INCENTIVES IN COOPERATIVE RESEARCH AND DEVELOPMENT

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A B S T R A C T

This paper provides an analysis of optimal contracts for a cooperative research and development programme. We first characterize the optimal decision rule, when the regulator has incomplete information about the firms' profitability and individual contributions are verifiable. We then turn to the case where the contractors' contributions cannot be publicly observed. It is shown that reward schemes can be chosen so that imperfect observability of the contractors' actions entails no additional welfare loss.

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Key words : - Research and development
- Adverse selection
- Moral hazard

RECHERCHE-DEVELOPPEMENT, INCITATION ET COOPERATION

R E S U M E

Cette note développe une analyse des contrats optimaux pour un programme de recherche-développement coopératif. Nous caractérisons d'abord la règle de décision optimale lorsque le planificateur a une information incomplète sur la profitabilité des entreprises concernées, mais que les contributions individuelles peuvent être vérifiées. Nous nous tournons ensuite vers le cas où la contribution de chaque contractant ne peut être identifiée et nous montrons que, par le choix d'un mécanisme incitatif adéquat, cette observabilité imparfaite des apports individuels ne conduit pas à une perte de bien-être.

Mots clefs : - Recherche-développement
- Antisélection
- Aléa moral

1. Introduction

In Europe, promoting cooperative research and development programmes has gained ground in the past few years and in the future it will probably become a major tool of the E.E.C. industrial policy. By cooperative research and development we mean the organisation of a project where a public regulator invites a number of oligopolistic firms or laboratories to participate in a joint research programme. In some respects, scientific knowledge is a public good which tends to be undersupplied ; cooperative research agreements are thus mostly viewed as a way of speeding invention and technical innovation by coordinating individual efforts, by making them less risky and by creating synergetic effects through exchange of information and combination of technological abilities.

The European Strategic Programme for Research and Development in Information Technology (ESPRIT) is a typical example of such cooperative research projects, whose coordination is organized by a public regulator. ESPRIT contracts bring industrial laboratories together in order to participate in a well defined applied (production for market) research in the field of information technology. Each laboratory bears a part of research costs and receives a subsidy which is equal to half its own costs. Furthermore, each laboratory may exploit all the results which have been achieved in the joint research programme. The EUREKA and RACE programmes are other typical examples of regulated cooperative R & D in Europe.[1]

Using cooperative programmes to promote research and development activities is something relatively new in Europe, but in Japan it is already an important tool of industrial policy : MITI takes vigorous initiative in organizing joint research projects among oligopolistic firms of the semiconductor and other high technology industries, in providing partial funding and assuming the role of an effective coordinator. As Yamamura (1986) writes : "Above all, MITI evidently has no doubt of the social desirability of joint research activities among selected oligopolistic firms. MITI officials, in their public pronouncements and interviews stress the necessity of pooling R & D efforts in order to make the most efficient use of scarce scientific manpower and research funds.

MITI clearly believes its joint research projects serve the national interest". This conception contrasts sharply with American perceptions of what should be the role of government in promoting technological progress, since American policymakers do not seem to reach a consensus as to the desirability of sponsoring joint research ventures among oligopolistic firms. In the United States, public sponsorship gives way to individual initiatives, possibly leading to privately organized joint research corporations such as the Microelectronics and Computer Consortium (M.C.C.).

Be that as it may, the purpose of this paper is not to provide a comparative analysis of respective merits of sponsored joint ventures and unregulated markets in R & D activities and the desirability of promoting a research joint venture will be considered here rather as a postulate[(2)]. Attention will be focused on the design of contractual rules which have to be decided upon if one aims at an efficient functioning of such a cooperative programme[(3)].

More precisely, we will consider a public regulator who is in charge of coordinating a joint research project and we will characterize socially optimal incentive schemes under imperfect information. It will be assumed that the regulator does not know precisely how much money each firm will be able to make from the patents and other statutory rights for inventions conceived by contractors in the execution of the cooperative programme. We will also consider informational constraints owing to the inability of the regulator to verify the actual contribution of each contractor, that is the actual effort levels when the programm is implemented.

Conforming to a terminology which is now familiar in the theory of incentives, the first type of the above-mentioned informational constraints refers to "hidden knowledge" and the second one corresponds to "hidden actions". Recent contributions have simultaneously considered both types of problems in a principal-(one) agent framework, where the observation of the agent's actions is garbled by some random noise. In particular results obtained by Laffont-Tirole (1986), Caillaud-Guesnerie-Rey (1986) and Picard (1987) show that, in most cases,unperfect observability of the agent's actions does not entail any additional welfare loss (by comparison with the pure adverse selection problem) when the principal and the agent are both income risk-neutral. They also emphasize the potential interest of simple and intuitive (linear or quadratic) reward schemes, which are quite robust to the nature of the garbling noise.

Here, we will consider an incentive problem where a principal (the regulator) enters into contractual relations with several agents (firms or industrial laboratories), so that strategic interactions bear on the agents' behaviour. A short methodological digression may be useful to clarify the following developments. First, in such a multi-agent setting, moral hazard can emerge from the fact that the regulator may be unable to identify individual contributions to an aggregate (observable) outcome : this gives rise to the free-rider problem in teams, as analyzed by Holmström (1982). On the other hand, optimal decision making under imperfect observability of the agents' characteristics has motivated a great deal of works on implementation of social choice rules. Informational assumptions differ among these works. In a first class of models, agents are supposed to have perfect information about others' characteristics and Nash-(or perfect Nash-) implementability is usually considered in that case. It turns out then that it is in general possible for a non-informed regulator to get all information revealed by proposing "denunciation" games (see for example Maskin (1977) and Moore-Repullo (1986) respectively for Nash- and perfect Nash-implementation). In a second class of models, agents have incomplete information. The Bayesian approach consists then in assuming that agents have prior beliefs about others' information. Once again, it may be the possible to extract the private information through with Bayesian incentive compatible mechanisms (see for example Maskin (1985), D'Aspremont-Gérard Varet (1979, 1986) and Bensaid (1987)) at least when beliefs are conviently correlated. However this approach requires strong assumptions and, in particular, prior beliefs must be common knowledge, which in some cases may not be an acceptable assumption. Moreover, once the information revealed, agents may desire to change their play. This stresses the question of the durability of these mechanisms [see for example Holmström-Myerson (1983) and Crawford (1985)] and of their robustness to pre-play communications [(Farell (1982), Forges (1986)] and renegotiations [(Cramton (1984), Dewatripont (1986), Hart-Tirole (1987)]. An alternative approach consists in assuming that agents have no information about others' characteristics and this leads to focusing on dominant strategies. Of course the decision rules which can be implemented in dominant strategies are usually much fewer than those which are Bayesian incentive compatible but, as a counterpart, implementation in dominant strategies is quite robust to "outside" communication or negotiation. From a different point of view, implementation in dominant strategies is a natural concept when the regulator has imperfect information about the agents' beliefs, since a decision rule is implementable in dominant strategies if and only if it is Bayesian implementable for any prior belief [see Dasgupta-Hammond-Maskin (1979].

A possible way of bringing these two approaches nearer is to add a posterior implementability condition to the Bayesian framework [see Green-Laffont (1987)] : a decision rule is said to be posterior implementable if, once all agents' plays are known, no agent has any regret about his initial strategy. When using direct truthful mechanisms in adverse selection models, posterior implementability coincides with implementability in dominant strategies. As we will show, this property generalizes to all types of mechanism, as soon as private information which is known to any given agent does not affect other agents' welfare.

Posterior implementabily will be assumed here and this choice is motivated by at least two reasons. First, posterior implementability considerably reinforce the strenghth of Bayesian implementability (as above-mentioned, in some cases, it even coincides with implementability in dominant strategies). Furthermore, posterior implementability makes sense when contractual agreements are formed in a two-step procedure. In a first step, there is a tatonnement during which agents announce the strategies they intend to play. This tatonnement terminates only when agents are satisfied with their play, taking into account the additional information which is transmitted in the messages announced by the other agents. Secondly, there is a signing step where no further information is exchanged and the agreement obtained at the first step is ratified. From this point of view, posterior implementability seems to be a reasonable condition for research and development joint ventures since oligopolistic firms will probably communicate in the process of forming such contractual arrangements.

From an abstract point of view, the purpose of this paper is to show that posterior implementable decision rules which are optimal when agents' actions are verifiable can still be (posterior) implemented in presence of moral hazard. We thus aim at generalizing results which are now reasonably will understood in the one-agent case to a multi-agent setting. More precisely, in our framework of cooperative R & D, we will first characterize the optimal posterior implementable decision rule, under incomplete information about private profitability : here contributions (effort levels) will be verifiable and can be contracted upon. We thus have a pure adverse selection problem, where ex post inefficiency results only from the inability of the regulator to observe contractors' private profits originated in the cooperative R & D.

Secondly, we will consider the case where individual contributions cannot be verified. It will be assumed that the aggregate outcome of the cooperative programme can be observed by the regulator (possibly with a random noise) but the effort levels of contractors are not publicly observed. In this framework, the following type of mechanism is developped. First, the regulator offers reward schemes, associated with profitability reports, which define individual subsidies as a function of the aggregate outcome. Firms then play a two stage game. At the first stage, each firm publicly announces its profitability report ; at the second stage firms choose their level of effort. Under reasonable assumptions, it will be shown that the reward schemes can be chosen so that effort levels and subsidies coincide with the optimal decision rule of the pure adverse selection case, for any truthful perfect equilibrium which satisfies the posterior implementability condition. In this sense, imperfect observability of the contractors' actions entails no additional welfare loss for the regulator.

Let us finally mention that, after writing the first draft of this paper, we became aware of a recent unpublished paper by McAfee and McMillan (1986), in which the same questions are approached in a different context. Connections and differences will be stated precisely at the end of this paper.

The paper is organized as follows. Section 2 presents the model and characterizes the complete information solution. Section 3 considers the case where the regulator has imperfect information about the firms' profitability. In section 4 this assumption is combined with unobservability of firms' contributions. All proofs are gathered in an appendix.

2. The basic framework

We consider n firms (laboratories) taking part in a joint research programme organized by a regulator (central planner). Let X be the outcome of this research joint venture (X can be thought of as the benefits resulting from a cost reduction in an elementary production process, owing to improvements conceived in the execution of cooperative R&D). This outcome depends on the firms' levels of effort (their individual contributions) and on a random disturbance. We will write :

$$X = R(a) + \epsilon$$

where :

- $a = (a_1, \ldots, a_n)$ and a_i denotes firm i's level of effort ($a_i \in \mathbb{R}_+$),
- $R(.)$ is a concave function, defined and differentiable on R^n_+ which satisfies, for i, j = 1 ... n :

$$R'_i = \frac{\partial R}{\partial a_i} > 0 \quad \text{and} \quad R''_{ij} = \frac{\partial^2 R}{\partial a_i \, \partial a_j} \geq 0 \quad i \neq j$$

- ε is a zero mean random variable with a variance σ^2.

Firm i's utility, taken as expected additional profits, attributable to the research programme, is given by[(4)] :

$$u_i(a, t_i, \theta_i) = E_\varepsilon\,[\, t_i - \lambda_i(a_i) + \theta_i X \,] = t_i - \lambda_i(a_i) + \theta_i R(a),$$

where t_i and $\lambda_i(.)$ respectively denote a transfer paid by the regulator to firm i and the monetary cost (disutility) to this firm of its effort. Function λ_i is defined and differentiable on R_+ and we assume : $\lambda'_i(a_i) > 0$ if $a_i > 0$, $\lambda'_i(0) = 0$, $\lambda'_i(a_i) \longrightarrow +\infty$ when $a_i \longrightarrow +\infty$, $\lambda''(a_i) > 0$ for all a_i.

In the previous formula $\theta_i X$ corresponds to additional cashflows for firm i , resulting from the exploitation of patents and statutory rights for invention or improvements conceived in the execution of the cooperative programme. The profitability parameter θ_i depends on the usefulness of the research programme for firm i. We assume :

$$\theta_i \in \Theta_i = [\theta_i^0 \,,\, \theta_i^1] \subset \mathbb{R} \,.$$

The regulator aims at maximizing the social surplus, which is supposed to be written as :

$$h(\theta)X - \sum_{i=1}^{n} t_i,$$

where $\theta = (\theta_1, \ldots, \theta_n)$ and $h(\theta) > 0$ for all θ in $\Theta = \prod_{i=1}^{n} \Theta_i$. $h(\theta)X$ stands for the effect of the outcome X on the social surplus.[5] A possible interpretation is to assume that the regulator maximizes the aggregate consumers' surplus on markets where firms $i = 1 \ldots n$ are present. The research programme modify firms' production costs and entails also an increase for the consumers' surplus. In that case, a functional relation can be derived between the consequences of the reseach programme for the aggregate consumers' surplus and for the firms' profits[6].

Transfers to firms are supposed to be paid by consumers through taxes and have to be deduced to obtain an evaluation of social benefits. More realistic objective functions could be introduced, for instance by assuming that the regulator maximizes a weighted sum of firms' profits and consumers' surplus, without any qualitative change in the result, at least when the weight is larger for the consumer surplus than for the firms' profits. However, we will confine ourselves to the previous formulation because of its siplicity.

Finally we will assume that the firms and the regulator are risk neutral.

In this framework an *allocation* $(a,t) = (a_1 \ldots a_n, t_1 \ldots t_n) \in \mathbb{R}_+^n \times \mathbb{R}^n$ specifies the levels of effort and the transfers. A *decision rule* $\varphi(.)$ is a function from Θ into $\mathbb{R}_+^n \times \mathbb{R}^n$, which associates an allocation $\varphi(\theta) = (a(\theta), t(\theta))$ to any possible list of profitability parameters $\theta \in \Theta$

If the regulator had perfect information about the profitability parameters θ_i and if he could verify the individual levels of effort, he would choose the first best allocation $\varphi^*(\theta) = (a^*(\theta), t^*(\theta))$ which maximizes the social surplus, subject to individual rationality constraints :

$$\underset{(a,t) \in \mathbb{R}_+^n \times \mathbb{R}^n}{\text{Maximize}} \quad h(\theta)\, R(a) - \sum_{i=1}^{n} t_i$$

$$\text{subject to : } u_i(a, t_i, \theta_i) \geq 0 \quad i = 1 \ldots n.$$

The first best allocation φ^* is thus characterized by :

$$\lambda_i'(a_i^*) = (h(\theta) + \sum_{j=1}^{n} \theta_j)\ R_i'(a^*) \qquad (1)$$

$$t_i^* = \lambda_i(a_i^*) - \theta_i\ R(a^*) \qquad (2)$$

From (1) the marginal disutility of effort $\lambda_i'(a_i^*)$ is equal to the sum of marginal profits $(\sum_j \theta_j)R_i'(a^*)$ and marginal social surplus $h(\theta)R_i'(a_i^*)$. From (2), costs $\lambda_i(a_i^*)$ – that is the monetary value of effort – minus expected cash flows $\theta_i\ R(a^*)$ are reimbursed.

3. The pure adverse selection case

We will now assume that the profitability parameters are private information (but in this section the levels of effort are still supposed to be verifiable). The characteristics are drawn from a joint distribution F, which is common knowledge, and each θ_i is only observed by the corresponding firm. In the following we will assume that the joint density f is strictly positive and continuous over the whole set Θ.

As familiar in incentives theory, we will define a ***mechanism*** as a n-uple of message (signal) spaces S_i and a measurable outcome function g from $S = \prod_{i=1}^{n} S_i$ to the set of allocations $\mathbb{R}_+^n \times \mathbb{R}^n$.

Given a mechanism (S,g), firms choose a signal $s_i \in S_i$ based on their beliefs, described by the conditional distributions $F_i(.|\theta_i)$ over the other firms' characteristics $\theta_{-i} = (\theta_1 \ldots \theta_{i-1}, \theta_{i+1}, \ldots \theta_n)$. A ***strategy*** for firm i is thus a measurable function σ_i from Θ_i into S_i. The outcome associated to the mechanism (S,g) and strategies $\sigma = (\sigma_1 \ldots \sigma_n)$ is given by the function φ, from Θ to $\mathbb{R}_+^n \times \mathbb{R}^n$, defined by :

$$\varphi(\theta) = g(\sigma_1(\theta_1) \ldots \sigma_n(\theta_n)).$$

and $\sigma = (\sigma_1 \dots \sigma_n)$ is a ***Bayesian equilibrium*** of the mechanism (S,g) if :

$$\forall\ i = 1,\dots n, \forall\ \theta \in \Theta, \sigma_i(\theta_i) \in \underset{s_i \in S_i}{\text{Arg Max}} \int_{\Theta_{-i}} u_i(g(s_i,\sigma_{-i}(\theta_{-i})),\theta_i)\ dF_i(\theta_{-i}|\theta_i),$$

where $\sigma_{-i}(\theta_{-i}) = (\sigma_1(\theta_1),\dots,\sigma_{i-1}(\theta_{i-1}),\sigma_{i+1}(\theta_{i+1}),\dots,\sigma_n(\theta_n))$.

Finally, an allocation $\varphi(.)$ is ***Bayesian implementable*** if there exists a mechanism (S,g) and a Bayesian equilibrium σ of this mechanism such that $\forall\theta \in \Theta : \varphi(\theta) = g(\sigma_1(\theta_1),\dots,\sigma_n(\theta_n))$. Following Green-Laffont (1987), $\varphi(.)$ is ***posterior implementable*** if $\varphi(.)$ is Bayesian implementable and it is common knowledge that firms will remain satisfied with their play given the play of all others. Let $\tilde{F}_i(\theta_{-i}|\theta_i,s_{-i})$ be the conditional distribution that firm i holds about θ_{-i} after having observed the others' choices $s_{-i} \in \Theta_{-i}$. Posterior implementability requires that equilibrium strategy rules satisfy :

$$\forall\ i = 1 \dots n,\ \forall\ \theta_i \in \Theta_i,\ \forall\ s_{-i} \in \sigma_{-i}(\Theta_{-i}) = \prod_{j\neq i} \sigma_j(\Theta_j):$$

$$\sigma_i(\theta_i) \in \underset{s_i \in S_i}{\text{Arg Max}} \int_{\Theta_{-i}} u_i(g(s_i,s_{-i}),\theta_i)\ d\tilde{F}_i(\theta_{-i}|\theta_i,s_{-i}).$$

Since the expected profit of firm i does not depend on θ_{-i} , the posterior implementability condition is equivalent to:

$$\sigma_i(\theta_i) \in \underset{s_i \in S_i}{\text{Arg Max}}\ u_i(g(s_i,s_{-i}),\theta_i)$$

for all s_{-i} in $\sigma_{-i}(\Theta_{-i})$; that is, $\sigma_i(\theta_i)$ is a dominant strategy for the mechanism $\{S',g\}$ with $S' = \prod_{i=1}^{n} S'_i$ and $S'_i = \sigma_i(\Theta_i) \subset S_i$. Thus, from the revelation principle, there exists a direct mechanism $\{\Theta, g'\}$ -i.e. here a mechanism where stragegies are profitability reports - for which truthtelling is a dominant strategy and which satisfies $g'(\theta) = \varphi(\theta)$ for all θ . In other words, to conclude this methodological digression, under our assumptions a decision rule that can be posterior implemented through a mechanism with abstract strategy sets can also be implemented truthfully through a direct mechanism. Thus there is no generality loss in restricting the design of decision rules by assuming that truthtelling is a dominant strategy in the corresponding direct mechanism. The individual rationality constraint has also to be introduced to ensure the feasibility of allocations. These conditions are summarized up in the following definition.

Definition 1

The decision rule $\hat{\varphi} : \theta \rightarrow (\hat{a}(\theta), \hat{t}(\theta))$ *is incentive compatible if* $\forall\, i = 1, \ldots, n\ ,\ \forall\, \theta \in \Theta$:

$$\theta_i \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}}\ u_i(\hat{\varphi}(\tilde{\theta}_i, \theta_{-i}), \theta_i) \tag{3}$$

and

$$u_i(\hat{\varphi}(\theta),\ \theta_i) \geq 0 \tag{4}$$

Incentive compatible decision rules are characterized in the following proposition.

Proposition 1 :

A continuous function $\hat{\varphi} : \Theta \longrightarrow R^n_+ \times R^n$ *is an incentive compatible decision rule if and only if* $\forall\, i = 1, \ldots, n,\ \forall\, \theta \in \Theta$:

$$\hat{t}_i(\theta) = \lambda_i(\hat{a}_i(\theta)) - \theta_i\, R(\hat{a}(\theta)) + \int_{\theta_i^0}^{\theta_i} R(\hat{a}(s_i, \theta_{-i}))\, ds_i + \pi_i(\theta_{-i}), \tag{5}$$

$$\sum_{j=1}^{n} R'_j(\hat{a}(\theta))\, \frac{\partial\, \hat{a}_j}{\partial\, \theta_i}(\theta) \geq 0 \qquad \text{a.e.,} \tag{6}$$

where $\hat{\varphi} = \{\hat{a}, \hat{t}\}$ *and* $\pi_i(.)$ *is an arbitrary function of* θ_{-i} *which satisfies*, $\forall\, \theta_{-i} \in \Theta_{-i}$: $\pi_i(\theta_{-i}) \geq 0$.

Such conditions are familiar from the literature on incentive compatibility in dominant strategies. Condition (5) defines the rent that the regulator has to pay to induce truthful revelation : the profit of firm i is equal to :

$$\int_{\theta_i^o}^{\theta_i} R(\hat{a}(s_i,\theta_{-i}))\, ds_i + \pi_i(\theta_{-i}),$$

so that high profitability firms get larger profits than low profitability ones. Furthermore from (6), the expected outcome function $R(\hat{a}(.))$ is weakly increasing.

The regulator is risk neutral and determines the optimal decision rule by maximizing the expected social surplus in the set of incentive compatible decision rules. A preliminary lemma will characterize the expected social surplus.

<u>Lemma</u>. *For any incentive compatible decision rule* $\hat{\varphi} = \{\hat{a}, \hat{t}\}$, *the expected social surplus* W *may be written as*

$$W = \int_{\Theta} \{\Psi(\theta) R(\hat{a}(\theta)) - \sum_{i=1}^{n} (\lambda_i(\hat{a}_i(\theta)) - \pi_i(\theta_{-i}))\}\, dF(\theta), \tag{7}$$

where $\pi_i(.)$ *satisfies* (5) *and* :

$$\Psi(\theta) = h(\theta) + \sum_{i=1}^{n} \theta_i - \sum_{i=1}^{n} \left[\frac{1-F_i(\theta_{-i}|\theta_i}{f_i(\theta_{-i}|\theta_i}\right] \tag{8}$$

<u>Proposition 2</u>.

If function $\Psi(\theta)$ *is strictly positive and weakly increasing, the optimal decision rule* $\hat{\varphi} = \{\hat{a}, \hat{t}\}$ *satisfies for all* θ *in* Θ *and* $i = 1 \ldots n$:

$$\Psi(\theta)\, R_i'(\hat{a}(\theta)) - \lambda_i'(\hat{a}_i(\theta)) = 0 \tag{9}$$

$$\hat{t}_i(\theta) = \lambda_i(\hat{a}_i(\theta)) - \theta_i\, R(\hat{a}(\theta)) + \int_{\theta_i^o}^{\theta_i} R(\hat{a}(s_i,\theta_{-i}))\, ds_i \tag{10}$$

and functions $\hat{a}_i(\theta)$, $i = 1,.. n$, *are strictly positive and weakly increasing.*

Conditions (8) and (9) define the optimal level of effort of firms which take part in the cooperative programme. The marginal disutility of effort λ_i' is now inferior to its marginal social value $(h(\theta) + \sum_j \theta_j)R'_i$, which shows that imperfect information entails an efficiency loss except when firms have the highest profitability parameters $(\theta_i = \theta_i^1, i = 1...n)$. From (10), profits are strictly positive except for firms with the lowest profitability parameter (i.e. $\theta_i = \theta_i^0$) . From (6) the expected outcome associated to any incentive compatible decision rule is a weakly increasing function defined over Θ. From proposition 2, this property is reinforced for the optimal decision rule, since all functions $\hat{a}_i(\theta)$ are weakly increasing.

For incentive compatible decision rules, profits of almost all firms are strictly positive. This informational rent marks up the cost of effort for the regulator and in turn, as stated in the following corollary, it leads to suboptimal levels of effort.

Corollary

The optimal decision rule $\hat{\varphi} = \{ \hat{a}, \hat{t} \}$ *satisfies* :

$$\hat{a}(\theta) = a^*(\theta) \text{ if } \theta = (\theta_1^1, ... \theta_n^1)$$

$$\hat{a}_i(\theta) < a_i^*(\theta) \textit{ for all } i\textit{, otherwise}$$

4. Adverse Selection and Moral Hazard.

We now assume that individual levels of effort are not verifiable. Only the outcome X is publicly known and can be contracted upon. However, we will show that the decision rule $\hat{\varphi}$ which was optimal when effort was verifiable can still be implemented through a two stage mechanism.

This mechanism can be described as follows. Let $T = (T_1,...,T_n)$ with $T_i = T_i(\tilde{\theta},X)$ be an incentive scheme which specifies the transfers received by each firm i as a function of profitability reports $\tilde{\theta} = (\tilde{\theta}_1,...,\tilde{\theta}_n)$ and ex-post outcome X . We consider a two stage sequential game. Each firm i sends a profitability report $\tilde{\theta}_i$ to the regulator at stage 1 and chooses a level of effort a_i at stage 2. The regulator commits to pay a transfer $T_i(\tilde{\theta},X) = T_i(\tilde{\theta},R(a)+\varepsilon)$.

In this sequential game, firm i's strategy is a pair (σ_i,α_i), where $\sigma_i : \Theta_i \to \Theta_i$ specifies the profitability report of the firm, as a function of its true characteristic, and $\alpha_i(\theta_i,\tilde{\theta}) : \Theta_i \times \Theta \to \mathbb{R}_+$ specifies its level of effort a_i as a function of its characteristic θ_i and of the reports $\tilde{\theta}$ observed at stage 1. Let $\tilde{F}_i(\theta_{-i} \mid \theta_i,\tilde{\theta}_{-i})$ be the posterior conditional distribution that firm i holds about θ_{-i}, after having observed the reports $\tilde{\theta}_{-i}$.

Strategies (σ, α) and posterior beliefs $\tilde{F}$, where $\sigma = (\sigma_1,\ldots,\sigma_n)$, $\alpha = (\alpha_1,\ldots,\alpha_n)$ and $\tilde{F} = (\tilde{F}_1,\ldots,\tilde{F}_n)$ define a *perfect Bayesian equilibrium* of the game induced by the incentive scheme $T(.)$ if, for all $i = 1,\ldots,n$, the following three conditions are satisfied :

<u>Condition 1</u> : σ is a Bayesian equilibrium of the game defined by $(T(.),\alpha)$ and prior beliefs F, i.e. $\forall\, i = 1,\ldots n$, $\forall\, \theta_i \in \Theta_i$:

$$\sigma_i(\theta_i) \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg max}} \int_{\Theta_{-i}} \Big\{ E_\varepsilon\Big[T_i(\tilde{\theta}_i,\sigma_{-i}(\theta_{-i}),R(\ldots,\alpha_j(\theta_j,(\tilde{\theta}_i,\sigma_{-i}(\theta_{-i}))),\ldots)+\varepsilon\Big]$$

$$-\lambda_i\Big(\alpha_i(\theta_i,(\tilde{\theta}_i,\sigma_{-i}(\theta_{-i})))\Big) + \theta_i R(\ldots,\alpha_j(\theta_j,(\tilde{\theta}_i,\sigma_{-i}(\theta_{-i}))),\ldots \Big\}\, dF_i(\theta_{-i} \mid \theta_i).$$

<u>Condition 2</u> : For all possible reports $\tilde{\theta}$, $((\alpha_1(.,\tilde{\theta}),\ldots,\alpha_n(.,\tilde{\theta}))$ is a Bayesian equilibrium of the corresponding subgame, for posterior beliefs $\tilde{F}$, i.e. $\forall\, i = 1,\ldots n$, $\forall\, \theta_i \in \Theta_i$, $\forall\, \tilde{\theta} \in \sigma(\Theta)$:

$$\alpha_i(\theta_i,\tilde{\theta}) \in \underset{a_i \in \mathbb{R}^n_+}{\text{Arg max}} \int_{\Theta_{-i}} \Big\{ E_\varepsilon\Big[T_i(\tilde{\theta},R(a_i,\ldots,\alpha_j(\theta_j,\tilde{\theta}),\ldots) + \varepsilon\Big]$$

$$-\lambda_i(a_i) + \theta_i R(a_i,\ldots,\alpha_j(\theta_j,\tilde{\theta}),\ldots)\Big\}\, d\tilde{F}_i(\theta_{-i} \mid \theta_i,\tilde{\theta}_{-i}).$$

<u>Condition 3</u> : Posterior beliefs $\tilde{F}$ are updated from prior F and are consistent with Bayes' rule and strategies σ.

We will say that $(\sigma,\alpha,\tilde{F})$ is a *posterior perfect Bayesian equilibrium* if it is a perfect Bayesian equilibrium and if it moreover satisfies :

<u>Condition 1 bis</u> :

$$\forall\, i = 1,\ldots n,\ \forall\, \theta_i \in \Theta_i, \forall\, \tilde{\theta}_{-i} \in \sigma_{-i}(\Theta_{-i}),$$

$$\sigma_i(\theta_i) \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg max}} \int_{\Theta_{-i}} \Big\{ E_\epsilon\Big[T_i((\tilde{\theta}_i,\tilde{\theta}_{-i}),R(\ldots,\alpha_j(\theta_j,(\tilde{\theta}_i,\tilde{\theta}_{-i})),\ldots) + \epsilon)\Big]$$

$$-\lambda_i(\alpha_i(\theta_i,(\tilde{\theta}_i,\tilde{\theta}_{-i}))) + \theta_i R(\ldots,\alpha_j(\theta_j, (\tilde{\theta}_i,\tilde{\theta}_{-i})),\ldots)\Big\}\ d\tilde{F}_i(\theta_{-i}|\theta_i,\tilde{\theta}_{-i}).$$

<u>Remark</u>

Condition 1 bis and 3 imply condition 1 (see the proof in the appendix).

The previous remark justifies the following definitions.

<u>Definition 2</u>

A vector of strategy rules (σ,α) *and beliefs* $\tilde{F}$ *define a posterior perfect Bayesian equilibrium if, for all* $i = 1,\ldots n$, *conditions 1 bis, 2 and 3 are satisfied.*

<u>Definition 3</u>

The incentive scheme T *is said to truthfully (posterior) implement the decision rule* $\varphi = \{\hat{a},\hat{t}\}$ *if it defines a sequential game for which :*

(a) *there exists a posterior perfect Bayesian equilibrium* $(\sigma,\alpha,\tilde{F})$ *such that* $\forall\, \theta \in \Theta : \sigma(\theta) = \theta$,

(b) *for any posterior perfect Bayesian equilibrium* $(\sigma,\alpha,\tilde{F})$ *such that* $\sigma(\theta) = \theta$, *we have* $\alpha_i(\theta_i,\theta) = \hat{a}_i(\theta)$ $\forall$ i = 1 ... n, $\forall\ \theta \in \Theta$.

(c) $\forall$ i = 1 ... n, $\forall\ \theta \in \Theta$: $E_\varepsilon T_i(\theta,R(\hat{a}(\theta))+\varepsilon) = \hat{t}_i(\theta)$.

Truthful posterior implementation means that there exists a posterior perfect Bayesian equilibrium such at stage 1 all firms tell the truth and at stage 2 they choose the levels of effort and they receive the expected transfers as specified by function $\hat{\varphi}$.

At first sight, turning one's attention to truthful equilibria may seem unduly restrictive here, since the revelation principle does not hold for the class of posterior implementable memchanisms - see Green and Laffont (1987) - . However, it will turn out that the optimal decision rule $\hat{\varphi}$ described in proposition 2 can actually be implemented through such a truthful equilibrium : there is thus no actual generality loss here. Of course, restricting attention to truthful equilibria does not mean that there does not exist other equilibria in the sequential game under consideration ; moreover, a non truthful equilibrium if any, might lead to suboptimal decisions. Such a possibility occurs in many models, even when the revelation principle applies, but it certainly sets limits to the relevance of our results.

Revelation problems here mix adverse selection and moral hazard. The following proposition confirms the intuition that the set of implementable allocations is a part of the set of the allocations which could be obtained in the pure adverse selection setting.

<u>Proposition 3</u>

If $\hat{\varphi}$ *is an individually rational decision rule which can be truthfully implemented through an incentive scheme* T(.) , *then* $\hat{\varphi}$ *is incentive compatible (in the sense of definition 1).*

In fact, we aim here at proving a symmetrical result : Under suitable assumptions, there exists an incentive scheme which truthfully posterior implements the optimal decision rule defined in proposition 2.

Proposition 4

Assume $\Psi(\theta)$ is positive and weakly increasing. Then there exists a nonpositive parameter $\bar{H}$ such that, for any $H \leq \bar{H}$, the optimal decision rule $\hat{\varphi} = \{\hat{a}, \hat{t}\}$ defined in proposition 2 is truthfully (posterior) implemented through quadratic incentive schemes $T^H = (T_i^H)$, defined by :

$$T_i^H(\tilde{\theta}, X) = \frac{H}{2}(X - \hat{R}(\tilde{\theta}))^2 + (\Psi(\tilde{\theta}) - \tilde{\theta}_i))(X - \hat{R}(\tilde{\theta})) + \hat{t}_i(\tilde{\theta}) - \frac{H\sigma^2}{2}$$

Furthermore, one can choose $\bar{H} = 0$ if $\Psi(\theta) - \sum_i \theta_i$ is weakly increasing and in that case, $\hat{\varphi}$ can be thruthfully (posterior) implemented through an incentive scheme which is linear with respect to the outcome X.

5. Comments and concluding remarks

Proposition 4, which is the main result of this paper, deserves a number of comments. First, it should be observed that this proposition extends results obtained by Laffont-Tirole (1986), Caillaud-Guesnerie-Rey (1986) and Picard (1987), in the framework of a one agent model. As in these papers, the optimal allocation can be implemented through a menu of linear or quadratic compensation schedules which define the agents' reward as a function of realized outcome. This family of compensation schedules takes the form of a simple penalty-bonus scheme, where $\hat{R}(\tilde{\theta})$ should be interpreted as the targeted outcome associated with vector of a messages $\tilde{\theta}$ sent to the regulator. These compensation schedules are thus fairly simple and, furthermore, they only require the knowledge of the variance of the random noise ε : it is thus unnecessary for the regulator to know the exact distribution of ε.

Under a more restrictive assumption - i.e. $\Psi(\theta) - \sum_i \theta_i$ weakly increasing - the optimal decision rule can even be implemented through a menu of linear compensation schedules. In that case, the agents' compensation includes a fixed fee $\hat{t}_i(\theta)$ and a variable transfer. From (5) the fixed fee satisfies:

$$\hat{t}_i(\tilde{\theta}) = \lambda_i(\hat{a}_i(\tilde{\theta})) - \tilde{\theta}_i R(\hat{a}(\tilde{\theta})) + \int_{\theta_i^0}^{\tilde{\theta}_i} R(\hat{a}(s_i, \tilde{\theta}_{-i}) ds_i.$$

The fixed fee includes the reimbursement of firm i's contribution $\lambda_i(\hat{a}_i(\tilde{\theta}))$ minus expected cashflows which result from the programme $\tilde{\theta}_i R(\hat{a}(\tilde{\theta})$. It also includes a term which increases with the announced profitability parameter $\tilde{\theta}_i$, in order to induce truthful revelation. The variable transfer $(\Psi(\tilde{\theta})-\tilde{\theta}_i)(X-\hat{R}(\tilde{\theta}))$ is proportional to the difference $X-\hat{R}(\tilde{\theta})$ between realized and expected outcome and the penalty coefficient $\Psi(\tilde{\theta})-\tilde{\theta}_i$ is higher for firms which announce a low profitability than for firms announcing a large profitability. Nevertheless penalty coefficients $\Psi(\tilde{\theta})-\tilde{\theta}_i$ are all increasing when any profitability announcement $\tilde{\theta}_j$ increase.

The proof of proposition 4 is rather tedious but a good intuition of results can be developped as follows. <u>Assume first that effort is verifiable</u>. Let $P_i = t_i - \lambda_i(a_i)$ be the "net transfer" to firm i. The optimal decision rule $\tilde{\varphi} = \{\hat{a},\hat{t}\}$ described in proposition 2 gives functions $\hat{P}_i(\theta)$, i = 1...n and $\hat{R}(\theta)$:

$$\hat{P}_i(\theta) = \hat{t}_i(\theta) - \lambda_i(\hat{a}_i(\theta)) \qquad i = 1\ldots n$$
$$\hat{R}(\theta) = R(\hat{a}(\theta))$$

For all i = 1,... n and for given reports $\tilde{\theta}_{-i}$, functions $\theta_i \longrightarrow \hat{P}_i(\theta_i,\tilde{\theta}_{-i})$ and $\theta_i \longrightarrow \hat{R}(\theta_i,\tilde{\theta}_{-i})$ define a smooth concave curve C_i in the $(\bar{X},P_i)$ plane, where $\bar{X}$ is the expected outcome (see figure 1). The expected profit of firm i writes as $u_i = P_i + \theta_i \bar{X}$ and isoprofit curves are straight lines with slope $-\theta_i$.

For firm i, announcing a profitability report $\tilde{\theta}_i$ amounts to select a point $A(\tilde{\theta}_i,\tilde{\theta}_{-i})$ on curve C_i. From incentive compatibility,profit maximisation leads firm i to choose $A(\theta_i,\tilde{\theta}_{-i})$ when the true profitability parameter is θ_i . Furthermore, optimal profits cancel out when $\theta_i = \theta_i^o$ and they are strictly positive when $\theta_i > \theta_i^o$.

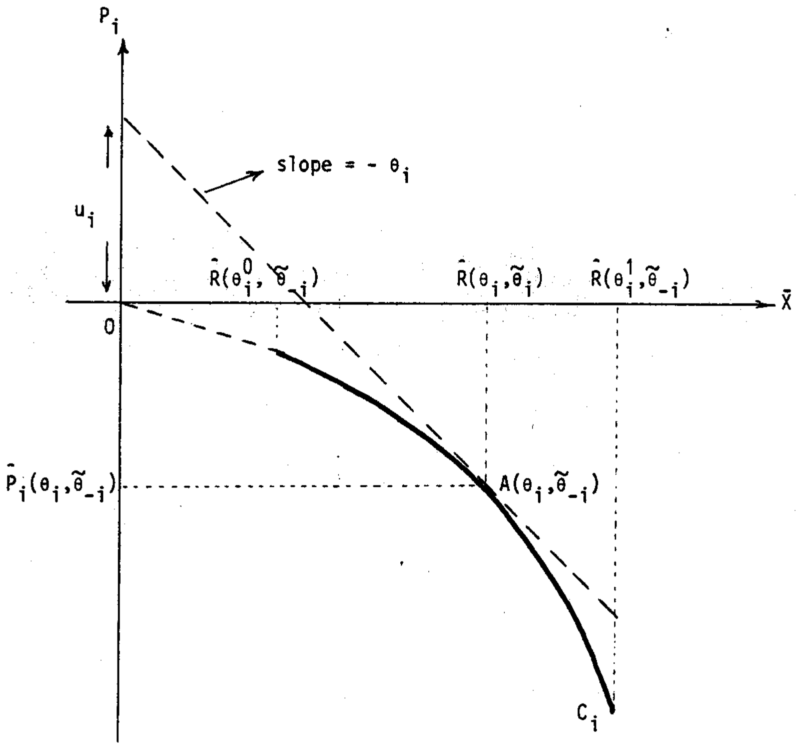


Figure 1.

Assume now that effort is not verifiable. Suppose that firm i *believes* that other firms abide by strategies (σ_j, α_j), $j \neq i$, which satisfy

$$(i) \quad \sigma_j(\theta_j) = \theta_j \quad , \quad \forall\, j \neq i \;, \; \forall\, \theta \in \Theta$$

$$(ii) \quad \alpha_j(\theta_j, \theta) = \hat{a}_j(\theta) \quad , \quad \forall\, j \neq i \;, \; \forall\, \theta \in \Theta$$

Now, let $\ell_i(\bar{X}, a_{-i})$ be defined by

$$R(\ell_i, a_{-i}) = \bar{X}$$

and let

$$\Gamma_i(\tilde{\theta}, \bar{X}) = E_\varepsilon\Big[T_i(\tilde{\theta}, \bar{X} + \varepsilon)\Big] - \lambda_i(\ell_i(\bar{X}, \hat{a}_{-i}(\tilde{\theta})))$$

Γ_i is a function from $\Theta \times R$ to R which specifies the expected net transfer to firm i as a function of profitability reports $\tilde{\theta}_1, \ldots \tilde{\theta}_n$ and of the expected outcome $\bar{X}$. When other firms announce $\tilde{\theta}_j$, $j \neq i$ at stage 1, firm i infers that these firms will choose levels of effort $\alpha_j(\tilde{\theta}_j,\tilde{\theta})$ with probability 1 at the second stage, since from *(i)* firm i thinks that other firms tell the truth at the first stage. From *(ii)*, we have

$$\alpha_j(\tilde{\theta}_j,\tilde{\theta}) = \hat{a}_j(\tilde{\theta}) \qquad \forall\ j \neq i$$

Then, from the definition of $\ell_i(.)$, $\ell_i(\bar{X},\hat{a}_{-i}(\tilde{\theta}))$ is the contribution which is viewed as necessary by firm i to achieve an expected outcome $\bar{X}$, when $\tilde{\theta}$ has been announced. Consequently, $\Gamma_i(\tilde{\theta},\bar{X})$ is the expected net transfer to firm i, from the point of view of this firm, when firm i chooses its own level of effort so as to achieve an expected outcome $\bar{X}$ after profitability reports $\tilde{\theta}_1, \ldots \tilde{\theta}_n$ have been announced at stage 1.

For quadratic or linear incentive schemes defined in proposition 4, the curve C_i and the image of function $\bar{X} \longrightarrow \Gamma_i(\theta_i,\tilde{\theta}_{-i},\bar{X})$ are tangent at point $A(\theta_i,\tilde{\theta}_{-i})$. For these incentive schemes, $\Gamma_i(\theta, \bar{X})$ is concave with respect to $\bar{X}$ and for $H \leqslant \bar{H}$, its curvature is large enough so that C_i is the upperenvelope of curves which correspond to the family $\Gamma_i(\theta_i,\tilde{\theta}_{-i},\bar{X})$, $\theta_i \in \Theta_i$. This family defines a menu of contracts offered to firm i, contingent on $\tilde{\theta}_{-i}$. Firm i has first to choose a contract by announcing $\tilde{\theta}_i$ and secondly firm i has to select an expected outcome $\bar{X}$. At stage 1, firm i's optimal choice consists in announcing $\tilde{\theta}_i = \theta_i$: $\sigma_i(\theta_i) = \theta_i$ is its optimal strategy. At stage 2, for any announcement $\tilde{\theta}$ firm i chooses a level of effort $\alpha_i(\theta_i,\tilde{\theta})$ which maximizes expected profits we thus have

$$\alpha_i(\theta_i,\tilde{\theta}) = \ell_i(v_i(\theta_i,\tilde{\theta}),\ \hat{a}_{-i}(\tilde{\theta})) \tag{11}$$

where

$$v_i(\theta_i,\tilde{\theta}) \in \operatorname*{Arg\,Max}_{\bar{X}} \left\{ \Gamma_i(\tilde{\theta},\bar{X}) + \theta_i\,\bar{X} \right\} \tag{12}$$

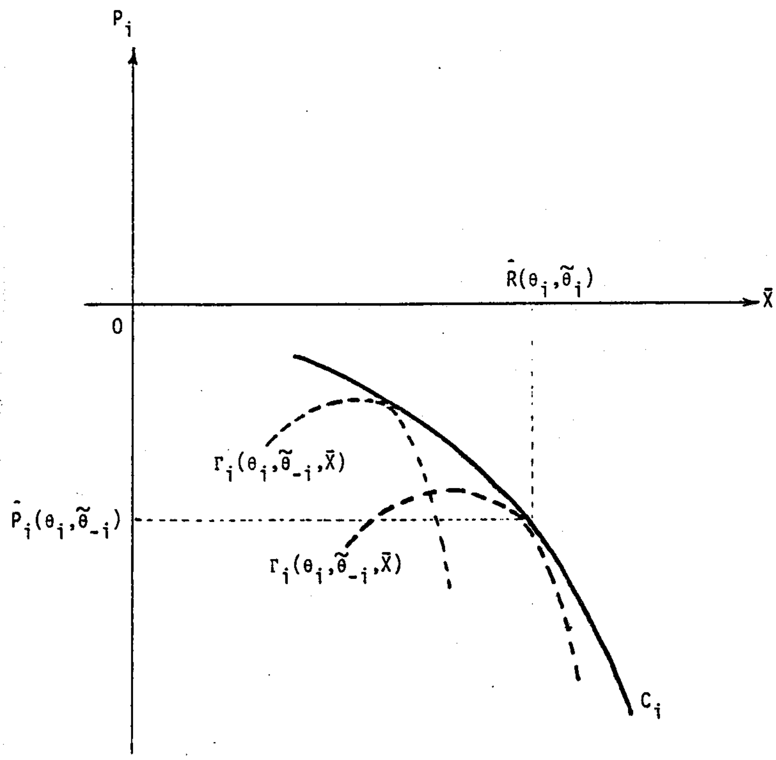
Since function v_i satisfies

$$v_i(\theta_i,\theta) = \hat{R}(\theta)$$

we have

$$\alpha_i(\theta_i,\theta) = \ell_i(\hat{R}(\theta),\ \hat{a}_{-i}(\theta)) = \hat{a}_i(\theta) \qquad (13)$$

Conditions *(i)* and *(ii)* are thus satisfied for firm i . The same argument holds for all firms. Consequently, telling the truth at stage 1, following the strategy $\alpha_i(.)$, i = 1 ... n defined in (11) and (12) at stage 2 and relying on "degenerated" updated beliefs, which assign probability one to announced parameters, define a posterior implementable perect Bayesian equilibrium.



<u>Figure 2</u>.

Let us finally state precisely the connection of proposition 4 with the work of McAfee and McMillan (1986). These authors analyse similar issues in a pure Bayesian setting, which thus differs from our posterior implementability requirement [8]. They exhibit conditions under which Bayesian incentive compatible decision rules can still be implemented through linear schemes, when individual contributions cannot be observed. However, they rule out by assumption correlation among the agents'characteristics. In that case, it turns out that adding the posterior implementability condition would not entail any welfare loss for the principal. In that sense, proposition 4 shows how their results can be extended to take into account correlation among characteristics. It moreover emphazises the potential usefulness of nonlinear schemes.

APPENDIX

Proof of proposition 1.

Let $\hat{\varphi}$ be a continuous incentive compatible decision rule and let $U_i(\theta_i,\tilde{\theta})$ be firm i's utility when its own characteristic is θ_i and reports are $\tilde{\theta}$. The first order condition associated to (3) writes as:

$$\frac{\partial U}{\partial \tilde{\theta}_i}\Big|_{\tilde{\theta}_i=\theta_i} = \frac{\partial \hat{t}_i}{\partial \theta_i}(\theta) - \frac{\partial}{\partial \theta_i}(\lambda_i(\hat{a}_i(\theta))) + \theta_i \frac{\partial}{\partial \theta_i}(R(\hat{a}(\theta))) = 0 \quad \text{a.e.},$$

which gives (5) by integrating. The local second order condition:

$$\frac{\partial^2 U_i}{\partial \tilde{\theta}_i^2}\Big|_{\tilde{\theta}_i=\theta_i} \leqslant 0$$

is equivalent to:

$$\frac{\partial^2 U_i}{\partial \tilde{\theta}_i \partial \theta_i}\Big|_{\tilde{\theta}_i=\theta_i} \geqslant 0,$$

which gives : $\sum_j R'_j(\hat{a}(\theta))\dfrac{\partial \hat{a}_j}{\partial \theta_i}(\theta) \geqslant 0 \quad$ a.e.

Lastly, we have: $\pi_i(\theta_i^o, \theta_i^o, \theta_{-i}) = \pi_i(\theta_{-i}) \geqslant 0$

Symmetrically, using the continuity of $\hat{a}(.)$, (3) results directly from (5) and (6). We then have :

$$U_i(\theta_i, \theta_i, \theta_{-i}) = \int_{\theta_i^o}^{\theta_i} R(\hat{a}(s_i,\theta_{-i}))\, ds_i + \pi_i(\theta_{-i}) \geqslant \pi_i(\theta_{-i}) \geqslant 0,$$

which shows that (4) is satisfied.

q.e.d.

Proof of the lemma

We have:

$$W = \int_{\Theta} \left\{ h(\theta)\ R(\hat{a}(\theta)) - \sum_i \hat{t}_i(\theta) \right\} dF(\theta)$$

Furthermore using (5) - which holds for any piecewise differentiable decision rule - We have:

$$\int_{\Theta} \hat{t}_i(\theta)\ dF(\theta) = \int_{\Theta} \left[\int_{\theta_i^o}^{\theta_i} R(\hat{a}(s_i,\theta_{-i}))\ ds_i \right] dF(\theta)$$

$$+ \int_{\Theta} \left\{ \lambda_i(\hat{a}_i(\theta)) - \theta_i\ R(\hat{a}(\theta)) + \pi_i(\theta_{-i}) \right\} dF(\theta)$$

The first term can be written as

$$\int_{\Theta_{-i}} \int_{\Theta_i} \int_{\theta_i^o}^{\theta_i} R(\hat{a}(s_i,\theta_{-i}))\ ds_i\ dF(\theta)$$

and integrating by parts we get

$$\int_{\Theta} \left([1-F_i(\theta_i|\theta_{-i})]\ R(\hat{a}(\theta))\ d\theta_i \right) \left(\int_{\Theta_i} f(s_i,\theta_{-i}) ds_i \right) d\theta_{-i}$$

(where $dF_i(\theta_i|\theta_{-i}) = \dfrac{f(\theta_i,\theta_{-i})}{\int_{\Theta_i} f(s_i,\theta_{-i})\ ds_i} d\theta_i = f_i(\theta_i|\theta_{-i})\ d\theta_i$)

The last expression may be written as

$$\int_{\Theta} \frac{1-F_i(\theta_i|\theta_{-i})}{f_i(\theta_i|\theta_{-i})} R(\hat{a}(\theta)) dF(\theta),$$

which gives the result.

Proof of proposition 2

From the lemma, maximizing W gives

$$\hat{a}(\theta) \in \underset{a \in R_+}{\text{Arg Max}} \left\{ \Psi(\theta)\ R(a) - \sum_{i=1}^{n} \lambda_i(a_i) \right\} \text{ for all } \theta \text{ in } \Theta$$

Assumptions on functions $R(.)$ and $\lambda_i(.)$, $i = 1 \ldots n$, guarantee that an interior solution is obtained, which gives (9). Furthermore from the maximum theorem, $\hat{a}(\theta)$ is continuous over Θ. Functions $\pi_i(\theta_{-i})$ are nonnegative and at the optimum we have for all θ_{-i} in Θ_{-i}: $\pi_i(\theta_{-i}) = 0$. Then (5) gives (10). It remains to check that functions $\hat{a}_i(\theta)$ are weakly increasing (which will imply that (6) is satisfied, so that the function $\hat{\varphi}$ defined in (9)-(10) is actually an incentive compatible decision rule). From (9) we have

$$\Psi(\theta)\ R'(\hat{a}) = I_n\ \lambda'(\hat{a})$$

where $R' = (R_i')$ and $\lambda' = (\lambda_i')$ are column vectors and I_n is the n-dimensional unit matrix. Differentiating gives

$$(\sum_i \Psi_i'\ d\theta_i)\ R'(\hat{a}) = (I_n\ \lambda'' - R'')\ d\hat{a} \qquad \text{a.e.}$$

and thus

$$d\hat{a} = (\sum_i\ \Psi_i'\ d\theta_i)\ [I_n\ \lambda'' - \Psi\ R'']^{-1}\ R'(\hat{a}) \qquad \text{a.e.}$$

The result follows from the fact that $(I_n\ \lambda'' - \Psi\ R'')^{-1}$ is nonnegative, since it is the inverse of a semi-definite positive matrix with nonpostive nondiagonal terms(9) and from the weak monotonicity of function $\Psi(\theta)$.

q.e.d.

Proof of the corollary

Let $x \in R_+$ and $\tilde{a}_i(x) \in R_n^+$, $i = 1 \ldots n$, be defined by the following equation

$$x\ R'(\tilde{a}) = I_n\ \lambda'(\tilde{a})$$

Using an argument similar to the one developped in the proof of proposition 2, $\tilde{a}_i(x)$, $i = 1 \ldots n$, are positive increasing functions. Furthermore, we have

$$\hat{a}(\theta) = \tilde{a}(\Psi(\theta)$$

$$a^*(\theta) = \tilde{a}(h(\theta) + \sum_{j=1}^{n} \theta_j)$$

for all θ in Θ. The result follows from the fact that

$$\Psi(\theta) = h(\theta) + \sum_{j=1}^{n} \theta_j \quad \text{if} \quad \theta = (\theta_1^1, \theta_2^1, \ldots, \theta_n^1)$$

$$\Psi(\theta) < h(\theta) + \sum_{j=1}^{n} \theta_j \quad \text{otherwise.}$$

<u>Proof of the remark</u>

Let $g_i(\tilde{\theta}_i, \theta_{-i}, \theta_i)$ and $h_i(\tilde{\theta}_i, \tilde{\theta}_{-i}, \theta_i, \theta_{-i})$ be respectively the functions under the integral in conditions 1 and 1 bis. We thus have

$$g_i(\tilde{\theta}_i, \theta_{-i}, \theta_i) = h_i(\tilde{\theta}_i, \sigma_{-i}(\theta_{-i}), \theta_i, \theta_{-i})$$

Assume that conditions 1 bis and 3 hold. Condition 1 bis writes as

$$\sigma_i(\theta_i) \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \int_{\Theta_{-i}} h_i(\tilde{\theta}_i, \tilde{\theta}_{-i}, \theta_i, \theta_{-i})\, d\tilde{F}_i(\theta_{-i} | \theta_i, \tilde{\theta}_{-i})$$

for all $i = 1 \ldots n$ and for all $\tilde{\theta}_{-i}$ in $\sigma_{-i}(\Theta_{-i})$.

Let $G_i(\tilde{\theta}_{-i} | \theta_i)$ be the conditional distribution of reports $\tilde{\theta}_{-i}$ which result from the conditional distribution of types $F_i(\theta_{-i} | \theta_i)$ and strategy rules σ_{-i}, i.e.

$$G_i(\tilde{\theta}_{-i} | \theta_i) = \int_{A(\tilde{\theta}_i)} dF_i(\theta_{-i} | \theta_i)$$

with

$$A(\tilde{\theta}_i) = \left\{ \theta_{-i} \in \Theta_{-i} \mid \sigma_{-i}(\theta_{-i}) \leqslant \tilde{\theta}_{-i} \right\}$$

We have

$$F_i(\theta_{-i} | \theta_i) = \int_{\Theta_{-i}} \tilde{F}_i(\theta_{-i} | \theta_i, \tilde{\theta}_{-i})\, dG_i(\tilde{\theta}_{-i} | \theta_i)$$

so that condition 1 writes as

$$\sigma_i(\theta_i) \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \int_{\Theta_{-i} \times \Theta_{-i}} h_i(\tilde{\theta}_i, \sigma_{-i}(\theta_{-i}), \theta_i, \theta_{-i})\, d\tilde{F}_i(\theta_{-i} | \theta_i, \tilde{\theta}_{-i})\, dG_i(\tilde{\theta}_{-i} | \theta_i)$$

But from condition 3, $\tilde{F}_i(.\ |\theta_i,\tilde{\theta}_{-i})$ assigns probability zero to types θ_{-i} such that $\sigma_i(\theta_{-i}) \neq \tilde{\theta}_{-i}$. Consequently, condition 1 can be rewritten as

$$\sigma_i(\theta_i) \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \int_{\Theta_{-i} \times \Theta_{-i}} \left[h_i(\tilde{\theta}_i,\tilde{\theta}_{-i},\theta_i,\theta_{-i})\ d\tilde{F}_i(\theta_{-i}|\theta_i,\tilde{\theta}_{-i})\right] dG_i(\tilde{\theta}_{-i}|\theta_i)$$

which is implied by condition 1bis.

<u>Proof of proposition 3</u>

Assume that T(.) implements $\varphi = (\hat{a},\hat{t})$ truthfully. Then, from (a), there exists a posterior perfect Bayesian equilibrium $(\sigma,\alpha,\tilde{F})$ of the game defined by T such that $\forall\ \theta \in \Theta : \sigma(\theta) = \theta$. For such an equilibrium, updated beliefs $\tilde{F}_i(\theta_{-i}|\theta_i,\tilde{\theta}_{-i})$ $(i = 1,\ldots,n)$ are degenerate distributions and condition 1 bis may be written as :

$$\forall\ \theta_i \in \Theta_i, \forall\ \tilde{\theta}_{-i} \in \Theta_{-i}, \theta_i \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \Big\{ E_\varepsilon [T_i((\tilde{\theta}_i,\tilde{\theta}_{-i}),R(\ldots,\alpha_j(\tilde{\theta}_i,\tilde{\theta}_{-i})\ldots) + \varepsilon)]$$

$$- \lambda_i(\alpha_i(\tilde{\theta}_i,(\tilde{\theta}_i,\tilde{\theta}_{-i}))) + \theta_i\ R(\ldots,\alpha_j(\tilde{\theta}_j,(\tilde{\theta}_i,\tilde{\theta}_{-i}))\ldots)\Big\},$$

and using (b) we have:

$$\theta_i \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \left\{E_\varepsilon T_i(\tilde{\theta},R(\hat{a}(\tilde{\theta}) + \varepsilon) - \lambda_i(\hat{a}_i(\tilde{\theta})) + \theta_i\ R(\hat{a}(\tilde{\theta}))\right\}.$$

Finally, using (c) we deduce:

$$\theta_i \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}} \left\{\hat{t}_i(\tilde{\theta}) - \lambda_i(\hat{a}_i(\tilde{\theta})) + \theta_i R(\hat{a}(\tilde{\theta}))\right\}$$

for all θ_i and $\tilde{\theta}$, which means that the individually rational decision rule $\hat{\varphi} = (\hat{a},\hat{t})$ is incentive compatible.

<u>Proof of proposition 4</u>

(a) Let $K_i(\tilde{\theta}) = \Psi(\tilde{\theta}) - \tilde{\theta}_i$ and

$$\alpha_i(\theta_i,\tilde{\theta}) \in \underset{a_i \in R^n_+}{\text{Arg Max}} \left\{ \frac{H}{2} (R(a_i,\hat{a}_{-i}(\tilde{\theta})) - \hat{R}(\tilde{\theta}))^2 \right.$$

$$\left. + (K_i(\tilde{\theta}) + \theta_i)R(a_i,\hat{a}_{-i}(\tilde{\theta})) - \lambda_i(a_i) \right\},$$

which implies: $\forall\, \theta \in \Theta,\ \forall\, H \leq 0,\ \alpha_i(\theta_i,\theta) = \hat{a}_i(\theta).$ (14)

Let $\sigma(\theta) = \theta \ \ \forall\theta \in \Theta$ and let $\tilde{F}_i(\theta_{-i}|\theta_i,\tilde{\theta}_{-i})$, $i = 1 \ldots n$, be degenerated distributions which assign probability one to $\theta_{-i} = \tilde{\theta}_{-i}$. We will show that $(\sigma,\alpha,\tilde{F})$ is a posterior perfect Bayesian equilibrium of the game defined by T. For such strategy rules and updated beliefs, condition 1 bis writes as

$$\theta_i \in \underset{\tilde{\theta}_i \in \Theta_i}{\text{Arg Max}}\ U_i(\theta_i, \tilde{\theta}_i,\tilde{\theta}_{-i},\tilde{\theta}_{-i}) = \bar{U}_i(\theta_i,\tilde{\theta})\ \forall\, \theta_i, \forall\, \tilde{\theta}_{-i} : \qquad (15)$$

where

$$U_i(\theta_i,\tilde{\theta}_i,\theta_{-i}, \tilde{\theta}_{-i}) = \frac{H}{2} (R(\ldots, \alpha_j(\theta_j,\tilde{\theta}),\ldots) - \tilde{R}(\tilde{\theta}))^2$$

$$+ (K_i(\tilde{\theta})+\theta_i)\ (R \ldots,\alpha_j(\theta_j,\tilde{\theta}),\ldots)$$

$$- K_i(\tilde{\theta})\hat{R}(\tilde{\theta}) + \hat{t}_i(\tilde{\theta}) - \lambda_i(\alpha_i(\theta_i,\tilde{\theta})) \qquad (16)$$

Using (14) we have:

$$\bar{U}_i(\theta_i,\tilde{\theta}) = \frac{H}{2} (R(\alpha_i(\theta_i,\tilde{\theta}), \hat{a}_{-i}(\tilde{\theta})) - \hat{R}(\tilde{\theta}))^2$$

$$+ (K_i(\tilde{\theta}) + \theta_i)\ (R(\alpha_i(\theta_i,\tilde{\theta}), \hat{a}_{-i}(\tilde{\theta}))$$

$$- K_i(\tilde{\theta})\ \hat{R}(\tilde{\theta}) + \hat{t}_i(\tilde{\theta}) - \lambda_i(\alpha_i(\theta_i,\tilde{\theta})). \qquad (17)$$

Using the first order condition which defines $\alpha_i(\theta_i,\tilde{\theta})$, we have:

$$\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) = \left\{ H(R(a^1) - R(a^2)) + K_i(\tilde{\theta}) \right\} \left\{ \sum_{j\neq i} R'_j(a^1)\frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} - \sum_j R'_j(a^2)\frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} \right\}$$

$$+ \frac{\partial K_i}{\partial \tilde{\theta}_i}(\tilde{\theta})\ (R(a^1) - R(a^2))$$

$$+ \frac{\partial \hat{t}_i}{\partial \tilde{\theta}_i}(\tilde{\theta}) + \theta_i \sum_{j\neq i} R'_j(a^1)\frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} \tag{18}$$

with $a^1 = (\alpha_i(\theta_i,\tilde{\theta}),\ \hat{a}_{-i}(\tilde{\theta}))$, $a^2 = \hat{a}(\tilde{\theta})$.

From (18) we get: $\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) = 0$ if $\tilde{\theta}_i = \theta_i$,and using the continuity of $\hat{a}(.)$, (15) will be satisfied if $\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta})$ is positive (respect. negative) when $\tilde{\theta}_i < \theta_i$ (respect. $\tilde{\theta}_i > \theta_i$).

Assume $\exists\, \tilde{\theta}_i > \theta_i$ such that $\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) > 0$. Then, we would have:

$$\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) > \frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\theta_i,\tilde{\theta}_{-i}) = \frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\tilde{\theta}_i,\tilde{\theta}) = 0 \tag{19}$$

We also have:

$$\frac{\partial^2 \bar{U}_i}{\partial \tilde{\theta}_i \, \partial \theta_i}(\theta_i,\tilde{\theta}) = H \left\{ \sum_{j \neq i} R_j'(a^1) \frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} - \sum_j R_j'(a^2) \frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} \right\} R_i'(a^1) \frac{\partial \alpha_i}{\partial \theta_i}$$

$$+ \left\{ H(R(a^1) - R(a^2)) + K_i(\tilde{\theta}) + \theta_i \right\} \sum_{j \neq i} R_{ji}''(a^1) \frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} \frac{\partial \alpha_i}{\partial \theta_i}$$

$$+ \frac{\partial K_i}{\partial \tilde{\theta}_i}(\tilde{\theta}) \; R_i'(a^1) \frac{\partial \alpha_i}{\partial \theta_i} + \sum_{j \neq i} R_j'(a^1) \frac{\partial \hat{a}_j}{\partial \tilde{\theta}_i} . \qquad (20)$$

From proposition 2, we know that $\forall\, i,j$: $\partial \hat{a}_j / \partial \tilde{\theta}_i \geq 0$. Then, one easily checks that the term which multiplies H in (20) is strictly negative. Thus, there exists $\hat{H}_i(\theta_i,\tilde{\theta})$ such that :

$$\frac{\partial^2 \bar{U}_i}{\partial \tilde{\theta}_i \, \partial \theta_i}(\theta_i,\tilde{\theta}) > 0 \text{ if } H \leq \hat{H}_i(\theta_i,\tilde{\theta}).$$

Define $\hat{H} = \min_{i=1...n} \operatorname{Inf} \left\{ \hat{H}_i(\theta_i,\tilde{\theta}) \mid \theta_i \in \Theta_i,\ \tilde{\theta} \in \Theta \right\}$. If we let $H \leq \hat{H}$, (19) cannot be satisfied and we have :

$$\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) < 0 \quad \text{if} \quad \tilde{\theta}_i > \theta_i.$$

Symmetrically, we also have:

$$\frac{\partial \bar{U}_i}{\partial \tilde{\theta}_i}(\theta_i,\tilde{\theta}) > 0 \quad \text{if} \quad \tilde{\theta}_i < \theta_i.$$

so that (15) is fulfilled in that case.

It can also be observed that a sufficient condition for $\partial^2\bar{U}_i/\partial\tilde{\theta}_i\partial\theta_i$, to be positive with $H \leq 0$ may be written as $\partial K_i/\partial\tilde{\theta}_i \geq 0$ which is satisfied if function $\Psi(\theta) - \sum_i \theta_i$ is weakly increasing.

Consequently, (15) is satisfied when $H \leq \bar{H}$, where $\bar{H} = 0$ if $\Psi(\theta) - \sum_i \theta_i$ is weakly increasing and $\bar{H} = \hat{H} \leq 0$ otherwise. Condition 1 bis is thus satisfied. Furthermore, conditions 2 and 3 are also satisfied, from the definition of strategy rules $\alpha_i(\theta_i, \tilde{\theta})$ and updated beliefs $\tilde{F}$. $\{\sigma, \alpha, \tilde{F}\}$ is thus a posterior perfect Bayesian equilibrium.

(b) We also have to prove that $\forall\ \theta$, $\forall\ i$, $\alpha_i(\theta_i, \theta) = \hat{a}_i(\theta)$ for any posterior perfect Bayesian equilibrium $\{\sigma, \alpha, \tilde{F}\}$ such that $\sigma(\theta) = \theta$. In that case, from condition 2 and 3 we have $\forall\ \theta_i \in \Theta_i$, $\forall\ \tilde{\theta} \in \Theta$, $\forall\ i = 1, \ldots, n$:

$$\alpha_i(\theta_i, \tilde{\theta}) \in \underset{a_i \in R_+^n}{\operatorname{Arg\,Max}} \left\{ \frac{H}{2}(R(a_i, \ldots, \alpha_j(\tilde{\theta}_j, \tilde{\theta}), \ldots) - \hat{R}(\tilde{\theta}))^2 \right.$$
$$\left. + (K_i(\tilde{\theta}) + \theta_i)\ R(a_i, \ldots, \alpha_j(\tilde{\theta}_j, \tilde{\theta}), \ldots) - \lambda_i(a_i) \right\}$$

and thus, for $\tilde{\theta} = \theta$:

$$\left\{ H(R(\ldots \alpha_j(\theta_j, \theta) \ldots) - \hat{R}(\theta)) + (K_i(\theta) + \theta_i) \right\} R_i'(\ldots, \alpha_j(\theta_j, \theta), \ldots)$$
$$- \lambda_i'(\alpha_i(\theta_i, \theta)) \leq 0$$
$$= 0 \text{ if } \alpha_i(\theta_i, \theta) > 0$$

which means that

$$(\alpha_1(\theta_1, \theta), \ldots, \alpha_n(\theta_n, \theta)) \in \underset{a \in R_+^n}{\operatorname{Arg\,Max}} \left\{ \frac{H}{2} (R(a) - \hat{R}(\theta))^2 + \Psi(\theta) R(a) - \sum_i \lambda_i(a_i) \right\},$$

which in turn implies:

$$(\alpha_1(\theta_1, \theta), \ldots, \alpha_n(\theta_n, \theta)) = \hat{a}(\theta).$$

(c) Lastly one immediately checks that:

$$\forall\ i,\ \forall\ \theta \in \Theta,\ E_\varepsilon T_i(\theta, \hat{R}(\theta) + \varepsilon) = \hat{t}_i(\theta).$$

q.e.d.

Footnotes

(1) On cooperative research in Europe, see Jacquemin and Spinoit (1986) and Jacquemin (1987).

(2) After developping potential advantages of cooperative R & D, Jacquemin and Spinoit (1986) writes : "A cooperative agreement is a compromise between a desire for collaboration and an intention of maintaining independence, so that the organizational structure reflecting such an ambiguity is usually complex. Various clauses of the legal structure express this concern and imply heavy transaction costs of negotiation, especially for transnational ventures. Partner selection and the possibility of defining well-balanced contributions is the first barrier... At the second stage, the management of existing cooperative agreements is also costly, especially in R & D where technological conditions are changing rapidly and unpredictably. As it is not possible to maintain complete control over the functioning of the cooperation, it is necessary to construct complex contracts containing explicit clauses concerning confidentiality and transmission of information... Finally, it is not easy to divide the benefits of a cooperative agreement in R & D, given that scientific knowledge has many aspects of a public good and that its results are not often easily incorporated... All of these elements suggest that even in the context of a very tolerant antitrust policy, without positive public actions, European cooperation in R & D will remain a limited and unstable phenomenon".

(3) For a different approach of cooperative R & D, more from an industrial organization standpoint, see Katz (1987).

(4) In the following $E_\varepsilon(.)$ denotes expectation with respect to the random variable ε .

(5) To simplify matters, the social value of the research project is supposed to be linear with respect to the oucome X but this assumption is not essential.

(6) Assume for instance that firms are monopolists in independent markets and, for simplicity, assume $n = 2$. Let $p_i(q_i) = b-q_i$, $C(q_i) = (c-k_iX)q_i$ for $i = 1, 2$, where $p(q_i)$, $C(q_i)$ and q_i are respectively the inverse demand function, the cost function and the production of firm i $(b > c)$. Firm i's profit π_i and consumers surplus S are respectively given by :

$$\pi_i = (b-c+k_iX)2/4$$
$$S = \left[(b-c+k_1X)^2 + (b-c+k_2X)^2\right]/8$$

Using a first order approximation, we have :

$$\theta_i = \frac{\partial \pi_i}{\partial X} \Big|_{X = o} = (b-c)k_i/2$$

$$h(\theta) = \frac{\partial S}{\partial X} \Big|_{X = o} = (b-c)\ (k_1+k_2)/4 = (\theta_1+\theta_2)/2$$

(7) More generally, one should allow for random mechanisms and strategies. It can however be shown that there is no gain here in considering such random policies.

(8) The modelling options also are slightly different : in their model, the unknown characteristics are productivity parameters.

(9) See for example Arrow and Hurwicz (1977, p.326 and 371). Using this property of semidefinite positive matrices with nonpositive nondiagonal terms in this revised version has been suggested to us by the paper of McAfee and McMillan (1986).

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