THE TREND OF THE ECONOMIC RATE OF RETURN IN
THE U.S. MANUFACTURING SINCE THE DEPRESSION∗

Gérard DUMÉNIL
and
Dominique LÉVY
N° 8732

*We thank Mark Glick for his help in the translation of this text into English.
RÉSUMÉ

LA TENDANCE DU TAUX DE RENDEMENT ÉCONOMIQUE DANS L'INDUSTRIE MANUFACTURIÈRE AMÉRICaine DEPUIS LA DÉPRESSION

Le but de cette étude est de vérifier si les objections de Fisher et McGovan à l'encontre de la mesure comptable traditionnelle des taux de profit, remettent en cause l'observation du trend à la baisse du taux de profit dans l'industrie manufacturière des États-Unis, notamment depuis la seconde moitié des années soixante. On donne une estimation du taux de rendement interne de l'investissement et du taux de profit sur le stock de capital avec amortissement économique au sens de Hotelling. On montre que ce taux manifeste la même tendance à la baisse, et que le taux de profit sur le stock de capital avec amortissement économique est très proche du taux de profit comptable avec amortissement linéaire.

ABSTRACT

THE TREND OF THE ECONOMIC RATE OF RETURN IN THE U.S. MANUFACTURING SINCE THE DEPRESSION

The purpose of this study is to determine whether the objections made by Fisher and McGovan to the traditional accounting measure of the rate of profit, applies to the downward trend in profitability which has been observed in U.S. Manufacturing since the Mid-sixties. An estimate of the economic rate of return on investment is provided, and compared with an estimate of the economic rate of return on the stock of capital with economic depreciation in the sense suggested by Hotelling. These two rates display a similar downward trend. Moreover, it can be shown that the movement of the economic rate of return on the stock of capital with economic depreciation is very close to that of the accounting rate of return with linear depreciation.

MOTS CLEFS : Taux de rendement, Taux de profit comptable, États-Unis

KEYWORDS : Economic rate of return, Accounting rate of profit, United States

J.E.L. Nomenclature : 020
It is well known that profitability can be measured in two distinct ways. The first concept is the "Economical rate of return on an investment," meaning the discount rate that equates the present value of the investment to the present value of future returns resulting from the investment. A famous example, in economic theory, is Keynes' computation of the marginal efficiency of capital. However, the "Accounting rate of return" is another concept. It is the ratio of a flow of profit in a given period to the accumulated stock of capital over time in a firm, industry, or economy. This accounting approach to profitability assessment has been widely used in two areas: 1) Industrial Organization literature, and 2) Historical studies of economy-wide profit averages.

Franklin M. Fisher and John J. McGovan pointed out in a recent study (1983) that accounting rates of return could not be used to infer monopoly profits. They suggested that only the economic rate of return on investment should be equalized in the long run, not accounting rates of return. Using numerical examples, Fisher and McGovan showed that accounting rates of return are not always good proxies for economic rates of return. Their demonstration questions the results of most, if not all, studies of comparative profitability.

The purpose of this analysis is to discuss the impact of this argument on the results of studies focusing on the historical movement of profitability in the U.S. economy since the Great Depression, especially in the post World War II period. An important issue is whether the decline in the rate of profit in the U.S. economy, which has been shown by many studies using accounting rates, holds if the economic rate on investment were used. The determination of the rate of return on investment is not a straightforward procedure, but we believe that an estimate of trend is possible. The calculation in this paper must be viewed as a first attempt at such an estimate, and will be limited to Manufacturing.

In the study of the evolution over time of profitability, the economic rate of return on investment is not the only relevant concept. The situation of an economy with regard to profitability should not be assessed exclusively at the margin (using the economic rate of return on investment in each year). This would be equivalent, in the study of a given year, to ignoring the burden of previous investments.

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1 A number of comments to Fisher and McGovan—by I. Horowitz, W.F. Long and D.J. Ravencraft, S. Martin, M.F. van Breda—have been published in the American Economic Review, Vol. 74, # 3, with a reply by Fisher (Fisher F.M. 1984).

investments realized earlier when prices and the situation of distribution were different— a burden which impacts very much on the situation of firms in the present. For this reason, we will also compute the rate of return on the total stock of capital existing in each year. In order to evaluate this stock of capital, it is necessary to estimate its depreciation. The only depreciation schedule which is theoretically consistent with the economic rate of return is the "economic depreciation" of capital, in the sense defined by Harold Hotelling (1925). It is well known that this measure of profitability is equivalent to the economic rate of return on investment in a stationary state, while, under ordinary circumstances (with technical progress, changes in prices and in distribution), it is not.

The main results reported below are the following (cf. figure 1):

1. In the American manufacturing industries, the trend of the economic rate of return on investment have been downward since the depression. This deterioration occurs before World War II and in the second half of the 1960s. The 1950s and the first half of the 1960s appear as a plateau.

2. The rate of return on the stock of capital with economic depreciation of capital has a profile very similar to traditional accounting rates of return, for both its trend and fluctuations (only the general level can be different, depending on the depreciation schedule).

3. The profile in the economic rate of return on investment is different from that of rates of return on the stock of capital, since the latter are subject to important fluctuations. All measures, however, have the same trend and, in particular, tend downward since the mid-1960s.

The crucial calculation in this study is that of the rate of return on investment. This rate corresponds to a given generation of capital: the fraction of the stock of capital which has been invested in one year. We assume that the technology is putty-clay. This assumption allows the derivation of the technical ratios (output/capital ratio, i.e., productivity of capital, productivity of labor) which characterize these generations. This description leads to the determination of the flow of returns for investments realized in each year, and therefore the computation of the rate of return on investment. In this study we will use a discrete time formalism and not a continuous time model as is usually the case. This is due to our empirical perspective (the data are annual).

The paper is divided as follows. The first part is devoted to the computation of the economic Rate of Return on Investment (RRI). The second part addresses the issue of the estimate of Rates of Return on the stock of Capital (RRK), focusing in particular on the economic depreciation of capital. Technical aspects of these issues are reserved for the appendix.

\[ ^3 \text{Often denoted IRR, for "Internal Rate of Return".} \]

THE RATE OF RETURN ON INVESTMENT (RRI)
I - THE RATE OF RETURN ON INVESTMENT (RRI)

In section A we describe the model for the marginal technology, i.e., the technology embodied in a given generation of investment. The RRI is defined in section B. Section C presents the methodology used in order to estimate the marginal technology. Section D briefly reviews a number of difficulties in the choice of the data series (for example, problems related to the existence of the so-called “Government Owned and Privately Operated” stock of capital). In Section E we present our results. Section F describes a group of variants of this basic calculation of the RRI.

A - THE MARGINAL TECHNOLOGY

The marginal technology in \( t \) is that technology which is embodied in the investment \( I(t) \) in period \( t \) (and changes over time). It is different from the average technology, corresponding to the average of all successive generations of capital invested prior to \( t \), and still in use (which also changes over time). Only the profile of the average technology can be directly derived from the observation of the available aggregate series on gross product, capital, and labor. In order to estimate the marginal technology on the basis of these series, it is necessary to build a model. We therefore make the following assumptions:

1. For each generation of investment, the technology is treated as putty-clay. When the decision to invest is made, enterprises choose a certain technology on the basis of the prevailing wages and prices. This state of the technology can not be modified in the subsequent years for this generation. We will attempt to estimate the evolution of this technology over time, but enterprises’ choices in this regard will be taken as given and not discussed in relation to the transformations of the production function. In a similar manner the formation of expectations is taken as given.

2. Two physical goods exist, fixed capital and output (gross product). In production, fixed capital is combined with labor. In a model in which three resources exist, the consideration of prices can be limited to two relative prices. We choose:

\[
\begin{align*}
\omega(t) &= \frac{\text{Price of labor}}{\text{Price of gross product}} \\
p(t) &= \frac{\text{Price of fixed capital}}{\text{Price of gross product}}
\end{align*}
\]

3. It is necessary to specify the fraction, denoted \( J(t, t') \), of each investment \( I(t) \), still in use after a number of years, at the dates \( t' = t+1, t+2, t+3, \ldots \). This profile of the productive survival of fixed capital is given by modified Winfrey retirement patterns (cf. B.E.A. 1985, Table D, p. 43). Winfrey assumes that the profile only depends on, \( \tilde{t} = (t' - t)/\bar{T} \), where \( \bar{T} \) is the average service life of capital. Winfrey’s coefficients,
which we denote \( \alpha(t) \), allow one to compute the series of \( J(t, t') \):

\[
J(t, t') = I(t) \left(1 - \alpha(t)\right) \quad \text{for} \quad t' = t, t+1, \ldots
\] (1)

\( \alpha(t) \) is an increasing function of its argument, which is equal to 0 for small values of \( t \), and equal to 1 for large values of \( t \).

In the B.E.A.'s computations, \( \bar{T} \) is independent from \( t \), but different for each type of capital (many such capitals are considered, cf. B.E.A. 1985, table C, p. 42). The average age of the stock of capital varies with time as a result of the modification of the proportion of each type of capital in the total stock. Basically, the age of capital diminishes, because of the increase in equipment relative to plant in the total stock (since the average service life of equipment is shorter than that of plants). In our model, in which only one capital good exists, this shortening of the life of fixed capital is accounted for by the assumption that \( \bar{T} \) varies with time: \( \bar{T} = \bar{T}(t) \).

4. For a given generation, we assume that the wear and tear of capital manifests itself only as a result of the dissipation of the stock of capital (given by the function \( J(t, t') \)), and that it is possible to define the gross productivities of capital and labor independently of the age of capital. We denote \( k(t) \) as the productivity of capital, and instead of considering the productivity of labor, we use the ratio of the two inputs, \( h(t) = \frac{\text{Input in labor}}{\text{Input in capital}} \). Using this notation, the gross product, resulting from the use of the remaining capital from \( I(t) \), in period \( t' \), is \( k(t)J(t, t') \). The quantity of labor used is \( h(t)J(t, t') \).

5. A further assumption is that an investment realized in \( t \) will only be productive in \( t+1 \) (an average lag of 6 months).

Under the previous assumptions, the gross stock of capital, \( K(t) \), the gross product, \( G(t) \), and the total quantity of labor, \( H(t) \), can be obtained by summing over the various generations still in use at this date:

\[
K(t) = \sum_{t'=-\infty}^{t} J(t', t) \tag{2}
\]

\[
G(t) = \sum_{t'=-\infty}^{t-1} k(t')J(t', t) \tag{3}
\]

\[
H(t) = \sum_{t'=-\infty}^{t-1} h(t')J(t', t) \tag{4}
\]

B - THE DEFINITION OF THE RRI

The RRI gives an ex ante measure of profitability, determined when the decision to invest is made. Thus, future circumstances must be anticipated. If we abstract

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from business cycle fluctuations⁴, it is primarily price expectations which must be considered. Two models of expectations will be studied here, myopic expectations and perfect expectations. In the first case, investors expect that, \( w(t) \) and \( p(t) \), the prices in the period in which they make their decision will be maintained. In the second case, they forecast prices at their actual values for the future.

Gross profit in \( t' \) corresponding to one unit of capital invested in \( t \), and still productive in \( t' \), is denoted \( \pi(t, t') \). Using this notation, the two models of expectations can be written:

\[
\begin{align*}
\pi^{(m)}(t, t') &= k(t) - w(t)h(t) \\
\pi^{(p)}(t, t') &= \max(k(t) - w(t')h(t), 0)
\end{align*}
\]

The "max" in the second equation is the expression of the fact that a rise in wages might result in a negative profit. In this case, the capital would no longer be used.

The RRI, denoted \( r \), is the discount rate which equalizes the value of the investment with the present value of the sequence of returns which will result from the investment. It is determined as the root of the following equation:

\[
\left\{ \begin{array}{c}
p(t)J(t) = \sum_{t'=t+1}^{\infty} \frac{J(t, t')\pi(t, t')}{(1 + r)^{t'-t}} \\
or, using the notation \( x = \frac{1}{1+r} \) and equation 1:
\end{array} \right.
\]

\[
p(t) = \sum_{t'=t+1}^{\infty} x^{t'-t} \left( 1 - \alpha(t) \right) \pi(t, t')
\]

Since \( \alpha \) is equal to 1 if its argument is large, the series is a polynomial whose order is equal to the maximum number of periods of use.

The proof of the existence of a unique positive root in the above equation (equation 6) is straightforward. From this, the existence and uniqueness of a RRI greater than \(-1\) both follow. The positiveness of the RRI is not guaranteed a priori.

C - THE DETERMINATION OF THE MARGINAL TECHNOLOGY

In this section we present the methodology used in order to derive the functions \( \overline{T}(t), k(t), \) and \( h(t) \) from equations 2, 3, and 4.

⁴ This is equivalent: 1) to assuming that enterprises, when they decide on an investment, expect a normal ratio of capacity utilization, 2) to abstracting from short-term fluctuations in \( p(t) \) and \( w(t) \) (regressing these variables on a polynomial in time and the ratio of capacity utilization).

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1. The Determination of \(k(t)\) and \(h(t)\)

It is important to keep in mind the actual purpose of the determination of the functions which describe the marginal technology. It is not the explanation of this technology which is at issue, but exclusively its representation. We model the transformations of the marginal technology by polynomials of small order. Functions \(k(t)\) and \(h(t)\) are represented by polynomials in \(t\):

\[
k(t) = \sum_{t=0}^{\infty} a_k t^t
\]
\[
h(t) = \sum_{t=0}^{\infty} a_h t^t
\]

We substitute these expressions in equations 3 and 4.

The variations of \(G(t)\) and \(H(t)\) in the short run do not reflect actual technical changes, but instead fluctuations related to business cycles. We assume that, \(u\), the ratio of capacity utilization is a good indicator of such fluctuations. We denote \(u_i\) as the average value of \(u\) over time. Variable \(u - \overline{u}\) is added in the description of \(G(t)\) and \(H(t)\):

\[
G(t) = \sum_{t=0}^{\infty} a^G b_t(t) + c^G (u(t) - \overline{u}) + e^G
\]
\[
H(t) = \sum_{t=0}^{\infty} a^H b_t(t) + c^H (u(t) - \overline{u}) + e^H
\]

in which: \(b_t(t) = \sum_{t'=\infty}^{t=1} t' J(t', t)\), and \(e^G\) and \(e^H\) are two random variables. Thus, parameters \(a_t\) can be determined using a simple linear regression. The main difficulty is the choice of the degree of the polynomial. This point is discussed in section C.1 of the appendix.

2. The determination of \(T(t)\)

We assume that \(T\) is a polynomial in \(t\): \(T(t) = \sum_{t=0}^{\infty} a^K t^t\). If the parameters in this polynomial were known, on the basis of the total investment series from B.E.A., and using the perpetual inventory method, equation 2 would allow the calculation of a series, \(K'(t)\), of the stock of gross capital. (The B.E.A series for \(K(t)\) is also estimated using the perpetual inventory method in a model with several goods and constant average service life.) We use a nonlinear regression in order to estimate the parameters \(a^K\) which minimize the distance between, \(K'(t)\), the estimated stock of capital and, \(K(t)\), the B.E.A.'s series. More precisely, we divide equation 2 by \(K(t)\) and the regression will determine the set of parameters \(a^K\) such that the ratio \(K'(t)/K(t)\) is as close as possible to 1.

D - DIFFICULTIES IN THE CHOICE OF THE SERIES

A first difficulty in the use of the available series concerns the choice of a particular definition of profits. In the reconstruction of the marginal technology, gross product

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must be considered globally including all of its components (depreciation allowances, taxes and interests). The consideration of profitability after taxes or interest would, of course, be possible, but these deductions from profits must be treated in a second step, after the determination of the technology as such. The scope of the present study is limited to the consideration of this broad definition of profit, before any deduction.

A second difficulty concerns the computation of the RRI with perfect expectations. In order to compute this ratio for the recent years, it is necessary to know the rate of wages after 1985. Consequently, it is impossible to determine this version of the RRI accurately. The assumption that \( w(85) \) is maintained in later years allows us to make the computation, but the results are increasingly biased for recent years.

A last difficulty is related to the measurement of capital. It is well known that during World War II and after, important amounts of fixed capital were purchased by the Government, but used by private corporations (cf. Gordon R. 1976). These stocks of fixed capital are called "Government Owned, Privately Operated" capitals (which we denote GOPO). In 1942, investment in GOPO represented three times the amount of private investment. In a study of the trend of the rate of return, it is important to consider this phenomenon. However, its treatment is difficult. Because of the war related nature of much of this equipment, it was discarded after the war at an exceptional rate. Consequently, it can not be considered jointly with other investments. The solution adopted is presented in section B.3 of the appendix.

E - RESULTS (RRI)

The results obtained for the profile of the RRI are presented in figure 1. For both models, curves (a) and (b), the trend is downward. The decline is interrupted during the 1950s and the first half of the 1960s by a plateau for about 15 years. The fall which begins in the second half of the 1960s is striking. The perfect expectation model, curve (b), yields a lower return because of the rise in wages. Progressively, the gap between the two measures is bridged, since myopic expectations are substituted for perfect expectations for wages after 1985, which are still unknown. Expectations are perfect until 1958 and are progressively biased in increasing proportions by myopic expectations. Consequently, the trend of the RRI with perfect expectations has no economic meaning.

The question raised in the introduction, whether the movement downward of the accounting rates of return could be confirmed using the economic return on investment, can now be answered positively.

F - ECONOMIC VARIANTS IN THE ESTIMATE OF THE RRI

In this section, we illustrate the robustness of the results. We test the impact
Figure 1 - The Rates of Return Before all Taxes and Interests
U.S. Manufacturing

This figure shows the estimations of the profile of the main rates of return in the U.S. manufacturing industries. (a) corresponds to the RRI with myopic expectations, and (b) to the RRI with perfect expectations. After 1958, perfect expectations are progressively transformed into myopic expectations, since the wage is not known after 1985 (in 1985, the two curves coincide). For this reason the curve has been represented by a dotted line. (c) is the ARRK with straight line depreciation, and (d) is the ERRK.

RRI: Rate of return on investment
ERRK: Economic rate of return on the stock of capital
ARRK: Accounting rate of return on the stock of capital

of assumptions which we made in the construction of the model, by presenting three variants.

1. Enterprises in manufacturing industries are not all incorporated. Although sole proprietors and partners are comparatively less important in manufacturing than in other industries, this difficulty can not be avoided. In order to take this phenomenon
This figure presents economic variants of the calculation of the RRI (myopic expectation model). (a) is the RRI as originally presented in figure 1. (b) describes the same RRI after incorporation of the self-employed in total employment. (c) corresponds to a model in which fixed capital is assumed to lose some of its productive power as time passes. (d) is a variant in which the ratio of capacity utilization is not assumed to be .90 in 1929, but instead determined by regression.

into account, one can aggregate self-employed to wage earners, and apply the average wage of salaried workers to this increased employment.

2. An assumption which we made in the construction of the model is that the remaining fraction of an investment after a number of years retains the same productive capacity as in its initial years. In order to assess the effect of this assumption on the results, we rebuilt the model with the alternative assumption that this productivity is exponentially decreasing with time. Functions \( k(t, t') \) and \( h(t, t') \) are substituted for functions \( k(t) \) and \( h(t) \). They describe in \( t' \) an investment realized in \( t \). A possible
form for this model is:

\[ k(t, t') = k(t) \exp(-b(t' - t)) \]

where \( k(t) \) would again be represented by a polynomial, and \( b \) a is new parameter. The same procedure is repeated for \( h(t) \) with the same parameter \( b \).

3. The ratio of capacity utilization is unknown prior to 1948. In section B.2 of the appendix, we explain how this ratio has been estimated, arbitrarily imposing the value of .90 in 1929. If this restriction is lifted, the ratio determined by the regression for 1929 is equal to 1.17. The estimation of the RRI can be done on this basis.

The results are displayed in figure 2. (a) is the original RRI. (b) represents the same ratio aggregating the self-employed to salaried workers. (c) tests the assumption of an accelerated loss of productive power of fixed capital. In (d) the ratio of capacity utilization is not set at .90 in 1929. We find that these variants do not produce important discrepancies in the resultant series. The more important difference concerns the third variant (c), which significantly alters only the general level of profitability.

It is obvious, however, that a number of other variants and corrections could be tested. For example, we assume a constant delay over time between the implementation of investments and their use for production. This delay might have been shortened or lengthened in the postwar years. The crucial problem here is the availability of relevant data. It is also evident that the restriction of invested capital to fixed capital (abstracting from circulating elements and inventories) should be lifted. Another important assumption is that the discards used in the determination of the stock of fixed capital results from Winfrey retirement patterns. The actual pattern of discards must certainly also reflect the movements of business cycle. It is also obvious that the assumption of a putty-clay technology is too strict. Previously installed equipment can be significantly altered along time. In our opinion, however, the main economic limitation of our study is related to the treatment of price changes which has been restricted to the consideration of wages and fixed capital in relation to output. Variations in the price of inputs (other than fixed capital) in the manufacturing sector of the economy, combined with variations in the price of the output in this sector, impact on the measurement of gross product and profits. Moreover, the prices of imports and exports can differ from prices of domestic goods. These examples serve only to demonstrate the dimension of the field which remains to be investigated.

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5Parameter \( b \) has been determined such that the productive capacities, \( k(t, t') \) and \( h(t, t') \), be divided by 2 at the end of the service life of an average investment.
II - THE RATES OF RETURN ON THE STOCK OF CAPITAL (RRK)

This part of the study is devoted to the calculation of the rates of return on the stock of capital. Section A discusses the distinction between the two notions: rates of return on investment and rate of return on the stock of capital. Section B presents the actual computations.

A - TWO CONCEPTS OF RATES OF RETURN

The investment of capital can be considered from two different points of view: that of the investment realized in a given period, and that of the total stock of capital formed by the accumulation of successive layers of investments, prior to their complete discard. The two types of return will be referred to as either the rate of return on investment (RRI) or the rate of return on the stock of capital (RRK) respectively.

The RRI measures the profitability of the investment over its entire service life. The returns are evaluated independently of any depreciation schedule (only service life assumptions), as gross returns. Their present value is computed using a rate of discount. The rate of discount which equalizes the value of the investment and the present value of this flow of return is the RRI.

In the case of the RRK, profitability is measured as the ratio of the net profit during the period to the value of the existing stock of capital. The crucial issue is that of depreciation (which conditions both the estimate of the net profit and the stock of capital). In our opinion, this problem has no other economically meaningful solution than the use of economic depreciation, which is the depreciation schedule such that the value of the capital is calculated as the present value of the stream of benefits remaining in it. We will denote the RRK determined in this manner as ERRK, the Economic Rate of Return on Capital. The present generation of investment does not raise problems of evaluation. Conversely, all previous generations must be reassessed according to their future yield. In the computation of the ERRK these old capitals are the object of a new evaluation as the present value of their sequence of future returns using the present RRI as a discount rate. Thus, if old capital is evaluated in this manner, a new investment and the purchase of a previously invested capital are equivalent from the point of view of profitability. The various generations of capitals yield the same rate of return, independently of their initial characteristics. It is, therefore, clear that the estimate of the RRK, with economic depreciation (ERRK), raises the same difficulties as the determination of the RRI. Ordinary accounting rates of return (ARRK) avoid the difficulties of economic evaluation using arbitrary depreciation schedules (Straight line, Sum-of-the-years’ digits, ...).

An important difference between our study and previous approaches to the determination of the RRI is that we do not assume a constant technology and constant

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prices (we do not assume a stationary state). This assumption could not be defended in an empirical study covering a period of half a century. In a model in which different generations of capital are considered, and in which the situation of distribution varies, RRI and ERRK are not equal, and both must be computed. This is due to the fact that the discount rate used in the computation of the present value of the various generations of capital is the RRI of investments realized in the present year, and not the original RRI of each generation.

To sum up, studies of the historical trend of profitability should rely, in our opinion, on two basic concepts and not one: The RRI, or the marginal profitability of capital, and the ERRK, or the profitability of the stock of capital. The ARRK is an accounting approximation of the ERRK avoiding the difficulties of economic estimate.

B - THE DETERMINATION OF THE RATES OF RETURN ON THE STOCK OF CAPITAL (RRK)

1. General Formalism

Independently of the depreciation schedule, $K^n(t)$, the stock of net capital in period $t$, is given by:

$$K^n(t) = K^n(t - 1) + I(t) - A(t)$$  \hspace{1cm} (9)

where $A(t)$ denotes the depreciation allowance in period $t$. Similar relationships can be established for each generation of capital. We denote $J^n(t, t')$ the net value in $t'$ of capital invested in $t$:

$$J^n(t, t') = I(t)$$

$$J^n(t, t') = J^n(t, t' - 1) - B(t, t')$$  \hspace{1cm} (10)

where $B(t, t')$ is the depreciation for period $t'$ for the capital invested in $t$. Thus:

$$A(t) = \sum_{t' = -\infty}^{t} B(t', t)$$  \hspace{1cm} (11)

$$K^n(t) = \sum_{t' = -\infty}^{t} J^n(t', t)$$  \hspace{1cm} (12)

All RRKs, $r^K(t)$, are determined as:

$$r^K(t) = \frac{\Pi(t) - p(t)A(t)}{p(t)K^n(t)}$$  \hspace{1cm} (13)

where $\Pi(t)$ denotes gross profits ($Gross\ product - Total\ compensation$).
We will now consider successively the determination of a set of ARRKs, using various accounting evaluations of depreciation, and of the ERRK corresponding to economic depreciation.

2. Accounting Rates of Return on Capital (ARRK)

We consider three depreciation schedules for which we give the expression of, $J^n(t, t')$, the net capital over time. The corresponding depreciation allowances can be computed using equation 10. On this basis, equations 11 to 13 give the corresponding values of the various RRKs:

1. Straight line depreciation on the average service life:

$$J^n(t, t') = (1 - (t' - t) / T(t)) \cdot I(t)$$

2. Sum-of-the-years’ digits (an accelerated depreciation schedule in comparison to 1):

$$J^n(t, t') = \frac{(T(t) - t' + t) (T(t) + 1 - t' + t)}{T(t) (T(t) + 1)} \cdot I(t)$$

3. We define the notion of an “End-of-life” depreciation schedule, in which depreciation allowances are equal to discards (the slowest accounting depreciation schedule):

$$J^n(t, t') = J(t, t')$$

3. Economic Rate of Return on Capital (ERRK)

In the estimate of the ERRK in period $t'$, the value of capital corresponding to the investment realized in period $t$ (for $t = t' - 1, t' - 2, \ldots$) is directly determined as the present value of the stream of future returns, using $r(t')$, the value of the RRI in period $t'$, as the discount rate:

$$p(t') J^n(t, t') = \sum_{r=t'+1}^{\infty} \frac{J(t, r) n(t, r)}{(1 + r(t'))^{t'-r}}$$

As a result of this evaluation, all generations of capital have, in $t'$, the same RRI (equal to $r(t')$).

4. Results (ARRK and ERRK)

An important difference between the RRI and all RRKs including the ERRK is that the latter are subject to business cycle fluctuations, whereas the RRI is not

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6 The formulas are given for integer values of $T$. The generalization is straightforward.

THE RATES OF RETURN ON THE STOCK OF CAPITAL (RRK) 13
In this figure, all RRKs are graphed. (a) is the ERRK, (b) is the ARRK with straight line depreciation schedule. (c) is the ARRK with the sum-of-the-years' digits depreciation schedule, and (d) is the ARRK with the "end-of-life" schedule. 

(since the RRI is a discounted measure of profitability over a considerable number of years).

The ARRK for straight line depreciation schedule is presented in figure 1 (curve (c)) for comparison with the RRI. The profile obtained is different from that of the RRI, for the reason mentioned above. The trend of the two ratios are similar, in particular, both have strong downward trends after the mid-1960s. The ERRK is presented in the same figure (curve (d)). Its profile is strikingly close to that of the ARRK with straight line depreciation, although the difference is greater in the early years.

Alternative rates of return on the stock of capital are presented in figure 3 (ratios already considered are reproduced). Curve (c) is the ARRK with the sum-of-the-years' digits depreciation schedule, and (d) is the ARRK with the "end-of-life" schedule.
the-years' digits depreciation schedule, and (d) is the ARRK with the "end-of-life" schedule. The fluctuation and trend are quite similar for all RRKs, only the general level is different. The value of the ratio is high when depreciation is fast and low when it is slow.

APPENDIX

A - DATA SOURCES

In what follows, ($) refers to "current dollars", and (82 $) to "1982 dollars". All variable names in 1982 dollars end with a 2.

1. National Income and Product Accounts

The series used are taken from the tape “National income and product accounts 1987”. They all refer to tables built on an establishment basis. Below we give series numbers and line numbers within parentheses.

- Basic series
  Gross product, Manufacturing (82 $) GPMA2 6.2(13)
  Gross product, Manufacturing ($) GPMA 6.1(13)
  Compensation of employees, Manufacturing (82 $) COMA2 6.4A(14), 6.4B(13)
  Full time equivalent employees, Manufacturing EEMA 6.7A(14), 6.7B(13)

- Series used in the estimate of the gross product before 1948
  Income without cap. cons. adj., Manufacturing ($) NIMA 6.3A(7), 6.3B(7)
  Depreciation, noncorporate manufacturing ($) DEMANC 6.15A(7), 6.15B(7)
  Depreciation, corporate manufacturing ($) DEMACO 6.24A(12), 6.24B(11)
  Gross product, nonfarm business ($) GPNF 1.7(4)
  Gross product, nonfarm business (82 $) GPNF2 1.8(4)
  Net product, nonfarm business ($) NPNF 1.12(4)
  Income, nonfarm business ($) NINF 1.12(15)

- Series used in the estimate of the wage-equivalent of self-employed
  Self-employed, Manufacturing SELF 6.9A(7), 6.9B(7)

2. Stock of Capital and Investment

Capital stock and investment series are taken from “B.E.A. Wealth data tape”. Only manufacturing is considered. No residential capital exists in these series. The first number in the parentheses refers to the section of the tape and the second number to the series. As before, the series are on an establishment basis:

Investment, total, Private (82 $) INPR2 (3,163)
Investment, total, GOPO, (82 $)
Gross capital, total, Private (82 $)
Gross capital, total, GOPO (82 $)
Gross capital, total, Private ($)

3. Other Series

The ratio of capacity utilization of fixed capital, $u$, is from The Business Condition Digest, BEA.

The modified Winfrey coefficients, $\alpha$, are from Survey of current business, BEA 1985, Table D, p. 43.

B - ESTIMATE OF MISSING SERIES AND CORRECTIONS

1. Gross Product in Manufacturing from 1929 to 1946

Prior to 1947, only the three following series are available for manufacturing: Net income without capital consumption adjustment, NIMA; Depreciation, noncorporate manufacturing, DEMANC; Depreciation, corporate manufacturing, DEMACO. These series allow for the computation of the gross income:

$$ GIMA = NIMA + DEMANC + DEMACO $$

In order to obtain the gross product, it is necessary to add indirect business tax (and nontax liability plus business transfer payments less subsidies) to GIMA. We define $\rho^{MA}$ as the ratio of gross product to gross income:

$$ GPMA = \rho^{MA} GIMA $$

(14)

The ratio $\rho^{MA}(t)$ is unknown prior to 1947 for manufacturing, but is available for Business nonfarm (NF):

$$ \rho^{NF} = GPNF/(GPNF - NPNF + NINF) $$

For the years prior to 1948, we use $\rho^{NF}(t)$ as a proxy for $\rho^{MA}(t)$. Since after 1947 a difference exists between the two series, $\rho^{MA}$ and $\rho^{NF}$, we estimate $\rho^{MA}(t)$ as follows:

$$ \rho^{MA}(t) = \rho^{NF}(t) \frac{\rho^{MA}(47)}{\rho^{NF}(47)} \quad \text{for} \quad t \leq 47 $$

(15)

Equations 14 and 15 allow for the estimation of GPMA prior to 1947.

No deflator for manufacturing exists in NIPA prior to 1947. Again we must use Nonfarm business. We denote $\delta_{NF}(t)$, the deflator for the gross product in year $t$, in
dollars of the year \(t_0\). We assume:

\[
\delta_{47}^{MA}(t) = \delta_{47}^{NF}(t) \quad \text{for} \quad t \leq 47
\]

This computation yields the following estimations for the gross product in manufacturing in 1982 dollars:

\[
GPMA2(t) = GPMA(t) \cdot \frac{GPNF2(t)}{GPNF(t)} \cdot \frac{GPMA2(47)}{GPNF2(47)} \cdot \frac{GPMA(47)}{GPNF(47)}
\]

2. Ratio of Capacity Utilization

The ratio of capacity utilization is only available since 1948. In order to estimate this ratio prior to this date, we build the following model:

1. We define the variable \(v(t)\) which is equal to \(u(t)\) from 1948 onward and equal to \(\bar{u}\) before.
2. We regress the productivity of the existing stock of capital from 1929 to 1985 on time and capacity utilization:

\[
G(t) = \sum_{i=0}^{L} a_i t^i + b (v(t) - \bar{u}) + \epsilon^G
\]

with \(G(t) = G(t)/K(t)\).
3. The residual in the regression yields an estimation of \(u\):

\[
u(t) = \left( \frac{\tilde{G}(t)}{\sum_{i=0}^{L} a_i t^i} - 1 \right) \frac{1}{b + \bar{u}} \quad \text{for} \quad t < 48\]

In fact this computation does not give an economically significant result, because of the exceptional character of the period 1929-1945. 1929 is still a year of high activity in spite of the onset of the depression in the last months. Then the economy swings between the depression and the war years of exceptionally high activity. With \(L = 3\), the regression gives \(u(29) = 1.17\) (see figure 4)—a figure which certainly overestimates the activity in 1929.

The RRI obtained with this estimation of \(u(29)\) is presented in figure 2, curve (d). Throughout the present study, a variant of this calculation has been used in which \(u(29)\) has been arbitrarily fixed at .90. This is equivalent to imposing the following

\section*{APPENDIX}
In (a), the ratio of capacity utilization has been estimated by regression. In (b), the same estimate is repeated imposing in the regression a ratio of .90 in 1929.

The ratio of capacity utilization obtained is presented in figure 4.

3. Government Owned Privately Operated Capital

In figure 5, the ratio of investment in GOPO to total investment, private and GOPO, is plotted (for manufacturing). In 1942, this ratio is equal to 72%, by 1953, it is still 18%. It is a combination of the GOPO and of the high ratio of capacity utilization which accounts for the high product during World War II. Clearly it is necessary to incorporate this stock of capital in a study of the return on investment.

A difficulty in this treatment of GOPO is that investment corresponds to specific fields such as armaments. After the war an important share of this stock has been discarded at an accelerated rate. In the fixed capital tape, discards for GOPO are not determined using Winfrey coefficients, but are specific. Thus private capital and...
In this figure, the ratio of GOPO investment to total investment (private and GOPO) has been plotted. The comparative importance of GOPO during the war is striking. Two generations in GOPO can be distinguished, prior to 1946 and after.

GOPO must be treated separately. Concerning fixed capital, the series are available. But for the gross product and employment, assumptions must be made to distinguish between the two types of capital. The following procedure was adopted:

1. We distinguish two generations of GOPO: between 1940 and 1946, and after 1946.
2. We aggregate the second generation with private capital. This is equivalent to assuming a same productivity and service life. The notion of an "extended private sector" is thus created, which is composed of the original private capital and of the second generation of GOPO.
3. Conversely, we consider that the specificity of GOPO during the first period requires a separate treatment. Since more than 80% of this generation has been invested between 1941 and 1943, we assume that the total generation can be characterized by a unique level of productivity of capital and ratio of employment to capital, equal to those of 1942 for private investment. We estimate the discards for the second generation using Winfrey coefficients, and subtract them from total discards for GOPO (which are known) in order to determine the discards for the first generation.

The series corresponding to the extended private sector will be denoted with the
This figure illustrates the assumptions which we made in the distinction between the two generations of GOPO. The continuous line plots the total stock of GOPO in Billion 1982 $. The dotted line (a) represents our estimation of the progressive erosion of the stock of GOPO of the first generation by discards. The second dotted line (b), correlative corresponds to the estimation of the growth of the stock of the second generation.

suffix PRX, instead of PR. The suffix GOX replaces GO to denote GOPO of the first generation (World War II).

The model constructed to account for the GOPO also requires a new definition of functions $b_1(t)$:

$$b_1(t) = \sum_{t'=-\infty}^{t-1} t'^i J(t', t) + (42)^2 GCGOX2$$

where the investment series INPRX2 has been used in the estimate of $J(t, t')$ (cf. equation 1).

The RRKs are computed for total manufacturing (private and GOPO). An estimate of the stock of capital for GOPO in the determination of the ERRK is still
required. For \( t \geq 46 \), we have:

\[
J^n(t) = \sum_{\tau=t+1}^{70} \frac{GCGOX2(\tau)\pi(42, \tau)}{(1 + r(t))^{r-t}} + \sum_{\tau=46}^{70} \frac{GCGOX2(t)\pi(42, \tau)}{(1 + r(t))^{r-t}}
\]

In order to determine \( J^n(t) \) for \( 40 \leq t \leq 45 \), we make the further assumption that GOPO of the first generation, invested in the different years, have been discarded only after 1946 and at the same rate. Thus:

\[
J^n(t) = \sum_{\tau=t+1}^{45} \frac{GCGOX2(t)\pi(42, \tau)}{(1 + r(t))^{r-t}} + \sum_{\tau=46}^{70} \frac{GCGOX2(t)GCGOX2(46)\pi(42, \tau)}{(1 + r(t))^{r-t}}
\]

C - NUMERICAL RESULTS OF THE REGRESSIONS AND TESTS

All estimations are for the years 1929-1985.

1. The Degree of the Polynomials

In this study, a number of variables have been represented by polynomial trends (marginal technology, prices). In general, these trends are not sensitive to the degree of the polynomials. Therefore, the results—in particular the RRI—do not depend in large part on these degrees.

For variable \( h(t) \) and to lesser extent for \( k(t) \), the two variables which account for the marginal technology, a certain fluctuation of the trends can be observed when the degrees of the polynomials are varied. This fluctuation only significantly affects the early and later years. It is for this reason that we limit the presentation of our results to the period 1935-1980, for which there is little sensitivity to this problem (see figure 7).

Since no existing econometric techniques can unambiguously determine the proper degree to be retained, we consider several converging indications. We raise the degree of the polynomials until:

1. The adjusted \( R^2 \), denoted \( \bar{R} \), can no longer be significantly increased, and the sum of squared residuals no longer decreases.
2. \( t \) statistics are no longer significant to the .001 level.

The first table shows the results of these tests for \( h(t) \).

2. Results of the Regressions

Ratios \( k(t) \) and \( h(t) \) are determined by equations 7 and 8, with: \( G(t) = GPMA2 \) and \( H(t) = EEMA \). The results of four variants have been presented in figure 2:

APPENDIX 21
Figure 7 - The choice of the degrees of the polynomials
The example of $h(t)$

All plots correspond to the same calculation of the RRI with the exception of the degree of the polynomial used to account for the trend of $h(t)$. The results are quite similar for the period 1935-1980. It is for this reason that the results have been limited to this period in the other figures. Note that the instability of the estimate for the recent years is not observed for $k(t)$.

I - Basic estimate
II - With correction for self-employed ($H(t) = EEMA + SELF$)
III - Accelerated loss of productive power
IV - $u(29)$ free

The results are given in the second and third tables for $k(t)$ and $h(t)$.

In the regression which determines $\overline{T}(t)$, the series INPRX2 and GCPRX2 are used. For prices we used $p(t) = (GCPR/GCPR2)/(GPMA/GPMA2)$ and $w(t) = COMA2/EEMA$. 
I. REGRESSION OF $H(T)$: THE DEGREE OF THE POLYNOMIAL

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* Student's t

II. VARIANTS: REGRESSION OF $k(t)$

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* Variant II only concerns $k(t)$

III. VARIANTS: REGRESSION OF $H(t)$

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* For variant I, cf. fourth degree in the first table

IV. REGRESSION OF THE OTHER VARIABLES

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<td>0.9965</td>
<td>0.9756</td>
<td>0.9756</td>
</tr>
</tbody>
</table>

* The estimate of $T$ is the result of a nonlinear regression

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