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# *DYNAMIC CONSISTENCY OF GOVERNMENT'S BEHAVIOR : A USER'S GUIDE(1)*

by

Daniel COHEN
CEPREMAP

Philippe MICHEL
Université Paris-I

N° 8605

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***ABSTRACT***

DYNAMIC CONSISTENCY OF GOVERNMENT'S BEHAVIOR : A USER'S GUIDE

The paper investigates a number of questions related to the dynamic inconsistency of government economic policy : when is the optimal progam likely to be time inconsistent ? How to select a time-consistent policy when the government cannot pre-commit itself for the future ? Does there always exist a rule which dominates the time-consistent policy ? These questions are answered in a stochastic linear quadratic framework and two examples dealing with the inflation-output trade-off and the consumption function are examined.



***RESUME***

COHERENCE TEMPORELLE DES CHOIX DU GOUVERNEMENT : UN GUIDE DE L'UTILISATEUR

Cet article analyse un certain nombre de questions liées à la cohérence temporelle des choix du gouvernement : Quand la politique optimale est-elle, en général, dynamiquement inconséquente ? Comment un gouvernement peut-il choisir sa politique temporellement conséquente ? Existe-t-il toujours une règle d'intervention qui domine strictement la politique temporellement conséquente ? Les réponses à ces questions sont données dans le cadre d'un modèle linéaire quadratique stochastique. Deux exemples sont traités qui portent sur la courbe de Phillips et la fonction de consommation.



MOTS CLEFS : Government Policy. Control Theory
Politique Economique. Théorie du Controle

J.E.L. Nomenclature : 300

## 1 - INTRODUCTION

It is well known since Kydland and Prescott (1977) and Calvo (1978) that a government selecting its optimal policy cannot use the tools of an engineer playing against nature. The reason is traditionnally explained as follows : a government, say, in 1985 would like to announce its policy for 1990 in order to influence the 1985 decisions of the private sector. When 1990 comes, however, the government does not care anymore about 1985 so that its optimal policy by then is "unlikely" to coincide with its original announcement. Its optimal policy is then said to be "time-inconsistent". In this paper, we investigate a number of questions related to this problem. We ask : how "likely" is it that the optimal policy of a government is time-inconsistent ? How could a government then select its policy when it cannot pre-commit itself for the future ? How should the econometrician help a time-consistent government to pick up its best policy ? Finally, does there always exist a rule which dominates the time-consistent equilibrium ? Even though there are some new results in this paper, we primarily view it as a summary guide to how one should cope with the time-inconsistency problem.

First, how likely is it that a government will face a time-inconsistency problem ? Think of a benevolent government, why should it like to misguide the private sector's beliefs ? It can only be due to the fact that even a benevolent government cannot always reach the "single controller" solution : ie, what the government would achieve if it could perfectly decide of what the private sector should do. In the framework we analyze, this will happen because of externalities, which we shall classify into two types : a purely private externality ("PPE") resulting from the effect of aggregate private sector actions onto the individual's choice ; a private externality onto the government's actions ("PGE"), a typical example of which being taxes which depend upon private decisions (ie, distortionary taxes). With none of these externalities, we shall see that a benevolent government does not face the time-inconsistency problem.

Now, let us assume that a new government is elected each period and that it optimizes its own criterion (the government is not necessarily benevolent). How should the government find its time-consistent policy ? It cannot use the first order conditions of the private sector : this would yield the time-inconsistent solution. We show that it only needs to know the equilibrium forecast rules of the private sector and postulate that they are independent of its own action. This is a simple technique of which we show the usefulness in an example dealing with price expectations in a Taylor (1980)-like model.

This technique almost seems the way econometric model users operate : they do postulate that the private sector equilibrium rules are independent of government's actions. However, this is where the Lucas (1976) Critique fits in. Even though a time-consistent government should not believe that it has the leverage to modify the private sector rules, these rules do depend (at equilibrium) upon the government's targets. To deal with a shift of these targets, we restrict our analysis to the simple case of the linear quadratic model (in which all future variables are viewed with subjective certainty ; see Sargent (1979) (1981) for a wide class of applications). In that case, we show that the equilibrium feed-back rules only vary in a limited sense : it is only the residuals and the constant terms which change, and not the parameters relating the private sector's actions to the state variables of the economy. For instance, in the case when government targets follow a random walk, the econometrician should only change ("once and for all") the constant term of his equation in order to cope with the Lucas Critique. We illustrate this point with an example dealing with the consumption function. (In our example, the government should postulate a keynesian consumption function in a case where the first order conditions of the private sector would yield a random walk consumption path).

Since the time-consistent equilibrium is sub-optimal, Kydland and Prescott argued in favor of rules against discretion. It is still, however, an open question to know if there always exists a fixed rule which strictly dominate the time-consistent solution. We show explicitly that there does exist such a superior rule. The rule we suggest has many interesting features : it corresponds to the "no-surprise" solution to the inflation-unemployment trade-off ; it induces a "multiplier" to government policy ; and furthermore it keeps the simple properties of the time-consistent equilibrium with respect to the Lucas Critique.

In section 2, we set up the model. In section 3 we analyze the sources of dynamic inconsistency. In section 4 we show how to calculate the time-consistent equilibrium. In section 5, we draw the implications of our analysis to the Lucas Critique. In section 6 we analyze the problem of finding a superior rule.

## *II - THE MODEL*

We deal with an economy which consists of a public and a private sector. Each sector minimizes a quadratic loss function, and all stochastic parameters are forecasted with subjective certainty because of the certainty equivalence principle. (See Miller and Salmon (1985) for a similar set up in a deterministic framework).

Each agent understands the behavior of the economy and expectations are rational in a sense we shall make more precise below. The private sector consists of infinitely many small agents and each of them takes as given the actions of the rest of the economy. On the other hand, the government does understand the impact of its decisions on the economy and takes it into account when formulating its policy.

We shall analyze two equilibria. One in which the government can pre-commit itself to follow in the future a set of pre-announced contingency plans. We shall call it "equilibrium with pre-commitment". The other equilibrium that we shall study, the "time-consistent equilibrium", is the equilibrium obtained by backward induction : at each period, the government selects its best move, when its future actions are also expected to obey the same principle. It is well known since Kydland and Prescott (1977) and Calvo (1978) that the equilibrium with pre-commitment is usually time-inconsistent : after some time, the government will not find it optimal to follow the plan initially announced. On the other hand, under the same circunstances, the time-consistent equilibrium will be sub-optimal.

a) - Private sector decisions

Each private agent is characterized by two vectors : his own state variable $x_i(t)$ and a set of decisions $u_i(t)$. He minimizes an expected quadratic loss function $E_0 \Sigma \beta^t Q [x_i(t), u_i(t), L_i z(t)]$ in which $z(t)$ is a vector of aggregate variable which we define below and $E_0$ is the expectational operator as of time 0. We assume quadratic loss function and linear evolutions equations. Each agent contemplates a linear law of motion :

(1) $\quad x_i(t+1) = A x_i(t) + B u_i(t) + C_i z(t)$ ; and some linear long run constraint $\lim_{t\to\infty} \Psi(t) x_i(t) = 0$

We assume all private agents to be identical except as for the influence of aggregate variables (on their loss function and on their law of motion).

The optimal decision of agent i can be written (see appendix 1) :

$$(2) \quad u_i(t) = N x_i(t) + \sum_{s=0}^{\infty} M_{is} E_t z(t+s)$$

Aggregating (1) and (2) for all agents i, one finds :

$$(3) \quad x(t+1) = x(t) + B u(t) + C z(t)$$

$$(4) \quad u(t) = N x(t) + \sum_{s=0}^{\infty} M_s E_t z(t+s)$$

in which we dropped the index i for all aggregate sums [eg., $x(t) \equiv \sum_i x_i(t), C = \Sigma C_i$, etcetera...)].

In this framework, the private agents' aggregate behavior can be characterized by a unique representative private agent who regards $z(t)$ as independent of his own actions. Conversely, one can disaggregate (3)-(4) into (1)-(2) (see appendix 1).

b) - Agregate variables and government's behavior

$z(t)$ is a vector of aggregate variables :

(5) $z(t) = [x(t), u(t), \bar{u}(t), g(t), \omega(t)]$

$[x(t), u(t)]$ is the aggregate vector appearing in (3)-(4). Note that the effect of these two variables on agent's i behavior implies an externality of the other agents' decisions. $\omega(t)$ is an information vector which follows a Markovian law of motion (this fits all Arima processes) :

(6) $\omega(t+1) = \Omega\ \omega(t) + \eta(t)$

in which $\eta(t)$ is a white noise. We assume that $\omega(t)$ is known at time t to all agents, including the government.

$\bar{u}(t)$ is the control variable of the government and $g(t)$ is the government's own state variable (say, government's debt). $g(t)$ follows a linear law of motion :

(7) $g(t+1) = Dz(t)$ ; $g(0)$ given

$\left\{\bar{u}(t), g(t)\right\}_{t\geq 0}$ are determined by the government so as to minimize its own loss function $\Sigma\ \bar{\beta}^t\ \bar{Q}\ [z(t)]$, in which $\bar{Q}$ is a quadratic form. We discuss below the various strategies that a government might chose.

Remark :

In many instances below, we shall only need the assumption that $u(t)$ can be written as in (4). In particular, when a government's policy depend upon the forecast of some foreward-looking agents, a structure such as (3), (4) might obtain even when the private sector behavior does result not from explicit intertemporal optimization (see example 1 below). In this sense, determining the equilibrium value of $\left\{u(t)\right\}_{t\geq 0}$ is equivalent to determine the rule used by the private sector to calculate $E_t\ z(t+s)$.

## *3 - GOVERNMENT'S OPTIMAL CHOICE AND THE THREE SOURCES OF DYNAMIC INCONSISTENCY*

When the government can pre-commit itself (say, by Law) to follow some strategy, it will usually pick up a time varying contingency plan $\left\{\bar{u}_s[x_s, g_s, \omega_s]\right\}$ to be played at each time s. In such a world, the policy regime is set once and for all (in particular, there is no Lucas Critique to examine). This optimum contingency plan usually yields a time-inconsistency problem : if the government were to re-optimize later on, it would not stick to the strategy it has initially chosen. This comes from the following reason : when the governments pre-commits itself at time zero to follow some strategy in the future, its choice takes into accounts its influence on $u_0$ , chosen by the private sector initially. If the government were to re-optimize at a later time $t > 0$, $u_0$ would be a bygone of the past, and the government's incentives would be changed.

Let us now ask the following question : when will it be the case that this time-inconsistency problem will <u>not</u> occur ? To deal with this question, let us define another concept : <u>the single controller solution</u> (1). It is the solution that would be achieved by a single controller who could pick up $\bar{u}(t)$ and $u(t)$ as his control variables, and minimize the government's loss function. Given this definition, we argue :

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(1) It is called the "team" equilibrium in Control Theory.

<u>Time inconsistency does not occur if and only if the government can achieve the single controller solution</u> (1).

It is clear that if the government can attain the single controller optimum, then, this optimum solves the backward recursion algorithm so that there is no reason why the government would revise its strategy as time passes. On the other hand, when the government cannot attain the single controller equilibrium, then it will always have an incentive to improve its pay -off by influencing private decision through the manipulations of the private sector's expectations : this is the seed of the time- inconsistency problem (see appendix 2).

Once this equivalence between the time-inconsistency problem and the single-controller equilibrium is recognized, we can ask : when will it be that the single controller equilibrium cannot be achieved by the government (even when it is allowed to pre-commit itself)? We shall distinguish three causes :

a) - <u>Purely private externalities ("PPE")</u>

Let us decompose the aggregate vector z(t) into :

(5') $z(t) = [z_0(t), z, (t)]$

where $z_0(t) = [x(t), u(t)]$ ; $z_1(t) = [\bar{u}(t), g(t), w(t)]$

Purely private externalities result from the effect of $z_0(t)$ onto the individual's behavior. This will occur in two instances : (a) when $z_0(t)$ appears in the law of motion of the individual's state variable, ie when $Cz(t) = C_0 z_0(t) + C_1 z_1(t)$ with $C_0 \neq 0$ in equation (3) ; (b) when $z_0(t)$

(1) See Hillier and Malcomson (1984) for a related analysis.

appears in the individual agent's objective function, ie when $Lz(t) = L_0\ z_0(t) + L_1\ z_1(t)$ and $L_0 \neq 0$ the definition of the private sector's loss function $Q[x(t),u(t),Lz(t)]$.

In both cases, it is easy to see that the the single controller equilibrium cannot be achieved in general (the first order conditions solved by each private sector are inappropriate).

b) - Private externalities onto government's actions ("PGE")

Let us now write the law of motion of the government's state variable : $g(t+1) = D_0\ z_0(t) + D_1\ z_1(t)$. When $D_0 \neq 0$, the private sector's actions interact with the government's state variable. One typical example will be the case when $g(t)$ is public debt and when $D_0\ z_0(t)$ measures (distortionary) tax collections (when $D_0 = 0$, all taxes are lump sum, when $D_0 \neq 0$, they depend upon the private agents' state or control variables).

In this "PGE" case again, externalities keep the government from achieving the single controller problem (the private sector' fails to recognize the influence of his decisions on the government's state variable).

c) - Government's non benevolence

A last source of dynamic inconsistency will come from the government's behavior itself. When the government's loss function does not coincide with the representative agent's criterion, ie when the government is not a "benevolent" government, then again the conflict between the private sector's first order conditions and the government's may keep the latter from achieving the single controller equilibrium.

When none of these three problems arise, we show (in appendix 2).

When there are no private externalities ("PPE" or "PPG") and when the government is benevolent, then the single controller equilibrium can be achieved and there is no time-inconsistency problem.

The reciprocal is a little more complex because the three sources of time inconsistency may neutralize their effects. Except in such cases, however, the converse will hold true (see appendix 2).

## 4 - *TIME-CONSISTENT EQUILIBRIUM*

Let us now restrict our analysis to the time-consistent equilibrium. We us assume that, each period, the government and the private sector chose their actions at the same moment (independently), and that the government has no leverage to pre-commit itself. In a finite horizon economy, the time-consistent equilibrium would be unambiguously determined by backward induction: the government at time t and the private sector know what governments at time t+1,... T will find it optimum to do. At time t, when the information $\omega(t)$ is disclosed, the private sector expects the government behavior to respond optimally to it. Taking as given these private sector's forecasts, the government plays its optimal move. The equilibrium is attained when the private sector's forecasts coincide with the government's actions.

In an infinite horizon model, the time-consistent equilibrium cannot be obtained by backward recursion (there is no last period to start the recursion with). Following Kydland and Prescott, we can then define the equilibrium as follows. Assume that the government is believed (both by itself and by the private sector) to follow a rule $u^f(x,g,\omega)$ in the future. Call $(Z\ u^f)\ (x,g,\omega)$ the rule that it would then follow to day in response. A time-consistent equilibrium is a fixed point of the mapping Z : in the linear quadratic case it is a rule $\bar{u}(x,g,\omega) = F_0\ x + F_1\ g + F_2\ \omega$ such that it will be applied to day if it is expected for to-morrow.

There is another (and equivalent) way to find the time-consistent equilibrium. Assume that the government is to play a time-invariant rule $\bar{u}[w,g',\omega]$ all the time. The dynamics of the system would then be described by :

$$
8)\begin{cases}
(8a) & x(t+1) = A\,x(t) + B\,u(t) + C\,z(t) \\
(8b) & u(t) = N\,x\,(t) + \sum_{s=0}^{\infty} M_s\,E_t\,z(t+s) \\
(8c) & g(t+1) = Dz(t) \\
(8d) & \omega(t+1) = \Omega\,\omega(t) + \eta(t) \\
(8e) & z(t) = [x(t), u(t), \bar{u}(x,g,\omega), g(t), \omega(t)]
\end{cases}
$$

The system is solved by finding the value u(t) which puts the system on the stable manifold (1). This is obtained for some linear rule :

$$u(t) = F_0^{\bar{u}}\,x(t) + F_1^{\bar{u}}\,g(t) + F_2^{\bar{u}}\,\omega(t)$$

Call $T_1 : \bar{u} \rightarrow u$ this mapping.

Now, conversely, assume that a rule u (  ) is followed by the private sector <u>independently of the government's decisions</u>. The government will then find it optimal to select a rule $\bar{u}(x,g,\omega)$ which minimizes its loss function under the postulated law of motion :

$$x(t+1) = A\,x(t) + B\,u(x,g,\omega) + C\,z(t)$$

(instead of (7-a).

Call this new transformation $T_2 : u \rightarrow \bar{u}$.

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(1) Which is also the solution of the private sector problem when it expects $\bar{u}(x,g,\omega)$ to be played by the government.

The time-consistent equilibrium is a fixed point of the mapping:

$$H = T_2 \circ T_1$$

(see appendix 3). (Note that H does not coïncide a priori with the mapping Z defined above). This result has the following consequence. When $u(x,g,\omega)$ is set at its equilibrium value, the time-consistent equilibrium is obtained by considering $\bar{u}(x,\omega) = [T_2\ u][x,g,\omega]$: The government selects its best response believing that the private sector rule is not influenced by its own actions (1). In view of the remark we did after equation (7), this is equilivalent to take as given the rules which are used by the private sector calculate $E_t\ z(t+s)$ as a function of $(x_t, g_t, \omega_t)$. We can summarize this result as follows :

<u>In order to reach the time-consistent equilibrium, the government only need to know the equilibrium rule of the private sector (or their forecast strategy) and must postulate that they are is independent of its own actions</u>.

This is not a very surprising result since the actions of a consistent government only depend upon the state variables. It is however a very useful one as we show in the following example.

<u>Example 1</u>

Consider the following price formation mechanism

$$(9)\quad \begin{cases} (9a)\ p(t) = a\ p(t-1) + b\ E_t\ p(t+1) + C\ y(t) + \omega(t) \\ (9b)\ \omega(t+1) = \omega(t) + \eta(t) \\ (9c)\ a + b = 1\ ;\ b < 1/2. \end{cases}$$

(1) The time-consistent looks like a "Closed Coop Nash" equilibrium. The difference comes from the private sector behavior : it results from the aggregate of infinitely many individual decisions whose influence onto aggregate variables are not taken into acount (as they would be in a "Closed Loop Nash" solution).

p(t) are prices at time t. They depend upon past and expected prices ; y(t) is a measure of excess demand which, we assume, the government can control ; ω(t) is a stochastic disturbance which follows a random walk. At time t, the government knows p(t-1) and ω(t) (which is the best predictor of all future ω(t+s), s > 0). (This price formation mechanism can be obtained from the Taylor (1980) staggered contract setting). It fits the format of our model when one sets

$$x(t) = p(t-1) \;;\; u(t) = E\,[p(t+1) \mid \omega(t), x(t)] \equiv E_t\, p(t+1) \;;\; \bar{u}(t) = y(t)$$

Now, let us assume that the government minimizes :

$$\sum_{t=0}^{\infty} \beta^t \left\{\pi(t)^2 + \emptyset\, y(t)^2\right\} \;;\; p(-1) \text{ given } ;\; \pi(t) = p(t) - p(t-1)$$

In this framework, the time-consistent government must take as given the forecast rule $E_t\, p(t+1)$ selected by the private agent. The reader will check that the equilibrium rule is given by :

$$(10) \qquad E_t\, p(t+1) = p(t-1) + \frac{2}{(1-2b+C^2/\emptyset)} \,.\, \omega(t)$$

to which the government responds by a contra-cyclical intervention rule :

$$y(t) = -\frac{C}{\emptyset}\,\pi(t)$$

which can be solved into :

$$y(t) = -\frac{C}{C^2+(1-2b)\emptyset}\,\omega(t)$$

We see that from (10) that the time-consistent equilibrium is obtained by contemplating a static trade-off between inflation and output which must be postulated to be :

$$\pi(t) = C\, y(t) + \frac{1-b+C^2/\emptyset}{1-2b+C^2/\emptyset}\, \omega(t)$$

(The time-inconsistent equilibrium, instead, would spread out the trade-off over the entire future).

We shall return to this example below and compare the time-consistent solution to a technique proposed by Chow (1980).

## *V - AN APPLICATION OF THE LUCAS CRITIQUE*

The Lucas Critique of econometric policy evaluation can be summarized as follows : the users of econometric models freely vary government policies assuming private sector behavior to be independent of these changes of policy actions. "Everything we know about dynamic economic theory indicates that this presumption is unjustified".

In our analysis above, we saw that the government should postulate that private sector rules are invariant to its own action. There is no contradiction with Lucas' point. In our framework the question is : which private rules should the government take as given ? and the Lucas Critique then becomes : how do these rules (believed to be invariant) vary with the government's objective? The crucial feature of our analysis above is that these changes cannot be exploited by the government even though they must be forecasted by the econometrician. (If they were exploited , they would yield a time inconsistent policy).

Let us now investigate in more details how does the Lucas Critique operate within our framework. To simplify the discussion,let us be more specific on the government's objective. Assume that the government's loss function can be written $\sum_{0}^{\infty} \beta^t\, \bar{Q}\, [z(t)]$, in which :

$$\bar{Q}[z(t)] = \bar{Q}_0[x(t)-x^*(t)] + \bar{Q}_1[u(t)-u^*(t)] + \bar{Q}_2[\bar{u}(t)-\bar{u}^*(t)]$$

$$+ \bar{Q}_3\ [g(t)-g*(t)]$$

where $\bar{Q}_i$ (i=0,1,2,3) are quadratic forms.

$\lambda^*(t)= [x^*(t),u_g^*(t),u^*(t),g*(t)]$ defines a set of target and ($\bar{Q}_0$, $\bar{Q}_1$, $\bar{Q}_2$ $\bar{Q}_3$) correspond to the weight the government attributes to these targets. By convenience, let us assume that $\lambda*(t)$ is part of the information vector $\omega(t)$ in a way such that one has :

$$\omega(t) = [\lambda*(t),\ \omega_0(t)]$$

$\omega_0(t)$ is a (sub-) vector of information.

a) - "Once and for all" shifts of government's targets

Let us first assume that $\lambda*(t)$ is a non-stochastic parameter which stays constant :

$$\lambda*(t) = \lambda$$

In Lucas' terms, a change of regime is a discrete "once and for all" shift of some policy parameters of the government. Consider such a shift in the value of $\lambda$. How should the econometric model user deal with it ? We see that he only needs to shift the constant terms of the feed-back rules of the private sectors. In effect the equilibrium feed back rule of the private sector can be written

$$u(t) = F_0\ \lambda + F_1\ x(t) + F_2\ g(t) + F_3\ \omega(t)$$

Whenever $\lambda$ is believed to stay constant, a "once and for all" shift of $\lambda$ therefore shifts "once and for all" the constant term of the private sector rules. This simple structure is a by-product of two important assumptions: (a) the Government acts consistently ; (b) only $\lambda$ is assumed to shift, and not $(\bar{Q}_0, \bar{Q}_1, \bar{Q}_2, \bar{Q}_3)$ which implies that $(F_0, F_1, F_2, F_3)$ are structural parameters.

b) - "Recurrent" shifts of regimes

As Sims (1982) noted, "once and for all shifts" of policy regime are, by definition, rare events. In what circunstances can one allow each government to think of its target vector $\lambda$ to be shifted forever ? This will happen when the stochastic vector $\lambda(t)$ follows a random walk. Because of the certainty equivalence, each government will hold with subjective certainty that future targets of future governements are identical to its own. In such a context, we can interpret any realization of the stochastic variable $\lambda*(t)$ as a shift of regime. Exactly as in the preceding paragraph, each government will only need to change the equilibrium feed-back rule once and for all. Because of the certainty equivalence,the parameters $(F_0,F_1,F_2,F_3)$ are exactly identical in the random walk case to those that would obtained if the world really consisted of "once and for all" shifts.

Assuming that the coefficients $(F_0,F_1,F_2,F_3)$ have been estimated, the task of the econometric model user becomes rather easy. The actions of the private sector and of the government can be summarized by two rules

$$\begin{cases} u(t) = F_0\,\lambda + F_1\,x(t) + F_2\,\omega(t) \\ \\ \bar{u}(t) = \bar{F}_0\,\lambda + \bar{F}_1\,x(t) + \bar{F}_2\,\omega(t) \end{cases}$$

Each period, the econometrician runs various policy experiments by shifting $\lambda$. By asking the government to pick up which of these simulations it prefers, the model user reveals the government's preferred target vector $\lambda$. Our model allows to deal with more general stochastic structure for the target vector $\lambda(t)$. In the Markovian case in which $\lambda(t) = D\ \lambda(t-1) + w(t)$ the econometrician can freely vary the innovation term $w(t)$ and spell out appropriately, in a time varying way, its consequences on $\lambda(t+s)(=D^S\ \lambda_t)$ (1). We now illustratre these techniques in the following example.

c) - Example 2 : The consumption function

Consider a closed economy with a simple linear technology of production : $y(t) = r\ K(t) + \omega(t)$. $y$ is total output, $K(t)$ is installed capital, $\omega(t)$ is a random walk. Assume no depreciation of capital and a one period delivery lag of investment so that $K(t+1)=K(t) + I(t)$, when $I(t)$ is current investment. Each consumer minimizes a quadratic loss function $\sum_0^\infty \beta^t [C_i(t) - C_i^*(t)]^2$, in which $C_i^*(t)$ is a target vector which follows a random walk. The government minimizes its own objective $\sum_0^\infty \bar{\beta}^t \left\{[C(t) - C_g^*(t)]^2 + \emptyset_g [G(t) - G^*(t)]^2\right\}$. We assume lump sum taxes so that we do not have to worry about government's debt. For instance, let us assume a constantly balanced budget. Private sector's aggregate wealth follows the law of motion :

$$W(t+1) = (1+r)\ W(t) - C(t) - G(t) + \omega(t)\ ;\ \lim_{t\to\infty} (1+r)^{-t}\ W(t) = 0$$

or equivalently $y(t+1) = (1+r)\ y(t) - - [C(t) + G(t)]$.

The first order conditions of the private agents can be aggregated into:

$$[E_t\ C(t+1) - C*(t)] = \beta(1+r)\ [C(t) - C*(t)]$$

(1) One the time-consistent policy is plugged into the law of motion of the economy, this becomes a result akin to Gooley and Prescott's (1973), quoted in Lucas (1976).

Consumption at time t can be written :

$$C(t) = C*(t) + \theta \left[ y(t) - r\, C*(t) - r \sum_{s \geq 0} \frac{E_t\, G(t+s)}{(1+r)^{s+1}} \right]$$

in which $\theta \equiv \frac{1}{r}\left[1 + r - \frac{1}{\beta(1+r)}\right]$ is the "true" marginal propensity to consume.

In order to find the time-consistent equilibrium, one can see that the government should postulmate a keynesian consumption function :

$$C(t) = F_0\, y(t) + F_1$$

$(F_0, F_1)$ being the solution of two quadratic equations. To this postulated consumption function, the government responds by a rule :

$$G(t) = \bar{F}_0\, y(t) + \bar{F}_1$$

<u>In this framework, one sees that a shift of government policy is simply a shift of $\bar{F}_1$</u>. In order to assess its impact on aggregate expenditure and investment one simply needs to calculate how does $F_1$ vary along with $\bar{F}_1$ . Aggregate investment $(=y(t) - C(t) - G(t))$ will increase, decrease or stay constant depending upon $F_1 + \bar{F}_1$ 1.

Explicit calculations show that $\bar{F}_1 + F_1$ can be written :

$$F_1 + \bar{F}_1 = -\gamma\, \bar{F}_1$$

in which $\gamma$ has the same sign as $[\beta(1+r) - 1]$. An increase in government's expenditures (an increase in $\bar{F}_1$) will therefore increase, reduce, or keep unchanged aggregate investment according to $\beta(1+r)$ smaller, equal or larger than one. In the particular case when $\beta = \bar{\beta} = \frac{1}{1+r}$, private consumption follows a random walk $(E_t((t+1) = C_t)$ as in Hall's model. Explicit calculations show that the government should then postulate :

$$C(t) = \frac{\emptyset_g(1+r)}{1 + \emptyset_g(1+r)} y(t) + \frac{1}{1 + (1+r)\,\emptyset_g} C^*_g - \frac{\emptyset_g(1+r)}{1 + \emptyset_g(1+r)} G*$$

and respond by a rule :

$$G(t) = \frac{1}{1 + \emptyset_g(1+r)} y(t) - \frac{1}{1 + (1+r)\,\emptyset_g} C^*_g + \frac{\emptyset_g(1+r)}{1 + \emptyset_g(1+r)} G^*$$

Here, any shift of government's spending leaves unchanged aggregate (private and public) consumption.

One sees in this particular case that the postulated consumption function does not reveal the "true" marginal propensity to consume.

## *VI - DOES THERE ALWAYS EXIST A RULE WHICH DOMINATES THE TIME-CONSISTENT EQUILIBRIUM ?*

a) - The optimal feed-back rule : introduction

Whenever a time-inconsistency problem arises in the choice of a government's optimal strategy, we know that the time-consistent equilibrium is sub-optimal. In response, Kydland and Prescott have argued that one should look for simple (time-invariant) rules which dominate the discretionary (time-consistent) equilibrium. At this stage, we do not know yet, however, if such rules always exist. If one simply looks for an optimal rule $\bar{u}(t) = \bar{F}\, z(t)$ and minimizes the corresponding loss function by some gradient method, the solution F* will usually depend on the initial value of the state variable $(x_0, g_0)$, so that there is no guarantee that F* will dominate the time-consistent equilibrium at all point of time (this point is already made in Kydland and Prescott). Chow has argued that dynamic programming could nevertheless be used to find an optimal feed-back rule which dominates the time-consistent equilibrium. Let us summarize his technique in the context of example 1 (which is formally very similar to the framework he sets up). Price formation were set according to :

(9-a) $p(t) = a\, p(t-1) + b\, E_t\, p(t+1) + c\, y(t) + \omega(t)$

Chow suggested to treat $E_t\, p(t+1)$ as an exogeneous stochastic disturbance, ie, to view price formation as following the law :

$$(11) \quad p(t) = a\, p(t-1) + c\, y(t) + \eta(t)$$

(in which $\eta(t) = \omega(t) + b\, E_t\, p(t+1)$ at the equilibrium). Under this postulated law of motion, government intervention is then determined by the following rule :

$$(12) \quad y(t) = - \frac{C}{\emptyset} \frac{1-\beta}{1-a\beta}\, \pi(t)$$

This rule, however, does not always dominate the time-consistent equilibrium . In our example, explicit calculation shows that the time-consistent equilibrium always yields a superior outcome b(1). The reason for this should be clear. A time-consistent government does not have enough leverage on the private sector's expectations to reach the time-inconsistent equilibrium. It does have some leverage, however, since it can move the state variable and influence expectations through that channel. This is why the dependency of price expectations upon past prices must be explicitly acknowledged.

Thus, the question is still open : does there always exist a rule which dominates the time-consitent equilibrium ? The answer is positive. Before we provide a proof, we first analyze a slightly modified version of our initial problem.

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(1) There might be some examples where this will not be true. See, eg, the calculations in Miller (1985) in which a simulation with "exogeneous competitiveness" can be interpreted as a counter-example" in which Chow's technique dominate the time-consistent equilibrium.

b) - The multiplier effect of a one period pre-commitment

In this subsection, let us consider the case when the government's current decision can be taken and observed before that the private sector choses its own action. Let us further assume that the vector of information $\omega(t)$ is already known to the government when it takes its decision. In this case, we see that we give the government a edge over the private sector : a one-period pre -commitment ($\bar{u}(t)$ is set before that $u(t)$ is selected) ; and an informational advantage ($\omega(t)$ is first given to the government). It is easy to see how the new equilibrium could be reached : instead of postulating an equilibrium rule $u(t) = F_0\ x(t) + F_1\ g(t) + F_2\ \omega(t)$, the time-consistent equilibrium is now obtained by postulating a rule depending on $x(t+1)$ (1) :

$$(13) \quad u(t) = F_0\ x(t) + F_1\ \bar{u}(t) + F_2\ g(t) + F_3\ x(t+1) + F_4\ \omega(t)$$

or equivalently to postulate a law of option :

$$(14) \quad x(t+1) = [I - B\ F_3]^{-1}[(A + B\ F_0)\ x(t) + (\bar{B} + B\ F_1)\bar{u}(t) + D'\ Z(t)]$$

We see that this postulated law of motion gives rise to a sort of "multiplier" to government's intervention.

When is it the case that this "multiplier" is effective, that is : when is it that a pre-commitment over one period does increase the effectiveness of government's policy ? In appendix 3, we show that the set of necessary (and

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(1) The rule is obtained by looking for a matrix F such that $\frac{\partial Q}{\partial u}\ [x(t),\ u(t),\ L\ z(t)] = F\ x(t+1)$, and such that F yields rational expectations of the private sector at the equilibrium.

sometimes sufficient) conditions are those which yield the time-inconsistency problem analyzed above. This shows that the seeds of dynamic inconsistency are really contained in the one-period model : <u>dynamic inconsistency arises whenever a one period precommitment strictly dominates the no-precommitment equilibrium</u>.

Let us call $\bar{u}'(t) = \bar{F}_0' \, x(t) + \bar{F}_2' \, g(t) + \bar{F}_3' \, \omega(t)$ the optimal rule that is followed by the government when it is given a one period pre-commitment leverage.

c) - <u>Dominating the time-consistent equilibrium</u>

Let us now return to our original framework. Assume that the government cannot play before the private sector does (either because the technology of government decisions keeps it from doing so, or because the information $\omega(t)$ is not available before the private sector plays).

We know that the solution $\bar{u}'(t)$ would dominate the time-consistent equilibrium if the government could pre-commit itself. Now assume that the government does play $\bar{u}'(t)$ all the time, even when it does not play first. If the rule $\bar{u}'(t)$ comes to be known and understood by the private sector, it is a rule which does dominate the time-consistent equilibrium all the time. Therefore, there does exist a rule $\bar{u}_1(t) = \bar{F}_0' \, x(t) + \bar{F}_1' \, g(t) + \bar{F}_2' \, \omega(t)$ which dominates the time-consistent equilibrium "whenever" there is a dynamic inconsistency problem (1).

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(1) The reader is urged to convince himself that the difference between these two equilibria is not a feature of our discrete time analysis. Even in a continuous time framework, the difference would remain (see below our discussion of the inflation-output trade-off).

For instance, in the example 1 above, the government should now postulate a private sector forecast rule $E_t\, p(t+1) = p(t) + \dfrac{1}{a + \dfrac{C^2}{a\,\emptyset}}\, \omega(t)$

which yields an equilibrium optimum inflation path

$\pi(t) = \dfrac{1}{1 + \dfrac{C^2}{a\,\emptyset}}\, \omega(t)\ \left[1 + b\, \dfrac{1}{a + C^2/\emptyset}\right] \omega(t)$ ; it only coincides with the solution before when $b = 0$ : ie, when there is no dynamic inconsistency problem.

It is interesting to note that many of Kydland and Prescott examples in favor of "rules versus discretion" do rely on $\bar{u}_1(t)$ versus $\bar{u}(t)$. The unemployment-inflation trade-off is maybe the best example. If the government could announce the money supply, each period, before that the private sector sets its price, there would be, say, no inflation. However if the government must play in the same time as the private sector, the time-consistent equilibrium would yield an equilibrium inflation up to the point where the marginal benefit of "surprise" inflation equals its disutility to the government (this is the essence of Barro and Gordon (1983)'s argument). Here, the best rule is what the solution $\bar{u}_1$, would predict : Let the Central Bank play as if it was playing first, even when this is not the case.

Now, does this superior rule $\bar{u}_1(z)$ invalidate our analysis of the Lucas Critique in the preceeding section ? The answer is negative : our techniques still apply : Since $\bar{u}_1(z)$ is obtained by backward recursion (of a new particular kind defined in sub-section (b) above), it is straightforward to check that any shift of governement's targets will have the same consequences on private sector rules as indicated in paragraph 5 above.

## *CONCLUSION*

In the paper we have examined various aspects of the dynamic inconsistency of government behavior. In the first place, we have shown that the optimal behavior of a benevolent government will be dynamically inconsistent if and only if there are externalities in private behavior.

Secondly, we have shown how a time-consistent policy could be attained : we require the government to take as given the feed-back rule of the private sector and to postulate that they do not depend upon its own behavior. Then, we have shown how this strategy should be applied by the econometrician (in the linear quadratic case) : we only require to change the constant terms and the residuals of the private sector rules. Last, we have shown how to dominate the time-consistent equilibrium : the government should act as if it could pre-commit itself one period ahead, even when it cannot.

## *APPENDIX 1 - PRIVATE AGENTS BEHAVIOR*

Each agent i minimizes his expected loss. At each time t, he minimizes

$$E_t \sum_{s=0}^{\infty} \beta^s \, Q(x_i(t+s),\ u_i(t+s),\ L_i z(t+s))$$

the evolution of $x_i$ being defined by

(1) $\quad x_i(t+s+1) = A\, x_i(t+s) + B\, u_i(t+s) + C_i\, z(t+s)$

We assume that the stochastic process $E_t\, z(t+s)$ allows convergence of the solution. From the certainty equivalence property, the optimal decision $u_i(t)$ is the solution of the deterministic problem obtained by viewing expectations with subjective certainty. If Q is definite positive with respect to $u_i$ , this solution is unique and can be written :

(2) $\quad u_i(t) = N\, x_i(t) + \sum_{s=0}^{\infty} M_{is}\, E_t\, z(t+s)$

N is identical to all agents because only the exogenous variables $L_i\, z$ and $C_i\, z$ are allowed to vary from one agent to the other. (see below for a more explicit proof). Aggregating (1) and (2) leads to

(3) $\quad x(t+1) = A\, x(t) + B\, u(t) + C\, z(t)$

(4) $\quad u(t) = N\, x(t) + \sum_{s=0}^{\infty} M_s\, E_t\, z(t+s)$

with $\quad x = \sum_i x_i,\ u = \sum_i u_i,\ C = \sum_i C_i$, and $M_i = \sum_i M_{is}$

Reciprocally, the solution of (3)-(4) determines uniquely each agent's behavior as a function of his initial state $x_i(0)$ : by induction, $u_i(t)$ is define by (2) and $x_i(t+1)$ by (1).

Another method is to characterize private agents' behavior by aggregation of their shadow price. Consider the optimal problem at time 0 : the first order conditions are :

(A1) $\partial Q/\partial u_i(t) + \pi_i(t)\, B = 0$

(A2) $\beta^{-1}\, \pi_i(t-1) = \partial Q/\partial x_i(t) + \pi_i(t)\, A$

with $\pi_i(t)$ the shadow price of agent i. If Q is definite positive with respect to $u_i$, the first order condition defines $u_i^e(t)$ as a linear function of $x_i^e(t)$, $D_i\, z\,(t)$ and $\pi_i(t)$ (calling $u_i^e(t) = E_0\, u_i(t)$, $x_i^e(t) = E_0\, x_i(t)$, ...) :

(A3) $u_i^e(t) = H(x_i^e(t),\, D_i\, z^e(t),\, \pi_i(t))$

By substituting (A3) into (A2), one finds :

$$\pi_i(t-1) = K(x_i^e(t),\, D_i\, z^e(t),\, \pi_i(t))$$

These two linear conditions may be aggregated :

$$u^e(t) = \sum_i u_i^e(t),\quad x^e(t) = \sum_i x_i^e(t),\quad \pi(t) = \sum_i \pi_i(t) \qquad D = \sum_i D_i$$

to yield :

(A4) $u^e(t) = H(x^e(t),\, D\, z^e(t),\, \pi\,(t))$

(A5) $\pi(t-1) = K(x^e(t),\, D\; z^e(t),\, \pi(t))$

We also have

(A6) $x^e(t+1) = A\, x^e(t),\ B\, z^e(t),\ C\, z^e(t))$

Conditions (A4), (A5) and (A6) (combined with budgetary of the tranversality condition) define the optimal decision at date 0, which may be written (by forward resolution of $\pi$) :

$$u(0) = u^e(0) = L\, x^e(0) + \sum_{t\geq 0} M\ \ z^e(t) = L\, x(0) + \sum_{t\geq 0} M_t\, E_0\, z(t)$$

The same logic applies at each period, and one obtains equation (4) defining private aggregate behavior. We also see that this behavior can be characterized be a <u>unique representative private agent</u> who minimizes

$$E_0 \sum \beta^t\, Q(x(t),\ u(t),\ L\, z(t))$$

subjet to $x(t+1) = A\, x(t) + B\, u(t) + C\, z(t)$ and who regards $z(t)$ as <u>independent</u> from his own decisions $u(t)$.

## APPENDIX 2 - TIME-CONSISTENCY AND THE SINGLE CONTROLLER SOLUTION

Assume that the optimal solution of a government with pre-commitment capability is time consistent. Then this rule is obtained by solving the following deterministic problem

$$\text{Min } \Sigma\, \beta^t\, \bar{Q}(x(t),\, u(t),\, \bar{u}(t),\, \omega(t)) \text{ subject to}$$

$$\text{(B1)} \quad \begin{cases} x(t+1) = A\, x(t) + b\, u(t) + C\, z(t) = f_1(x(t),\, u(t),\, u(t),\, \omega(t)) \\ \omega(t+1) = \Omega\, \omega(t) \\ g(t+1) = D\, z(t) = f_2\, [X(t),\, u(t),\, u(t),\, g(t),\, \omega(t)] \end{cases}$$

$$\text{(B2)} \quad \begin{cases} \beta^{-1}\pi(t-1) = \partial Q/\partial x(t) + \pi(t)\, A \\ \partial Q/\partial u(t) + \pi(t)\, B = 0 \\ \lim\limits_{t\to\infty} \Psi\, [t,x(t)] = 0 \end{cases}$$

Conditions (B2) are the aggregtion of the first-order conditions of the private sector. Let $q_1(t)$ and $q_2(t)$ be the shadow price of the two first equations in (B2).

Because the initial value $\pi(0)$ is free, optimality implies $q(0) = 0$ (see Pontryagin and al.). Then the assumption of time-consistency of the solution implies : $q\,(t) = 0$ for all t : otherwise, reoptimizing at time t a latter would give a different solution (we exclude the trivial case in which other optimality conditions are independent of $q(t)$). In addition, the evolution of $q_1(t)$ : $q_1(t+1) = A\, q_1(t) + B\, q_2(t)$ implies : $B\, q_2(t) = 0$ for all t.

<u>Assuming B to be injective</u>, one then obtain : $q_2(t) = 0$ for all t. In this case, the other conditions of optimality are the following

$$
\text{(B3)} \quad \begin{cases}
\partial \left[ \bar{Q} + p_1(t)\ f_1 + p_2(t)\ f_2 \right] / \partial \bar{u}(t) = 0 \\
\partial \left[ \bar{Q} + p_1(t)\ f_1 + p_2(t)\ f_2 \right] / \partial u(t) = 0 \\
\beta^{-1} p_1(t-1) = \partial \left[ \bar{Q} + p_1(t)\ f_1 + p_2(t)\ f_2 \right] / \partial x(t) \\
\beta^{-1} p_2(t-1) = \partial \left[ \bar{Q} + p_1(t)\ f_1 + p_2(t)\ f_2 \right] / \partial g(t)
\end{cases}
$$

These are precisely the optimality condition of the single controller problem. Therefore, if the government solution is time consistent, it achieves the single controller solution. The reciprocal is trivial.

Assume now that there are no private externalities (PPE or PPG) and that the government is benevolent. No PPE imply :

$$\partial f_1/\partial u = B \quad ; \quad \partial f_1/\partial x = A$$

$$\partial Q/\partial u = \partial Q_1/\partial u \quad ; \quad \partial Q/\partial x = \partial Q_1/\partial x$$

with $Q_1[x,u,u,g,\omega] = Q[x,u,Lz]$

no PPE imply

$$\partial f_2/\partial u = 0 \quad ; \quad \partial f_2/\partial x = 0$$

Government's benevolence imply :

$$\partial \bar{Q}/\partial u = \partial Q_1/\partial u \quad , \quad \partial \bar{Q}_1/\partial x = \partial Q_1/\partial x$$

All these properties and condition (B3) imply the private sector optimality conditions (B2) (with $\pi(t) = p_1(t)$). Thus the single controller solution is an equilibrium, and the government can achieve this solution by playing, the decision rule $u[x,g,\omega]$ of the single controller solution : this rule being chosen, the single controller problem becomes the one of the representative private agent (1).

Conseversely, it is sufficient to modify one of the relations obtained by the assumptions of no PPE, no PGE, and a benevolent government, in order to obtain the incompatibility between the private sector optimality condition (B2) and the single controller optimality conditions (B3). Changing more than one relation will lead to the same conclusion, except if these changes neutralize their effects (this is the case for instance when there are PPE in the private objective function and when the government is not benevolent in such a way that its objective precisely ignores the effect of the PPE.

(1) This argument shows that the equivalence between time-consistency of the model and achieving the single controller solution does not depend on the linear-quadratic form of the model.

## *APPENDIX 3 - TIME-CONSISTENT EQUILIBRIUM AND THE EQUILIBRIUM WITH ONE PERIOD PRECOMMITMENT*

The state of the economy is the vector $u(t) = [x(t), g(t), \omega(t)]$. The time consistent solution is defined by solving the problem backwards ; in infinite horizon, this requires to define the expected future loss function. For agent i,

$$\text{(C1)} \quad V_i(x_i(t), y(t)) = \min_{u_i} E_t\Big\{Q(x_i(t), u_i, L_i z(t)) + \beta V_i(x_i(t+1), y(t+1))\Big\}$$

And for the government

$$\text{(C2)} \quad \bar{V}(y(t)) = \min_{\bar{u}} E_t\Big\{ \bar{Q}(z(t)) + \bar{\beta} \bar{V}(y(t+1))\Big\}$$

Looking for linear rules, we assume that $V_i$ and $\bar{V}$ are quadratic functions. The time-consistent equilibrium is defined by solving (C1) and (C2) by. After agregation of private agents' variables, this determines u(t) and $\bar{u}(t)$ as linear functions of the current state of the economy. In this analysis, each agent (private or public) does not observe the other agents' decisions and makes rational expectations on the current decisions of the others.

The analysis is different when the private agents observe the current government decision $\bar{u}(t)$ before taking their own decisions. In this case, private decision u(t) directly depend upon $\bar{u}(t)$ and the government will take into account this immediate effect. The government knows the linear relation

$$\text{(C3)} \quad \frac{\partial Q}{\partial u}(x(t), u(t), Lz(t)) = S\, E_t\, y(t+1)$$

resulting from aggregating the optimality conditions of the optimality conditions of the private sector (in relations (C1)). Through this relation, the private sector's decision u(t) does depend on u(t), and it is this relation which the government takes into account by solving (C2). This equilibrium is what we call equilibrium with one period precommitment. Not that there is no symmetry in the model : each private agent always consider the decisions of the other agents as given.

In the <u>Time-consistent equilibrium</u> (with no precommitment at all) the government minimizes with respect to u(t) its expected loss

$$\Psi(z(t)) = \bar{Q}(z(t)) + \bar{\beta}\, E_t\, \bar{V}(y(t+1))$$

and we have : $\partial\Psi(z(t))/\partial u(t) = 0$. In the equlibrium with one period precommitment, the government minimizes $\Psi(z(t))$ taking into account the effet of $\bar{u}(t)$ on u(t) given by relation (C3), and we then have :

$$\frac{\partial\Psi(z(t))}{\partial\bar{u}(t)} + \frac{\partial\Psi(z(t))}{\partial u(t)}\,\frac{\partial u(t)}{\partial\,\bar{u}(t)} = 0$$

Consider the case when both equilibria coincide and assume that(1)

$$\frac{\partial\Psi(z(t))}{\partial u(t)}\,.\,\frac{\partial u(z(t))}{\partial\bar{u}(t)} = 0 \Rightarrow \frac{\partial\Psi(z(t))}{\partial\, u(t)} = 0$$

Then the time-consistent equilibrium verifies : $\partial\Psi(z(t))/\partial\bar{u}(t) = 0$ and $\partial\Psi(z(t))/\partial u(t) = 0$, i.e the equilibrium is defined by minimizing $\Psi(z(t))$ with respect to both the private sector's decision u(t) and the governmnent decision $\bar{u}(t)$. Doing this at each period and defining in this way the expected loss of the government implies that the time-consistent equilibrium also coincides with the single controller solution, i.e there is no time-inconsistency problem at all.

(1) Condition (C4) means that the government has sufficiently many efficient instruments.

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