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ON THE DESIGN OF INCENTIVE SCHEMES UNDER MORAL HAZARD AND ADVERSE SELECTION

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MECANISMES INCITATIFS AVEC HASARD MORAL ET SELECTION ADVERSE

Résumé

Cet article caractérise les mécanismes incitatifs optimaux dans un modèle simple de type principal-agent où sélection adverse et hasard moral interviennent simultanément. Dans un cas simple, on montre que la solution optimale peut être obtenue en utilisant des schémas incitatifs où la rétribution de l'agent dépend linéairement du résultat observé. La solution optimale est également caractérisée dans le cas général et on démontre que celle-ci peut être approximée aussi précisément que l'on veut par des schémas incitatifs quadratiques. Enfin, le modèle est appliqué à différents problèmes d'incitations : politique de régulation d'entreprises, dont les coûts de production sont observables, contrats de services bancaires, planification décentralisée par objectifs.

Mots clefs : Incitations, sélection adverse, hasard moral.

ON THE DESIGN OF INCENTIVE SCHEMES UNDER MORAL HAZARD AND ADVERSE SELECTION

ABSTRACT

This paper aims at characterizing optimal incentive mechanisms in a simple principal-agent model with both adverse selection and moral hazard. When a monotonic hazard rate property is satisfied it is shown that an optimal solution is using incentive schemes where the agent's reward depends linearly on observed outcome. The optimal solution is also characterized in the general case and we show that this solution can be approximated as closed as desirable by means of quadratic incentive schemes. Lastly, the model is applied to a number of incentives problems including the regulatory policy for firms under cost observability, optimal investment banking contracts or decentralized planning with production targets.

Nomenclature JEL : 020

I. INTRODUCTION.

Moral hazard and adverse selection are both fundamental features of principal-agent relationships. Moral hazard results from the inability of the principal to monitor agent's actions while adverse selection corresponds to the inability of observing agent's private information.

Many principal-agent problems involve simultaneously moral hazard and adverse selection. For instance in the owner-manager relationship, the owner may be unable to observe the effort level of the manager and, simultaneously, some profitability parameters may be private knowledge to the manager. Likewise, an insurer may be unable to identify high risk individuals and low risk individuals and he will not observe the level of care taken by the insured individuals...

Although considerable attention has been paid to understanding the principal-agent problem under either moral hazard or adverse selection, few researches have focused on the interactions between these two sources of inefficiency in ressource allocation. Worthy exceptions include the income tax model of Mirlees (1971), the literature on the new soviet incentive scheme (Weitzman, 1976) and more recent papers by Baron and Holmstrom (1980), Baron (1982), Melumad and Reichelstein (1984, 1985) and Laffont and Tirole (1985).

In particular, Laffont and Tirole (1985) have analysed the design of an optimal regulatory policy for private or public firms when a cost parameter is private knowledge to the firm and an unobservable effort variable is introduced. Under suitable assumptions (and, in particular, assuming risk-neutrality and, a well behaved distribution function for the cost parameter) they show that inducing truthful revelation of the firm's private information prevents the attainment of a full optimum. They also characterize an optimal incentive scheme which is linear in ex-post cost.

This paper aims at extending these results.

First, an optimal incentive scheme is characterized in the framework of a principal-agent model where moral hazard and adverse selection are combined in a simple way. For a given cost report from the agent to the principal, this incentive scheme defines the agent's reward as a function of observed outcome. As in the Laffont-Tirole's paper, there exists an optimal linear incentive scheme when the distribution function of the cost parameter satisfies a monotonic hazard rate property. However, the model highlights also possible discontinuities of coefficients of this linear scheme because of an eventual non convexity of the principal's objective function.

Secondly, the optimal agent's decision is characterized for any distribution function of the cost parameter. It is shown that this optimal solution does not depend on random disturbances and can be approximated as closed as desirable by using incentive schemes which are quadratic in expost outcome. Both linear and quadratic incentive schemes include a fixed transfer (which is higher for low cost agents then for high cost agents) and a bonus which depend (linearly or non linearly) on the difference between expected and observed outcomes.

Lastly, a number of simple extensions of the basic model are proposed, including the control of regulated firms, the design of investment banking contracts and decentralized planning with production targets.

The paper is organized as follows. Section 2 presents a principal-agent model which may be formally viewed as a simplified version of the Laffont-Tirole (1985) model. The principal's optimization problem is developped in section 3. Section 4 solves for the optimal linear incentive scheme and the general case is developped in section 5. Extensions of the model are presented in section 6 and some concluding comments are given in the final section.

II. THE BASIC FRAMEWORK

We consider a simple principal-agent model which can be described as follows. The agent's decision is a level of effort. Effort is supposed to be an unobservable variable which cannot be contracted upon. Effort creates a direct disutility for the agent and, simultaneously with a random state of nature, determines a monetary outcome. It will be assumed that the distribution of the outcome depends also on a cost parameter which is unknown to the principal but perfectly known to the agent. The principal can neither monitor the agent's level of effort nor observe the cost parameter so that both moral hazard and adverse selection are simultaneously considered in this paper.

A simple application of this model is to the case of the relationship between the owner and the manager of a firm. The owner is the principal and the manager the agent and the owner delegates the running of the firm to the manager. The intrinsic profitability of the firm is not perfectly known to the owner and is characterized by the cost parameter. For a given state of nature, profits depend simultaneously on the manager's level of effort and on the cost parameter.

Formally, let $x \in R$, $a \in R^+$ and $\theta \in \Delta = [\theta_0,\theta_1]$ denote respectively the monetary outcome of the principal-agent relationship, the agent's level of effort and the cost parameter. We will assume that the outcome writes as

$$x = a - \theta + \varepsilon \qquad (1)$$

where ε is a random variable with zero mean and a compact support $\Omega = [-\varepsilon_0,\varepsilon_1]$ with $\varepsilon_0 > 0$, $\varepsilon_1 > 0$. Let σ^2 be the variance of ε and $g(\varepsilon)$ be a density function for ε .

The principal's utility is $x - t(\tilde{\theta},x)$ where $t(\tilde{\theta},x) : \Delta \times R \to R \cup \{-\infty\}$ denotes the agent's compensation.(1)

(1) In what follows we restrict ourselves to incentive schemes $t(\tilde{\theta},x)$ which are continuously differentiable almost everywhere over the set $\{(\tilde{\theta},x) \in \Delta \times R$ such that $t(\tilde{\theta},x) \neq -\infty\}$.

This compensation depends on the outcome x and on a cost report $\tilde{\theta}$ from the agent to the principal, which may differ from the true cost parameter θ. Implicitly $t(\tilde{\theta},x) = -\infty$ means that an agent whose cost report is $\tilde{\theta}$ precommits to yield an outcome different from x.

The principal and the agent are both supposed to be risk neutral. When the incentive scheme $t(\tilde{\theta},x)$ is used, the principal's expected utility $\hat{w}_t$ writes as

$$\hat{w}_t(\theta,\tilde{\theta},a) = a - \theta - \int_\Omega t(\tilde{\theta},a - \theta + \varepsilon)\ g(\varepsilon)\ d\ \varepsilon \tag{2}$$

The agent's utility is written as $t(\tilde{\theta},x) - \psi(a)$ where $\psi(a)$ denotes the disutility of effort. Function ψ is defined over R^+ and is twice continuously differentiable and satisfies

$$\psi'(a) > 0 \quad \text{if} \quad a > 0$$

$$\psi'(0) = 0$$

$$\psi''(a) > 0 \quad \text{for all} \quad a$$

The agent's expected utility $\hat{u}_t$ writes as

$$\hat{u}_t(\theta,\tilde{\theta},a) = \int_\Omega t(\tilde{\theta},a - \theta + \varepsilon)\ g(\varepsilon)\ d\ \varepsilon - \psi(a) \tag{3}$$

and we have(1)

$$\hat{u}_t(\theta,\tilde{\theta},a) = a - \theta - \psi(a) - \hat{w}_t(\theta,\tilde{\theta},a) \tag{4}$$

When reporting his cost parameter and choosing his level of effort the agent behaves strategically. However, further developments are highly simplified by the Revelation Principle which allows to restrict the design of incentive schemes to mecanisms where truthtelling belongs to the set of optimal

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(1) If $t(\tilde{\theta}, a - \theta + \varepsilon) = -\infty$ for some ε in Ω we have $\hat{w}_t(\theta,\tilde{\theta},a) = +\infty$ and $\hat{u}_t(\theta,\tilde{\theta},a) = -\infty$. Furthermore, under previous assumptions, functions $\hat{w}_t(\theta,\tilde{\theta},a)$ and $\hat{u}_t(\theta,\tilde{\theta},a)$ are differentiable over the set $\{(\theta,\tilde{\theta},a) \in \Delta \times \Delta \times R^+$ such that $t(\tilde{\theta}, a - \theta + \varepsilon) \neq -\infty$ for all ε in $\Omega\}$

strategies of the agent. To define completely feasible decentralisation processes an individual rationality constraint has also to be introduced to be sure that the agent is willing to participate. These conditions will be summed up in a definition : we will say that an incentive scheme $t(\theta,x)$ implements an effort function $a(\theta)$ if two conditions are satisfied : first, the agent is willing to tell the truth when reporting his cost parameter and he finds optimal to choose $a(\theta)$ if his parameter equals θ . Secondly, the agent's expected utility is non negative so that he is willing to participate. Formally, we have the following definition

DEFINITION. The incentive scheme $t(\theta,x) : \Delta \times R \to R$ implements the effort function $a(\theta) : \Delta \to R_+$ if for all θ in Δ

(i) $$(\theta,a(\theta)) \in \underset{\substack{\tilde{a}\in R_+ \\ \tilde{\theta}\in\Delta}}{\text{Arg Max}} \left\{\int_\Omega t(\tilde{\theta},\tilde{a} - \theta + \varepsilon)\, g(\varepsilon)\, d\varepsilon - \psi(\tilde{a})\right\}$$

(ii) $$\int_\Omega t(\theta,a(\theta) - \theta + \varepsilon)\, g(\varepsilon)\, d\varepsilon - \psi(a(\theta)) \geqq 0$$

From (i), the agent is willing to report truthfully his cost parameter θ and he picks the level of effort $a(\theta)$ when his cost parameter equals θ . From (ii) his expected utility is non negative.

In what follows we will say that an effort function $a(\theta)$ and an incentive scheme $t(\theta,x)$ define together a mecanism. Let $u_{a,t}(\theta)$ - respect. $w_{a,t}(\theta)$ - denote the agent's - respect. the principal's - expected utility for the mecanism $\{a(.),t(.)\}$ when $t(.)$ implements $a(.)$ and the cost parameter equals θ . We have

$$u_{a,t}(\theta) = \hat{u}_t(\theta,\theta,a(\theta)) = \int_\Omega t(\theta,a(\theta) - \theta + \varepsilon)\, g(\varepsilon)\, d\varepsilon - \psi(a(\theta)) \tag{5}$$

and

$$\begin{aligned} w_{a,t}(\theta) &= \hat{w}_t(\theta,\theta,a(\theta)) = a(\theta) - \theta - \int_\Omega t(\theta,a(\theta) - \theta + \varepsilon)\, g(\varepsilon)\, d\varepsilon \\ &= a(\theta) - \theta - \psi(a(\theta)) - u_{t,a}(\theta) \end{aligned} \tag{6}$$

Conditions (i) and (ii) can then be rewritten as

(i) $(\theta,a(\theta)) \in \underset{\substack{\tilde{a}\in R^+ \\ \tilde{\theta}\in\Delta}}{\text{Arg Max}}\ \hat{u}_t(\theta,\tilde{\theta},\tilde{a})$

(ii) $u_{a,t}(\theta) \geqq 0$

Lastly it will be assumed that the principal has a subjective prior probability distribution for the unknown parameter θ . We let $f(\theta)$ and $F(\theta)$ be repectively the density function and the cumulative distribution function for θ , with $f(\theta) > 0$ for all θ in Δ . The principal's expected welfare W is thus written

$$W = \int_\Delta w_{a,t}(\theta)\ f(\theta)\ d\theta \tag{7}$$

We are now in position to characterize the principal's problem and to derive his optimal strategy.

III. THE PRINCIPAL'S OPTIMIZATION PROBLEM.

Two preliminary lemma will be useful to define the principal's optimization problem. First, lemma 1 shows that a simple relation lies the effort function $a(.)$ and the agent's expected utility function $u_{a,t}(.)$ when $t(.)$ implements $a(.)$.

LEMMA 1. For any mecanism $\{a(.),t(.)\}$ *which satisfies*

(i) $(\theta,a(\theta)) \in \text{Arg Max}\ \{\hat{u}_t(\theta,\tilde{\theta},\tilde{a})\ ,\ \tilde{a}\in R_+\ ,\ \tilde{\theta}\in\Delta\}$

(iii) $t(\theta,a(\theta) - \theta + \varepsilon) \neq -\infty$ *for all* ε *in* Ω

we have

$$u_{a,t}(\theta) = u_{a,t}(\theta_1) + \int_\theta^{\theta_1} \psi'(a(s))\ ds \quad \text{for all } \theta \text{ in } \Delta \tag{8}$$

Proof. Assume that (i) and (iii) are satisfied. From (iii), $\hat{u}_t(\theta,\tilde{\theta},\tilde{a})$ is differentiable at $\tilde{\theta} = \theta$ and $\tilde{a} = a(\theta)$ for all θ and (i) implies

$$\frac{\partial \hat{u}_t}{\partial a}(\theta,\theta,a(\theta)) = 0 \quad \text{for all} \quad \theta \tag{9}$$

$$\frac{\partial \hat{u}_t}{\partial \tilde{\theta}}(\theta,\theta,a(\theta)) = 0 \quad \text{for all} \quad \theta \tag{10}$$

From (3) we have

$$\frac{\partial \hat{u}_t}{\partial \theta}(\theta,\tilde{\theta},\tilde{a}) = -\frac{\partial \hat{u}_t}{\partial a}(\theta,\tilde{\theta},\tilde{a}) - \psi'(\tilde{a})$$

and thus from (9)

$$\frac{\partial \hat{u}_t}{\partial \theta}(\theta,\theta,a(\theta)) = -\psi'(a(\theta)) \quad \text{for all} \quad \theta \tag{11}$$

Differentiating $u_{a,t}(\theta) = \hat{u}_t(\theta,\theta,a(\theta))$ and using (9),(10) and (11) give

$$\frac{du_{a,t}}{d\theta}(\theta) = -\psi'(a(\theta))$$

which implies (8).

q.e.d.

The next lemma will characterize the principal's expected welfare.

LEMMA 2. *If* $t(.)$ *implements* $a(.)$ *, the principal's expected welfare writes as*

$$W = \int_\Delta (a(\theta) - \theta - \psi(a(\theta)) - z(\theta)\,\psi'(a(\theta)))\,f(\theta)\,d\theta - u(\theta_1) \tag{12}$$

with $z(\theta) = \frac{F(\theta)}{f(\theta)}$.

Proof. From (6),(7) and (8) we have

$$W = \int_\Delta (a(\theta) - \theta - \psi(a(\theta)))\,f(\theta)\,d\theta - \int_\Delta \left(\int_\theta^{\theta_1} \psi'(a(s)\,ds\right) f(\theta)\,d\theta - u(\theta_1) \tag{13}$$

and (12) is obtained by integrating by parts the second integral in (13).

q.e.d.

The principal's problem is to choose a mecanism $\{a(.),t(.)\}$ such that $t(.)$ implements $a(.)$ so as to maximize the expected welfare W . Using lemma 2, this problem writes as

$$\underset{a(.),t(.)}{\text{Maximize}} \int_\Delta (a(\theta) - \theta - \psi(a(\theta)) - z(\theta)\,\psi'(a(\theta)))\,f(\theta)\,d\theta - u_{t,a}(\theta_1)$$

subject to

(i) $\{\theta,a(\theta)\} \in \underset{\substack{\tilde{a}\in R^+ \\ \tilde{\theta}\in\Delta}}{\text{Arg Max}}\ \hat{u}_t(\theta,\tilde{\theta},\tilde{a})$ for all θ in Δ .

(ii) $u_{a,t}(\theta) \geqq 0$ for all θ in Δ

From lemma 1, $u_{a,t}(\theta)$ is nonincreasing if (i) and (iii) are satisfied. As (ii) implies (iii), conditions (i) - (ii) are equivalent to (i) - (iii) and $u_{a,t}(\theta_1) \geqq 0$.

Furthermore, one easily checks that $u_{a,t}(\theta_1) = 0$ for the optimal mecanism(1) so that the principal's problem can be written as

$$\underset{a(.),t(.)}{\text{Maximize}} \int_\Delta (a(\theta) - \theta - \psi(a(\theta)) - z(\theta)\,\psi'(a(\theta)))\,f(\theta)\,d\theta$$

subject to

(i) $\{\theta,a(\theta)\} \in \underset{\substack{\tilde{a}\in R^+ \\ \tilde{\theta}\in\Delta}}{\text{Arg Max}}\ \hat{u}_t(\theta,\tilde{\theta},\tilde{a})$ for all θ in Δ

(ii') $u_{a,t}(\theta_1) = 0$

(iii) $t(\theta,\theta + a(\theta) + \varepsilon) \neq -\infty$ for all ε in Ω , for all θ in Δ .

In what follows we will say that a mecanism $\{a(.),t(.)\}$ is <u>efficient</u> if conditions (i),(ii') and (iii) are satisfied, that is

- the incentive scheme $t(.)$ implements the effort function $a(.)$
- the agent's expected utility equals zero when the cost parameter is at the

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(1) Consider a mecanism $\{a(.),t_1(.)\}$ which satisfies (i),(iii) and $u_{a,t_1}(\theta_1) > 0$. Let $t_2(.)$ be defined as $t_2(\theta,x) = t_1(\theta,x) - u_{a,t_1}(\theta_1)$ for all θ . The mecanism $\{a(.),t_2(.)\}$ satisfies (i),(iii) and $u_{a,t_2}(\theta_1) \geqq 0$ and provides a higher welfare level then $\{a(.),t_1(.)\}$ to the principal.

highest level θ_1 .

From previous developments, the optimal mecanism $\{a^*(.),t^*(.)\}$ necessarily belongs to the set of efficient mecanisms. Clearly "efficiency" is restricted here to a class of incentive compatible mecanisms and no ambiguity should arise from this terminology : if the cost parameter were common knowledge, such "efficient mecanisms" would be dominated by other decision rules.

Usually if $\{a(.),t_1(.)\}$ is an efficient mecanism, there exists another incentive scheme $t_2(.)$ such that $\{a(.),t_2(.)\}$ is also efficient and both mecanisms provide the same expected utility to the principal. However, of particular interest is the case of incentive scheme which are linear in x and this case is considered in the following section.

IV. OPTIMALITY OF LINEAR INCENTIVE SCHEMES.

Efficient mecanisms with linear incentive scheme will be characterized in a first proposition. In a second proposition, we will show that using linear incentive schemes is indeed an optimal strategy when function $z(\theta)$ is non decreasing.

PROPOSITION 1. Let $t(\theta,x) = K(\theta)x + G(\theta)$. The mecanism $\{a(.),t(.)\}$ is efficient if and only if

(a) $a(\theta)$ is nonincreasing for all θ

(b) $K(\theta) = \psi'(a(\theta))$

(c) $G(\theta) = \psi(a(\theta_1)) - K(\theta_1)\ (a(\theta_1) - \theta_1) + \int_\theta^{\theta_1} K'(s)(a(s) - s)\, ds$

Proof. Since $t(\theta,x) = K(\theta)\ x + G(\theta)$, we have

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) = K(\tilde{\theta})\ (\tilde{a} - \theta) + G(\tilde{\theta}) - \psi(\tilde{a}) \qquad (14)$$

$$u_{a,t}(\theta) = K(\theta)\ (a(\theta) - \theta) + G(\theta) - \psi(a(\theta)) \qquad (15)$$

1/ Assume first that $\{a(.),t(.)\}$ is efficient. Conditions (i),(ii') and (iii) are thus satisfied. (i) implies

$$\hat{u}_t(\theta,\tilde{\theta},a(\tilde{\theta})) \leq u_{a,t}(\theta) \quad \text{for all } \theta \text{ and } \tilde{\theta}$$

which gives using (14) and (15)

$$u_{a,t}(\theta) - u_{a,t}(\tilde{\theta}) \geq K(\tilde{\theta})\ (\tilde{\theta} - \theta) \quad \text{for all } \theta \text{ and } \tilde{\theta} \tag{16}$$

and symmetrically

$$u_{a,t}(\tilde{\theta}) - u_{a,t}(\theta) \geq K(\theta)\ (\theta - \tilde{\theta}) \quad \text{for all } \theta \text{ and } \tilde{\theta} \tag{17}$$

(16) and (17) imply together

$$K(\theta)\ (\theta - \tilde{\theta}) \leq u_{a,t}(\tilde{\theta}) - u_{a,t}(\theta) \leq K(\tilde{\theta})\ (\theta - \tilde{\theta})$$

which proves that $K(\theta)$ is nonincreasing and $u'_{a,t}(\theta) = -K(\theta)$. Since $u'_{a,t}(\theta) = -\psi'(a(\theta))$ from lemma 1, conditions (a) and (b) are satisfied.

Lastly, differentiating (15) and using lemma 1 give (c).

2/ Conversely, assume that conditions (a),(b) and (c) are fulfilled. Let us prove (i),(ii') and (iii).

Function $a(\theta)$ is nonincreasing and thus differentiable almost everywhere. Differentiating (15) and using (b) and (c) yield

$$u'_{a,t}(\theta) = -K(\theta) \qquad \text{a.e.} \tag{18}$$

Moreover, using (b) and $\psi'' > 0$ gives

$$\hat{u}_t(\theta,\tilde{\theta},a(\tilde{\theta})) \geq \hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \quad \text{for all } \tilde{a},\tilde{\theta},\theta$$

and thus from (14)

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\tilde{\theta}) + (\tilde{\theta} - \theta)\ K(\tilde{\theta}) \quad \text{for all } \tilde{a},\tilde{\theta},\theta \tag{19}$$

Using (18) and (19), we deduce

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\theta) - \int_{\theta}^{\tilde{\theta}} K(s)\ ds + (\tilde{\theta} - \theta)\ K(\tilde{\theta})$$

and thus

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\theta) - \int_{\theta}^{\tilde{\theta}} (K(s) - K(\tilde{\theta}))\, ds \quad \text{for all} \quad \theta,\tilde{\theta},\tilde{a} \tag{20}$$

- if $\tilde{\theta} \geq \theta$, (a) and (b) imply $K(s) \geq K(\theta)$ for all s in $[\theta,\tilde{\theta}]$ and thus from (20) $\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\theta)$
- if $\tilde{\theta} < \theta$, we have from (20)

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\theta) + \int_{\tilde{\theta}}^{\theta} (K(s)-K(\tilde{\theta}))\, ds \leq u_{a,t}(\theta)$$

which proves (i).

Furthermore, (ii') results from (c) and (15). Lastly $K(\theta)$ and $G(\theta)$ are finite for all θ and (iii) is satisfied.

q.e.d.

Proposition 1 yields a simple characterization of efficient mecanisms with linear incentive schemes : for such mecanisms, $a(\theta)$ is nonincreasing. Furthermore, there exists a single linear incentive scheme associated to a given nonincreasing effort function and this incentive scheme is defined by conditions (b) and (c).

We will show now that using a linear incentive scheme is indeed optimal when function $z(\theta)$ is nondecreasing.

<u>PROPOSITION 2</u>. *When $z(\theta)$ is nondecreasing for all θ , an optimal mecanism $\{a^*(.),t^*(.)\}$ is defined as*

$$a^*(\theta) \in \underset{a \in R^+}{\text{Arg Max}} \{a - \psi(a) - z(\theta)\, \psi'(a)\} \quad \text{for all} \quad \theta \tag{21}$$

$$t^*(\theta,x) = K^*(\theta)x + G^*(\theta) \tag{22}$$

with

$$K^*(\theta) = \psi'(a^*(\theta)) \tag{23}$$

$$G^*(\theta) = \psi(a^*(\theta_1)) - K^*(\theta_1)(a^*(\theta_1) - \theta_1) + \int_{\theta}^{\theta_1} K^{*\prime}(s)(a^*(s) - s)\, ds \tag{24}$$

Proof. An optimal mecanism maximizes the principal's expected welfare W over the set of efficient mecanisms. For any efficient mecanism we have

$$W = \int_{\Delta} (a(\theta) - \theta - \psi(a(\theta)) - z(\theta)\,\psi'(a(\theta)))\, f(\theta)\, d\theta \tag{25}$$

Let $\{a^*(.),t^*(.)\}$ be defined by conditions (21) to (24). Function $a(.)$ maximizes the integral (25) so that proposition 2 will be proved if $\{a^*(.),t^*(.)\}$ is efficient and thus (using proposition 1) if $a^*(\theta)$ is nonincreasing. Let us show that $a^*(\theta)$ is actually a nonincreasing function if $z(\theta)$ is non-decreasing.

Let $\theta^i \in \Delta$, $i = 1 = 1,2$. We have from (21)

$$a^*(\theta^i) - \psi(a^*(\theta^i)) - z(\theta^i)\,\psi'(a^*(\theta^i)) \geqq a^*(\theta^j) - \psi(a^*(\theta^j)) - z(\theta^i)\,\psi'(a^*(\theta^j)) \tag{26}$$

and

$$a^*(\theta^j) - \psi(a^*(\theta^j)) - z(\theta^j)\,\psi'(a^*(\theta^j)) \geqq a^*(\theta^i) - \psi(a^*(\theta^i)) - z(\theta^j)\,\psi'(a^*(\theta^i)) \tag{27}$$

(26) and (27) imply together

$$\begin{aligned} z(\theta^j)\,[\psi'(a^*(\theta^j)) - \psi'(a^*(\theta^i))] &\leqq a^*(\theta^j) - a^*(\theta^i) - \psi(a^*(\theta^j)) + \psi(a^*(\theta^i)) \\ &\leqq z(\theta^i)\,[\psi'(a^*(\theta^j)) - \psi'(a^*(\theta^i))] \end{aligned} \tag{28}$$

Since ψ'' is positive and $z(\theta)$ is nondecreasing, (28) gives

$$a^*(\theta^j) > a^*(\theta^i) \Rightarrow z(\theta^j) < z(\theta^i) \Rightarrow \theta^j < \theta^i$$

which implies that function $a^*(\theta)$ is nonincreasing.

q.e.d.

Proposition 2 characterizes the optimal solution of the principal's problem when function $z(\theta)$ is nonincreasing over Δ. This assumption is satisfied for a number of usual probability distributions (for example the uniform or the exponential law). Let us proceed to a brief analysis of this optimal solution.

From (21), the optimal effort function $a^*(\theta)$ satisfies the first order optimality condition

$$\left.\begin{array}{l} 1 - \psi'(a^*(\theta)) - z(\theta)\,\psi''(a^*(\theta)) \leq 0 \\ \qquad\qquad\qquad\qquad = 0 \quad \text{if} \quad a^*(\theta) > 0 \end{array}\right\} \qquad (29)$$

and (29) implies

$$\psi'(a^*(\theta)) < 1 \quad \text{if} \quad \theta > \theta_0$$

At any level of the cost parameter except the lowest, the marginal disutility of effort is inferior to the expected marginal return of effort (which is equal to 1). Equivalently, the optimal level of effort $a^*(\theta)$ inferior to the full information solution $\psi'^{-1}(1)$.

We also have from condition (b)

$$\theta \leq K^*(\theta) = \psi'(a^*(\theta)) < 1 \quad \text{of} \quad \theta > \theta_0$$

The agent receives a part of the outcome $K^*(\theta)$ and a fixed fee $G^*(\theta)$. As $a^*(\theta)$ is nonincreasing, $K^*(\theta)$ is nondincreasing : the higher is the cost parameter, the lower is the proportion which goes to the agent.

Straightforward calculations show also that t^* can be rewritten as

$$t^*(\theta,x) = \psi'(a^*(\theta))\ (x - x^e(\theta)) + \psi(a^*(\theta)) + \int_{\theta}^{\theta_1} \psi'(a^*(s)) \qquad (30)$$

where $x^e(\theta) = a^*(\theta) - \theta$ is the optimal expected outcome : the agent's compensation includes a fixed fee which decreases with the cost parameter and a variable transfer which is a parentage of the difference between realized and expected outcomes.

This optimal incentive scheme can be compared to the solution that would prevail if the cost parameter were common knowledge . In this case, an optimal solution is to use a linear incentive scheme $\hat{t}(\theta,x) = x + \psi(\psi'^{-1}(1)) - \psi'^{-1}(1) + \theta$. When facing this incentive scheme, the agent finds optimal to choose the efficient level of effort $\psi'^{-1}(1)$ and the individual rationality constraint is binding for all θ .

For illustrative purpose consider the following example. Assume $\psi(a) = \frac{a^2}{2}$. Assume also that θ is uniformly distributed over $\Delta = [0,1]$ so that $z(\theta) = \theta$. We get $a^*(\theta) = 1 - \theta$, $t^*(\theta,x) = (1 - \theta)\, x + \frac{1-\theta^2}{2}$ while the full information solution is $\hat{a}(\theta) = 1$ and $\hat{t}(\theta,x) = x + \theta - \frac{1}{2}$.

Let us come back to the general result of proposition 2. From (21), the optimal effort function $a^*(\theta)$ is deduced from

$$a^*(\theta) \in \underset{a \in R^+}{\text{Arg Max}}\ \varphi_\theta(a)$$

with $\varphi_\theta(\hat{a}) = a - \psi(a) - z(\theta)\, \psi'(a)$.

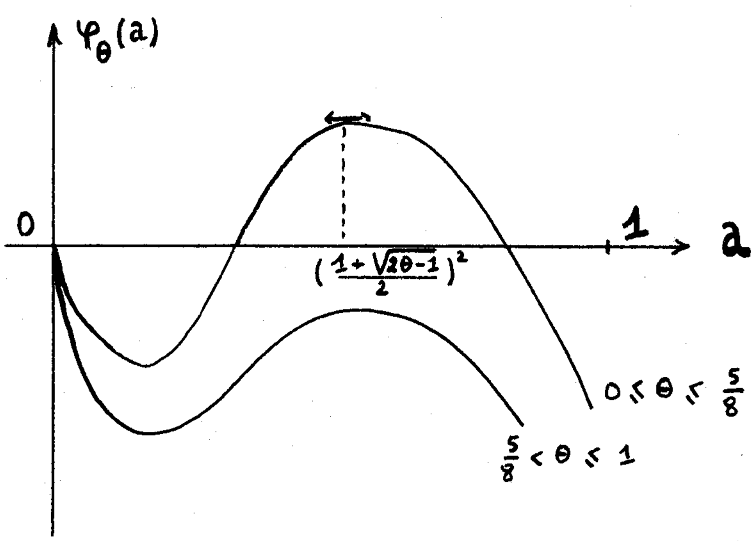
It is worthwhile to notice that function $\varphi_\theta(a)$ may not be concave and the first order optimality condition (29) is not sufficient to characterize the optimal effort function $a^*(\theta)$. Function $a^*(\theta)$ may be discontinuous and the following example will illustrate this eventuality.

Assume $\psi(\theta) = \frac{2}{3}\, a^{\frac{3}{2}}$ and, as in the previous example $\Delta = [0,1]$ and $z(\theta) = \theta$. We have

$$\varphi_\theta(a) = a - \frac{2}{3}\, a^{\frac{3}{2}} - \theta\, a^{\frac{1}{2}}$$

The graph of φ_θ is drawn on figure 1, by distinguishing the cases $\theta \leqq \theta < \frac{5}{8}$ and $\frac{5}{8} < \theta \leqq 1$

FIGURE 1



Function $\varphi_\theta(a)$ is negative over R^+ when $\frac{5}{8} < \theta \leq 1$ while it can be strictly positive if $0 \leq \theta < \frac{5}{8}$. We get

$$a^*(\theta) = \left(\frac{1+\sqrt{2\theta - 1}}{2}\right)^2 \quad \text{if} \quad \theta < \frac{5}{8}$$

$$a^*(\theta) = 0 \quad \text{if} \quad \theta \geq \frac{5}{8}$$

so that function $a^*(\theta)$ - and consequently $K^*(\theta)$ and $G^*(\theta)$ - are discontinuous at $\theta = \frac{5}{8}$.

Proposition 2 has shown that a linear incentive scheme is optimal if $z(\theta)$ is non increasing for all θ. We now turn to the more general case where this assumption is not necessarily satisfied. In such a case, using a non linear incentive scheme may be an optimal strategy for the principal.

V. OPTIMALITY OF NON LINEAR INCENTIVE SCHEMES.

Considering the general case where function $z(\theta)$ may be increasing, proposition 3 will provide a simple characterization of efficient mecanisms. To simplify matters, attention will be limited to continuous effort functions.

PROPOSITION 3. *For any continuous effort function $a(.)$, there exists an incentive scheme $t(\theta,x)$ such that $\{a(.),t(.)\}$ is efficient if and only if the function $\theta \to a(\theta) - \theta$ is nonincreasing.*

Proof.

1/ Assume first that $\{a(.),t(.)\}$ is an efficient mecanism. Conditions (i), (ii') and (iii) are thus satisfied. From (i) we have

$$\hat{u}_t(\theta,\tilde{\theta},\tilde{a}) \leq u_{a,t}(\theta) \quad \text{for all} \quad \theta,\tilde{\theta},\tilde{a}$$

and in particular

$$u_{a,t}(\theta) \geq \hat{u}_t(\theta,\tilde{\theta},a(\tilde{\theta}) - \tilde{\theta} + \theta) \tag{31}$$

Since

$$\hat{u}_t(\theta,\tilde{\theta},a(\tilde{\theta}) - \tilde{\theta} + \theta) = u_{a,t}(\tilde{\theta}) + \psi(a(\tilde{\theta})) - \psi(a(\tilde{\theta}) - \tilde{\theta} + \theta)$$

(31) gives

$$u_{a,t}(\theta) - u_{a,t}(\tilde{\theta}) \geqq \psi(a(\tilde{\theta})) - \psi(a(\tilde{\theta}) - \tilde{\theta} + \theta) \tag{32}$$

and symmetrically

$$u_{a,t}(\tilde{\theta}) - u_{a,t}(\theta) \geqq \psi(a(\theta)) - \psi(a(\theta) - \theta + \tilde{\theta}) \tag{33}$$

Let

$$\eta(\theta,\tilde{\theta}) = \psi(a(\theta) - \theta + \tilde{\theta}) - \psi(a(\theta) - \psi(a(\tilde{\theta})) + \psi(a(\tilde{\theta}) - \tilde{\theta} + \theta)$$

From (32) and (33), we have

$$\eta(\theta,\tilde{\theta}) \geqq 0 \quad \text{for all } \theta \text{ and } \tilde{\theta} \text{ in } \Delta \tag{34}$$

We also have

$$\eta(\theta,\theta) = 0$$

$$\frac{\partial\eta}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) = \psi'(a(\theta) - \theta + \tilde{\theta}) - \psi'(a(\tilde{\theta}))\, a'(\tilde{\theta}) + \psi'(a(\tilde{\theta}) - \tilde{\theta} + \theta)(a'(\tilde{\theta}) - 1)$$

at any point where function $a(\theta)$ is defferentiable. We thus have

$$\frac{\partial\eta}{\partial\tilde{\theta}}(\theta,\theta) = 0 \quad \text{a.e.}$$

For (34) to be realized, a necessary local second order conditions must hold and this condition writes as

$$\frac{\partial^2\eta}{\partial\tilde{\theta}^2}(\theta,\theta) = 2\,\psi''(a(\theta))\,(1 - a'(\theta)) \geqq 0 \quad \text{a.e.}$$

which gives $1 - a'(\theta) \geqq 0$. Since $a(\theta)$ is supposed to be continuous, function $a(\theta) - \theta$ is nonincreasing.

2/ Let us assume now that function $\theta \to a(\theta) - \theta$ is nonincreasing.

Consider the following incentive scheme.

$$t(\theta,x) = \begin{cases} s(\theta) & \text{if } x \in a(\theta) - \theta + \Omega \\ -\infty & \text{otherwise} \end{cases}$$

we will show that function $s(\theta)$ can be chosen so as to satisfy (i), (ii') and (iii).

Let $\alpha(\theta,\tilde{\theta})$ denote the level of effort chosen by the agent when his cost report is $\tilde{\theta}$ and the true cost parameter is θ, i.e.

$$\alpha(\theta,\tilde{\theta}) \in \underset{\tilde{a} \in R^+}{\text{Arg Max}}\ u_t(\theta,\tilde{\theta},\tilde{a}) \tag{35}$$

If $\alpha(\theta,\tilde{\theta})$ were different from $a(\tilde{\theta}) - \tilde{\theta} + \theta$, we would have $\alpha(\theta,\tilde{\theta}) - \theta + \varepsilon \notin a(\tilde{\theta}) - \tilde{\theta} + \Omega$ for some ε in Ω and thus $u_t(\theta,\tilde{\theta},\alpha(\theta,\tilde{\theta})) = -\infty$. But we have

$$u_t(\theta,\tilde{\theta},a(\tilde{\theta}) - \tilde{\theta} + \theta) = u_t(\tilde{\theta},\tilde{\theta},a(\tilde{\theta})) - \psi(a(\tilde{\theta}) - \tilde{\theta} + \theta) + \psi(a(\tilde{\theta})) > -\infty$$

which contradicts (35). We thus have

$$\alpha(\theta,\tilde{\theta}) = a(\tilde{\theta}) - \tilde{\theta} + \theta \tag{36}$$

Let
$$\begin{aligned} U_t(\theta,\tilde{\theta}) &= \text{Max}\ \{u_t(\theta,\tilde{\theta},\tilde{a}),\tilde{a} \geq 0\} \\ &= u_t(\theta,\tilde{\theta},a(\tilde{\theta}) - \tilde{\theta} + \theta) \\ &= s(\tilde{\theta}) - \psi(a(\tilde{\theta}) - \tilde{\theta} + \theta) \end{aligned}$$

$U_t(\theta,\tilde{\theta})$ is the agent's optimal expected utility when the cost parameter θ and the cost report is $\tilde{\theta}$. Let us derive a function $s(\theta)$ such that

$$\theta \in \underset{\tilde{\theta} \in \Delta}{\text{Arg Max}}\ U_t(\theta,\tilde{\theta}) \quad \text{for all } \theta \text{ in } \Delta \tag{37}$$

which will imply (i).

We have at any point of differentiability

$$\frac{\partial U_t}{\partial \tilde{\theta}} (\theta,\tilde{\theta}) = s'(\tilde{\theta}) - \psi'(a(\tilde{\theta}) - \tilde{\theta} + \theta)(a'(\tilde{\theta}) - 1) \tag{38}$$

and a first order condition for (37) to be realized writes as

$$s'(\theta) - \psi'(a(\theta)) (a'(\theta) - 1) = 0 \quad \text{a.e.}$$

which gives

$$s(\theta) = - \int_{\theta}^{\theta_1} \psi'(a(s)) (a'(s) - 1) \, ds + s(\theta_1)$$

we then have

$$\frac{\partial U_t}{\partial \tilde{\theta}} (\theta,\tilde{\theta}) = (\psi'(a(\tilde{\theta}) - \psi'(a(\tilde{\theta}) - \tilde{\theta} + \theta)) (a'(\tilde{\theta}) - 1) \tag{39}$$

Since function $a(\theta) - \theta$ is nonincreasing, (39) gives

$$\tilde{\theta} \lesseqgtr \theta \text{ if } \frac{\partial U_t}{\partial \tilde{\theta}} (\theta,\tilde{\theta}) \gtrless 0 \tag{40}$$

Since $a(\theta)$ is continuous, (40) implies (37). Condition (i) is thus satisfied. Furthermore, we have

$$u_{a,t}(\theta_1) = s(\theta_1) - \psi(a(\theta_1))$$

and (ii') is satisfied if $s(\theta_1) = \psi(a(\theta_1))$. Lastly we have

$t(\theta,\theta + a(\theta) + \varepsilon) = s(\theta)$ for all ε in Ω and (iii) is also satisfied.

q.e.d.

Proposition 3 deserves a number of comments. Let us observe first that the characterization of effort functions which correspond to efficient mecanism is quite independent of the distribution of the random disturbance ε . In particular, this characterization includes the case of no uncertainty, that is $\varepsilon = 0$ with probability 1. So, proposition 3 shows that the principal can obtain the same welfare level when the relation between the level of effort and the outcome is stochastic and when it is noiseless[(1)].

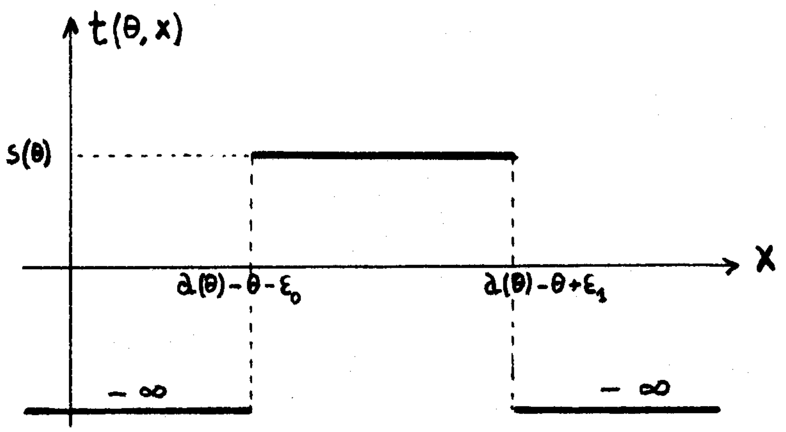
In the noiseless model we have $a = x + \theta$: if the agent reports truthfully his cost parameter θ observing the outcome x is equivallent to observing directly the level of effort a . The problem is then reduced to a pure adverse selection problem where inefficiency results only from the inability of the principal to observe the cost parameter. Consersely, when there is a random disturbance, different levels of effort correspond to the same outcome and observing the outcome provides an imperfect information about the agent's level of effort. However, proposition 3 shows that the same welfare level is obtained in both cases.

Secondly, a step of the proof of proposition 3 is using the following incentive scheme :

$$t(\theta,x) = \begin{cases} s(\theta) & \text{if } x \in a(\theta) - \theta + \Omega \\ -\infty & \text{otherwise} \end{cases}$$

with $s(\theta) = -\int_{\theta}^{\theta_1} \psi'(a(s))(a'(s) - 1)\, ds + \psi(a(\theta_1))$

FIGURE 2



(1) This neutrality of the introduction of noise has been demonstrated in a different framework by Melumad and Reichelstein (1984). Laffont and Tirole (1985) argue that this property holds in their model for a well-behaved distribution function ($z(\theta)$ non decreasing). More generally proposition 3 shows that the neutrality of noise holds for any distribution function of the unknown parameter.

In the noiseless-model, this incentive scheme coincides with the "knife-edge" transfer schedule introduced by Laffont and Tirole (1985). If a random disturbance ε is introduced, using the previous incentive scheme requires a precise knowledge of the distribution of ε. The problem is highly simplified here by assuming that the support of ε is compact. Nevertheless, the principal is supposed to know exactly parameters ε_0 and ε_1 which define the support $\Omega = [\varepsilon_0, \varepsilon_1]$. If the principal makes a small error when estimating parameters ε_0 and ε_1, the incentive scheme is no more efficient. This unpleasant property justifies seeking for an incentive scheme which requires less information about the distribution pf the random disturbance. In this respect, Proposition 4 will show that any efficient mecanism can be approximated as closed as desirable by using a quadratic incentive schemes the coefficients of which depend only on σ^2 and θ.

<u>*PROPOSITION 4*</u>. *If $\{a(.), t(.)\}$ is efficient and $a(.)$ is continous, for all $\eta > 0$ there exists an efficient mecanism $\{\bar{a}(.), \bar{t}(.)\}$ such that*

$$(\alpha) \quad |\bar{a}(\theta) - a(\theta)| \leqq \eta \quad \text{for all } \theta \text{ in } \Delta \tag{41}$$

$$(\beta) \quad \bar{t}(\theta,x) = \frac{H}{2}\left((x - \bar{x}^e(\theta))^2 - \sigma^2\right) + \psi'(\bar{a}(\theta))(x - \bar{x}^e(\theta)) + \psi(\bar{a}(\theta)) + \int_\theta^{\theta_1} \psi'(\bar{a}(s))\, ds \tag{42}$$

where $\bar{x}^e(\theta) = \bar{a}(\theta) - \theta$ denotes the expected outcome. Furthermore parameter H is nonpositive and depends only on function $a(.)$ and parameter η.

The proof of proposition 4 is rather tedious and is therefore developped in appendix.

Observe that $\bar{t}(\theta,x)$ may be interpreted as a bonus-penalty system including a fixed transfer (which is higher for low cost agents than for high cost agents) and a bonus (or a penalty) which depends non linearly on the difference between observed and expected outcomes.

Function $\bar{t}(\theta,x)$ coincides with the optimal linear incentive scheme of the previous section if H can be chosen equal to zero, which is the case when $\bar{a}(\theta)$ is nonincreasing. In the general case, we have $\frac{\partial \bar{t}}{\partial x}(\theta,\bar{x}^e(\theta)) = \psi'(a(\theta)$: when the realized outcome equals the expected one, the quadratic incentive scheme $\bar{t}$ and the linear incentive scheme developped in proposition 1 coïncide and their slope equals the marginal disutility of effort. When x is greater (respect. lower) than $\bar{x}^e(\theta)$, the marginal reward $\frac{\partial \bar{t}}{\partial x}(\theta, x)$ is lower (respect. greater) than $\psi'(\bar{a}(\theta))$ and may even be negative for large values of x. We also have by total differentiation

$$\frac{d\bar{t}}{d\theta}(\theta,\bar{x}^e(\theta)) = \psi'(\bar{a}(\theta))\ (\bar{a}'(\theta) - 1) \leq 0$$

When the cost report increases, the quadratic incentive scheme decreases or remains unchanged, at least at $x = \bar{x}^e(\theta)$: inducing truthful revelation requires using an incentive scheme which is less favorable to high-cost agents than to low-cost agents.

Lastly it is worth observing that the quadratic incentive scheme $\bar{t}$ is much more robust to errors on the distribution of ε than the discontinuous function of proposition 3. Assuming that an upper bound $\hat{\sigma}$ for σ is known to the principal and $\hat{\sigma}$ taking the place of σ in $\bar{t}$, the resulting incentive scheme still implements $\bar{a}(.)$ and the involuntary increase in the agent's expected reward is $-\frac{H}{2}(\hat{\sigma}^2 - \sigma^2)$.

Using previous results, the principal's problem writes as.(1)

$$\underset{a(.),t(.)}{\text{Maximize}} \int_\Delta (a(\theta) - \theta - \psi(a(\theta)) - z(\theta)\ \psi'(a(\theta)))\ f(\theta)\ d\theta$$

$$\text{subject to : } a(\theta) - \theta \text{ is nonincreasing for all } \theta$$

$$a(\theta) \geq \theta \text{ for all } \theta$$

---

(1) In what follows the optimal effort function is supposed to be continuous.

This problem is formally similar to a number of usual problem in the theory of incentives. In particular, it belongs to the class of principal-agent problems studied by Guesnerie and Laffont (1984) and the optimal effort function can be explicitly obtained by means of the algorithm developped by these authors : the function $a(\theta) - \theta$ is locally constant over a finite number of intervalls where "bunching" occurs and it is strictly increasing for other values of θ .

From a different standpoint, we will show that using a method similar to Baron-Myerson's (1982) provides the optimal solution, at least when function $\psi(a)$ is quadratic. Let

$$m(\phi) = z\,[F^{-1}(\phi)] + F^{-1}(\phi)]$$

for any ϕ between 0 and 1 . Let

$$M(\phi) = \int_0^{\phi} m(\tilde{\phi})\, d\tilde{\phi}$$

and let $\overline{M}(\phi)$ be the convex hull of M , that is $\overline{M}(\phi)$ is the highest convex function on $[0,1]$ such that $M(\phi) \leq M(\phi)$ for all ϕ in $[0,1]$. Let

$$\overline{m}(\phi) = \overline{M}'(\phi)$$

extending $\overline{m}$ by right continuity when $\overline{M}'$ is not defined [1].

Finally let

$$\overline{z}(\theta) = \overline{m}[F(\theta)] - \theta$$

we then have the following lemma.

<u>*LEMMA 3.*</u> *There exists a continuous function* $\Gamma(\theta) : \Delta \to R$ *such that* $\Gamma(\theta) \geqq 0$ *for all* θ , $\overline{z}(\theta) + \theta$ *is locally constant whenever* $\Gamma(\theta) > 0$ *and*

$$\int_\Delta a(\theta)\, z(\theta)\, f(\theta)\, d\theta = \int_\Delta a(\theta)\, \overline{z}(\theta)\, f(\theta)\, d\theta - \int_\Delta \Gamma(\theta)\, d(a(\theta) - \theta)$$

$$+ \int_\Delta \theta(z(\theta) - \overline{z}(\theta))\, d\theta \qquad (44)$$

(1) Observe that $\overline{M}$ is convex and thus differentiable almost every where.

for any function $a(\theta)$ *such that* $a(\theta) - \theta$ *is non increasing. Furthermore,* $\bar{z}(\theta) + \theta$ *is a non decreasing function of* θ *, and if* $z(\theta) + \theta$ *is a non decreasing function of* θ *then* $\bar{z}(\theta) = z(\theta)$ *for all* θ *.*

Proof. The function $\Gamma(\theta)$ in the lemma is $\Gamma(\theta) = M(F(\theta)) - \bar{M}(F(\theta))$ and the proof is quite similar to the proof ofBaron-Myerson's lemma 3 (1982).

Using lemma 3, we are now in position to derive the optimal solution of the principal's problem in a simple case.

PROPOSITION 5. *If* $\psi(a) = \frac{ka^2}{2}$, $k > 0$ *and* $\bar{z}(\theta) \leq \frac{1}{k}$ *for all* θ *, the optimal effort function is*

$$a^*(\theta) = \frac{1-k\,\bar{z}(\theta)}{k} \quad \textit{for all } \theta$$

Proof. From lemma 3, the principal's expected welfare writes as

$$W = \int_\Delta (a(\theta) - \theta - k\frac{a(\theta)^2}{2} - k\,\bar{z}(\theta)\,a(\theta))\,f(\theta)\,d\theta$$
$$+ k\int_\Delta \Gamma(\theta)\,d(a(\theta) - \theta) - k\int_\Delta \theta(z(\theta) - \bar{z}(\theta))\,d\theta \tag{45}$$

$a^*(\theta) = \frac{1-k\,\bar{z}(\theta)}{k}$ maximizes the first integral in (45) and $a^*(\theta)$ is non-negative since $\bar{z}(\theta) \leq \frac{1}{k}$. Furthermore, since $\bar{z}(\theta) + \theta$ is nondecreasing, $a^*(\theta) - \theta$ is nonincreasing. From lemma 3, $\Gamma(\theta)$ is nonnegative so that the second integral in (45) is nonpositive for any function $a(\theta)$ such that $a(\theta) - \theta$ is nonincreasing. Since $\bar{z}(\theta) + \theta$ is locally constant whenever $\Gamma(\theta) > 0$, we have

$$\int_{\Delta} \Gamma(\theta)\, d(a(\theta) - \theta) = - \int_{\Delta} \Gamma(\theta)\, d(\bar{z}(\theta) + \theta) = 0$$

Since the third integral in (45) does not depend on $a(\theta)$, functions $a^{*}(\theta)$ maximizes W over the set of nonnegative function $a(\theta)$ such that $a(\theta) - \theta$ is nonincreasing.

Q.E.D.

To illustrate proposition 5 assume $k = 1$, $\Delta = [0,1]$ and $f(\theta) = \frac{1}{2}$ if $0 \leq \theta < \frac{1}{2}$, $f(\theta) = \frac{3}{2}$ if $\frac{1}{2} \leq \theta \leq 1$. We have $z(\theta) = \theta$ if $0 \leq \theta < \frac{1}{2}$ and $z(\theta) = \theta - \frac{1}{3}$ if $\frac{1}{2} \leq \theta < 1$. Computations give

$$M(\phi) = \text{Min } \{2\phi^2 , \frac{2\phi^2}{3} + \frac{\phi}{3}\} \quad \text{for all } \phi \text{ in } [0,1]$$

and

$$\bar{M}(\phi) = 2\phi^2 \quad \text{if } 0 \leq \phi \leq \frac{3+\sqrt{3}}{24}$$

$$\bar{M}(\phi) = \frac{3+\sqrt{3}}{6}\phi - 2\left(\frac{3+\sqrt{3}}{24}\right)^2 \quad \text{if } \frac{3+\sqrt{3}}{24} < \phi \leq \frac{1+\sqrt{3}}{8}$$

$$\bar{M}(\phi) = \frac{2\phi^2}{3} + \frac{\phi}{3} \quad \text{if } \frac{1+\sqrt{3}}{8} < \phi \leq 1$$

we obtain

$$\bar{z}(\theta) = \theta \text{ and } a^{*}(\theta) = 1 - \theta \quad \text{if } 0 \leq \theta \leq \frac{3+\sqrt{3}}{12}$$

$$\bar{z}(\theta) = \frac{3+\sqrt{3}}{6} - \theta \text{ and } a^{*}(\theta) = \frac{3-\sqrt{3}}{6} + \theta \quad \text{if } \frac{3+\sqrt{3}}{12} \leq \theta \leq \frac{5+\sqrt{3}}{12}$$

$$\bar{z}(\theta) = \theta - \frac{1}{3} \text{ and } a^{*}(\theta) = \frac{4}{3} - \theta \quad \text{if } \frac{5+\sqrt{3}}{12} \leq \theta \leq 1$$

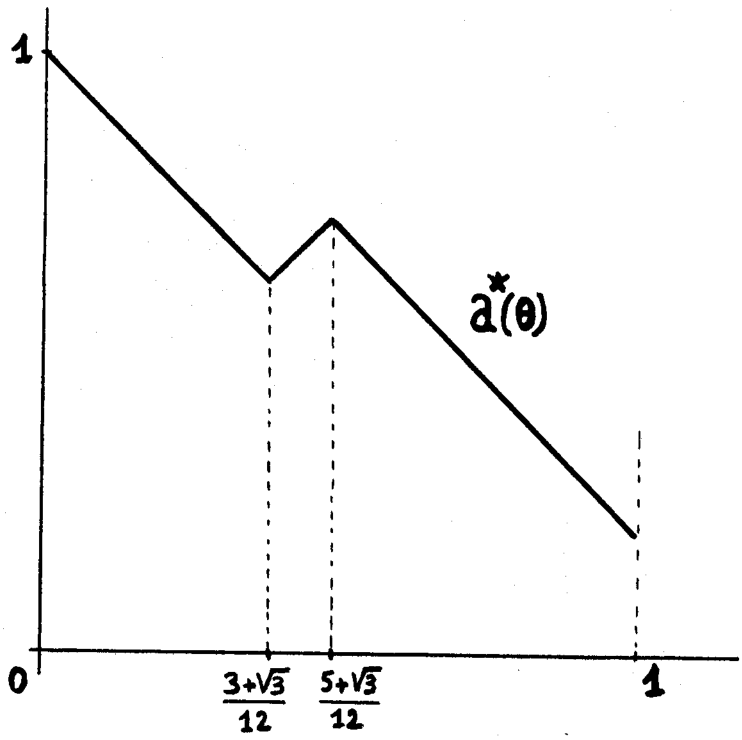


FIGURE 3

VI. EXTENSIONS.

Assume now that the agent's decision process includes picking a level of effort and also choosing the value of an observable variable y . This variable will be relevant of a number of applications.

When reporting a cost parameter $\tilde{\theta}$, the agent commits to take decision $y(\tilde{\theta})$.[1] The agent's utility does not depend on y and writes still as $t(\tilde{\theta},x) - \varphi(a)$ with $x = a - \theta + \varepsilon$.

The principal's utility is supposed to write as

$$\lambda(y) + \mu(y)x - t(\tilde{\theta},x)$$

---

(1) Implicitly, $t(\tilde{\theta},x) = -\infty$ for all x if $y \neq y(\tilde{\theta})$.

and functions $\lambda(y)$ and $\mu(y)$ satisfy $\lambda''(y) \leqq 0$, $\mu''(y) \leqq 0$. A mecanism will describe the observable decision $y(\theta)$, the level of effort $a(\theta)$ and the agent's reward $t(\theta,x)$. A mecanism $\{y(.),a(.),t(.)\}$ is said to be efficient if $\{a(.),t(.)\}$ satisfies (i),(ii') and (iii).

The principal's expected welfare writes as

$$W = \int_{\Delta} (\lambda(y(\theta)) + \mu(y(\theta))(a(\theta) - \theta) - \psi(a(\theta)) - z(\theta)\ \psi'(a(\theta)))\ f(\theta)\ d\theta \qquad (46)$$

Ignoring implementability constraints and maximizing W gives the following necessary conditions at an interior optimum $y^*(\theta),a^*(\theta)$ (1) :

$$\lambda'(y^*) + \mu'(y^*)\ (a^* - \theta) = 0 \qquad (47)$$

$$\mu(y^*) - \psi'(a^*) - z(\theta)\ \psi''(a^*) = 0 \qquad (48)$$

and a local second order condition implies that the determinant of

$$A = \begin{pmatrix} \lambda''(y^*) + \mu''(y^*)(a^* - \theta) & \mu'(y^*) \\ \mu'(y^*) & -\psi''(a^*) - z(\theta)\ \psi'''(a^*) \end{pmatrix}$$

is non negative.

Differentiating (47) and (48) gives

$$\frac{da^*}{d\theta} = \frac{z'(\theta)\ \psi''(a^*)(\lambda''(y^*) + (a^* - \theta)\ \mu''(y^*)) - \mu'(y^*)^2}{\det\ (A)}$$

and a sufficient condition for a^* to be non increasing is $z'(\theta) \geqq 0$ (2) . Then $\{a^*,t^*\}$ satisfies (i), (ii') and (iii) if t^* is a linear incentive scheme

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(1) As in the basic model of previous sections, the integrand in (46) is not concave so that first order conditions are not sufficient to define an optimum y^* , a^* .

(2) We assume $a^*(\theta) - \theta > 0$ for all θ in Δ .

defined as in proposition 2 by conditions (22),(23),(24) or (30). We thus have proved :

PROPOSITION 6. If $z(\theta)$ is non decreasing, an optimal mecanism $\{y^*(.),a^*(.),t^*(.)\}$ satisfies conditions (22),(23),(24) -(30),(47),(48) and (49).

This extension of our basic model is relevant of various applications and examples are sketched in what follows. Function $z(\theta)$ is supposed to be non decreasing so that proposition 6 holds. If this assumption were not satisfied, using a technique similar to section 5's would provide the optimal mecanism. In particular, using a quadratic incentive scheme would allow to approximate the optimal mecanism as closed as desirable.

a/ Regulating firms under cost observability.

Regulation procedures for firms when the planner can observe cost but cannot monitor effort have been studied by Laffont and Tirole (1985) and fundamental results of these authors can be obtained as consequences of proposition 6.

Assume that the agent is the manager of a regulated firm which produces a public good q at cost $C = - x q = (\theta - a - \varepsilon)q$. We have here $y = q$. The level of effort a decreases the initial marginal cost $\theta - \varepsilon$. The principal is a public regulator who observes and reimburses the cost C and pays in addition a net monetary transfert t . The public good provides a consumer surplus $S(q)$ $(S' > 0, S'' < 0)$ and the principal's welfare is $S(q) - (t + C) = S(q) + x q - t$. We thus have $\lambda(q) = S(q)$ and $\mu(q) = q$. Conditions (47),(48) and (30) become respectively

$$S'(q^*) = \theta - a^* \tag{47'}$$

$$q^* = \psi'(a^*) + z(\theta)\ \psi''(a^*) \tag{48'}$$

$$t^*(\theta,C) = \frac{\psi'(a^*(\theta))}{q^*(\theta)}\ (C^*(\theta) - C) + \psi(a^*(\theta)) + \int_{\theta}^{\theta_1} \psi'(a^*(s))\ ds \tag{30'}$$

where $C^*(\theta) = (\theta - a^*(\theta)\ q^*(\theta)$ is the expected production cost. (q^*,a^*)

can be compared to the solution $(\bar{q},\bar{a})$ which would prevail if the cost parameter θ were common knowledge, i.e.

$$S'(\bar{q}) = \theta - \bar{a}$$

$$\bar{q} = \psi'(\bar{a})$$

which implies $q^*(\theta) < \bar{q}(\theta)$ and $a^*(\theta) < \bar{a}(\theta)$ for all θ (except θ_0) .

(48') implies $0 < \frac{\psi'(a^*)}{q^*} < 1$ and from (30') a variable fraction of realized cost is reimbursed to the firm.(1)

The analysis applies if the product is sold on a market at a price $P(q)$. The consumer's gain is the consumer's surplus minus the firm's subsidy, i.e.

$$\int_0^q P(\tilde{q})\, d\tilde{q} + x\, q - t$$

and in this case we have $\lambda(q) = \int_0^q P(\tilde{q})\, d\tilde{q}$ and $\mu(q) = q$. (47) now become

$$P(q^*) = \theta - a^*$$

so that marginal cost pricing is optimal but costs are calculated for a suboptimal level of effort.

b/ <u>Investment banking contracts for new issues</u>.

Another application of the model is to the case of the relationship between an investment banker and an issues of new securities as studied by Baron (1982) and Baron and Holmstrom (1980). This problem is rather specific but may be viewed as an example of producer-retailer contracts.

When placing a new security issue the banker obtains private information about the capital market through preselling activities. Furthermore, the banker's distribution effort may, in some extend, generate demand for the issue. As underlined by Baron and Holmstrom, in such a framework, "the task

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(1) Under suitable assumptions on function ψ , it could be shown that the fract of reimbursed costs - which is equal to $1 - \frac{\psi'(a^*)}{q^*}$ - decreases with the cost parameter (see Laffont-Tirole (1985) for details).

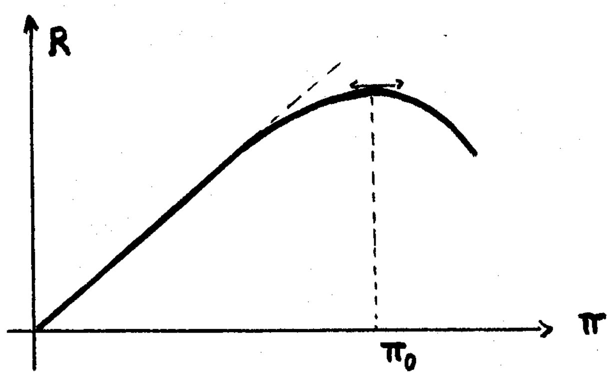
of the issuer is to design a contract that both induces the banker to use (his) information to the issuer's advantage and provides a disincentive for the banker to price the issue too low in order to reduce the effort required to sell the issue".

Consider a as the banker's level of distribution effort and θ as a parameter negatively correlated with the demand for the issue. Assume that the proceeds from the sale of the issue depend simultaneously on the offer price π and on the stochastic parameter $x = a - \theta + \varepsilon$. Proceeds R will be written as

$$R = k\,\pi + x\,\gamma(\pi) \qquad \gamma'' \leqq 0$$

Proceeds may be linear for offer prices such that the issue is oversubscribed. They will be increasing and then decreasing for higher offer prices. Furthermore, the higher is the parameter x , the higher is the offer price π_0 which maximizes proceeds and the higher are corresponding proceeds.(1)

FIGURE 4



We have here $y = \pi$ and $\lambda(\pi) = k\,\pi$, $\mu(\pi) = \gamma(\pi)$. Conditions (47), (48) and (30) become respectively

$$k = (\theta - a^*)\,\gamma'(\pi^*) \tag{47"}$$

$$\gamma(\pi^*) = \psi'(a^*) + z(\theta)\,\psi''(a^*) \tag{48"}$$

$$t^*(\theta,R) = \frac{\psi'(a^*(\theta))}{\gamma(\pi^*(\theta))}(R - R^*(\theta)) + \psi(a^*(\theta)) + \int_\theta^{\theta_1} \psi'(a^*(s))\,ds \tag{30"}$$

---

(1) Baron (1982) studies a similar model but does not derive the optimal incentive scheme.

while the first best solution $(\bar{\pi},\bar{a})$ satisfies

$$k = (\theta - \bar{a})\ \gamma'(\bar{\pi})$$

$$\gamma(\bar{\pi}) = \psi'(\bar{a})$$

which implies $\bar{\pi}^*(\theta) < \pi(\theta)$ and $a^*(\theta) < \bar{a}(\theta)$ for all θ (except θ_0) : because of the asymmetric information, the issuer lowers the offer price and the banker lowers the level of effort. Furthermore, the banker receives a fixed payment and a part of the proceeds from the sale of the issue. When θ converges to θ_0 , the incentive scheme converges to a commitment contract in which the issuer receives a fixed payment independent of realized proceeds.

c/ Two-level planning with production targets.

Another example combining adverse selection and moral hazard is described in the litterature on incentives in central planned economy (see Weitzman (1976) on the new Soviet Incentive Scheme).

Assume that the principal is a central planner who allocates a scarce ressource (say labour) to decentralized firms. For any firm, using ℓ units of labour provides a net output $Y = x\ h(\ell)$ with $h' > 0$, $h'' < 0$. Assume that $f(\theta)$ reflects the objective distribution of the cost parameter over the set of decentralized firms. A feasibility constraint is

$$\int_{\Delta} \ell(\theta)\ f(\theta)\ d\theta \leq L \tag{50}$$

where L is the available labour force. The planner's objective is to maximize the agregate output net of transfers to firms. Under incomplete information this problem writes as

$$\text{Maximize} \int_{\Delta} ((a(\theta) - \theta)\ h(\ell(\theta)) - \psi(a(\theta)) - z(\theta)\ \psi'(a(\theta)))\ f(\theta)\ d\theta$$

subject to the feasibility constraint (50). Introducing a Kuhn-Tucker multiplier optimality conditions are given by (47)-(48) with $y = \ell$, $\lambda(\ell) = -\beta\ \ell$ and $\mu(\ell) = h(\ell)$. An optimal interior mecanism satisfies

$$(a^* - \theta)\ h'(\ell^*) = \beta \tag{47"'}$$

$$h(\ell^*) = \psi'(a^*) + z(\theta)\ \psi''(a^*) \tag{48"'}$$

$$t^*(\theta,Y) = \frac{\psi'(a^*(\theta))}{h(\ell^*(\theta))}\ (Y - Y^*(\theta)) + \psi(a^*(\theta)) + \int_\theta^{\theta_1} \psi'(a^*(s))\ ds \tag{30"'}$$

where $Y^*(\theta) = (a^*(\theta) - \theta)\ h(\ell^*(\theta))$ is a production target which depends on the cost report θ.

Since $a^*(\theta)$ is nonincreasing, (47"') show that $\ell^*(\theta)$ and thus $Y^*(\theta)$ are both nonincreasing functions of θ.

From (30"'), firms receive a fixed transfer $\psi(a^*) - \int_\theta^{\theta_1} \psi'(a^*)\ ds$ which is higher for low cost firms than for high cost firms. They also receive a bonus or pay a penalty which is proportionnal to the difference between realized and targeted output levels.(1)

To evaluate the consequences of incomplete information on the allocation of labour, consider the following example. Assume that θ is uniformly distributed on $\Delta = [-2,-1]$ (2) , $\psi(a) = \frac{a^2}{2}$ and $h(\ell) = \ell^{1/2}$.

The full information optimal solution $\hat{\ell}(\theta), \hat{a}(\theta)$ is given by

$$\text{Maximize} \int_{-2}^{-1} ((a - \theta)\ \ell^{1/2} - \frac{a^2}{2})\ d\theta$$

subject to

$$\int_{-2}^{-1} \ell\ d\theta \leqq L$$

which yields $\hat{\ell}(\theta) = \frac{3L}{7}\theta^2$ and $\hat{a}(\theta) = -\theta\sqrt{\frac{3L}{7}}$

Under asymmetric information, the principal's problem writes as

$$\text{Maximize} \int_{-2}^{-1} ((a - \theta)\ \ell^{1/2} - \frac{a^2}{2} - (\theta + 1)a)\ d\theta$$

---

(1) One easily cheks that a sufficient condition for the proportionality coefficient $\frac{\psi'(a^*)}{h(\ell^*)}$ to be a nonincreasing function of θ is that function ψ is quadratic.

(2) Observe that $\Delta \subset R^-$ gives $a(\theta) - \theta > 0$ for any positive effort function so that the expected output $(a(\theta) - \theta)\ h(\ell)$ is positive for all ℓ.

subject to

$$\int_{-2}^{-1} \ell \, d\theta \leq L$$

and we obtain $\ell^*(\theta) = \frac{3L}{13}(2\theta + 1)^2$ and $a^*(\theta) = -(2\theta + 1)\sqrt{\frac{3L}{13}} - (\theta + 1)$

In this simple case, the labour allocation of low cost firms is larger under asymmetric information than under complete information and the reverse is true for high cost firms. Moreover, under incomplete information, low cost firms yield a higher level of effort than under full information while the contrary holds for high cost firms.

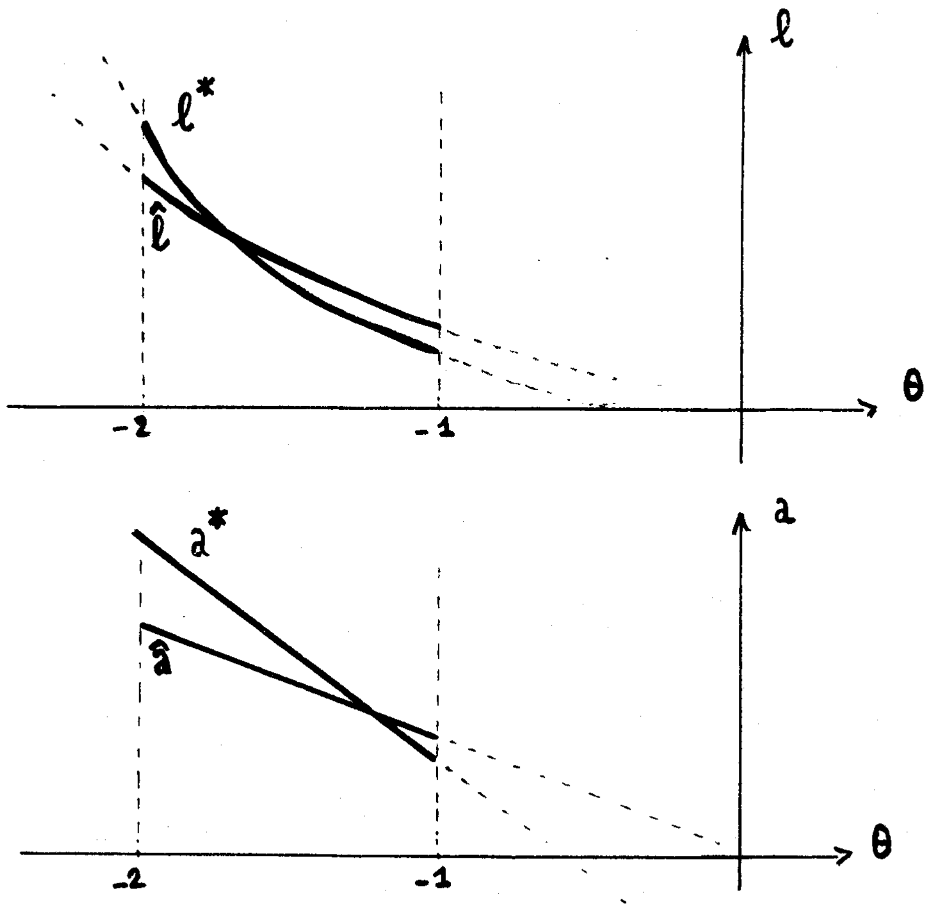


FIGURE 5

VII. CONCLUSION.

The purpose of this paper has been to state properties of a principal-agent model combining moral hazard and adverse selection. The optimal design of incentive schemes as well as a number of applications of the model have been presented.

However, the framework in which these results are derived neglects important issues which would require further research. In particular, it would be important to relax the risk-neutrality assumption (for instance to study the interaction between moral hazard and risk aversion in the design of insurance contracts). Likewise, the separability assumption for the expected outcome $a - \theta$ makes the model tractable but is quite restrictive.

APPENDIX : Proof of proposition 4.

Assume that $\{a(.),t(.)\}$ is efficient and $a(.)$ is continuous. From proposition 3 we have $a'(\theta) \leqq 1$ for all θ . Let $\eta > 0$ and

$$\bar{a}(\theta) = a(\theta) - \frac{\eta}{2} \left(\frac{\theta - \theta_0}{\theta_1 - \theta_0} - 1\right)$$

Condition (α) is satisfied and we have $\bar{a}'(\theta) - 1 \leqq - \frac{\eta}{2(\theta_1-\theta_0)}$ for all θ. Consider the following quadratic incentive scheme (where H does not depend on θ) :

$$\bar{t}(\theta,x) = \frac{H}{2} x^2 + K(\theta)\, x + G(\theta)$$

we have

$$u_{\bar{t}}(\theta,\tilde{\theta},a) = \int_{\Omega} \bar{t}(\tilde{\theta},a - \theta + \varepsilon)\, g(\varepsilon)\, d\, \varepsilon - \psi(a) \tag{A-1}$$

$$= \frac{H}{2} (\sigma^2 + (a - \theta)^2) + K(\tilde{\theta})(a - \theta) + G(\tilde{\theta}) - \psi(a) \tag{A-2}$$

We will show that the coefficient H and functions $K(\theta)$ and $G(\theta)$ can be chosen so as to satisfy conditions (i),(ii') and (iii), i.e.

(i) $u_{\bar{t}}(\theta,\theta,\bar{a}(\theta)) \geqq u_{\bar{t}}(\theta,\tilde{\theta},\tilde{a})$ for all $\tilde{a}$ in R_+, for all θ and $\tilde{\theta}$ in Δ

(ii') $u_{\bar{t}}(\theta_1,\theta_1,\bar{a}(\theta_1)) = 0$

(iii) $\bar{t}(\theta, \mathbf{a(\theta) - \theta} + \varepsilon) \neq - \infty$ for all ε in Ω , for all θ in Δ .

Let $\alpha(\theta,\tilde{\theta})$ denote the optimal level of effort when the cost report in $\tilde{\theta}$ and the true cost parameter is θ . Function α is defined by

$$\alpha(\theta,\tilde{\theta}) \in \underset{\tilde{a} \in R^+}{\text{Arg Max}} \{u_t(\theta,\tilde{\theta},\tilde{a})) \tag{A-3}$$

For all θ in Δ assume

$$K(\theta) = \psi'(\bar{a}(\theta)) - H(\bar{a}(\theta) - \theta) \tag{A-4}$$

$$H \leqq 0 \tag{A-5}$$

(A-4) and (A-5) yield together a sufficient condition for

$$\alpha(\theta,\theta) = \bar{a}(\theta) \quad \text{for all} \quad \theta \tag{A-6}$$

Using (A-4) and (A-5), condition (i) will be fulfilled if truthtelling is an optimal strategy for the agent.

Let

$$U_{\bar{t}}(\theta,\tilde{\theta}) = \text{Max}\ \{u_{\bar{t}}(\theta,\tilde{\theta},\tilde{a})\ ,\ \tilde{a} \in R^{+}\}$$

$$= u_{\bar{t}}(\theta,\tilde{\theta},\alpha(\theta,\tilde{\theta}))$$

$U_{\bar{t}}(\theta,\tilde{\theta})$ is the optimal expected utility of the agent when his cost parameter is θ and he reports $\tilde{\theta}$. We will derive sufficient conditions for truthtelling to be an optimal agent's strategy, that is

$$\theta \in \underset{\tilde{\theta} \in \Delta}{\text{Arg Max}}\ U_{\bar{t}}(\theta,\tilde{\theta}) \quad \text{for all} \quad \theta \tag{A-7}$$

From (A-2) and the "enveloppe theorem", we have

$$\frac{\partial U_{\bar{t}}}{\partial \tilde{\theta}}(\theta,\tilde{\theta}) = K'(\tilde{\theta})\ (\alpha(\theta,\tilde{\theta}) - \theta) + G'(\tilde{\theta}) \tag{A-8}$$

Using (A-6) and (A-8), a first order condition for (A-7) writes as

$$K'(\theta)\ (\bar{a}(\theta) - \theta) + G'(\theta) = 0 \tag{A-9}$$

or equivalently for a continuous function $G(\theta)$:

$$G(\theta) = G(\theta_1) + \int_{\theta}^{\theta_1} K'(s)\ (\bar{a}(s) - s)\ ds \tag{A-10}$$

From now we assume that $H, K(\theta)$ and $G(\theta)$ satisfy (A-4), (A-5) and (A-10) and we will derive a local second order condition for (A-7) .

At points of differentiability of function α we have

$$\frac{\partial^2 U_{\bar{t}}}{\partial \tilde{\theta}^2}(\theta,\tilde{\theta}) = K'(\tilde{\theta})\ \frac{\partial \alpha}{\partial \tilde{\theta}}(\theta,\tilde{\theta}) + K''(\tilde{\theta})\ (\alpha(\theta,\tilde{\theta}) - \theta) + G''(\tilde{\theta}) \tag{A-11}$$

From the maximum theorem, if functions $K(\theta)$ and $G(\theta)$ are continuous, function $\alpha(\theta,\tilde{\theta})$ is also continuous. As $a(\theta)$ is positive, $\bar{a}(\theta)$ is strictly positive and (A-6) implies that $\alpha(\theta,\tilde{\theta})$ is strictly positive if $\tilde{\theta}$ is not too different from θ . From (A-2) and (A-3) we have then

$$H(\alpha(\theta,\tilde{\theta}) - \theta) + K(\tilde{\theta}) - \psi'(\alpha(\theta,\tilde{\theta})) = 0 \tag{A-12}$$

and from (A-6) we have

$$\frac{\partial\alpha}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) = \frac{K'(\theta)}{\psi''(\bar{a}(\theta)) - H} \quad \text{if } \tilde{\theta} = \theta \tag{A-13}$$

Furthermore, differentiating (A-9) gives

$$K''(\theta)\,(\bar{a}(\theta) - \theta) + G''(\theta) = -K'(\theta)\,(\bar{a}'(\theta) - 1) \tag{A-14}$$

and (A-6),(A-11),(A-13) and (A-14) simultaneously give

$$\frac{\partial^2 U_{\bar{t}}}{\partial\tilde{\theta}^2}(\theta,\tilde{\theta}) = K'(\theta)\left[\frac{K'(\theta)}{\psi''(a(\theta)) - H} - \bar{a}'(\theta) + 1\right] \quad \text{if } \tilde{\theta} = \theta \tag{A-15}$$

Differentiating (A-4) we have

$$K'(\theta) = \psi''(\bar{a}(\theta))\ \ a'(\theta) - H(a'(\theta) - 1) \tag{A-16}$$

(A-15) and (A-16) give

$$\frac{\partial^2 U_{\bar{t}}}{\partial\tilde{\theta}^2}(\theta,\tilde{\theta}) = \frac{\psi''(\bar{a}(\theta))(\psi''(a(\theta))\ \bar{a}'(\theta) - H(\bar{a}'(\theta) - 1)}{\psi''(\bar{a}(\theta)) - H} \quad \text{if } \tilde{\theta} = \theta$$

and the local second order condition

$$\frac{\partial^2 U_t}{\partial\tilde{\theta}^2}(\theta,\tilde{\theta}) \leq 0 \quad \text{if } \tilde{\theta} = \theta$$

is satisfied if

$$H \leq -\frac{\psi''(\bar{a}(\theta))\ \bar{a}'(\theta)}{1 - \bar{a}'(\theta)} \tag{A-17}$$

Since $\bar{a}'(\theta) \leq 1 - \frac{\eta}{2(\theta_1 - \theta_0)}$ for all θ , (A-17) will be satisfied if

$$H \leq - \frac{(2(\theta_1-\theta_0)-\eta)\rho}{\eta} \qquad \text{(A-18)}$$

with $\rho = \text{Max}\ \{\psi''(\bar{a}(\theta)),\ \theta \in \Delta\}$.

We will show now that (A-18) is sufficient for (A-7) to be satisfied. Since $U_{\bar{t}}(\theta,\tilde{\theta})$ is continuous, if (A-7) were not satisfied there would exist $\tilde{\theta}$ such that either $\tilde{\theta} > \theta$ and $\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) > 0$ or $\tilde{\theta} < \theta$ and $\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) < 0$. In the first case, we would have $\tilde{\theta} > \theta$ and

$$\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) < \frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\theta) = \frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\tilde{\theta},\tilde{\theta}) = 0 \qquad \text{(A-19)}$$

But (A-8) and (A-9) imply

$$\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta}) = K'(\tilde{\theta})\ (\alpha(\theta,\tilde{\theta}) - \theta + \tilde{\theta} - \bar{a}(\tilde{\theta}))$$

which implies that $\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta})$ is a continuous function of θ with

$$\frac{\partial^2 U_{\bar{t}}}{\partial\tilde{\theta}\partial\theta} = K'(\tilde{\theta})\ (\frac{\partial\alpha}{\partial\theta}(\theta,\tilde{\theta}) - 1) \qquad \text{(A-20)}$$

If $\alpha(\theta,\tilde{\theta})$ is locally strictly positive, we have from (A-12)

$$\frac{\partial\alpha}{\partial\theta}(\theta,\tilde{\theta}) = \frac{H}{H - \psi''(\alpha(\theta,\tilde{\theta}))} \qquad \text{(A-21)}$$

which gives using (A-17), (A-20) and (A-21) (1)

$$\frac{\partial^2 U_{\bar{t}}}{\partial\tilde{\theta}\partial\theta}(\theta,\tilde{\theta}) = K'(\tilde{\theta})\ \frac{\psi''(\alpha(\tilde{\theta},\theta))}{H - \psi''(\alpha(\theta,\tilde{\theta}))} > 0$$

If $\alpha(\theta,\tilde{\theta})$ is locally equal to zero, we have

$$\frac{\partial^2 U_{\bar{t}}}{\partial\tilde{\theta}\partial\theta}(\theta,\tilde{\theta}) = -\ K'(\tilde{\theta}) > 0$$

Hence, $\frac{\partial U_{\bar{t}}}{\partial\tilde{\theta}}(\theta,\tilde{\theta})$ is an increasing function of θ which contradicts (A-19) and $\tilde{\theta} > \theta$.

The proof is symmetrical when $\tilde{\theta} < \theta$.

---

(1) observe that (A-16) and (A-17) imply together $K'(\theta) < 0$ for all θ .

So, we have proved that (A-4),(A-5),(A-10) and (A-18) yield together a sufficient condition for condition (i) to be fulfilled.

(ii) will be satisfied if

$$\frac{H}{2}(\sigma^2 + (\bar{a}(\theta_1) - \theta_1)^2) + K(\theta_1)(\bar{a}(\theta_1) - \theta_1) + G(\theta_1) = \psi(a(\theta_1)) \qquad \text{(A-22)}$$

which gives $G(\theta_1)$. Lastly, condition (iii) is obviously satisfied.

To sum up, sufficient conditions for the mecanism $\{a(.),t(.)\}$ to be efficient are

$$H \leq \text{Min}\ \{0, -\frac{(2(\theta_1-\theta_0)-\eta)}{\eta}\} \qquad \text{(A-5),(A-18)}$$

$$K(\theta) = \psi'(\bar{a}(\theta)) - H(\bar{a}(\theta) - \theta) \qquad \text{(A-4)}$$

$$G(\theta) = G(\theta_1) + \int_{\theta}^{\theta_1} K'(s)\ (\bar{a}(s) - s)\ ds \qquad \text{(A-10)}$$

$$G(\theta_1) = -\frac{H}{2}(\sigma^2 + (\bar{a}(\theta_1) - \theta_1)^2) - K(\theta_1)(\bar{a}(\theta_1) - \theta_1) + \psi(\bar{a}(\theta_1)) \qquad \text{(A-22)}$$

Straighforward computations show that $\bar{t}$ can be rewritten as

$$\bar{t}(\theta,x) = \frac{H}{2}((x - \bar{x}^e(\theta))^2 - \sigma^2) + \psi'(\bar{a}(\theta))\ (x - x^e(\theta)) + \int_{\theta}^{\theta_1} \psi'(\bar{a}(s))\ ds + \psi(\bar{a}(\theta))$$

with $\bar{x}^e(\theta) = \bar{a}(\theta) - \theta.$

q.e.d.

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