THE OBJECTIVE DEMAND CURVE IN GENERAL EQUILIBRIUM WITH PRICE MAKERS

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WITH PRICE MAKERS

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A B S T R A C T

Objective demand curves in general equilibrium with explicitly modelled price makers have been defined so far in a number of specific cases. This paper gives a general definition of such an objective demand curve. The associated general equilibrium concept is defined and sufficient existence conditions are given.

keywords : Objective demand curve, Imperfect competition, General equilibrium.

Journal of Economic Literature Classification Number : 021

LA COURBE DE DEMANDE OBJECTIVE DANS UN MODELE D'EQUILIBRE

GENERAL AVEC FIXATION ENDOGENE DES PRIX

R E S U M E

Jusqu'ici la courbe de demande "objective" en équilibre général et avec fixation explicite des prix par des agents décentralisés n'avait été définie que pour quelques cas particuliers. Cet article en donne une définition générale, construit le concept d'équilibre général associé, et donne des conditions d'existence simples pour ce type d'équilibre.

Mots clefs : Demande objective, Concurrence imparfaite, Equilibre général.

Codes J.E.L. : 021.

THE OBJECTIVE DEMANDE CURVE IN GENERAL EQUILIBRIUM WITH PRICE MAKERS (*)

1. INTRODUCTION

The purpose of this paper is to do for models of competition by prices what was done in their seminal paper by Gabszewicz-Vial (1972) for the Cournot-Nash model of competition by quantities (1). Namely we want to construct in a framework of general equilibrium with actual price makers a concept of an objective demand curve, which imbeds all the feedback effects of the price decisions of the various competitors, and study the associated general equilibrium concept.

The theory of general equilibrium with explicit price making behavior was first elegantly developed by Negishi (1961) in a framework of subjective demand curves : Each price maker has a subjective perception of the demand curve, which is correct at the equilibrium point but which may be different elsewhere. By contrast the idea of an objective demand curve is that it should be correct everywhere, and not only at the equilibrium point.

The first concepts of equilibrium with objective demand curves were built in the pioneering works of Marschak-Selten (1974) and Nikaido (1975). As we shall see below these concepts were developed in particular cases where price makers do not sell to each other (Marschak-Selten) or for a Leontief economy (Nikaido), and the definitions of objective demand curves are valid only in a subset of the price space. Though a number of later writers have used the idea of an equilibrium with objective demand curves for various applied purposes (2), at this stage there is no definition of an objective demand curve and of the associated general equilibrium concept which matches the generality of that found in Cournotian analysis (3).

So what we shall do in this paper is to address first the problem of the proper definition of an objective demand curve in a general equilibrium model where prices are strategic variables. As might be expected, this definition will involve an equilibrium concept, and we shall thus be concerned with the problem of existence of such an objective demand curve. We shall find out that it exists for the whole domain of strictly positive prices under traditional assumptions. We shall then show how agents set prices with such objective demand curves, and define an associated concept of general equilibrium. Finally simple sufficient existence conditions will be given.

2. THE GENERAL FRAMEWORK

We shall thus consider a general equilibrium setting, with a set of ℓ goods $h \in H$, and an additional good, called money for convenience, which serves both as a numéraire and medium of exchange. The price of good h in terms of this numéraire is p_h. The vector of these prices is denoted by $p \in R^{\ell}$.

The agents in the economy are firms and households. Firms are indexed by $j \in J$. Households are indexed by $i \in I$. We shall denote by $A = I \cup J$ the set of all agents, firms and households together, indexed by $a = 1 \ldots n$.

Firm j has a production vector y_j which must belong to a production set $Y_j \subset R^{\ell}$. Its objective is to maximize profits $\pi_j = py_j = - pz_j$, where z_j, the net trade vector of firm j, will be equal to $- y_j$ since we assume the firm carries no inventories.

Households are indexed by $i \in I$. Household i has an initial endowment of goods $\omega_i \in R^{\ell}_{+}$ and money $\bar{m}_i \geq 0$, and owns shares θ_{ij} of firms j . Households' total shares of each firm of course sum to one :

$$\sum_{i \in I} \theta_{ij} = 1 \qquad \forall j \in J$$

Household i carries a vector of net trades $z_i \in R^{\ell}$ and maximizes a utility function $U_i(x_i, m_i)$ where $x_i = \omega_i + z_i$ is the vector of final holdings of goods and the final quantity of money m_i is given by the budget constraint :

$$pz_i + m_i = \bar{m}_i + \sum_{j \in J} \theta_{ij} \pi_j$$

As for price making, the basic institutional setting, as in usual in most models with explicit price makers, is that on each market one side consists of price makers, the other of price takers (4) . We shall moreover identify goods by their physical characteristics <u>and</u> the agent who sets their price. In that way each good has its price controlled by one and only one of the agents, and each price maker is alone on his side of the market.

Call H_a the (possibly empty) subset of the goods whose price is controlled by agent a . Subdivide H_a into H^d_a (goods demanded by a) and H^s_a (goods supplied by a). Agent a appears, at least formally, as a monopolist on markets $h \in H^s_a$, as a monopsonist on markets $h \in H^d_a$, and we have :

$$H_a \cap H_b = \{\emptyset\} \quad \text{if } b \neq a$$

We shall denote by p_a the set of prices controlled by agent a and by p_{-a} the set of prices controlled by the other agents (i.e. the rest of prices) :

$$p_a = \{p_h \mid h \in H_a\}$$

$$p_{-a} = \{p_b \mid b \neq a\} = \{p_h \mid h \notin H_a\}$$

3. A QUICK REVIEW

We shall now very quickly review the previous literature, concentrating especially on the model by Marschak-Selten (1974), which is the most suited for a general equilibrium framework.

The Marschak-Selten model [5]

In that model, goods are subdivided into non produced goods, sold by households to firms, and produced goods, sold by firms to households. There are no intermediary products. Furthermore it is assumed that firms are the sole price setters

$$H_i = \{\emptyset\} \qquad \forall\, i \in I$$

A main import of these assumptions is that no price maker buys from or sells to other price makers. This will allow to base directly the concept of objective demand curve on the Walrasian demand of the household sector, as we shall see below. A further important assumption is made : All price makers serve whatever demand or supply is addressed to them. Under this assumption, each household i can satisfy his Walrasian demand, given by the solution in z_i to the following program :

$$\text{Maximize } U_i(\omega_i + z_i \;,\; m_i) \qquad \text{s.t.}$$

$$m_i + pz_i = \bar{m}_i + \sum_{j \in J} \theta_{ij}\, \pi_j$$

The Walrasian demand of household i is denoted functionally as $\xi_i(p,\pi)$, where π is the vector of all firms' profits.

Consider now a good $h \in H_j$ controlled by firm j . This good is only sold to, or purchased from households. Moreover by the assumption that households are never rationed, they will be able to achieve their Walrasian demands and supplies on all markets. Consequently the objective demand (or supply) on market h is simply the sum of the Walrasian net demands of the household sector, i.e. :

$$\sum_{i \in I} \xi_{ih}(p,\pi)$$

With this definition of objective demand, we can now give the definition of an equilibrium :

<u>Definition 1</u> : An equilibrium with price-making firms consists of a set of p_j^* , y_j^* , π_j^* and z_i^* such that :

(a) $z_i^* = \xi_i(p^*,\pi^*)$ $\qquad i \in I$

(b) p_j^* and y_j^* are solutions of :

Maximize $p\, y_j$ s.t.

$$\begin{cases} y_j \in Y_j & \\ y_{jh} = \sum_{i \in I} \xi_{ih}(p,\pi^*) & h \in H_j \\ p_h = p_h^* & h \notin H_j \end{cases}$$

(c) $\pi_j^* = p^* y_j^*$ $\qquad j \in J$

Conditions (a) and (c) are self-evident. Condition (b) says that the firm j chooses its price p_j and production plan y_j to maximize profits, assuming that it serves all demand and supply on markets $h \in H_j$, and taking all profits as given.

<u>A few problems</u>

As we just saw, the definition of the objective demand curve is fundamentally based on the Walrasian demands of the household sector. We shall now see that such a definition, and thus the associated equilibrium concept, have serious limitations.

The first problem of the above definition is the absence of "quantity feedback" effects. As we underlined above, a very important feature of the model is that no price maker sells to another price maker. If instead such was not the case, then the various demand functions should have as arguments some quantities, which themselves would be functions of other quantities, etc ... This problem, which was noted by Marschak-Selten (1974) themselves, has never been solved, except in the case of a Leontief economy by Nikaido (1975). In such a case the quantity feedback effects could be resolved simply by matrix inversion, but of course that method is very particular to the Leontief economy, and does not generalize readily to other cases.

The second problem one encounters with this definition is that of the domain of definition of the objective demand curve : This curve was constructed under the maintained assumption that all price makers will serve all demands and supplies addressed to them. But this is feasible only if the demands and supplies given by the objective demand and supply curves correspond to feasible production vectors. In mathematical terms the domain of definition of the objective demand curve is at the very most the set :

$$\{(p,\pi) \mid \sum_{i \in I} \xi_i(p,\pi) \in \sum_{j \in J} Y_j\}$$

Obviously if the price-profits vector falls outside this set, not all demands and supplies can be satisfied, and some rationing must occur. We may further note that the actual domain of validity of the objective demand curve is actually quite smaller than the above set. Indeed the production plan implied by the condition of satisfying demands and supplies may very well be feasible, but imply negative profits, in which case a firm will not want to satisfy demand eventhough the corresponding production is physically possible. The consequence of such a limited domain of validity is that the optimal price strategies derived under the above definition will be at best local optima, and that taking instead into account the "true" objective demand curve may have a substantial impact on the existence and characteristics of an equilibrium[(6)].

Of course what we would like, and shall work on in the following sections, is a definition of an objective demand curve which is valid on the whole price domain, and moreover can handle a larger set of quantity feedback effects.

4. THE QUESTION AND SOME BASIC CONCEPTS

The question

The concept of equilibrium we shall work with is that of a general imperfectly competitive equilibrium where agents use prices as strategic variables, thus some kind of "Bertrand-Nash" equilibrium (as opposed to a "Cournot-Nash" equilibrium where agents would use quantities as strategic variables). In such a model an objective demand curve is a function (or a correspondence) which indicates for each price choice of his competitors p_{-a} and for each of his own price choices p_a the total demand forthcoming to an agent a on all markets.

Our analysis will proceed in a manner quite symmetrical to that used by Gabszewicz-Vial (1972) for the Cournotian case : In the same way as the Cournotian analysis of an objective demand curve involves the study of a Walrasian equilibrium concept for given quantities, the definition of an objective demand curve with price makers will involve an equilibrium concept with given prices. At this stage the reader might be surprised that concepts, where rationing and quantity signals play an important role, should be an important building block for a theory where prices are endogenous. But we should note that the "traditional" approach is also based, though much more implicitly,on a "rationing scheme" whereby each price maker, whatever his own preferred transactions, is forced to purchase and sell whatever is supplied to or demanded from him on the market he controls. We saw however that such a rationing scheme led to internal contradictions, and we shall thus now make these features more explicit, and consistent, at a general equilibrium level.

Some basic concepts

A basic idea of non Walrasian analysis is that not all agents may be able to trade what they want on all markets, and that accordingly they receive quantity signals which tell them the maximum quantity they can trade. This is expressed by the following transaction rules :

$$d^*_{ah} = \min(\tilde{\tilde{d}}_{ah}, \bar{d}_{ah})$$

$$s^*_{ah} = \min(\tilde{\tilde{s}}_{ah}, \bar{s}_{ah}) \qquad (1)$$

where $\tilde{\tilde{d}}_{ah}$ and $\tilde{\tilde{s}}_{ah}$ are agent a's demand and supply on market h , d^*_{ah} and s^*_{ah} his purchase and sale (i.e. his actual transactions), and $\bar{d}_{ah}$ and $\bar{s}_{ah}$ quantity signals representing the maximum quantities agent a can respectively buy or sell on market h . We may note already that such quantity signals very naturally relate to the idea of an objective demand curve, since objective demand for a given price vector precisely represents the maximum quantity a price maker can sell at that price. As we shall see below, these quantity signals depend on all demands and supplies expressed on the market. They have, however, a particularly simple and natural form for price makers on the markets they control : Indeed, since they are alone on their side of these markets, their quantity constraints have the simple form :

$$\bar{s}_{ah} = \sum_{b \neq a} \tilde{\tilde{d}}_{bh} \qquad h \in H_a^s$$

$$\bar{d}_{ah} = \sum_{b \neq a} \tilde{\tilde{s}}_{bh} \qquad h \in H_a^d \tag{2}$$

i.e. the maximum quantity that price setter a can sell is the total demand of the others, and conversely if he is a buyer.

Now our ultimate purpose is to find out how these constraints can be "manipulated" by prices, once all feedback effects have been taken into account. For that we need to describe in more detail the interrelations of the various quantities in a fixprice equilibrium.

<u>Fixprice equilibrium</u>[7]

To shorten notation, we shall work in what follows with net demands and transactions :

$$\tilde{\tilde{z}}_{ah} = \tilde{\tilde{d}}_{ah} - \tilde{\tilde{s}}_{ah} \qquad z^*_{ah} = d^*_{ah} - s^*_{ah}$$

Consider first a market h . Assume there are n agents (firms and households) who have expressed net demands $\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}$. The transactions realized will be given by a rationing scheme :

$$z^*_{ah} = F_{ah}(\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}) \qquad a = 1,\ldots,n \tag{3}$$

and quantity signals $\bar{d}_{ah}$, $\bar{s}_{ah}$ will also be functions of the demands and supplies of the market :

$$\bar{d}_{ah} = G^d_{ah}(\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh})$$

$$\bar{s}_{ah} = G^s_{ah}(\tilde{\tilde{z}}_{1h},\ldots,\tilde{\tilde{z}}_{nh}) \tag{4}$$

The functions G^d_{ah} and G^s_{ah} actually do not depend on $\tilde{\tilde{z}}_{ah}$ if agent a is rationed. An agent thus cannot "manipulate" his maximum purchases and sales by increasing his demand or supply. He can however do so by changing the price if he is a price maker, as we shall see below.

Now call $\tilde{\tilde{z}}_a$, z^*_a , $\bar{d}_a$, $\bar{s}_a$ the vectors of effective demands, transactions and quantity constraints for agent a . Equations (3) and (4) can be rewritten in vector form as :

$$z^*_a = F_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A \tag{5}$$

$$\bar{d}_a = G^d_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A$$

$$\bar{s}_a = G^s_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A \tag{6}$$

Each agent a is thus faced with a vector of price signals p (part of which, p_a , is determined by himself) and quantity signals $\bar{d}_a$, $\bar{s}_a$. As a function of these signals he expresses net demands $\tilde{\tilde{z}}_a$, which are determined as follows :

Firms j's effective demand for good h , $\tilde{\tilde{z}}_{jh}$, is the solution of the following program :

$$\text{Maximize } -pz_j \quad \text{s.t.}$$

$$\begin{cases} -z_j \in Y_j \\ -\bar{s}_{jk} \leqslant z_{jk} \leqslant \bar{d}_{jk} \qquad k \neq h \end{cases}$$

Repeating the operation for all goods h , we obtain a vector of net demands :

$$\tilde{\tilde{z}}_j = \tilde{\tilde{\xi}}_j(p,\bar{d}_j,\bar{s}_j) \qquad j \in J \tag{7}$$

Household i's effective demand for good h , $\tilde{\tilde{z}}_{ih}$, is solution in z_{ih} of the following program :

$$\text{Maximize } U_i(\omega_i+z_i,m_i) \quad \text{s.t.}$$

$$\begin{cases} m_i = \bar{m}_i - pz_i + \sum_{j\in J} \theta_{ij}\,\pi_j \\ -\bar{s}_{ik} \leqslant z_{ik} \leqslant \bar{d}_{ik} \qquad k \neq h \end{cases}$$

Repeating the operation for all $h \in H$, we obtain a vector of net demands :

$$\tilde{\tilde{z}}_i = \tilde{\tilde{\xi}}_i(p,\bar{d}_i,\bar{s}_i,\pi) \qquad i \in I \tag{8}$$

where $\pi = \langle \pi_j \mid j \in J \rangle$ is the vector of all firm's profits.

We are now ready to give the definition of a fixprice equilibrium :

Definition

A fixprice equilibrium associated to p is a set of $\tilde{\tilde{z}}_a$, z^*_a, $\bar{d}_a$, $\bar{s}_a$, π^*_j such that :

$$\tilde{\tilde{z}}_i = \tilde{\tilde{\xi}}_i(p,\bar{d}_i,\bar{s}_i,\pi^*) \qquad i \in I$$

$$\tilde{\tilde{z}}_j = \tilde{\tilde{\xi}}_j(p,\bar{d}_j,\bar{s}_j) \qquad j \in J$$

$$z^*_a = F_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A$$

$$\bar{d}_a = G^d_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A$$

$$\bar{s}_a = G^s_a(\tilde{\tilde{z}}_1,\ldots,\tilde{\tilde{z}}_n) \qquad a \in A$$

$$\pi^*_j = -\,pz^*_j \qquad j \in J$$

A fixprice equilibrium exists for all strictly positive prices, provided the households' utility functions are strictly concave and the firms' production sets strictly convex (Benassy 1975, 1976, 1982). A fixprice equilibrium is globally unique under fairly reasonable assumptions (Schulz 1983).

We shall denote by $\tilde{\tilde{Z}}_a(p)$, $\tilde{\tilde{D}}_a(p)$, $\tilde{\tilde{S}}_a(p)$, $Z^*_a(p)$, $D^*_a(p)$, $S^*_a(p)$, $\bar{D}_a(p)$, $\bar{S}_a(p)$ respectively the values of $\tilde{\tilde{z}}_a$, $\tilde{\tilde{d}}_a$, $\tilde{\tilde{s}}_a$, z^*_a , d^*_a , s^*_a , $\bar{d}_a$, $\bar{s}_a$ at a fixprice equilibrium associated to p .

5. THE OBJECTIVE DEMAND CURVE, PRICE MAKING AND EQUILIBRIUM

Definition of the objective demand curve

Consider a vector $p = (p_a, p_{-a})$. As indicated above the objective demand curve should give the total demand forthcoming once all feedback effects have been taken into account. This objective demand (or supply if the price maker is a demander) is thus on the markets controlled by a :

$$\sum_{b \neq a} \tilde{\tilde{D}}_{bh}(p) \qquad h \in H_a^s$$

$$\sum_{b \neq a} \tilde{\tilde{S}}_{bh}(p) \qquad h \in H_a^d$$

In view of equations (2) the objective demand curve is alternatively written :

$$\bar{D}_{ah}(p) \qquad h \in H_a^d$$

$$\bar{S}_{ah}(p) \qquad h \in H_a^s$$

This form shows quite well how the price allows to "manipulate" the demand or supply constraints faced by price maker a on the markets $h \in H_a$ he controls.

We may also note that agent a perceives as well constraints $\bar{D}_{ah}(p)$ and $\bar{S}_{ah}(p)$ on markets $h \notin H_a$ (even though these constraints are not binding whenever the other price makers satisfy demand or supply). Therefore the objective demand and supply curves will consist of the whole vectors $\bar{S}_a(p)$ and $\bar{D}_a(p)$ respectively (note that the objective demand curve is a constraint on a's supply, and symmetrically).

Existence and uniqueness

We see immediately that the objective demand curve exists for all prices for which a fixprice equilibrium exists. Standard results (Benassy, 1975, 1982) show that a fixprice equilibrium as defined above exists for all strictly positive prices and all continuous nonmanipulable rationing schemes. The objective demand curve thus exists on the whole domain of strictly positive prices.

Similarly the objective demand curve will be unique (i.e. a function) if the fixprice equilibrium is globally unique. Schulz (1983) has given some sufficient conditions for global uniqueness. Intuitively the basic condition is that changes in quantity constraints "spill over" onto the other markets by less than hundred per cent in value terms. A traditional exemple is that marginal propensity to consume should be less than hundred per cent. We shall assume in all that follows that the Schulz conditions hold, and thus that the objective demand curve is actually a function.

Price making

With the above definition of the objective demand curve, it is now easy to describe the price making behavior of firms and households.

The optimal price of firm j is obtained by maximizing profits subject to technological constraints, and to the fact that trades are limited by the objective demand and supply curves $\bar{S}_j(p)$ and $\bar{D}_j(p)$. This optimal price is thus the solution in p_j of the following program P_j (in both p_j and z_j) :

$$\text{Maximize } - pz_j \qquad \text{s.t.}$$

$$\begin{cases} - z_j \in Y_j \\ - \bar{S}_j(p) \leqslant z_j \leqslant \bar{D}_j(p) \end{cases} \qquad (P_j)$$

which yields :

$$p_j = \psi_j(p_{-j})$$

We should note that at most one of the constraints will be binding for each market.

Consider now household i . He receives profits in the amount :

$$\pi_i(p) = - \sum_{j \in J} \theta_{ij} \, p \, Z^*_j(p)$$

The program P_i yielding the optimum price p_i (and z_i) is :

$$\text{Maximize } U_i(\omega_i + z_i \, , m_i) \qquad \text{s.t.}$$

$$\begin{cases} m_i = \bar{m}_i - pz_i + \pi_i(p) \\ - \bar{S}_i(p) \leqslant z_i \leqslant \bar{D}_i(p) \end{cases} \qquad (P_i)$$

which yields :

$$p_i = \psi_i(p_{-i})$$

<u>Equilibrium with price makers</u>

It is now easy to define an equilibrium with price makers :

<u>Definition 2</u> : An equilibrium with price makers is characterized by a set of p^*_i , $i \in I$ and p^*_j , $j \in J$ (and of course the associated $\tilde{z}_a$, z^*_a, $\bar{d}_a$, $\bar{s}_a$, $a \in A$) such that :

$$p_i^* \in \psi_i(p_{-i}^*) \qquad \forall\, i \in I$$

$$p_j^* \in \psi_j(p_{-j}^*) \qquad \forall\, j \in J$$

Before giving an existence theorem for such an equilibrium in the next section, we shall make a few comments and give an example.

A few comments

We may now comment on how our concept of objective demand curve and equilibrium allows to lift the two main objections to previous definitions, concerning the absence of feedback effects and the incomplete domain of definition.

First the previous definitions, using Walrasian demand as a basic building block, had to eliminate any kind of quantity feedback, notably by not allowing situations where a price maker sells to another, and also by treating profit incomes as parametric (whereas they should be endogenous in a full general equilibrium treatment). Contrarily to this our concept of objective demand curve, being based on a concept of fixprice equilibrium where all quantity feedbacks are by definition taken into account, places no such restrictions as to who sells to whom and fully takes into account these feedbacks. As a result, in the equilibrium concept each price maker only treats as parametric the prices set by others, as should be in a Nash equilibrium in prices.

The second problem is that of the domain of definition. The traditional definitions of the objective demand curve were based on the assumption that price makers would satisfy all demand and supply addressed to them. Ignoring for the moment the above mentioned feedback effects, this means that the traditional definition would be valid only in the subregion of all prices where price makers are actually willing to satisfy demands and supplies, which corresponds to :

- excess supply for goods h belonging to $\bigcup_a H_a^s$
- excess demand for goods h belonging to $\bigcup_a H_a^d$

For example if the suppliers set prices, the definition based on the traditional assumption would be valid only in the zone of excess supply on all markets, a fairly restricted domain. We already mentioned above the problems related to such a limited domain.

Instead, our definition of objective demand is valid even in that part of the price domain where some demands and supplies cannot be fulfilled, because the corresponding rationing is fully taken into account.

An example

Consider two agents A and B in an Edgeworth box (Figure 1). They exchange one good (measured horizontally) against money. Point O represents initial endowments, point W the Walrasian equilibrium. Agent A is demander of the good, agent B is supplier and sets the price.

Figure 1

The effective demand $\tilde{\tilde{d}}_A$ of agent A is given by the locus of tangency points between various budget lines and A's indifference curves. The "demand curve" corresponds to the curvilinear line OMWD (which actually goes beyond point D). The set of feasible points, from B's point of view, are (a) points on the curve OMWD (b) points between O and a point on OMWD ... This corresponds to the shaded area.

The equilibrium point is then simply point M, the tangency point of this shaded area with B's indifference curve, which yields agent B the highest possible utility, given A's objective demand behavior.

6. AN EXISTENCE THEOREM

As an illustration we shall give here an existence theorem for the case, most often considered in the literature, with produced and nonproduced goods, and no intermediary products, as this will substantially lighten notation (8). The set of produced goods will be denoted as H_p, the set of nonproduced goods, i.e. factors of production owned by the households, by H_f. Of course :

$$H = H_f \cup H_p$$

In order to prove existence, we shall first need a natural continuity assumption :

Assumption A1. For all agents $a \in A$, $Z^*_a(p)$ is a continuous function.

In order to obtain boundedness of produced goods prices, we shall make the fairly traditional assumption that for all produced goods, the households have a "reservation price" bounded above, which we shall express as follows :

Assumption A2. If good h is a produced good ($h \in H_p$), there exists $\beta_h > 0$ such that :

$$\frac{\partial U_i/\partial x_{ih}}{\partial U_i/\partial m_i} \leqslant \beta_h \qquad \forall\ i \in I$$

We shall make the symmetric assumption that suppliers of factors of production have a "reservation price" bounded below, which we shall express as

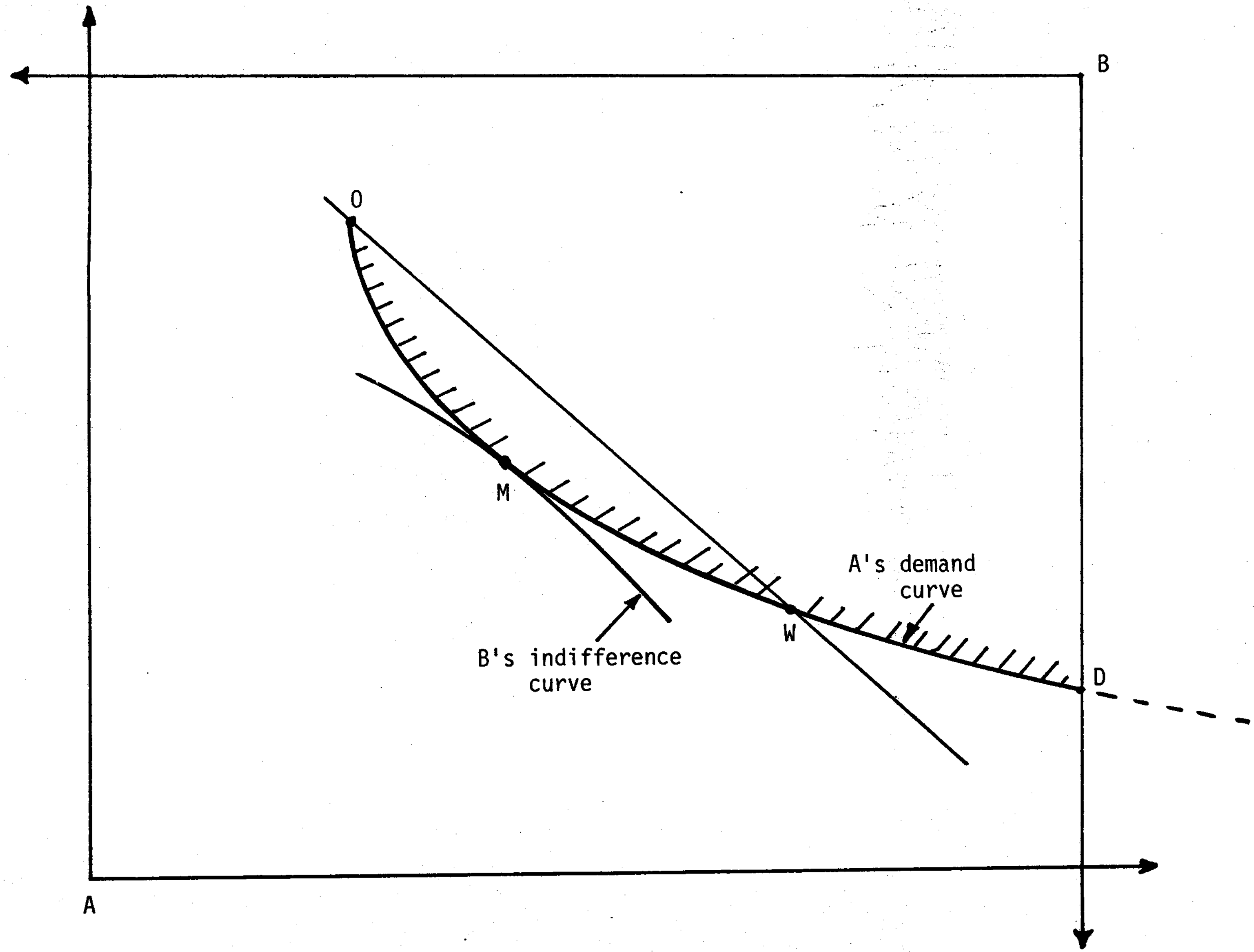

Figure 1

follows :

Assumption A3. If good h is a factor of production ($h \in H_f$) there exists an $\alpha_h > 0$ such that

$$\frac{\partial U_i/\partial x_{ih}}{\partial U_i/\partial m_i} \geq \alpha_h \qquad \forall\ i \quad \text{such that } \omega_{ih} > 0$$

We shall also assume that all factor productivities are bounded :

Assumption A4. If good h is a factor of production ($h \in H_f$) and good k a produced good ($k \in H_p$) , then there exists a $\gamma_{hk} > 0$ such that :

$$\frac{\partial y_{jk}}{\partial z_{jh}} \leq \gamma_{hk} \qquad \forall\ j \in J$$

where y_{jk} is firm j's output of good k and z_{jh} is firm j's input of factor h . The partial derivative must of course be taken at an efficient point, all other inputs and outputs being maintained constant.

We shall finally make the usual assumption that the optimal prices chosen by each firm, given the others' prices, form a convex valued set (9) :

Assumption A5. For all a , $\psi_a(p_{-a})$ is convex valued.

We can now state and prove the existence theorem.

Theorem. Under A1, A2, A3, A4 and A5 an equilibrium with price makers exists.

Proof. The equilibrium will be constructed as a fixed point of a mapping $\psi(p)$ from the set of prices into itself, consisting of the following submappings :

$$p_i \longrightarrow \psi_i(p_{-i}) \qquad i \in I$$

$$p_j \longrightarrow \psi_j(p_{-j}) \qquad j \in J$$

In order to reduce the domain of this mapping to a bounded set of prices, we want first to show boundedness of prices.

Consider first a produced good $h \in H_p$. In view of Assumption A2, the effective demand of households for this good is zero if $p_h > \beta_h$. Therefore the price of that good is bounded above by β_h.

Symmetrically consider a factor of production $h \in H_f$. In view of A3 the effective supply of households for this good is zero if $p_h < \alpha_h$, so that the price p_h will be bounded below by α_h.

We shall now look for an upper bound for the price p_h of a factor of production $h \in H_f$. Consider the number β_h defined as :

$$\beta_h = \max_{k \in H_p} \gamma_{hk} \beta_k > 0$$

Clearly if $p_h > \beta_h$ the demand for that factor will be zero. Indeed by Assumption A4, it would be profitable for every firm j to decrease its input of factor h in the production of any produced good $k \in H_p$ to zero, and factor h would not be demanded. Therefore p_h is bounded above by the β_h just defined.

Consider symmetrically a produced good $h \in H_p$ and the number α_h defined by :

$$\alpha_h = \min_{k \in H_f} \frac{\alpha_k}{\gamma_{kh}} > 0$$

Again by assumption A4, the supply of that good by firms will be zero if

$p_h < \alpha_h$. Therefore p_h is bounded below by this α_h .

We shall now take as the domain of the above mapping ψ the product of the closed intervals $[\alpha_h, \beta_h]$, $h \in H$, which is a convex compact set. Note that if for some h this interval is empty because $\alpha_h > \beta_h$, then the price p_h can be fixed arbitrarily at any value between β_h and α_h . The corresponding market will be inactive.

We want now to show the continuity of the mapping ψ , and for that purpose we shall characterize optimal prices $\psi_i(p_{-i})$ and $\psi_j(p_{-j})$ in a slightly different way as above. We may note indeed that the above maximization programs P_i and P_j are programs in (p_i, z_i) and (p_j, z_j) respectively. We know from the theory of non Walrasian equilibria that the solutions in z_i and z_j for a given p are $Z_i^*(p)$ and $Z_j^*(p)$ respectively. Since what we are directly interested in are p_i and p_j , we can replace z_i and z_j by these values (unique by virtue of A1) and rewrite programs P_i and P_j more compactly as P'_i and P'_j :

For firm j :

$$\text{Maximize } - p\, Z_j^*(p) \qquad (P'_j)$$

yielding $p_j = \psi_j(p_{-j})$

For household i :

$$\text{Maximize } U_i[\omega_i + Z_i^*(p)\ ,\ m_i - p\, Z_i^*(p) + \pi_i(p)] \qquad (P'_i)$$

yielding $p_i = \psi_i(p_{-i})$

Because of the continuity of $Z_a^*(p)$ for all a (by A1), the maximands of programs P'_i and P'_j are continuous functions, and therefore by the theorem of the maximum $\psi_i(p_{-i})$ and $\psi_j(p_{-j})$ are upper semi continuous correspondences.

The mapping ψ is thus an upper semi continuous correspondence with convex values from a compact convex set into itself. By Kakutani's theorem it has a fixed point and an equilibrium exists. Q.E.D.

Comments

At this point it may be useful to comment a little on the theorem and assumptions, and further avenues for research.

The most delicate assumption in our theorem is evidently A5, which is not derived from households' tastes or firms' technologies. Though such an assumption is always used under one form or the other in the literature, it would be worthwhile to investigate more basic conditions for existence, at it is known that this assumption may not hold for non pathological demand functions (See for example Roberts-Sonnenschein, 1977). A few steps have already been taken in this direction : H. Dierker (1986), working in the traditional framework, investigated the role of the distribution of consumers' preferences. Benassy (1986b), in a framework closer to the one of this article, showed that both the number of competitors and the degree of substituability between goods, obviously two fundamental parameters in imperfect competition, played a crucial role in existence of an equilibrium of this sort. All this should be a worthwhile topic for future research.

Another fruitful area of research would be to investigate different forms of strategic price interaction between price makers. In the "Chamberlinian" tradition of monopolistic competition, we adopted here the so called "Bertrand-Nash" conjectures that each price maker expects the other price makers not to change their prices as a response to his own moves. While this may be a good assumption in markets with many competitors, there may be some oligopolistic situations where strategies allowing more feedback effects on prices could be a more suitable representation [10]. This too should be the subject of future research.

7. CONCLUSIONS

We have given in this paper a definition of an objective demand curve in the context of a general equilibrium with price makers. This definition is valid on the full price domain and takes into account all the general equilibrium feedback effects of price decisions. Both features represent a clearcut progress over previous definitions, and accordingly the associated general equilibrium concept has also a wider applicability.

We may note that an instrumental element of this progress was the use of the methods of non-Walrasian theory. This, which had already been used earlier in the framework of subjective demand curves (Benassy, 1976, 1977, 1982), appears thus as a powerful tool to analyze price making by decentralized agents in the absence of an auctioneer.

FOOTNOTES

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(1) This Cournot-Nash model has spanned a very abundant literature. For a survey, see for example Mas-Colell (1982).

(2) See for example the applications to unemployment theory in d'Aspremont-Dos-Santos, Gérard-Varet (1986), Dehez (1985), Silvestre (1986), Snower (1983), Weitzman (1985), or in Benassy (1987) which uses the framework of this article.

(3) For example the quite recent survey by Hart (1985) basically uses the Marschak-Selten definition.

(4) This formalization thus applies to markets in which buyers and sellers are well identified, and not to markets, such as securities markets, where people regularly shift from buying to selling, etc ... For these markets a more symmetric formulation is called for. See for example Benassy (1986a), and references therein, for a formalization of symmetric Nash equilibria with strategic price quoting.

(5) We actually give here a slightly simplified version of the Marschak-Selten model, as presented by Hart (1985). In their original contribution Marschak-Selten considered the feedback of a firm's own profits on its objective demand.

(6) See Benassy (1986b) for a further elaboration in a more partial equilibrium framework.

(7) What follows is borrowed from Benassy (1975), (1976), (1982), to which the reader is referred for further developments. An alternative concept of equilibrium with rigid prices is found in Drèze (1975).

(8) Note that this framework does not preclude the main conceptual diffi-

culty, price makers selling to price makers. One may think for example of the case (Benassy, 1987) where workers set wages and firms set product prices. Then every price maker sells to other price makers.

(9) Actually the most usual form of this assumption is that the relevant payoff functions, for example the firms' profit functions, are quasi concave in the choice variables, which immediately leads to a convex valued best response.

(10) For early attempts in this direction, see notably Marschak-Selten (1974), or Hahn's (1978) idea of rational conjectures.

(11) See Benassy (1976, 1977, 1982).

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