NON WALRASIAN EQUILIBRIA,

MONEY AND MACROECONOMICS

by Jean-Pascal BENASSY

N° 8725

December 1986

Revised December 1987

Prepared for the Handbook of Monetary Economics, edited by F.H. Hahn and B. Friedman, forthcoming, North-Holland.

NON WALRASIAN EQUILIBRIA,

MONEY AND MACROECONOMICS

by Jean-Pascal BENASSY

A B S T R A C T

This article gives a synthetic account of the theory on non-Walrasian equilibria and of some applications to macroeconomics, while insisting on the specific role of money. After a description of the basic microeconomic notions, various concept of equilibria with rigid prices and their optimality properties are studied, followed by concepts where prices are endogenously set in a framework of imperfect competition. After an exposition of the role of expectations in the theory, two macroeconomic models, with rigid and flexible prices respectively, are studied with a special focus on problems of economic policy.

Keywords : Non-Walrasian equilibria, General equilibrium, Macroeconomics, Money.

Journal of Economic Literature Classification Numbers : 021, 022, 023, 311.

EQUILIBRES NON WALRASIENS,

MONNAIE ET MACROECONOMIE

R E S U M E

Cet article donne un exposé synthétique de la théorie des équilibres non walrasiens et de quelques applications à la macroéconomie, en insistant sur le rôle de la monnaie. Après une description des notions microéconomiques de base, on étudie divers concepts d'équilibres à prix rigides et leurs propriétés d'optimalité, puis des concepts à prix endogènes, fixés dans un cadre de concurrence imparfaite. Après un exposé du rôle des anticipations dans la théorie, on étudie deux modèles macroéconomiques, respectivement à prix rigides et flexibles, en examinant particulièrement les problèmes de politique économique.

Mots clefs : Equilibres non walrasiens, Equilibre général, Macroéconomie, Monnaie.

Codes J.E.L. : 021, 022, 023, 311.

CONTENTS

1. *INTRODUCTION*

This chapter will be concerned with non-Walrasian equilibria, money and macroeconomics. We shall thus naturally begin this introduction with a brief description of the salient characteristics of non-Walrasian equilibria, give a few motivations why such equilibria should be considered, and show what role money has in such a theory. A summary of subsequent sections will end the introduction.

1.1. Non-Walrasian equilibria

Non-Walrasian equilibria, also often called equilibria with rationing, are a wide class of equilibrium concepts which generalize the traditional notion of Walrasian equilibrium by allowing markets not to clear (i.e. aggregate demand to differ from aggregate supply) and therefore quantity rationing to be experienced. Their scope is best described by examining first Walrasian equilibrium as a reference.

In a Walrasian equilibrium by definition all markets clear, that is, total demand equals total supply for each good. This consistency of the actions of all agents is achieved by price movements solely in the following manner : All private agents receive a price signal (i.e. prices for every market), and assume that they will be able to exchange whatever they want at that price system. They express Walrasian demands and supplies which are functions of this price signal only. A Walrasian equilibrium price system is a set of prices for which total demand and total supply are equal on all markets. Transactions are equal to the demands and supplies at this price system. No rationing is experienced by any agent since demand and supplies match on all markets.

Two characteristics of the Walrasian model deserve to be stressed : all private agents receive price signals and make rational quantity decisions with respect to them. But no agent makes any use of the quantity signals sent to the market. Also no agent actually sets prices, the determination of which is left to the "invisible hand" or to the implicit Walrasian auctioneer. Of course there are some real world markets for which the equality between demand and supply is ensured institutionally, for example the stock market, which inspired Walras. But for other markets where no auctioneer is present, there is, as Arrow (1959) noted",... a logical gap in the usual formulations of the theory of the perfectly competitive economy, namely, that there is no place for a rational decision with respect to prices as there is with respect to quantities", and more specifically "each individual participant in the economy is supposed to take prices as given and determine his choice as to purchases and sales accordingly ; there is no one left over whose job is to make a decision on price".

Non-Walrasian theory takes this strong logical objection quite seriously, and its purpose is to build a consistent theory of the functioning of decentralized economies when market clearing is not axiomatically assumed. An almost immediate corollary is that rationing may appear, and that quantity signals will have to be introduced together with price signals. Using this enlarged set of signals allows to generalize the concept of Walrasian equilibrium in the following directions :

- More general price formation schemes can be considered, ranging from full rigidity to full flexibility, with intermediate forms of imperfect competition. Moreover each market may have its own price determination scheme.
- Demand and supply theory must be substantially modified to take into account these quantity signals. One thus obtains a theory of effective demand, generalizing Walrasian demand which only takes price signals into account.
- Price theory must also be amended in a way which integrates the possibility of non clearing markets, the presence of quantity signals, and makes agents themselves responsible for price making.
- Equilibrium in the short run is achieved by quantity adjustments as well as by price adjustments.
- Finally expectations, which in market clearing models are concerned with price signals only, must now include quantity signals expectations as well.

1.2. Why study non-Walrasian equilibria

The set of equilibria just outlined is clearly more general than the concept of Walrasian equilibrium. But this does not imply per se that they should be more relevant. There are however a number of reasons which make the use of non Walrasian equilibria relevant in various institutional contexts.

The most obvious one is that there are in the world a number of planned (or partially planned) economies where the determination of prices is not left to "market forces", but where prices are fixed for some length of time by a central planning authority. In such economies non-Walrasian theory is clearly a relevant tool of analysis.

But we shall now argue that this theory is also quite relevant for free market economies : First, and from an institutional point of view, we already noted that, except for a very limited number of markets, there is no real world auctioneer to carry out the "Walrasian tatonnement". Prices are determined by the interaction of decentralized agents in the various markets. Clearly the modelling of price and quantity determination in such markets must involve a different representation than the "auctioneer" one. Notably the quantity signals sent by the traders, which only the Walrasian auctioneer used in Walrasian theory, must play a role in such representations. We shall actually see a few examples below.

Secondly some recent theoretical developments in industrial economics, labor economics or the study of some financial markets such as credit markets all point out to the possibility of non-market clearing situations, due either to the game-theoretic nature of the price making process, or to various informational imperfections. We thus need a theory that can accomodate in a general equilibrium setting the possibility of these non market clearing situations.

Thirdly, and though a significant body of macroeconomic literature has developed recently to show that employment fluctuations could be made consistent with market clearing models, many real world phenomena such as the massive unemployment or the idleness of productive capacities observed in the latest years may seem difficult to reconcile with a general market clearing view of the world. In the absence of conclusive empirical tests, it is preferable to stick to a more general level of analysis.

1.3. The role of Money

Having explained the nature of non-Walrasian equilibria, we now have to say a few things about the role of money in the theory. As it turns out, and will appear below, almost all formalizations of non Walrasian equilibria have been made in the framework of <u>monetary</u> economies, where money plays its three traditional roles of numeraire, medium of exchange and reserve of value. Besides its convenience in formalization, the use of a monetary framework is not entirely fortuitous. Non Walrasian theory aims at representing the functioning of market economies. Actual market economies are characterized in general by the famous problem of "absence of double coincidence of wants", and in such a case it is well known that in the absence of a medium of exchange, the coordination of trading poses immense, and usually insolvable coordination problems. It is thus natural that a theory which aims at describing decentralized economies be set in the framework of a monetary economy [(1)]. We may note that the issue of the absence of double coincidence of wants will appear in some applications below.

A final specific feature of money should be stressed : Money can be interpreted as commodity money (such as gold). But the recent evolution of money tends to privilege the interpretation of money as fiat money, i.e. an (almost) costlessly produced legal medium of exchange. In this last case money (or rather the quantity of money) becomes a policy variable. We shall also scrutinize money in this respect, and notably try to link our results with the traditional debates on the "neutrality" of monetary policy.

Of course the monetary nature of the economies studied explains that there exists a most natural link with macroeconomics, which also studies monetary economies. This link will be particularly explored in the last two sections of this chapter.

1.4. Historical Antecedents

Equilibria with rationing have a double ancestry : On one hand Walras (1874) developed a model of general equilibrium with interdependent markets where adjustment was made through prices. On the other hand Keynes (1936) built, at the macroeconomic level, a concept of equilibrium where adjustment was made by quantities (the level of national income) as well as by prices. The Keynesian model was chiefly used under the form of the IS-LM model, as developed by Hicks (1937). The Walrasian model was elaborated into a very rigorous and beautiful construction, notably in Hicks (1939), Arrow-Debreu (1954), Debreu (1959), Arrow-Hahn (1971), and it became the basic concept in microeconomics. As a result of these evolutions, a growing gap developed between microeconomics and macroeconomics.

A few isolated contributions in the postwar period made some steps towards modern non-Walrasian theories. Samuelson (1947), Tobin-Houthakker (1950) studied the theory of demand under conditions of rationing. Bent Hansen (1951) introduced the ideas of active demand, close in spirit to that of effective demand, and of quasi-equilibrium where persistent excess demand created steady inflation. Patinkin (1956, ch. 13) considered the situation where the firms might not be able to sell all their "notional" output. Hahn and Negishi (1962) studied non-tatonnement processes where trade could take place before a general equilibrium price system was reached. Hicks (1965) discussed the "fixprice" method as opposed to the flexprice method.

A main impetus came from the stimulating works of Clower (1965) and Leijonhufvud (1968). Both were concerned with the microeconomic foundations of Keynesian theory. Clower showed that the Keynesian consumption function made no sense unless reinterpreted as the response of a rational consumer to a rationing of his sales on the labor markets. He introduced the "dual-decision" hypothesis, a precursor of modern effective demand theory, showing how the consumption function could have two different functional forms, depending on whether the consumer was rationed on the labor market or not. Leijonhufvud (1968) insisted on the importance of short-run quantity adjustments to explain the establishment of an equilibrium with involuntary unemployment. These contributions were followed by the macroeconomic model of Barro-Grossman (1971), (1976), integrating the "Clower" consumption function and the "Patinkin" employment function in the first non-Walrasian macroeconomic model.

The main subsequent development was the construction of rigorous microeconomic concepts of non-Walrasian equilibria. These truly bridged the gap between traditional microeconomics and macroeconomics. On one hand they generalized the Walrasian general equilibrium concept in the multiple ways we described in subsection 1.1. above. One the other hand they allowed to derive macromodels which synthesized and generalized hitherto disjoint macrotheories within a general unified framework. These concepts of non Walrasian equilibria, and a few macroeconomic applications, will be our main concern in this chapter.

1.5. Scope and outline of the chapter

As the title of this part suggests, we shall be concerned here exclusively with non Walrasian models which have been developed at the General equilibrium level. This means that we shall not cover some recent theories of price determination [(2)] which, though non Walrasian, have been only developed so far in partial equilibrium contexts. Subject to this limitation, we shall try to give a fairly broad outline of the available microeconomic concepts and their properties.

Section 2 deals with the functioning of nonclearing markets in a monetary economy and the formation of quantity signals, obviously a fundamental element for all that follows. Section 3 exposes the main concepts of equilibrium in the case polar to that of Walrasian equilibrium, that with fixed prices. This gives a very broad class of equilibria, and section 4 studies their optimality (or rather suboptimality) properties. Section 5 introduces endogenous price making by agents, in a framework akin to that of imperfect competition ; two concepts, with objective and subjective demand curves, are presented. Section 6 introduces expectations into the non-Walrasian framework. The last two sections are devoted to simple macroeconomic applications of the microeconomic concepts : Section 7 gives a simple exposition of the famous three goods - three regimes fixprice macroeconomic model. Finally section 8 presents a micro-macro model of imperfect competition and unemployment. These models allow to apply all the concepts seen previously in a framework which allows to deal with such macroeconomic problems as unemployment and the effectiveness of government policy.

At the end of every section, a short subsection indicates the origins of the material in that section, and gives some brief notes on supplementary references.

2. MARKETS, MONEY AND QUANTITY SIGNALS

2.1. A monetary Economy

As we indicated above, we shall describe the various concepts of non Walrasian economics in the framework of a monetary economy. One good, called money, serves as numéraire, medium of exchange and reserve of value. Assume that there are ℓ active markets in the period considered. On each of these markets one of the ℓ non monetary goods, indexed by $h = 1,\dots,\ell$, is exchanged against money at the price p_h . Call p the ℓ-dimensional vector of these prices (Money being the numeraire, its price is of course equal to one).

To simplify the exposition, we shall consider in the following sections a pure exchange economy. The agents in this economy are indexed by $i = 1,\dots,n$. At the beginning of the period agent i has a quantity of money $\bar{m}_i \geqslant 0$ and holdings of nonmonetary goods represented by a vector ω_i , with components $\omega_{ih} \geqslant 0$ for each good.

Consider an agent i in market h . He may make a purchase $d_{ih} > 0$, for which he pays $p_h d_{ih}$ units of money, or a sale $s_{ih} > 0$, for which he receives $p_h s_{ih}$ units of money. Define $z_{ih} = d_{ih} - s_{ih}$ his net purchase of good h and z_i the ℓ-dimensional vector of these net purchases. Agent i's final holdings of nonmonetary goods and money, x_i and m_i , are respectively :

$$x_i = \omega_i + z_i$$

$$m_i = \bar{m}_i - pz_i$$

Note that the last equation, which describes the evolution of money holdings, is simply the conventional budget constraint. At this stage we may also note that the use of a monetary structure of exchange tremendously simplies the formalization. Indeed if we considered for example a barter structure of exchange between the ℓ nonmonetary goods, there would be $\ell(\ell-1)/2$ markets, one for each pair of goods, a number quite greater than ℓ for an economy with many goods. Furthermore having a monetary structure allows us to speak unambiguously of <u>the</u> demand or supply for good h , the implicit counterpart of such demands and supplies being always money. In a barter economy such demands and supplies would not be meaningful, since each good would be traded on $\ell-1$ markets against $\ell-1$ different counterparts.

2.2. Walrasian Equilibrium

Having described the basic institutional structure of the economy, we shall now describe its Walrasian equilibrium in order to contrast it with the non Walrasian equilibrium concepts that will follow. We must still describe the preferences of the agents, and we shall thus assume that agent i has a utility function $U_i(x_i, m_i) = U_i(\omega_i + z_i , m_i)$, which we shall assume throughout strictly concave in its arguments. Note that money appears in the utility function, in a way which will be made clearer below (Section 6).

As indicated above, each agent is assumed to be able to exchange as much as he wants on each market. He thus transmits demands and supplies which maximize his utility subject to the budget constraint ; the Walrasian net demand function $z_i(p)$ is the solution in z_i of the following program :

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \quad \text{s.t.}$$

$$pz_i + m_i = \bar{m}_i$$

This yields a vector of Walrasian net demands $z_i(p)$. Note that there is no "demand for money" since there is no such thing as a market for money. A Walrasian equilibrium price vector p^* is defined by the condition that all markets clear, i.e. :

$$\sum_{i=1}^{n} z_i(p^*) = 0$$

the vector of transactions realized by agent i is equal to $z_i(p^*)$.

Walrasian equilibrium allocations possess a number of good properties. By construction they are consistent both at individual and market level. Furthermore they are Pareto-optimal, i.e. it is impossible to find an allocation which would be at least as good for all agents, and strictly better for at least one. We shall later contrast this with the suboptimality properties of non-Walrasian equilibria.

2.3. Demands, transactions and rationing schemes

Since we shall be dealing with nonclearing markets, we must now make an important distinction, that between demands and supplies on the one hand, and the resulting transactions on the other. Demands and supplies, denoted $\tilde{\tilde{d}}_{ih}$ and $\tilde{\tilde{s}}_{ih}$, are signals transmitted by each agent to the market (i.e. the other agents) before exchange takes place. They represent as a first approximation the exchanges agents wish to make on each market, and thus do not necessarily match on a specific market. Transactions, i.e. purchases or sales of goods, denoted respectively as d^*_{ih} or s^*_{ih}, are exchanges actually made, and must thus identically balance on each market :

$$\sum_{i=1}^{n} d^*_{ih} = \sum_{i=1}^{n} s^*_{ih} \qquad \forall\, h$$

The exchange process must generate consistent transactions from any set of possibly inconsistent demands and supplies. Some rationing will necessarily occur, which may take various forms, such as uniform rationing, queueing, priority systems, proportional rationing, depending on the particular organization of each market. We shall call rationing scheme the mathematical representation of each specific organization. To be more precise define :

$$\tilde{\tilde{z}}_{ih} = \tilde{\tilde{d}}_{ih} - \tilde{\tilde{s}}_{ih} \qquad z^*_{ih} = d^*_{ih} - s^*_{ih}$$

A rationing scheme in a market h is described by a set of n functions :

$$z^*_{ih} = F_{ih}(\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}) \qquad i = 1,\ldots,n \tag{1}$$

such that :

$$\sum_{i=1}^{n} F_{ih}(\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}) = 0 \qquad \text{for all } \tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}$$

We shall generally assume that F_{ih} is continuous, nondecreasing in $\tilde{\tilde{z}}_{ih}$ and nonincreasing in the other arguments. Before examining further the possible properties of these rationing schemes, we shall give an example, that of a queue (or priority system).

In a queueing system the demanders (or the suppliers) are ranked in a predetermined order, and served according to that order. Let there be n-1 demanders, ranked in the order $i = 1,\ldots, n-1$, each having a demand $\tilde{\tilde{d}}_{ih}$, and a supplier indexed by n who supplies $\tilde{\tilde{s}}_{nh}$. When the turn of demander i comes, the maximum quantity he can obtain is what demanders before him, i.e. agents $j < i$, have not taken, i.e. :

$$\tilde{\tilde{s}}_{nh} - \sum_{j<i} d^*_{jh} = \max(0, \tilde{\tilde{s}}_{nh} - \sum_{j<i} \tilde{\tilde{d}}_{jh})$$

The level of his purchase is simply the minimum of this quantity and his demand, i.e. :

$$d^*_{ih} = \min[\tilde{\tilde{d}}_{ih}, \max(0, \tilde{\tilde{s}}_{nh} - \sum_{j<i} \tilde{\tilde{d}}_{jh})]$$

As for the supplier, he sells the minimum of his supply and of total demand :

$$s^*_{nh} = \min(\tilde{\tilde{s}}_{nh} , \sum_{j=1}^{n-1} \tilde{\tilde{d}}_{jh})$$

It is clear that total purchases equal total sales on the market h , whatever demands and supplies.

2.4. Properties of rationing schemes

We shall now study three possible and important properties that a rationing scheme may satisfy : voluntary exchange, market efficiency, nonmanipulability.

There is voluntary exchange in market h if no agent can be forced to purchase more than he demands, or to sell more than he supplies, which is expressed by :

$$d^{*}_{ih} \leq \tilde{d}_{ih} \qquad s^{*}_{ih} \leq \tilde{s}_{ih} \qquad \text{for all } i$$

or equivalently in algebraic terms :

$$|z^{*}_{ih}| \leq |\tilde{z}_{ih}| \qquad z^{*}_{ih} \cdot \tilde{z}_{ih} \geq 0 \qquad \text{for all } i$$

Most markets in reality meet this condition (except perhaps some labor markets) and we shall henceforth assume that it always holds. This allows to classify agents in two categories : Unrationed agents for which $z^{*}_{ih} = \tilde{z}_{ih}$, and rationed agents who trade less than they wanted.

The second property we shall study here is that of market efficiency, or absence of friction, which corresponds to the idea of exhaustion of all mutually advantageous exchanges : A rationing scheme is efficient, or frictionless, if one cannot find simultaneously a rationed demander and a rationed supplier in the corresponding market. The intuitive idea behind it is that in an efficiently organized market a rationed buyer and a rationed seller should be able to meet, and would exchange until one of the two is not rationed. Together with the voluntary exchange assumption, it implies the "short-side" rule, according to which agents on the short side of the market can realize their desired transactions :

$$\left(\sum_{j=1}^{n} \tilde{z}_{jh}\right) \cdot \tilde{z}_{ih} \leq 0 \implies z^{*}_{ih} = \tilde{z}_{ih}$$

This rule also implies that the global level of transactions on a market settles at the minimum of aggregate demand and supply. We may note that this market efficiency assumption is quite acceptable if one considers a small decentralized market where each demander meets each supplier (like the queue described above). It becomes less acceptable if one considers a fairly wide and decentralized market, because some buyers and sellers might not meet. We may note in particular that the market efficiency property is usually lost by aggregation of submarkets (whereas on the contrary the voluntary exchange property remains intact in this aggregation process). As a result the global level of transactions may be smaller than either total demand or supply, Figure 1 actually depicts how the aggregation of two frictionless submarkets yields an inefficient aggregate market, at least in some price range. The market efficiency assumption is very useful for constructing simple macroeconomic models, but we must keep in mind that it may not always hold. Fortunately this hypothesis is not necessary for the microeconomic concepts presented in the next sections.

Figure 1

We shall now consider a third important property of rationing schemes, that of nonmanipulability : A rationing scheme is nonmanipulable if an agent, once rationed, cannot increase the level of his transactions by increasing his demand or supply, as shown in Figure 2. A nonmanipulable scheme can be written under the form :

$$d^*_{ih} = \min(\tilde{\tilde{d}}_{ih}, \bar{d}_{ih})$$

$$s^*_{ih} = \min(\tilde{\tilde{s}}_{ih}, \bar{s}_{ih})$$

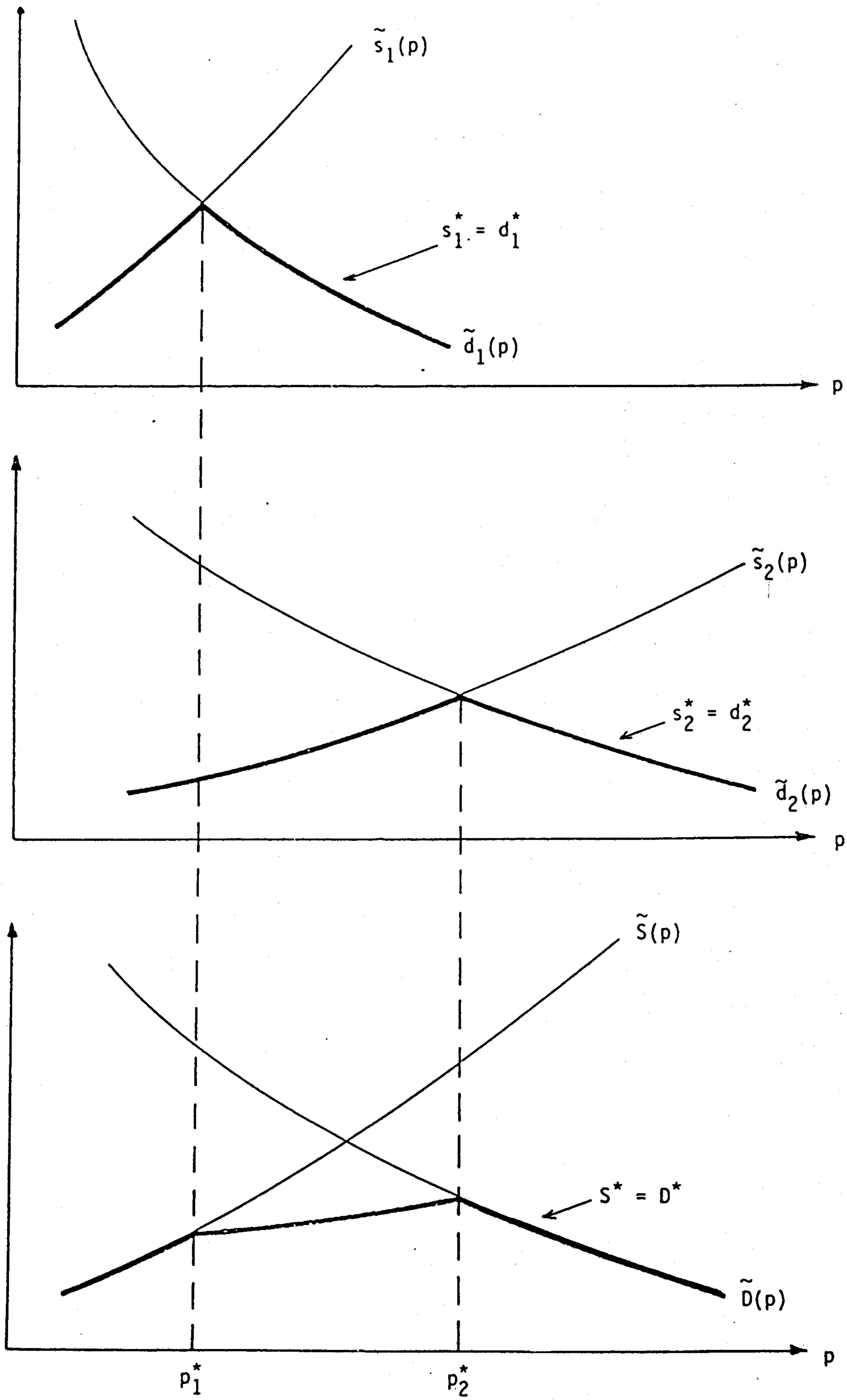
$\tilde{s}_1(p)$
$s_1^* = d_1^*$
$\tilde{d}_1(p)$
p
$\tilde{s}_2(p)$
$s_2^* = d_2^*$
$\tilde{d}_2(p)$
p
$\tilde{S}(p)$
$S^* = D^*$
$\tilde{D}(p)$
p
p_1^*
p_2^*

Figure 1

where the bounds $\bar{d}_{ih}$ and $\bar{s}_{ih}$ depend only on the net demands of the other agents (and are thus independent of $\tilde{\tilde{z}}_{ih}$). Otherwise the scheme is manipulable. For example the queueing system described above is nonmanipulable, and can be written under the above form, with :

$$\bar{d}_{ih} = \max(0, \tilde{\tilde{s}}_{nh} - \sum_{j<i} \tilde{\tilde{d}}_{jh}) \qquad 1 \leq i \leq n-1$$

$$\bar{s}_{nh} = \sum_{j=1}^{n-1} \tilde{\tilde{d}}_{jh}$$

Figure 2

On the contrary a proportional rationing scheme, for example, is manipulable. To characterize the nonmanipulability property more formally, it is convenient to separate $\tilde{\tilde{z}}_{ih}$ from the other net demands, and thus to write a rationing scheme under the form :

$$z^*_{ih} = F_{ih}(\tilde{\tilde{z}}_{ih}, \tilde{\tilde{z}}_{-ih}) \qquad i = 1,\ldots,n$$

with $$\tilde{\tilde{z}}_{-ih} = \{\tilde{\tilde{z}}_{jh} \mid j \neq i\}$$

where $\tilde{\tilde{z}}_{-ih}$ is the set of net demands on market h , except for agent i's demand. The rationing scheme on market h is nonmanipulable if it can be written under the form :

$$F_{ih}(\tilde{\tilde{z}}_{ih}, \tilde{\tilde{z}}_{-ih}) = \begin{cases} \min [\tilde{\tilde{z}}_{ih}, G^d_{ih}(\tilde{\tilde{z}}_{-ih})] & \text{if } \tilde{\tilde{z}}_{ih} \geq 0 \\ \max [\tilde{\tilde{z}}_{ih}, -G^s_{ih}(\tilde{\tilde{z}}_{-ih})] & \text{if } \tilde{\tilde{z}}_{ih} \leq 0 \end{cases}$$

with

$$G^d_{ih}(\tilde{\tilde{z}}_{-ih}) \geq 0 \qquad G^s_{ih}(\tilde{\tilde{z}}_{-ih}) \geq 0$$

2.5. Price and quantity signals

We just saw how transactions would occur in a nonclearing market. In such a market each agent receives, in addition to the traditional price signal, some quantity signals. In what follows we shall concentrate on markets which satisfy the properties of voluntary exchange and nonmanipulability [(3)]. These may be represented as :

$$z^*_{ih} = \begin{cases} \min(\tilde{\tilde{z}}_{ih}, \bar{d}_{ih}) & \tilde{\tilde{z}}_{ih} \geq 0 \\ \max(\tilde{\tilde{z}}_{ih}, -\bar{s}_{ih}) & \tilde{\tilde{z}}_{ih} \leq 0 \end{cases}$$

or more compactly by :

$$z^*_{ih} = \min(\bar{d}_{ih}, \max(\tilde{\tilde{z}}_{ih}, -\bar{s}_{ih}))$$

with

$$\bar{d}_{ih} = G^d_{ih}(\tilde{\tilde{z}}_{-ih}) \qquad \bar{s}_{ih} = G^s_{ih}(\tilde{\tilde{z}}_{-ih}) \tag{2}$$

The quantities $\bar{d}_{ih}$ and $\bar{s}_{ih}$, which we shall call henceforth the perceived constraints, are the quantity signals which agent i receives in market h in addition to the price p_h.

To make notation more compact, we can represent the rationing functions and perceived constraints concerning an agent i (Equations 1 and 2) as vector functions :

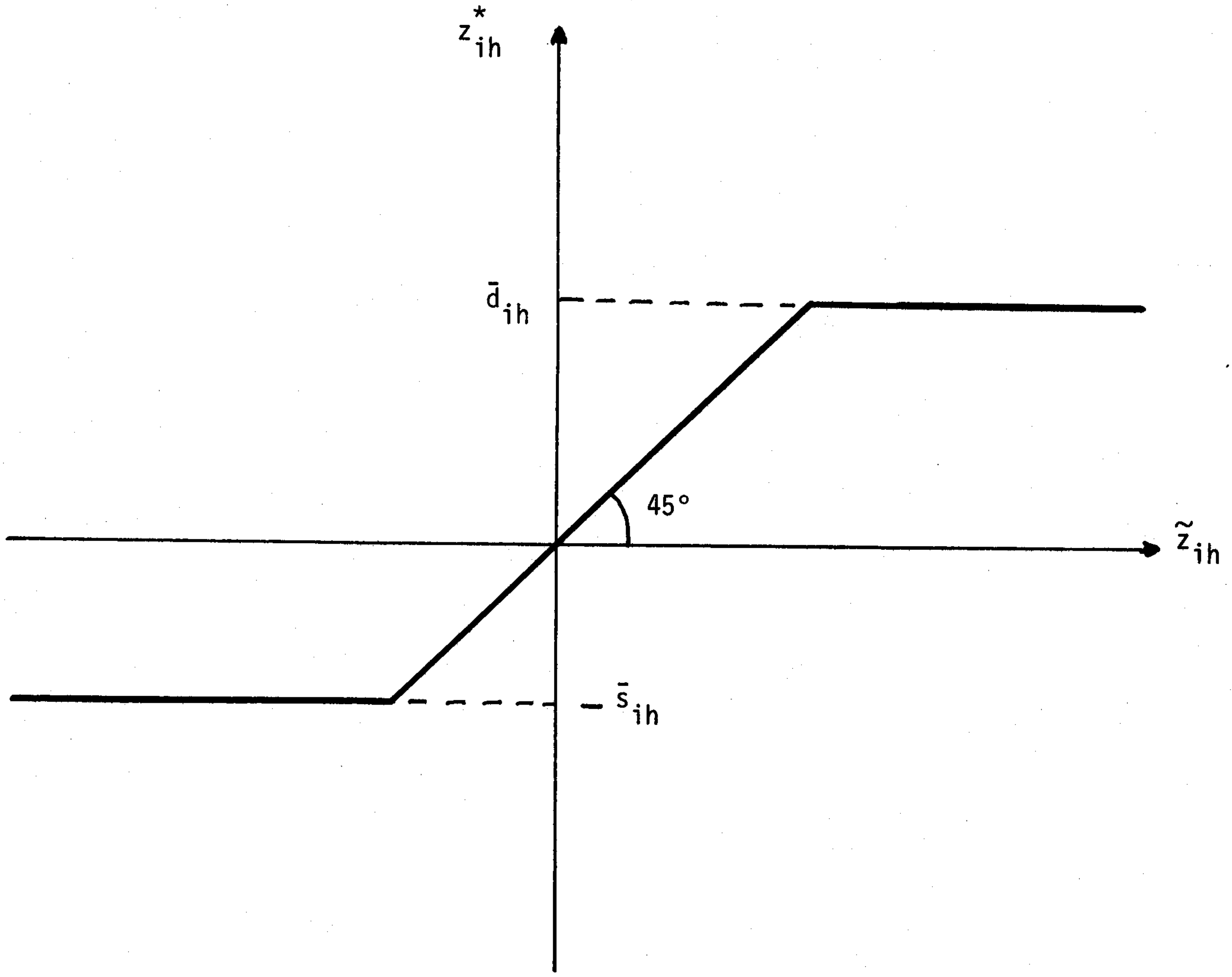

Figure 2

$$z_i^* = F_i(\tilde{\tilde{z}}_i, \tilde{\tilde{z}}_{-i}) \tag{3}$$

$$\begin{aligned} \bar{d}_i &= G_i^d(\tilde{\tilde{z}}_{-i}) \\ \bar{s}_i &= G_i^s(\tilde{\tilde{z}}_{-i}) \end{aligned} \tag{4}$$

where $\tilde{\tilde{z}}_i$ is the vector of demands expressed by agent i and $\tilde{\tilde{z}}_{-i}$ is the set of all such vectors for all agents, except agent i .

We shall now see in the following sections that the introduction of quantity signals plays a fundamental role in both demand and price theory, and that their consideration allows to enlarge the set of possible equilibria in a very substantial manner.

2.6. Notes and references

The distinction between monetary and barter economies is of course a traditional one. A recent restatement is in Clower (1967). The informational problems encountered in nonmonetary economies are stressed in Ostroy (1973), Ostroy-Starr (1974), Veendorp (1970).

The representation of rationing schemes and quantity signals in this section are taken from Benassy (1975a),(1977b),(1982). An alternative approach to rationing and quantity signals is that of Drèze (1975), which will be seen in the next section.

The voluntary exchange and market efficiency properties have been also discussed under various forms in Clower (1960, 1965), Hahn-Negishi (1962), Barro-Grossman (1971), Grossman (1971), Howitt (1974). The problem of manipulability was studied in Benassy (1977b).

3. *FIXPRICE EQUILIBRIA*

We shall now start with a first concept of non Walrasian equilibrium, that of fixprice equilibrium. This concept is of interest for several reasons : First it will give us a very general structure of non Walrasian equilibria, since we shall find that under some very mild conditions fixprice equilibria exist for every positive price system and every set of rationing schemes. Secondly, as we shall see in section 5, they are a very useful building block in constructing other non Walrasian equilibrium concepts with flexible prices. Third, the study of their optimality (or rather suboptimality) properties will also be useful for the subsequent concepts.

We shall thus assume that the price system p is given. As we indicated we assume that the rationing schemes on all markets are nonmanipulable. Accordingly transactions and quantity signals are generated on all markets according to the formulas seen above (Equations 3 and 4)

$$z_i^* = F_i(\tilde{\tilde{z}}_i, \tilde{\tilde{z}}_{-i})$$

$$\bar{d}_i = G_i^d(\tilde{\tilde{z}}_{-i})$$

$$\bar{s}_i = G_i^s(\tilde{\tilde{z}}_{-i})$$

We immediately see that all that remains in order to obtain a fix price equilibrium concept, is to determine how demands themselves are formed, a task to which we now turn.

3.2. Effective demand and supply

Demands and supplies are signals that agents send to the "market" (i.e. to the other agents) in order to obtain the best transactions. As we saw above the Walrasian demands and supplies are the best responses to a price signal under the assumption (which is actually verified ex-post in a Walrasian equilibrium) that each agent can buy or sell as much as he demands or supplies. We shall now consider an agent i faced with a price vector p and vectors of perceived constraints $\bar{d}_i$ and $\bar{s}_i$, and we shall see how he can choose a vector of effective demands $\tilde{\tilde{z}}_i$ which leads him to the best possible transaction.

Let us first start with the determination of this optimal transaction. Agent i knows that on market h his transactions are limited to the interval given by the perceived constraints :

$$- \bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih}$$

so that his best transaction is the solution in z_i of the following program :

$$\text{Maximize } U_i(\omega_i + z_i , m_i) \qquad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i \\ - \bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih} \qquad\qquad h = 1,\ldots,\ell \end{cases}$$

Since the function U_i is strictly concave, the solution is unique and we shall denote it as $\xi^*_i(p,\bar{d}_i,\bar{s}_i)$. However in our system an agent does not announce directly a vector of transactions, but sends a vector $\tilde{\tilde{z}}_i$ of effective demands and supplies in order to obtain this optimal transaction. If agent i sends a demand $\tilde{\tilde{z}}_{ih}$ on market h , he will obtain a transaction equal to :

$$\min[\bar{d}_{ih}, \max(\tilde{\tilde{z}}_{ih}, -\bar{s}_{ih})]$$

so that a demand leading to the optimal transaction must be solution in $\tilde{\tilde{z}}_{ih}$ of:

$$\min[\bar{d}_{ih}, \max(\tilde{\tilde{z}}_{ih}, -\bar{s}_{ih})] = \zeta^{*}_{ih}(p,\bar{d}_i,\bar{s}_i)$$

This equation has actually an infinity of solutions if ξ^{*}_{ih} is equal to any of the bounds $\bar{d}_{ih}$ or $-\bar{s}_{ih}$. In order to work in what follows with demand functions, and not correspondences, we shall make a selection in the above set of solutions, and define the effective demand of agent i on market h , denoted as $\tilde{\tilde{\xi}}_{ih}(p,\bar{d}_i,\bar{s}_i)$, as the solution in z_{ih} of the following program :

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i \\ -\bar{s}_{ik} \leqslant z_{ik} \leqslant \bar{d}_{ik} \qquad\qquad k \neq h \end{cases}$$

In words, the effective demand on a market h corresponds to the exchange which maximizes utility, taking into account the constraints on the <u>other</u> markets. Because of the strict concavity, we obtain a function. Repeating this program for all markets h, we obtain a vector function of effective demands $\tilde{\tilde{\xi}}_i(p,\bar{d}_i,\bar{s}_i)$. This vector of effective demands has a double property. First, as is easy to check, it leads to the best transaction vector $\xi^{*}_i(p,\bar{d}_i,\bar{s}_i)$ (See for example Benassy 1977b, 1982). Secondly, whenever a constraint is binding on a market h , the corresponding demand (or supply) is greater than the constraint or the transaction, which thus "signals" to the market that the agent trades less than he would want.

We shall immediately give an illustrative example of this definition, due to Patinkin (1956) and Barro and Grossman (1971), that of the employment function of the firm. Consider indeed a firm with a production function $y = F(\ell)$ exhibiting diminishing returns, and faced with a price p on the output market and a wage w on the labor market. The Walrasian demand for labor, which results from the maximization of profit $py - w\ell$ subject to the production function $y = F(\ell)$, is equal to $F'^{-1}(w/p)$. Assume now that the firm faces a constraint $\bar{y}$ on its sales of output. According to the above definition the effective demand for labor $\tilde{\tilde{\ell}}^d$ is the solution in ℓ of the following program :

$$\text{Maximize } py - w\ell \quad \text{s.t.}$$

$$\begin{cases} y = F(\ell) \\ y \leq \bar{y} \end{cases}$$

which yields an effective demand for labor :

$$\tilde{\tilde{\ell}}^d = \min(F'^{-1}(w/p) \; , \; F^{-1}(\bar{y}))$$

We see that the effective demand for labor may have two forms : the Walrasian demand just seen above if the sales constraint is not binding, or, if this constraint is binding, a more "Keynesian" form equal to the quantity of labor just necessary to produce the output demand. We see immediately on this example that effective demand may have various functional forms, which intuitively explains why non Walrasian models generally have multiple regimes (see for example the three goods, three regimes model in section 7 below).

3.3. Fixprice equilibrium

With the above definition of effective demand, we are now ready to give the definition of a fixprice equilibrium, found in Benassy (1975a, 1982) which we shall call for short K-equilibrium :

Definition 1 : A K-equilibrium associated with a price system p and rationing schemes represented by functions F_i , i = 1,...,n , is a set of effective demands $\tilde{\tilde{z}}_i$, transactions z_i^* and perceived constraints $\bar{d}_i$ and $\bar{s}_i$ such that :

(a) $\tilde{\tilde{z}}_i = \tilde{\tilde{\zeta}}_i(p,\bar{d}_i,\bar{s}_i)$ $\quad i = 1,\ldots,n$

(b) $z_i^* = F_i(\tilde{\tilde{z}}_i,\tilde{\tilde{z}}_{-i})$ $\quad i = 1,\ldots,n$

(c) $\bar{d}_i = G_i^d(\tilde{\tilde{z}}_{-i})$ $\quad i = 1,\ldots,n$

$\bar{s}_i = G_i^s(\tilde{\tilde{z}}_{-i})$ $\quad i = 1,\ldots,n$

We see that in a fixprice K-equilibrium the quantity constraints $\bar{d}_i$ and $\bar{s}_i$ from which the agents construct their effective demands (equation a) are the ones which will be generated by the exchange process (equation c). At equilibrium the agents thus have a correct perception of these quantity constraints.

Equilibria defined by these equations exist for all positive prices and rationing schemes satisfying voluntary exchange and nonmanipulability (Benassy 1975a, 1982). The "exogenous" data consist of the price system and the rationing schemes in all markets, F_i , i = 1,...n. One may wonder whether for given such exogenous data the equilibrium is likely to be unique. A positive answer has been given by Schulz (1983) who showed that the equilibrium is unique, provided the spillover effects [(4)] from one market to the other are less than hundred per cent in value terms. For example, in the simplest traditional Keynesian model, this would amount to assume a propensity to consume smaller than one.

We shall denote by $\tilde{\tilde{Z}}_i(p)$, $Z_i^*(p)$, $\bar{D}_i(p)$, $\bar{S}_i(p)$, the values of $\tilde{\tilde{z}}_i$, z_i^* $\bar{d}_i$, $\bar{s}_i$ at a fixprice equilibrium.

As a simple example of fixprice equilibrium, consider the traditional Edgeworth box example (Figure 3), which represents a single market where agents A and B exchange a good (measured horizontally) against money (measured vertically). Point 0 corresponds to initial endowments, DC is the budget line of the two agents at price p , points A and B are the tangency points of the indifference curves with this budget line.

<u>Figure 3</u>

Measuring the level of exchanges along the line OC, we see that A demands a quantity 0A, B supplies a quantity 0B. They exchange the minimum of these two quantities, i.e. 0A, and agent B is rationed. Perceived constraints are respectively 0A for agent B and 0B for agent A. Agent B is constrained on his supply, while A is not constrained.

We may now say a few words about the properties of the allocations at a fixprice K-equilibrium. First, considering a particular market h , the transactions of the various agents are by construction mutually consistent, since they result from rationing schemes :

$$\sum_{i=1}^{n} z^*_{ih} = 0 \qquad \forall h$$

Demands and supplies need not balance, however, and in a particular market one may have three different categories of agents :

- Unrationed agents such that $z^*_{ih} = \tilde{\tilde{z}}_{ih}$

- Rationed demanders such that $\tilde{\tilde{z}}_{ih} > z^*_{ih} = \bar{d}_{ih}$

- Rationed suppliers such that $\tilde{\tilde{z}}_{ih} < z^*_{ih} = - \bar{s}_{ih}$

Note that since our concept permits inefficient rationing schemes, one may have both rationed demanders and rationed suppliers on the same market. If however the rationing scheme on the market considered is frictionless, at most one side of that market is rationed.

If we now consider a particular agent i , we see that his transactions vector z^*_i is the best, taking account of the perceived constraints on all markets, because effective demand has been constructed so as to yield precisely this optimal trade. Mathematically z^*_i is the solution in z_i of the following program, already seen above :

$$\text{Maximize } U_i(\omega_i + z_i \ , m_i) \qquad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i \\ - \bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih} \qquad h = 1,\ldots,\ell \end{cases}$$

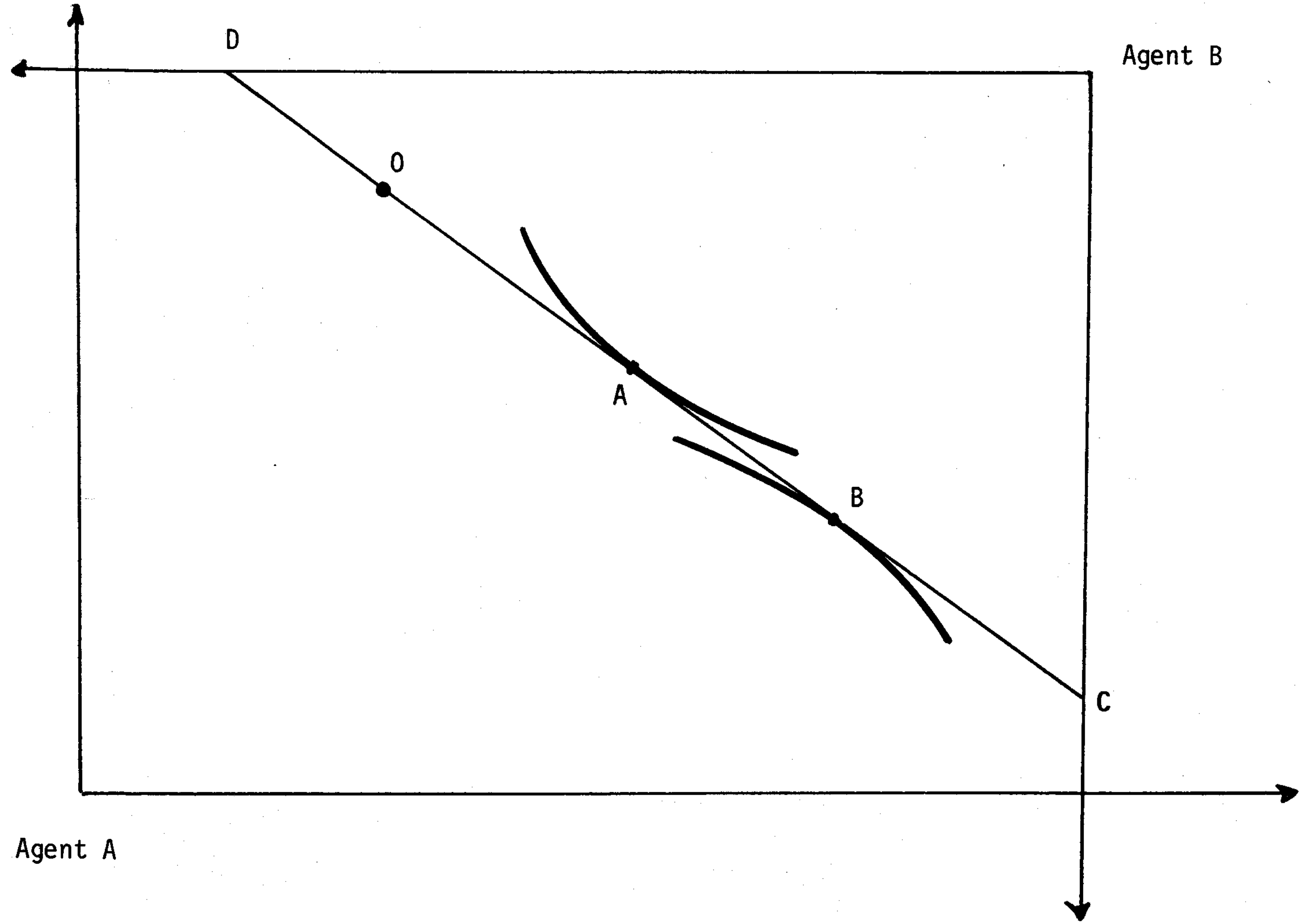

Figure 3

3.4. An alternative concept

We shall now present an alternative concept of fixprice equilibrium, due to Drèze (1975),[5] which we shall recast using our notations. That concept deals directly with the vectors of transactions z_i^* and quantity constraints $\bar{d}_i$ and $\bar{s}_i$. The original concept by Drèze actually dealt with uniform rationing, so that the vectors $\bar{d}_i$ and $\bar{s}_i$ were the same for all agents.

Definition 2 : <u>A fixprice D-equilibrium for a given set of prices</u> p <u>is defined as a set of vectors of transactions</u> z_i^* <u>and quantity constraints</u> $\bar{d}_i$ <u>and</u> $\bar{s}_i$ <u>such that</u> :

(a) $$\sum_{i=1}^{n} z_{ih}^* = 0 \qquad \forall\, h$$

(b) The vector z_i^* is solution in z_i of

$$\begin{cases} \text{Maximize } U_i(\omega_i + z_i\,,\, \bar{m}_i)\,, \quad \text{s.t.} \\ pz_i + m_i = \bar{m}_i \\ -\bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih} \qquad h = 1,\ldots,\ell \end{cases}$$

(c) $$\forall\, h \quad z_{ih}^* = \bar{d}_{ih} \text{ for some } i \text{ implies } z_{jh}^* > -\bar{s}_{jh} \quad \forall\, j$$
$$z_{ih}^* = -\bar{s}_{ih} \text{ for some } i \text{ implies } z_{jh}^* < \bar{d}_{jh} \quad \forall\, j$$

Let us now interpret these conditions. Condition (a) is the natural requirement that transactions should balance on each market. Condition (b) says that transactions must be individually rational, i.e. they must maximize utility subject to the budget constraint and the quantity constraints on all markets. In the notations of the preceding subsection, $z_i^* = \xi_i^*(p,\bar{d}_i,\bar{s}_i)$. We should note at this stage that using quantity constraints under the form of maximum upper bounds and lower bounds on trades implicitly assumes rationing schemes which exhibit voluntary exchange and nonmanipulability, as we saw in section 2.

Condition (c) basically says that rationing may affect either supply or demand, but not both simultaneously [(6)] . We recognize here with a different formalization the condition of market efficiency which is thus built in this definition of equilibrium, whereas it was not in the previous definition.

Drèze (1975) proved that an equilibrium according to definition 2 exists for all positive price systems and for uniform rationing schemes under the traditional concavity assumptions for the utility functions. The concept is easily extended to some nonuniform bounds (Grandmont-Laroque 1976, Greenberg-Müller 1979), but in this last case it is not specified in the concept how shortages are allocated among rationed demanders or rationed suppliers. Because of this there will be usually an infinity of fixprice equilibria corresponding to a given price, as soon as there are two rationed agents on one side of a market.

As we noted in the preceding sections the two concepts are, implicitly or explicitly, based on a representation of markets under the form of rationing schemes satisfying voluntary exchange and nonmanipulability. This suggests that if in the first definition we further assume that all rationing schemes are efficient the two definitions should yield similar sets of equilibrium allocations for a given price system. This was indeed proved by Silvestre (1982) (1983) for both exchange and production economies.

3.5. Notes and references

The two concepts of fixprice equilibrium described in this chapter have been developed in Benassy (1975a),(1977b),(1982) and Drèze (1975) respectively. An alternative concept which does not use quantity signals is due to Younès (1975). An early model of equilibrium with nonclearing labor market is due to Glustoff (1968). The relations between various concepts have been explored by Silvestre (1982), (1983), D'Autume (1985). Fixprice equilibrium has been modelled as a Nash equilibrium in Böhm-Levine (1979), Heller-Starr (1979). However this approach yields a large number of unwanted equilibria (including the no trade one) for reasons described in Benassy (1982).

Various extensions of these concepts have been studied : Drèze (1975) constructed equilibria where some prices are subject to exogenous bounds, or indexed on each other, an issue pursued in Dehez-Drèze (1984). The problems of manipulable rationing schemes were studied in Benassy (1977b), (1982). Fixprice equilibrium in a barter economy was modelled in Benassy (1975b). Section 5 below describes endogenous price making in various frameworks of monopolistic competition.

The issue of uniqueness of fixprice equilibrium has been studied by Schulz (1983) who gives sufficient conditions for global uniqueness in the Benassy model. We indicated that the Drèze equilibrium is usually not unique, but uniqueness results can be obtained in some specific cases (Laroque 1981).

4. OPTIMALITY

One of the great virtues of Walrasian equilibrium is the property of Pareto optimality. So it is quite natural to inquire whether the large class of fixprice equilibria possesses either the Pareto optimality property, or even a weaker optimality property taking into account the fact that agents trade at given prices. As we shall see these two questions are usually answered in the negative. Before going to each of these criteria, we shall first characterize our equilibria in a differential way.

4.1. Characterization of equilibria

As we saw above, transactions at a fixprice equilibrium are the solutions in z_i to the following program :

$$\text{Maximize } U_i(\omega_i + z_i , m_i) \qquad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i \\ -\bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih} \qquad \forall\, h \end{cases}$$

Call λ_i and δ_{ih} the Kuhn-Tucker multipliers of the above constraints. The Kuhn-Tucker conditions for this programme can be written, assuming an interior maximum :

$$\frac{\partial U_i}{\partial m_i} = \lambda_i$$

$$\frac{\partial U_i}{\partial z_{ih}} = \lambda_i p_h + \delta_{ih}$$

The multiplier λ_i can be interpreted as the marginal utility of income. δ_{ih} is an index of rationing for agent i on market h :

$\delta_{ih} = 0$ if i is unconstrained on market h ($z^*_{ih} = \tilde{\tilde{z}}_{ih}$)

$\delta_{ih} > 0$ if i is constrained in his demand for good h ($0 \leq z^*_{ih} < \tilde{\tilde{z}}_{ih}$)

$\delta_{ih} < 0$ if i is constrained in his supply of good h ($0 \geq z^*_{ih} > \tilde{\tilde{z}}_{ih}$)

Let us call "shadow price" of good h the ratio $(\partial U_i/\partial z_{ih})/(\partial U_i/\partial m_i)$. The above relations allow us to compute the shadow price of good h for agent i as :

$$\frac{\partial U_i/\partial z_{ih}}{\partial U_i/\partial m_i} = p_h + \frac{\delta_{ih}}{\lambda_i} \tag{5}$$

We may thus note that the shadow price of good h is greater than, equal to, or smaller than p_h depending on whether i is constrained on his demand, unconstrained, or supply constrained, in market h .

We shall now use the above results to study the optimality properties of fixprice equilibria. Before starting we must make one further remark and assumption : It is clear that it would be too easy to show that fixprice equilibria are inefficient if some markets were functioning inefficiently themselves. We shall thus assume in what follows that all markets are frictionless. This immediately implies that the numbers δ_{ih} will be zero for all least all agents on the short side of every market h .

4.2. Pareto-optimality

We shall now inquire whether fixprice allocations have the property of Pareto-optimality. From a heuristic point of view, we should first note that the constraint of trading at fixed prices does not necessarily rule out Pareto optimality, as Figure 4 shows. Indeed there is a point on the budget line, point P, which is a Pareto optimum. But it clearly differs from the fixprice equilibrium A. Continuing with the Edgeworth box example, it is easy to see that this situation is fairly general. Indeed the set of Pareto-optima is the contract curve, whereas the set of fixprice equilibria, when we vary the price, is the lentil shaped curve (Figure 5). We see that these two curves intersect only at the Walrasian equilibrium point.

<u>Figure 4</u>

An interesting way to look at this problem is to note (cf. Figure 4) that at point P trader A would exchange more than he wants at that price. This leads to the intuition that efficiency would require forced trading, or that voluntary trading would generally imply inefficiency. This has been studied and made rigorous in Silvestre (1985) who showed that Pareto optimality and voluntary exchange were satisfied together only at the Walrasian allocation.

<u>Figure 5</u>

We can also see in a direct manner why fixprice equilibria generally are not Pareto optima. For that let us first recall the differential characterization of a Pareto optimum : Such an optimum can be obtained by maximizing a weighted sum of agents' utilities, subject to the material "adding up" constraints :

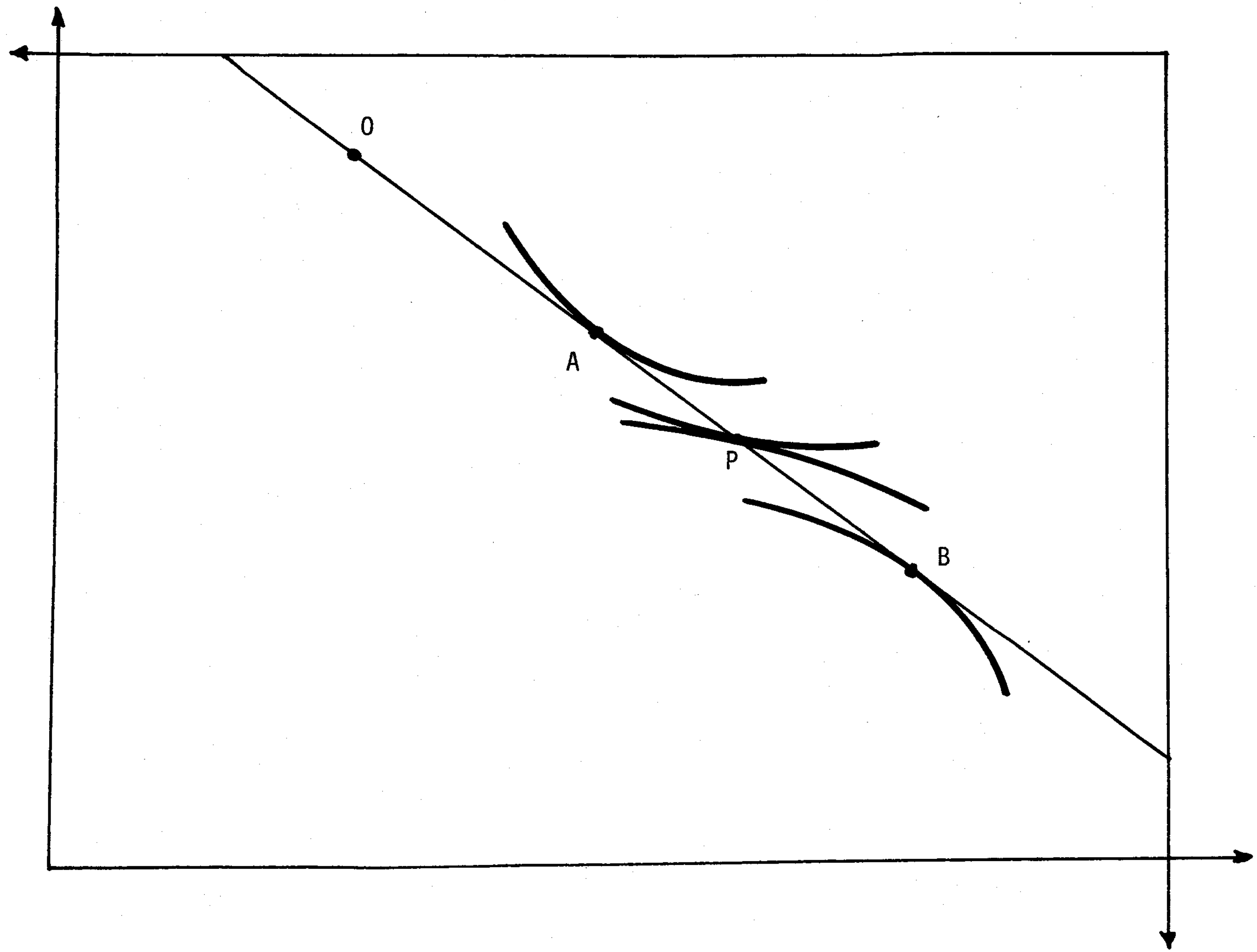

Figure 4

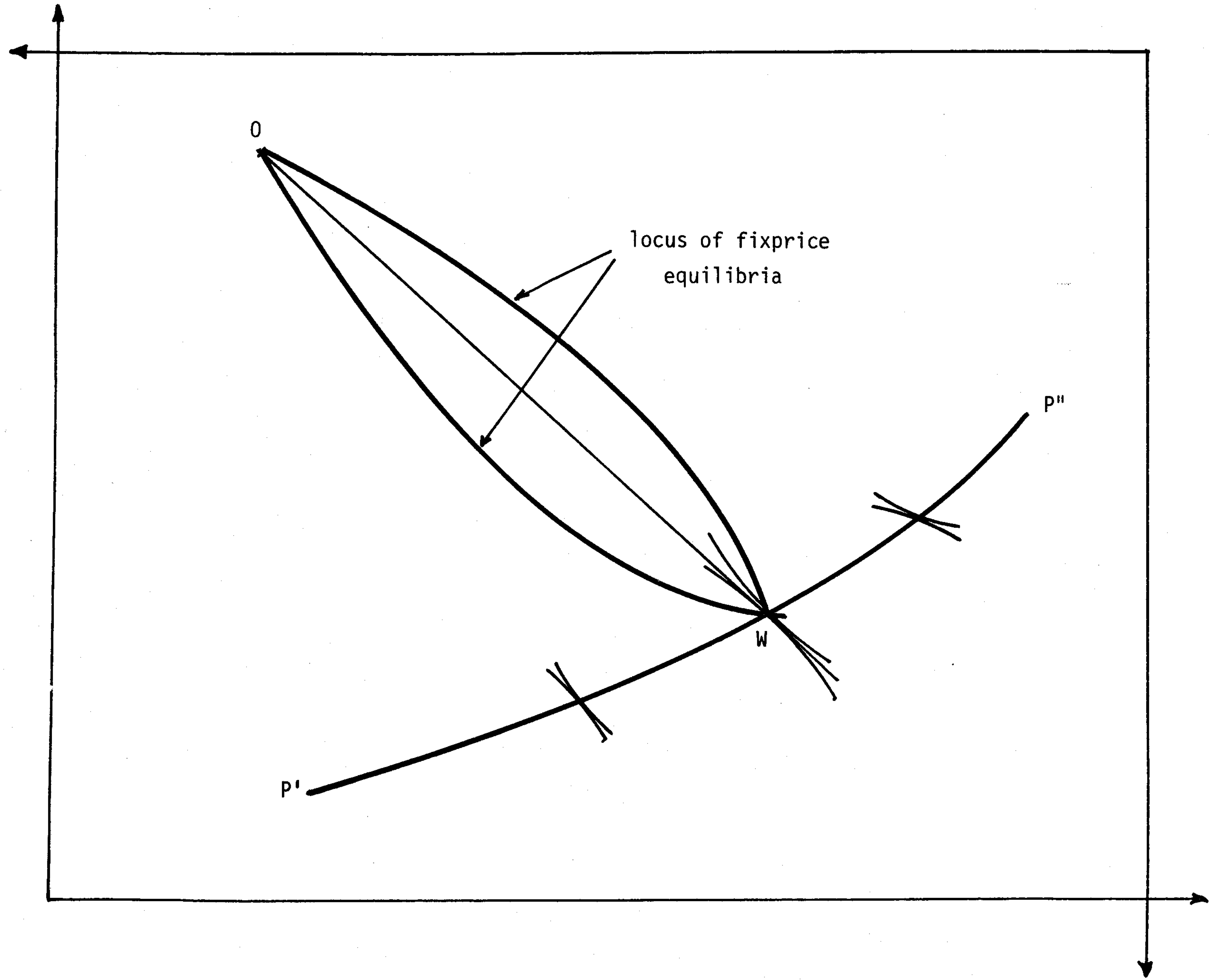

Figure 5

$$\text{Maximize} \quad \sum_{i=1}^{n} v_i \, U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$

$$\begin{cases} \sum_{i=1}^{n} z_{ih} = 0 \qquad h = 1,\ldots,\ell \\ \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} \bar{m}_i \end{cases}$$

The Kuhn-Tucker conditions for such a program can be rewritten under the traditional form :

$$\frac{\partial U_i / \partial z_{ih}}{\partial U_i / \partial m_i} = \rho_h \qquad i = 1,\ldots,n$$

that is, the "shadow price" of every good h must be the same for all agents.

Now we can see why fixprice equilibria are usually not Pareto optimal : In a market h , there are always some agents who are unconstrained (namely those on the short side). For these agents, in view of the above characterizations (Equation 5), the shadow price of good h is equal to p_h . If the allocation is a Pareto optimum the shadow price must be the same for all agents, and thus equal to p_h for all agents, which implies :

$$\delta_{ih} = 0 \qquad \forall\, i \;,\; \forall\, h$$

This means that all agents are unconstrained on all markets, i.e. that we are in a Walrasian equilibrium.

4.3. Constrained Pareto-optimality

We shall now investigate a less demanding notion of optimality, which takes into account the fact that trades take place at a given price system.

We shall thus say that an allocation is a constrained Pareto optimum if there is no allocation which (a) satisfies the physical feasibility conditions, (b) satisfies the budget constraints for the given price vector p, (c) Pareto dominates the allocation considered. In the one market Edgeworth box example of Figure 4 , we see that the set of constrained-Pareto optima is the segment AB, and the fixprice equilibrium A is one of them. We shall now see, however, that this property does not generally extend to the multimarket case.

Indeed, coming back to the general framework, we see that a constrained-Pareto-optimal allocation can be obtained as a solution of the following program :

$$\text{Maximize } \sum_{i=1}^{n} v_i U_i(\omega_i + z_i, m_i) \quad \text{s.t.}$$

$$\begin{cases} \sum_{i=1}^{n} z_{ih} = 0 & \forall\, h \\ pz_i + m_i = \bar{m}_i & \forall\, i \end{cases}$$

Note that we do not need to write the feasibility constraint for money, which follows from the other equalities.

The Kuhn-Tucker conditions for this program can be written :

$$\frac{\partial U_i/\partial z_{ih}}{\partial U_i/\partial m_i} = p_h + \mu_i \delta_h \tag{6}$$

where μ_i is positive, and represents somehow the degree to which agent i is constrained, as compared to the others. The numbers δ_h can have any sign. For example in the case of the Edgeworth box of Figure 4, which displays excess supply for the single good, the δ_h will be negative. In general the numbers μ_i

and δ_h will depend in a more complex manner both on the price system p and on the weigths v_i chosen for each agent.

If we now compare the shadow prices at a constrained Pareto optimum (Equations 6) with those in a fixprice equilibrium (Equations 5), we see that a constrained Pareto optimum will obtain only under very special circumstances. Indeed formula (6) above shows that an agent i should be either constrained in all markets where δ_h is different from zero (if $\mu_i \neq 0$) or constrained in none (if $\mu_i = 0$). A particular case where this occurs is the situation of classical unemployment in the three goods model (Section 7 below), but this type of situation will occur infrequently if many markets do not clear.

Indeed there are other cases where constrained optimality structurally cannot hold. Consider for example a situation of generalized excess supply. There typically each agent is constrained on the goods he sells ($\delta_{ih} < 0$) and unconstrained for the goods he purchases ($\delta_{ih} = 0$). In such a case there is no way the shadow prices at a fixprice equilibrium can be written under the form given in equations (6), and the corresponding equilibrium will be suboptimal, even taking into account the constraint that trades must me made at given prices. A symmetric situation occurs with generalized excess demand. Particularly striking examples of this suboptimality will be given in sections 7 and 8.

More generally, we may expect suboptimality to occur when there are several markets with excess demands of the same sign (a situation which also leads to multiplier effects ; see Benassy 1975a, 1982 for a study of the relation between the two). As a typical example, consider the case, traditional in Keynesian theory, of an excess supply in several markets, and consider two markets h and k in excess supply. Consider two agents i and j , i being a rationed supplier in k and an unrationed demander in h , j being a rationed supplier in h and an unrationed demander in k . Then the conditions seen above immediately lead to :

$$\frac{1}{p_h} \frac{\partial U_i}{\partial z_{ih}} = \frac{\partial U_i}{\partial m_i} > \frac{1}{p_k} \frac{\partial U_i}{\partial z_{ik}}$$

$$\frac{1}{p_k} \frac{\partial U_j}{\partial z_{jk}} = \frac{\partial U_j}{\partial m_j} > \frac{1}{p_h} \frac{\partial U_j}{\partial z_{jh}}$$

One thus sees that i and j would both be interested in exchanging goods h and k directly against each other at the prices p_h and p_k , which suggests a simple Pareto improving trade. Of course in general, in the absence of double coincidence of wants, such Pareto improving trades will be quite more complex, making them quite difficult to achieve by decentralized agents. The existence of unrealized exchange possibilities at the economy level therefore suggests that in such cases government intervention may improve the situation of the private sector. Whether this actually holds will be investigated in the examples of sections 7 and 8.

4.4. Notes and references

The suboptimality properties of non-Walrasian equilibria have been particularly studied in Benassy (1973, 1975a, 1975b, 1982), Drèze-Muller (1980), Silvestre (1985), Younès (1975).

The problem of the Pareto optimality of non Walrasian allocations has been notably scrutinized in Silvestre (1985).

The notion of constrained Pareto optimality used in this section is due to Uzawa (1962). Another quite interesting form of constrained optimality, based on pairwise exchanges, is due to Arrow-Hahn (1971). Whether fixprice allocations are constrained Pareto optimal has been investigated in Benassy (1973, 1975a, 1982), Younès (1975). The treatment in this section follows the former. Drèze-Muller (1980) have shown that constrained optimality may be obtained through "coupons rationing".

The inefficiency of multiplier states had been discussed in Clower (1965) and Leijonhufvud (1968). This was developed formally in Benassy (1973), (1975a), (1977a), (1982). The role of expectations is discussed in Benassy (1982, 1986), Persson-Svensson (1983). Properties of monetary and barter economies are compared in Benassy (1975b, 1982).

5. *PRICE MAKING AND EQUILIBRIUM*

At this stage, the theory is still in need of a description of decentralized price making by agent internal to the system. We shall describe in this section a few concepts dealing with that problem, and we shall see that, just as in demand and supply theory, quantity signals play a prominent role. It is actually quite intuitive that quantity signals must be a fundamental part of the competitive process in a truly decentralized economy. Indeed, it is the inability to sell as much as they want that leads suppliers to propose, or to accept from other agents, a lower price, and conversely it is the inability to buy as much as they want that leads demanders to propose, or accept, a higher price. Various modes of price making integrating these aspects can be envisioned. We shall deal here with a particular organization of the pricing process where agents on one side of the market (most often the suppliers) quote prices and agents on the other side act as price takers. The general idea relating the concepts in this section to those of the previous ones is that price makers change their prices so as to "manipulate" the quantity constraints they face (that is, so as to increase or decrease their possible sales or purchases). As we shall see, this model of price making is quite reminiscent of the imperfect competition line : Chamberlin (1933), Robinson (1933), Triffin (1940), Bushaw-Clower (1957), Arrow (1959), and more particularly of the theories of general equilibrium with monopolistic competition, as developed notably by Negishi (1961) (1972).

5.1. The general framework

We shall thus now assume that each agent i controls the prices of a (possibly empty) subset H_i of the goods. Goods are distinguished both by their physical characteristics and by the agent who sets their price (we thus consider two goods sold by different sellers as different goods, a fairly usual assumption in microeconomic theory since these goods differ at least by location, quality, etc...), so that :

$$H_i \cap H_j = \{\emptyset\} \qquad i \neq j$$

Each price maker is thus alone on his side of the market. We can further subdivide H_i into H_i^d (goods demanded by i) and H_i^s (goods supplied by i). Agent i appears, at least formally, as a monopolist on markets $h \in H_i^s$, as a monopsonist on markets $h \in H_i^d$.

We shall denote by p_i the set of prices controlled by agent i and by p_{-i} the set of prices controlled by the other agents (i.e. the rest of prices) :

$$p_i = \{p_h \mid h \in H_i\}$$

$$p_{-i} = \{p_j \mid j \neq i\}$$

Each agent will choose a price vector p_i taking the other prices p_{-i} as given ; the equilibrium structure is thus that of a Nash-equilibrium in prices corresponding to an idea of monopolistic competition. The basic idea behind the modelling of price making itself in such models is, as we indicated above, that each price maker uses the prices he controls to "manipulate" the quantity constraints he faces. In particular, because the price makers are alone on their side of the markets they control, their quantity constraints on these markets have the simple form:

$$\bar{s}_{ih} = \sum_{j \neq i} \tilde{\tilde{d}}_{jh} \qquad h \in H_i^s$$

$$\bar{d}_{ih} = \sum_{j \neq i} \tilde{\tilde{s}}_{jh} \qquad h \in H_i^d$$

i.e. the maximum quantity that price setter i can sell is the total demand of the others, and conversely if he is a buyer.

In order to be able to pose the problem of the choice of prices by price makers as a standard decision problem, all we now need to know how the constraints faced by an agent on all markets vary as a function of the prices he sets.

We shall now describe two approaches and equilibrium concepts dealing with that problem : One based on objective demand curves, the other on subjective demand curves.

5.2. Objective demand curves

The implicit idea behind the objective demand curve approach is that each price maker knows well enough the economy to be able to compute under all circumstances the actual quantity constraints he would face. Since we are considering a Nash equilibrium, he must be able to perform this computation for any set p_{-i} of prices set by the other agents, and any prices p_i set by himself, i.e. he must be able to compute his constraints for any vector of prices, once all feedback effects have been accounted for.

But on the other hand we know, from section 3 above, that for a given organization of the economy (i.e. in particular rationing schemes) and for a given set of prices p a fixprice equilibrium is characterized by net demands $\tilde{\tilde{Z}}_i(p)$, transactions $Z_i^*(p)$ and perceived constraints $\bar{D}_i(p)$ and $\bar{S}_i(p)$. If the agent has full knowledge of the parameters of the economy (a strong assumption of course, but which is embedded in the notion of an objective demand curve), then he knows this and the "objective demand curve" will be given by the two vector functions $\bar{D}_i(p)$ and $\bar{S}_i(p)$.

Accordingly the price p_i maximizing the utility of agent i is the solution in p_i of the following program :

$$\text{Maximize } U_i(\omega_i+z_i, m_i) \quad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i \\ -\bar{S}_i(p) \leqslant z_i \leqslant \bar{D}_i(p) \end{cases}$$

which yields the optimum price of agent i as a function of the prices of the other agents :

$$p_i = \psi_i(p_{-i})$$

We can now give the definition of an equilibrium :

Definition 3 : <u>An equilibrium with price makers is characterized by a set of</u> p_i^* , $\tilde{\tilde{z}}_i$, z_i^* , $\bar{d}_i$, $\bar{s}_i$, $i \in I$, <u>such that</u> :

(a) $p_i^* = \psi_i(p_{-i}^*) \quad \forall\, i$

(b) $\tilde{\tilde{z}}_i$, z_i^* , $\bar{d}_i$, $\bar{s}_i$ form a fixprice equilibrium for the price vector p^* , i.e. they are equal respectively to $\tilde{\tilde{Z}}_i(p^*)$, $Z_i^*(p^*)$, $\bar{D}_i(p^*)$, $\bar{S}_i(p^*)$ for all i .

Further description and conditions for existence can be found in Benassy (1988). We shall see in section 8 below a macroeconomic application of this concept. Before that let us give a simple example in the Edgeworth box already considered above (Figure 6).

Assume now that agent B (the seller) sets the price. The "objective demand curve" is then agent A's demand, and corresponds thus to the locus of tangency points between various budget lines and A's indifference curves. This is depicted as the curved line OMW in Figure 6, where W is the Walrasian point. The equilibrium point is then simply point M, the tangency point of this curve with B's indifference curve, which yields agent B the highest possible utility, given A's objective demand behavior.

Figure 6

5.3. Subjective demand curves

Of course assuming every agent knows the "true" objective demand curve as defined above is a very strong assumption in view of all the parameters involved. Another approach, the subjective demand curve approach, consists thus in having each price maker estimate the demand and supply curves. These estimated curves are called the subjective, or perceived demand or supply curves.

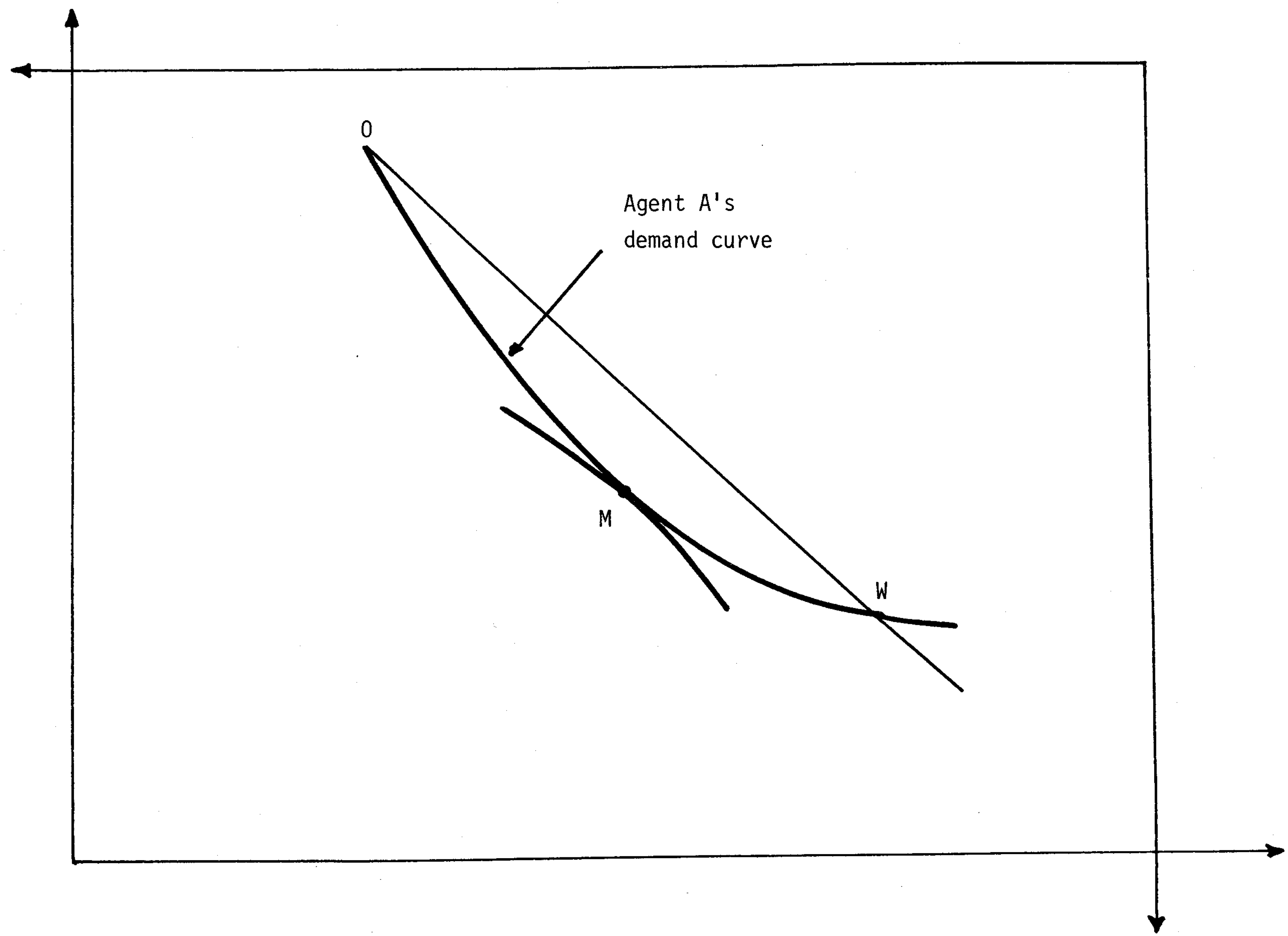

Figure 6

Formally the perceived demand curve links the maximum quantity a price maker i can sell on a market $h \in H_i^s$ he controls, i.e. $\bar{s}_{ih}$, to the prices p_i he sets. It will be denoted as :

$$\bar{S}_{ih}(p_i\ ,\ \theta_i)$$

where θ_i is a vector of parameters (for example the position or elasticity of the perceived demand curve),themselves estimated on the basis of price-quantity signals, as be shall see below. We shall assume that $\bar{S}_{ih}$ is nonincreasing in p_h . Symmetrically, the perceived supply curve, which indicates the maximum quantity a price maker i can purchase on a market $h \in H_i^d$ he controls, will be denoted as :

$$\bar{D}_{ih}(p_i\ ,\ \theta_i)$$

and we shall assume it is nondecreasing in p_h .

Now the parameters θ_i are not arbitrary but must be estimated so as to fit the price-quantity observations. We shall thus assume that the estimated parameter is a function of current signals (and of course, but this is left implicit, of all past signals) :

$$\theta_i = \theta_i(\bar{p}\ ,\ \bar{d}_i\ ,\ \bar{s}_i)$$

Because we are going to consider an equilibrium concept for the current period, this estimation procedure must be such that perceived demand and supply curves "go through" the observed point (Bushaw-Clower, 1957) i.e. :

$$\bar{D}_{ih}[p_i\ ,\ \theta_i(\bar{p}\ ,\ \bar{d}_i\ ,\ \bar{s}_i)] = \bar{d}_{ih} \qquad \text{for } p_i = \bar{p}_i$$

$$\bar{S}_{ih}[p_i\ ,\ \theta_i(\bar{p}\ ,\ \bar{d}_i\ ,\ \bar{s}_i)] = \bar{s}_{ih} \qquad \text{for } p_i = \bar{p}_i$$

As an example imagine agent i is a price setter for a good h he sells, and the perceived demand curve has the form :

$$\theta_{ih}\ p_h^{-\varepsilon}$$

where the elasticity ε may have been estimated from earlier observations. Then the above consistency equation yields :

$$\theta_{ih}\ p_h^{-\varepsilon} = \bar{s}_{ih}$$

$$\theta_{ih} = p_h^{\varepsilon}.\bar{s}_{ih}$$

We can now make explicit the procedure of price formation. Agent i , facing a price p_h and constraints $\bar{d}_{ih}$ and $\bar{s}_{ih}$ on markets $h \notin H_i$ will choose his price so as to maximize his utility, i.e. the solution in p_i to the program

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$

$$\begin{cases} pz_i + m_i = \bar{m}_i & \\ -\bar{s}_{ih} \leqslant z_{ih} \leqslant \bar{d}_{ih} & h \notin H_i \\ -\bar{S}_{ih}(p_i, \theta_i) \leqslant z_{ih} \leqslant \bar{D}_{ih}(p_i, \theta_i) & h \in H_i \end{cases}$$

which yields a function $p_i = \mathcal{P}_i(p, \bar{d}_i, \bar{s}_i)$ since the parameters θ_i are themselves functions of the signals $p, \bar{d}_i, \bar{s}_i$. We can now define an equilibrium as a situation where quantities are optimal given prices and no price maker has interest in changing his price i.e. :

Definition 4 : <u>An equilibrium with price makers consists of a set of prices</u> p_i^*, <u>demands</u> $\tilde{\tilde{z}}_i$, <u>transactions</u> z_i^* <u>and quantity signals</u> $\bar{d}_i$ <u>and</u> $\bar{s}_i$ <u>such that</u> :

(a) $\quad p_i^* = \mathcal{P}_i(p^*, \bar{d}_i, \bar{s}_i) \qquad \forall\ i$

(b) $\quad$ <u>The quantities</u> $\tilde{\tilde{z}}_i$, z_i^* , $\bar{d}_i$, $\bar{s}_i$ <u>form a fixprice equilibrium for</u> p^* .

A further description of this concept and its properties, as well as existence proofs, can be found in Benassy (1976b) (1982). We may now make a few remarks, comparing notably the two concepts with subjective and objective demand curves.

The first and obvious one is that the two categories of objective and subjective demand curves are not antagonistic : The objective demand curve would arise as a particular case of the subjective demand curve approach, if the family of subjective demand curves happens to contain the objective one. In fact the most natural and fruitful way to interpret subjective demand curves is as a temporary state in an ongoing learning process. Indeed the theory developed here gives a natural way of "learning" about the demand curve : Each realization p , $\bar{d}_i$, $\bar{s}_i$ in a period is a point on the "true" curve of that period. Using the sequence of these observations, plus any extra informations available, the price makers can use statistical techniques to yield an estimation of the "objective" demand curve. Thus the parameters of the family of subjective demand curves in one period correspond to whatever has been learned up to that period. Whether such learning would lead to the "true" demand curve is still an unresolved problem.

5.4. Notes and references

The two concepts described in this section are taken from Benassy (1988) for the objective demand curve approach and Benassy (1976b), (1982) for the subjective demand curve approach. The studies in general equilibrium with subjective demand curves, originate in the seminal work of Negishi (1961), (1972). The "consistency condition" for subjective demand curves had been given by Bushaw-Clower (1957). The theory of general equilibrium with objective demand curves was first developed in the Cournotian "quantity setting" framework by Gabszewicz-Vial (1972), then in some specific models with price makers by Marschak-Selten (1974) and Nikaido (1975). The general treatment in this section follows Benassy (1988).

We may note that we have described the two "extremes" in terms of consistency with the data : The "objective" demand curve, on the one hand, is the correct one, and is determined uniquely by the data of the economy. On the other hand subjective demand curves are much more numerous as they are only required to satisfy the minimal consistency requirement of "going through" the observed point on the "true" curve. One may think of various less demanding requirements in between the two extremes. For example one may assume that the price makers know, maybe by local experiments, the slope of the "true" demand curve at the point considered (Silvestre, 1977).

We should also note that we have maintained throughout the assumption of a Nash equilibrium in prices, investigating the consistency of the quantity responses to these prices. Another direction of research is to abandon the "monopolistic competition" assumption and to go to a more oligopolistic framework where agents should conjecture the price responses of competitors to their own actions (Hahn 1978, Marschak-Selten 1974).

6. EXPECTATIONS

6.1. The problem

Up to now we have dealt with an equilibrium structure in the period considered, implicitly a short-run one, but of course the economy extends further in the future, as we are reminded at least by the presence of money as a store of value. More generally the presence of stocks (inventories, capital goods, financial assets) makes necessary to form expectations.

We shall now show that expectations are actually present in the theory, and how they can be integrated explicitly. This integration of expectations goes through an evaluation of the utility of stocks, which are the physical link between present and future. The construction of such indirect utilities allows one to convert expectations on future exchanges into effective demands and supplies for current goods.

The general method can be sketched as follows : Each agent actually plans for the current and future periods. Expectations for future periods take the form of prices and quantity constraints (for price takers) or expected demand curves (for price makers). These may be deterministic or stochastic. These expectations are formed via expectations schemes, which link future price-quantity expectations to all price-quantity signals received in past and current periods. This formulation is thus quite general and covers any expectations scheme, "rational" or not, based on actually available information.

By a standard dynamic programming technique (Bellman 1957), one can reduce the multiperiod problem to a single period one, where the valuation of all stocks (and notably money) depends upon future expectations, and thus, via the expectations schemes, upon the current and past price-quantity signals. We are thus formally back to the one period formulation used in the section 3, with the only difference that current and past quantity signals must be added in the valuation functions.

6.2. The indirect utility of money

We shall now show how to derive the indirect utility of money from expectations in a framework similar to that seen in section 3. However instead of assuming that agent i has a one period utility function $U_i(x_i, m_i)$ we shall assume that he plans for two periods, current and future (the method would extend without difficulty to any finite number of periods). Variables relative to the future period are denoted by a superindex e , for expected. Agent i has a utility function for current and future consumptions

$$V_i(x_i, x_i^e)$$

assumed strictly concave, where :

$$x_i = \omega_i + z_i \qquad x_i^e = \omega_i^e + z_i^e$$

At the beginning of the current period, the agent holds an initial quantity of money $\bar{m}_i$. He transfers to the second period a quantity m_i equal to :

$$m_i = \bar{m}_i - pz_i \geq 0$$

The expected transactions in the second period must satisfy :

$$p^e z_i^e \leqslant m_i$$

Each agent must furthermore form expectations about the price and quantity constraints he will face in the future period. We shall denote these as :

$$\sigma_i^e = (p^e, \bar{d}_i^e, \bar{s}_i^e)$$

Assume that agent i has consumed $x_i = \omega_i + z_i$ in the first period, and transfers a quantity of money m_i to the second period. Given some expectations $\sigma_i^e = (p^e, \bar{d}_i^e, \bar{s}_i^e)$, his expected second period transactions are those which maximize his utility under the budget constraint, and taking all quantity signals into account. Thus z_i^e is solution of the followig program :

$$\text{Maximize } V_i(\omega_i + z_i \ , \ \omega_i^e + z_i^e) \qquad \text{s.t.}$$

$$\begin{cases} p^e z_i^e \leqslant m_i \\ -\bar{s}_i^e \leqslant z_i^e \leqslant \bar{d}_i^e \end{cases}$$

We shall denote functionally the vector z_i^e solution of this program as :

$$Z_i^e(z_i, m_i, p^e, \bar{d}_i^e, \bar{s}_i^e) = Z_i^e(z_i, m_i, \sigma_i^e)$$

One can then write the level of utility expected in the first period as :

$$U_i^e(\omega_i + z_i, m_i, \sigma_i^e) = V_i[\omega_i + z_i, \omega_i^e + Z_i^e(z_i, m_i, \sigma_i^e)]$$

Under this form the indirect utility function depends explicitly on expectations of future signals σ_i^e . We see that we have a utility function having ω_i+z_i and m_i as arguments, but "parameterized" by the expectations σ_i^e . We may note at this stage that this utility function has the natural property of homogeneity of degree zero with respect to m_i and p^e .

If the expectations were given, the analysis would actually stop here. But of course in general the expectations will depend upon all information available up to the current period. We can make in particular these expectation explicitly dependent upon current period signals, which we will write, if expectations are deterministic :

$$\sigma_i^e = \phi_i(\sigma_i)$$

and the utility function will then be :

$$U_i(\omega_i+z_i, m_i, \sigma_i) = U_i^e[\omega_i+z_i, m_i, \phi_i(\sigma_i)]$$

If expectations are stochastic, and described by a cumulative probability distribution $\Psi_i(\sigma_i^e|\sigma_i)$ then the indirect utility will be :

$$U_i(x_i, m_i, \sigma_i) = \int U_i^e(x_i, m_i, \sigma_i^e)\, d\Psi_i(\sigma_i^e|\sigma_i)$$

With these indirect utility functions, we can proceed to the same equilibrium analysis as that performed in section 3 or 5.

Note also that for price makers what should be forecasted is expected demand curves, not price and quantity signals.

6.3. An example

Let us consider an agent living two periods, working ℓ and ℓ^e, consuming c and c^e in these two periods. We assume this agent has a utility function

$$V(c,\ell,c^e,\ell^e)$$

which we shall assume, for the simplicity of exposition, separable in its arguments.

The second period plan of this agent, given c and ℓ, will be given by the following maximization problem :

$$\text{Maximize } V(c,\ell,c^e,\ell^e) \quad \text{s.t.}$$

$$\begin{cases} p^e c^e \leqslant m + w^e \ell^e \\ c^e \leqslant \bar{c}^e \\ \ell^e \leqslant \bar{\ell}^e \end{cases}$$

Consider first the case where the agent is unconstrained in the second period. Then we obtain the second period Walrasian levels of consumption and employment :

$$c^{ew}(m,w^e,p^e) \qquad \ell^{ew}(m,w^e,p^e)$$

where these two functions are homogeneous of degree zero in m , w^e , p^e. The indirect utility is written :

$$V[c,\ell,c^{ew}(m,p^e,w^e) \ , \ \ell^{ew}(m,p^e,w^e)]$$

which depends only on expected price signals, p^e and w^e , just as in Walrasian theory.

Consider now the case where the agent expects to be labor constrained in the second period, and thus $\ell^e = \bar{\ell}^e$. Then clearly his second period consumption will be equal to :

$$c^e = \frac{m + w^e \bar{\ell}^e}{p^e}$$

and the indirect utility function is :

$$V\left[c\ ,\ \ell\ ,\ \frac{m + w^e \bar{\ell}^e}{p^e}\ ,\ \bar{\ell}^e\right]$$

We first remark that an expected quantity signal, $\bar{\ell}^e$, now enters the indirect utility function together with the expected price signals. Secondly, as the marginal utility of future consumption c^e decreases with its level, the marginal utility of money will be increasing as $\bar{\ell}^e$ decreases, i.e. as employment perspectives become dimmer, which highlights the role of money as precautionary savings against expected inability to sell labor.

Finally consider now the case where the agent does not expect to be constrained in his labor supply in the second period, but in his consumption purchases, so that $c^e = \bar{c}^e$. Then it is logical for him to supply just that quantity of labor necessary to pay for the consumption, i.e. :

$$\ell^e = \max\left\{0\ ,\ \frac{p^e \bar{c}^e - m}{w^e}\right\}$$

so that in that case the utility function is :

$$V\left[c \ , \ell \ , \bar{c}^e \ , \ \max \left\{ 0 \ , \ \frac{p^e \bar{c}^e - m}{w^e} \right\} \right]$$

Here again an expected quantity signal, $\bar{c}^e$, enters the utility function ; furthermore the marginal utility of money will decrease as $\bar{c}^e$ decreases, becoming even zero if $m \geqslant p^e \bar{c}^e$. We see this time that expected inability to purchase goods may induce some kind of "flight from money".

Which of the three utility functions will be relevant is easily seen from Figure 7. We may note that in all three cases the indirect utility function is homogeneous of degree zero in m , p^e , w^e , eventhough expected quantity signals have been added.

Figure 7

6.4. Notes and references

A treatment of expected quantity signals in a macroeconomic setting appears in Grossman (1972) which formalizes the investment accelator.

The integration of quantity expectations into the theory of non Walrasian equilibria was made in Benassy (1973), (1975a).

Subsequent applications in micro or macroeconomic frameworks are found in Grandmont-Laroque (1976), Hildenbrand-Hildenbrand (1978), Muellbauer-Portes (1978), Benassy (1982, 1986), Neary-Stiglitz (1983), Persson-Svensson (1983). The particular utility function example of this section is borrowed from Benassy (1976c), Muellbauer-Portes (1978).

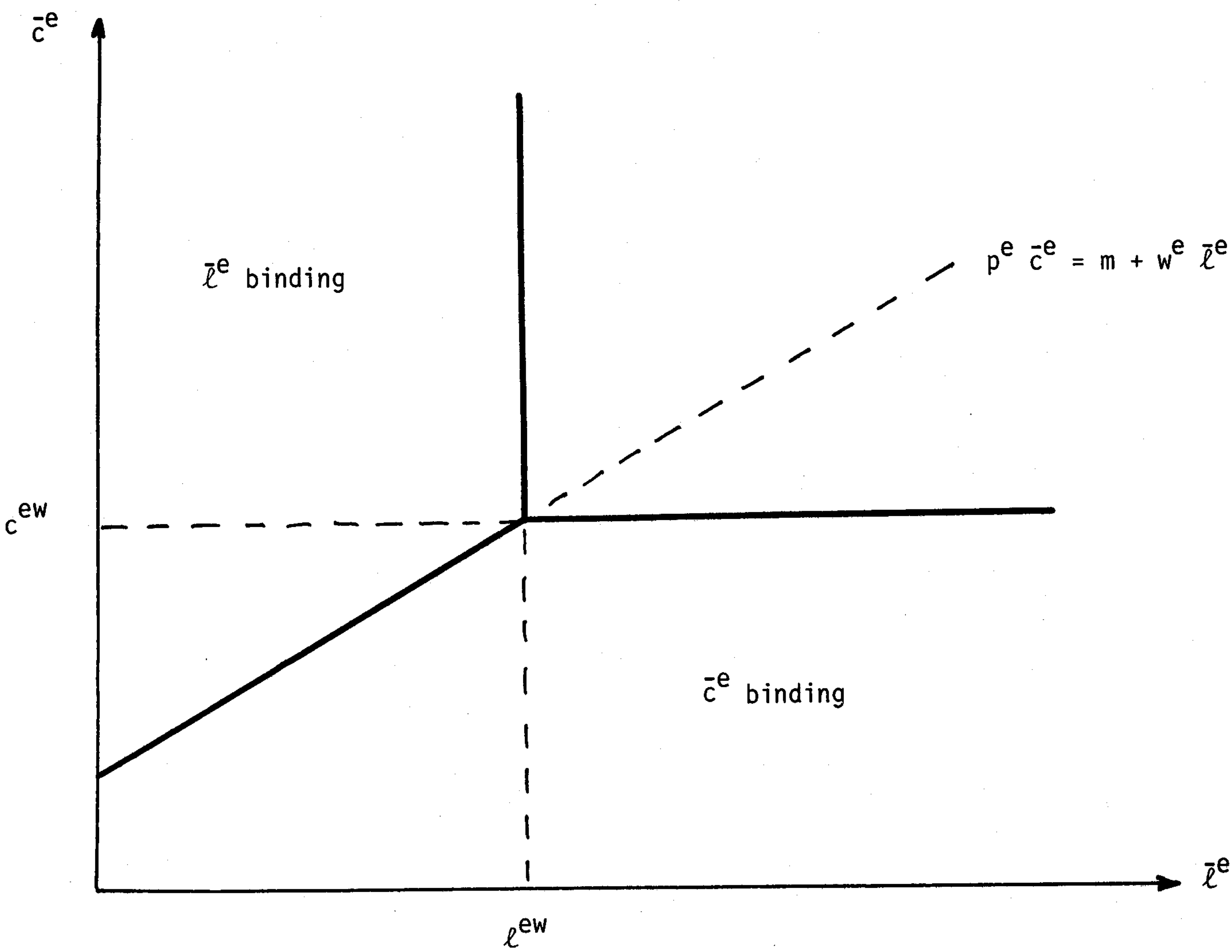
$\bar{c}^e$
$\bar{\ell}^e$ binding
$p^e \bar{c}^e = m + w^e \bar{\ell}^e$
c^{ew}
$\bar{c}^e$ binding
$\bar{\ell}^e$
ℓ^{ew}

Figure 7

We may note that non-Walrasian concepts are fully consistent with perfect foresight (Neary-Stiglitz 1983). In that case the structure of the model is formally identical to that of a one period model.

7. A MACROECONOMIC MODEL

We shall now describe a simple version of the wellknown fixprice macromodel of Barro-Grossman (1971) (1976). This will allow us to see that non Walrasian models may have a variety of "subregimes", and moreover that the same policy variable (which we shall take here to be government spending or a simple form of monetary policy) may have completely different effects on activity, employment and welfare depending on the particular regime the economy is in.

7.1. The model

We shall consider here a simple aggregate monetary economy with three goods (money, output and labor) and three agents : An aggregate firm, an aggregate household and the government. Accordingly there are two markets where output and labor are exchanged against money, respectively at the price p and wage w . These are assumed rigid in the period considered. Each market is assumed to function without frictions so that the transaction on each is equal to the minimum of supply and demand. The transactions on output and labor will be denoted by y and ℓ .

The firm has a production function $F(\ell)$ which we assume strictly concave. The firm does not hold any inventories, maximizes current profits $py - w\ell$, which it fully redistributes to the household.

The household has an initial endowment of labor ℓ_0 and of money $\overline{m}$. It consumes c , works ℓ and its budget constraint is :

$$pc + m = w\ell + \pi + \mu\bar{m}$$

The variable μ is one of the policy parameters of the government, who can multiply the initial quantity of money by μ . This particular, but popular, policy variable has been chosen because, as we shall see below, it is neutral in Walrasian equilibrium. The household's utility function will be taken to be of the general form :

$$U(c, \ell_o - \ell, m/p^e)$$

which, as we saw in the preceding section, can be derived from an intertemporal utility maximization problem. In order to make the exposition more compact we shall actually in what follows use the particular logarithmic specification :

$$U = \alpha_c \operatorname{Log} c + \alpha_\ell \operatorname{Log}(\ell_o - \ell) + \alpha_m \operatorname{Log}(m/p^e)$$

with $$\alpha_c + \alpha_\ell + \alpha_m = 1$$

Note that, because of this particular form, we shall not need in what follows to specify how p^e is determined, which will simplify the exposition.

Finally the government can choose, besides the parameter μ , its level of output purchases g , which it pays by monetary creation (taxation could be also added without difficulty, but is omitted for the sake of expositional simplicity). As a result of the different government policies, the final quantity of money in the economy will be $\mu\bar{m} + pg$.

7.2. Walrasian equilibrium

Before studying its non Walrasian regimes, we shall briefly describe the Walrasian equilibrium of this economy.

The Walrasian demands and supplies of the firm are obtained by maximizing profit $py - w\ell$ subject to the production function $y = F(\ell)$. This immediately yields :

$$y^s = F[F'^{-1}(w/p)]$$

$$\ell^d = F'^{-1}(w/p)$$

The household maximizes utility subject to his budget constraint

$$pc + m = \mu\bar{m} + \pi + w\ell$$

which yields :

$$c^d = \alpha_c\left(\frac{\mu\bar{m} + \pi + w\ell_o}{p}\right)$$

$$\ell^s = \ell_o - \alpha\left(\frac{\mu\bar{m} + \pi + w\ell_o}{w}\right)$$

The Walrasian equilibrium is obtained by solving the equations :

$$\begin{cases} \ell^* = \ell^s = \ell^d \\ y^* = y^s = c^d + g \end{cases}$$

with in addition $\pi = py^* - w\ell^*$.

It is easy to check that in this Walrasian equilibrium money is neutral, i.e. that, other things equal, the equilibrium price and wage are proportional to μ , whereas quantities traded are independent of μ . Changes in g however, are obviously non neutral. More precisely an increase in g leads to a reduction in w/p , an increase in employment and production, but a reduction in private consumption. There is thus a crowding out effect of g , though of less than hundred per cent, due to the fact that the labor supply is elastic.

7.3. The regimes

We shall now assume that p and w are given and derive the level of transactions for the various subregimes. Anticipating what follows we shall see that there are in this model three possible regimes :

- Keynesian unemployment with excess supply for both output and labor
- Classical unemployment with excess supply of labor and excess demand for goods
- Repressed inflation with excess demand for labor and output

7.4. Keynesian unemployment

This regime is characterized by excess supply on both markets. In particular the household faces a (binding) constraint $\bar{\ell}^s$ on the labor market. Its effective demand for consumption $\approx\atop c$ is thus the solution in c of the program :

$$\text{Maximize } \alpha_c \text{ Log } c + \alpha_\ell \text{ Log}(\ell_0 - \ell) + \alpha_m \text{ Log}(m/p^e) \qquad \text{s.t.}$$

$$\begin{cases} pc + m = \mu\bar{m} + \pi + w\ell \\ \ell \leqslant \bar{\ell}^s \end{cases}$$

where the last constraint is binding. The solution is :

$$\overset{\approx}{c} = \frac{\alpha_c}{\alpha_c + \alpha_m} \left[\frac{\mu\bar{m} + \pi + w\bar{\ell}^s}{p} \right]$$

a traditional Keynesian consumption function, with $\alpha_c/(\alpha_c + \alpha_m)$ as the marginal propensity to consume out of income. Because $\bar{\ell}^s$ is binding, it is equal to the household's transaction on labor ℓ^*, so that, given the definition of π, $\tilde{\tilde{c}}$ can be written in the even more familiar Keynesian form :

$$\tilde{\tilde{c}} = \frac{\alpha_c}{\alpha_c + \alpha_m}\left[\frac{\mu\bar{m}}{p} + y\right]$$

Because of the excess supply of goods, transactions on the goods market are demand determined, so that y is equal to total output demand, i.e. $\tilde{\tilde{c}}$ plus government demand for output $\tilde{\tilde{g}}$:

$$y = \tilde{\tilde{c}} + \tilde{\tilde{g}} = \frac{\alpha_c}{\alpha_c + \alpha_m}\left[\frac{\mu\bar{m}}{p} + y\right] + \tilde{\tilde{g}}$$

yielding :

$$y^* = \frac{\alpha_c}{\alpha_m} \cdot \frac{\mu\bar{m}}{p} + \frac{(\alpha_c + \alpha_m)}{\alpha_m}\tilde{\tilde{g}} = y_k \tag{7}$$

We recognize here a traditional Keynesian multiplier, with $(\alpha_c + \alpha_m)/\alpha_m$ being the multiplier.

Labor transactions ℓ^* are equal to the firm's demand for labor, which, since the firm is constrained in its output sales, is equal to the quantity just necessary to produce y_k, i.e. :

$$\ell^* = F^{-1}(y_k) = \ell_k \tag{8}$$

We may also compute private consumption $c^* = y^* - \tilde{\tilde{g}}$:

$$c^{*} = \frac{\alpha_c}{\alpha_m}\left(\frac{\mu\bar{m}}{p} + \tilde{\tilde{g}}\right)$$

We can now make a few observations : The first is that money is not any more neutral, but that an increase in μ will increase production, employment and private consumption. These will also be increased by increases in government spending. The traditional results of Keynesian "multiplier" analysis are thus valid in this regime.

Maybe the most striking fact is that an increase in government spending will increase not only production and employment, but <u>also</u> private consumption. There is thus no crowding out, quite in the contrary, since eventhough the government collects real output from the private sector, more of it is available for private consumption, a quite remarkable by-product of the inefficiency of the Keynesian multiplier state.

7.5. <u>Classical unemployment</u>

In this case there is excess supply of labor and excess demand for goods. The firm, being on the "short" side in both markets, is able to carry out its Walrasian plan, so that :

$$\ell^{*} = F'^{-1}(w/p) = \ell_c \qquad (9)$$

$$y^{*} = F[F'^{-1}(w/p)] = y_c \qquad (10)$$

We immediately see that "Keynesian" policies have no impact on employment and production. In fact it is easy to check that their main effect is to aggravate excess demand on the goods market. If we further assume that government has priority in the goods market, then the actual consumption is equal to :

$$c^* = y^* - \tilde{\tilde{g}} = y_c - \tilde{\tilde{g}}$$

There is a hundred per cent crowding out effect of government expenditures, which thus appears as an undesirable policy instrument. Note that this time crowding out does not occur through prices, but by a direct rationing mechanism.

The only thing which affects the level of employment and production is the level of real wages, thus validating the "classical" presumption that there is unemployment because "real wages are too high".

7.6. Repressed inflation

In this regime there is excess demand in both markets. In particular the consumer is faced to a binding constraint $\bar{c}$ (actually equal to his purchases c^*) in the goods market. His supply of labor $\tilde{\tilde{\ell}}^s$ is given by the solution in ℓ to the following program :

$$\text{Maximize } \alpha_c \text{ Log } c + \alpha_\ell \text{ Log}(\ell_o - \ell) + \alpha_m \text{ Log}(m/p^e) \qquad \text{s.t.}$$

$$\begin{cases} pc + m = \mu\bar{m} + w\ell + \pi \\ c \leqslant \bar{c} \end{cases}$$

where the last constraint is binding. The solution to this program is :

$$\tilde{\tilde{\ell}}^s = \ell_o - \frac{\alpha_\ell}{\alpha_\ell + \alpha_m} \left[\frac{\mu\bar{m} + \pi + w\ell_o - p\bar{c}}{w} \right]$$

We see on this formula that rationing in the goods market leads the household to reduce his supply of labor. Because there is excess demand of labor, the actual transaction ℓ^* is equal to this supply $\tilde{\tilde{\ell}}^s$. Now let us assume again that government has priority on the goods market. Then

$$\bar{c} = c^* = y^* - \tilde{\tilde{g}}$$

and using again the definition of profits π , the above equation is rewritten :

$$\ell^* = \ell_o - \frac{\alpha_\ell}{\alpha_\ell + \alpha_m} \left[\frac{\mu\bar{m} + w\ell_o - w\ell^* + p\tilde{\tilde{g}}}{w} \right]$$

yielding :

$$\ell^* = \ell_o - \frac{\alpha_\ell}{\alpha_m} \left(\frac{\mu\bar{m} + p\tilde{\tilde{g}}}{w} \right) = \ell_r \qquad (11)$$

Output transactions y^* are equal to the supply, which, because the firm is constrained in its labor purchases, is equal to the maximum quantity producible with ℓ_r , i.e. :

$$y^* = F(\ell_r) = y_r \qquad (12)$$

We now may note that the economic policy variables have an effect completely opposite to that in the Keynesian regime. In particular increases in the quantity of money or in government spending diminish employment and production.

Furthermore private consumption is, assuming again government priority :

$$c^* = y^* - \tilde{\tilde{g}} = y_r - \tilde{\tilde{g}}$$

Since an increase in $\tilde{\tilde{g}}$ reduces y_r , there is a more than hundred per cent crowding out effect. An increase in $\tilde{\tilde{g}}$ reduces c^* by an even greater quantity ! The mechanism at work in this regime is some kind of "supply multiplier" (Barro-Grossman, 1974), by which a reduction in the amount of goods available for consumption reduces the supply of labor, which itself reduces further the amount of goods produced ...

7.7. The general picture

As we have seen above, the three regimes of our model display strikingly different properties concerning the determination of employment and activity and the response to policy variables. It is therefore important to know for which values of the "exogenous" parameters μ , $\bar{m}$, p , w , $\tilde{\tilde{g}}$, each of the three regimes obtains. As the reader may easily check, both the employment level and the nature of the regime are determined by finding out the lowest of the three possible employment levels computed above (Cf. Equations 7 to 12), i.e. :

$$\ell^* = \min(\ell_k, \ell_c, \ell_r)$$

$$y^* = \min(y_k, y_c, y_r)$$

It is convenient to depict the three regimes in the space $(\mu\bar{m}/p \ , w/p)$, $\tilde{\tilde{g}}$ being parametric. Figure 8 has been drawn for $\tilde{\tilde{g}}$ equal to zero. As $\tilde{\tilde{g}}$ increases point W will move southwest. The triangles are iso-employment or iso-output lines. The highest level of employment and production occurs at W , the Walrasian equilibrium point.

Figure 8

On this diagram we see particularly well that the spillover effects from the goods market to the labor market do matter a lot : Even if the real wage is "right" (i.e. equal to its Walrasian equilibrium level, which corresponds to the horizontal linegoing through W), we may have inefficiently low values of employment due to insufficient (or excessive) demand in the output market.

7.8. Efficiency properties

We want now to highlight in this particular macroeconomic example the inefficiencies which we described in the more general model of section 4. We noted particularly that in cases of generalized excess demand or supply, which lead to the existence of multiplier effects, there were most likely potential Pareto improving trades at the given price system. We shall now verify this in the two regimes concerned, those of Keynesian unemployment and repressed inflation. Since all mutual gains from trade have been exhausted in trades of money versus output or labor, we shall investigate direct trades of labor against output.

Let us start with the interior of the Keynesian regime. Because $\ell_k < \bar{\ell}_c$, we have

$$F'(\ell) < \frac{w}{p}$$

so that the firm would increase its real profit by purchasing directly labor for output. Consider now the household. Since it is unconstrained in the goods market and constrained on the labor market, we have

$$\frac{1}{p}\,\frac{\partial U}{\partial c} = \frac{\partial U}{\partial m} > \frac{1}{w}\,\frac{\partial U}{\partial(\ell_0 - \ell)}$$

so that the household would also gain in selling directly labor for output.

Symmetrically in the interior of the repressed inflation regime we have:

$$F'(\ell) < \frac{w}{p}$$

$$\frac{1}{p}\,\frac{\partial U}{\partial c} > \frac{\partial U}{\partial m} = \frac{\partial U}{\partial(\ell_0 - \ell)}$$

so that again the firm and household could profitably exchange directly output for labor.

We should at this point make a caveat : The above inefficiency results might suggest that the monetary nature of exchange is the problem, and that allowing barter trades to be carried out between households and firms would allow to alleviate the market failure. This would be missing the fundamental issue, which is that both the market failure and monetary exchange are caused by the lack of double coincidence of wants. Such an issue cannot be discussed adequately in this very aggregated model, but will be tackled in the more disaggregated model of the next section.

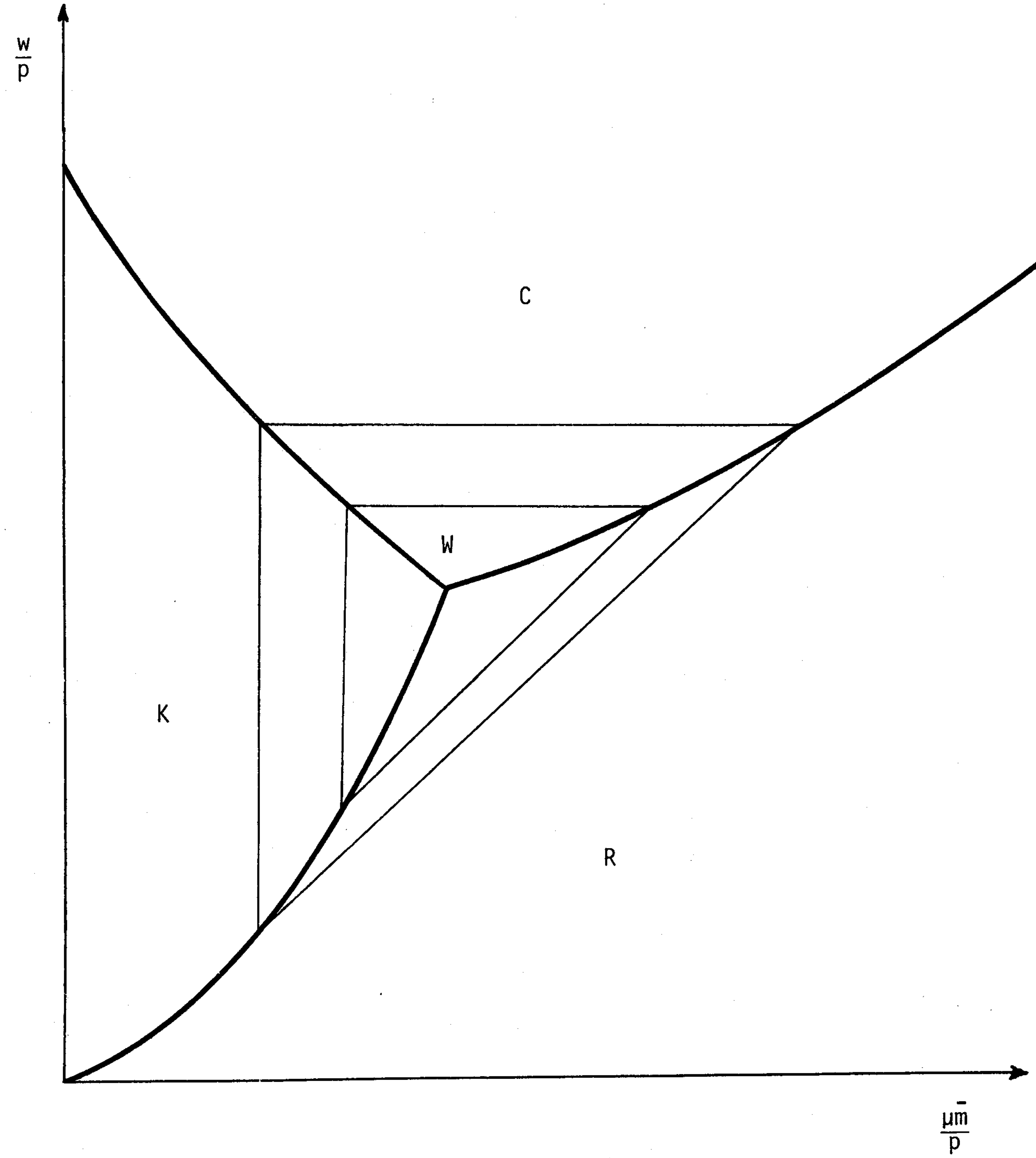

Figure 8

7.9. Notes and references

The model of this section was originally developped by Barro-Grossman (1971), (1974), (1976). Elements of it can be found in Solow-Stiglitz (1968), Younès (1970). The particular adaptation in this section is taken from Benassy (1976a), (1977a). Subsequent adaptations can be found in Malinvaud (1977), Hildenbrand-Hildenbrand (1978), Muellbauer-Portes (1978). The Classical-Keynesian unemployment terminology appears in Malinvaud (1977).

Various extensions of this basic macroeconomic model have been considered : Introduction of a bonds market (Barro-Grossman 1976, Hool 1980, Benassy 1983), consideration of foreign trade (Dixit 1978, Neary 1980, Cuddington-Johansson-Lofgren 1984), capital accumulation (Ito 1980, Picard 1983), fluctuations (Benassy 1984), etc ... A number of macroeconomic applications with varying assumptions on price rigidities are found in Benassy (1986). These concepts have also been applied to socialist economies (Cf. notably Portes 1981, or see also Kornai 1980, 1982, for an alternative formulation). Finally a number of applied econometric models have been developed. See for example the surveys in Quandt (1982), Sneessens (1981).

8. A MODEL WITH IMPERFECT COMPETITION AND UNEMPLOYMENT

We shall now use the developments of the preceding sections to construct a simple "micro-macro" model of imperfect competition adapted from Benassy (1987), displaying endogenous price making resulting from explicit utility or profit maximization by agents. Within this framework we shall study such problems as the efficiency properties of the equilibrium, the existence of unemployment or the effectiveness of simple governmental monetary policies. We shall notably try to emphasize the similarities (or dissimilarities) of this model with the Walrasian one, which is the main general equilibrium alternative with flexible prices.

8.1. The model

We shall consider a monetary economy with three types of goods : Money, different types of labor indexed by $i = 1,\ldots,n$, and consumption goods indexed by $j = 1,\ldots,m$. There are three types of agents : Households indexed by $i = 1,\ldots,n$, firms indexed by $j = 1,\ldots,m$, and government. Consumer i is the only one to be endowed by labor of type i , firm j is the only one to produce good j . We shall call w_i the money wage for type i labor, p_j the price of good j , w and p the corresponding vectors :

$$p = \langle p_j \mid j = 1,\ldots,m \rangle \qquad w = \langle w_i \mid i = 1,\ldots,n \rangle$$

Firm j produces a quantity of output y_j according to a production function :

$$y_j = F_j(\ell_j) \qquad \ell_j = \langle \ell_{ij} \mid i = 1,\ldots,n \rangle$$

where ℓ_{ij} is the quantity of labor i used by firm j (and thus purchased from household i). We shall assume F_j strictly concave in its arguments. Firm j maximizes its profits π_j :

$$\pi_j = p_j y_j - w \ell_j = p_j y_j - \sum_{i=1}^{n} w_i \ell_{ij}$$

Household i has initial endowments ℓ_{i0} of type i labor, and $\bar{m}_i$ of money. It consumes a vector

$$c_i = \langle c_{ij} \mid j = 1,\dots,m \rangle$$

Its budget constraint is, in the absence of government intervention :

$$pc_i + m_i = w_i \ell_i + \bar{m}_i + \sum_{j=1} \theta_{ij} \pi_j$$

where m_i is the final quantity of money and θ_{ij} the share of firm j owned by household i . The quantity of labor ℓ_i is equal to :

$$\ell_i = \sum_{j=1}^{m} \ell_{ij} \leq \ell_{i0}$$

We shall actually assume, as in the preceding section, that the government can manipulate by policy action the initial money holdings, increasing them proportionately by a factor μ , so the the initial holdings of household i become $\mu\bar{m}_i$ and the budget constraint :

$$pc_i + m_i = w_i \ell_i + \mu\bar{m}_i + \sum_{j=1}^{m} \theta_{ij} \pi_j$$

This particular policy has again been chosen because it is known to be "neutral" in Walrasian equilibrium [(7)], which will allow an easy comparison.

Household i maximizes a utility function of the form :

$$U_i(c_i , \ell_i , m_i , \mu)$$

which is assumed to be strictly quasi-concave in c_i , $-\ell_i$ and m_i and separable in the three first arguments. The presence of the first two arguments is obvious. The arguments m_i and μ represent the indirect utility of money, and we shall assume that U_i is homogeneous of degree zero in m_i and μ . As we saw in the preceding section, the indirect utility of money should be homogeneous of degree zero in money and expected prices. The implicit idea behind our assumption is thus that, other things equal, future prices are expected to move proportionately to μ . As we shall see in subsection 8.6 below, this proportionality of prices to μ is indeed consistent with the working of the model.

This model has a particular market structure where each good (labor or consumer good) is sold by a single agent to a multitude of other agents : Labor of type i is sold by household i to all firms $j = 1,\dots,m$. Output j is sold by firm j to all households $i = 1,\dots,n$. We shall assume, as is traditional in such models, that the price is decided upon by the single seller, who appears thus formally as a "monopolist". So firm j sets price p_j , household i sets wage w_i , taking all other prices and wages as given. We shall assume that they do so using objective demand curves, as described in section 5 above. The equilibrium is thus a Nash equilibrium in prices and wages, conditionally on these objective demand curves.

8.2. Objective demand curves

Each seller sells only one good, and, as we shall see below, sets its price high enough so as to be willing to satisfy all demand for that good. In equilibrium each agent will thus be constrained only on his sales, and we shall have somehow a situation of "general excess supply". We shall now compute the objective demand curves in this zone of general excess supply. These objective demand curves will be functions of the vectors p and w , and of the policy parameter μ . So for given p , w and μ we must find out which demands for goods and labor types will arise once all feedback effects have been taken into account. As indicated in section 5, this boils down to finding total demand for goods i and j at a fixprice K-equilibrium corresponding to p , w and μ .

For given (p,w,μ) firm j is constrained on its sales of output j . It thus solves the following program in ℓ_j :

$$\text{Maximize } p_j y_j - w \ell_j \quad \text{s.t.}$$

$$y_j \leqslant F_j(\ell_j)$$

where y_j is a binding constraint exogenous to the firm. The solution is a set of labor demands $L_{ij}(y_j,w)$, and a cost function $\chi_j(y_j,w)$.

Similarly consider a household i which solves the following program :

$$\text{Maximize } U_i(c_i , \ell_i , m_i , \mu) \quad \text{s.t.}$$

$$pc_i + m_i = \mu\bar{m}_i + w_i\ell_i + \sum_{j=1}^{m} \theta_{ij} \pi_j$$

where p , w , μ , ℓ_i and the profits π_j are given. The solution is a set of consumption demands :

$$C_{ij}(\mu\bar{m}_i + w_i \ell_i + \sum_{j=1}^{m} \theta_{ij} \pi_j , p , \mu)$$

Consider now the following mapping :

$$y_j \rightarrow \sum_{i=1}^{n} C_{ij}(\mu\bar{m}_i + w_i \ell_i + \sum_{j=1}^{m} \theta_{ij} \pi_j , p , \mu)$$

$$\ell_i \rightarrow \sum_{j=1}^{m} L_{ij}(y_j , w)$$

$$\pi_j \rightarrow p_j y_j - \chi_j(y_j,w)$$

Assuming a unique fixed point, this yields functions $Y_j(p,w,\mu)$, $L_i(p,w,\mu)$ and $\Pi_j(p,w,\mu)$ which represent respectively the objective demand for good j , for labor i , and the associated profits of firm j .

Secondly let us note that the functions L_{ij} are homogeneous of degree zero in w , the functions C_{ij} are homogeneous of degree zero in μ , w_i , p , and π_j , and profits themselves are homogeneous of degree one in p_j and w . From that we deduce Y_j and L_i are homogeneous of degree zero, and Π_j homogeneous of degree one, in the arguments p , w and μ .

8.3. Equilibrium : Definition and characterization

Following section 5, we shall describe the equilibrium as a Nash-equilibrium in prices and wages, conditional on the objective demand curves. The quantities are those corresponding to a fixprice equilibrium associated with the prices and wages. We shall now derive the optimal price and wage responses of the agents.

Consider first firm j ; it will solve the following profit maximization program A_j in p_j , y_j and ℓ_j :

$$\text{Maximize } p_j y_j - w \ell_j \quad \text{s.t.}$$

$$\begin{cases} y_j \leqslant F_j(\ell_j) \\ y_j \leqslant Y_j(p,w,\mu) \end{cases} \qquad (A_j)$$

We assume this program has a unique solution, which thus yields optimal price p_j as a function of the other prices and wages :

$$p_j = \psi_j(p_{-j},w,\mu)$$

where $p_{-j} = \langle p_k \mid k \neq j \rangle$.

In order to characterize the equilibrium quantities simply, we shall also need to characterize the optimal production plan of the firm (y_j,ℓ_j) as a function of the same variables, so that we shall write this optimal plan functionally as :

$$(y_j,\ell_j) = \varphi_j(p_{-j},w,\mu)$$

Consider now household i . It chooses the wage w_i , labor sales ℓ_i and consumption vector c_i so as to maximize utility according to the following program A_i in w_i , ℓ_i and c_i :

$$\text{Maximize } U_i(c_i,\ell_i,m_i,\mu) \quad \text{s.t.}$$

$$\begin{cases} pc_i + m_i = \mu\bar{m}_i + w_i\ell_i + \sum_{j=1}^{m} \theta_{ij} \Pi_j(p,w,\mu) \\ \ell_i \leqslant L_i(p,w,\mu) \\ \ell_i \leqslant \ell_{i0} \end{cases} \qquad (A_i)$$

which yields the functions :

$$w_i = \psi_i(w_{-i},p,\mu)$$

$$(\ell_i,c_i) = \varphi_i(w_{-i},p,\mu)$$

where $w_{-i} = \langle w_k \mid k \neq i \rangle$.

We shall assume in all that follows that the disutility of labor becomes high enough near ℓ_{i0}, so that the last constraint in the above program A_i is never binding, and we shall accordingly ignore it henceforth.

We can now define our equilibrium with imperfect competition as a Nash equilibrium as follows :

(a) $\quad w^*_i = \psi^*_i(w^*_{-i},p^*,\mu) \qquad \forall\ i$

(b) $\quad p^*_j = \psi_j(p^*_{-j},w^*,\mu) \qquad \forall\ j$

(c) $\quad (\ell^*_i,c^*_i) = \varphi_i(w^*_{-i},p^*,\mu) \qquad \forall\ i$

(d) $\quad (y^*_j,\ell^*_j) = \varphi_j(p^*_{-j},w^*,\mu) \qquad \forall\ j$

The first two sets of equalities express the Nash property in prices and wages, and determine p^* and w^* . The last two sets of equations give the optimal production, work and consumption plans of the agents, which correspond to their transactions on all markets. Note that because of the definition of the objective demand curves, which is based on a fixprice equilibrium notion, the consistency of transactions is automatically ensured, so that we do not need to add the traditional equations stating the equality between purchases and sales on every market.

Of course the equilibrium will change as μ changes. In what follows we shall assume that to a given government policy is associated a unique equilibrium. In order to study its various properties, we shall now characterize it by deriving a number of relevant partial derivatives.

Consider first firm j , and recall the program yielding its optimal actions :

$$\text{Maximize } p_j y_j - w\ell_j \quad \text{s.t.}$$

$$\left\{ \begin{array}{l} y_j = F_j(\ell_j) \\ \\ y_j \leqslant Y_j(p,w,\mu) \end{array} \right. \qquad (A_j)$$

Let us assume an interior solution, so that in particular all components of ℓ_j are strictly positive. Then the Kuhn-Tucker conditions associated to this program yield immediately :

$$\frac{\partial F_j}{\partial \ell_{ij}} = \frac{w_i}{p_j} \frac{1}{(1-1/\varepsilon_j)} \qquad (13)$$

where $\varepsilon_j = -(p_j/y_j)\, \partial Y_j/\partial p_j$ is the absolute value of the own price elasticity of objective demand. At an equilibrium ε_j is greater than 1.

These conditions can also be rewritten, using the cost function $\chi_j(y_j,w)$ of firm j :

$$\frac{\partial \chi_j}{\partial y_j} = p_j \left(1 - \frac{1}{\varepsilon_j}\right) \qquad (14)$$

which is the traditional "marginal cost equals marginal revenue" equation. Let us turn now to the optimal program of household i :

$$\text{Maximize } U_i(c_i, \ell_i, m_i, \mu) \qquad \text{s.t.}$$

$$\begin{cases} pc_i + m_i = \mu\overline{m}_i + w_i\ell_i + \sum_{j=1}^{m} \theta_{ij}\,\Pi_j(p,w,\mu) \\ \ell_i \leqslant L_i(p,w,\mu) \end{cases}$$

We shall assume an interior solution so that in particular all consumptions are positive. Call λ_i the "marginal utility" of wealth, i.e. the Kuhn-Tucker multiplier of the budget constraint. We obtain the following conditions[(8)] :

$$\frac{\partial U_i}{\partial m_i} = \lambda_i \qquad \frac{\partial U_i}{\partial c_{ij}} = \lambda_i\, p_j \tag{15}$$

$$\frac{\partial U_i}{\partial \ell_i} = -\lambda_i\, w_i\left(1 - \frac{1}{\varepsilon_i}\right) \tag{16}$$

where $\varepsilon_i = -(w_i/\ell_i)\,\partial L_i/\partial w_i$. Again, at equilibrium $\varepsilon_i > 1$.

With the help of these differential characterizations, we can now describe some salient properties of our equilibrium.

8.4. Underemployment, underproduction and inefficiency

We shall now show that, eventhough prices are fully flexible and rationally decided upon by agents, the equilibrium allocation has properties which strongly differentiate it from those of a Walrasian equilibrium.

First we see that at equilibrium there is both underemployment and underproduction. Indeed equation (14) shows that at the going price and wage firm j would be happy to produce and sell more, if the demand was forthcoming, thus displaying underproduction. Symmetrically equation (16) shows that household i would like to sell more of its labor if the demand was present, thus displaying underemployment.

We shall further show that employment and production throughout the economy are inefficiently low in the following strong sense : it is possible to find increases in production and employment which would increase all firms' profits and all households' utilities at the equilibrium prices and wages.

Consider indeed at the given price and wage system p^* and w^* some small arbitrary increases $d\ell_{ij} > 0$. These yield extra amounts of employment and production :

$$d\ell_i = \sum_{j=1}^{m} d\ell_{ij} > 0$$

$$dy_j = \sum_{i=1}^{n} \frac{\partial F_j}{\partial \ell_{ij}} d\ell_{ij} > 0$$

Consider first the profit variation for firm j :

$$d\pi_j = p_j \, dy_j - \sum_{i=1}^{n} w_i \, d\ell_{ij}$$

In view of equations (13), this is easily found equal to :

$$d\pi_j = \frac{p_j \, dy_j}{\varepsilon_j} > 0 \tag{17}$$

Now we shall assume that the extra productions in the economy are redistributed to households in such a way that the value of the extra consumptions for each household sum up to the value of his extra labor and profit incomes, which is written for household i :

$$\sum_{j=1}^{m} p_j \, dc_{ij} = w_i \, d\ell_i + \sum_{j=1}^{m} \theta_{ij} \, d\pi_j \tag{18}$$

The increment in utility is, since m_i and μ do not change :

$$dU_i = \sum_{j=1}^{m} \frac{\partial U_i}{\partial c_{ij}} dc_{ij} + \frac{\partial U_i}{\partial \ell_i} d\ell_i$$

which, using equations (15), (16), (17) and (18) becomes :

$$dU_i = \lambda_i \left[\frac{w_i \, d\ell_i}{\varepsilon_i} + \sum_{j=1}^{m} \frac{\theta_{ij} \, p_j \, dy_j}{\varepsilon_j} \right] > 0 \tag{19}$$

Equations (17) and (19) clearly show that the incremental employment and production will increase all agents' utilities or profits. We may note that this inefficiency result is quite stronger than Pareto inefficiency since we have constrained the incremental trades to be consistent with the equilibrium price wage system, whereas such a constraint is not required to prove Pareto inefficiency. We should also note that these inefficiencies are quite similar to those observed in "Keynesian type" general excess supply states (see for example Benassy 1977a, 1982).

We shall now investigate in the following subsections whether these inefficiencies can be alleviated, either in a decentralized manner or by governmental policy.

8.5. Decentralized trading and the problem of double coincidence of wants

We shall first investigate whether the inefficiencies observed in our equilibrium can be overcome in a decentralized manner, and we shall see that the problem of double coincidence of wants is crucial in that respect. Though we did not say anything explicit on that topic, we implicitly assumed some kind of double coincidence of wants by assuming interior solutions in the maximization programs A_i and A_j above. Indeed this was equivalent to assuming that at equilibrium :

$$\ell^*_{ij} > 0 \qquad c^*_{ij} > 0 \qquad \forall\ i\ ,\ j$$

which means that each firm employs every household and each household consumes some of every good. It is easy to see that under such circumstances agents themselves could undertake the Pareto improving trades mentioned in the preceding section by making simple propositions to their usual trading partners.

Consider for example firm j . It can decide to employ extra quantities of labor $d\ell_{ij} > 0$, $i = 1,\dots,n$, producing an extra output $dy_j > 0$. Assume now that firm j distributes (in kind) to each household i the supplementary quantity of good j :

$$\frac{w_i \, d\ell_{ij}}{p_j} + \frac{\theta_{ij} \, d\pi_j}{p_j} = dc_{ij}$$

where $d\pi_j$ is equal to :

$$d\pi_j = p_j \, dy_j - \sum_{i=1}^{n} w_i \, d\ell_{ij}$$

It is easy to check that these quantities are summing to dy_j , and that every household is better off both as a worker-consumer (because of equations 15-16) and as a shareholder (because of equations 13-14).

We can further note that such Pareto improving trades can be made by only pairs of trading partners : Consider indeed any pair formed of a household i and a firm j , and the following trade proposition : The household i works more for firm j by $d\ell_{ij}$, thus allowing an extra production of (by formula 13):

$$dy_j = \frac{\partial F_j}{\partial \ell_{ij}} \, d\ell_{ij} = \frac{\varepsilon_j}{\varepsilon_j - 1} \, \frac{w_i}{p_j} \, d\ell_{ij}$$

The firm pays in kind (i.e. with output j) the equivalent of his wage to household i , i.e. the quantity $dc_{ij} = w_i \, d\ell_{ij}/p_j$ of output j , so that the increment of utility to household i is (not counting possible profit receipts as a shareholder) :

$$dU_i = \frac{\partial U_i}{\partial c_{ij}} \, dc_{ij} + \frac{\partial U_i}{\partial \ell_i} \, d\ell_{ij}$$

$$= \frac{\lambda_i \, w_i \, d\ell_{ij}}{\varepsilon_i} > 0$$

The firm thus retains as a surplus a positive quantity, equal to :

$$dy_j - dc_{ij} = \frac{1}{\varepsilon_j - 1} \frac{w_i}{p_j} d\ell_{ij}$$

which it can distribute to its shareholders, making all of them better off.

We see that under our implicit assumptions the equilibrium could be fairly easily upset by simple decentralized trade propositions, as the workers and the firms' shareholders would unanimously want to carry the above production and exchange plans.

We shall now see that such will not be the case if we have <u>absence of double coincidence of wants</u>. This last condition means that no consumer purchases the goods from the firms he works for, no firm employs households to which it sells goods, i.e. :

$$\ell^*_{ij} . c^*_{ij} = 0 \qquad \forall\ i,j \tag{20}$$

This could happen, for example, if we had the following condition :

$$\left(\frac{\partial F_j}{\partial \ell_{ij}}\right) \left(\frac{\partial U_i}{\partial c_{ij}}\right) = 0 \qquad \forall\ i,j$$

i.e. either household i has no utility for output j , or it is not productive for firm j (of course less stringent conditions could also lead to condition 20).

In such a case of absence of double coincidence of wants, the solutions to programs A_i and A_j are not interior, and the marginal conditions must be rewritten. In particular conditions (13) and (15) are rewritten as :

$$\frac{\partial F_j}{\partial \ell_{ij}} \leq \frac{w_i}{p_j} \frac{1}{1-1/\varepsilon_j} \quad \text{with equality if } \ell^*_{ij} > 0 \tag{21}$$

$$\frac{\partial U_i}{\partial c_{ij}} \leq \lambda_i p_j \quad \text{with equality if } c^*_{ij} > 0 \tag{22}$$

Then generally an exchange between a household i and a firm j such as we described above will not be Pareto improving, either because the household is not productive enough for firm j (condition 21 with strict inequality) or because the household does not derive enough utility from good j (condition 22 with strict inequality).

We thus see that in case of absence of double coincidence of wants propositions of trade involving usual trading partners will not usually lead to Pareto improvements. The Pareto improving trades of section 8.4 still exist (we shall exhibit an example in section 8.7 below), but they are typically quite complex, since workers must be allocated only to firms where they are productive, and goods only to consumers who actually enjoy them. These trades will normally involve groups of agents, some of which are not usual trading partners. Finding such Pareto improving chains of trades may thus involve some very complex calculations, quite beyond the computing capacities of private agents. On the other hand this task cannot be delegated to a central authority, as it is difficult to envision in a free market economy that such an authority could dictate private agents incremental production activities and trades, even if these would turn up to be Pareto improving, and anyway there is no reason to believe that this central authority would have enough information on the private sector to find these Pareto-improving trades. So such improvements have rather been sought via more impersonal policy actions, in particular "demand" policies. In the next section we shall investigate one such policy, monetary policy.

8.6. Neutrality of monetary policy

We have seen in the preceding subsections that our imperfect competition equilibrium displayed some inefficiency properties very akin to traditional Keynesian excess supply states, but that, because of the absence of double coincidence of wants, these inefficiencies could not be expected to be remedied by the actions of private agents. We shall thus now investigate a traditional "Keynesian" policy to cure such inefficiencies, a monetary policy, and we shall now see that, in spite of its "Keynesian" characteristics, the system described above reacts to monetary policy in a more "Walrasian" than "Keynesian" manner. Namely, monetary policies taking the form of proportional increases in initial money holdings (i.e. $\mu > 1$) are ineffective, or "neutral", just as in Walrasian models. As a response to such a policy, production, employment and utilities do not change. Prices, wages and profits are multiplied by μ .

The proof of that result is actually quite trivial in view of the homogeneity properties of our system. Let us look indeed at the programs A_j and A_i yielding the optimal actions of agents ; from program A_j it is clear that :

$$\psi_j(\mu p_{-j},\mu w,\mu) = \mu\, \psi_j(p_{-j},w,1)$$

$$\varphi_j(\mu p_{-j},\mu w,\mu) = \varphi_j(p_{-j},w,1)$$

And similarly program A_i yields :

$$\psi_i(\mu w_{-i},\mu p,\mu) = \mu\ \psi_i(w_{-i},p,1)$$

$$\varphi_i(\mu w_{-i},\mu p,\mu) = \varphi_i(w_{-i},p,1)$$

In view of these homogeneity properties, y_j^* , ℓ_j^*, ℓ_i^*, c_i^* are homogeneous of degree 0 in μ , whereas p^* and w^* are homogeneous of degree one in μ .

8.7. The possibility of effective policies

We just saw that our system reacted to monetary stimulus like a Walrasian system, i.e. by a proportional increase in prices and wages with no impact on quantities, inspite of the inefficiencies. One should however not jump to the conclusion that our system will react similarly to a Walrasian one for any choice of policy, even including monetary policy, and we shall now prove this by considering a different policy, namely a price-wage freeze coupled with a small monetary expansion $d\mu > 0$. To carry out the exercise we shall make the further assumption of "stabilized" money holdings", i.e. that at the equilibrium considered $m_i^* = \overline{m}_i$. Such will be the case if the economy is in a state of long-run equilibrium to start with. This also contains as a particular case the situation, most often considered in the literature, where the model is symmetrical.

Consider thus, at fixed prices and wages, an increase $d\mu$ in money holdings. The basic intuition is that multiplier effects will provoke increases in the levels of employment and production, and this will increase profits and utilities.

To make this intuition rigorous, let us first note that in view of equations (14) and (16), the firms will be willing to make small expansions of production and the households will willingly increase their labor sales if the demand is forthcoming. Let us now compute the profit and utility increments associated with new trades : using again equations (14) and (15) we find :

$$d\pi_j = \frac{p_j\, dy_j}{\varepsilon_j} \tag{23}$$

Turning now to households we find :

$$dU_i = \sum_{j=1}^{m} \frac{\partial U_i}{\partial c_{ij}} dc_{ij} + \frac{\partial U_i}{\partial \ell_i} d\ell_i + \frac{\partial U_i}{\partial m_i} dm_i + \frac{\partial U_i}{\partial \mu} d\mu$$

Homogeneity of degree zero in m_i and μ implies (remember we start from μ equal to one) :

$$\frac{\partial U_i}{\partial \mu} = - \frac{m_i^*}{\mu} \frac{\partial U_i}{\partial m_i} = - m_i^* \frac{\partial U_i}{\partial m_i}$$

Using equations (16) and (17) the above utility increment is rewritten :

$$dU_i = \lambda_i \left[\sum_{j=1}^{m} p_j\, dc_{ij} + dm_i - w_i \left(1 - \frac{1}{\varepsilon_i}\right) d\ell_i - m_i^* d\mu \right]$$

Let us differentiate the budget constraint :

$$\sum_{j=1}^{m} p_j\, dc_{ij} + dm_i = \overline{m}_i\, d\mu + w_i\, d\ell_i + \sum_{j=1}^{m} \theta_{ij}\, d\pi_j$$

and combine with the preceding equation so that :

$$dU_i = \lambda_i \left[\frac{w_i \, d\ell_i}{\varepsilon_i} + \sum_{j=1}^{m} \theta_{ij} \, d\pi_j + (\bar{m}_i - m_i^*) \, d\mu \right]$$

In view of the assumption of stabilized money balances, the last term disappears[9] and finally, using (23) :

$$dU_i = \lambda_i \left\{ \frac{w_i \, d\ell_i}{\varepsilon_i} + \sum_{j=1}^{m} \theta_{ij} \frac{p_j \, dy_j}{\varepsilon_j} \right\} \qquad (24)$$

Note that formulas (23) and (24) look very much like what we found in section 8.3, but now the increases dy_j and d_i are not arbitrary but must result from voluntary trading. The Appendix shows that such increases can actually be engineered at fixed wages and prices via the monetary increase $d\mu$ through traditional multiplier effects, yielding increases of the form :

$$p_j \, dy_j = k_j \, d\mu \qquad k_j > 0$$

$$w_i \, d\ell_i = k_i \, d\mu \qquad k_i > 0$$

Feeding these into equations (23) and (24), we find :

$$d\pi_j = \frac{k_j}{\varepsilon_j} d\mu > 0$$

$$dU_i = \lambda_i \left[\frac{k_i}{\varepsilon_i} + \sum_{j=1}^{m} \theta_{ij} \frac{k_j}{\varepsilon_j} \right] d\mu > 0$$

The policy considered thus leads to strict increases in all firms' profits and all households' utilities. We should now emphasize that the reaction of the economy to that particular policy is very different from the reaction of the same economy starting from a Walrasian equilibrium, where there would be no additional employment and production following a price-wage freeze and a monetary expansion, since in particular firms would not want to produce more, being already at their Walrasian employment-production plan.

Now we should of course end this section with a strong caveat on how its results should, and should not, be interpreted. The main purpose was to show that, although monetary policy has the same neutrality properties as those found in Walrasian models, other policies may yield substantially different results in the two settings. The particular policy studied here was chosen for convenience of the demonstration, and <u>not</u> as an advice to block all prices and wages and reflate to cure unemployment. Indeed it is clear that in this model the ultimate cause of inefficiency lies in the fact that partial market power allows agents to set prices and wages "too high". The Nash structure and resulting spillovers create the inefficiencies. A more constructive approach would be to investigate how to reduce the "monopoly power" of the agents on their respective markets, an investigation which is beyond the scope of this chapter, but should be the subject of future research.

8.8. Notes and references

The model in this section is based on Benassy (1987). Earlier models linking imperfect competition and non Walrasian equilibria are found in Benassy (1976b, 1977a, 1982), Negishi (1977, 1979), Hahn (1978), Hart (1982), Snower (1983), Weitzman (1986).

APPENDIX

We shall in this appendix briefly show how to compute the multipliers in the zone of general excess supply. Let us recall the equations determining quantities in this region (cf. section 8.2).

$$y_j = \sum_{i=1}^{n} C_{ij}(\mu \bar{m}_i + w_i \ell_i + \sum_{j=1}^{m} \theta_{ij} \pi_j , p , \mu) \qquad (25)$$

$$\ell_i = \sum_{j=1}^{m} L_{ij}(y_j , w) \qquad (26)$$

$$\pi_j = p_j y_j - \chi_j(y_j , w) \qquad (27)$$

Differentiating the consumption functions, the labor demand functions and equation (27), we obtain, in value terms :

$$p_j dc_{ij} = \alpha_{ij}(w_i d\ell_i + \sum_{j=1}^{m} \theta_{ij} d\pi_j) + \gamma_{ij} d\mu \qquad (28)$$

$$w_i d\ell_{ij} = \beta_{ij} p_j dy_j \qquad (29)$$

$$d\pi_j = \beta_{oj} p_j dy_j \qquad (30)$$

We assume that all goods, including money, are "normal" in production and consumption, so that

$$\alpha_{ij} \geq 0 \qquad \beta_{ij} \geq 0 \qquad \gamma_{ij} \geq 0 \qquad (31)$$

and :

$$\sum_{j=1}^{m} \alpha_{ij} < 1 \qquad (32)$$

and by definition

$$\sum_{i=1}^{n} \beta_{ij} + \beta_{oj} = 1 \qquad (33)$$

Note that $\beta_{oj} = 1/\varepsilon_j$ at the equilibrium point, and is positive in the excess supply zone.

Combining equations (28) (29) (30) with the equalities

$$dy_j = \sum_{i=1}^{n} dc_{ij} \qquad d\ell_i = \sum_{j=1}^{m} d\ell_{ij}$$

we obtain the following equation in output variations :

$$p_k \, dy_k = \sum_{j=1}^{m} \rho_{kj} \, p_j \, dy_j + v_j \, d\mu \qquad (34)$$

where

$$\rho_{kj} = \sum_{i=1}^{n} \alpha_{ik} (\beta_{ij} + \theta_{ij} \, \beta_{oj})$$

$$v_k = \sum_{i=1}^{n} \gamma_{ik}$$

Call ρ the square matrix with elements ρ_{kj} . Straightforward manipulation using properties (31) (32) (33) shows that :

$$\sum_{k=1}^{m} \rho_{kj} < 1$$

The matrix $I - \rho$ has thus a dominant diagonal with positive on-diagonal terms and negative off-diagonal terms. Call η the inverse matrix. It has only positive terms. From (34) we find :

$$p_j \, dy_j = \sum_{k=1}^{m} \eta_{jk} \, v_k \, d\mu = k_j \, d\mu \qquad (35)$$

We assume that each good is consumed by at least one household, so that all v_k

are strictly positive, and thus $k_j > 0$ for all j .

Inserting (35) into equation (29) and summing over j we find :

$$w_i \, d\ell_i = \sum_{j=1} \beta_{ij} \, p_j \, dy_j$$

$$= \left(\sum_{j=1}^{m} \beta_{ij} \, k_j \right) d\mu = k_i \, d\mu \qquad (36)$$

If each household i's work type is demanded by at least one firm, $\beta_{ij} > 0$ for at least one j , and so all k_i are strictly positive.

BIBLIOGRAPHY

ARROW, K.J. (1959), "Towards a Theory of Price Adjustment", in M. Abramowitz, (Ed), The Allocation of Economic Resources, Stanford University Press, Stanford.

ARROW, K.J., and G. DEBREU (1954), "Existence of an Equilibrium for a Competitive Economy", Econometrica, 22, 265-290.

ARROW, K.J., and F.H. HAHN (1971), General Competitive Analysis, Holden-Day, San Francisco.

BALASKO, Y. (1979), "Budget-constrained Pareto-efficient Allocations", Journal of Economic Theory, 21, 359-379.

BARRO, R.J., and H.I. GROSSMAN (1971), "A General Disequilibrium Model of Income and Employment", American Economic Review, 61, 82-93.

BARRO, R.J., and H.I. GROSSMAN (1974), "Suppressed Inflation and the Supply Multiplier", Review of Economic Studies, 41, 87-104.

BARRO, R.J., and H.I. GROSSMAN (1976), Money, Employment and Inflation, Cambridge University Press, Cambridge.

BELLMAN, R. (1957), Dynamic Programming, Princeton University Press, Princeton, New Jersey.

BENASSY, J.P. (1973), "Disequilibrium Theory", Ph.D. Dissertation and Working Paper, University of California, Berkeley. Hungarian translation in Szygma (1974).

BENASSY, J.P. (1975a), "Neo-Keynesian Disequilibrium Theory in a Monetary Economy", Review of Economic Studies, 42, 503-523.

BENASSY, J.P., (1975b), "Disequilibrium Exchange in Barter and Monetary Economies", Economic Inquiry, 13, 131-156.

BENASSY, J.P. (1976a), "Théorie néokeynésienne du déséquilibre dans une économie monétaire", Cahiers du Séminaire d'Econométrie, 17, 81-113.

BENASSY, J.P. (1976b), "The Disequilibrium Approach to Monopolistic Price Setting and General Monopolistic Equilibrium", Review of Economic Studies, 43, 69-81.

BENASSY, J.P. (1976c), "Théorie du déséquilibre et fondements microéconomiques de la macroéconomie", Revue Economique, 27, 755-804.

BENASSY, J.P. (1977a), "A Neokeynesian Model of Price and Quantity Determination in Disequilibrium", in G. Schwödiauer (Ed), Equilibrium and Disequilibrium in Economic Theory, D. Reidel Publishing Company, Boston.

BENASSY, J.P. (1977b), "On Quantity Signals and the Foundations of Effective Demand Theory", The Scandinavian Journal of Economics, 79, 147-168.

BENASSY, J.P. (1978), "Cost and Demand Inflation Revisited : A Neo-Keynesian Approach", Economie Appliquée, 31, 147-168.

BENASSY, J.P. (1982), The Economics of Market Disequilibrium, Academic Press, New York.

BENASSY, J.P. (1983), "The Three Regimes of the IS-LM Model : A Non-Walrasian Analysis", European Economic Review, 23, 1-17.

BENASSY, J.P. (1984), "A Non-Walrasian Model of the Business Cycle, Journal of Economic Behavior and Organization, 5, 77-89.

BENASSY, J.P. (1986), Macroeconomics : An Introduction to the Non-Walrasian Approach, Academic Press, New York.

BENASSY, J.P. (1987), "Imperfect Competition, Unemployment and Policy", European Economic Review, 31, 417-426.

BENASSY, J.P. (1988), "The Objective Demand Curve in General Equilibrium with Price Makers", CEPREMAP Working Paper N° 8719, forthcoming, Economic Journal Supplement, March 1988.

BOHM, V., and P. LEVINE (1979), "Temporary Equilibria with Quantity Rationing", Review of Economic Studies, 46, 361-377.

BUSHAW, D.W., and R. CLOWER (1957), Introduction to Mathematical Economics, Richard D. Irwin, Homewood, Illinois.

CHAMBERLIN, E.H. (1933), The Theory of Monopolistic Competition, Harvard University Press, Cambridge.

CLOWER, R.W. (1960), "Keynes and the Classics : A Dynamical Perspective", Quarterly Journal of Economics, 74, 318-323.

CLOWER, R.W. (1965), "The Keynesian Counterrevolution : A Theoretical Apraisal", in F.H. Hahn and F.P.R. Brechling 'Eds), The Theory of Interest Rates, Macmillan, London.

CLOWER, R.W. (1967), 'A Reconsideration of the Microfoundations of Monetary Theory", Western Economic Journal, 6, 1-9.

CUDDINGTON, J.T., P.O. JOHANSSON, and K.G. LOFGREN (1984), Disequilibrium Macroeconomics in Open Economies, Basil Blackwell, Oxford.

D'AUTUME, A., (1985), Monnaie, Croissance et Déséquilibre, Economica, Paris.

DEBREU, G. (1959), Theory of Value, Wiley, New York.

DEHEZ, P., and J.H. DREZE (1984), "On Supply Constrained Equilibria", Journal of Economic Theory, 33, 172-182.

DIXIT, A. (1978), "The Balance of Trade in a Model of Temporary Equilibrium with Rationing", Review of Economic Studies, 45, 393-404.

DREZE, J.H. (1975), "Existence of an Exchange Equilibrium under Price Rigidities", International Economic Review, 16, 301-320.

DREZE, J.H., and H. MULLER (1980), "Optimality Properties of Rationing Schemes", Journal of Economic Theory, 23, 131-149.

FITOUSSI, J.P. (1983), "Modern Macroeconomic Theory : An Overview", in J.P. Fitoussi (Ed), Modern Macroeconomic Theory, Basil Blackwell, Oxford.

GABSZEWICZ, J.J., and J.P. VIAL (1972), "Oligopoly "à la Cournot" in a General Equilibrium Analysis", Journal of Economic Theory, 4, 381-400.

GARY-BOBO, R. (1987), "Locally Consistent Oligopolistic Equilibria are Cournot-Walras Equilibria", Economic Letters, 23, 217-221.

GLUSTOFF, E. (1968), "On the Existence of a Keynesian Equilibrium", Review of Economic Studies, 35, 327-334.

GRANDMONT, J.M. (1983), Money and Value, Cambridge University Press, Cambridge.

GRANDMONT, J.M., and G. LAROQUE (1976), "On Keynesian Temporary Equilibria", Review of Economic Studies, 43, 53-67.

GROSSMAN, H.I. (1971), "Money, Interest and Prices in Market Disequilibrium", Journal of Political Economy, 79, 943-961.

GROSSMAN, H.I. (1972), "A Choice-Theoretic Model of an Income Investment Accelerator", American Economic Review, 62, 630-641.

HAHN, F.H. (1978), "On Non-Walrasian Equilibria", Review of Economic Studies, 45, 1-17.

HAHN, F.H., and T. NEGISHI (1962), "A Theorem on Non Tatonnement Stability", Econometrica, 30, 463-469.

HANSEN, B. (1951), A Study in the Theory of Inflation, Allen and Unwin, London.

HART, O. (1982), "A Model of Imperfect Competition with Keynesian Features", Quarterly Journal of Economics, 97, 109-138.

HELLER, W.P., and R.M. STARR (1979), "Unemployment Equilibrium with Myopic Complete Information", Review of Economic Studies, 46, 339-359.

HENIN, P.Y. and MICHEL, P. (Eds) (1982), Croissance et Accumulation en Déséquilibre, Economica, Paris.

HICKS, J.R. (1937), "Mr. Keynes and the Classics : A Suggested Interpretation", Econometrica, 5, 147-159.

HICKS, J.R. (1939), Value and Capital, Clarendon Press, Oxford, 2nd Edition, 1946.

HICKS, J.R. (1965), Capital and Growth, Oxford University Press, London.

HILDENBRAND, K., and W. HILDENBRAND (1978), "On Keynesian Equilibria with Unemployment and Quantity Rationing", Journal of Economic Theory, 18, 255-277.

HONKAPOHJA, S. (1979), "On the Dynamics of Disequilibria in a Macro Model with Flexible Wages and Prices", in M. Aoki and A. Marzollo (Eds), New Trends in Dynamic System Theory and Economics, Academic Press, New York.

HOOL, B. (1980), "Monetary and Fiscal Policies in Short-Run Equilibria with Rationing", International Economic Review, 21, 301-316.

HOWITT, P.W. (1974), "Stability and the Quantity Theory", Journal of Political Economy, 82, 133-151.

ITO, T. (1980), "Disequilibrium Growth Theory", Journal of Economic Theory, 23, 380-409.

IWAI, K. (1974), "The Firm in Uncertain Markets and its Price, Wage and Employment Adjustments", Review of Economic Studies, 41, 257-276.

KEYNES, J.M. (1936), The General Theory of Money, Interest and Employment, Harcourt Brace, New York.

KEYNES, J.M. (1937), "Alternative Theories of the Rate of Interest", Economic Journal, 47, 241-252.

KORNAI, J. (1980), Economics of Shortage, North-Holland, Amsterdam.

KORNAI, J. (1982), Growth, Shortage and Efficiency, Basil Blackwell, Oxford.

LAROQUE, G. (1981), "On the Local Uniqueness of the Fixed Price Equilibria", Review of Economic Studies, 48, 113-129.

LEIJONHUFVUD, A. (1968), On Keynesian Economics and the Economics of Keynes, Oxford University Press, Oxford.

MALINVAUD, E. (1977), The Theory of Unemployment Reconsidered, Basil Blackwell, Oxford.

MARSCHAK, T., and SELTEN R. (1974), General Equilibrium with Price Making Firms, Springer-Verlag, Berlin.

MUELLBAUER, J., and R. PORTES (1978), "Macroeconomic Models with Quantity Rationing", Economic Journal, 88, 788-821.

NEARY, J.P. (1980), "Nontraded Goods and the Balance of Trade in a Neo-Keynesian Temporary Equilibrium", Quarterly Journal of Economics, 95, 403-430.

NEARY, J.P., and J.E. STIGLITZ (1983), "Towards a Reconstruction of Keynesian Economics : Expectations and Constrained Equilibria", Quarterly Journal of Economics, 98, Supplement, 199-228.

NEGISHI, T. (1961), "Monopolistic Competition and General Equilibrium", Review of Economic Studies, 28, 196-201.

NEGISHI, T. (1972), General Equilibrium Theory and International Trade, North-Holland, Amsterdam.

NEGISHI, T. (1977), "Existence of an Under Employment Equilibrium", in G. Schwödiauer (Ed), Equilibrium and Disequilibrium in Economic Theory, D. Reidel Publishing Company, Boston.

NEGISHI, T. (1979), Microeconomic Foundations of Keynesian Macroeconomics, North-Holland, Amsterdam.

NIKAIDO, H. (1975), Monopolistic Competition and Effective Demand, Princeton University Press, Princeton.

OSTROY, J. (1973), "The Informational Efficiency of Monetary Exchange", American Economic Review, 63, 597-610.

OSTROY, J., and R. STARR (1974), "Money and the Decentralization of Exchange", Econometrica, 42, 1093-1114.

PATINKIN, D. (1956), Money, Interest and Prices, Row, Peterson and Company, 2nd edition, Harper and Row, New York 1965.

PERSSON, T., and L.E.O. SVENSSON (1983), "Is Optimism Good in a Keynesian Economy ?", Economica, 50, 291-300.

PICARD, P. (1983), "Inflation and Growth in a Disequilibrium Macroeconomic Model", Journal of Economic Theory, 30, 266-295.

PICARD, P. (1985), Théorie du déséquilibre et politique économique, Economica, Paris.

PORTES, R. (1981), "Macroeconomic Equilibrium and Disequilibrium in Centrally Planned Economies", Economic Inquiry, 19, 559-578.

QUANDT, R.E. (1982), "Econometric Disequilibrium Models", Econometric Review 1, 1-63.

ROBINSON, J. (1933), The Economics of Imperfect Competition, Macmillan, London.

SAMUELSON, P.A. (1947), Foundations of Economic Analysis, Harvard University Press, Cambridge.

SCHULZ, N. (1983), "On the Global Uniqueness of Fixprice Equilibria", Econometrica, 51, 47-68.

SILVESTRE, J. (1977), "A Model of General Equilibrium with Monopolistic Behavior", Journal of Economic Theory, 16, 425-442.

SILVESTRE, J. (1982), "Fixprice Analysis of Exchange Economies", Journal of Economic Theory, 26, 28-58.

SILVESTRE, J. (1983), "Fixprice Analysis in Productive Economies", Journal of Economic Theory, 30, 401-409.

SILVESTRE, J. (1985), "Voluntary and Efficient Allocations are Walrasian", Econometrica, 53, 807-816.

SILVESTRE, J. (1987), "Undominated Prices in the Three Goods Model", European Economic Review, forthcoming.

SNEESSENS, H. (1981), Theory and Estimation of Macroeconomic Rationing Models, Springer Verlag, New York.

SNOWER, D.J. (1983), "Imperfect Competition, Underemployment and Crowding Out", Oxford Economic Papers, 35, 569-584.

SOLOW, R.M., and J. STIGLITZ (1968), "Output, Employment and Wages in the Short Run", Quarterly Journal of Economics, 82, 537-560.

TOBIN, J. (1952), "A Survey of the Theory of Rationing", Econometrica, 20, 521-553.

TOBIN, J., and H.S. HOUTHAKKER (1950), "The Effects of Rationing on Demand Elasticities", Review of Economic Studies, 18, 140-153.

TRIFFIN, R. (1940), Monopolistic Competition and General Equilibrium Theory, Harvard University Press, Cambridge.

UZAWA, H. (1962), "On the Stability of Edgeworth's Barter Process", International Economic Review, , .

VEENDORP, E.C.H. (1970), "General Equilibrium Theory for a Barter Economy", Western Economic Journal, 8, 1-23.

WALRAS, L. (1874), Eléments d'économie politique pure, Corbaz, Lausanne. Definitive edition translated by W. Jaffé : Elements of Pure Economics, Allen and Unwin, London, 1954.

WEITZMAN, M.L. (1986), "The Simple Macroeconomics of Profit Sharing", American Economic Review, 75, 937-953.

YOUNES, Y. (1970), "Sur les notions d'équilibre et de déséquilibre utilisées dans les modèles décrivant l'évolution d'une économie capitaliste", CEPREMAP, Paris.

YOUNES, Y. (1975), "On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium", Review of Economic Studies, 42, 489-501.

FOOTNOTES

(1) Of course this does not mean that equilibria with rationing cannot be modelled in nonmonetary economies. See for example Benassy (1975b).

(2) Examples are the theory of contracts, efficiency wages, theories of trade-unions, bargaining, and many others.

(3) Manipulable schemes lead to a perverse phenomenon of "overbidding" which prevents the establishment of an equilibrium (cf. Benassy 1977b, 1982).

(4) There is a spillover effect when a (binding) constraint in one market modifies the effective demand in an other market.

(5) The original concept of Drèze actually deals with the more general case of prices variable between given limits.

(6) The original paper also stipulated that one good should not be rationed at all, a condition aimed at suppressing trivial equilibria such as that where all agents would be constrained to trade nothing. Identifying this unrationed good as the medium of exchange allows for a more symmetric presentation.

(7) See for example Grandmont (1983) for a thorough discussion of this issue in Walrasian models.

(8) We actually assume to simplify that the influence of w_i on household i's profit income is negligible, which will be the case if there are many households, and each owns negligibles shares of each firm.

(9) Note that alternatively additional balanced money transfers $(m_i^* - \bar{m}_i)\,d\mu$ would also eliminate the last term and take care of these distributional problems.