C.E.P.R.E.M.A.P. September 1986

LABOR VALUES

AND THE IMPUTATION OF LABOR CONTENTS*

G. DUMÉNIL

D. LÉVY

N° 8620

* *We thank S. Bernstein for her aid in the translation of this text into English*

RÉSUMÉ

LA VALEUR TRAVAIL ET L'IMPUTATION DES CONTENUS EN TRAVAIL

Dans un article récent (Econometrica 1983), P. Flaschel a donné une formalisation de procédures empiriques de calcul des valeurs travail dans un modèle de production jointe, fondées sur la désagrégation des procédés de production. Il a utilisé le concept de "valeur de marché" (valeur moyenne) pour calculer les valeurs sur la base de la technique simple obtenue par désagrégation.
Le but du présent article est double. Dans une première partie, on applique directement le formalisme de la valeur de marché au cas des productions jointes, et on discute les conditions d'existence de solutions positives. On démontre le caractère très peu restrictif de ces conditions, qui ne sont liées à aucune conditions d'exitence d'un quelconque équilibre. Dans cette analyse, on substitue le concept de "non-réductivité" à la notion traditionnelle de productivité. La seconde partie de l'article traite des problèmes liés aux procédures de désagrégation. On considère d'abord leurs propriétés générales. Puis on suggère une procédure alternative à celle de Flaschel, qui possède des propriétés plus avantageuses vis-à-vis de certaines applications empiriques. Enfin, on propose une généralisation de la définition traditionnelle des valeurs, par désagrégation, qui résout les problèmes de négativité.

ABSTRACT

LABOR VALUES AND THE IMPUTATION OF LABOR CONTENTS

In a recent article (Econometrica 1983), Peter Flaschel presented a formalization of empirical procedures for the calculation of labor values in joint production. These procedures were based on a disaggregation of production processes. The concept of "market value" (average value) was then used to determine values on the basis of the single technology which resulted from the disaggregation.
The purpose of the present article is twofold. In the first part, the formalism of market value is directly applied to the case of joint production, and the conditions for the existence of positive solutions are discussed. We demonstrate that these conditions are quite general, and are not related to conditions concerning the existence of any equilibrium. In this analysis, we substitute the concept of "nonreductivity" for the traditional notion of productivity of the technology. The second part of the article considers problems involved in procedures of disaggregation. We first examine their general properties. A new procedure is suggested as an alternative to Flaschel's, which possesses advantages for a number of empirical applications. Lastly, we propose a generalization of the traditional notion of values, using disaggregation, which solves the problem of negativity.

MOTS CLEFS : Théorie de la valeur, Production jointe

KEYWORDS : Theory of value, Joint production

J.E.L. Nomenclature : 020

INTRODUCTION

Since the publication of *Capital* the labor theory of value[1] has been responsible for much controversy and, more precisely, for two debates:

1. The relationship between values and prices of production was at the origin of the famous "transformation" debate. Marx's transformation was deemed impossible, due to the fact that the relations stipulated by Marx on total profits and total prices could not hold simultaneously. This debate evolved into the formulation of the "Fundamental Marxian Theorem" which attempts to salvage Marx's relation between labor value and exploitation from critics who claim that the transformation of values into prices is impossible (MORISHIMA M. 1973).
2. More recently, the issue of the computation of values in relation to the theory of exploitation, independently of the link between values and prices, has been the object of new discussions in models of joint production. In linear models of joint production (analysed in particular by Sraffa (SRAFFA P. 1960)), it is possible to obtain positive prices of production, where "values" — in a certain sense — are negative (STEEDMAN I. 1975). Therefore, it was demonstrated that positive profits could coincide with a negative rate of exploitation. The Fundamental Marxian Theorem seemed, thus, untenable.

The foundations of Marx's theories of value and exploitation were again unsettled by this second debate on "negativity", as they had been previously shaken by the "transformation" controversy. A first reply was formulated by Morishima (MORISHIMA M. 1976)[2] who suggested that since commodities can be produced in different manners in a system of joint production, the mode of computation of values must be modified. He defined values as the *minimum* amount of labor time necessary for the production of a commodity and referred to them as "true values". A first step forward had thus been accomplished by Morishima who recognized the origin of the difficulty: *the possibility of producing the same commodity under different conditions of production.* The generalization of the General Marxian Theorem came from Roemer (ROEMER J.

[1] *The law of value — "...the value of a commodity is determined by the quantity of labor expended to produce it..." (*MARX K. 1867, *p. 129) — must not be confused with the law of exchange which prevails in noncapitalist commodity producing economies: commodities exchange at their "values". The basic relationship that Marx established between value and prices is fully general in commodity economies. It does not depend on the prevalence of any particular sets of prices, such as the above (exchange of commodities at their values). Such a concept of value, which is independent of the definition of relative prices, is specific to the works of the classics and to Marx's analysis in particular. Outside of Marxian economics, such a notion is usually regarded as irrelevant.*

[2] *See also* MORISHIMA M., CATEPHORES G. 1978 *and* WOLFSTETTER E. 1976.

1981), who used the same definition of values in a more general framework. In 1983, new progress was made which, in our opinion, rendered previous approaches irrelevant. Flaschel (FLASCHEL P. 1983) contended that the distinction made by Marx between individual and market values could solve the difficulty posed above, if applied to the case of joint production. In Marx's sense of the expression, the *individual value* of a commodity is the value of a commodity taking into account the specific conditions of production in one unit of production. The *market value* of a commodity, still following Marx, is its average value corresponding to its average conditions of production in the various units of production. This new interpretation emphasizes the average value of commodities (as opposed to minimum values) and certainly fits more tightly with Marx's analysis. It presently defines the most advanced stage of the debate.

In our opinion, although Flaschel's contribution represents definitive progress in the debate, it is still limited in two important regards:

1. Flaschel's study does not directly apply the formalism of individual and market values to the case of *joint* production. Instead, he first defines a procedure to compute a single production technology based on a disaggregation of joint production processes. He then applies the formalism of individual and market values to *single* production. By failing to introduce the formalism of individual and market values directly into joint production, Flaschel cannot provide the foundation for the development of a general theory of value in a model of joint production. We believe that once such a framework has been set out, it is possible to state very general conditions for the positivity of values, and thereby contribute to the debate over the negativity of values.

2. In fact, Flaschel's main objective is to solve the problem of the empirical determination of values on the basis of traditional input-output matrices, rather than to address a theoretical issue. However, even as an empirical computation, the main justification of Flaschel's disaggregation is only its simplicity. The choice of this particular procedure remains arbitrary. In fact, it is possible to define other disaggregations which have the same properties and also possess other qualities. The choice between methods cannot be determined in general, but should depend on the purpose of the empirical study.

The main purpose of the present study is to overcome these two important limitations and thus progress further down the avenues opened by Flaschel. We will also show that, using Flashel's idea to disaggregate joint production processes, it is possible to provide an elegant formal solution to the problem of the negativity of values within the traditional conception of values (which does not takes into account the concepts of individual and market values). The possibility of this demonstration should not be mistaken as a theoretical justification for this traditional view.

The study is divided into four parts. *Part I* defines individual and market values in a model of joint production.[3] *Part II* presents the properties common to all disaggregations and recalls Flaschel's procedure. *Part III* introduces an alternative method which extends the application of disaggregation to the study of the relationship between actual prices and long term equilibrium prices. *Part IV* is devoted to the generalization of the traditional definition of values which does not rely on the distinction between individual and market values, and provides a solution to the problem of the negativity of values in this perspective.

I - INDIVIDUAL AND MARKET VALUES

Throughout the present study, we use the same framework and notation as that defined by Flaschel. Lower case letters denote vectors (a subscript k refers to the kth component of this vector) and upper case letters denotes matrices. A prime (') denotes transposition. Untransposed vectors are column vectors. $\mathbb{R}^n_+$ denotes the subspace of nonnegative elements (≥ 0) of $\mathbb{R}^n$. Semipositivity (strict positivity) is characterized by > 0 ($\gg 0$). E is the identity matrix. A_i, A^j indicate the ith row and the jth column of a matrix A. As a result of such conventions, the vectors $E^1, \ldots, E^n$ represent the canonical basis of $\mathbb{R}^n$. The number of physically distinguished commodities is equal to $n+1$, n produced and one nonproduced, labor power (assumed homogeneous). There are m capitalists ($j = 1, \ldots, m$), each facing a set of production possibilities $P^j \subset \mathbb{R}^{2n+1}$ which contains 0, the possibility of inaction. Vectors $\alpha^j \in P^j$ are written as:

$$\alpha^j = (-\alpha_0^j, -\underline{\alpha}^j, \overline{\alpha}^j)$$

where we denote: $\alpha_0^j \in \mathbb{R}_+$, the inputs of direct labor, $\underline{\alpha}^j \in \mathbb{R}^n_+$, the inputs of produced commodities for capitalist j and, $\overline{\alpha}^j \in \mathbb{R}^n_+$, the outputs of produced commodities for capitalist j. Net product $\tilde{\alpha}^j$ is defined by $\overline{\alpha}^j - \underline{\alpha}^j$. The symbols $\alpha = (-\alpha_0, -\underline{\alpha}, \overline{\alpha})$ and $\tilde{\alpha}$ refer to the economy as a whole, i.e., are obtained by summation over j. The vector of commodity prices is $p \in \mathbb{R}^n$. Prices are normalized since the wage rate is taken equal to one. An economy, $\mathcal{E}$, is a set of m processes of production and a set of strictly positive prices:

$$\mathcal{E} = \{\alpha^j \in P^j \quad j = 1, \ldots, m \quad ; \quad p \in \mathbb{R}^n \quad / \quad p \gg 0\}$$

Notice that, in this definition, endowments are not taken into account (initial endowments or inventories transmitted during the occurence of production).

---

[3] *The analysis in Part I extends to a more general formalism some results obtained in* DUMÉNIL G., LÉVY D. 1986(b) *in a linear formalism with constant return to scale.*

Part I is divided into four sections. Section A introduces the initial definitions and assumptions. Section B sets out the formalism for the equations of individual and market values. Section C states the conditions for the existence of positive values. Finally, Section D specifies the formalization and conditions of existence of values for single production.

A - DEFINITIONS AND ASSUMPTIONS

In order to simplify the coming discussion, the following assumption will be made which excludes from the formalism all inactive capitalists and all commodities which are not produced:

Assumption 1 : *All capitalists have a productive activity ($\alpha^j \neq 0$). All commodities are produced ($\overline{\alpha}_k \neq 0$).*

We introduce the following definition and the corresponding assumption:

Definition 1 : Nonreductivity[4]. *An economy is nonreductive if the prices of all the net products are strictly positive:*

$$p'\widetilde{\alpha}^j > 0 \quad for \quad j \in \{1, \ldots, m\}$$

---

[4] *In* DUMÉNIL G., LÉVY D. 1986(b) *— as a result of the different character of the formalism: a linear model of joint production — we used a slightly different notion, under the same name. (A, B) are two matrices with m rows and n columns modelling m joint processes with constant returns to scale. Z is a row vector with m components denoting the levels of activity of these processes. The definition of reductivity was:*

$$\exists Z \in \mathbb{R}^m \quad such\ that \quad Z > 0 \quad and \quad Z(B - A) \leq 0$$

The use of the term "reductivity" corresponded in this framework to the fact that, in a reductive technology, it is possible for given levels of activity Z to activate processes of production without producing anything, but instead "reducing" the bundle of inputs in all of its components. This definition is strictly equivalent (see GALE D. 1960 *and* DUMÉNIL G., LÉVY D. 1984*) to:*

$$\nexists\, p \in \mathbb{R}^n \quad such\ that \quad p \geq 0 \quad and \quad (B - A)p \gg 0$$

This latter approach differs from that adopted in the present study in the sense that the set of prices now considered, which yields a positive price of the net product in each process, is not any set of prices, but the actual set of prices. Although the interpretation of reductivity given above in terms of quantities produced cannot be extended to a formalism in which returns are not constant, we conserve the term. Nonreductivity is not equivalent to the traditional notion of "productivity".

Assumption 2 : *The economy is nonreductive.*

The assumption of nonreductivity is very weak for any commodity economy (an economy in which production is undertaken for the explicit purpose of sale). In any instance it is reasonable to disregard a process of production in which the price of the outputs does not even cover the price of the physical inputs without generating any income for labor or capital. No assumption, other than Assumptions 1 and 2 will be made. In particular, we make no assumptions concerning:

1. The behavior of economic agents (whether they use procedures of optimization or adaptation.[5])
2. The definition of equilibrium (Walrasian equilibrium, equilibrium in the sense of ROEMER J. 1980, long term equilibrium corresponding to prices of production,...).
3. The state of the economy concerning equilibrium. The economy can be in disequilibrium.

B - THE EQUATIONS OF *VALUES*

Following Marx, whenever commodities can be produced under different conditions of production, it is necessary to distinguish between the individual and market (or average) values of a commodity:

> *"Besides this, however, there is always* a market value *(of which more later), as distinct from the individual value of particular commodities produced by the different producers. The individual value of some of these commodities will stand below the market value (i.e., less labor time has been required for their production than the market value expresses), the value of the others above it. Market value is to be viewed on the one hand as the average value of the commodities produced in a particular sphere, and on the other hand as the individual value of commodities produced under average conditions in the sphere in question, and forming the great mass of these commodities."*
> (MARX K. 1863, p.279)

Flaschel applied this analysis to single production (equations 1 and 2 in FLASCHEL P. 1983), but not directly to joint production. He first disaggregates each joint production process into a set of single production processes and then computes the market values for this single production technology. Below, we directly apply the definitions to joint production.

---

[5] *In the sense given to this term in* DUMÉNIL G., LÉVY D. 1986(a).

We denote $v^j \in \mathbb{R}_+^n$, the vector of individual values for process j. The individual value of commodity k, when produced by process j, is v_k^j. $v \in \mathbb{R}_+^n$ is the vector of market values. D_{jk} is the share of capitalist j in the production of commodity k: $D_{jk} = \overline{\alpha}_k^j / \overline{\alpha}_k$.

Individual and market values are defined by the following set of equations:

$$\underbrace{v'^j \overline{\alpha}^j}_{\substack{\text{Individual}\\ \text{values of}\\ \text{the outputs}}} = \underbrace{v' \underline{\alpha}^j}_{\substack{\text{Market}\\ \text{values of}\\ \text{the inputs}}} + \underbrace{\alpha_0^j}_{\substack{\text{Newly}\\ \text{incorporated}\\ \text{labor}}} \quad for \quad j \in \{1, \ldots, m\} \tag{1}$$

$$\underbrace{v_k}_{\substack{\text{Market}\\ \text{values}}} = \underbrace{\sum_j v_k^j D_{jk}}_{\substack{\text{Average of}\\ \text{individual}\\ \text{values}}} \quad for \quad k \in \{1, \ldots, n\} \tag{2}$$

Equations 1 and 2 define *values* in the strict sense, which must be opposed to values *in the traditional sense* v^*. It is well known that values are traditionally computed in a different manner[6]:

$$v^{*\prime} \overline{\alpha}^j = v^* \underline{\alpha}^j + \alpha_0^j \tag{3}$$

It is interesting to notice that, when values in the traditional sense exist, and are strictly positive, they are a particular case of *values*, in the sense that $v^j = v = v^*$ are solutions of equations 1 and 2.

Individual values, v^j, can be interpreted as the traditional values of a technology derived from the initial joint technology in which we consider that two commodities are distinct if they are produced by different processes. To show how this derivation is obtained, we define the following notations:
$C = \{\ (j,k) \quad / \quad \overline{\alpha}_k^j \neq 0\}$ is the set of all commodities actually produced
$C(j) = \{\ k \quad / \quad \overline{\alpha}_k^j \neq 0\}$ is the set of commodities actually produced by process j.
$c(j)$ and c are the number of elements of $C(j)$ and C
One has: $c = \sum_j c(j)$ and $m \leq c \leq mn$. i is the index of the commodities of the derived technology: $i \in \{1, \ldots, c\}$. $k(i)$ denotes the physical product corresponding to

[6] *Using the notations introduced in the footnote above for linear formalisms, one usually writes:*

$$Bv^* = Av^* + L$$

commodity i and, $j(i)$, the unique process producing i. Notice that the commodities whose outputs are zero have been eliminated from this formalism.

We substitute the value of v given by equation 2 in equation 1 and obtain:

$$w^{*\prime}\overline{\beta}^j = w^{*\prime}\underline{\beta}^j + \beta_0^j \quad for \quad j \in \{1, \ldots, m\} \tag{4}$$

where w^* is the vector of values, in the traditional sense, for the derived technology (with c components), and $\beta^j = (-\beta_0^j, -\underline{\beta}_0^j, \overline{\beta}^j)$ defines the jth process of production of the derived technology:

$$\begin{cases} w_i^* = v_{k(i)}^{j(i)} & \\ \beta_0^j = \alpha_0^j & \\ \underline{\beta}_i^j = D_{j(i)k(i)}\underline{\alpha}_{k(i)}^j & \\ \overline{\beta}_i^j = 0 & \text{if } j \neq j(i) \\ \overline{\beta}_i^j = \overline{\alpha}_{k(i)}^{j(i)} & \text{if } j = j(i) \end{cases}$$

The formalism of values in the traditional sense (equation 3) is applied to the case of the derived technology in equation 4. The number of equations is equal to m, and the number of unknowns is c. *Therefore, in the general case, individual values as well as market values are not determinate in a unique manner.*

In spite of this indeterminacy, it is possible to state the following proposition which is important for the interpretation of the relation established by Marx between *values* and prices[7] (the proof of this proposition is evident from the definition of $\widetilde{\alpha}$ and equations 1 and 2):

Proposition 1 : *The market* value *of the net product of the whole economy is determinate in a unique manner and equal to the number of hours expended during the period:*

$$v'\widetilde{\alpha} = \alpha_0$$

C - THE CONDITIONS FOR THE EXISTENCE OF *VALUES*

We can state the general condition of positivity of individual and market values:

Proposition 2 : *If the economy is nonreductive, then strictly positive individual and market values exist.*

---

[7] *If one follows the Duménil-Foley's interpretation of the transformation problem which puts the emphasis on the net product (*DUMÉNIL G. 1980 *and* 1984, FOLEY D. 1982*).*

As a preliminary step to the proof of this proposition, we establish the following lemma:

Lemma:

If we denote M, the $m \times n$ matrix of the net products: $M = (\tilde{\beta}^1, \ldots, \tilde{\beta}^m)$ and N, the matrix M to which the line of inputs in labor power have been added: $N = \begin{pmatrix} \tilde{\beta}^1 & ,\ldots & ,\tilde{\beta}^m \\ -\beta_0^1 & ,\ldots & ,-\beta_0^m \end{pmatrix}$, then:

$$\exists z \in \mathbb{R}_+^m \quad \text{such that} \quad Mz \leq 0 \Leftrightarrow \exists y \in \mathbb{R}^m \quad \text{such that} \quad Ny \leq 0$$

Proof of the lemma:

It is evident that the second condition can be derived from the first one (for $y = z$).

We assume that the second condition is verified and define the two vectors y^+ and y^- by: $y^+ \in \mathbb{R}_+^m$, $y^- \in \mathbb{R}_+^m$, $y = y^+ - y^-$ and for all j: $y_j^+ y_j^- = 0$. From inequality $Ny \leq 0$, we derive:

$$My^+ \leq My^- \tag{5}$$

We can show that it is possible to choose $z = y^+$, i.e., that $My^+ \leq 0$. If this were not the case, one value of $i \in \{1, \ldots, c\}$ would exist for which $(My^+)_i > 0$. Consequently we would have $y_{j(i)}^+ > 0$, since commodity i is produced only by process $j(i)$. As a result of inequality 5, we also have $(My^-)_i > 0$, which, for the same reason, would entail $y_{j(i)}^- > 0$. This last derivation contradicts the condition $y_j^+ y_j^- = 0, \forall j$.

Proof of proposition 2:

If we define $x = \begin{pmatrix} w^* \\ 1 \end{pmatrix}$, equation 4 can be rewritten:

$$x'N = 0 \tag{6}$$

The problem under consideration is to demonstrate that this equation has a strictly positive solution. If the economy is nonreductive, i.e., if $p'\tilde{\alpha}^j > 0$, then there exists $p^* \in \mathbb{R}^c$ strictly positive and verifying $p^{*\prime}M \gg 0$: it is sufficient to choose $p_i^* = p_{k(i)}$. From this condition, we can infer that no vector z exists such that:

$$z \in \mathbb{R}_+^m, Mz \leq 0$$

If such a vector existed, we would have:

$$\begin{aligned} p^{*\prime}Mz &= (p^{*\prime}M)z > 0 \\ &= p^{*\prime}(Mz) \leq 0 \end{aligned}$$

The preliminary lemma establishes that the existence of this vector z is equivalent to the nonexistence of a vector y such that:

$$y \in \mathbb{R}^m, Ny \leq 0$$

A well known proposition about linear equations with positive solutions (cf. for example, Corollary 2 p. 49 in GALE D. 1960*) states that such a condition entails the existence of a strictly positive vector x which verifies equation 6, i.e., that individual values exist. Market values are also strictly positive (cf. equation 2).*

The nonreductivity of the economy is a far more general condition than any assumption concerning equilibrium. Without questioning the economic relevance of any notion of equilibrium one can contend that, if the economy is in equilibrium, positive values always exist (with the exception of situations in which both wages and profits are equal to zero). Indeed, if prices of production exist with a positive rate of profit, *values* in the above sense always exist. This solves the paradox of negative *values* as raised by Steedman.

D - *VALUES* IN SINGLE PRODUCTION

In a model of single production, each process of production produces only one commodity. We denote $i(j)$ the commodity produced by process j. We use, $v(j)$, the same notation as Flaschel for, $v^j_{i(j)}$, the value of this commodity. Thus, the output bundle of process j becomes: $\overline{\alpha}^j = \overline{\alpha}^j_{i(j)} E^{i(j)}$. Using those symbols, equations 1 and 2 are exactly equivalent to equations 1 and 2 in Flaschel's article:

$$v(j)\overline{\alpha}^j_{i(j)} = v'\underline{\alpha}^j + \alpha^j_0 \qquad (1')$$

$$v_k = \sum_{j=1}^{m} v(j) D_{jk} \qquad (2')$$

Proposition 2 can be applied to the case of single production. The number of equations is now equal to that of equations $(m+n)$. The uniqueness of the solution can be expected. In proposition 3 below, a new proof of the existence and uniqueness of values in single production is established. The concept of *average technology* is introduced in definition 2. An average technology provides what is usually called an input-output matrix:

Definition 2 : Average Technology. *Considering a set of m processes of single production, we call average technology the set of processes of production, α^{*k} ($k =$*

$1,\ldots,n)$:

$$\alpha^{*k} = \sum_{j=1}^{m} D_{jk}\alpha^{j} \tag{7}$$

Proposition 3 : *In single production, if the economy is nonreductive, a unique set of individual and market values exists. The market values are the values of the average technology.*

Proof: It is possible to eliminate the $v(j)$ in equations (1') and (2'). The market values v verify:

$$v_k \overline{\alpha}_k^{*k} = v' \underline{\alpha}^{*k} + \alpha_0^{*k} \tag{8}$$

Since the economy is nonreductive, the traditional condition concerning the productivity of the technology is also verified, and equation 8 has a unique positive solution.

II - DISAGGREGATION AND VALUES

The application of the notions of individual and market values to the case of joint production leads to the indeterminacy of values. Although this property is no obstacle to the development of the labor theory of value, it can be a problem with regard to empirical studies in which labor contents are considered. For this reason Flaschel has suggested a specific procedure in order to avoid such indeterminacy. Recall that this procedure involves two successive steps:

1. Each joint process of production is disaggregated into single processes (as a cost accountant would do).
2. Individual and market values are computed on the basis of the technology of single production which has been obtained (equations (1') and (2')).

There is nothing which can be deemed arbitrary in the second step, which is a direct application of the definition of market values to the case of single production technologies. However, the option taken in the first step is fully arbitrary: this procedure must be seen as one possibility among many. In Part II, we consider the properties common to all such disaggregations. In Section A, we investigate the properties of values imputed to particular commodities by disaggregation in the general case. Section B considers such properties for particular sets of disaggregations.

A - IMPUTED VALUES

Each joint process, α^j, is disaggregated into a number of single production processes equal to that of commodities actually produced in this process, i.e., equal to $c(j)$, the number of elements in $C(j)$.

Definition 3 : Disaggregation. *We call disaggregation of a state of the economy* $\mathcal{E}$, *a set of processes of single production* $\alpha^{jk}, (j,k) \in C$ *which verify:*

$$\overline{\alpha}^{jk} = \overline{\alpha}^j_k E^k \quad for \quad (j,k) \in C \tag{9.a}$$

$$\sum_{k \in C(j)} \alpha^{jk} = \alpha^j \quad for \quad j \in \{1, \ldots, m\} \tag{9.b}$$

Equality 9.b expresses the fact that the production of $\overline{\alpha}^j$ requires the same inputs, whether one uses the processes obtained by disaggregation or the original joint production process.

Disaggregation allows the computation of a unique vector of individual and market values. We just have to apply the method described in Section I.D. Values obtained in this manner, denoted v^I will be named "imputed values". A direct consequence of the definitions used is:

Proposition 4 : *Imputed values are a particular case of values.*

A procedure of disaggregation can depend on prices, as in the case of Flaschel's study: $\alpha^{jk} = \alpha^{jk}(p)$. Of course, this fact entails that imputed values also depend on prices:

$$v^I = v^I(p) \tag{10}$$

B - PARTICULAR CASES OF DISAGGREGATION

In this section we consider two particular cases of disaggregation relevant to the issue of negativity and recall Flaschel's procedure as an example of such cases.

1. Disaggregations Which Conserve Nonreductivity

The conditions concerning nonreductivity of the economy can be extended to disaggregations.

Definition 4 : Disaggregation Conserving Nonreductivity. *If, whenever the initial economy is nonreductive, the prices of the net products of the single processes obtained by disaggregation are positive, we say that a disaggregation conserves nonreductivity:*

$$p'\tilde{\alpha}^j > 0 \Rightarrow p'\tilde{\alpha}^{jk} > 0 \quad for \quad (j,k) \in C$$

It is possible to state the following proposition for the case of disaggregations which conserve nonreductivity:

Proposition 5 :
1. If the economy is nonreductive, and if the disaggregation conserves nonreductivity, then, imputed values exist.
2. When disaggregation depends on prices, imputed values, $v^I(p)$, have the same properties of regularity (continuity, differentiability...) with regard to prices as, $\alpha^{jk}(p)$, the disaggregation itself.

Proof:

1. The first point is a consequence of the definition of imputed values and of proposition 3.

2. Imputed values are the solutions of a linear nonsingular set of equations (equations 8), whose coefficients are linearly derived from $\alpha^{jk}(p)$ through equation 7. Therefore, they have the same properties of regularity with regard to prices as functions $\alpha^{jk}(p)$ themselves.

2. Disaggregations Which Conserve the Rate of Profit

For any process of production α^j and for any set of prices $q \in \mathbb{R}^n_+$ (where q is a priori different from, p, the actual set of prices), a rate of profit can be computed:

$$1 + r^j(q) = \frac{q'\overline{\alpha}^j}{q'\underline{\alpha}^j + \alpha_0^j} \tag{11}$$

In a similar manner, it is possible to calculate rates of profit, $r^{jk}(q)$, for the disaggregated processes α^{jk}.

Definition 5 : Disaggregations Conserving the Rate of Profit. *We say that a procedure of disaggregation conserves the rate of profit, if the rate of profit of the initial process of production is equal to those of the single processes in which it has* *been disaggregated for the actual set of prices:*

$$r^{jk}(p) = r^j(p) \quad for \quad (j,k) \in C$$

Notice that, in general, if $q \neq p$, then, $r^{jk}(q) \neq r^j(q)$.

The following proposition can be derived in a straightforward manner from the definitions:

Proposition 6 : *A disaggregation which conserves the rate of profit has the following properties:*

1. It conserves nonreductivity if, for all j, the rate of profit r^j defined in equation 11 is strictly positive.

2. If traditional values, v^*, exist (definition 3), and if, p, the actual prices are equal to these values, then, imputed values are also equal to these traditional values: $v^I(v^*) = v^*$.

3. If imputed values are equal to actual prices, then traditional values exist and are equal to imputed values: $v^I(p) = p \Rightarrow v^* = v^I(p) = p$.

3. An Example: Flaschel's Disaggregation

An example of disaggregation which conserves the rate of profit is that presented by Flaschel. It is defined by the two following properties:

1. It conserves the rate of profit for the actual prices.
2. The inputs (physical inputs and labor power) of the single processes must be proportional to those of the initial joint processes.

One obtains a proportional imputation of costs using prices as weights:

$$\underline{\alpha}^{jk} = \lambda \underline{\alpha}^j \quad and \quad \alpha_0^{jk} = \lambda \alpha_0^j \quad with \quad \lambda = \frac{p_k \overline{\alpha}_k^j}{p' \overline{\alpha}^j}$$

This disaggregation conserves nonreductivity, since:

$$p' \tilde{\alpha}^{jk} = p_i \overline{\alpha}_i^j - \lambda p' \underline{\alpha}^j = \lambda p' \tilde{\alpha}^j$$

Therefore values "à la Flaschel" exist if the economy is nonreductive (proposition 5).

Although the very general condition of nonreductivity guarantees the positivity of values "à la Flaschel", Flaschel resorts to much more restrictive conditions: the existence of "reproducible solutions" in the sense of Roemer's General Equilibrium Model (ROEMER J. 1980). The existence of reproducible solutions implies that the economy is nonreductive (see Lemma 2, p. 441, in FLASCHEL P. 1983), but the reciprocal does not hold. Moreover, in order to define values or to demonstrate that they exist, it is quite irrelevant to make an assumption concerning equilibrium (whatever the definition of equilibrium, classical, Walrasian, or in Roemer's sense). Only very fundamental conditions related to the incorporation of labor in production are material.

III - DISAGGREGATION AND PRICES

Concerning the imputation of labor to the various commodities for empirical purposes, it is not possible to define a criterium for the selection of a particular procedure of disaggregation. All imputations correspond to a specific vector of value within the general set (proposition 4). Flaschel's procedure has the advantage of being simple and seems as good as another one in this regard in order to realize studies which require the determination of labor contents.

The determination of an input-output matrix can, however, have other empirical purposes. One such purpose is the analysis of the formation of prices. For example, one central question which arises from the works of the classics is whether long term equilibrium prices (classical prices of production, which guarantee to each industry an evenly distributed rate of profit) are empirically relevant. In other words, the issue is whether actual prices "gravitate" around long term equilibrium prices ? The problem in such a study is whether the substitution of prices of production computed on the basis of the disaggregated technology for prices of production computed on the basis of the initial joint technology is of any consequence with regard to the measurement of the distance between actual prices and prices of production.

A - IMPUTED PRICES OF PRODUCTION

In what follows, we call "imputed prices of production", prices which result in equalized rates of profit for the disaggregated technology. Prices of production are denoted p^*, and imputed prices of production are denoted p^{I*}.

The issue of the existence of prices of production will not be discussed in the present study (see DUMÉNIL G., LÉVY D. 1984). We will assume that they exist:

Assumption 3 : *A set of prices of production, p^*, exists for the rate of profit r:*

$$\exists p^* \in \mathbb{R}^n \quad \textit{such that} \quad p^* \gg 0 \quad \textit{and} \quad r^j(p^*) = r$$

The following proposition which relates prices of production to imputed prices of production can be established for any disaggregation which conserves the rate of profit (definition 5). The proof of this proposition is straightforward:

Proposition 7 : *Any disaggregation which conserves the rate of profit has the following properties:*

1. If the difference between prices of production and actual prices is infinitely small, then the difference between prices of production and imputed prices of production is of the same order:

$$p^* - p^{I*} = \mathcal{O}(p^* - p)$$

2. If the difference between imputed prices of production and actual prices is infinitely small, then the difference between prices of production and actual prices is of the same order:

$$p^* - p = \mathcal{O}(p^{I*} - p)$$

Empirical investigations reveal that actual prices are not very different from imputed prices of production computed on the basis of an input-output matrix (OCHOA E. 1984). Therefore, from proposition 7.2, we should expect that actual prices are also close to prices of production p^*. Empirical evidence supporting this conclusion can be found in EHRBAR H., GLICK M. 1986. This study measures directly industrial rates of profit over a period of 30 years, and demonstrates that they are nearly equalized in the long run.[8] These results show the proximity of actual prices to equilibrium prices p^* and, a posteriori, justify the use of input-output matrices in the study of prices of production (cf. proposition 7.1).

The interest of these first conclusions concerning all disaggregations which conserve the rate of profit, is to show that the distance between actual and long term equilibrium prices, implies a similar distance between imputed prices of production and the original prices of production. However, it is possible to greatly improve this result by choosing among this set a procedure far better adapted to the study of prices. Using this specific procedure, which we call "the tight disaggregation", the same proximity of actual prices to long term equilibrium prices yields an approximate equality between imputed and original prices of production.

---

[8] *Empirical investigation also reveals that imputed values, which can be regarded as a particular case of prices of production for a zero rate of profit, are not very different from actual prices (see* OCHOA E. 1985*). Consequently, traditional values are not very different from the actual prices.*

B - THE TIGHT DISAGGREGATION

The "tight disaggregation" is defined by the two following conditions:

1. It conserves the rate of profit for actual prices.
2. The rate of profit in the single processes must be equal to that of the initial joint process, not only for, p, the actual prices, but for all prices in the vicinity of p.

The first condition, $r^{jk}(p) = r^j(p)$, defines $p'\underline{\alpha}^{jk} + \alpha_0^{jk}$:

$$p'\underline{\alpha}^{jk} + \alpha_0^{jk} = p'\underline{\alpha}^j + \alpha_0^j$$

The second condition can be written:

$$\frac{\mathrm{d}}{\mathrm{d}q}\left(r^j(q) - r^{jk}(q)\right)\bigg|_{q=p} = 0 \qquad (12)$$

These conditions allows the determination in a unique manner of $\underline{\alpha}^{jk}$ and α_0^{jk}:[9]

$$\begin{aligned}\underline{\alpha}^{jk} &= \frac{p^k\overline{\alpha}_k^j}{p'\overline{\alpha}^j}\underline{\alpha}^j + \frac{\alpha_k^j}{1+r^j(p)}\left(E^k - \frac{p^k}{p'\overline{\alpha}^j}\overline{\alpha}^j\right)\\ \alpha_0^{jk} &= \frac{p^k\overline{\alpha}_k^j}{p'\overline{\alpha}^j}\alpha_0^j\end{aligned}$$

This disaggregation conserves nonreductivity, therefore the imputed values thus obtained are positive if the initial economy is nonreductive. However, our interest in this specific disaggregation lies in an additional property it possesses in relation to prices:

Proposition 8 : *If the difference between prices of production, p^*, and actual prices, p, is an infinitely small magnitude of the first order, then the difference between prices of production and, p^{I*}, the imputed prices of production computed on the basis of the tight disaggregation, is an infinitely small magnitude of the second order:*

$$p^* - p^{I*} = O^2(p^* - p)$$

---

[9] *The first condition is identical to that used in the disaggregation "à la Flaschel". The second condition is substituted for Flaschel's requirement concerning proportionality. α_0^{jk} is the same as used by Flaschel. The first term $\underline{\alpha}^{jk}$ is also identical. In our formula, some elements of the single production matrix may be negative, due to the effect of the second term (for "very joint" technologies). This negativity does not ruin the validity of propositions 5 and 8.*

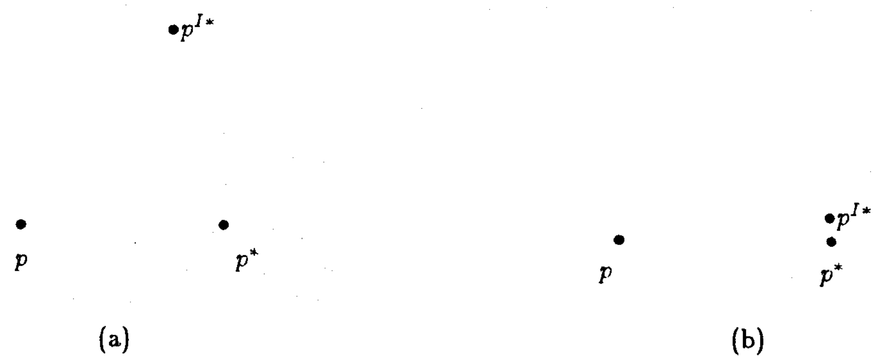

Figure 1 - A Comparison Between Two Disaggregations

Figure (a) corresponds to the case of a disaggregation "à la Flaschel". According to proposition 7, if the distance between prices of production and actual prices is small, the distance between actual prices and imputed prices of production is also small. Figure (b) corresponds to the case of the tight disaggregation (proposition 8). The same distance between actual prices and imputed prices of production implies that prices of production are infinitely closer to the imputed prices of production. Consequently, the distance obtained between actual prices and imputed prices of production is a good approximation for the distance between actual prices and prices of production.

Proof: From condition 12, we derive:

$$r^{jk}(p^*) = r^j(p^*) + O^2(p^* - p)$$

The definition of p^* implies that $r^j(p^*) = r$. Thus the $r^{jk}(p^*)$'s are equal to r, for all j and k, with an infinitely small approximation of the second order. The same result holds for the rates of profit of the average technology: $\overline{r}^k(p^*) = r + O^2(p^* - p)$. Since all the functions are continuously differentiable, one obtains: $p^{I*} = p^* + O^2(p^* - p)$.

The tight disaggregation can be used in order to compute imputed prices of production. Since we know from empirical investigation that actual prices are close to prices of production, we can deduce that prices of production are themselves very close to imputed prices of production (proposition 8). Therefore, when we compute the distance between actual prices and imputed prices of production, we obtain a very good approximation of the distance between actual prices and original prices of production as suggested in figure 1 in relation to the approximation obtained using Flaschel's disaggregation.

IV - DISAGGREGATION AND TRADITIONAL VALUES

Although we believe that the theoretically correct definition of value in joint production is that considered so far which refers to the concepts of individual and market values, this final part of the study is devoted to the definition of a set of disaggregations which generalize the traditional conception of values. We recall that, by "traditional conception", we mean a method of computing values which does not rely on the distinction between individual and market values (equation 3). In what follows, such generalized values will be called "imputed traditional values" and denoted v^{I*}.

This generalization mathematically solves the problem of the negativity of values within the traditional conception of value:

1. When values in the traditional sense exist, the imputed traditional values are equal to them.
2. When values in the traditional sense do not exist (which is the general case if $m \neq n$, and can be the case if $m = n$), the imputed traditional values are still defined and positive, under general conditions.

On the basis of any method of disaggregation based on prices $\alpha^{jk} = \alpha^{jk}(p)$, it is possible to construct another disaggregation which does not refer to prices, but to, v^{I*}, the unknown values themselves:

$$\alpha^{jk} = \alpha^{jk}(v^{I*}) \tag{13}$$

It is possible to apply to this disaggragation the general formalism as defined in Section II.A. Imputed traditional values verify (cf equation 10):

$$v^{I*} = v^{I}(v^{I*}) \tag{14}$$

Prices are not taken into account in this disaggregation and thus these values only depend on the technology.

The difficulty in the determination of these values is that they must be determined simultaneously with the disaggregation. Consequently, the existence of the disaggregation and that of values must be proved simultaneously.

The two following propositions can be established. They correspond to the two properties of imputed traditional values mentioned above. The first one shows that imputed traditional values generalize traditional values. The second one sets out the conditions for the existence of imputed traditional values.

Proposition 9 : *We consider a technology defined by α^j $j = 1, \ldots, m$, and a* *disaggregation $\alpha^{jk}(p)$ which conserves the rate of profit. When the technology α^j possesses a set of traditional values (equation 3), imputed traditional values are equal to these traditional values.*

Proof: We disaggregate the joint processes using disaggregation $\alpha^{jk}(p)$ for the vector of prices v^{I*} : $\alpha^{jk} = \alpha^{jk}(v^{I*})$. Since the disaggregation conserves the rate of profit, the rates of profit for each process are equal to that of the initial process of production : $r^{jk}(v^{I*}) = r^j(v^{I*})$ which, because of the definition of values in the traditional sense are equal to zero: $r^j(v^{I*}) = 0$. The rates of profit of the average technology are also equal to zero, since they are equal to the average of rates of profit of the single processes. Consequently, v^{I*} is the vectors of values of the average technology, i.e., the vector of values corresponding to the disaggregation defined by equation 13. Thus v^{I*} is the vector of imputed traditional values.

Proposition 10 : *If $\tilde{\alpha}$, the net product of the economy is strictly positive, v^{I*}, the imputed traditional values exist.*

Proof: We denote X the set of vectors $\lambda \in \mathbb{R}^n_+$, such that their scalar product with $\tilde{\alpha}$ equals α_0 :

$$X = \{\lambda \in \mathbb{R}^n_+ \quad / \quad \lambda'\tilde{\alpha} = \alpha_0\}$$

The set X is convex, closed, and is bounded (since $\tilde{\alpha} \gg 0$). Thus it is compact. The mapping $p \rightarrow v^I(p)$ (equation 10) is continuous (see proposition 5.b), and leaves X invariant (see proposition 1). Therefore Brouwer's theorem can be applied and imputed traditional values exist which are a fixed point of the mapping (equation 14).

In order to use Brouwer's theorem, X must be a closed set. Thus, some components of v^{I*} can be equal to zero.

This generalization of traditional values seems interesting, since it corresponds to a self-contained definition of values. Its use in empirical research is limited by the difficulty of computing the fixed point in $p \rightarrow v^I(p)$.[10]

---

[10] *This study can be extended to prices of production. The disaggregation is performed using prices of production themselves : $\alpha^{jk} = \alpha^{jk}(p^*)$, in which the prices of production are computed on the basis of the average technology which results from the disaggregation itself. It is possible to show that such imputed prices of production (self-imputed) exist and generalize prices of production. In this case, the fixed point must be computed for each value of r. The average technology obtained also depends on r.*

CONCLUSION

In the present study we first clarify the nature of the problem of calculating values in models of joint production and propose a formal solution to this difficulty by applying Marx's concepts of individual and market values directly to the formalism of joint production. We argue that, most, if not all, of the difficulties in the controversy over the positivity of values in joint production arise from the lack of distinction between two qualitatively different definitions of "values", which correspond to two theoretically distinct fields. The original definition of values in the labor theory of value (the value of a commodity is determined by the quantity of labor expended to produce it) has been confused with Marx's use of the word "values" to mean a specific set of equilibrium prices which would guarantee the equal remuneration to individual producers in a noncapitalist commodity economy. The former definition belongs to the law of value. The latter belongs to the theory of exchange, and is of the same nature as the concept of prices of production in a capitalist economy, which guarantee an equal remuneration to capitalists (uniform rate of profit).

In joint production, the conditions for the existence of a set of equilibrium prices in either a capitalist or noncapitalist commodity economy do not coincide with those necessary for the determination of positive values in the labor theory of value sense, and are, in fact, much more restrictive. This difference is easy to grasp if one considers the case of the production of one commodity by two processes of single production of which one is technically very inferior. It is possible to compute the average labor time necessary for the production of one unit of this commodity, but it is not possible to find a unique price yielding the same remuneration to the two producers. Within a joint production technology the same type of conflicting situation can manifest itself in more complex configurations.

In the case of single production technologies when the number of processes exceeds the number of products, as well as in a joint production economy, the traditional formalism for the determination of values (equation 3) should not be used to calculate value in the labor theory of value sense, because it cannot capture the fact that if different production processes exist for the same commodity, they need not require the same amount of labor time. In both the above cases, *values* must be defined as average values in order to take into account these differences in the conditions of production.

This heterogeneity in the conditions of production implies the reference to Marx's concepts of individual and market values which have been devised to account for such heterogeneity. In part I of this study we define individual and market values in a model of joint production and state the conditions for their positivity. In this framework it appears that the positivity of *values* is obtained provided that the economy is *nonreductive*. This new concept of nonreductivity is very general. In a reductive economy, the set of prices which exists does not yield a positive net product (corresponding to wages and profits taken together) in every activity. If we abtract from the problem of returns, reductivity means that it is possible in such an economy to expend labor in a set of activities and to "reduce", i.e. diminish in all of its components, the bundle of inputs — indeed, a possibility which conflicts with the notion of production that underlies the labor theory of value. The strict application of the notions of individual and market values to the case of joint production solves the problem of the negativity of values. However, it must be stressed that in this framework, individual and market values remain are not determinate in a unique manner.

Although the property of indeterminacy poses no obstacle to the labor theory of value, it can be a problem with regard to empirical studies. The second group of results obtained address the usefulness of three different types of disaggregations for different empirical problems:

1. If the scope of the investigation is limited to the determination of labor contents of individual commodities, a large number of procedures for the imputation of labor time can be defined. An example of such method is that proposed by Flaschel (FLASCHEL P. 1983). We demonstrate that the properties which Flaschel's disaggregation possesses are shared by a large set of other procedures. His method does have the evident advantage of simplicity.

2. However, if the interest is not limited to values, but extended to prices and, in particular, to the analysis of competition, it is clear that other methods can be proposed. We have defined such a method called "tight disaggregation", which illustrates this possibility. The disaggregation obtained in this manner produces the important result that, if prices are close to long term equilibrium prices (with a uniform rate of profit), equilibrium prices computed on the basis of the disaggregated technology are *very* close to the original long term equilibrium prices determined on the basis of the initial nondisaggregated technology.

A last result is the generalization of the traditional definition of values by disaggregation, which allows the computation of values (which do not depend on prices), which are *equal* to traditional values when they exist, and *positive* under very general conditions, if these traditional values do not exist.

REFERENCES

DUMÉNIL G. 1980, *De la valeur aux prix de production*, Économica, Paris.

DUMÉNIL G. 1984, *The So-Called "Transformation Problem" Revisited: A Brief Comment*, Journal of Economic Theory, Vol. 33, # 2, pp. 340-348.

DUMÉNIL G., LÉVY D. 1984, "The Unifying Formalism of Domination: Value, Price, Distribution and Growth in Joint Production," *Zeitschrift für Nationalökonomie, Journal of Economics*, Vol.44, # 4, pp. 349-371.

DUMÉNIL G., LÉVY D. 1986(a), The Dynamics of Competition : A Restoration of the Classical Analysis, Cambridge Journal of Economics, Forthcoming.

DUMÉNIL G., LÉVY D. 1986(b), Values and Natural Prices Trapped in Joint Production Pitfalls, CEPREMAP, Paris.

EHRBAR H., GLICK M. 1986, An Econometric Model of Industrial Profit Rate Adjustment: The United States and Four European Countries, University of Utah, Salt Lake City.

FLASCHEL P. 1983, "Actual Labor Values in a General Model of Production," *Econometrica*, Vol. 51, # 2, pp. 435-454.

FOLEY D. 1982, "Value of Money, the Value of Labor Power and the Marxian Transformation Problem," *Review of Radical Political Economy*, Vol. 14, pp. 37-47.

GALE D. 1960, *The Theory of Linear Economic Models*, McGraw-Hill Book Company, inc., New York, Toronto, London.

MARX K. 1863, *Capital, Volume III*, First Vintage Book Edition, New York, 1981.

MARX K. 1867, *Capital, Volume I*, First Vintage Book Edition, New York, 1981.

MORISHIMA M., CATEPHORES G. 1978, *Value, Exploitation and Growth; Marx in the Light of Modern Economic Theory*, McGraw-Hill Book Company (UK) Limited, London.

MORISHIMA M. 1973, *Marx's Economics, A Dual Theory of Value and Growth*, Cambridge University Press, Cambridge.

MORISHIMA M. 1976, "Positive Profits with Negative Surplus Value: A comment," *The Economic Journal*, Vol. 86, Septembre, pp. 599-603.

OCHOA E. 1984, Labor Values and Prices of Production: an Interindustry Study of the U.S. Economy, 1947-1972, Dissertation, New School for Social Research, New York.

OCHOA E. 1985, The Convergence of Prices and Labor-Values, U.S. Evidence and Theoretical Implications, California State University, Los Angeles.

ROEMER J. 1980, "A General Equilibrium Approach to Marxian Economics," *Econometrica*, Vol. 48; pp. 505-530.

ROEMER J. 1981, *Analytical Foundations of Marxian Economic Theory*, Cambridge University Press, Cambridge.

SRAFFA P. 1960, *Production of Commodities by Means of Commodities, Prelude to a Critique of Political Economy*, Cambridge University Press, Cambridge.

STEEDMAN I. 1975, "Positive Profits with Negative Surplus Value," *The Economic Journal*, March, pp. 114-123.

WOLFSTETTER E. 1976, "Positive Profits with Negative Surplus Value: A Comment," *The Economic Journal*, Vol. 86, pp. 864-876.

CONTENTS