly equal to one in the long run –cf. eq.S8, β_{13}= .929–, revealing that their implicit utility function does not include only consumption, or that they are constraint in a way not modelled. Clearly, something is missing here, like a monetary (and financial) equation or block[10] , as non-walrasian theory has shown to play an essential role .

The "rest of the world" agent interacts through exports and imports. We first note that the relative price elasticity of X is equal to 1.107 while the –implicit– correspondent for M amounts to .414. This could be rationalized by an oligopolistic competition argument similar to the one used for domestic market. However these two values look rather low for the long period, and contribute to make the "aggregate domestic demand" Q^d almost horizontal.

Another point of great importance, as far as foreign trade is concerned, is the absence of balance of payments constraint and of flexibility of exchange rate, which both are rather inconsistent with the notion of long run.

At last, another potentially essential constraint is not present in the maquette, the government budget constraint. This feature comes from the fact that government is taken as "above" the play and hence exogenous, which is not tenable in the long run.

2.4 - Dynamic structure of METRIC

We now turn to a structural analysis of the dynamic properties of our core version of METRIC. As the methodology used is well known and rather standard, we will focuse mainly on the transition between short run and long run, aspect not generally tackled when studying macroeconometric models.

We will proceed as follows : after introductory simulations of the model, over 400 periods (§1), it can be represented by a two blocks system to a large extend working independently (§2 and §3). A fourth paragraph is devoted to the so-called "linear analysis" –Kuh et Al. (1985)– to get more precise statements about its quantitative time structure.

1. The following figures 3 to 6 give the dynamic and long run multipliers in percentage variations of production Q, prices p, importations M and employment L, associated with a sustained shock on public expenditures (1 % of Q).

FIGURES 3 to 6 about here

We observe that Q, after a slight peak, increases smoothly, from 0.8 to 3.4 (this maximum value being obtained in about 25 years),and then decreases toward its long run value,which seems far from being reached after 100 years (400 periods)... There is no tendency of oscillatory behaviour of any length.

Prices first drop significantly, and cross the horizontal axis after a surprisingly long period (17 years). This is clearly the consequence of the so-called productivity cycle-combination of long lags in employment and in prices. Furthermore we observe that the subsequent monotonic increase, however strong, is just able to lead prices half way from the long run in 100 years...

The evolution of imports, first magnifies the one of production, then looks more and more influenced by prices, preventing the asymptotic decrease.

Figure 6 confirms the previous finding that employment is fixed in the long run, and furthermore exhibits a seemlingly quicker convergence than the other variables.

2. The apparent complexity of a macro model is strongly linked to its non-recursive nature and a first attempt at enlightening its short run structure consists in exhibiting an economically meaningful "feed-back set" or "cohesion set". By this it is meant a subset as small as possible of the endogenous variables of the system such that their exogeneization leads to a purely recursive system.

Several algorithms implementing this idea are available, especially one built by Gilli (1984) and accessible within TROLL. The use of this last for

METRIC resulted in several possible subsets of three variables, among which we chose the subset : (W, p, Q). This finding is in a sense very satisfactory : feed-back variables may be in general interpreted as signals necessary for the agents to make their plans. In a Keynesian working of the economy, we know that, beside prices, agents need quantity signals, summarized by Q.

So, qualitatively, in the short run, METRIC can be written a :

(12) $(W, p, Q) = \Psi\ (W, p, Q, [\text{past}])$,

other contemporaneous variables, considered as intermediate variables being substituted. We then add to (12) a vector of "pure shocks" (null on the reference equilibrium), $\Delta\bar{W}$, $\Delta\bar{p}$, $\Delta\bar{Q}$. By (log) linearisation around the reference –steady state–, we can associate to (12) a quantitative correspondent :

$$(13)\qquad \begin{pmatrix} \Delta W \\ \Delta p \\ \Delta Q \end{pmatrix} = \begin{bmatrix} -.00003 & .105 & .0017 \\ .105 & -.1645 & -.039 \\ .086 & -.033 & -.089 \end{bmatrix} \begin{pmatrix} \Delta W \\ \Delta p \\ \Delta Q \end{pmatrix} + \begin{pmatrix} \Delta\bar{W} \\ \Delta\bar{p} \\ \Delta\bar{Q} \end{pmatrix}$$

Everything is measured in percent points, so the matrix is directly interpretable is terms of elasticities.

The resolution of (13) leads to the "ex post" matrix

$$(14)\qquad \begin{pmatrix} \Delta W \\ \Delta P \\ \Delta Q \end{pmatrix} = \begin{bmatrix} 1.009 & .091 & -.002 \\ .089 & .867 & -.031 \\ .079 & -.018 & .919 \end{bmatrix} \begin{pmatrix} \Delta\bar{W} \\ \Delta\bar{P} \\ \Delta\bar{Q} \end{pmatrix}$$

It appears that the diagonal effects are highly dominant, but that cross-effects between wages and prices are non negligible. We tried several approximations of (13) by more or less decomposable systems, and found that the system is almost perfectly represented by a configuration in which prices and wages are coupled but predetermined with respect to production, leading to the reduced form :

$$(15)\qquad \begin{pmatrix} \Delta W'' \\ \Delta p'' \\ \Delta Q'' \end{pmatrix} = \begin{bmatrix} 1.010 & .091 & 0 \\ .091 & .867 & 0 \\ .077 & -.010 & .918 \end{bmatrix} \begin{pmatrix} \Delta \bar{W} \\ \Delta \bar{p} \\ \Delta \bar{Q} \end{pmatrix}$$

In other words the system is almost hierarchical in the short run in conformity with disequilibrium theory. It can be verified that the predetermination of quantities relatively to prices, is a worse approximation. This outcome coincides with the hierarchy found by Kuh et Al. (1985) on the american model MQEM.

This implies that when computing the effects on production of shocks on quantities, we can neglect those effects going through the prices-wages' block, and conversely for prices and wages[11].

3. We now extend the previous analysis in order to see if this near hierarchical structure between prices and quantities vanishes quickly in dynamics.

The following table 5 presents the dynamic multipliers associated with maintained "pure" mutiplicative shocks on p, W and Q, over 80 periods as well as the long run values, for the whole model (table 5a), and for the model with disconnection between production and prices (table 5b).

TABLE 5 about here

From this table it is clear that the real block works almost perfectly in isolation from prices during 8 to 10 quarters. As for the price-wage loop, the integration is slower, –about 20 quarters–, corroborating, up to an horizon of five years, the assertion that prices are rather predetermined with respect to quantities.

We furthermore observe that the wages-prices loop, taken in isolation, (figure 5b) generates monotonic divergent dynamics as opposed to the rather "rapid" convergence of the real block. However the apparent very slow "rapid" convergence of the real block. However the apparent very slow convergence of the full model –figure 5a– as we will confirm below, and its non-monotonic behaviour, shows that integration between prices and quantities generates effects asymptotically at total variance with the conclusions we would be tempted to draw at the light of a study bounded to a medium run horizon.

4. Another fruitful synthetic insight into the dynamic behaviour of the model is given by the computation of its eigenvalues and of the so called "sensitivity analysis". This is now a routine procedure, and made especially easy by the disponibility of LIMO software from TROLL –cf.Kuh et Al.(1985)–.

Let us recall briefly that this technique amounts to work on a linearized stationnary approximation of the system –formula (11)–, and to study its endogenous dynamics $-\Delta\hat{x}(t) = 0$–, that is, to put it under the form :

$$(16) \qquad \Delta\hat{y}(t) = A\ \Delta\hat{y}(t-1)\ ,\ \text{with } A = (I-D)^{-1}\ E$$

The n eigenvalues λ_i of the matrix A are then computed as well as the sensitivity coefficients:

$$(17) \qquad \varepsilon_i(\beta_j) = \frac{\partial|\lambda_i|}{\partial\beta_j}\ \frac{\beta_i}{|\lambda_i|},$$

elasticities of these eigenvalues with respect to the various structural coefficients β_j of the model imbedded in the state matrix A[12].

The following table 6 synthetizes the informations grasped from the computation of eigenvalues and sensitivity coefficients.

TABLE 6 about here

The first two columns give the magnitude and the periodicity of the 23 non-negligible eigenvalues, among the theoretical 34, of which 7 are real and positive, 2 real negative and 14 correspond to seven complex pairs. All lie within the unit circle, implying the –local– dynamic stability of the system. They are far from equally distributed : almost one half lies between .8 and 1.We observe further that no complex eigenvalue, but eventually number 7, is able to generate business cycle like oscillations not too quickly vanishing through time.

The third column exhibits the dying out lag, more precisely the date T such that $\lambda_i^T = .05$. It is striking that the first eigenvalue overcomes the whole dynamics after 200 periods but stays active during 1000 more periods.

The two last columns summarize the result of the sensitivity analysis as assignments of eigenvalues to lagged state variables. Without getting into somehow technical details, let us consider the elasticities :

$$\varepsilon_{i,j,k} = \frac{\partial|\lambda_i|}{\partial a_{j,k}} \frac{a_{j,k}}{|\lambda_i|},$$

sensitivities of the different eigenvalues with respect to the coefficients of the state matrix A and compute $\eta_{i,k} = \underset{j}{\mathrm{Max}} \, |\varepsilon_{i,j,k}|$ summing up the overall influence of the k^{th} lagged variable. Finally only the $\eta_{i,k}$ higher than .2 are selected (13) and splitted into column 4 or 5 according as the relevant lagged state variable concerns a quantity or a price.

(i) Few eigenvalues are influenced simultaneously by the two blocks. These are the complex pairs 2,4 and 10.

(ii) For real eigenvalues, whose assignment is easy to get, there exists a quasi-identity between eigenvalue and autoregressive term of the relevant variable as can be shown by the following table :

Eigenvalue	λ_3=.9625	λ_6=.8371	λ_9=.7175	λ_{11}=.3986	λ_{12}=.3003	λ_{13}=.27
Variable	Q*	C	PI	PM	NS	ND
Autoregression	δ' =.968	α_{17}=.832	α_{39} =.717	α_{46}=.399	α_6 =.295	α_2 =.27

(iii) Let us denote by XPR and MPR the dynamic terms of relative price influence on respectively exports and imports :

$$\begin{cases} XPR = \varphi_x(L) \text{ Log } (p_e/p_x) \\ MPR = \varphi_m(L) \text{ Log } (p_m/p\ ) \end{cases}$$

which can be written as (cf.table 2) :

$$\begin{cases} XPR = 1.533 \,.\, XPR_{-1} - .389\ XPR_{-2} - .165\ XPR_{-3} + .0227 \text{ Log } (\underline{p}_e/p_x) \\ MPR = 1.612\ MPR_{-1} - .677\ MPR_{-2} + 4.4610^{-3} \text{ Log}(p_m/p) \end{cases}$$

If the autoregressive components corresponded exactly to eigenvalues λ_5 and λ_6, for XPR and λ_7,for MPR,respectively,as table 6 suggests, they would result in the following expressions :

$$\begin{cases} XPR = 1.537\ XPR_{-1} - .391\ XPR_{-2} - .165\ XPR_{-3} \\ MPR = 1.606\ MPR_{-1} - .672\ MPR_{-2} \end{cases}$$

(iv) At last, the two first eigenvalues govern the wages-prices loop. Indeed the second one is almost a multiple real root mixing separate real and nominal phenomena. The working in isolation of the wage-price dynamics –eq.D18 and D24 of Table 1–, may be shown to imply generically an unitary eigenvalue and another smaller and very close to one linked to the speed of prices adjustment and of the degree of indexation of wages to prices –see e.g.Deleau and Al. (1984)–. As a consequence the interrelations quantities/prices in the maquette are stabilizing but in a very tiny way.

5. A conclusion emerges clearly from this section : the dynamics of our core model is extremely sluggish, and for a number of variables the own dynamics is highly dominant. So it is not at all surprising to find that the

quantities and wages-prices blocks work almost in isolation during more than 10 periods. We can illustrate this by the following qualitative stylized two variables following model :

$$(17) \quad \begin{cases} x = \rho_1 \, x_{-1} + \sum\limits_{i=0}^{n} a_i \, y_{-i} + \bar{x} \\ \\ y = \rho_2 \, y_{-1} + \sum\limits_{0}^{m} b_i \, x_{-i} + \bar{y} \end{cases}$$

with $\rho_2 < \rho_1 < 1$ and $(\sum\limits_{0}^{n} a_i) \times (\sum\limits_{0}^{m} b_i)$ non negligible, but the a_i's and b_i's all very small.

In the line of the concept of "near decomposability" developped by Ando-Fisher and Simon (1963), we can define an horizon H_1 before which x and y may be analyzed separately, and another horizon H_2 from which the own dynamics of x is stabilized making x a passive variable. Between H_1 and H_2 interactions become more and more important, but the lag structure can be simplified.

So it is possible to build a whole set of quite simple approximations, of the maquette, valid for different time spans and giving quite different views of the working of the economy.

General conclusion

The method for analysing a given macroeconometric model presented here aimed at enlightening its theoretical foundations hidden behind the apparently numerous and complex equations. The basic idea was to operate on the model as if it was a theoretical disequilibrium growth model, but with numerical parameters instead of formal ones. The procedure involved two steps: first, determination and analysis of the long run solution in terms of steady state growth path, second, analysis of the dynamics on the basis of a stationary representation.

For the case of METRIC, we discovered that the formalization it gives of the long run equilibrium working of the economy is not Walrasian, but a kind of monopolistic equilibrium, with a rather ambiguous determination of employment. Some elements are however missing, related to stock-flow relations (consumers and government budget constraints, balance of payments constraint), or to the monetary aspects of exchanges. Some inconsistencies emerge between parts of the model as price elasticities of various supplies and demands.

The dynamic investigation of METRIC focused mainly on its potential block structure. A strong decomposability, lasting for a surprisingly long time, was found between prices and quantities. This feature results from the very weak interaction between variables as opposed to the very strong autoregressive structure of this model, thus creating rigidities almost everywhere. It is however important to keep in mind that the study dealt with a reduction of a specific version of METRIC, and is not problably be as pronounced on the newer versions. It would be highly instructive to "follow" precisely. how the model has evolved in that respect.

FOOTNOTES

1) Theses growth rates look rather high as compared to the performances of the eighties ; but they are consistent with the historical estimation period of the model.

2) This result is rather general —see for instance Goldberger (1959) for the Klein-Goldberger model or Kuh et Al.(1985) for the MQEM model.

3) There are indeed several versions of METRIC. Our reference is the version published in Artus et Al.(1980).

4) The differences in the specifications between exports and imports are casual and do not mean differences in the thoery.

5) The symbol $\hat{x}$ means the rate of growth $(x_t-x_{t-1})/x_{t-1}$ of the variable x.

6) See Deleau, Le Van, Malgrange (1987) for more considerations on this point.

7) The fixation of r does not come from economic policy but from the hypothesis of small open economy.

8) Furthermore the degree of capacity utilization U_c is less than 100%, but this is rationnalizable, in the present context, by uncertainty arguments —see e.g. Sneessens (1987), Malinvaud (1987).

9) The work of Sneessens (1987) after Weitzman (1984) supplies a very interesting step toward reconciling monopolistic competition in the short run, and perfect competition in the long run for disequilibrium macroeconomic models.

10) We recall that the original core version of METRIC involves a monetary financial block, however almost inactive –see footnote 2–.

11) We assume here that we are considering only "pure" shocks, perturbing ex ante only one block. But economically meaningflul shocks may consist in a combination of several pure shocks from different blocks. On typical example is a shock on exchange rate or foreign prices which ex-ante influences exports and imports as well as relative prices, thus generating both real and prices perturbations.

12) These coefficients are computed formally from the eigenvalues and the associated eigenvectors –see Kuh and Al. (1985)–.

13) This is common practice –See e.g. Deleau-Malgrange (1978), Kuh and Al. (1982, 1985), and Schoonbeek (1984) for a rationalization–.

TABLE 1 : STATIONARY DYNAMIC VERSION AND LONG RUN STATIC VERSION OF METRIC

1 $N^d = \alpha_1 + \alpha_2 N^d_{-1} - \sum_0^2 \alpha_{3,i} L_{-i} + \alpha_4 \underline{N}$	S1 $N^d = \beta_1 - \beta_2 L + \beta_3 \underline{N}$	Unfilled demands for employment
2 $N^s = \alpha_5 + \alpha_6 N^s_{-1} - \alpha_7 L_{-1} + \alpha_8 L^*$	S2 $N^s = \beta_4 - \beta_5 L + \beta_6 L^*$	Employment vacancies
3 $U_n = \text{Log}(N^d/N^s)$	S3 $U_n = \text{Log}(N^d/N^s)$	Measure of labor market disequilibrium
4 $L = \alpha_9 + \alpha_{10} L_{-1} + \alpha_{11} L^*$	S4 $L = \beta_7 + \beta_8 L^*$	Actual employment
5 $L^* = \alpha_{12}(CC/W)^b (Q-\delta' Q_{-1}) + \delta'' L^*_{-1}$	S5 $L^* = \beta_9 (CC/W)^b Q$	Optimal employment
6 $I = \alpha_{13} I^* + \alpha_{14} I_{-1}$	S6 $I = \beta_{10} I^*$	Actual fixed investment
7 $I^* = \alpha_{15} (W/CC)^{1-b} (Q-\delta' Q_{-1})$	S7 $I^* = \beta_{11}(W/CC)^{1-b} Q$	Optimal fixed investment
8 $C = \alpha_{16} + \alpha_{17} C_{-1} + \alpha_{18} R$	S8 $C = \beta_{12} + \beta_{13} R$	Consumption
9 $R = (\alpha_{19} WL + \alpha_{20} pQ)/p_c$	S9 $R = (\beta_{14} WL + \beta_{15} pQ)/pc$	Real disposable income
10 $IS = \alpha_{21} + \alpha_{22} IS_{-1} + \sum_0^2 \alpha_{23,i}(D_{-i} - \delta D_{-i-1})$	S10 $IS = \beta_{16} + \beta_{17} D$	Inventory investment
11 $M = (D+IS)(\alpha_{24} + \alpha_{25} U_c - \varphi_m \text{Log}(p_m/p))$	S11 $M = (D+IS)(\beta_{18}+\beta_{19}U_c-\beta_{20}(L)\text{Log}(p_m/p))$	Imports
12 $X = \underline{D}_e^{\alpha_{26}} \exp(\alpha_{27} - \alpha_{28} U_c + \varphi_x(L)\text{Log}(\underline{p}_e/p_x))$	S12 $X = D_e^{\beta_{21}} \exp(\beta_{22} - \beta_{23} U_c)(\underline{p}_e/p_x)^{\beta_{24}}$	Exports
13 $Q = C + I + IS + \underline{G} + X - M$	S13 $Q = C + I + IS + \underline{G} + X - M$	Actual production
14 $D = C + I + \underline{G} + X$	S14 $D = C + I + \underline{G} + X$	Aggregate demand out of inventory investment
15 $U_c = Q/Q^*$	S15 $U_c = Q/Q^*$	Degree of capacity utilization
6 $Q^* = \alpha_{29}(Q-\delta' Q_{-1}) \,.\, I/I^* + \delta' Q^*_{-1}$	S16 $Q^* = \beta_{25} Q \,.\, I/I^*$	Optimal production capacity
7 $TC = \alpha_{30} WL + \alpha_{31} W + \alpha_{32} p_{-1} Q_{-1}$	S17 $TC = \beta_{26} WL + \beta_{27} W + \beta_{28} pQ$	Total costs of firms
8 $p_c = \alpha_{33}(TC/\sum_0^3 Q_{-i})^{\alpha_{34}} p_{-1}^{1-\alpha_{34}}$	S18 $p = \beta_{29} TC/Q$	General price level
9 $p = \alpha_{35} p_{c-1}^{\alpha_{36}} p^{\alpha_{37}} p_m^{1-\alpha_{36}-\alpha_{37}}$	S19 $p_c = \beta_{30} p^{\beta_{31}} p_m^{1-\beta_{31}}$	Consumption price
0 $p_i = \alpha_{38} p_{i-1}^{\alpha_{39}} p^{\alpha_{40}} p_m^{1-\alpha_{39}-\alpha_{40}}$	S20 $p_i = \beta_{32} p^{\beta_{33}} p_m^{1-\beta_{33}}$	Investment price
1 $p_x = \alpha_{41} p_{x-1}^{\alpha_{42}} \underline{p}_e^{\alpha_{43}} p^{\alpha_{44}} p_m^{1-\alpha_{42}-\alpha_{43}-\alpha_{44}}$	S21 $p_x = \beta_{34} p^{\beta_{35}} \underline{p}_e^{\beta_{36}} p_m^{1-\beta_{35}-\beta_{36}}$	Export price
2 $p_m = \alpha_{45} p_{m-1}^{\alpha_{46}} \underline{p}_e^{1-\alpha_{46}}$	S22 $p_m = \beta_{37} \underline{p}_e$	Import price
3 $CC = p_i\, r$	S23 $CC = p_i .\, r$	Cost of capital
4 $\hat{W} = \alpha_{47} + \sum_0^2 \alpha_{48,i} \hat{p}_{c-i} - \sum_0^3 \alpha_{49,i} U_{n-i}$	S24 $U_n = \beta_{38}$	Phillips relation
5 $r = \alpha_{50} + \alpha_{51} r_{-1} + \alpha_{52}(r_c - \sum_0^2 \alpha_{53,i} \hat{p}_{-i})$	S25 $r = \beta_{39} + \beta_{40} \underline{r}_c$	Long run real interest rate

TABLE 2 : VALUES OF COEFFICIENTS

$\alpha_1 = -2\ 836$	$\alpha_{28} = 1.27$	$\alpha_{53,1} = 120$	$\beta_{18} = -.0547$
$\alpha_2 = .275$	$\alpha_{29} = 1.08$	$\alpha_{53,2} = 80$	$\beta_{19} = .281$
$\alpha_{3,0} = .764$	$\alpha_{30} = .571$	$b = .55$	$\beta_{20} = .0686$
$\alpha_{3,1} = -.217$	$\alpha_{31} = 2\ 120$	$\sigma = .008$	$\beta_{21} = 1.074$
$\alpha_{3,2} = .124$	$\alpha_{32} = .329$	$\varrho = .02$	$\beta_{22} = 4.707$
$\alpha_4 = .558$	$\alpha_{33} = 4.27$	$n = .002$	$\beta_{23} = 1.27$
$\alpha_5 = 380.$	$\alpha_{34} = .141$	$\delta = .02$	$\beta_{24} = 1.107$
$\alpha_6 = .295$	$\alpha_{35} = 1.015$	$\delta = .99$	$\beta_{25} = 1.082$
$\alpha_7 = .0315$	$\alpha_{36} = .720$	$\delta' = .968$	$\beta_{26} = .571$
$\alpha_8 = .0165$	$\alpha_{37} = .213$	$\delta'' = .976$	$\beta_{27} = 2\ 120$
$\alpha_9 = 58{,}6$	$\alpha_{38} = 1.014$	$\varphi_m(L) = \dfrac{4.46X10^{-3}}{1-1.612L+.677L^2}$	$\beta_{28} = .329$
$\alpha_{10} = .947$	$\alpha_{39} = .717$		$\beta_{29} = 1.084$
$\alpha_{11} = .0483$	$\alpha_{40} = .227$	$\varphi_x(L) = \dfrac{.0227}{1-1.533L+.389L^2+.165L^3}$	$\beta_{30} = 1.055$
$\alpha_{12} = .0773$	$\alpha_{41} = 1.092$		$\beta_{31} = .761$
$\alpha_{13} = .121$	$\alpha_{42} = .733$	$\beta_1 = -3\ 911$	$\beta_{32} = 1.050$
$\alpha_{14} = .88$	$\alpha_{43} = .180$	$\beta_2 = .926$	$\beta_{33} = .802$
$\alpha_{15} = 2.403$	$\alpha_{44} = .069$	$\beta_3 = .770$	$\beta_{34} = 1.388$
$\alpha_{16} = 1\ 477$	$\alpha_{45} = 1.366$	$\beta_4 = 539$	$\beta_{35} = .258$
$\alpha_{17} = .832$	$\alpha_{46} = .399$	$\beta_5 = .0447$	$\beta_{36} = .670$
$\alpha_{18} = .156$	$\alpha_{47} = .0162$	$\beta_6 = .0234$	$\beta_{37} = 1.680$
$\alpha_{19} = .782$	$\alpha_{48,0} = .50$	$\beta_7 = 1\ 103$	$\beta_{38} = 1.771$
$\alpha_{20} = .0843$	$\alpha_{48,1} = .33$	$\beta_8 = .912$	$\beta_{39} = 2.691$
$\alpha_{21} = -630$	$\alpha_{48,2} = .17$	$\beta_9 = .102$	$\beta_{40} = 1.136$
$\alpha_{22} = .591$	$\alpha_{49,0} = 1.86\times10^{-3}$	$\beta_{10} = 1.014$	
$\alpha_{23,0} = .139$	$\alpha_{49,1} = 1.40\times10^{-3}$	$\beta_{11} = .0762$	
$\alpha_{23,1} = .114$	$\alpha_{49,2} = .93\times10^{-3}$	$\beta_{12} = 8\ 792$	
$\alpha_{23,2} = .331$	$\alpha_{49,3} = .46\times10^{-3}$	$\beta_{13} = .929$	
$\alpha_{24} = -.0377$	$\alpha_{50} = .654$	$\beta_{14} = .782$	
$\alpha_{25} = .281$	$\alpha_{51} = .757$	$\beta_{15} = .0843$	
$\alpha_{26} = 1.074$	$\alpha_{52} = .276$	$\beta_{16} = -1\ 540$	
$\alpha_{27} = 4.707$	$\alpha_{53,0} = 200$	$\beta_{17} = .0142$	

TABLE 3 - LONG RUN LEVELS, 1979-1

Variable	Symbol	Equilibrium level (1979-1)	Percentage deviation from actual (1979)	Units FF
Consumption	C	161116	- 6.55	10^6FF70
Aggregate Demand	D	292169	- 4.38	"
Foreign Demand	$\underline{D}_e$	1789	-	"
Government Spending	$\underline{G}$	41748	-	"
Actual investment	I	28969	-10.95	"
Optimal investment	I*	28573	- 8.91	"
Inventory investment	IS	2624	-50.77	"
Actual employment	L	13201	- 4.27	10^3units
Optimal employment	L*	13261	- .45	"
Imports	M	61933	- .69	10^6FF70
Labor force	$\underline{N}$	22915	-	10^3units
Unfilled demands for employment	N^d	1508	+21.67	"
Employment vacancies	N^s	256	+62.24	"
General price level	p	2.101	+ 3.94	1 in 70
Consumption price	p_c	2.146	+ 4.26	"
Foreign price	$\underline{p}_e$	1.102	-	"
Investment price	p_i	2.153	+ 4.00	"
Import price	p_m	1.851	- .55	"
Export price	p_x	1.875	ε	"
Actual production	Q	232860	- 5.93	10^6FF70
Optimal production	Q*	255512	- 6.39	"
Disposable income	R	163879	- 8.59	"
Short run interest rate	r_c	1.893	-	"
Long run interest rate	$\bar{r}$	4.843	+ 7.74	"
Total costs of firms	TC	451421	- 3.15	10^6FF70
Diseq.on goods market	U_c	.9113	+ .43	units
Diseq.on labor market	U_N	1.771	-41.19	percent
Wage rate	W	30.06	ε	10^3FF
Exports	X	60337	+ 1.60	10^6FF70

TABLE 4 : COEFFICIENTS OF THE COMPACT MODEL

ε_1 = 8 792	ε_8 = .014	ε_{15} = .101
ε_2 = .608	ε_9 = 23.30	ε_{16} = 3 295
ε_3 = .0656	ε_{10} = 1.074	ε_{17} = .887
ε_4 = .239	ε_{11} = 1.107	ε_{18} = -8 480
ε_5 = .0722	ε_{12} = .201	ε_{19} = .946
ε_6 = .198	ε_{13} = .0687	ε_{20} = 2.691
ε_7 = -1 518	ε_{14} = 1 103	ε_{21} = 1.136

TABLE 5 : DYNAMIC INTERACTIONS

	$\Delta\overline{W}$	$\Delta\overline{p}$	$\Delta\overline{Q}$
ΔW	1.009	.91	-.002
Δp	.089	.867	-.031
ΔQ	.077	-.018	.919
ΔW	2.051	.300	-.004
Δp	.259	1.680	-.044
ΔQ	.225	-.050	.964
ΔW	4.273	.997	-.022
Δp	.810	3.192	-.160
ΔQ	.756	-.084	1.058
ΔW	9.372	2.737	-.125
Δp	2.544	5.860	-.415
ΔQ	2.355	-.094	1.190
ΔW	29.07	6.326	-.324
Δp	10.42	11.36	-.913
ΔQ	9.327	-2.106	1.705
ΔW	69.25	5.805	.288
Δp	25.28	14.17	-.912
ΔQ	19.70	-6.635	2.631
ΔW	176.5	-1.731	3.089
Δp	61.44	12.82	.429
ΔQ	33.39	-12.11	3.912
ΔW	391.	-23.06	15.32
Δp	353.	-3.867	12.20
ΔQ	-21.18	-11.90	2.777

(a) Whole model

$\Delta\overline{W}$	$\Delta\overline{p}$	$\Delta\overline{Q}$
1.010	.091	0
.091	.867	0
0	0	.919
2.042	.300	0
.267	1.678	0
0	0	.963
4.215	.996	0
.852	3.184	0
0	0	1.048
9.106	2.728	0
2.856	5.831	0
0	0	1.140
27.35	6.108	0
13.58	10.69	0
0	0	1.410
58.83	4.282	0
34.80	10.20	0
0	0	1.709
96.25	.853	0
58.78	8.975	0
0	0	2.046
∞	∞	0
∞	∞	0
0	0	2.418

(b) Blocks disconnected

TABLE 6 : EIGENVALUES OF METRIC

N°	Magnitude	Periodicity	Dying out lag	Quantities	Prices-wages
1	.9975	∞	1200	-	p, W
2	.9851	473	200	Q, L*, Q*, I	W
3	.9625	∞	78	Q, Q*, L*, L	-
4	.9452	112	53	Q*, I, Q, L	W
5	.8767	158	23	-	XPR, XPR_{-1}
6	.8371	∞	17	C	-
7	.8202	30	15	-	MPR, MPR_{-1}
8	.7343	7100	10	-	PX, R, PC
9	.7175	∞	9	-	PI
10	.5964	97	6	IS, Q, I, Q*	PC
11	.3986	∞	3.25	-	PM
12	.3003	∞	2.50	NS, Q	-
13	.2760	∞	2.32	ND	-
14	.2673	(2)	2.27	I, Q, D, D_{-1}, D_{-2},IS	-
15	.2636	15.3	2.25	I, Q, IS, D_{-2}, D,D_{-1}	-
16	.2152	(2)	1.95	-	XPR_{-2}, XPR_{-1}

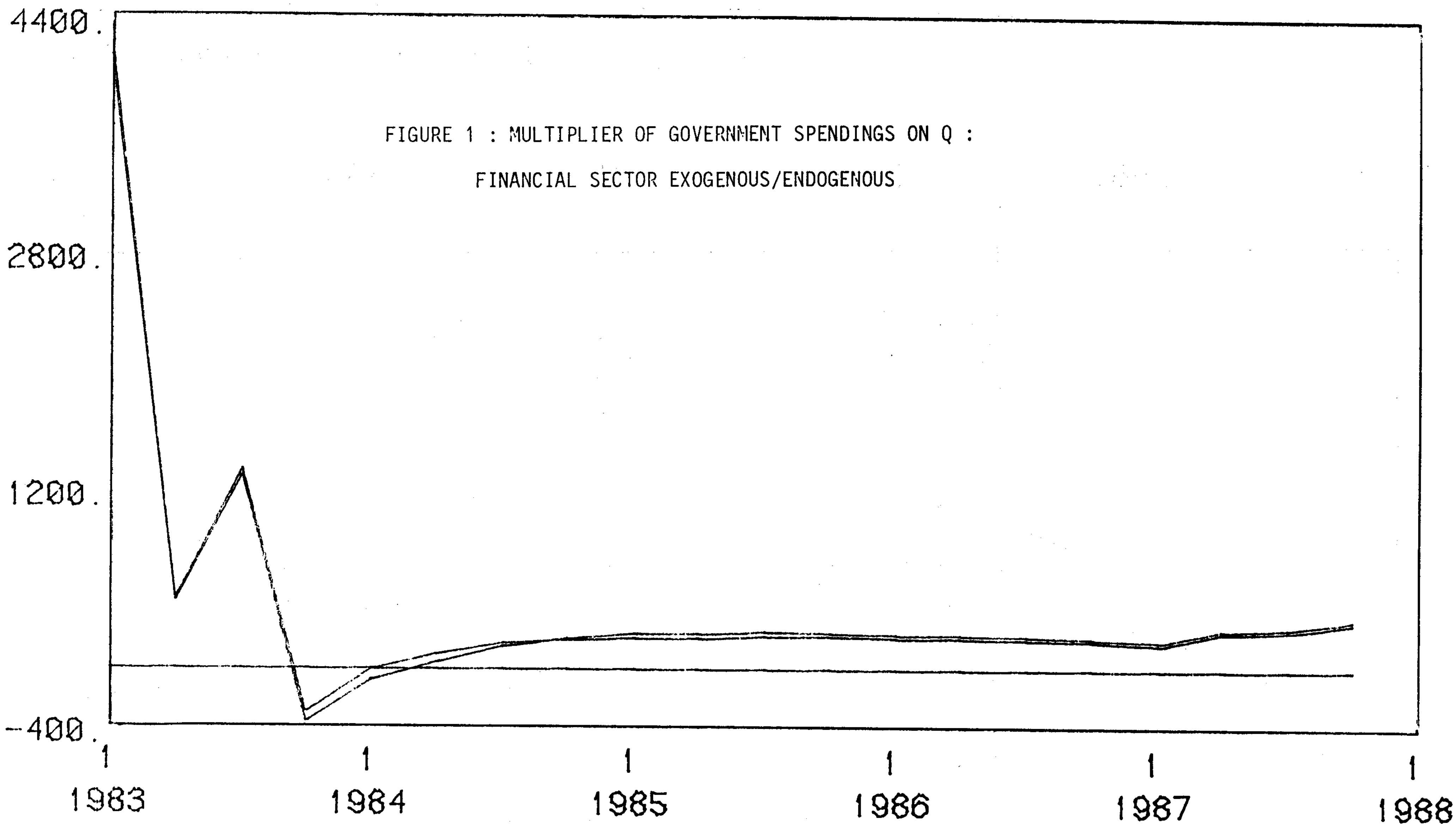
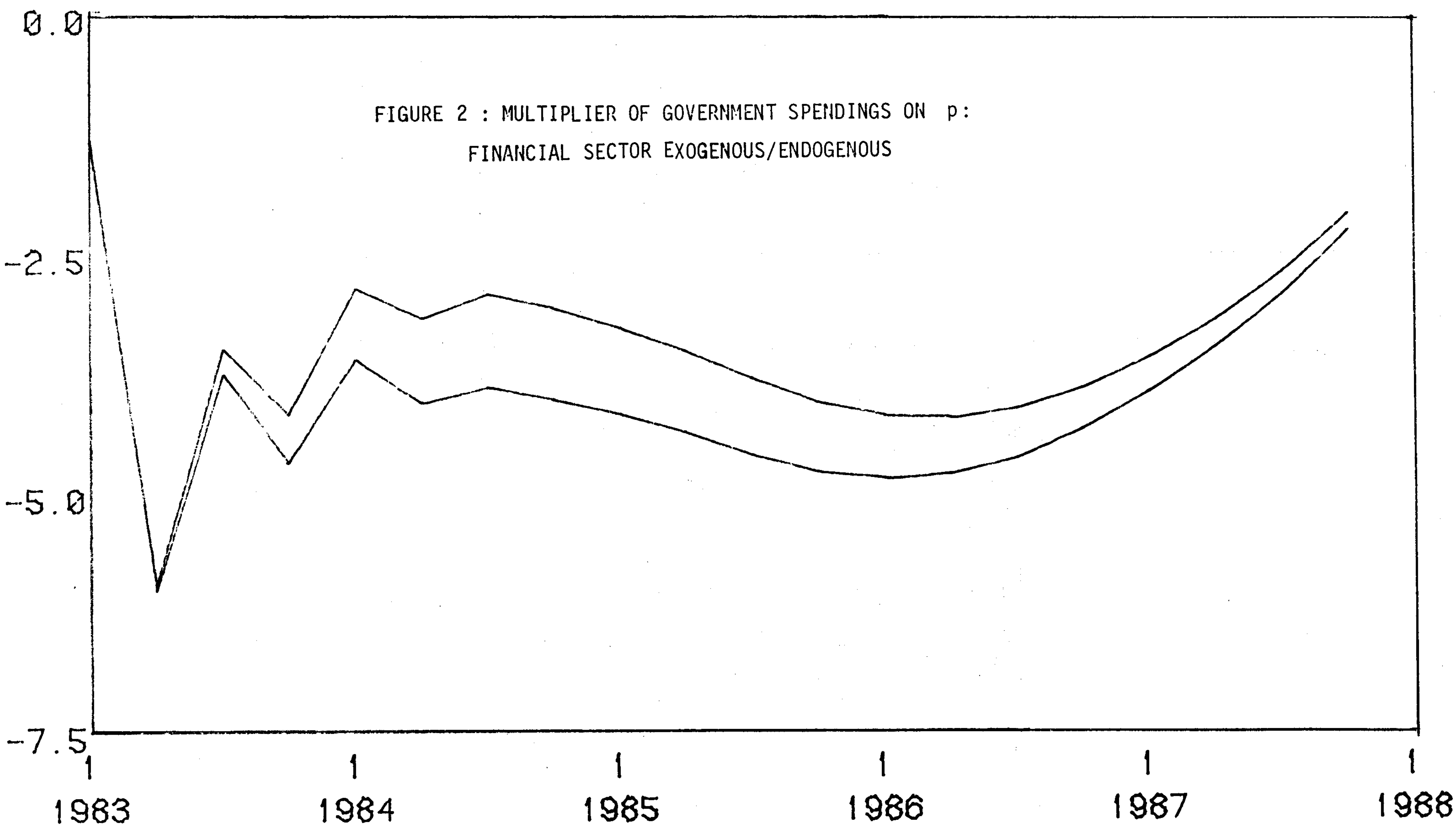


FIGURE 1 : MULTIPLIER OF GOVERNMENT SPENDINGS ON Q :

FINANCIAL SECTOR EXOGENOUS/ENDOGENOUS

TIME BOUNDS: 1983 1ST TO 1987 4TH



TIME BOUNDS: 1983 1ST TO 1987 4TH

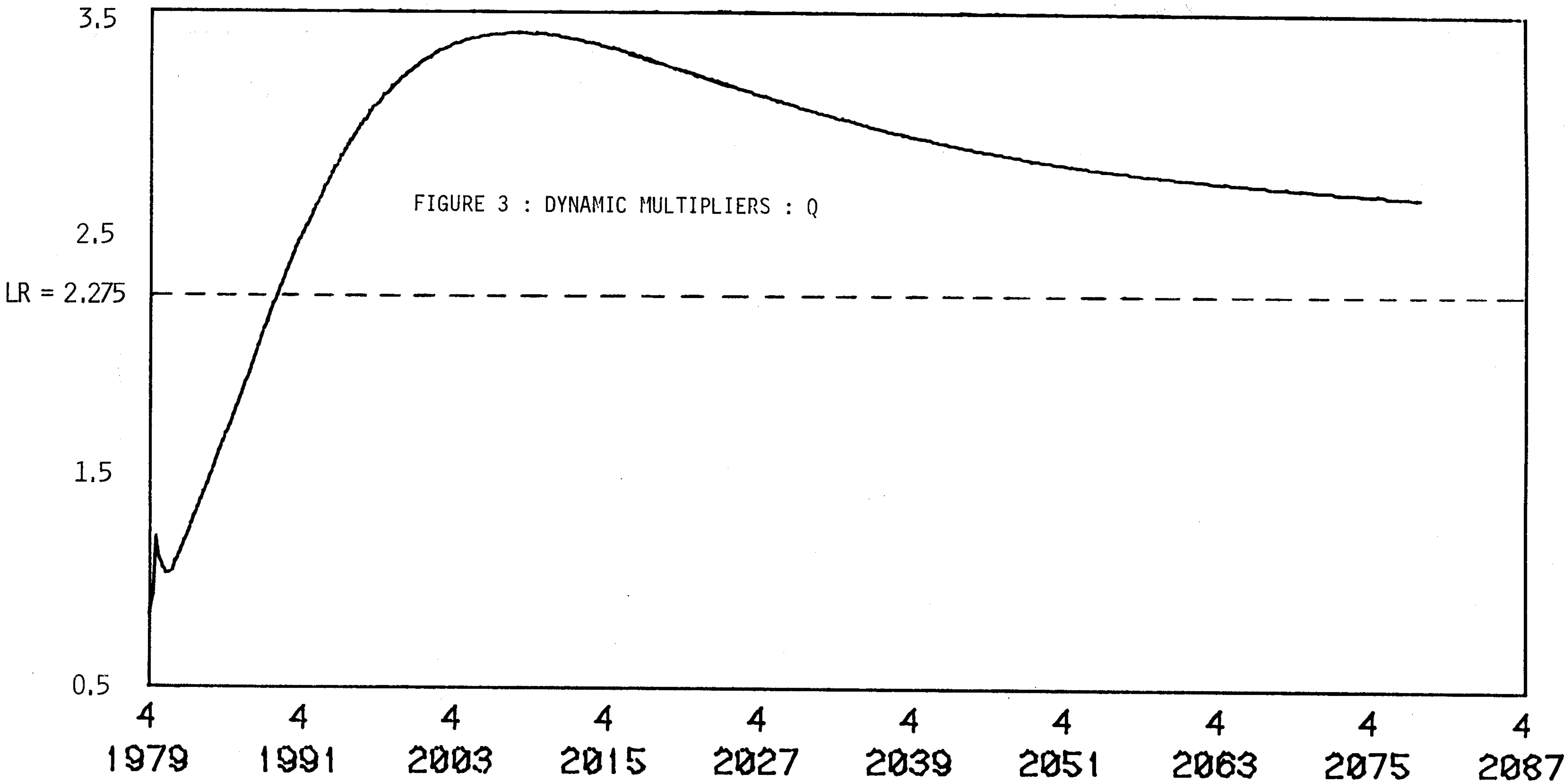


FIGURE 3 : DYNAMIC MULTIPLIERS : Q

TIME BOUNDS: 1979 4TH TO 2079 3RD

DATA NAMES : Q

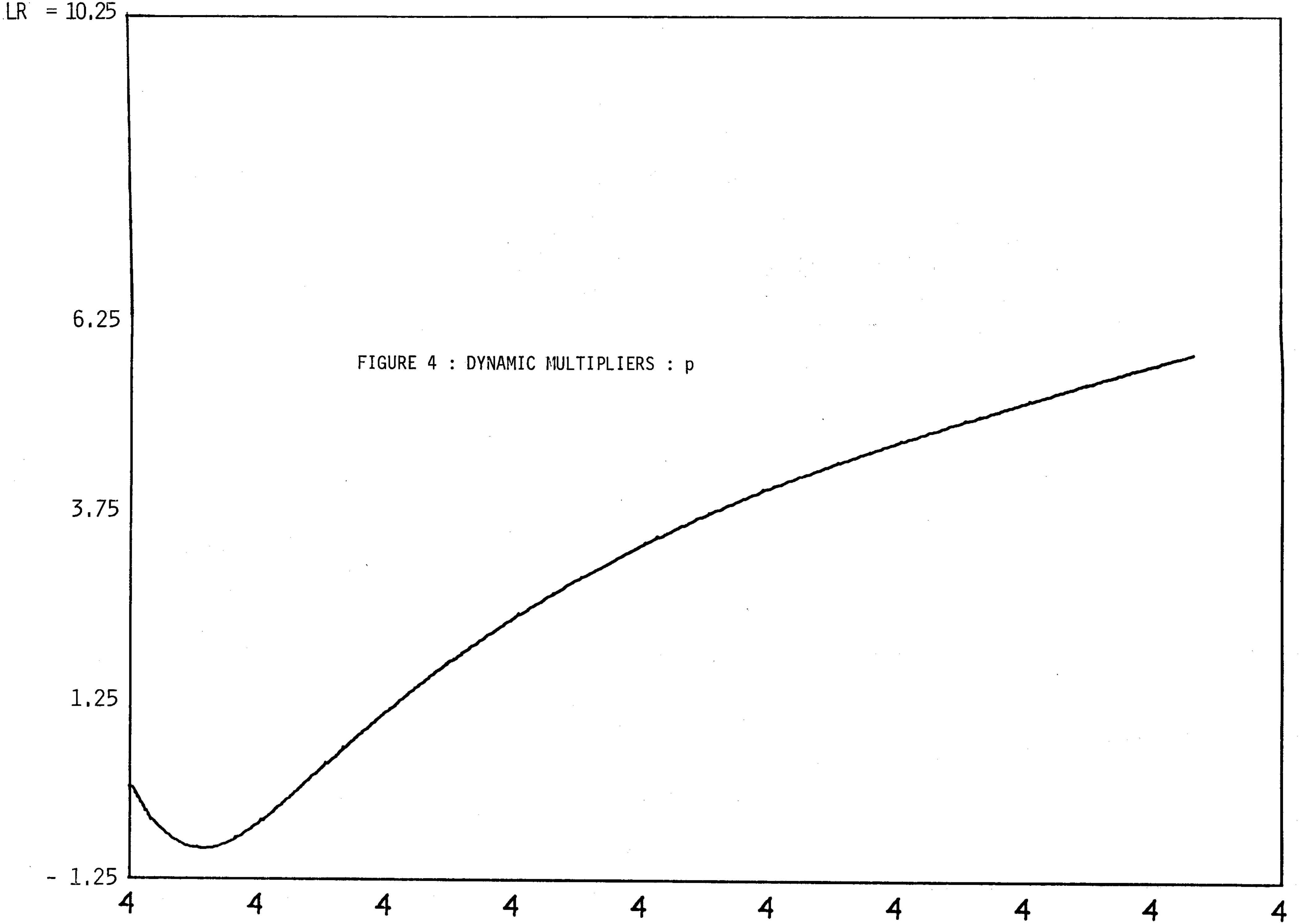
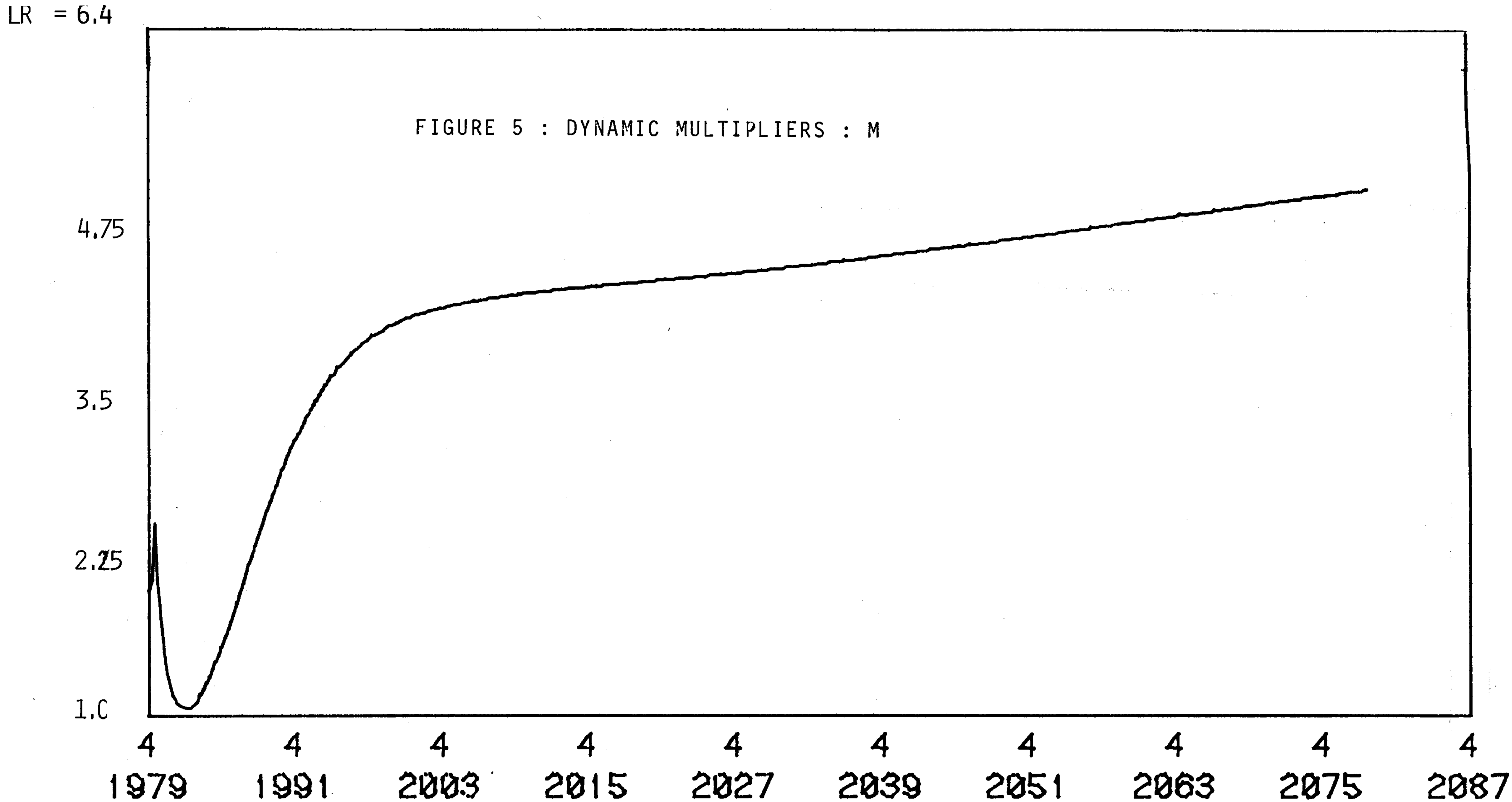


FIGURE 4 : DYNAMIC MULTIPLIERS : p



TIME BOUNDS: 1979 4TH TO 2079 3RD

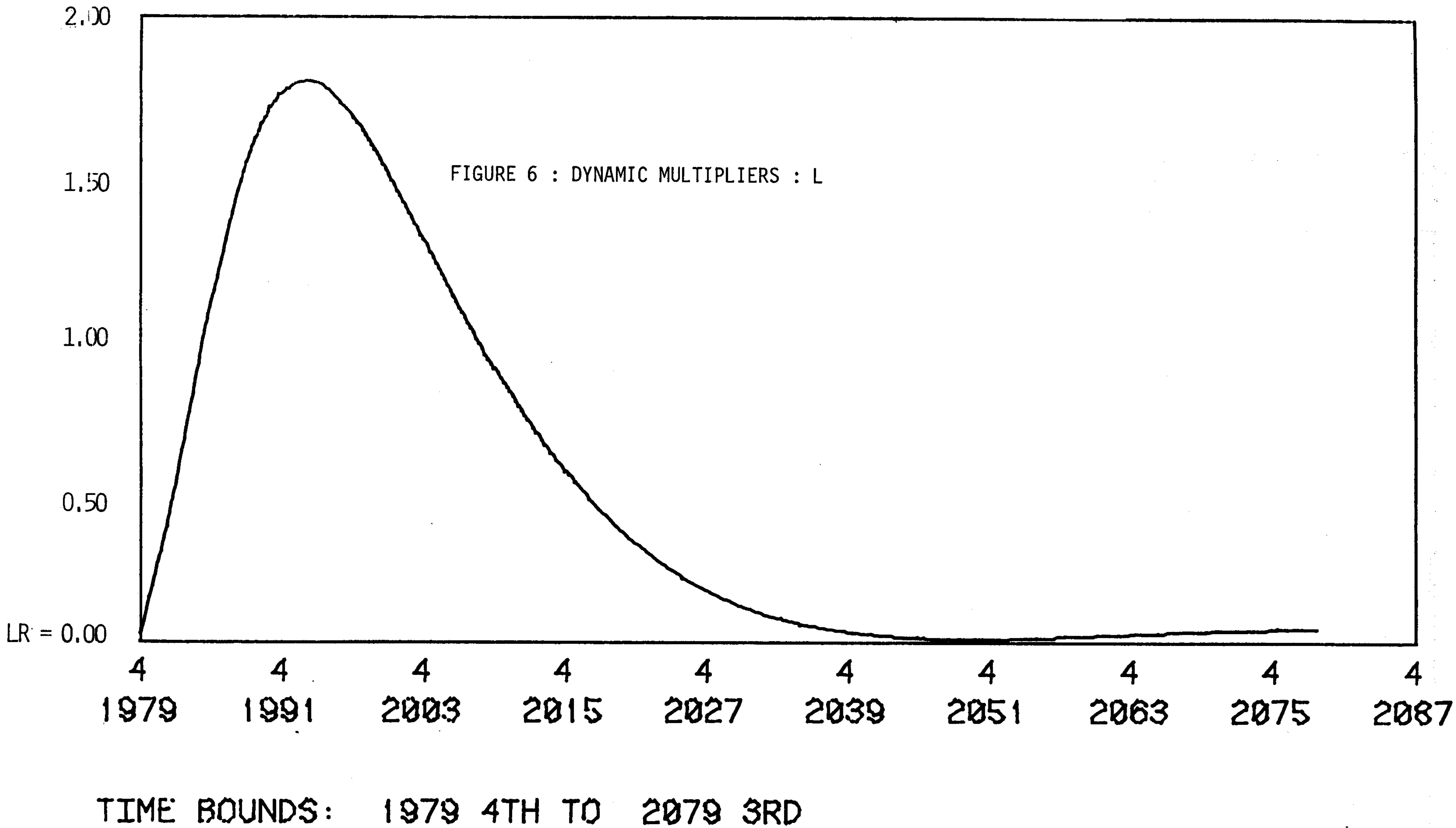


FIGURE 6 : DYNAMIC MULTIPLIERS : L
2.00
1.50
1.00
0.50
LR = 0.00
4 1979
4 1991
4 2003
4 2015
4 2027
4 2039
4 2051
4 2063
4 2075
4 2087
TIME BOUNDS: 1979 4TH TO 2079 3RD

References

Ando, A., F. M. Fisher, H. Simon (1963)- *Essais on the Structure of Social Science Models*, MIT Press, Cambridge.

Artus, P., P. Morin, A. Pacaud, C. Peyroux, M. Sterdyniak, R. Teyssier (1981)- *METRIC, modèle trimestriel de conjoncture*, Publication de l'INSEE, Paris.

Benassy, J. P. (1976)- "The Disequilibrium Approach to Monopolistic Price Setting and General Monopolistic Equilibrium", *Review of Economic Studies*, 43, 69-81.

Benassy, J. P. (1987)- "Imperfect Competition, Unemployment and Policy", *European Economic Review*,31,417-426.

Chow, G., P. Corsi, Eds. (1982)- *Evaluating the Reliability of Macro-economic Models*, J. Wiley, New-York.

Deleau, M., C. Le Van, P. Malgrange (1987)- "Le long terme des modèles macroéconomiques", *CEPREMAP Working Paper*, 8729.

Deleau, M., P. Malgrange (1978)- *L'analyse des modèles macroéconomiques quantatifs*, Economica, Paris.

Deleau, M., P. Malgrange, P. A. Muet (1984)- "A Study of Short-Run and Long-Run Properties of Macroeconometric Models by Means of an Aggregative Core Model", In Malgrange, Muet Eds., *Contemporary Macroeconomic Modelling* Blackwell, Oxford,215-246

Gillli, M. (1982)- "TROLL Program CAUSOR: A Program for the Analysis of Recursive and Interdependent Causal Structures" *Technical Report 34*, MIT.

Goldberger, A. S. (1959)- *Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model*, North-Holland, Amsterdam.

Hickman, B.G. (1972)- *Econometric Models of Cyclical Behaviour*, NBER, Studies in Income and Wealth, Columbia University Press, 36.

Kuh, E., C. Le Van, P. Malgrange (1984)- "Une étude de la dynamique structurelle des modèles macroéconomiques" , *CEPREMAP Working Paper*.

Kuh, E, J. Neese, (1982)- "Econometric Model Diagnostics", in G. C. Chow and P. Corsi, Eds., *Evaluating the Reliability of Macro-economic Models*, J. Wiley, New York,119-146.

Kuh, E, J. Neese, P. Hollinger (1985)- *Structural Sensivity in Econometric Models*, J. Wiley, New York.

Kmenta, J., J.B. Ramsey, Eds (1981)- *Large-Scale Macro-Econometric Models*, North-Holland, Amsterdam.

Maccini, L. J. (1981)- " On the Theory of the Firm Underlying Empirical Models of Aggregate Price Behaviour", *International Economic Review*, 22, 609-624.

Malgrange, P. (1983)- "Steady Growth Paths in a Short Run Dynamic Model : The Case of the French Quarterly Model METRIC" , *CEPREMAP Working Paper*, 8321.

Malinvaud,E (1987)- "Capital productif, incertitudes et profitabilité", *Annales d'Economie et de Statistique* ,5,1-36.

Schoonbeek, L. (1984)- "Coefficient Values and the Dynamic Properties of Econometric Models", *Economic Letters*, 16, 303-308.

Sims, C. (1980)- " Macroeconomics and Reality", *Econometrica*, 48, 1-48.

Sneessens, H. (1987)- "Investment and the Inflation-Unemployment Trade-off in a Macroeconomic Rationing Model with Monopolistic Competition", *European Economic Review*, 31, 781-815.

Wallis, K.F., Ed. (1984)- *Models of the UK Economy: AReview by the ESRC Macroeconomic Modeling Bureau*, Oxford University Press, Oxford.

Weitzman, M. (1984)- "The Simple Macroeconomics of Profit Sharing", *American Economic Review*, 75, 937-953.