C.E.P.R.E.M.A.P Juillet 1984

THE DYNAMICS OF COMPETITION : A RESTORATION OF THE CLASSICAL ANALYSIS*

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N° 8416

* We thank M. Glick for the translation of this text into English.

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SUMMARY

This article seeks to build a formal model of the dynamics of competition on the foundations laid by the classical economists : Smith, Ricardo, and Marx. The analysis of such a model reveals that with certain assumptions market prices will converge to prices of production and the economy will stabilize on a path of homothetical growth. The adjustment process of market prices involves both a price and quantity component, where inventories play an important role. The most striking result is that convergence is reached in three stages where the economy displays the features of equilibrium in some regards and disequilibrium in others. First, for each commodity a unique price is reached ; second, the rates of profit in each industry are equalized (even where multiple techniques exist), and last, the most profitable technique eventually prevails. The model is robust enough to allow for the investigation of numerous "cases". For instance, we analyse the results where "rationed" markets exist.

UNE RESTAURATION DE L'ANALYSE CLASSIQUE DE LA DYNAMIQUE CONCURRENTIELLE

RESUME

L'objet de cet article est de construire un modèle de la dynamique concurrentielle classique et d'en explorer les propriétés. On montre qu'un tel modèle peut être élaboré et qu'il garantit la convergence des prix de marché vers les prix de production et la stabilisation de l'économie sur un sentier de croissance homothétique équilibrée. Dans ce modèle, l'existence de stocks d'invendus joue un rôle central et l'ajustement se fait par un double mécanisme prix/quantité. Le résultat obtenu est en fait une triple convergence, successivement : formation d'un prix unique par bien, uniformisation des taux de profit moyen de branches à procédés multiples, sélection de la technique optimale. Plusieurs variantes du modèle sont proposées traitant notamment de la possibilité du rationnement sur les marchés.

INTRODUCTION

This article is an initial contribution in a much broader project devoted to the restoration of the classical analysis of competition in a capitalist economy [1]. The project stems from the conviction that quite another political economy could have eventually developed from the pioneering works of Smith, Ricardo, and Marx if the "marginalist Revolution" of just over a century ago had not triumphed in the academic world. The "inner contradictions" and theoritical difficulties of the great post-Ricardian school which culminated in the work of Alfred Marshall left clear field for the domination of Walrasian general equilibrium. Today, Walrasian microeconomics appears as the neo-classical impregnable stronghold in the aftermath of the Keynesian revolution of the 1930's and more recent neo-Ricardian attacks. However, our present aim is not so much to criticize the foundations of the modern development of this Walrasian approach as it is to show that the abandonment of classical economics was in fact detrimental to the advance of economic knowledge. We believe, in fact, that there exists a basis for powerful and coherent "classical microeconomics" within the classical analysis. Dynamics, which is a very weak point for the Walrasian approach, has a very sound footing in the classics upon which we will elaborate.

This article is divided into three sections. In the first section the basic principles governing the construction of a dynamic model of the convergence of market prices to prices of production are established. The second and third sections then reveal the results we obtain applying these principles in a specific set of models [2].

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(1) Duménil-Lévy : 1983 (a), 1983 (b)

(2) At the end of this article appear four appendixes : a list of the symbols used (Appendix A), two diagrams illustrating the structure of the model (Appendix B) and two further technical appendixes.

I - CONVERGENCE OF PRICES TOWARD PRICES OF PRODUCTION.

A. The Classical Analysis of Competition [1].

One of the striking features of classical analysis is its emphasis on dynamic and disequilibrium phenomena. A distinction is always made between two sets of prices : market prices (disequilibrium prices) and natural prices (equilibrium prices [2]).The latter would have the possibility to exist if a capitalist economy did not continually display disturbing forces. If these perturbations were temporarily suspended the inner competitive mechanisms described by the classics would drive the economy toward equilibrium. Dispite these constant disturbances Ricardo refers to equilibrium prices, as "natural prices". Marx conceived such equilibrium prices as "prices of production". They can be computed by marking up cost with a profit proportional to the capital advanced. In a situation of known technology and distribution these prices are immediately calculable [3]. He, like Smith and Ricardo, defines "market prices" as those prices which gravitate around prices of production and are determined by the interaction of supply and demand.

The classical competitive mechanism further entails the adjustement of the quantities produced to those specific values which Marx called the "social need" for each commodity (Smith's effective demand). These social needs correspond to the quantity of each good which can be sold when market prices are equal to prices of production.

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[1] A number of studies have already been devoted to classical dynamics. Consult for instance : Arena : 1979, Garegnani : 1981, Semmler : 1983 (a) and (b).

[2] A concept which will be more accurately specified later as "quasi-equilibrium".

[3] This does not mean that prices of production are completely independent of the quantities produced. Such a dependency exists indirectly through the determination of the technology which itself can depend on quantities, for instance if scarce ressources such as land exist. Moreover, if the situation of distribution is not given by the rate of profit or the purchasing power of workers for a fixed bundle of commodities, prices are also functions of the quantities (one can for instance indicate the share of wages in the total net product).

The classical conception of this dynamic mechanism is that of a decentralized market adjustment process which does not require the mediation of any central agency such as an auctioneer or any central bargaining prior to exchange [1]. Instead the process unfolds in a more realistic pattern. If production in one industry yields a relatively high rate of profit, some capital will be shifted to this industry. Conversely, if profitability is subnormal, some capital will exit this activity. Associated with these flows of capital, are changes in market prices which respond to the divergence of supply and demand and eventually will equalize them.

A description of a supply and demand mechanism exists in every important work by the classics. They make explicit that an increase in quantity sold is only possible when price is diminished and that an increase in price corresponds to smaller sales outlets. This dependency of demand on price is especially clear in Marx where only those frightened by neo-classical connotations would fail to identify an explicit "demand function" in his analysis.

The above investment and price mechanisms jointly move the economy toward equilibrium in a step by step fashion. If the shift of capital into a profitable activity is excessive it provokes a large increase in supply, thus decreasing the price. Eventually, the low price leads to a diminished profit rate which ends the inflow of investment. This double adjustment which includes both prices and quantities results in the tendency toward the classical equilibrium [2].

Because of the existence of constant perturbation and/or inner imperfections in the competitive mechanism the classics contended that the convergence forces do not lead to an equilibrium position but instead result in the gravitation of prices and quantities around a reference situation. However, some precision of terminology is important. This reference situation can be presented as a long run equilibrium in the sense that in the absence of perturbations and with certain assumptions concerning the parameters, the economy would tend to reach this reference target asymptotically. As for the short run, it can be contended

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(1) Here we refer to a Walrasian tatonnement in the first instance, and game theories in the second.

(2) Such behavior is of course premised on a number of assumptions. The dynamic process in the classics is described for a situation of normal accumulation, capitalists must be willing to engage their funds and not disinvesting as in a recessionary process.

that only for the classics does disequilibrium exist, since stockpiling, shortages, multiple techniques, and unequal rates of profit prevail [1]. However, the modern usage of the terms "short run equilibrium" and "long run equilibrium" is unsuitable for the classical account of gravitation because equilibrium in the classics is a state which is in fact never reached. Disequilibrium can not be defined as a short run phenomenon since it may exist permanently as a result of uninterrupted perturbation. The "normal" state of the classics should be viewed more as a reference in the description of the fundamental competitive forces of the system than an actual achievable point.

Beyond the distinction "short run" and "long run", the modern concept of "equilibrium" itself raises some difficulties when applied to the classical conceptions. A more suitable term would be "quasi-equilibrium" since the prefix "quasi" allows for certain restrictions. In Marx's analysis of prices of production with multiple techniques only equalized profit rates between industries are required for the "normal" state, and not between individual capitals. In our framework this interesting property plays a prominent role.

Further difficulties arise in the specification of the economic agents involved in the classical mechanism. The focus of the classical perspective is on the decisions of capital [2]. Four areas are of importance :

1. The allocation of capital.
2. The distribution of profit destined for consumption.
3. The determination of output.
4. The determination of prices.

The latter two, the determination of prices and outputs are managerial decisions. But some ambiguity exists concerning the agents responsible for the first two.

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(1) In modern neo-classical economics three sorts of conditions can be responsible for establishing a short run equilibrium none of which exist in the classics :
- An auctioneer computes equilibrium prices in advance ;
- Economic agents possess perfect knowledge ;
- Economic agents in some unspecified manner are capable of solving a system of equations, automatically equating total supply and total demand.

(2) As for workers, when the analysis of the labor market is excluded they influence the determination of prices and output exclusively through their decisions as consumers.

The allocation of capital is complex and can take a number of forms :

- Firms perform the function of allocating capital when they make decisions concerning where their retained earnings will be invested.

- Other examples of agencies which allocate capital include banks which make lending decisions among several clients, holding companies which select among subsidiaries for expansion, and individuals who make portfolio decisions.

Similarly, these same agents perform the function of the distribution of profit in various institutionally specific forms such as dividends, bonuses, expense accounts ...

B. General Principles involved in the formalization of competition.

A number of "classical" models of capitalist competition already exist [1]. Rather than carry out an exhaustive survey here we intend to present a set of general and necessary principles the totality of which the existing studies have so far ignored.

1. A model of competition must be general.

Although the classical analysis of competition differs from the general equilibrium perspective in the sense that the classical analysis is a disequilibrium analysis, they both share the property that all economic variables are considered as interdependent in a general framework. This general framework is essential precisely because of the interdependency of the variables in the adjustment process. This principle is violated in models in which prices are derived from costs through a constant mark-up procedure because no account is taken of the quantity side of the adjustment process. Similarly, it goes without saying that a one sector model is inadequate for the study of convergence. No analysis is possible of commodity prices since these prices depend on prices and outputs in other sectors,as well as on incomes distributed elsewhere -- a rate of profit is a function of numerous other prices which cannot be assumed to be constant.

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(1) Egidi : 1975, Nikaido : 1977, 1983, Benetti : 1981, Cartelier : 1981, Steedman : 1982, Flaschel: 1983, Franke : 1983, 1984, Filippini : 1983.

2. *The adjustment process must be decentralized and executed in disequilibrium.*

This requirement is clearly violated by the famous Walrasian tatonnement. In the tatonnement process the auctioneer presents a vector of prices and agents respond by announcing what transactions they plan to perform. However no exchanges actually occur until the auctioneer, using the information supplied by the agents, modifies the price vector so that all intended supplies equal their respective demands. Once these equilibrium prices are reached, transactions are performed and markets are cleared (1).

A decentralized market adjustment procedure differs in at least two regards :

- Outputs are decided and productsare priced by sellers before demand is known.
- Transactions actually take place at these prices. Without perfect knowledge markets are not cleared. Instead the usual case will be excess inventories or shortages (2).

An important difference exists between equilibrium and disequilibrium models. In the Walrasian equilibrium a unique price is obtained in advance for each type of commodity which guarantees the equality of supply and demand.

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(1) In the General Intertemporal Equilibrium framework (Debreu : 1969, Malinvaud : 1968, Arrow - Hahn : 1971) the tatonnement process appears in a highly unrealistic form. The auctioneer settles in advance the course of the economy into the infinite future or until doomsday. In the Temporary Equilibrium Model (Hick : 1968, Grandmont : 1977) the auctioneer appears in each period to mediate transaction. Neo-classical attempts to construct a "non-tatonnement" procedure with production have so far lacked success (Arrow - Hahn : 1971, Fisher : 1976).

(2) Stockpiling is the earmark of genuine disequilibrium model, the "theory of disequilibrium" is, in fact, a theory of equilibrium with fixed prices. Equilibrium cannot be achieved without the aid of the autioneer (Benassi : 1982).

Therefore, the actual structure of exchanges between individual agents becomes irrelevant. Conservely, in the existing "disequilibrium" models where prices are not determined in advance and in the classical perspective, a model of the market is necessary. This formalisation of the market is achieved by the use of "strategic outcome functions" or "rationing schemes" [1]. They indicate :

- The allocation of sales among several sellers if total demand is smaller than total supply. This allocation depends on the different prices proposed by individual sellers for the same commodity.
- The allocation of output among several buyers if total supply is smaller than total demand, i.e. when rationing takes place [2].

3. *Information is imperfect but nevertheless ajustment occurs on the basis of the agent's understanding of the working of the market.*

A model in which economic agents possess a perfect knowledge of demand is an unrealistic characterization of capitalist competition. However, the classical perspective assumes that economic agents have at least some understanding of the market mechanism in which they participate. They know that they must adapt to a demand which is negatively sloped. This knowledge prevents them from continuously trying to increase their prices or moving their entire capital when a more profitable investment opportunity manifests itself. Therefore the reactions of capitalist are continuous functions of their arguments.

If one capitalist discovers a relative lack of profitability in the activity in which he is engaged, he is induced to move a portion of his capital into some other activity. However, this does not imply the complete abandonment of the original line. Capitalists know that the sharp decline in supply in this field created by the capital departure may result in a rise in the price and thus higher profits. Similarly a massive entry into the new field may produce a glut.

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(1) Strategic outcome functions are generalized rationing schemes with a multiplicity of prices for the same good. These functions have been used in game theoretic models as an attempt to replace the auctioneer.

(2) In this article the word rationing is used only on the buyer side. Inventories which are a form of rationing on the seller's side are a basic characteristic of the model and a necessary component of the capitalist competitive system.

The modelling of reactions in which the whole social capital switches to the field which yields the highest rate of profit is not an accurate interpretation of the classics. Such a discontinuous reaction can only take place with the guaranties of an auctioneer adjusting supply and demand in advance but not in a decentralized economy.

C. The formalization of Individual Behavior with Imperfect Knowledge.

With these remarks in mind some initial comments concerning the formalization of the model can now be proposed. First, the variables of the model must be specific to individual agents. Global variables such as total supply or total demand can not model the behavior of individuals. Second, equilibrium variables such as the uniform rate of profit, prices of production or equilibrium outputs cannot be involved in the dynamic equations since their value are unknown to individual agents. Third, individual agents can only affect a subset of their own variables. A unit of production, j, for instance, can only determine its own output Y^j and its own "price tag" p^j. These are the only variables which can be derived from the behavioral equations :

$$p_t^j = p_t^j\ (\ldots,\ldots,\ldots)$$

$$Y_t^j = Y_t^j\ (\ldots,\ldots,\ldots)$$

Concerning the arguments for such functions, as we have already implied, the information which individual agents possess is strictly limited. What they know is their technique, "T^j" and the past values of their own variables p^j, Y^j, the inventories, S^j, the value of the capital, K^j, and the prices of other products, $\vec{p}$. When the decision of production is taken, K_t^j, Y_{t-1}^j, S_{t-1}^j, $\vec{p}_{t-1}$ are known as well as the values of the same variables in previous periods (See Appendix B, figure 5) :

$$Y_t^j = Y_t^t(T^j, K_t^j, Y_{t-1}^j, S_{t-1}^j, \vec{p}_{t-1}, \ldots) \qquad (1)$$

and for prices :

$$p_t^j = p_t^j(T^j, K_t^j, Y_t^j, S_t^j, \vec{p}_{t-1}, \ldots) \qquad (1')$$

Similarly, a consumer's information is restricted to prices, his present income, as well as his previously realized transactions. Concerning the agents which allocatecapital, they possess information limited to the set of rates of profit in the activities which they consider as potential investments.

D. Hypotheses and Methods Common to all Models Proposed in this Article [1].

In many regards the construction of a model requires that precision be given to the classical principles. Below we try to present our specification of their views :

1. No barriers to entry and exit exist.

Except in one model proposed in subsection II-E, we always assume that only one center of allocation of capital operates. All considerations concerning the existence of several centers of allocation are concentrated in subsection II-E.

2. Fixed capital is not taken into account.

3. Two goods are produced which can be used alternatively for either production or consumption.

4. Technology is defined by the A matrix of input coefficients (where each row represents a productive process and each column a product input ; A is not necessarily square), the $\vec{L}$ column vector of labor requirements and $\overleftarrow{Y}$ the activity vector. The jth process is represented $(\overleftarrow{A}^j, L^j, Y^j)$, the jth row of A and the jth components of $\vec{L}$ and $\overleftarrow{Y}$.

5. Each enterprise, j, adjusts its price, p^j, according to the changes [2] in its inventories as a portion of its production :

$$p_t^j = p_{t-1}^j \; g^j\left(\frac{\Delta S_t^j}{Y_{t-1}^j}\right) \qquad (2)$$

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(1) The operations of the models are schematically diagrammed in appendix B.

(2) The changes in inventories and not the inventories themselves are considered. This choice must be regarded more as a simplifying assumption rather than a theoretical contention.

where $\Delta S_t^j = S_t^j - S_{t-1}^j$, g^j is a strictly positive increasing function, and $g^j(0) = 1$. Since the only inventories considered are those of j, equation (2) can be considered a particular case of equation (1').

The prominent role played by inventories in our model corresponds to the idea of the confrontation of supply and demand as proposed by the classicals. From the point of view of the individual enterprises in a decentralized economy, the difference between supply and demand is equal to the changes in inventories.

The strategic outcome function must express this equality of treatment. When several suppliers present the same commodity at different prices, no buyer will be discriminated against. This means that no one enjoys a privileged position allowing him/her to buy at the lower price.

For example, suppose that two prices prevail for a given commodity. Consumers are assumed to purchase from both sellers in the same proportions (at both prices). It follows then that each consumer buys at the average price. The vector of average prices is denoted $\vec{\bar{p}}$ with the number of components equal to the number of goods (Recall $\vec{p}$ has components equal to the number of processes of production). $\vec{\bar{p}}$ is taken into account in the determination of demand, wages, and the prices of inputs (eq. (3), (9), (11), (12) and (14)).

Sellers who maintain prices which are too high must be penalized. In order to obtain such a result, the model assumes that the ratio of the realized transactions to the amount brought to the market is a decreasing function of prices. When a unique price exists this ratio is equal for all sellers (proportional rationing).

7. Total profit is given by the following equation [1] :

$$\Pi_{t-1} = \overleftarrow{Y}_{t-1}\,\vec{p}_{t-1} - \overleftarrow{Y}_{t-1}(A\,\vec{\bar{p}}_{t-1} + \vec{L}\,w_{t-1}) \qquad (3)$$

Profit is then distributed according to proportions defined by the parameters α and α'. "$\alpha\,\Pi_{t-1}$" is alloted to the final consumption of those who receive profit in the form of income. $\alpha'\Pi_{t-1}$ accrues to accumulation. If "k" denotes total capital then :

$$k_t = k_{t-1} + \alpha'\Pi_{t-1}. \qquad (4)$$

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(1) Inputs are evaluated according to contemporary prices, so that the total amount of profit corresponds to its actual purchasing power when it is distributed. Similarly, in (4) and (7), k_{t-1} and K_{t-1}^j must be re-evaluated. Such a correction is usually done in empirical studies.

The remainder which is hoarded may be zero if :

$$\alpha + \alpha' = 1 \qquad (5)$$

or different from zero if :

$$\alpha + \alpha' < 1 \qquad (5')$$

A consequence of (5) is that the value of total inventories remains constant at prices which prevail onthe market at the time of the change :

$$\overleftarrow{S}_{t-1} \overrightarrow{p_{t-1}} = \overleftarrow{S}_t \overrightarrow{p_{t-1}} \qquad (6)$$

In this case we approach a type of Say's law in the macro-economy : total demand equals total income. This assumption applies throughout most of this article. In the second case (5'), the value of total inventories increases at the same rate as the total amount of capital. This growth is financed by the portion of profits $(1 - \alpha - \alpha')\Pi_{t-1}$.

8. Capital is allocated to the various activities in proportion to their profitability. In the period $t-1$ each production unit has capital in amounts K^j_{t-1}. The augmented capital in period t is distributed according to an increasing function of the rate of profit :

$$\frac{\dfrac{K^j_t}{K^j_{t-1}}}{f(r^j_{t-1})} = \mu_t, \qquad (7)$$

μ_t is a scalar independent of j, and is determined by the aggregate amount of capital available for production :

$$\sum_j K^j_t = k_t.$$

f is a strictly positive and increasing function of the rates of profit. This function is unique and thus identical for each activity. This identity expresses the indifference of capitalists to the particular line which valorises their capital.

9. If we limit ourselves to periods of normal accumulation enterprises can be assumed to use all of their capital. Given (4) it follows that [1] :

$$Y_t^j = \frac{K_t^j}{\bar{A}^j \vec{p}_{t-1} + L^j w_{t-1}} \qquad (9)$$

This equation is a specification of (1). From equations (7) and (9), we can obtain :

$$Y_t^j = \mu_t \, Y_{t-1}^j \, f(r_{t-1}^j) \qquad (10)$$

In equation (7) and (10) the rate of profit taken into account is an anticipated rate not the past rate at a previous period. The value of inventories do not play a role in its computation. When the amount of capital K_t^j is determined, it should be anticipated that it will be used (this is actually what equation (9) means), and further that the entire output will be sold [2]. If not, a smaller output and a smaller amount of capital would be allocated.

The future prices at which output is sold are ignored when output decisions are made. Prices depend on the level of demand manifested in the market (t-1/t). The rate of profit in (7) and (10) is therefore estimated by the following formula :

$$1 + r_{t-1}^j = \frac{p_{t-1}^j}{\bar{A}^j \overrightarrow{p_{t-1}} + L^j w_{t-1}} \qquad (11)$$

10. Wages are determined by the purchasing power of the hourly wage on $\overleftarrow{d}$ a bundle of commodities, whether the workers actually purchase this particular bundle or another one.

$$w_t = \overleftarrow{d} \, \vec{p}_t \qquad (12)$$

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(1) Except in model II-C.

(2) For reasons that have already been mentioned demand cannot be known in advance. Instead it is revealed on the market when the output is sold. Therefore, outputs must be decided on the basis of anticipations.

This purchasing power is assumed to be fixed over time, i.e. it is independent of the parameters t, $\overleftarrow{Y}$ and $\vec{p}$. Other specifications however are possible. For instance, workers could be allocated a constant share of the net product, i.e. a variable bundle of commodities :

$$\overleftarrow{d}_t = s\,\frac{\overleftarrow{Y}_t(I-A)}{\overleftarrow{Y}_t\,\vec{L}}$$

It is further assumed that wage earners spend their entire income $w_{t-1}\,\overleftarrow{Y}_t\,\vec{L}$.

11. Labor is always available.

12. In some models rationing among buyers is set aside. This means that for good j initial inventories are fixed so that total supply (inventories plus output) remains greater than demand for the succession of periods. In such models without rationing, S_t^j, the amount of inventories held by enterprise j, is given by :

$$S_t^j = S_{t-1}^j + Y_{t-1}^j - D_{t-1}^{(j)} \tag{13}$$

where $D^{(j)}$ is the demand facing enterprise j [1]. This inventory must be kept positive.
In models which allow rationing to occur, the same inventory S_t^j is given by :

$$S_t^j = \mathrm{Max}(0\ ,\ S_{t-1}^j + Y_{t-1}^j - D_{t-1}^{(j)}) \tag{13'}$$

and is positive by definition.

13. Total demand is a sum of three components :

a. Demand for production, $\overleftarrow{Y}_t\,A$;

b. Final demand of workers, $\overleftarrow{D}^w(w_{t-1}\,\vec{Y}_t\,\vec{L},\,\vec{\bar{p}}_{t-1})$;

c. Final demand from recipients of profit, $\overleftarrow{D}^k(\alpha\Pi_{t-1},\vec{\bar{p}}_{t-1})$.

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[1] One must distinguish $D^{(j)}$, demand facing enterprise j and D^j, the ith component of $\overleftarrow{D}$, the total demand for good i.

Both $\overleftarrow{D}^w$ and $\overleftarrow{D}^k$ are homogeneous of degree zero for both variables and homogeneous of degree one for income. No autonomous demand exists, therefore :

$$\overleftarrow{D} = \overleftarrow{Y}_t A + \overleftarrow{D}^w + \overleftarrow{D}^k \tag{14}$$

At the point of departure of the analysis there exists a set of variables (prices, outputs, and inventories) which prevail in the period t-1 (or in preceeding periods).The model which we elaborate must determine the values for these same variables in the next period. More precisely a recursive relationship must be built which can account for the evolution of the basic variables from one period to the next. The asymptotical behavior of this relationship must then be studied both from the point of view of its equilibrium values and stability in the neighborhood of equilibrium.

Two methods prove highly useful for such an analysis : analytical and numerical methods.Analytics studies the issue from a mathematical point of view and in this regard stability proves to be a much more difficult problem than equilibrium [1]. The scope of our presentation is restricted to the issue of local stability. Since the recursion is not linear it is necessary to study its linear development in the vicinity of equilibrium. This requires the formulation of the conditions for all the eigenvalues of the matrix which expresses the recursion to have a modulus smaller than one. Since the number of eigenvalues equals the number of variables the analysis grows in complexity as the number of variables increases. For this reason numerical methods are also important in the study of the model's properties. Using simulation analysis with the aid of a computer, properties of the model can be inspected visually and the size of the stability domain can be explored.

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(1) Hirsch - Smale : 1974

II - VARIOUS MODELS OF CONVERGENCE

In the five subsections which immediately follow it is assumed that each good is produced by a single producer. More specifically we consider models where two enterprises and two goods exist. It follows that only one price will prevail for each good and the demand faced by each enterprise j will be equal to the total demand for good j. These assumptions will be abandoned in subsection F.

A. A basic Model.

In this subsection the least complex convergence model is constructed by way of the following simplifications :

1. Demand is equal to total income $(\alpha + \alpha' = 1)$;
2. The function g^j is identical for both enterprises ;
3. No rationing exists.

Using equations (2) to (5) and (10) to (14), we can obtain a six variable recursion corresponding to the six components of the vectors $\vec{p}$, $\overleftarrow{Y}$ and $\overleftarrow{S}$. The number of variables can then be reduced to two :

$$x = \frac{p^2}{p^1} \quad \text{and} \quad y = \frac{Y^2}{Y^1}$$

with the condition of positivity of inventories :

$$\overleftarrow{S}_t \geq \overleftarrow{0}.$$

A model with these conditions allows for a complete analytical treatment (Duménil - Lévy : 1983 (a)).

Concerning equilibrium values the following results are obtained. Equilibrium prices are prices of production which satisfy the following equation :

$$\vec{p}^{\,*} = (1 + r^*) \, (A + \overleftarrow{L} \otimes \overleftarrow{d})\vec{p}^{\,*}.$$

The rate of profit, r^*, is uniform and capital movements are thus discontinued. x^* is the equilibrium value of x.

Under such conditions the economy moves along an homothetical growth path with a rate of growth : $\rho^* = \alpha' r^*$, and proportions of output y^*. All of these equilibrium values, x^*, r^*, y^*, and ρ^*, do not depend on the function f and g nor on any other of the initial parameters $\vec{p}_0$, $\overleftarrow{Y}_0$ or $\overleftarrow{S}_0$. When the economy reaches this balanced growth path inventories will be constant in absolute terms and their ratios to output will tend toward zero $(\frac{S^J}{Y^J} \to 0)$ (1).

In order to address the issue of stability we must develop in the vicinity of the equilibrium the non-linear recursion which governs the transformation of (x_{t-1}, y_{t-1}) to (x_t, y_t) (cf. Appendix C). To facilitate this task we define the following two variables :

$$\bar{x}_t = x_t - x^*$$

$$\bar{y}_t = y_t - y^*$$

The below recursion is then obtained :

$$\begin{bmatrix} \bar{x}_t \\ \bar{y}_t \end{bmatrix} = M \begin{bmatrix} \bar{x}_{t-1} \\ \bar{y}_{t-1} \end{bmatrix}$$

M is a two dimensional square matrix which is a function of β,γ,ω and θ, four parameters which summarize the workings of the economic system in the vicinity of equilibrium (cf. Appendix C). Two of these parameters, γ and β, account for the local behavior of the functions f and g. β models the reaction of prices to quantities produced while γ models the reaction of quantities to price changes (through the rates of profit). ω expresses the degree of substitution in demand as prices change and θ represents the difference between the two processes of production.

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(1) This is not true in the case of simple reproduction where this ratio tends toward some constant value.

Convergence can be guaranteed under two conditions. The first is that the product $\beta\gamma$ is in the region of one which insures acceptable dimensions for the indirect reactions of prices to prices and quantities to quantities. The second crucial condition is that $\gamma \simeq \frac{\omega}{2}$. The meaning of this condition is that the speed of capital movements must be proportioned to the intensity of substitution. If either of these conditions cannot be met the process will diverge either because of an excess or a lack in the intensity of reactions. When this double condition is satisfied prices will tend toward prices of production and quantities toward homothetical growth (inventories tend toward a finite limit) (1).

The analytical demonstration of stability is limited to the vicinity of equilibrium. However computer simulations have allowed further exploration concerning the size of the convergence region.

The determination of prices is more "robust" in a certain sense than that of outputs. Changes in technology or distribution disrupt all of the equilibrium values. Conversely changes in the rate of accumulation or in the size or proportions of capitalist consumption do not modify the equilibrium values of prices. If these parameters change repeatedly then prices will continue to gravitate around the same prices of production but outputs will drift from one homothetical path to another. However, the strict existence of prices of production requires the fine tuning of outputs to the homothetical path.

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(1) F. Hahn has directed our attention to the fact in models which take into account several capital goods and in which periods are linked to one another through clearing markets, the homothetical growth path will be a saddle point (and thus unstable). These models, however, greatly differ from the classical model where both a price and quantity mechanism exists and where stockpiling is possible. As for the Intertemporal General Equilibrium over an infinite horizon, it "converges" in some sense toward prices of production only because of perfect foresight and the intervention of the auctioneer (Duménil-Lévy : 1983 (b), Dana-Lévy : 1984).

The classics never associated homothetical growth to convergence toward prices of production. However, it is impossible to escape the fact that the very mechanism they describe leads to this conclusion. Once normal outputs (Marx's "social needs") are associated with normal prices, only one theoretical step remains before homothetical growth is accepted. It only needs to be assumed that reproduction continues on an extended scale with a constant rate of growth.

Another important difference between our model and the classical analysis is that our model is a model of convergence while the classics conceived of a process of gravitation. One way of obtaining a gravitation model would be to introduce stochastic variables. In such a model market prices would not converge toward prices of production but would converge only "in the average", i.e. gravitate around them. However, the construction of a model of convergence appears to be a prerequisite to the construction of a gravitation model.

As indicated above,the reaction of the capitalists to profit rate differentials and to changes in inventories in the vicinity of equilibrium have been represented in the model by the two parameters γ and β. It can be demonstrated that there exists particular values of these parameters $\tilde{\gamma}$ and $\tilde{\beta}$ for which the speed of convergence is at its maximum [1] (See Appendix C).

Fig. 1 reveals the set of values of γ and β around their optimal values for which convergence is guaranteed (zone 1).

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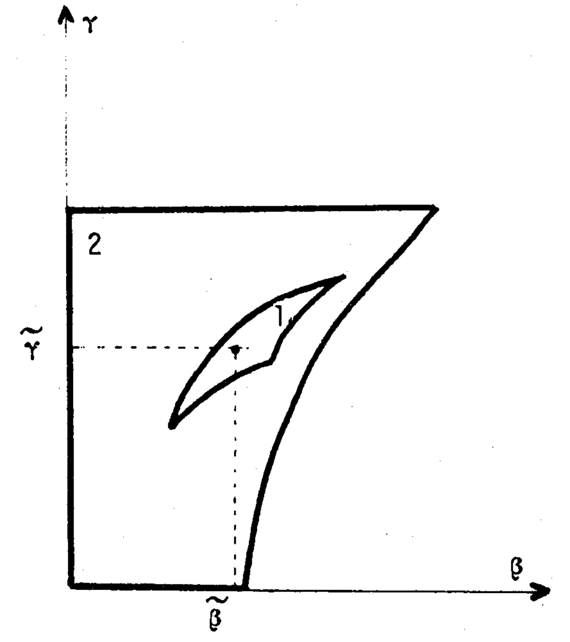
(1) For such values in the linear approximation of the model, convergence occurs in two periods since $(M)^2 = 0$.

Figure 1



β models the intensity of the change of prices as a result of disequilibrium of supply and demand. γ models the intensity of the reaction of capitalists to profitability differentials. In zone 1 convergence is insured. In zone 2 the convergence of prices and quantities is guaranteed but the change in the levels of inventories is not limited to finite numbers. Beyond zone 2 capitalists overreact and the model diverges.

For this set of values of γ and β, prices and quantities converge toward their equilibrium values while changes in inventories tend toward zero and inventories themselves tend to a finite limit. This last property ensures that the adjustment can he achieved provided that the initial inventories have been chosen at a sufficient level. If the positivity of inventories is not required then the conditions for the convergence (of prices and quantities exclusively) are considerably weakened. Convergence in this sense is ensured for values of γ and β which differ considerably from $\tilde{\gamma}$ and $\tilde{\beta}$. This is depicted in the second zone of figure 1. In this second zone convergence is verified for both prices and quantities but increasing changes occur for inventories.

The ratio of inventories to outputs does tend toward zero but the dimension of inventories has no upper or lower limit. It is impossible under such conditions even with sufficient initial inventories to complete the convergence process without some shortages [1]. Indeed a convergence model with rationing appears as a necessary and important elaboration of this basic model.

B. Competition with Rationing.

In a rationing model some economic agents may be unable to realize their purchasing plans because of the unavailability of certain goods. Money stocks must be accounted for, and the possible and the desired levels of production must be distinguished. The elaboration of such a model of convergence becomes very complex, and some simplifying assumptions are necessary. In order to avoid inventories in raw materials (goods purchased but not used in production), it will be assumed that the first good is strictly a production good, and that the second good is strictly a consumption good. It is further postulated that rationing is proportional (transactions are proportional to demands). As a result of these assumptions the proportions of possible and desired production are identical and the analysis of demand from final consumers can be approached globally (the demand and the monetary reserves of the different final consumers can be aggregated).

In spite of these simplifications the rationing model involves a recursion incorporating nine variables (eight components of four vectors and one scalar)

$\overleftarrow{Z}$: desired production

$\overleftarrow{Y}$: possible production

$\overleftarrow{S}$: unsold goods in inventories

$\vec{p}$: prices

N : stock of purchasing power held by final consumers.

---

(1) The discussion above is dependent on our choice of assumption (5), rather than (5').

These nine variables can be reduced to six : $p^2/p^1, Y^2/Y^1, N/Y^2p^2, S^1/Y^1, S^2/Y^2$ and $\overleftarrow{Z}\ \overrightarrow{L}/\overleftarrow{Y}\ \overrightarrow{L}$.

The analytical treatment of this model raises mathematical difficulties. Although the calculation of the six roots of the characteristic equation for the 6×6 matrix is complex, the real problem is posed by the presence of the "Max" and "Min" in the recursion which is thus not analytical (cf. Equation (13')). There are four analytical expressions corresponding to the excess supply or demand of each good. It is nevertheless possible to establish the conditions for convergence in each of the four zones. However to our knowledge a theorem of "assemblage" when the recursion involves more than two variables does not as yet exist [1].

Computer simulations reveal the following conclusions. The model with rationing does converge regardless of the origin of the rationing, either unavoidable deficiency of initial inventories (zone 2) [2] or the possible deficiency (zone 1). When convergence without rationing is possible and occurs (zone 1), it generates in equal periods greater outputs than in the rationing case. The asymptotical growth rate is the same, but a lag is created when rationing manifests itself.

The possibility of rationing considerably slackens the conditions for convergence. In particular, convergence occurs for small values of the parameters modelling capitalist functions. However overreactions, (γ, β beyond zone 2) still lead to divergences through progressively increasing oscillations.

C. Direct Control of Outputs.

Previously it was assumed that enterprises react to changes in inventories by price adjustments. It may also be possible that this adjustment is simultaneously made with regard to price and to quantity. That is, output is modified without waiting for the effect of price changes on profitability. This possibility may implicitely belongs at the very core of the classical conception.

---

(1) Laroque has demonstrated such a theorem only for differential equation with two variables (Laroque : 1981).

(2) However the convergence zone is not strictly identical to zone 2.

Two models can be considered. We can substitute for the function $f(r^j_{t-1})$ in equation (7) a similar equation with two arguments, $f(r^j_{t-1}, S^j_{t-1}/Y^j_{t-1})$ in which f is a decreasing function of the second variable. A second possibility is to replace equation (9) by :

$$Y^j_t = \frac{K^j_t}{\overleftarrow{A}^j \, \overrightarrow{p}_{t-1} + L^j w_{t-1}} \, h^j\left(\frac{S^j_{t-1}}{Y^j_{t-1}}\right)$$

with $h^j \leq 1$.

In the first instance changes in inventories becomes a factor in the allocation of capital. In the second case the unit of production scales down its level of activity when inventories are inflated.

The change in the dynamics of the model does not modify the equilibrium values. Again the same outcome is revealed. Capitalists must not overreact to economic signals for convergence to be insured, but the size of the convergence region is extended.

There is a common aspect in the direct control mechanism and rationing. Both are compatible with a centrally planned economy and not specific to capitalism. Both fulfill symmetrical functions. If shortages exist then the productive system is slowed. If involuntary stockpiling takes place then the level of activity is scaled down for its reabsorption. The specificity of the capitalist mode of production is the regulating roles played by the rate of profit and relative prices.

D. Convergence with a Constant Deficiency in Demand.

Say's law in the sense specified previously can be eliminated as a prerequisite for convergence. With $\alpha + \alpha' < 1$ convergence can still be obtained in the basic model for both prices and quantities. The homothetical proportions of output are modified in this case. Inventories increase at the same speed as production. Since the basic relation $\rho^* = \alpha' r^*$ holds, the smaller α' implies a diminished rate of growth.

E. Several Independent Capital Allocation Centers.

In the previous models all the accumulated capital is centralized by a unique institution (enterprise, bank, holding, investor) which allocates it among the various units of production according to their comparative profitabilities. The uniqueness of this capital allocation center is one of the fundamental simplifying assumptions of our basic model (II-A) which naturally entails the uniqueness of the function $f(r)$ and the parameters μ_t and α.

The reality of capitalism is that of a multiplicity of such centers.We therefore now intend to lift this assumption and propose a model which contains two independent centers.We retain the existence of two units of production. The two centers can invest in these two units of production that they jointly own. We suppose that the centers receive a share of the total profit of the period in proportion to their share of the property of each unit. Their investment behavior is identical to that of model II-A, but the functions $f(r)$, and the parameters μ and α are now specific. We introduce a new parameter $\eta^{j,\ell}$ which denotes the share of the unit of production j owned by center ℓ. The η's verify :

$$\sum_{\ell} \eta^{j,\ell} = 1.$$

Since the two centers do not necessarily have the same investment behavior with regard to α and $f(r)$, the η's may depend on t.

Let's begin the description of the general mechanism with the appropriation of profit by the units of production. Profit of unit j is :

$$\Pi^{j}_{t-1} = Y^{j}_{t-1}[p^{j}_{t-1} - (\overleftarrow{A}^{j}\,\vec{p}_{t-1} + L^{j}w_{t-1})]$$

This profit is then transmitted to the centers. Center ℓ receives :

$$\pi^{\ell}_{t-1} = \sum_{j} \eta^{j,\ell}_{t-1}\,\Pi^{j}_{t-1}$$

This profit is divided into two fractions :

- $\alpha^{\ell}\,\pi^{\ell}_{t-1}$ is distributed to consumers ;

- $(1-\alpha^{\ell})\,\pi^{\ell}_{t-1}$ is accumulated by center ℓ.

By k^{ℓ} we denote the total amount of capital held by center ℓ. Thus :

$$k_t^{\ell} = k_{t-1}^{\ell} + (1 - \alpha^{\ell})\, \pi_{t-1}^{\ell}$$

Center ℓ allocates its capital between units 1 and 2 : $K^{1,\ell}$ to unit 1, and $K^{2,\ell}$ to unit 2. This behavior is modelled by :

$$\frac{\dfrac{K_t^{j,\ell}}{K_{t-1}^{j,\ell}}}{f^{\ell}(r_{t-1}^{j})} = \mu_t^{\ell} \qquad \text{with } j = 1,2.$$

The μ_t^{ℓ} parameter is given by :

$$\sum_j K_t^{j,\ell} = k_t^{\ell}.$$

The total amount of capital allocated to unit j is :

$$K_t^{j} = \sum_{\ell} K_t^{j,\ell}.$$

It is used to produce good j at level Y_t^j (cf. Equations (9)).
The new values of the $\eta^{j,\ell}$ parameters are :

$$\eta_t^{j,\ell} = \frac{K_t^{j,\ell}}{K_t^{j}}.$$

In the numerical study we limit ourselves to the case of a uniform α. As a result of this assumption, no center is eliminated in the asymptotical position. Convergence is obtained for a set of parameters. Indeed our results show that convergence can be obtained even with a high degree of decentralization as is premised in the classical model.

F. Multiplicity of Producers and the Choice of Technology.

In the previous models only one process of production existed and a single price for each product. We will now consider multiple producers and multiple technologies thus allowing for the study of technical progress. Consequently, we extend the numbers of production processes to three by the introduction of an additional process producing product 2. Thus in the first activity a unique producer and seller of good 1 exists with output Y^1 ; A second production process, $j = 21$, produces Y^{21} units of good 2 and sets its own price ; the third process, $j = 22$, also produces the second commodity in quantity Y^{22} according to its own technology and establishes its price p^{22} . The determination of prices is modelled by equation (2) in which ΔS_t^j is given by equation (14), (recall that the g^j functions depend on j).

With two sellers of good 2 and two prices on the market, a strategic outcome function must be specified as indicated in I C 6. Such a function allocates total demand among the two sellers taking into account their price differences. If the two sellers set the same price then the quantities sold will be assumed to be proportional to the output (Proportional rationing). If such an assumption is not made then one seller receives more demand for no apparent reason. Various types of functions can model the situation of different prices as long as more demand is received for the lower price. However we find that our results do not depend on the particular form of the function chosen.

The model with different techniques has nine variables, three prices, three outputs, and three inventories. The equations are the same as in the basic model but the above mentioned strategic outcome function must be added. The inventories can be excluded and the introduction of reduced form variables leads to a recursion involving only four variables : $\frac{p^2}{p^1}$, $\frac{p^{22}}{p^{21}}$, $\frac{Y^2}{Y^1}$ and $\frac{Y^{22}}{Y^{21}}$, in which

$Y^2 = Y^{21} + Y^{22}$ denotes the total production of good 2, and

$p^2 = (Y^{21}\, p^{21} + Y^{22}\, p^{22})/Y^2$ denotes the average price of good 2.

Two states of technology exist according to the process considered for the production of good 2 :

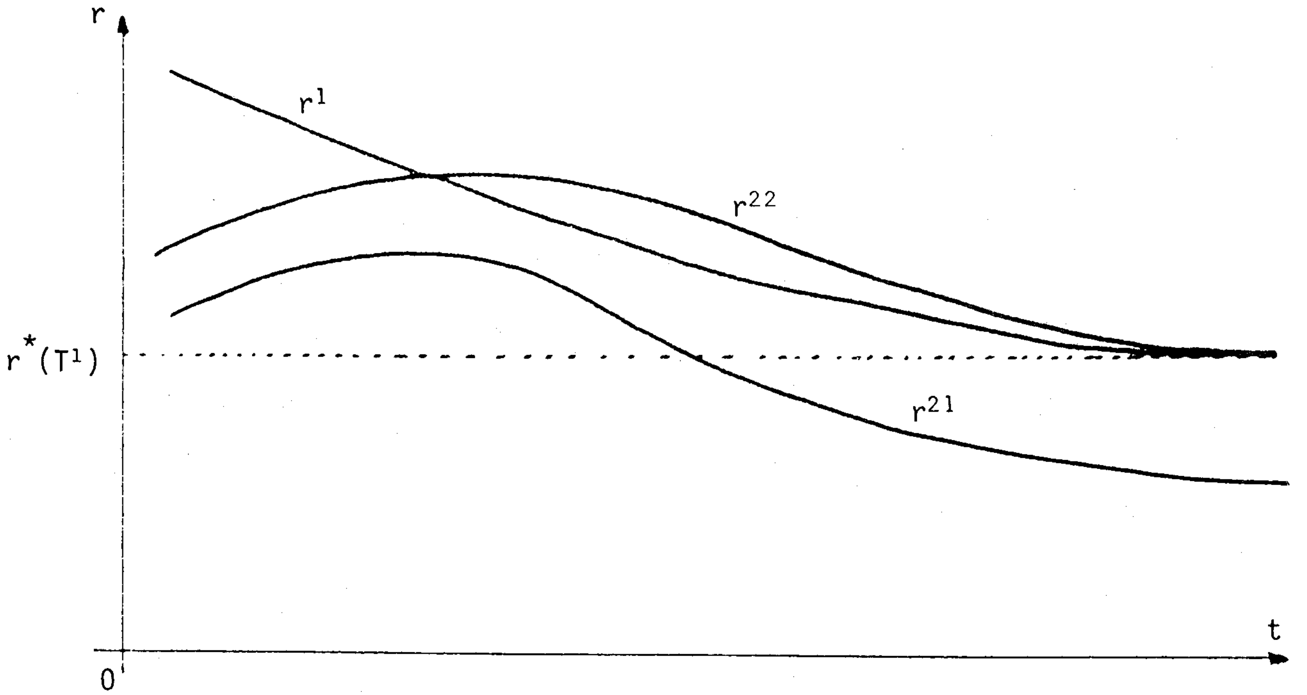
T^1 = (Process 1, Process 21)

T^2 = (Process 1, Process 22)

There corresponds to each of these technologies an equilibrium rate of profit ($r^*(T^1)$ and $r^*(T^2)$). The "dominating technology" is that technology with the higher rate of profit and the "dominated process" is the process of production for good 2 which is not used in the dominating technology.

Our simulation results show that prices and outputs in the multiple technique model converge asymptotically toward the equilibrium values of the dominating technology. The dominated process is eliminated (its level of activity tends toward zero).[1] This result is in conformity with the choice procedure described by P. Sraffa (Sraffa : 1961, ch. 13). The result holds even in cases of reswitching Figure 2 illustrates the patterns over time for the rates of profit. Indeed, one of the favorable qualities of classical dynamics is its capability to be generalized. A wide range of complexity can be built on the basic model of II-A.

Figure 2



(1) When $r^*(T^1) = r^*(T^2)$ the proportions of prices of production are identical (Pasinetti : 1977). Asymptotically, prices converge toward these common values and no process is eliminated. The particular case when process 21 and 22 are identical is of this type.

The investigation of the stability of this model has been carried out numerically. This choice of methodology is confirmed by the fact that the analytical approach has so far been limited to asymptotical behavior, when, as we shall see in part III, some of the most interesting results of this model evolve much earlier where the analytics are even more sophisticated.

III - THE STAGES OF THE COMPETITIVE PROCESS.

The previous sections have presented a series of models of capitalist competition based on the descriptive analysis of this process found in the classics. We have shown that these models do indeed lead to the convergence hypothesized by the classics. The conditions for convergence are rather strict in the first model. However the conditions can be slackened once the possibility of shortages is allowed for (II-B) as well as the possibility of"direct control" of outputs (II-C).

A. The Insights of the Classics.

In Smith and Ricardo we find a rather simple analysis of competition and it would be easy to claim that we fulfilled our obligations to their theoretical legacy. However this same confidence can not be extended to the works of Karl Marx. We reserve for a future study an explanation of how Marx preceived the results of the competitive process without the use of formalism. We will limit ourselves to one aspect of Marx's demonstration in Chapter 10 of Volume III of Capital : how the competitive process unfolds through several stages. He contends that :

> "What competition brings about, first of all in one sphere, is the establishment of a uniform market value and market price out of the various individual values of commodities. But it is only the competition of capital in different spheres that brings forth the production price that equalizes the rate of profit between those spheres" (Marx : 1863-67).

Here Marx identifies two separate processes at work : the formation of a unique price for each commodity and the establishment of a uniform rate of profit among industries. In the first sentence he mentions individual values of different magnitudes, indicating that he is discussing industries with multiple techniques. Thus when he refers to the equalization of rates of profit between industries in the second sentence he certainly means that the average rates of profit of industries are equalized. This study of the average return to each industry justifies the introduction of the concept of "market value" already briefly considered in volume I. Market value is the average of "individual values [1].

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(1) The concept of market value is at the origin of a number of ambiguities. Problems related to translation have worsened the situation. Consult in this regard Duménil : 1977 (p. 115-121).

In this third section we consider in more depth these two processes described by Marx, using the model with multiple techniques of section II-F. However, it is now necessary to study its pre-asymptotical stages toward equilibrium. At the end of this section we will consider the concept of "quasi-equilibrium" and try to summarize some of the previous results.

B. Two pre-Asymptotical Stages.

1. The formation of a unique price for each industry.

In order to distinguish carefully the problems of the determination of a unique price and that of multiple techniques it is necessary to treat the two processes referred to by the subscripts 21 and 22 as identical. In such a model the convergence unfolds in two stages. In the first the two prices set for commodity 2 by enterprises 21 and 22 quickly tend to uniformity and the ratio of the two levels of activity Y^{22}/Y^{21} tend to stabilize. We can denote p^2 as this unique price and $Y^2 = Y^{21} + Y^{22}$ as the total production of good 2.

In the second stage, the model continues on its asymptotical path as if only one firm was producing good 2.

It is evident that without the slightest modification classical dynamics leads to the first result considered.

2. Uniform average rates of profit by industries.

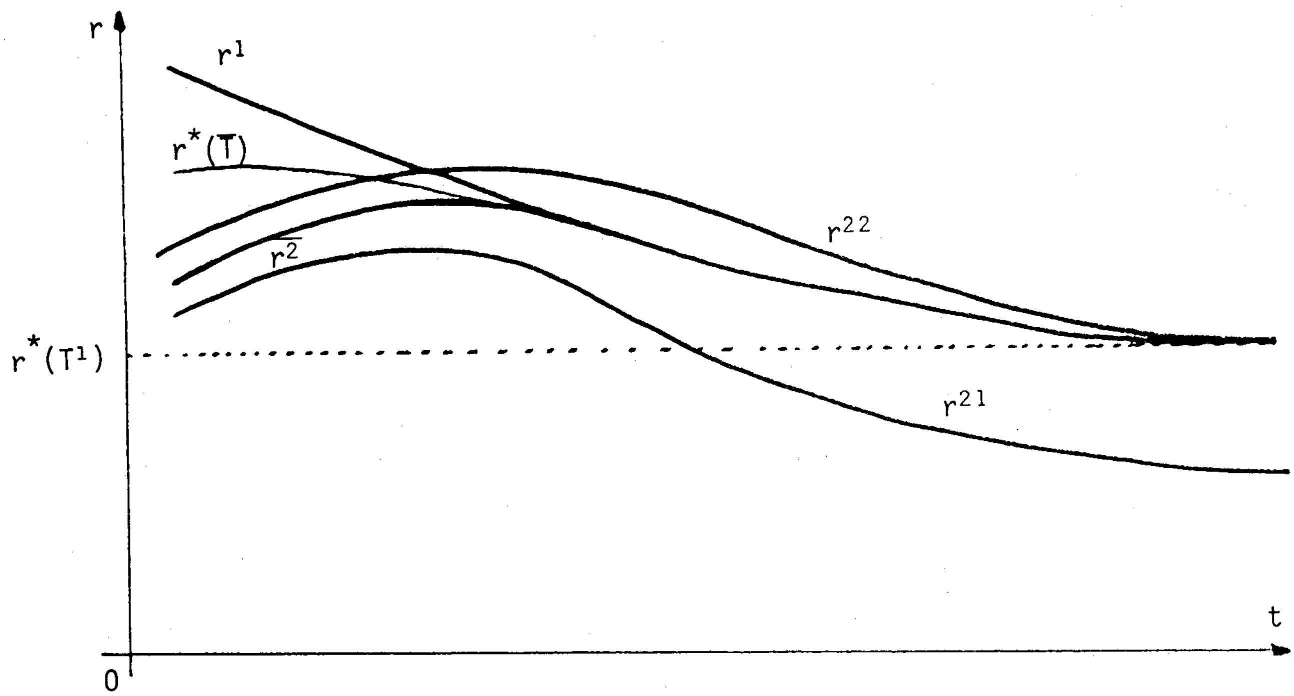
Let us now assume that processes 21 and 22 are distinct in the II-F model. We postulate that technology (1,21) is the dominant technology, while technology (1,22) is dominated. For simplicity we further postulate that a unique price exists for commodity 2. This is easily obtained by choosing the same function g for both 21 and 22 while beginning the convergence process with p^{21} equal to p^{22}. Since a unique price entails proportional rationing, this property is perpetuated along the course of the adjustment. At any point along the convergence path the average technology is defined as the average of the two technologies weighted by their respective levels of activity :

$$\bar{T} = \left(\text{Process } 1, \ \frac{Y^{21}(\text{Process } 21) + Y^{22}(\text{Process } 22)}{Y^{21} + Y^{22}} \right)$$

The equilibrium rate of profit for the average technology is then denoted $r^*(\bar{T})$. The average rate of profit for industry 1 remains r^1 while the average rate of profit for the second industry where two techniques prevail is now $\bar{r}^2$. Both rates are calculated according to the prevailing market prices.

Figure 3 reproduces the rates of profit r^1, r^{21}, and r^{22} previously exhibited in Figure 2. In the long run the two dominating processes 1 and 21 yield an equal rate of profit while the permanently inferior profitability of technique 22 is established. In the medium run, r^1 and $\bar{r}^2$ (the average rate of profit for industry 2) converge toward $r^*(\bar{T})$ (the rate of profit corresponding to the prices of production of the average technology).

Figure 3



Market prices are quickly adjusted to prices to production of the average technology. Then the average technology and prices of production drift toward their asymptotical values. Quantities also in the medium run adjust to the homothetical path of the average technology.

The two speeds of convergence are explained by the fact that the rates of profit are equalized by successive changes in relative prices. This adjustment process has a limited impact on processes producing the same good, whereas it has a more powerful effect on industries producing different commodities.

Indeed Marx's comments concerning the formation of an average rate of profit is very relevant here since Marx claimed that prices of production were prices which yielded an equal remuneration in multitechnic industries, independent of the generalization of an optimal technology !

3. The general case.

It is possible to combine the results obtained in the last two subsections. Enterprises 21 and 22 share the market for good 2 produced with two distinct processes. The two prices for this good are decided independently according to the difficulty of sales. A three stage process is set in motion. First a uniform price for good 2 is established. Then the price system is stabilized and industry profit rates converge to the social average while market prices converge to prices of production. However at this stage heterogeneous techniques and profit rates still prevail within the industries. Finally the dominated process of production is progressively abandoned.

Thus, the study of convergence cannot be limited to the asymptotical path. A global analysis leads to a process which unfolds through two pre-asymptotical stages whose properties are far from evident even in the simple models we have considered.

C. A Reinterpretation of Classical Dynamics

At the origin of the convergence process are four reduced form variables which are interacting (see II-F) : $\frac{p^2}{p^1}$, $\frac{p^{22}}{p^{11}}$, $\frac{Y^2}{Y^1}$ and $\frac{Y^{22}}{Y^{21}}$.

We abstract from the general level of prices and outputs, and disregard inventories. The above four variables evolve along the convergence path at different speeds. The variable $\frac{p^{22}}{p^{21}}$ moves at the greatest speed during the adjustment process while the other three can be regarded initially as constant.

When $\frac{p^{22}}{p^{21}}$ is close to one, only three variables remain significant :

$\frac{p^2}{p^1}, \frac{Y^2}{Y^1}, \frac{Y^{22}}{Y^{21}}$. During the second adjustment there is a quasi-convergence of prices

and outputs toward the equilibrium values for the average technology $\bar{T}$:

$$\frac{p^2}{p^1} \to \frac{p^{2*}}{p^1}(\bar{T}) \quad \text{and} \quad \frac{Y^2}{Y^1} \to \frac{Y^{2*}}{Y^1}(\bar{T}).$$

When this phase is concluded only one variable remains in motion : $\frac{Y^{22}}{Y^{21}}$, a variable which now determines the average technology. This variable slowly tends toward its asymptotical value of zero and the average technology tends toward the dominating technology. Thus, the classical analysis provides us with an aggregation procedure which could be developed in the direction of new micro-foundations for macro-economics.

Or basic model can be regarded as a model of Marx's prices of production with two simplifying assumptions. The first is that the initial convergence is completed and a unique price for each good exists. The second is that the last convergence is slow so that $\frac{Y^{22}}{Y^{21}}$ can be considered a constant parameter. Moreover the issue of inventories can be avoided. Recall that the rationing model can solve these difficulties and extend the range of acceptable values for the parameters modelling capitalist behavior [1]. Therefore the concept of prices of production "à la Marx" can be reinterpreted in the framework of our basic model with little loss of rigor.

D. Quasi-equilibrium : a new concept for a new approach.

From a methodological point of view an important result of our research may be a sounder understanting of these intermediate equilibrium states where prices of production exist along side multiple techniques. This notion of stable intermediate states, "quasi-equilibrium", equilibrated in some regards and disequilibrium in others (like under-equilibria in the theory of disequilibrium) may have a broad field of application. We believe that this concept could be important in

(1) The interesting application of models II-C and II-D lies in their use in the analysis of crisis periods whose study we leave aside for the present.

such diverse fields as the labor theory of value and what we have referred to as the micro-foundations of macro-economics. But such developments transcend the scope of the present work.

CONCLUSION.

By way of conclusion we reiterate some of the main results of our research :

1. It is indeed possible to construct a decentralized dynamic model of competition without an auctioneer, based on the classical perspective. It must be emphasized that in our model the determination of prices and the allocation of capital is performed in a decentralized fashion. No central institution determines in advance equilibrium values before the beginning of transactions and production :

- Each enterprise adjusts its own price according to the variations of its own inventory and according to an individual reaction function ;

- The same property characterizes the allocation of capital when several independent centers of allocation exist and react according to specific functions.

2. There exists a realistic set of parameters which model the behavior of economic agents and ensures convergence.

3. In models with multiple techniques for a single good a unique price is formed in the first stage of the adjustment process.

4. In models with multiple techniques,the second stage of convergence involves the formation of a uniform average rate of profit between industries the formation of prices of production in the sense described by Marx.

5. Eventually the dominated processes are eliminated.

6. The proportions of outputs produced are fixed into patterns which correspond either to the average technology (4.)or the optimal technology (5.).

7. In spite of the number of variations on the basic model (rationing, direct control, several capital allocation centers, etc.), this basic model displays the main features of capitalist competition as presented by the classics. Again, if we ignore the issue of inventories which can be handled in an appropriate rationing model, and consider the technology as the average technology, the basic model displays the fundamental properties of a system of prices of production. A complete analytical treatment can be made with a two variable recursion.

8. Finally the concept of "quasi-equilibrium" appears to be quite fruitful. The most interesting states which can be simulated with the help of modelling are not full-fledged equilibrium states (the asymptotical behavior of the model), but rather the states of limited stabilization, equilibrated in some regards, disequilibrated in others. Indeed, it is in the investigation of these sorts of states of the economy, but from a static approach, that some streams of contemporary economic thought seek the micro-foundations of macro-economics.

The dimension of the field which remains to be investigated is quite large. Even before macro applications can be made many areas remain to be explored : the basic justification of the behavioral functions, the application to crisis scenarios, the study of adjustment from initial situations far from equilibrium, the analysis of imperfections in competition, divergence, the gravitational model, and the list goes on !

APPENDIX A

Notations

Column vectors are identified by arrows pointing to the right $\rightarrow$; row vectors, with an arrow pointing to the left $\leftarrow$. A subscript is always an index of time. A superscript refers to a good (i) or an enterprise (j). For instance Y_t^j the jth component of vector $\overleftarrow{Y}$ represents the level of activity of a production unit j, at period t. A star* indicates that a variable is at its equilibrium value.

List of variables and paragraphs where they are introduced

I	Identity matrix.
A	Matrix of input coefficients (I-C-4).
$\vec{L}$	Quantities of incorporated labor (I-C-4).
$\overleftarrow{Y}$	Levels of activity (I-C-4).
ρ	Rate of growth (II-A-I).
$\overleftarrow{S}$	Magnitudes of inventories (I-C-5).
$\overleftarrow{\Delta S}$	Changes in inventories (I-C-5).
$\vec{p}$	Prices set by enterprises (I-C-5).
$\vec{\bar{p}}$	Average purchasing prices (I-C-6).
w	Hourly wages (I-C-13).
$\overleftarrow{d}$	Fixed basket determining the hourly wages (I-C-13).
r	Rate of profit (I-C-7).
Π	Total profit (I-C-7).
α, α'	Share of profit allocated to consumption, to accumulation (I.C.7).
k	Total capital (I-C-7).
$\overleftarrow{K}$	Capitals by enterprises (I-C-8).
μ	Scalar determining the general level of production (I-C-8).
$\overleftarrow{D}^w, \overleftarrow{D}^k$	Workers, capitalistsdemands (I-C-13).
$\overleftarrow{D}$	Total demand (I-C-13).
$f(r)$	Function modelling the allocation of capital (I-C-8).
$g(\frac{\Delta S}{Y})$	Function modelling the correction of prices (I-C-5).

When the number of goods is limited to two (for two enterprises), one can also define :

$x = \frac{p^2}{p^1}$ relative price (II-A).

$y = \frac{Y^2}{Y^1}$ proportion of outputs (II-A).

In the model with rationing, we need :

N Money stock held by consumers.

$\overleftarrow{Z}$ Levels of desired production.

In subsection II-E where two centers of allocation are considered the following notations are introduced :

ℓ	Superscript of the centers ($\ell = 1,2$).
Π^j	Profit of enterprise j.
π^ℓ	Profit distributed to center ℓ.
k^ℓ	Total capital owned by center ℓ.
$K^{j,\ell}$	Capital allocated to enterprise j by center ℓ.
$\eta^{j,\ell}$	Share of the total capital of enterprise j owned by center ℓ.
K^j	Total capital allocated to enterprise j.
$f^\ell(r)$	Function modelling the allocation of capital by center ℓ.

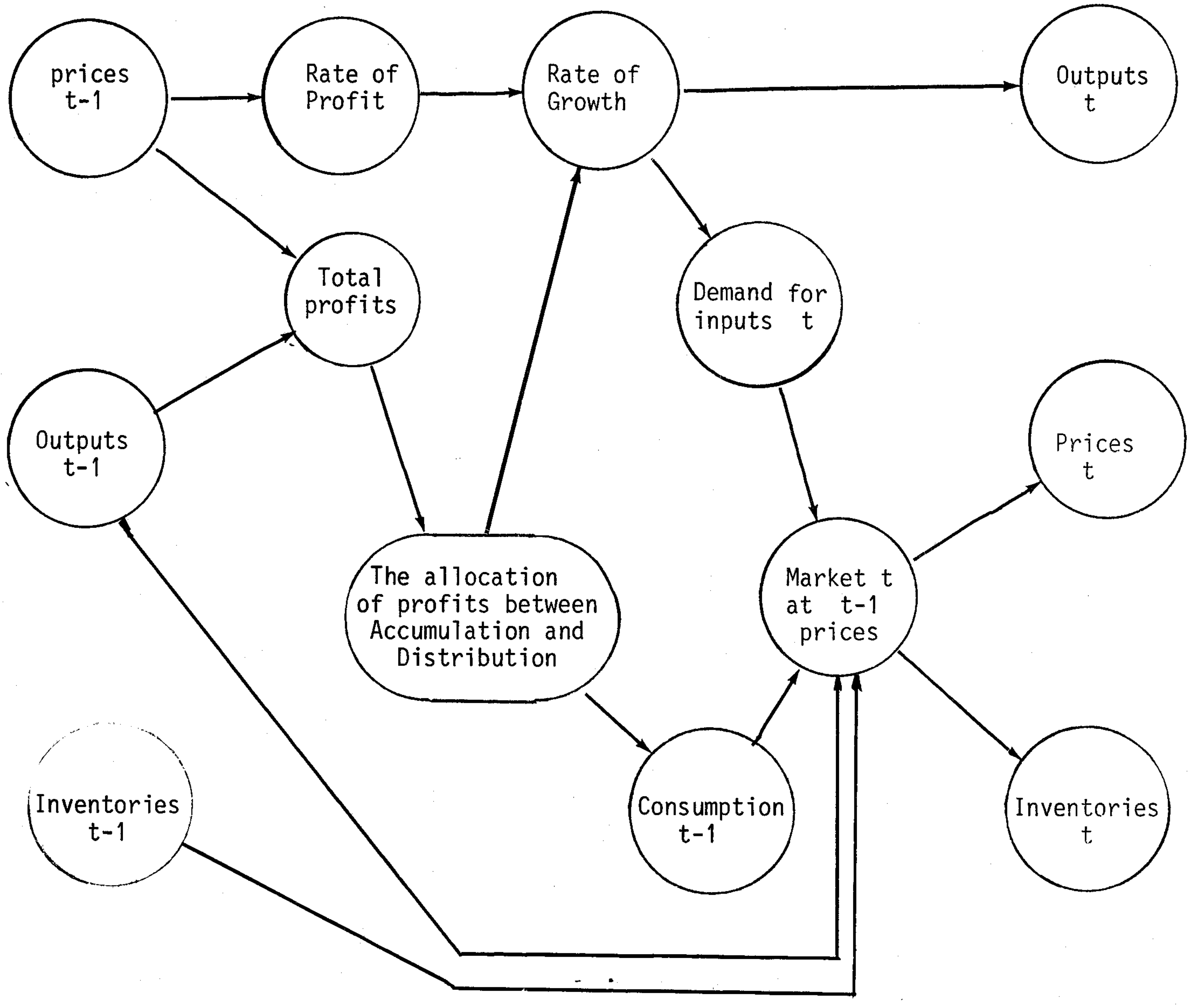
Variables $\bar{x},\bar{y},M,\omega,\theta,\beta$ and γ concern the analytical study of stability of the model II-A and are defined in the appendix C.

APPENDIX B

General structure of the models

Figure 4 presents a general diagram of the functioning of the various models. Figure 5 indicates, in detail, the chronological progression of events, in order to show that no central institution (auctioneer, invisible hand,...) is necessary.

Figure 4. *Diagram of the general functioning of the models*



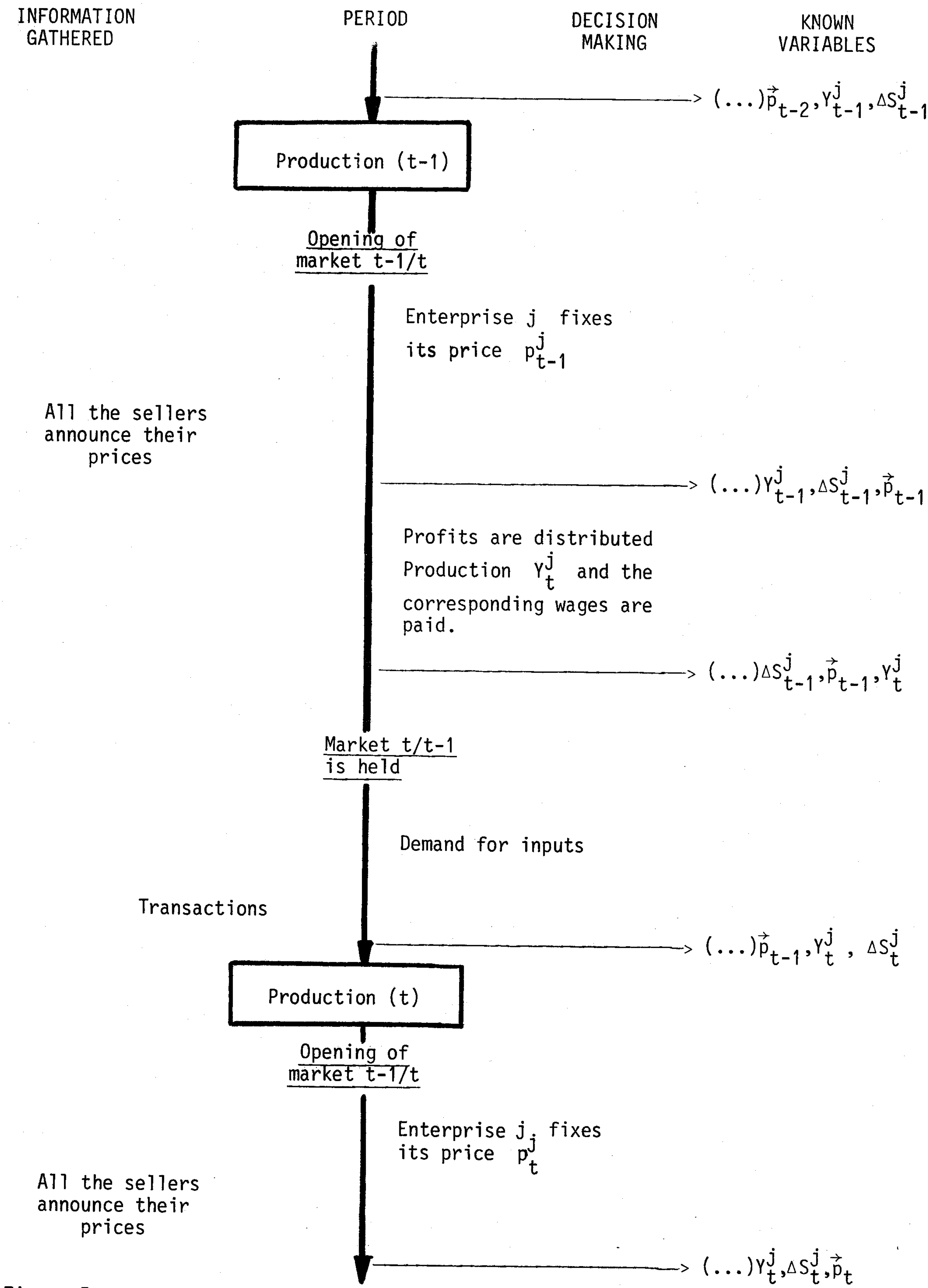


Figure 5. *Sequence of the activities of enterprise* j.

APPENDIX C

Definition and values of the variables taken into account in the analytical study of convergence in model II-A.

$\bar{x}$, $\bar{y}$ Distance of variables x and y to their equilibrium values

M Matrix of the linear approximation of the recursion

$$M = \begin{bmatrix} 1 + \beta(\gamma\theta - \omega) & -\beta(1-\theta)\frac{x^*}{y^*} \\ \gamma \frac{y^*}{x^*} & 1 \end{bmatrix}$$

ω Parameter characteristic of the degree of substitution in total demand when prices change

$$\omega = \frac{x^*}{y^*}(1-\alpha)\frac{1+r^*}{1+\rho^*}(y^* - (1+r^*)(a_1^2 + a_2^2\, y^*))\frac{d^2 - d^1 y^*}{d^1 + d^2 x^*}$$

$$- \alpha \frac{r^*}{1+r^*}\frac{x^*}{y^*}\frac{1 + x^* y^*}{1 + x^* \frac{D^{k,2}}{D^{k,1}}}\frac{d}{dx}\left(\frac{D^{k,2}}{D^{k,1}}\right)$$

$$- (1+\rho^*)\frac{x^*}{y^*}(L^1 + L^2 y^*)(1 + x^* y^*)\frac{(d^1)^2}{d^1 + d^2 x^*}\frac{d}{dx}\left(\frac{D^{w,2}}{D^{w,1}}\right)$$

θ Parameter characteristic of the technical difference between the two productive processes.

$$\theta = (1+r^*)(1+\rho^*)\det(A + \vec{L} \otimes \overleftarrow{d})$$

One can show that :

$$0 \leq \omega$$
$$-1 < \theta < 1.$$

β Absolute magnitude of the derivative of g at equilibrium

$$\beta = -g'(o)$$

γ Scalar proportional to the logarithmic derivative of f at equilibrium

$$\gamma = (1+r^*)(2 - (1+r^*)\ \mathrm{tr}(A + \vec{L} \otimes \overleftarrow{d}))\ \frac{f'(r^*)}{f(r^*)}$$

$\tilde{\beta}, \tilde{\gamma}$ Values of β, γ for which convergence occurs in the shortest time span :

$$\tilde{\beta} = \frac{1}{\omega}\,\frac{2-\theta}{1-\theta}$$

$$\tilde{\gamma} = \omega\,\frac{1}{2-\theta}$$

a^i_j Components of matrix $A + \vec{L} \otimes \overleftarrow{d}$

$$A + \vec{L} \otimes \overleftarrow{d} = \begin{pmatrix} a^1_1 & a^2_1 \\ a^1_2 & a^2_2 \end{pmatrix}$$

APPENDIX D

Examples of functions used in the numerical studies performed by computer

- *Reaction functions*

$$f(r) = (1+r)^{\gamma}$$

$$g^j(\frac{\Delta S}{Y}) = (1 - \frac{\Delta S}{Y})^{\beta^j}$$

Functions f and g^j are strictly positive. f is increasing, g^j is decreasing.

- *Consumption functions*

Consumption proportional to fixed bundles have been used in general :

$$\overleftarrow{D}^w = (\overleftarrow{Y}_t \overrightarrow{L})\overleftarrow{d}$$

$$\overleftarrow{D}^k = \frac{\alpha \; \pi_{t-1}}{\overleftarrow{d}^k \overrightarrow{p}_{t-1}} \overleftarrow{d}^k$$

We also considered functions allowing for a certain degree of substitution (controlled by parameter ε).

$$\overleftarrow{D}^k = \frac{\alpha \; \pi_{t-1}}{\overleftarrow{d}^k \overrightarrow{p}_{t-1} + 2\varepsilon \sqrt{p^1_{t-1} \; p^2_{t-1}}} (\overleftarrow{d}^k + \varepsilon(x^{\frac{1}{2}}_{t-1}, x^{-\frac{1}{2}}_{t-1}))$$

- *Strategic outcome functions*

Two enterprises are considered whose outputs are Y^1 and Y^2 and which set prices p^1 and p^2. The problem is to determine the demand facing each of them. This determination is done in several steps :

1. We first compute the market share of each enterprise. We denote σ the share of enterprise 1 (the share of enterprise 2 is $1-\sigma$), x the relative price, $x = \frac{p^2}{p^1}$, and y the proportion of outputs, $y = \frac{Y^2}{Y^1}$. The following function can now be defined :

$$\sigma(x,y) = \frac{1}{2}\left(1 + \left(\frac{1}{1+y}\right)^{x^{\phi}} - \left(\frac{y}{1+y}\right)^{x^{-\phi}}\right)$$

σ is an increasing function of x, a decreasing function of y and is always limited to the $[0,1]$ interval. If the two prices are equal, we have :

$$\sigma(1,y) = \frac{1}{1+y} = \frac{Y^1}{Y^1+Y^2}$$

Put differently σ, enterprise 1's share of market, is equal to its share in total output $Y^1/(Y^1 + Y^2)$: rationing is proportional.

2. The average price $\bar{p}$ is obtained by summing the prices weighted by the market shares of each seller :

$$\bar{p} = \sigma p^1 + (1-\sigma)p^2$$

3. Total demand is computed (cf. I-C-13).

4. Demand facing a single unit of production is the product of the total demand and the market share of the unit.

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