

INTERNATIONAL COOPERATION  
AND  
EXCHANGE RATE STABILIZATION

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## 1. INTRODUCTION

In this paper we will develop the argument that, when countries do not coordinate their economic policies, they do not give enough weight to the stabilization of the exchange rate. Consequently, the exchange rate fluctuates more than it would in the case of international cooperation.

The reason lies in the "public good" feature of exchange rate stabilization. To reduce exchange rate fluctuations may be in itself beneficial to all countries, but each country would like the others to take care of it ; so that its policy instruments would still be completely available for other purposes, such as the stabilization of domestic output, or price, etc. The case of a fixed exchange rate system may illustrate this last point. When policymakers of a given country peg the exchange rate, they lose the instrument which is used for that pegging. For example, under perfect capital mobility and substitutability of domestic and foreign assets, in which case sterilized intervention has no effect, the exchange rate can be pegged through non-sterilized interventions ; but this means that monetary policy is directed toward the exchange rate goal and cannot be used for the stabilization of the domestic economy. In that case, in order to keep an independent monetary policy, each country would prefer that intervention be done by the other countries.

We will consider a two country world and assume that each country has one policy instrument at its disposal, and two kinds of objectives: the first is the stabilization of some internal variable, like domestic output or the price level ; the second is the stabilization of the exchange rate. Because there are fewer instruments than objectives, some trade-off has to be found, and the result depends on the relative weight given by policymakers of the various countries to each objective. Using a quite general reduced form model of the world economy, we will compare two kinds of solutions : the first is a Nash non cooperative equilibrium where each country takes the policy of the other country as given ; the second is the set of Pareto optima<sup>1</sup>. We will show that at the Nash equilibrium policymakers of countries do not give enough weight to the objective of exchange rate stabilization and, therefore, that the exchange rate fluctuates more than at any Pareto optimum.

This result is compatible with both positive and negative transmission mechanisms. For example, if the policy instrument is monetary policy and the internal objective is output, then, under a flexible exchange rate system an expansionary monetary policy in one country, which raises the output of that country, is allowed to either increase or decrease the output of the other country. We only make the assumption that the absolute value of the effect is smaller on the foreign variable than on the domestic variable. However, it may be worth investigating how the magnitude of the gap between non cooperative and cooperative solutions depends on the positivity or negativity of the transmission mechanism. In fact, we show that a positive transmission mechanism tends to increase that gap.

A key feature of our analysis is that, besides the stabilization of some internal variables such as output or the price level, policymakers find it desirable to reduce the fluctuations of the exchange rate. This seems to be compatible with the concerns shown by policymakers of different countries when they are confronted to large fluctuations of the exchange rate, as those which have been observed in the last decade. But, indeed, there are reasons for such a concern. Exchange rate fluctuations are usually associated with changes in the relative prices of goods of different countries : the terms of trade or the real exchange rate fluctuations closely follow that of the nominal exchange rate. These relative price variations are likely to have detrimental effects on the economies of countries, because of adjustment costs, of the uncertainty thus created and their potential adverse effects on international trade. Obviously, one might have tried to model these aspects and introduced them among the objectives of policymakers. However, such a strategy would probably unnecessarily complicate the analysis for the issue at hand. Therefore, we will more directly consider that policymakers have some objective of exchange rate stabilization.

The model is developed in section 2. In section 3 we examine the set of Pareto optima, while in section 4 we consider the Nash equilibrium and we compare it to the optima. The issue of the role of a positive versus a negative transmission mechanism is dealt with in section 5. Finally, section 6 summarizes the results and draws some conclusions.

## 2. THE MODEL

Consider a two country world economy and suppose that in each country policymakers have two kinds of objectives : first, they want to stabilize some internal objective, such as unemployment, output or price, around some target level ; second, they want to reduce the fluctuations of the exchange rate . For that, in each country one policy instrument is available. Under a flexible exchange rate, the world economy is represented by a linear static reduced form model :

$$(1a) \quad x = \alpha_1 m + \alpha_2 m^* - s$$

$$(1b) \quad x^* = \alpha_2 m + \alpha_1 m^* - s^*$$

$$(1c) \quad e = \beta(m - m^*) + \theta \quad \beta \neq 0$$

In these equations  $x$  and  $x^*$  represent the deviations around their target levels of the internal objectives of country 1 and 2 respectively. The variable  $e$  is the gap between the logarithm of the exchange rate (measured as the value of one unit of country 2 currency in terms of country 1 currency) and the logarithm of the level of the exchange rate around which each country tries to reduce the fluctuations of this variable, and which is assumed to be the same in the two countries. The policy instrument of country 1 is  $m$ , and that of country 2 is  $m^*$ . The variables  $s$ ,  $s^*$  and  $\theta$  represent shocks which affect the world economy. These shocks are assumed to be known to policy makers<sup>2</sup>.

We make the simplifying assumption that the two countries are similar in size and structure, and have the same type of objectives and instruments. Therefore, the effects of  $m$  on  $x$  and  $x^*$  are respectively the same as the effect of  $m^*$  on  $x^*$  and  $x$  ; also  $m$  and  $m^*$  have opposite effects on  $e$ . However, the variables  $s$  and  $s^*$  are generally not equal because the shocks affecting the two countries are not identical. The assumption  $\beta \neq 0$  is innocuous and is simply required in order to consider the issue at hand : otherwise the policy instruments would have no effect on the exchange rate.

The objective functions of the countries are given by :

$$(2a) \quad V = x^2 + \varphi e^2$$

$$(2b) \quad V^* = x^{*2} + \varphi e^2$$

where policymakers of country 1 minimize  $V$  and those of country 2 minimize  $V^*$ . The relative weight of the exchange rate objective is  $\varphi/1+\varphi$ , and, as a simplifying assumption, is also taken to be the same for the two countries.

We will make the following assumption :

$$(3) \quad |\alpha_2| < |\alpha_1|$$

Inequality (3) means that the effect of the policy instrument of any country is greater in absolute value on the internal variable of this country than on the internal variable of the other country. Note that this assumption is compatible both with a positive transmission mechanism ( $\alpha_1$  and  $\alpha_2$  of the same signs) and with a negative transmission mechanism ( $\alpha_1$  and  $\alpha_2$  of opposite signs). If we interpret  $m$  and  $m^*$  as being the money supplies, and  $x$  and  $x^*$  as being output or the price level, the assumption seems to be adequate ; however, our analysis will be valid for any couple (instrument-internal objective) which satisfies this assumption.

Assumption (3) is equivalent to  $\alpha_1 \neq 0$  and to the two inequalities :

$$(4) \quad 1 - \frac{\alpha_2}{\alpha_1} > 0$$

$$(5) \quad 1 + \frac{\alpha_2}{\alpha_1} > 0$$

### 3. OPTIMA

We will consider Pareto optima. We can obtain them by minimizing the following world objective function  $U$  :

$$(6) \quad U = k V + (1-k) V \quad 0 < k < 1$$

where the parameter  $k$  is the relative weight given to the objective of country 1. An optimum can be considered as a cooperative solution where the parameter  $k$  is the result of the negociation. Obviously, because the two countries have been taken to have similar characteristics, the case  $k = 1/2$  may appear more natural. Nonetheless, we will more generally consider all values of  $k$ , and in fact we will find that if our result holds for  $k$  equal to  $1/2$ , it will a fortiori hold for any value of  $k$ .

#### 3.1 - Special cases

As a first step of analysis let us consider two special cases defined for extreme values of the weight  $\varphi$  given to the stabilization of the exchange rate in (2). First take the case  $\varphi = 0$  where the objective of each country is to reduce the fluctuations of its internal objective. Then, because we have two objectives and two instruments, these objectives can actually be attained : internal objectives can be completely stabilized. The optimum is given by :

$$\begin{cases} x = 0 \\ x^* = 0 \end{cases}$$

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From (1a) and (1b) this can respectively be written :

$$(7a) \quad \alpha_1 m + \alpha_2 m^* - s = 0 \quad (D)$$

$$(7b) \quad \alpha_2 m + \alpha_1 m^* - s^* = 0 \quad (D^*)$$

These are the equations of the two straight lines (D) and (D\*) in figure 1. (D) is the locus of (m, m\*) such that the internal objective x of country 1 is completely stabilized, while (D\*) is the corresponding locus for the internal objective of country 2<sup>3</sup>.

From (7), this optimum verifies :

$$(8a) \quad \hat{m}_0 - \hat{m}_0^* = \frac{1}{\alpha_1 - \alpha_2} (s - s^*)$$

$$(8b) \quad \hat{m}_0 + \hat{m}_0^* = \frac{1}{\alpha_1 + \alpha_2} (s + s^*)$$

Then, the value of the exchange rate  $\hat{e}_0$  is given by (1c) :

$$(9) \quad \hat{e}_0 = \beta(\hat{m}_0 - \hat{m}_0^*) + \theta$$

The optimum given by (8) is an optimum for all values of the parameter k defined in (6) because, in that very special case  $\varphi = 0$ , both V and V\* are minimized (and equal to 0).

Second, consider the other extreme case where each country would only be concerned with stabilizing the exchange rate (obtained for  $\varphi$  going to infinity). Then, the optimum would be any fixed exchange rate system given by :

$$e = 0$$

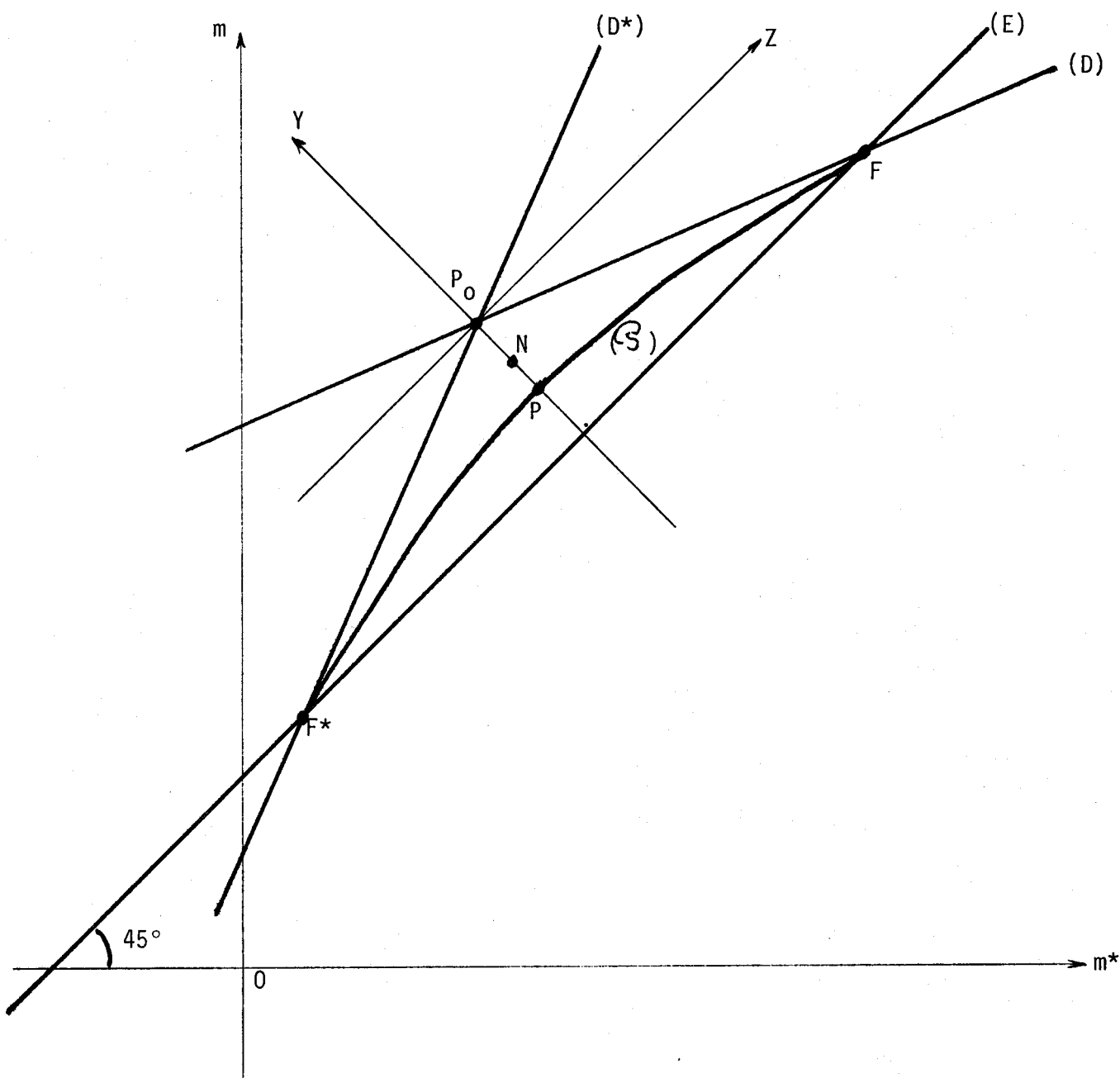


FIGURE 1



From equation (1c) this can be written :

$$(10) \quad \beta(m-m^*) + \theta = 0 \quad (E)$$

Fixed exchange rate systems are represented by the straight line (E) in figure 1. The fixity of the exchange rate imposes a constraint on the possible policies of the countries.

### 3.2 - General case

Take the more general case where countries give weight to both kinds of objective ( $\varphi$  positive and finite). First consider what happens in the two limit cases where  $k$ , defined in (6), goes to 0 or 1. When  $k$  goes to 1 we look for the optimum from the point of view of country 1. In figure 1 this is given by point F which is the intersection of (D) and (E), because at this point the two objectives of country 1 are attained : we have  $x = 0$  and  $e = 0$ . This optimum can be obtained in a system where the exchange rate is pegged by country 2 : country 1 does not lose any policy independence and, therefore, can set  $m$  at the level  $m_F$  ; on the other hand, country 2 completely loses its policy independence and, given that country 1 has the policy  $m_F$ , must set  $m$  at the level  $m_F^*$  in order to peg the exchange rate. In the same way, point F\* in figure 1 is the optimum from the point of view of country 2 ( $k=0$ ), and can be obtained in a system where the burden of pegging the exchange rate would be completely born by country 1.

Now consider any value of  $k$ . Using (1), and  $|\alpha_1|^2 \neq |\alpha_2|^2$  from (3), we see that the world objective function  $U$  defined by (6) is a strictly convex quadratic function of the variables  $m$  and  $m^*$ . Consequently, for each value of the parameter  $k$  an optimum exists and is unique, and first order conditions are necessary and sufficient. These can be written :

$$\left\{ \begin{array}{l} k \frac{\partial V}{\partial m} + (1-k) \frac{\partial V^*}{\partial m} = 0 \\ k \frac{\partial V}{\partial m^*} + (1-k) \frac{\partial V^*}{\partial m^*} = 0 \end{array} \right.$$

Using (1) and (2) this gives :

$$\begin{cases} k \alpha_1 x + (1-k) \alpha_2 x^* + \varphi \beta e = 0 \\ k \alpha_2 x + (1-k) \alpha_1 x^* - \varphi \beta e = 0 \end{cases}$$

Adding and subtracting these two equations we obtain :

$$(11a) (\alpha_1 + \alpha_2)[k x + (1-k) x^*] = 0$$

$$(11b) (\alpha_1 - \alpha_2)[k x - (1-k) x^*] + 2 \varphi \beta e = 0$$

As, from (3),  $\alpha_1 + \alpha_2 \neq 0$ , equation (11a) can also be written :

$$(12a) k x + (1-k) x^* = 0$$

Consider first the special case where each country receives the same weight in the world objective function ( $k = 1/2$ ). Equations (12a) and (11b) become, respectively :

$$(13a) x + x^* = 0$$

$$(13b) (\alpha_1 - \alpha_2)(x - x^*) + 4 \varphi \beta e = 0$$

We introduce new variables given by the coordinates along the axes  $P_0 Z$  and  $P_0 Y$  in figure 1 :

$$(14a) Z = m + m^* - (\hat{m}_0 + \hat{m}_0^*)$$

$$(14b) Y = m - m^* - (\hat{m}_0 - \hat{m}_0^*)$$

Then, from (1), and using (9) we have :

$$(15a) x - x^* = (\alpha_1 - \alpha_2) Y$$

$$(15b) x + x^* = (\alpha_1 + \alpha_2) Z$$

$$(15c) \quad e = \beta Y + \hat{e}_0$$

Therefore conditions (13) become (using the fact that, from (3),  $\alpha_1 + \alpha_2 \neq 0$ ).

$$(16a) \quad Z = 0$$

$$(16b) \quad [(\alpha_1 - \alpha_2)^2 + 4 \varphi \beta^2] Y = - 4 \varphi \beta^2 (\hat{e}_0 / \beta)$$

Equation (16b) gives :

$$(17) \quad Y = - Q \frac{\hat{e}_0}{\beta}$$

Where we define Q by :

$$(18) \quad Q = \frac{4 \varphi \beta^2}{(\alpha_1 - \alpha_2)^2 + 4 \varphi \beta^2}$$

and, consequently, we have :

$$(19) \quad 0 < Q < 1$$

then, from (15c) and (17), we obtain :

$$(20) \quad \hat{e} = (1-Q) \hat{e}_0$$

Therefore, we have :

$$(21) \quad \hat{e}^2 = (1-Q)^2 \hat{e}_0^2$$

From (18), coefficient Q is positive and is an increasing function of  $\varphi$ . Therefore  $\hat{e}^2$  decreases when  $\varphi$  increases, a result which seems natural : at the optimum (for  $k=1/2$ ), the more weight policymakers give to the stabilization of the exchange rate, the less this exchange rate fluctuates.

In figure 1 such an optimum is represented by point P. This point is located along the  $P_0Y$  axis between point  $P_0$  and the straight line (E), being closer to (E) when  $\varphi$  gets larger.

Now, consider the set of Pareto optima. As we have shown it has to go through points P, F and  $F^*$ . Such a set is represented by the curve (S) in figure 1. We see that at point P the exchange rate fluctuates more than at F and  $F^*$  (where it is completely stabilized). In fact it can be shown (cf. the appendix) that, as it is represented on figure 1, when  $k$  gets further away from  $1/2$  the exchange rate fluctuation is smaller<sup>4</sup> :

$$(22) \quad \hat{e}^2 - \hat{e}^2(k) > 0 \text{ when } k \neq 1/2, \text{ and } \hat{e}^2 - \hat{e}^2(k) \text{ increases when } |k - \frac{1}{2}| \text{ increases}$$

where  $\hat{e}(k)$  is the exchange rate at the optimum obtained for a given value of  $k$ , and where, as before,  $\hat{e} = \hat{e}(k)$  when  $k = 1/2$ .

Consequently, as we are going to show that at the Nash equilibrium the exchange rate fluctuates more than at an optimum, it will be sufficient to show that the result holds for  $k = 1/2$ . Then, because of inequality (22) it will a fortiori hold for any Pareto optimum.

#### 4. NASH EQUILIBRIUM

Country 1 takes the policy  $m^*$  of country 2 as given and chooses its policy  $m$  in order to minimize  $V$ . Because  $V$  is a strictly convex quadratic function of  $m$ , there is a unique response  $m$  to each value of  $m^*$ , and the reaction function is given by the first order condition :

$$(23a) \quad \frac{\partial V}{\partial m} = 0$$

In the same way, country 2 takes  $m$  as given and chooses  $m^*$  in order to minimize  $V^*$ , and its reaction function is given by :

$$(23b) \quad \frac{\partial V^*}{\partial m^*} = 0$$

Using (1) and (2) conditions (23a) and (23b) become :

$$(24a) \alpha_1 x + \varphi \beta e = 0$$

$$(24b) \alpha_1 x^* - \varphi \beta e = 0$$

A Nash equilibrium is a solution to system (24). Adding and subtracting (24a) and (24b) we obtain :

$$\alpha_1 (x+x^*) = 0$$

$$\alpha_1 (x-x^*) + 2 \varphi \beta e = 0$$

Because  $\alpha_1 \neq 0$  and  $\alpha_1 - \alpha_2 \neq 0$  (from our assumption that  $|\alpha_2| < |\alpha_1|$ ), these conditions are equivalent to :

$$(25a) x + x^* = 0$$

$$(25b) (\alpha_1 - \alpha_2)(x-x^*) + \frac{2(\alpha_1 - \alpha_2)\varphi}{\alpha_1} \beta e = 0$$

If we compare system (25) and system (13) we see that there exists a unique Nash equilibrium which has the same solutions for  $m$  and  $m^*$  as the optimum which would be obtained for a value  $k = 1/2$  and a weight  $\varphi'$  equal to

$$(26) \quad \varphi' = \frac{1}{2} \left( 1 - \frac{\alpha_2}{\alpha_1} \right) \varphi$$

As  $(1 - \alpha_2/\alpha_1)$  is positive (from (4)),  $\varphi'$  is positive<sup>5</sup>. From (26) we have :

$$(27) \quad \varphi - \varphi' = \frac{1}{2} \left( 1 + \frac{\alpha_2}{\alpha_1} \right) \varphi$$

Therefore, because of inequality (5), when  $\varphi \neq 0$  we have :

$$(28) \quad \varphi' < \varphi$$

Inequality (28) indicates that at the Nash equilibrium, policymakers do not give enough weight to the exchange rate stabilization objective. We have seen that, at the optimum for  $k = 1/2$ ,  $\hat{e}^2$  increases when  $\varphi$  decreases. Therefore, inequality (28) implies :

$$(29) \quad e_N^2 > \hat{e}^2$$

and, because of inequality (22), we have for any optimum  $\hat{e}(k)$

$$(30) \quad e_N^2 > \hat{e}^2(k)$$

where  $e_N$  is the exchange rate at the Nash equilibrium.

Therefore, at the Nash equilibrium the exchange rate fluctuation is greater than at any Pareto optimum. In figure 1, the Nash equilibrium is at point N between  $P_0$  and P along the  $P_0 Y$  axis<sup>6</sup>.

The reason is the following. At the Nash equilibrium policymakers do not take into account the effect of their policies on the objective function of the other country. Then, first consider the case  $\alpha_2 = 0$  where the policy instrument has no effect on the internal objective of the other country, and, consequently, affects the objective function of the other country only through its influence on the exchange rate. In that case, at the Nash equilibrium, countries do not take into consideration the positive effect that a greater reduction of exchange rate fluctuation will have on the welfare of the other country, and, therefore, do not give enough weight to the exchange rate objective. We see that, when  $\alpha_2$  is equal to 0, the result comes from the fact that exchange rate stabilization is a public good, and that at the Nash equilibrium the amount of public good produced is not large enough. When  $\alpha_2$  is not equal to zero there is a further channel through the direct effect of the policy instrument on the internal objective of the other country. Our assumption  $|\alpha_2| < |\alpha_1|$  puts a limit on the importance of that channel, so that the public good effect always dominates<sup>7</sup>. The issue of whether the effect going through this second channel reinforces the public good effect or dampens it, is examined in section 5 below.

Note that when  $\varphi=0$ , we also have  $\varphi'=0$  and, therefore, the optimum and the Nash equilibrium are the same. Indeed, this was obvious from the analysis of section 3.1 : at point  $P_0$ , which is the optimum for all  $k$  (when  $\varphi=0$ ), both  $V$  and  $V^*$  are minimized and there is no incentive for any country to depart from this point :  $P_0$  is also a Nash equilibrium in the case  $\varphi = 0$ . We must have more objectives than policy instruments in order for the Nash equilibrium not to be optimal.

The Nash equilibrium concept is really meaningful only if we also have stability (or at least local stability). Here, we can see that it is the case. Consider the reaction functions (24a) and (24b) of country 1 and 2 respectively. Using (1) these reaction functions can be written :

$$(31a) (\alpha_1^2 + \varphi \beta^2) m + (\alpha_1 \alpha_2 - \varphi \beta^2) m^* - \alpha_1 s + \varphi \beta \theta = 0 \quad (R)$$

$$(31b) (\alpha_1 \alpha_2 - \varphi \beta^2) m + (\alpha_1^2 + \varphi \beta^2) m^* - \alpha_1 s^* - \varphi \beta \theta = 0 \quad (R^*)$$

They are represented by the straight lines (R) and (R\*) in figure 2<sup>8</sup>. The slopes of (R) and (R\*) may be positive or negative but, because of our assumption  $|\alpha_2| < |\alpha_1|$ , the slope of (R) is less than 1 and that of (R\*) is greater than 1, in absolute values. Therefore, the Nash equilibrium N is (globally) stable. Reciprocally, we can see that, in order for the Nash equilibrium to be stable for any value of  $\varphi$ , we must have  $|\alpha_2| < |\alpha_1|$ . Therefore our assumption  $|\alpha_2| < |\alpha_1|$  can also be considered as a necessary and sufficient condition for the stability of the Nash equilibrium.

A policy implication is that, in a non-cooperative world, an optimum could be attained through international monetary rules which would penalize exchange rates variations. If we wanted to reach the optimum P, this could be done by explicit penalties that each country would have to pay when the exchange rate fluctuates. Our results indicates that the amount of this

penalty should be such that it would be equivalent to add a term  $\Psi e^2$  to the objective function of each country. The objective function of the countries would become :

$$\tilde{V} = V + \Psi e^2 = x^2 + (\varphi + \Psi) e^2$$

$$\tilde{V}^* = V^* + \Psi e^2 = x^{*2} + (\varphi + \Psi) e^2$$

According to (26), the weight  $\Psi$  should satisfy :

$$\varphi = \frac{1}{2} \left( 1 - \frac{\alpha_2}{\alpha_1} \right) (\varphi + \Psi)$$

which gives :

$$(32) \quad \Psi = \frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} \varphi$$

The weight  $\Psi$  can take any positive value and increases with  $\alpha_2/\alpha_1$ . This last point is considered in more details in the next section.

## 5. POSITIVE AND NEGATIVE TRANSMISSION MECHANISMS

Equality (27) suggests that the greater  $(\alpha_2/\alpha_1)$  is, the more important the difference between the Nash equilibrium and the optimum is. In particular, this would mean that going from a negative transmission mechanism to a positive one would widen the difference. In this section, we will examine such an issue.

From (18) and (21) and our results of section 4 we can write :

$$\hat{e}^2 = (1-Q)^2 \hat{e}_0^2$$

$$e_N^2 = (1-S)^2 \hat{e}_0^2$$



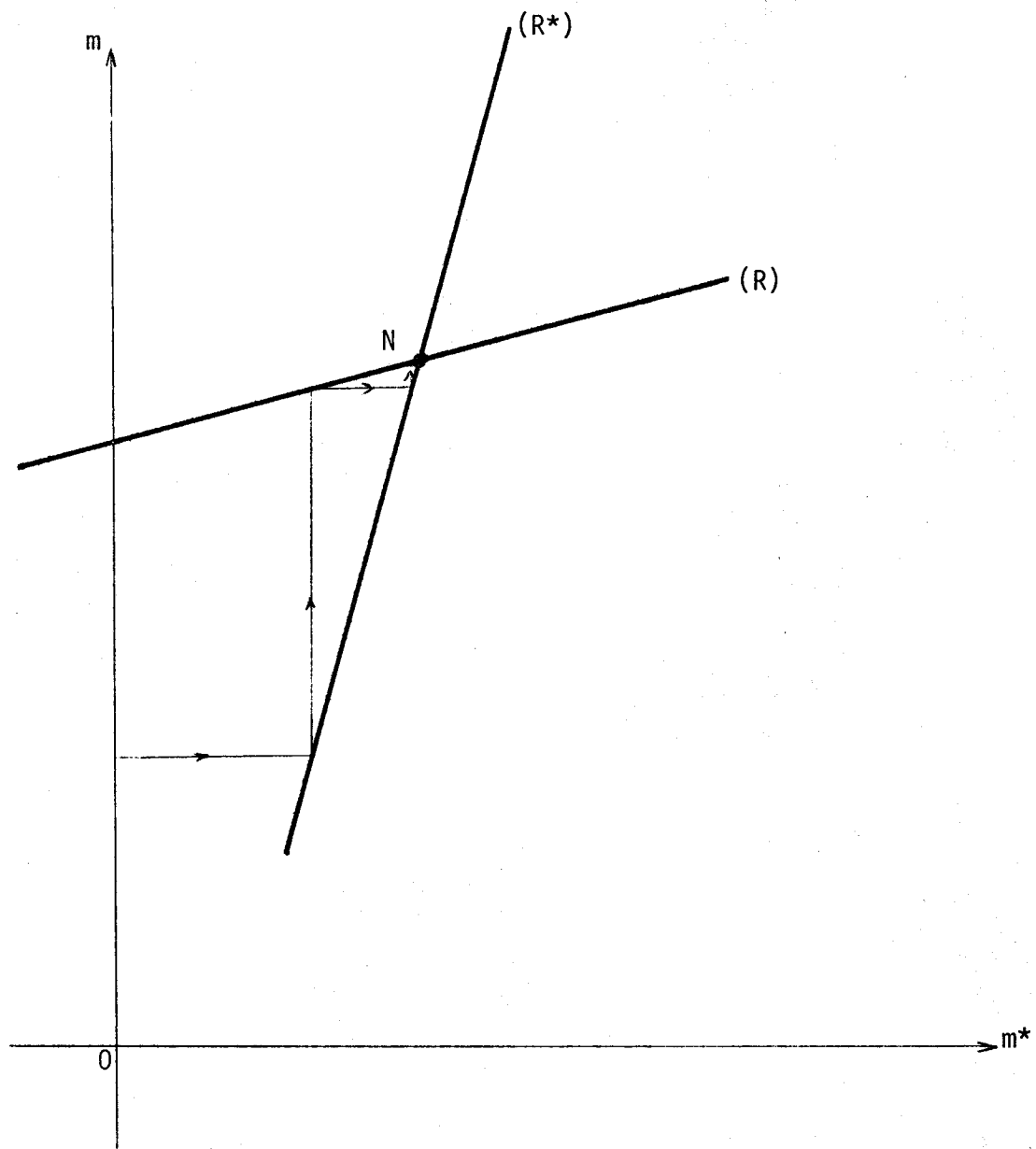


FIGURE 2

where we define S by :

$$(33) \quad S = \frac{4 \varphi' \beta^2}{(\alpha_1 - \alpha_2)^2 + 4 \varphi' \beta^2}$$

In these expressions,  $\hat{e}_0$  and Q have been defined by (9) and (18) and  $\varphi'$  is given by (26). Therefore, we have :

$$(34) \quad \frac{e_N^2 - \hat{e}^2}{\hat{e}^2} = \left( \frac{1-S}{1-Q} \right)^2 - 1$$

From (18), (26) and (33), we obtain :

$$\frac{1-S}{1-Q} = \frac{\alpha_1^2 \mu^2 + 4 \varphi \beta^2}{\alpha_1^2 \mu^2 + 2 \varphi \beta^2 \mu}$$

Where  $\mu$  is defined by :

$$\mu = 1 - \frac{\alpha_2}{\alpha_1}$$

Because of our assumption  $|\alpha_2| < |\alpha_1|$ ,  $\mu$  verifies the inequality :

$$(35) \quad 0 < \mu < 2$$

We want to show that  $(e_N^2 - \hat{e}^2)/\hat{e}^2$  increases when  $\alpha_2/\alpha_1$  increases. This will be the case if

$$(36) \quad \frac{\partial(1-S/1-Q)}{\partial \mu} < 0$$

Calculating this derivative we obtain that its sign is given by the sign of :

$$\alpha_1^2 \mu(\mu-4) - 4 \varphi \beta^2$$

Because of inequality (35) this sign is actually negative, and inequality (36) holds.

Therefore, when  $\alpha_2/\alpha_1$  increases the relative gap  $(e_N^2 - \bar{e}^2)/\bar{e}^2$  increases, which implies that, *ceteris paribus*, positive transmission mechanisms create more excessive fluctuations of the exchange rate than negative transmission mechanisms.

In order to get a more intuitive understanding of the result, let us consider the situation where, at the Nash equilibrium N, the internal objective of country 1 is below its target ( $x < 0$ ) while that of country 2 is above it ( $x > 0$ ), and where the exchange rate is depreciated relative to its target level ( $e > 0$ ). With the convention signs<sup>9</sup>  $\alpha_1 \geq 0$  and  $\beta \geq 0$  this situation is that depicted in figure 1. Then, to go from the Nash equilibrium N to the optimum P would require that country 1 decrease  $m$  and country 2 increase  $m^*$ , so that the exchange would be less depreciated. Now, starting from the Nash equilibrium N, consider the effect of a marginal decrease in  $m$  ( $dm < 0$ ). By definition of the Nash equilibrium, this should keep the value of the objective function of country 1 unchanged (at the first order of magnitude) : the favorable effect of the implied lower variation (depreciation) of the exchange rate, is just compensated by the defavorable effect of the further decrease in  $x$  ( $x < 0$  and  $dx < 0$ ).

As emphasized in section 4, the effect of  $dm$  on the value of the objective function of country 2 goes through two channels. First, there is a favorable effect due to the lower variation of the exchange rate. Second, the decrease in  $m$  has an effect on the internal objective of country 2. In the case of a positive transmission mechanism ( $\alpha_2 > 0$ ), this second effect is also favorable because  $x^*$  gets closer to its target level ( $x^* > 0$  and  $dx^* < 0$ ). Then, the two channels work into the same direction. In the case of a negative transmission mechanism, however, the second effect is defavorable because  $x^*$  gets further away from its target level ( $x^* > 0$  and  $dx^* > 0$ ), and the two channels work into opposite directions. Nonetheless, even in this last case the total net effect on the objective function of country 2 is still favorable because, as we assumed  $|\alpha_2| < |\alpha_1|$ , the first channel dominates. Therefore, in both cases there is a positive effect on the welfare of country 2, and, consequently, a marginal decrease in  $m$  would be a Pareto improvement. The fact that country 1 does not take into account this favorable effect on

country 2 of a decrease in  $m$ , is at the source of the non-optimality of the Nash equilibrium. As the effects of the two channels add up in the first case, and subtract in the second case, the gap between the Nash equilibrium  $N$  and the optimum  $P$  is widened when we go from a negative to a positive transmission mechanism.

## 6. CONCLUSION

We have considered a two country world where policymakers of each country have at their disposal one policy instrument and want to reduce the fluctuations of two variables : an internal variable, such as domestic output or price, on the one hand, and the exchange rate, on the other hand. Using a quite general symmetric reduced form model, we have shown that at the non cooperative Nash equilibrium policymakers do not give enough weight to the exchange rate stabilization objective and, therefore, that the exchange rate fluctuates more than what it would at any Pareto optimum. The basic reason is that, because exchange rate stabilization can be considered as a public good, each country would prefer that the other take care of this objective, thus keeping its policy instrument more completely available for the stabilization of the internal objective.

We have examined how the result depends on the existence of a positive versus a negative transmission mechanism. The qualitative result is valid in both cases as long as the effect of a policy instrument of a given country is greater in absolute value on the internal objective of this country than on that of the other country. However, the magnitude of the effect on exchange rate fluctuations increases when we go from a negative to a positive transmission mechanism.

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Our result indicates that in the absence of international cooperation the exchange rate fluctuates more than it should (ie. more than at any Pareto optimum). This would leave room for international monetary rules which would provide for more fixity of the exchange rate. However, our analysis also points out that fixed exchange rate system are generally not optimal and may even be worse than the non cooperative solution<sup>10</sup>. Thus, there is some rationale for international monetary agreements, such that the European Monetary System, which are not completely fixed exchange rate system, but put limits to exchange rate variations. From this point of view, we stress the importance of having non negligible fluctuation margins in the EMS. Our analysis underlines that we should look for international monetary rules which give the optimal greater fixity of the exchange rate.

F O O T N O T E S

1. Following the initial papers of Hamada (1974 and 1976), in the recent years there has been a large use of game theory for the issue of international cooperation in macroeconomic policies. Among them we have Canzoneri and Gray (1983), de Macedo (1983), Johansen (1982), Melitz (1984), Sachs (1983).
2. The case where these shocks are unknown is examined in Laskar (1984) where the literature on optimal foreign exchange intervention (Boyer (1978), Canzoneri (1982), Roper and Turnovsky (1980), Turnovsky (1984)) is extended to an explicit two country framework.
3. The slopes of (D) and (D\*) can either be negative or positive, depending on the sign of  $\alpha_2/\alpha_1$ . Because of our assumption (3) the absolute value of the slope of (D) is always less than 1, and that of (D\*) greater than 1.
4. The intuition of the result is the following. Consider the case  $k > 1/2$ . Then, as country 1 receives more weight, at the corresponding optimum its internal objective is more stabilized than when  $k = 1/2$ . Therefore, keeping the same trade-off for the two objectives of country 1 would require the exchange rate also to be more stabilized than in the case  $k = 1/2$ . However, because the internal objective of country 2 is less stabilized, the same reasoning would imply that, looking at country 2's trade off, the exchange rate, on the contrary, should fluctuate more. But, as the weight of country 1 is greater, the first aspect dominates the second aspect, so that the exchange rate should actually fluctuate less than when we have  $k = 1/2$ . The further  $k$  is away from  $1/2$ , the greater the gap is. If we had started from  $k < 1/2$ , by the same argument, we would have obtained exactly the same result.

- <sup>5</sup>.  $\varphi'$  also increases with  $\varphi$ . Therefore, the greater the weight  $\varphi$  given to the exchange rate objective, the lower the fluctuation of the exchange rate at the Nash equilibrium is.
- <sup>6</sup>.  $N$  is between  $P_0$  and the straight line (E) because first, as indicated below in the text,  $P_0$  is a Nash equilibrium when  $\varphi = 0$ ; and second (cf. fn. 5 above), at the Nash equilibrium, when  $\varphi$  increases, the exchange rate fluctuates less.
- <sup>7</sup>. This occurs because  $m$  and  $m^*$  have the same effect in absolute value on the exchange rate. We can write  $e = \beta_1 m + \beta_2 m^* + \theta$ . We have  $|\beta_1| = |\beta_2|$  from (1c)

When we say that our result comes from the fact that exchange rate stabilization is a public good, we must not be misled by such a statement because there are actually two channels through which the policy instrument of a given country affects the objective function of the other country. The statement, however, is basically correct because our assumption implies that the public good channel dominates.

- <sup>8</sup>. In figure 1 the straight lines (R) and (R\*) go through points F and F\* respectively.
- <sup>9</sup>. Without loss of generality we can always choose appropriate signs for the variables  $x$  and  $m$  in order to have  $\alpha_1 \geq 0$  and  $\beta \geq 0$ .
- <sup>10</sup>. First, in our analysis, fixed exchange rate system are never optimal when we give weight to both countries in the world objective function  $U$  defined in (6). Only F and F\* in figure 1 where the point of view of only one of the two countries is taken ( $k = 0$  or  $k = 1$ ) can be optimal.

Second, obviously, when  $\varphi$  is small, the Nash equilibrium  $N$  is close the optimum  $P$  and, therefore, dominates a whole set of fixed exchange rate systems.

## A P P E N D I X

### O P T I M A

Consider the general case where  $k$  is any number  $0 < k < 1$ . The corresponding optimum is given by equations (12a) and (11b) in the text.

Using (12a), equation (11b) becomes :

$$(A1) \quad (\alpha_1 - \alpha_2) k x + \varphi \beta e = 0$$

From (15a) and (15b) we have :

$$(A2) \quad 2 x = (\alpha_1 + \alpha_2) Z + (\alpha_1 - \alpha_2) Y$$

But using (15a) and (15b), equation (12a) may be rewritten as :

$$(A3) \quad (\alpha_1 + \alpha_2) Z + (2k - 1)(\alpha_1 - \alpha_2) Y = 0$$

Therefore, from (A2) and (A3) we have :

$$(A4) \quad 2 x = 2(1 - k)(\alpha_1 - \alpha_2) Y$$

Using (A4) and (15c), equation (A1) becomes :

$$(A5) \quad [k(1 - k)(\alpha_1 - \alpha_2)^2 + \varphi \beta^2] Y + \varphi \beta^2 (\hat{e}_0 / \beta) = 0$$

and therefore, at the optimum :

$$(A6) \quad Y = - T(k) \frac{\hat{e}_0}{\beta}$$

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where we define :

$$T(k) = \frac{\varphi \beta^2}{k(1-k)(\alpha_1 - \alpha_2)^2 + \varphi \beta^2}$$

We have :

$$0 < T(k) < 1 \quad \text{for all } k \quad 0 < k < 1$$

Of course when  $k$  is equal to  $1/2$  we have  $T(k) = Q$ , where  $Q$  is defined by (18).

From A6 and (15c) we get :

$$\hat{e}(k) = [1-T(k)] \hat{e}_0$$

and

$$\hat{e}^2(k) = [1-T(k)]^2 \hat{e}_0^2$$

$T(k)$  reaches its minimum when  $k(1-k)$  is maximum, which occurs for  $k = 1/2$ . Therefore,  $\hat{e}^2(k)$  takes its maximum value when  $k$  is equal to  $1/2$  :

$$\hat{e}^2(k) < \hat{e}^2(1/2)$$

When  $k$  gets further apart from  $1/2$  on either side,  $k(1-k)$  decreases, and consequently, as  $T(k)$  increases,  $\hat{e}^2(k)$  decreases. This is the result of inequality (22) in the text.

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## A B S T R A C T

### INTERNATIONAL COOPERATION AND EXCHANGE RATE STABILIZATION

We consider a two country world where policymakers of each country have at their disposal one policy instrument and want to reduce the fluctuations of two variables : an internal variable, such as domestic output or price, on the one hand, and the exchange rate, on the other hand. Using a quite general symmetric reduced form model, we show that at the non cooperative Nash equilibrium policymakers do not give enough weight to the exchange rate stabilization objective and, therefore, that the exchange rate fluctuates more than what it would at any Pareto optimum. The magnitude of the gap between the two types of solutions increases when we go from a negative to a positive transmission mechanism.

## R E S U M E

### COOPERATION INTERNATIONALE ET STABILISATION DU TAUX DE CHANGE

On considère un monde à deux pays où les décideurs politiques de chaque pays ont à leur disposition un instrument de politique économique et veulent réduire les fluctuations de deux variables : une variable interne, telle que l'output ou le niveau des prix, d'une part, et le taux de change d'autre part. A partir d'un modèle symétrique sous forme réduite de nature assez générale, on montre qu'à l'équilibre non-coopératif de Nash les décideurs politiques n'accordent pas suffisamment de poids à l'objectif de stabilisation du taux de change et que, par conséquent, le taux de change y fluctue davantage qu'à n'importe quel optimum de Pareto. L'écart entre ces deux types de solutions s'accroît lorsque l'on passe d'un mécanisme de transmission négatif à un mécanisme de transmission positif.