DISTRIBUTIONS OF PREFERENCES AND THE "LAW OF DEMAND"

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"LES DISTRIBUTIONS DE PREFERENCES ET LA LOI DE LA DEMANDE

RESUME

Cet article donne des conditions suffisantes pour que la matrice Jacobienne de la demande concurrentielle agrégée soit définie négative. Cette propriété forte implique en particulier que l'axiome faible de la préférence révélée est satisfait par la demande agrégée, et que la demande totale pour chaque bien augmente lorsque son prix décroit. Ces conditions suffisantes portent sur la distribution des préférences et sont indépendantes de la distribution des revenus. L'approche repose sur la notion de transformation homothétique d'une relation de préférence. On discute également la portée pour ce type de problème de la notion de transformation affine d'une préférence, qui introduit une structure algébrique intéressante sur l'espace des préférences. On montre également comment le résultat est lié à celui obenu par W. Hildenbrand (Econometrica (1983)) en posant des restrictions sur la distribution des revenus.

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"DISTRIBUTIONS OF PREFERENCES AND THE LAW OF DEMAND"

ABSTRACT

This paper gives sufficient conditions for the Jacobian matrix of aggregate competitive demand to be negative definite. This strong property implies that the weak axiom of revealed preference is satisfied in the aggregate, and that total demand for each good is a decreasing function of its own price. These sufficient conditions involve restrictions on the distribution of preferences and are independent of the distribution of income. The approach relies upon the notion of a homothetic transformation of a preference relation. The possible usefulness for aggregation problems of more general affine transformations, which induce a nice algebraic structure on the space of preferences, is also pointed out. It is shown how the result of the paper is related to a similar result obtained by W. Hildenbrand (Econometrica (1983)) who imposed restrictions on the distribution of income.

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A la mémoire d'André Nataf

Economic theory is plagued by quite a few embarrassing results. An obvious example is social choice theory with Arrow's famous impossibility theorem. No less important is the Debreu-Sonnenschein claim that summation over consumers does not place any other restrictions on competitive aggregate excess demand than Walras' law on arbitrary compact sets of prices. That sort of results – which is by no means confined to the two specific areas just mentioned -- should be quite disquieting at times where in particular, it is increasingly recognized that macroeconomics must be appropriately rooted in microeconomic theory.

The principle of a possible solution to the problem has been known for some time, but has not yet been implemented much successfully. It is to put restrictions not so much on the support of the distribution of the agents' characteristics but on its shape. An early example of this approach in social choice was the result that aggregation is indeed possible through majority voting whenever the distribution of the voters' preferences has nice symmetry properties (Tullock [1967], Davis, de Groot and Hinich [1972], Grandmont [1978]). Other examples were the finding that suitably dispersed distributions over the space of consumers' characteristics (preferences, wealth) lead to a nice "smoothing" of competitive aggregate demand (see e.g., E. Dierker, H. Dierker and W. Trockel [1984] with references to earlier works).

A strong result along this line was obtained recently by W. Hildenbrand [1983] in demand analysis. He shows in particular that if the conditional distribution of income, or expenditure, among individuals who have the same tastes, has a continuous nonincreasing density, then competitive aggregate demand has a negative definite Jacobian matrix (implying in particular that the weak axiom of revealed preference is satisfied in the aggregate, and that aggregate partial demands are decreasing functions of their own price), and this independently of the distribution of preferences in the society. The present work is in a sense complementary to the latter, since it aims at getting a similar outcome by placing restrictions on the shape of the distribution of preferences rather than on the income distribution.

The difficulty when trying to speak about the "shape" of the distribution of preferences is of course that one needs an algebraic structure on the space of preferences under consideration, e.g., as in Grandmont [1978] or Dierker et al. [1984]. This is achieved here by employing homothetic transformations of preferences, that are particular instances of similar, more general "affine" transformations employed in related contexts by A. Mas-Colell and W. Neuefeind [1977, p. 597] or by Dierker et al. [1984, pp 15-16] (see also the notion of replicas used by M. Jerison (1982, Remark 5)). More precisely, for every preference relation R defined on the nonnegative orthant of the commodity space and every income w, we generate a new pair (R_α, w) involving the same income but where the new preferences R_α is derived from the original one through an homothecy of center 0 (the origin of the commodity space) with ratio e^α, α being an arbitrary real number. It is shown that under suitable regularity assumptions, there is a large class of distributions on the parameter α, including specific gamma distributions, that is independent of the particular pair (R,w) under consideration, such that, given (R,w), competitive aggregate demand has a negative definite Jacobian matrix. Since this property is preserved through addition, the result is of course still valid when the agents' characteristics are distributed over the pairs (R,w) in an arbitrary way (1).

It turns out that the class of distributions of preferences considered in the present paper can be obtained nicely from the class of income distributions considered by W. Hildenbrand in [1983] through a simple change of variable. The first section of the paper deals briefly with a slight generalization of Hildenbrand [1983, Theorem 3]. The second part introduces the notion of an homothetic transformation of a preference and gives sufficient conditions on the distribution of preferences that lead to negative definiteness of the Jacobian matrix of competitive aggregate demand. We briefly comment in the concluding section on the prospects of using in this context more general "affine" transformations, as in Mas Colell and Neuefeind [1977] and Dierker <u>et al.</u> [1984]. We indicate there that, as pointed out to us by Dale Jorgenson, this concept is in fact identical to the notion of <u>household equivalence scales</u> introduced in demand analysis by A.P. Barten [1964], and subsequently used in econometric work (see A. Deaton and J.S. Muellbauer [1980], D.W. Jorgenson and D.T. Selsnick [1984], J.S. Muellbauer [1980]).

1. *INCOME DISTRIBUTIONS.*

We first state and prove a slight generalization of W. Hildenbrand (1983, Th. 3), exhibiting a class of income distributions that ensure negative definiteness of the Jacobian matrix of competitive aggregate demand, independently of the distribution of preferences in the society.

We consider preference relations R , i.e. complete and transitive binary relations on the consumption set $X = \mathbb{R}^{\ell}_{+}$, $\ell \geqq 2$, with the understanding that xRy means "x is preferred or equivalent to y" . We assume R to be <u>continuous</u> (its graph is closed), <u>strictly convex</u> and <u>locally nonsatiated</u>.(2) The corresponding competitive demand function $\xi(R,p,w)$ is then well defined for all price systems p in $P = \mathrm{Int}\,\mathbb{R}^{\ell}_{+}$ (the set of positive vectors of $\mathbb{R}^{\ell}$) and all nonnegative income levels w . We assume that R is such that the demand function is <u>regular</u> in the sense of W. Hildenbrand (1983, Def. 3, p. 1006), i.e. ξ is a continuously differentiable function of (p,w) on $P \times \mathbb{R}_+$, and the rank of its Slutsky matrix is $\ell-1$ for all p in P and all $w > 0$. Let $\mathcal{R}^*$ be the set of preferences that satisfy these regularity requirements.

In the present context, the <u>characteristics</u> of a consumer are his preference R in $\mathcal{R}^*$ and his income $w \geqq 0$, although this viewpoint is somewhat restrictive since it means that income is independent of prices. A distribution on characteristics is then defined by a probability measure λ on preferences — which we may assume, to simplify, to have a finite support $\langle R^1, \dots, R^m \rangle$ in $\mathcal{R}^*$ — and for each R^i, a <u>conditional</u> income distribution that is represented by a probability measure ν_i on $\mathbb{R}_+$. Aggregate demand per capita or <u>mean demand</u> is then given by $\Sigma_i \lambda_i \bar{\xi}_i(p)$, where λ_i is the weight of the preference R^i and

$$\bar{\xi}_i(p) = \int \xi(R^i, p, w) \quad \nu_i(dw)$$

Since $\xi_h(R,p,w) \leqq w/p_h$, the above integral is well defined when ν_i has a finite mean. The question is to find a class of income distributions ν_i that ensures negative definiteness of the Jacobian matrix of the mean demand, independently of the distribution λ of preferences. Obviously, since negative definiteness of the Jacobian matrix is preserved through addition, one may without loss of generality assume that λ is concentrated on a single preference R, and consider accordingly a single income distribution ν.

W. Hildenbrand (1983) showed that it is enough to assume that the income distribution ν has a support with a finite upper bound (3), and admits a density that is continuous, nonincreasing on $\mathbb{R}_+$. The intuition behind the result is that demand is "well behaved" when income is low enough (all goods are then normal). In a sense, putting more weight on low incomes makes the aggregate income effects to have the "right sign". A (minor) extension of his result is provided by the following.

PROPOSITION 1. Consider a preference R in $\mathcal{R}^*$ and a probability distribution v over $\mathbb{R}_+$. If the support of v has a finite upper bound, i.e. $b = \text{Max supp } v < +\infty$, the mean demand

$$\bar{\xi}(p) = \int \xi(R,p,w) \quad v(dw)$$

is continuously differentiable and

$$\frac{\partial \bar{\xi}_h}{\partial p_k}(p) = \int \frac{\partial \xi_h}{\partial p_k}(R,p,w) \quad v(dw)$$

Assume that, in addition, v has a nonincreasing density $f(w)$ such that $\lim_{w \to 0} w^2 f(w) = 0$. Then $\bar{\xi}(p)$ has a negative definite Jacobian matrix.

Imposing continuity of the density function f over $\mathbb{R}_+$ implies $f(b) = 0$ and that f is bounded. The above proposition allows for densities that are discontinuous, with $\lim_{w \uparrow b} f(w) > 0$, and more importantly, that diverge to infinity, but less fast than w^{-2}, when income goes to zero. In the sequel, we shall assume without loss of generality that f is left-continuous at $w = b$, i.e. $f(b) = \lim_{w \uparrow b} f(w)$.

The first part of the result, i.e. continuous differentiability and the fact that one can differentiate under the integral sign when $b < +\infty$, follows from the dominated convergence theorem. The proof that $\bar{\xi}$ has a negative definite Jacobian matrix is then a straightforward adaptation of W. Hildenbrand's proof (1983, p. 1018).

According to his argument, by decomposing as usual $\frac{\partial \xi_h}{\partial p_k}(R,p,w)$ into income and substitution terms through the Slutsky equation, one finds that it suffices to show that the matrix of aggregate income terms

$$\bar{A}_{hk}(p) = \int \xi_k(R,p,w) \frac{\partial \xi_h}{\partial w}(R,p,w) \quad \nu(dw)$$

is positive semidefinite, that is

$$\Sigma_{h,k} v_h v_k \bar{A}_{hk}(p) \geqq 0$$

for all vectors v of $\mathbb{R}^{\ell}$, with strict inequality when $v = rp$, $r \neq 0$. Now, given v, using the notation

$$z(w) = \Sigma_h v_h \xi_h(R,p,w)$$

one obtains

$$2 \Sigma_{h,k} v_h v_k \bar{A}_{hk}(p) = \int 2\, z(w)\, z'(w) \quad \nu(dw) \tag{1.1}$$

If $v = rp$, $r \neq 0$, one has $z(w) = rw$ and the above expression is equal to $2 r^2 \int w\, \nu(dw)$ which is positive when ν has a density, since then $b > 0$. If ν is arbitrary, using the assumption that ν has a nonincreasing density f yields that (1.1) is the limit as c tends to zero of the integral $\int_c^b (z^2(w))'\, f(w)\, dw$. One obtains then from the "second mean value theorem" (Dieudonné, 1969, p. 169) that there exists $c \leqq \eta_c \leqq b$ such that

$$\int_c^b (z^2(w))' f(w)\, dw = f(c) \int_c^{\eta_c} (z^2(w))'\, dw + f(b) \int_{\eta_c}^b (z^2(w))'\, dw$$

$$= f(b)\, z^2(b) - f(c)\, z^2(c) + \left[f(c) - f(b)\right] z^2(\eta_c)$$

$$\geqq f(b)\, z^2(b) - f(c)\, z^2(c)$$

Remark next that $|z(c)| \leqq c \;\; \Sigma\, |v_h|/p_h$ and therefore that $\lim_{c \to 0} f(c)\, z^2(c) = 0$.

One gets accordingly

$$2\, \Sigma_{h,k}\, v_h\, v_k\, \bar{A}_{hk}(p) \geqq f(b)\, z^2(b) \geqq 0$$

which completes the proof.

2. DISTRIBUTIONS OF PREFERENCES.

We show in this section that there are restrictions on the shape of the distribution of preferences that yield negative definiteness of the Jacobian matrix of competitive aggregate demand, independently of the income distribution. Central to the approach is the notion of an homothetic transformation of a preference, which is presented now.

Consider a preference R on $X = \mathbb{R}^{\ell}_{+}$. For any real number α , we define a new preference R_α by

(2.1) $\qquad (e^{\alpha} x)\; R_\alpha\; (e^{\alpha} y) \quad$ if and only if $\quad xRy$

The indifferences surfaces of R_α are obtained from those of R through an homothecy of center 0 and ratio e^{α} , see Figure 1.a. It is well known (Debreu [1964]) that R is continuous if and only if it has a continuous real-valued representation u . Then $u(e^{-\alpha}x)$ is a representation of R_α and it is continuous. Thus R_α is continuous, strictly convex, locally nonsatiated if and only if R satisfies these properties. Note that R is homothetic if and only if $R_\alpha = R$ for all α .(4)

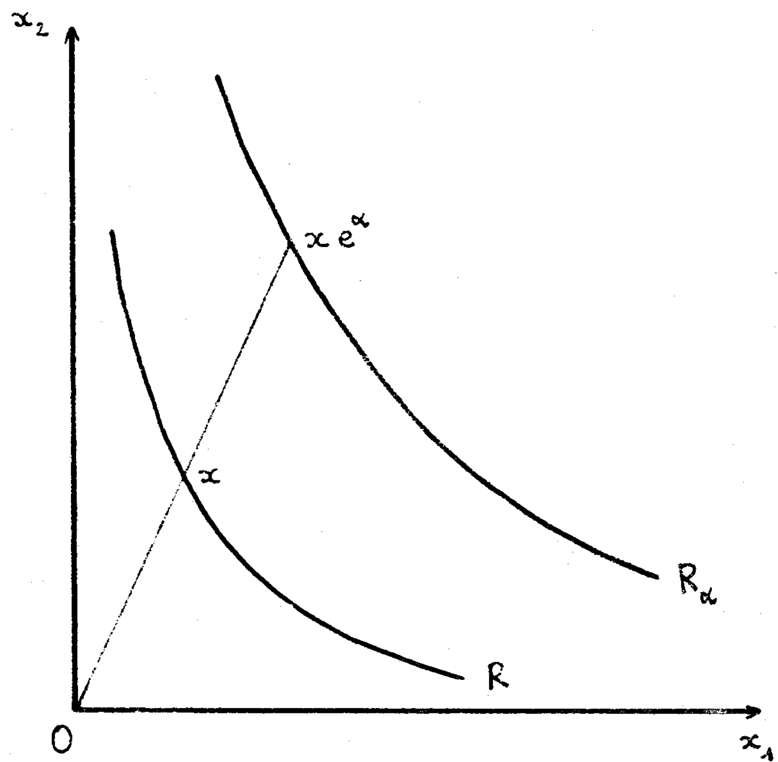
Consider next the effect of the transformation on demand. The demand function $\xi(R_\alpha,p,w)$ is obtained by maximizing $u(e^{-\alpha}x)$ under the constraints $x \in X$, and $p.x \leq w$, or equivalently, $p.(xe^{-\alpha}) \leq we^{-\alpha}$. Hence

$$(2.2) \qquad \xi(R_\alpha,p,w) = e^{\alpha}\xi(R,p,we^{-\alpha})$$

The effect of the homothetic transformation on the demand function is shown in Figure 1.b, in the case where $e^{\alpha} = 2$. The curve OBAC is the Engel curve of the preference R corresponding to the price system p . The point A represents the demand for R at (p,w) . To obtain the demand $\xi(R_\alpha,p,w)$, one considers first the point of the Engel curve of the <u>original</u> preference R at the income $we^{-\alpha}$, i.e., the point B , then one "rescales" it to get back to the original budget set, which yields B′ . It is then clear that R_α belongs to $\mathcal{R}^*$ if and only if R does, for all α .

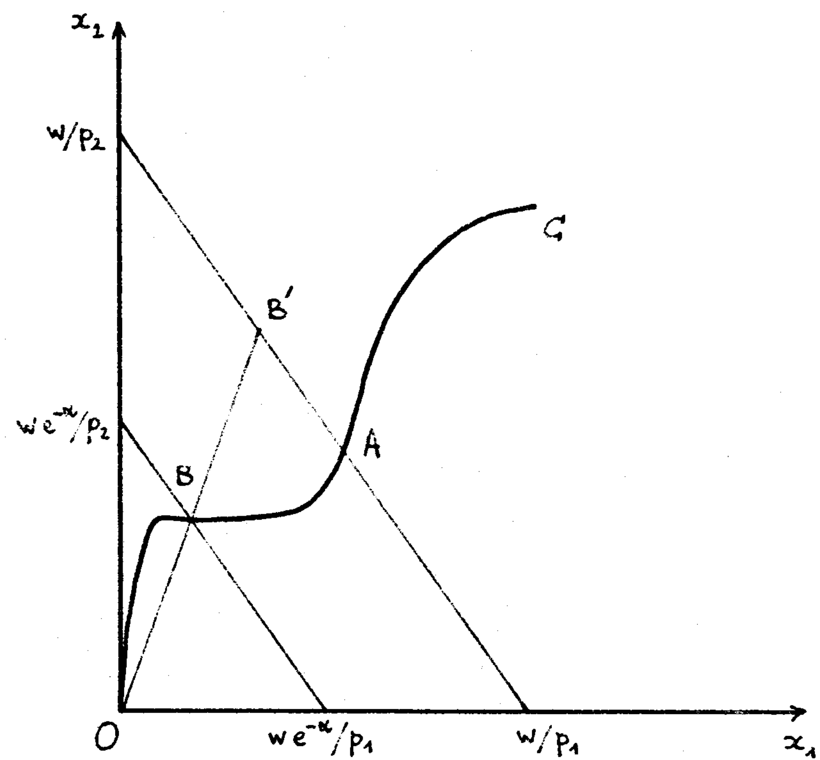
<u>Fig. 1.a</u> <u>Fig. 1.b</u>

Here again, we say that the characteristics of a consumer are his preference R in $\mathcal{R}^*$ and his income $w > 0$. We define a distribution over the agents' characteristics in the following way(5) . There is first a probability γ on characteristics, with finite support, say $((R^1,w^1),\dots,(R^m,w^m))$. The key assumption is that for each (R^i,w^i) there is a whole distribution of



x_2
$x e^{\alpha}$
x
R_α
R
O
x_1

Fig. 1.a



x_2
w/p_2
B'
G
$we^{-\alpha}/p_2$
A
B
O
$we^{-\alpha}/p_1$
w/p_1
x_1

Fig. 1.b

individuals who have the same income w^i but preferences that are homothetic transforms R^i_α of R^i, in the above sense. In order to specify this distribution, we need only to specify the conditional probability distribution of the corresponding parameter α, say μ_i. The conditional probabilities μ_i together with the probability γ yield an overall probability distribution over the space of characteristics.

Aggregate demand per capita or <u>mean demand</u> is then given by $\Sigma_i \gamma_i \bar{\xi}_i(p)$, where γ_i is the weight of (R^i, w^i) and

$$\bar{\xi}_i(p) = \int \xi(R^i_\alpha, p, w^i)\ \mu_i(d\alpha)$$

The problem to be studied is then to find conditions on the distributions μ_i which ensure that the mean demand has a negative definite Jacobian matrix, independently of the particular distribution γ over generators. Here again, since negative definiteness of the Jacobian matrix is preserved through addition, we may assume without loss of generality that γ is concentrated on a single generator (R,w) and consider accordingly a single conditional probability μ. By using (2.2), the mean demand is then

$$\bar{\xi}(p) = \int \xi(R_\alpha, p, w)\ \mu(d\alpha)$$

$$= \int \xi(R, p, we^{-\alpha})\ e^{\alpha}\ \mu(d\alpha)$$

This formulation makes clear that integration over the parameter α is in fact equivalent to integration over income through a suitable change of variable. Let μ' be the measure over the real line whose density with respect to the probability μ is e^{α}, and let ν be the measure over $\mathbb{R}_+$ that is the image of μ' through the change of variable $v = we^{-\alpha}$. One gets

$$\bar{\xi}(p) = \int \xi(R,p,we^{-\alpha})\ \mu'(d\alpha)$$
$$= \int \xi(R,p,v)\ \nu(dv)$$

The problem at hand is thus equivalent to the problem of the preceding section, provided that the measure ν or μ' is <u>finite</u>, i.e. when e^{α} is μ-integrable. One can then apply Proposition 1 above. Since the change of variable $v = we^{-\alpha}$ reverses orientation, the support of ν has a finite upper bound whenever the support of μ has a finite lower bound $a = \text{Min supp}\ \mu > -\infty$. The measure ν has a density $f(v)$ if and only if μ has a density $g(\alpha)$, and the two densities are related by

$$g(\alpha) = we^{-2\alpha} f(we^{-\alpha})$$

Thus f is nonincreasing if and only if $e^{2\alpha}\ g(\alpha)$ is nondecreasing, and $\lim_{v\to 0} v^2\ f(v) = 0$ whenever $\lim_{\alpha\to+\infty} g(\alpha) = 0$. To sum up, we have obtained

PROPOSITION 2. <u>Consider a preference</u> R <u>in</u> $\mathcal{A}^*$ <u>and an income</u> $w > 0$. <u>Let</u> μ <u>be a probability distribution over the real line</u>. <u>Assume that the support of</u> μ <u>has a finite lower bound</u>, <u>i.e.</u>, $-\infty < a = \text{Inf supp}\ \mu$. <u>Then if</u> e^{α} <u>is</u> μ-<u>integrable, the mean demand</u>

$$\bar{\xi}(p) = \int \xi(R_{\alpha},p,w)\ \mu(d\alpha)$$

<u>is continuously differentiable and</u>

$$\frac{\partial\bar{\xi}_h}{\partial p_k}(p) = \int \frac{\partial\xi_h}{\partial p_k}(R_{\alpha},p,w)\ \mu(d\alpha)$$

<u>Assume that in addition</u> μ <u>has a density</u> $g(\alpha)$ <u>such that</u> $e^{2\alpha} g(\alpha)$ <u>is nondecreasing and</u> $\lim_{\alpha \to +\infty} g(\alpha) = 0$. <u>Then</u> $\bar{\xi}(p)$ <u>has a negative definite Jacobian matrix</u>.

<u>Remarks</u>. 1. The class of densities defined by the Proposition includes when a = 0 , <u>gamma distributions</u>

$$g(\alpha) = \frac{s^{r+1}}{\Gamma(r+1)} e^{-s\alpha} \alpha^{r}$$

with $r \geqq 0$ and $1 < s \leqq 2$.

2. Making the change of variable $\alpha' = e^{\alpha}$ yields distributions over the real line whose support have a finite lower bound and that have a finite mean, with a density $h(\alpha')$ such that $(\alpha')^{3} h(\alpha')$ is nondecreasing and $\lim_{\alpha' \to +\infty} \alpha' h(\alpha') = 0$.

3. One can pursue further the approach by considering, for a given preference R in $\mathcal{R}^{*}$, a <u>joint</u> probability distribution δ over $\mathbb{R}_{+} \times \mathbb{R}$ on the variables w and α , which yields the mean demand

$$\bar{\xi}(p) = \int \xi(R_{\alpha}, p, w)\, \delta(dw, d\alpha)$$

$$= \int \xi(R, p, we^{-\alpha})\, e^{\alpha}\, \delta(dw, d\alpha)$$

and by making the change of variable $v = we^{-\alpha}$. The details are left to the reader.

3. CONCLUSION.

The result presented in this note does not go very far. The main limitations come from the fact that income has been treated as independent of prices, which precludes apparently any meaningful application to, say, general equilibrium theory. There may be possible applications of the approach, however, to models of competition with spatially separated markets or differentiated products. Another topic of interest would be to investigate whether or not the theory presented here, or some simple versions of it, has testable implications for demand analysis.

From a purely theoretical viewpoint, the most interesting feature of the analysis seems to be that it exploits a particularly simple algebraic structure of the space of preferences. In this respect, it is perhaps worthwhile to draw attention to a notion contained in the papers by A. Mas-Colell and W. Neuefeind, and by E. Dierker, H. Dierker and W. Trockel, already mentioned. These authors introduce indeed a group of "affine" transformations acting on the space of preferences, which seems to be quite promising for aggregation purposes.

A version of this group is the following. Let R be a preference relation on $X = \mathbb{R}^{\ell}_{+}$. If x is a commodity bundle, and t any vector of $\mathbb{R}^{\ell}$, one defines a new commodity bundle by

$$x*t = (x_1\, e^{t_1}, \ldots, x_{\ell}\, e^{t_{\ell}}) \tag{3.1}$$

The new commodity bundle x*t is obtained through a sequence of affine transformations. Then one may define a transformed preference R_t by

$$(x*t)\ R_t\,(y*t) \quad \text{if and only if} \quad xRy \tag{3.2}$$

The indifference surfaces of R_t are obtained here too from those of R through a sequence of affine transformations. The particular case of an homothetic transformation that has been considered in the present work arises when t belongs to the diagonal of $\mathbb{R}^\ell$, i.e., $t = (\alpha,\dots,\alpha)$. If u(x) is a (continuous) representation of R , then

$$u(x*(-t)) = u(x_1 e^{-t_1},\dots,x_\ell e^{-t_\ell})$$

is a continuous representation of R_t .

Therefore (3.2) defines consistently a map (<u>transformation</u>) σ_t from, say, the space $\mathcal{R}$ of continuous preferences on X into itself by $\sigma_t(R) = R_t$. These transformations form a group. Given R , we have $R_o = R$, $(R_t)_s = R_{t+s}$, and thus

$$\sigma_o = \mathrm{id} \ , \ \sigma_t \circ \sigma_s = \sigma_{t+s}$$

where id stands for the identity map of $\mathcal{R}$ into itself [(6),(7)]. Equivalently, one may view (3.2) as defining a <u>dynamical system</u> (<u>flow</u>) acting on the space of preferences, in which the vector t plays the role of "time".

Assuming that R is continuous, strictly convex, one sees easily that the effect on demand of these transformations is particularly simple, since it is given by $\xi(R_t,p,w) = \xi(R,p*t,w)*t$, or equivalently for every h

$$\xi_h(R_t,p,w) = e^{t_h}\,\xi_h(R,p_1 e^{t_1},\dots,p_\ell e^{t_\ell},w)$$

Affine transformations, of course, can be defined for preferences on arbitrary consumption sets X . It suffices indeed to define R_t through (3.2) on the transformed consumption set $X_t = X*t$.

Affine transformations generate a simple neat algebraic structure on the space of preferences. We have not used in this note the more general multidimensional structure of this Section for it does not add anything for the treatment of the problem at hand, i.e., the analysis of the probability distributions on α , or on t , that give rise to negative definiteness of the aggregate Jacobian matrix, independently of the distribution over generators (R,w) . This is so because any vector t of $\mathbb{R}^{\ell}$ has a unique representation of the form $t = \alpha \bar{1} + \beta$, where $\bar{1} = (1,\ldots,1)$ and β verifies $\Sigma_h \beta_h = 0$. We would not have gained by considering probability distributions over $\mathbb{R}^{\ell}$ on the whole vector t , for in the end, through an application of the Fubini theorem, what matters is the shape of the conditional distributions of the variable α : they should meet the requirements of Proposition 2. But it seems likely that multidimentional affine transformations should play a useful role when dealing with aggregation issues. This feeling is reinforced by the remark made to us by Dale Jorgenson, that the concept of an affine transformation is in fact identical to the notion of a <u>household equivalent scale</u> introduced by A. Barten [1964] in applied demand analysis to account for differences in preferences, and subsequently used in econometric work (see A. Deaton and J.S. Muellbauer [1980], J.S. Muellbauer [1980], D.W. Jorgenson and D.T. Slesnick [1984]). The hypothesis is there that all individuals have the same preference up to a rescaling of the units of measurement of commodities. This is exactly the same as saying that preferences are affine transformations of each other, in the sense of the present section.

FOOTNOTES

(*) I wish to thank very much Philippe Aghion, Dale Jorgenson, Andreu Mas-Colell and Walter Trockel for every helpful conversations. I am much indebted to Werner Hildenbrand and Mike Jerison who, after having read the first version of this paper, pointed out to me that the class of distributions of preferences considered in the paper could be obtained from the class of income distributions considered by Werner Hildenbrand (1983) through a simple change of variable that had escaped me. I owe also to Werner Hildenbrand the reference to the work of Walter Trockel given in footnote 1 below.

(1) After this work has been completed, I learned that Walter Trockel had obtained, in handwritten notes circulated in 1983 at the University of Bonn, a result going in the same direction, exhibiting a class of distributions of preferences ensuring that aggregate demand for each good was a decreasing function of its own price. Although I have not seen W. Trockel's work, I am glad to acknowledge his intellectual priority.

(2) R is strictly convex if for every x_1, x_2, y in X with x_i Ry, $x_1 \neq x_2$ then $\lambda x_1 + (1 - \lambda) x_2$ Py for all λ in $(0,1)$ (xPy stands for "not yRx"). Local nonsatiation means that for any x and any neighborhood V of x, there exists y in V with yPx.

(3) This condition does not appear explicitly in the statement of Werner Hildenbrand (1983, Th. 3) but is used in his proof.

(4) We used the parameter α instead of $\alpha' = e^{\alpha}$, which might appear more "natural" at first sight, for it will make much easier in the next section the presentation of the group of "affine" transformations of preferences. See also Remark 2 below.

(5) This way is directly inspired from Dierker et al [1984].

(6) Dierker et al. [1984] work directly with the vectors $(e^{t_1},\dots,e^{t_\ell})$ which form a multiplicative group. The present formulation seems more natural. In particular, the Haar measure that they consider is apparently the image of the Lebesgue measure of $\mathbb{R}^\ell$ by the map

$(t_1,\dots,t_\ell) \to (e^{t_1},\dots,e^{t_\ell})$.

(7) A lot of subsets of the space of preferences are left invariant by these affine transformations (continuous, (strictly) convex, homothetic, etc..) – in particular the space $\mathcal{R}^*$ considered in this note. It is amusing to note that a preference that has a Cobb-Douglas utility representation is invariant, i.e., satisfies $R_t = R$ for all t . It is not known if other preferences have this property.

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