CLASSICAL AND KEYNESIAN UNEMPLOYMENT IN THE IS-LM MODEL

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The recent literature on equilibrium with quantity rationing has cast a new light on the origins and the cures of unemployment. By reconsidering the traditional Keynesian elementary multiplier model, this approach has in particular brought to the forefront of the analysis the important distinction between Keynesian unemployment, which results from a lack of demand, and Classical unemployment, which comes from too high a real wage (the first attempts to model these different sorts of unemployment are due to Solow and Stiglitz (1968), Barro and Grossman (1971, 1976), Younès (1970, 1975), Benassy (1973, 1975, 1977, 1982a). The term Classical unemployment has been coined by Malinvaud (1977) in his thorough study of a similar model).

Although there has been some work towards incorporating investment and/or a bond market into the analysis (Benassy (1982b), Danthine and Peytrignet (1980), Fourgeaud, Lenclud and Michel (1981), Gelpi and Younès (1977), Hool (1980), Malinvaud (1978, 1980), Neary and Stiglitz (1979), Sneessens (1981), Varian (1977)), a comparable study of the implications of this approach in the traditional framework of the Keynesian IS-LM model, is still needed. The present paper is a preliminary report on a research currently going on, that attempts to make progress in that direction.

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1. THE INSTITUTIONAL SET-UP.

The purpose of this section is to make precise the framework and the notations of the model.

We consider an economy within a single period — the "current" period —, which involves four "commodities". A (nonstorable) *good* which may be used in current consumption, or may be aggregated to the existing capital stock to increase productive capacity for the next periods (investment). The current money price of this good is p . *Labour* is assumed to be homogeneous, the current nominal wage rate being denoted w . *Money* (fiat money and short term deposits), whish is taken as the numéraire, is assumed — to simplify — to bear no interest. There are in addition *bonds* or perpetuities. A unit of bond is a claim to one unit of money in each and every period. The current money price of bonds is q ; the current nominal interest rate r is then, by definition, the reciprocal of the price of bonds, $r = 1/q$.

Three sectors are distinguished in the model : the productive sector, the households' sector, and the Government.

The *productive sector* will be represented by a single firm. It will be convenient, however, to view the firm as composed of three departments. The production department decides the level of output $Y \geqq 0$, and the amount of labour employed $E \geqq 0$, subject to a production function, $Y = F(E)$, and to the quantitative constraints that is perceives on its good supply, $Y \leqq \bar{Y}$, as well as on its demand for labour, $E \leqq \bar{E}$. The investment department decides how much to invest, i.e. the quantity $I \geqq 0$ that it demands on the good market in order to increase — for the future —

the firm's existing productive capacity. This department may face a constraint on its demand on the good market, $I \leqq \bar{I}$. Finally, there is a finance department. It receives the current profit of the production department, $pY - wE$, and borrows by selling on the bond market the amount $\Delta B^S \geqq 0$ of perpetuities, at the price $q = 1/r$. By contrast with the other markets, we shall assume that the bond market is always cleared by movements of the current nominal rate r , so that there are no quantitative constraints on that market. The firm is assumed to hold no money. On the other hand, the finance department pays interest to the holders of the stock of perpetuities that it has issued in the past, say $B_{of} \geqq 0$ (Note that $B_f^S = B_{of} + \Delta B_f^S$ is then the stock of perpetuities that the firm supplies in the current period). Finally, it pays dividends to the shareholders, say D , finances the Investment Department's expenditure on the good market, pI , and pays taxes T_f to the Government [1]. Of course, expenditures and receipts of the Finance Department must balance :

$$B_{of} + pI + D + T_f = pY - wE + (\Delta B_f^S/r) \qquad (1.1)$$

We shall consider two classes of *households*, i.e. workers and "rentiers" with index a . This particular assumption is made to take into account differences in propensities to consume out of wages and of profits, while keeping the model simple. Workers will be represented by a single household, with $a = 1$. The worker must choose in the current period his

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[1] Note that dividends are not restricted to be nonnegative. This — admittedly unrealistic — assumption is made, for simplicity, to avoid the problems associated with the firm's insolvency.

supply of labour L, his consumption C_1, his demand for money and for bonds M_1^d and B_1^d, all these variables being nonnegative. The labour supply cannot exceed $L^* > 0$. The worker owns at the outset of the period the quantities $M_{o1} > 0$ and $B_{o1} > 0$ of money and of bonds. The worker's current income is composed of his wage income wL and of the share $\theta_1 D$ of the firm's dividends ($\theta_1 \geqq 0$). He pays the taxes T_1 to the Government. The worker's current budget constraint reads accordingly

$$pC_1 + M_1^d + (B_1^d/r) = wL + \theta_1 D + M_{o1} + (1 + \frac{1}{r}) B_{o1} - T_1 \qquad (1.2)$$

We assume that labour involves no disutility. The worker's problem is then to maximize the utility of current consumption and the (expected) utility of his future consumption, subject to the current budget constraint (1.2), to the physical constraint $L \leqq L^*$, to the quantitative constraints he may perceive on the labour and the good markets, $L \leqq \bar{L}$ and $C_1 \leqq \bar{C}_1$, and subject to the anticipations he has concerning these constraints for the future.

"Rentiers" will be represented by a single household too, with $a = 2$. The rentier's problem is the same as the worker's with the only difference that he does not work, i.e. $L = 0$ (of course, one assumes $\theta_1 + \theta_2 = 1$).

We shall denote by C, M^d, B^d, the households' aggregate demands for consumption, for money and for bonds. By virtue of (1.2), and of its equivalent for the rentier, one has

$$pC + M^d + (B^d/r) = wL + D + M_o + (1 + \frac{1}{r}) B_o - T_h \qquad (1.3)$$

where M_o and B_o are the households' aggregate initial stocks of money and of perpetuities, while $T_h = T_1 + T_2$.

Finally, the *Government* will be assumed to have a target demand for the good $G^d > 0$. The Government's actual purchase of the good, G , will be the minimum of G^d and of the constraint $\bar{G}$ he may face on its demand on that market. The Government finances its expenditure pG by levying the taxes $T = T_f + T_h$, by issuing the quantity ΔB_g^s of bonds and by creating the quantity ΔM of money. Lastly, the Government pays interest to the holders of the bonds that he has issued in the past, say B_{og}(remark that $B_g^s = B_{og} + \Delta B_g^s$ is the stock of bonds that is supplied by the Government in the current period) [2]. The Government's budget constraint will read

$$pG + B_{og} = \Delta M + T + (\Delta B_g^s/r) \tag{1.4}$$

If we denote by $M = M_o + \Delta M > 0$ the final stock of money in the current period, equilibrium of demand and supply on every market will require :

(good) $$C + I + G = Y \tag{1.5}$$

(labour) $$E = L \tag{1.6}$$

(Money) $$M^d = M \tag{1.7}$$

(Bonds) $$B^d = B_f^s + B_g^s \tag{1.8}$$

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[2] We assume of course $B_o = B_{of} + B_{og}$. This means simply that the bond market has been equilibrated in the past.

In what follows, we shall assume that p and w are temporarily fixed at the outset of the period. The Government's target demand G^d , the level of its taxes T as well as its repartition T_f , T_1 , T_2 , the amount of money ΔM created (and thus the final money stock M) will be taken as exogenous. As a counterpart, the nominal interest rate r will be free to vary.

Equilibrium of the good and the labour markets at the fixed configuration (p,w) will be achieved as usual by quantity rationing, and we shall assume that only the long side of the market is rationed. The foregoing equations will serve to determine the equilibrium values of the interest rate r and of the quantitative constraints that the agents face on the various markets. As it is well known, the agents' budget constraints (1.1), (1.3) and (1.4) imply that the foregoing system satisfies *Walras Law*. One may delete accordingly one of the equations. We shall follow the standard practice in macroeconomic textbooks, by ignoring the bond market, and shall focus attention on the equations for the good, for labour, and for money. [3]

There are four possible equilibrium situations, depending on the type of disequilibrium that obtains on the good and the labour market. We shall use the now traditional terminology. *Keynesian unemployment* will stand for a situation where excess supply prevails on these two markets.

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[3] Since there are several demanders on the good market, one should complete the model by specifying a rationing scheme on that market. We shall be more specific about this point later on.

Classical unemployment will designate the case where there is an excess supply of labour, while there is an excess demand on the good market. *Repressed Inflation* will mean that there is an excess demand on both markets. *Under Consumption* will stand for a situation where there is an excess demand for labour but an excess supply of the good. In what follows, we shall focus attention on the Keynesian and the Classical Unemployment regimes.

2. BEHAVIOURAL ASSUMPTIONS.

Before analysing the different equilibrium regimes that may arise in the present model, it is useful to introduce the analogue, in our framework, of the textbook consumption and investment functions, and of the demand for money.

The Firm

Let us first consider the behaviour of the firm. Given (p,w), its production department seeks to maximize the short run profit $pY - wE$ subject to the production function $Y = F(E)$. We shall assume that F has all good "neoclassical" properties :

(a) *F is increasing, strictly concave, continuously differentiable, with* $F(0) = 0$. *The marginal productivity of labour* $F'(E)$ *decreases from* $+\infty$ *to* 0 *when* E *varies from* 0 *to* $+\infty$.

If there were no other constraint than the production function, the production department would choose the output level such that $\frac{w}{p} = F'(E)$. We shall denote $Y(\frac{w}{p})$ this optimum production level, and $E(\frac{w}{p})$ the corresponding demand for labour. In fact the production department faces constraints on the good and on the labour markets $Y \leqq \bar{Y}$ and $E \leqq \bar{E}$. The optimum output level will be thus the minimum of $Y(\frac{w}{p})$, of $\bar{Y}$ and of $F(\bar{E})$.

As for the finance department, we have to make precise how are determined dividends and the firm's issue of perpetuities. To keep things simple, we shall assume that investment is financed by an issue of bonds

$$pI = (\Delta B_f^s/r) \qquad (2.1)$$

In view of the firm's budget constraint (1.1), this implies that dividends are given by

$$D = (pY - wE) - B_{of} - T_f \qquad (2.2)$$

Remark again that dividends may be negative : the part of interest payments and of taxes that are not covered by current profits is passed on to the shareholders.

To conclude with the firm, we must specify how investment is chosen. Microeconomic theories of investment determination[4] teach us that the firm's desired level of investment is a function of the aggregate effective demand, of the "real" interest rate and of the real wage that are anticipated for the future periods. The desired investment level should then depend on the signals perceived by the firm in the current period, through their influence on expectations : the current price

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[4] See, e.g. Malinvaud (1980).

system (p , w , r) , the current level of effective aggregate demand $\bar{Y}$, the quantitative rations $\bar{E}$, $\bar{I}$, and the firm's information about the Government's current policy. We shall assume, for simplicity, that the ration $\bar{I}$ does not influence the firm's expectations, and thus investment behaviour. On the other hand, it should be clear that in equilibrium, the ration $\bar{E}$ will be always equal to the maximum labour supply L^* . Its influence can be therefore omitted. If we keep implicit investment's dependence on the Government's policy parameters, we can write the desired level of investment as $I(\bar{Y} , p , w , r)$. Actual investment I will be then the minimum of $I(\bar{Y} , p , w , r)$ and of the ration $\bar{I}$ that is perceived by the investment department on the good market.

One may think that expected aggregate demand is positively associated with the current constraint on sales $\bar{Y}$. Thus desired investment should increase with $\bar{Y}$. Specifically, we shall assume

(b) $I(\bar{Y} , p , w , r)$ *is differentiable, and the marginal propensity to invest* $\frac{\partial I}{\partial \bar{Y}}$ *is such that* $0 \leqq \frac{\partial I}{\partial \bar{Y}} \leqq \alpha < 1$.

On the other hand, an increase of the nominal rate r raises the cost of investment and should reduce it accordingly (in so far, however, as the increase of r does not induce, say, an increase of the firm's expected rate of inflation that would make the "real" interest rate go down). We shall postulate accordingly

(c) $I(\bar{Y} , p , w , r)$ *is a decreasing function of the nominal interest rate* r .

Finally, the consequences of an increase of w or p are ambigous. For instance an increase of w alone is likely to raise the real wage that is expected by the firm. This reduces on the one hand the (expected) profitability of investment. But on the other hand, that may raise the demand for investment by substitution between capital and labour. The effect of increase of p is likewise ambiguous for the same reasons. However, a proportional increase of p and w should lead to a reduction of the demand for investment, if we admit that such a change does not alter significantly the firm's expect real wage, nor the nominal levels of the expected price and wage.

Households

We wish now to make precise the households' behaviour. We shall take here as our primitive concept each household's *effective demand*, that is, the demand he would express on the good market, ignoring the quantitative constraint $\bar{C}_a$ he may face on that market, but taking into account the ration $\bar{L}$ that he may face on his labour supply. If one considers an household's intertemporal decision problem, all the signals perceived by an household should influence his effective demand — in particular through their impact on the household's expectations. In order to keep the model simple, and to facilitate comparisons with standard macroeconomic models, we shall write each household's effective demand as a function of the current price p , the nominal interest rate r , and of his current real wealth R_a , say $C_a(p , r , R_a)$, (a = 1,2) , with

$$pR_1 = wL + \theta_1 D + M_{o1} + (1 + \frac{1}{r}) B_{o1} - T_1 \quad ,$$

L being the worker's labour supply, i.e., the minimum of L^* and of the ration $\bar{L}$, and

$$pR_2 = \theta_2 D + M_{o2} + (1 + \frac{1}{r}) B_{o2} - T_2$$

Each household's corresponding demand for money will depend on the same variables and will be denoted $M_a^d(p , r , R_a)$, $a = 1,2$. [5]

Remark. Dividends may be negative, as we already noted, but whenever the firm's profit $pY - wE$ is non negative, they are greater than or equal to $-(B_{of} + T_f)$. We shall henceforth assume $M_{oa} + B_{oa} > T_a + \theta_a(B_{of} + T_f)$, $a = 1,2$, to keep each household's current money wealth positive.

Actual demands will of course take into account the rations $\bar{C}_a$. Indeed each household's actual demand for the good C_a will be the minimum of $\bar{C}_a$ and of $C_a(p , r , R_a)$. On the other hand, each household's demand for money M_a^d , *taking into account now the constraint* $\bar{C}_a$, will depend on that variable. Let us denote it $M_a^d(p , r , R_a , \bar{C}_a)$. Of course, one should have

$$M_a^d(p , r , R_a , \bar{C}_a) = M_a^d(p , r , R_a) \quad ,$$

when the ration $\bar{C}_a$ is not binding, i.e. when $\bar{C}_a \geqq C_a(p , r , R_a)$. When this ration is binding ($\bar{C}_a < C_a(p , r , R_a)$) , one should expect

$$M_a^d(p , r , R_a , \bar{C}_a) \geqq M_a^d(p , r , R_a)$$

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[5] Strictly speaking, this formulation is valid only if a variation of the signals currently perceived by the agent (others than p and r) leaves unaltered his expectations.

In that case, there are "forced savings", and that should increase both the demands for money and for bonds [6]. As a matter of fact, this argument shows that one should expect $M_a^d(p , r , R_a , \bar{C}_a)$ to be a decreasing function of $\bar{C}_a$ whenever this constraint is binding.

We shall need in the sequel the following assumptions, for each $a = 1,2$:

(d) *(Properties of the function* $C_a(p , r , R_a)$*)* .

(i) The function C_a *is continuously differentiable, increases with* R_a *, and tends to* $+\infty$ *when* R_a *goes to infinity. The marginal propensity to consume is such that* $0 < \frac{\partial C_a}{\partial R_a} \leqq \beta < 1$ *, with* $\alpha + \beta < 1$ *, where* α *is the upper bound of the marginal propensity to invest given in (b)* .

(ii) C_a *is a decreasing function of* r *and of* p .

The first part of this assumption needs almost no comment. The condition $\alpha + \beta < 1$ will ensure that the sum of the marginal propensities to consume and to invest out of current real income is less than unity, a condition which is common to most keynesian macroeconomic models. [7] On the other hand, an increase of r , current real wealth being fixed, should make savings more attractive and thus should reduce current consumption (this supposes, as the reader will easily check, some inelasticity

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[6] Demands for bonds are obtained immediately from the households' budget constraints.

[7] A well known exception is Kaldor's model (1940).

of the households' price and interest rate expectations). Finally, if an increase of p does not affect significantly the household's expectations, it must reduce his expected rate of inflation, and thus increase the "real" yield on money and perpetuities. This should, again, decrease current consumption.

(e) *(Properties of the functions* $M_a^d(p , r , R_a)$ *and* $M_a^d(p , r , R_a , \bar{C}_a)$*).*

(i) One has $M_a^d(p , r , R_a , \bar{C}_a) = M^d(p , r , R_a)$ *whenever* $\bar{C}_a \geqq C_a(p , r , R_a)$, *and* $M_a^d(p , r , R_a , \bar{C}_a) \geqq M_a^d(p , r , R_a)$ *when* $\bar{C}_a < C_a(p , r , R_a)$. $M_a^d(p , r , R_a , \bar{C}_a)$ *is a non-increasing function of* $\bar{C}_a$ *when* $\bar{C}_a < C_a(p , r , R_a)$.

(ii) Both functions are continuously differentiable. They increase with R_a *and tend to* $+\infty$ *when* R_a *goes to infinity. They decrease with the nominal interest rate, and tend to* 0 *as* r *goes to infinity. They both increase with the current price* p .

The first part *(i)* of this assumption was already discussed previously. As for *(ii)*, its rationale comes essentially as before, from the implicit assumption that variations of p , r , R_a do not influence significantly expectations. A particular comment is in order about the assumption that the demands for money decrease with r . A rise of the nominal interest rate, if it is does not affect much expectations, makes savings in bonds more attractive. That should reduce the current demand for consumption, as we already noted. In that case both demands for perpetuities and for money should increase (at least when the ration

$\bar{C}_a$ is not binding). But an increase of r makes at the same time bonds more attractive relatively to money, and should accordingly generate a substitution between the two assets. The above assumption means simply that this substitution effect is predominant. Furthermore, if the nominal rate r is large, nobody should be willing to hold money.

Finally, an increase of p which does not influence much expectations, should generate, current real wealth being unchanged, an intertemporal substitution effect which should reduce current consumption demand and raise savings — in particular the demand for real balances (the households' expected rate of inflation goes down) and, *a fortiori*, that for nominal balances.

Keynesian Consumption and Investment Functions

We introduce now a few concepts that will be useful in the sequel, and that should make the connection between our framework and the traditional Keynesian apparatus more apparent.

Given p , w , r , consider an output level $Y \geqq 0$ which does not exceed full employment output $Y^* = F(L^*)$ nor the profit maximizing output $Y(\frac{w}{p})$. To this production level is associated a demand for labour $E = F^{-1}(Y) \leqq L^*$. If we assume that employment L adjusts — through rationing — to this level, the households' current real wealths R_a are given by

$$pR_1 = wF^{-1}(Y) + \theta_1(pY - wF^{-1}(Y) - B_{of} - T_f) + M_{o1} + (1 + \frac{1}{r}) B_{o1} - T_1 ,$$

$$pR_2 = \theta_2(pY - wF^{-1}(Y) - B_{of} - T_f) + M_{o2} + (1 + \frac{1}{r}) B_{o2} - T_2 .$$

(Remark that the assumptions we made imply that both R_1 and R_2 are positive as long as Y does not exceed $Y(\frac{w}{p})$) . Reporting these values into the demands $C_a(p , r , R_a)$ and $M_a^d(p , r , R_a)$ and summing over the two households yields two functions which depend on Y , p , w and r (and on taxes, which will be left implicit for convenience), and which are very much alike the consumption function and the demand for money that can be found in the traditional Keynesian model. We shall note them $C(Y , p , w , r)$ and $M^d(Y , p , w , r)$, respectively.

The assumptions that we made imply immediately a few properties of these functions, which we shall review presently.

First, when Y increases while remaining less than or equal to $Y(\frac{w}{p})$, real wage income and real profit both increase. Each household's current real wealth R_a thus rises. By assumptions *(d)* and *(e)*, $C(Y , p , w , r)$ and $M^d(Y , p , w , r)$ increase with the level of production Y . It is further easy to verify that $\frac{\partial C}{\partial Y} \leqq \beta < 1$.

Let us turn now to the consequence of a variation of the nominal interest rate. An increase of r decreases the values of the households' initial holdings of bonds B_{oa}/r , and thus their current real wealth R_a . On account of *(d)* and *(e)* , therefore, both $C(Y , p , w , r)$ and $M^d(Y , p , w , r)$ are decreasing functions of r , and they tend to $+ \infty$ as r goes to 0 . Furthermore $M^d(Y , p , w , r)$ tends to 0 as the nominal interest rate increases indefinitely.

The only effect of a rise of the nominal wage rate w is to increase the worker's current real wealth R_1 and to decrease the "rentier"'s current real wealth by the same amounts. We shall assume that the marginal propensity to consume of the worker is greater than the "rentier's". We shall postulate accordingly that $C(Y , p , w , r)$ is an increasing function of w , and that $M^d(Y , p , w , r)$ is, on the contrary, decreasing with that variable.

A rise of the current price p decreases the real value of the expression $M_{oa} + (1 + \frac{1}{r}) B_{oa} - T_a - \theta_a(B_{of} + T_f)$, which is positive by assumption. On the other hand, it decreases $\frac{w}{p} F^{-1}(Y) + \theta_1(Y - \frac{w}{p} F^{-1}(Y))$, while it increases $\theta_2(Y - \frac{w}{p} F^{-1}(Y))$ of the same amount. On balance, since we assume the worker's marginal propensity to save to be greater than the rentier's, $C(Y , p , w , r)$ is a decreasing function of p . A similar argument would show that $M^d(Y , p\ w , r)$ should be an increasing function of p . The reader will check that a proportional increase of p and w generates the same effects. To sum up, we have with obvious notation

$$C(\overset{+}{Y} , \overset{-}{p} , \overset{+}{w} , \overset{-}{r}) \quad \text{and} \quad M^d(\overset{+}{Y} , \overset{+}{p} , \overset{-}{w} , \overset{-}{r}) .$$

It will be convenient to consider at some point the function

$$S(Y , p , w , r) = Y - C(Y , p , w , r) - I(Y , p , w , r)$$

for $Y \leqq \text{Min}(Y^* , Y(\frac{w}{p}))$, which can be interpreted as the aggregate private excess supply function associated with the output level Y . This function is increasing with both Y and r . Its variation with p and w is more ambiguous since investment is not clearly influenced either way be these variables.

Lastly, we wish to introduce another function which will be quite useful later on . Given Y , p , w , consider the equation

$$M^d(Y , p , w , r) = M$$

We have seen that M^d is a decreasing function of r , which tends to + ∞ as r goes to 0 , and which tends to 0 when r increases indefinitely. There is accordingly one and only one value of r which satisfies the foregoing equation. Reporting this value — which depends on Y , p , w and M — in the functions C(Y , p , w , r), I(Y , p , w , r) and S(Y , p , w , r), yields functions which we shall denote $C^*(Y , p ,w)$, $I^*(Y , p , w)$ and $S^*(Y , p , w)$ respectively (we keep the influence of M implicit for notational convenience) . It is easily checked that all three functions are increasing with Y . The function C^* is decreasing with p and increasing with w . The consequences of a variation of p or w on I^* , and thus on S^* , are ambiguous.

3. KEYNESIAN UNEMPLOYMENT.

We wish to study in this section the case where there is an excess supply on the labour and the good markets at a given configuration (p , w) .

In such a case, the firm perceives a binding constraint on its sales. In other words, the ration $\bar{Y}$ and $\bar{E}$ which the firm's production department faces must satisfy

$$\bar{Y} < F(\bar{E}) \quad \text{and} \quad \bar{Y} < Y(\frac{w}{p}) \ .$$

In that case the firm's supply of good and its labour demand are given by

$$Y = \bar{Y} \quad , \quad E = F^{-1}(\bar{Y}) \quad ,$$

while dividends D are equal to $p\bar{Y} - wD^{-1}(\bar{Y}) - B_{of} - T_f$. On the other hand, the investment department is not constrained on the good market. Its demand for investment is accordingly $I(\bar{Y} , p , w , r)$, and the ration $\bar{I}$ is greater than this quantity.

Consider now the household sector. Since there is an excess supply on the labour market, the worker faces a binding constraint on his labour supply , $\bar{L} < L^*$. On the other hand, none of the rations $\bar{C}_a$ is binding (a = 1,2) . The constrained demands and supplies of both households are thus given by :

$$L = \bar{L} \ , \ C_1 = C_1(p , r , R_1) \ , \ M_1^d = M_1^d(p , r , R_1) \quad ,$$

and $$C_2 = C_2(p , r , R_2) \ , \ M_2^d = M_2^d(p , r , R_2) \quad ,$$

where $$pR_1 = w\bar{L} + \theta_1 D + M_{o1} + (1 + \frac{1}{r}) B_{o1} - T_1 \quad ,$$

$$pR_2 = \theta_2 D + M_{o2} + (1 + \frac{1}{r}) B_{o2} - T_2 \quad .$$

Finally, the ration $\bar{G}$ that the Government faces on the good market is not binding. Actual public consumption G is thus equal to the target demand $G^d < \bar{G}$.

All that precedes concerned the agents' behaviour, the rations and the interest rate that they face being treated parametrically. We turn now to the determination of the equilibrium values of these variables. This will be achieved, as we said in Section 1 , by considering the equilibrium conditions for the good, for labour and for money.

Let us look first at the labour market. We must write the equality of the supply and demand of labour, L and E . On the other hand, the ration $\bar{E}$ that is perceived by the firm will be naturally equal to the worker's effective labour supply, i.e. to L^* . All this yields :

$$\bar{L} = F^{-1}(\bar{Y}) < L^* \qquad \text{and} \qquad \bar{E} = L^* \tag{3.1}$$

As for the good market, we have to write the equality of the firm's output Y with aggregate demand C + I + G . If one uses the relation $\bar{L} = F^{-1}(\bar{Y})$, the households' demand is easily seen to be $C(\bar{Y}, p, w, r)$.

Equilibrium of the good market requires accordingly that the ration $\bar{Y}$ satisfies :

$$\bar{Y} = C(\bar{Y}, p, w, r) + I(\bar{Y}, p, w, r) + G^d$$

or, more compactly,

$$S(\bar{Y}, p, w, r) = G^d \tag{3.2}$$

Of course, the ration $\bar{Y}$ must be less than $F(\bar{E})$ — which is, by (3.1), equal to full employment output Y^* — and than $Y(\frac{w}{p})$:

$$\bar{Y} < Y^* \qquad \text{and} \qquad \bar{Y} < Y(\frac{w}{p}) \tag{3.3}$$

The equilibrium condition for money states the equality of the households' aggregate demand for money M^d with the exogenous money supply M . Using again the relation $\bar{L} = F^{-1}(Y)$, M^d is easily seen to be equal to $M^d(\bar{Y}, p, w, r)$. One gets then

$$M^d(\bar{Y}, p, w, r) = M \tag{3.4}$$

To sum up, the determination of a Keynesian equilibrium associated to the given configuration (p,w) amounts to finding a pair ($\bar{Y}$, r) that satisfies (3.2), (3.3) and (3.4). The equilibrium rations $\bar{L}$ and $\bar{E}$ are then in turn given by (3.1). As for the (nonbinding) rations $\bar{C}_a$, $\bar{I}$ and $\bar{G}$ that the demanders face on the good market, they are determined by the rationing scheme in that market. Since in a Keynesian equilibrium the ration $\bar{Y}$ is binding and is thus equal to the equilibrium output level Y , we can state :

(1) *The Keynesian equilibrium levels of output* Y *and of the interest rate* r *that are associated with* (p , w) *are the solutions of the system*

$$\begin{cases} S(Y , p , w , r) = G^d \\ M^d(Y , p , w , r) = M \\ Y < Y(\frac{w}{p}) \ , \ Y < Y^* \ . \end{cases}$$

The first two equalities are of course those of the traditional Keynesian IS-LM model. The novelty of the present approach is the last two inequalities.

In order to find the conditions under which the above system has indeed a solution, we first solve the LM equation in r , for a given output level Y . This is always possible since by assumption $M^d(Y , p , w , r)$ is a decreasing function of r , which tends to + ∞ as r goes to 0 , and to 0 as r increases towards infinity. Reporting this value of r in S yields the system

$$\begin{cases} S^*(Y , p , w) = G^d \\ Y < Y(\frac{w}{p}) \ , \ Y < Y^* \ . \end{cases}$$

We are almost done. Indeed our assumptions imply, as we have seen in Section 2, that S^* is an increasing function of Y . It is clearly nonpositive, and thus less than G^d for $Y = 0$. The foregoing system has thus a solution — which is then unique — if and only if G^d is less than the value of S^* when Y is equal to the minimum of $Y(\frac{w}{p})$ and Y^* . When $\frac{w}{p} \geqq F'(L^*)$, this means $S^*(Y(\frac{w}{p}), p, w) > G^d$. If on the contrary, $\frac{w}{p} \leqq F'(L^*)$, the conditions reads $S^*(Y^*, p, w) > G^d$. To sum up,

(2) *A Keynesian unemployment equilibrium exists at the configuration* (p, w) *if and only if* $S^*(Y(\frac{w}{p}), p, w) > G^d$ *whenever* $\frac{w}{p} \geqq F'(L^*)$, *and* $S^*(Y^*, p, w) > G^d$ *when* $\frac{w}{p} \leqq F'(L^*)$. *The equilibrium is then unique.*

The foregoing result shows that a Keynesian equilibrium does not obtain for all configurations (p, w) . Figure 1 represents the region of the plane (p, w) where a Keynesian equilibrium does obtain.

Fig. 1

There, the line L_1 is given by the equation $\frac{w}{p} = F'(L^*)$, while the lines L_2 and L_3 correspond to the equations

$$(L_2) \qquad S^*(Y(\tfrac{w}{p}), p, w) = G^d , \quad \frac{w}{p} \geqq F'(L^*)$$

$$(L_3) \qquad S^*(Y^*, p, w) = G^d , \quad \frac{w}{p} \leqq F'(L^*) .$$

It is readily seen that a point of intersection of any two of the three curves L_1 , L_2 or L_3 corresponds to a situation where all markets clear without rationing, i.e. to a Walrasian equilibrium. Under the assumption that a proportional increase of p and w reduce both the demands for consumption and for investment, and increases the aggregate demand for money, such a short run Walrasian equilibrium is easily shown to be unique, whenever it exists. Figure 1 is drawn under this assumption . The region of Keynesian unemployment equilibria is then on the right of the curves L_2 and L_3 .

The size of the disequilibrium which obtains on the labour market, at a given (p , w) in the Keynesian region, is measured by the difference $L^* - F^{-1}(Y)$, where Y is solution of $S^*(Y , p , w) = G^d$. The locus of all (p , w) in the Keynesian region that are associated to a Keynesian equilibrium which involves a disequilibrium of size $\delta = L^* - F^{-1}(Y)$ on the labour market are thus given by the equation

$$S^*(F(L^* - \delta) , p , w) = G^d$$

This yields a curve that is similar to L_3 , but on its right (Fig. 2.a). Curve L_3 corresponds to full employment of the labour force.

Similarly, disequilibrium on the good market may be measured by the difference between the firm's *effective* supply of good — which is given by $Y(\frac{w}{p})$ when $\frac{w}{p} \geqq F'(L^*)$ and by Y^* when $\frac{w}{p} \leqq F'(L^*)$ — and the equilibrium level of output Y . The locus of all (p , w) in the Keynesian region which are associated with a constant disequilibrium of a given size $\delta > 0$ on the good market, is thus described by the curves of equations

$$S^*(Y(\frac{w}{p}) - \delta , p , w) = G^d \quad \text{when} \quad \frac{w}{p} \geqq F'(L^*) ,$$

and

$$S^*(Y^* - \delta , p , w) = G^d \quad \text{when} \quad \frac{w}{p} \leqq F'(L^*) .$$

This locus is represented by the dotted curves in Fig. 2.b, which are "translations" towards the right of the curves L_2 and L_3 . The latter curves correspond to the case where there is an equilibrium, without rationing, on the good market ($\delta = 0$) .

Finally, we may note that the sign of the slopes of the curves L_2 and L_3 is ambiguous. Under our assumptions (see Section 2), the function $C^*(Y , p ,w)$ is increasing with w and decreasing with p . But the investment function, and thus $I^*(Y , p , w)$, is not clearly influenced either way by variations of w or of p , owing to their conflicting consequences on capital-labour substitution and on the profitability of investment. In all cases however, the slope of L_3 at the Walrasian equilibrium W is greater than the slope of L_2 .

Policy Shifts

Consider a pair (p , w) which lies in the Keynesian region. By (1), The system that defines the equilibrium output and interest rate is identical to the usual Keynesian IS-LM equations. Thus, all policy conclusions of the IS-LM model hold equally well here. We review them briefly for the sake of completeness.

A rise of public spending G^d generates an increase of output and employment, while the equilibrium interest rate goes up. The multiplier dY/dG^d will be greater than 1 when $\gamma = \left|\frac{\partial S}{\partial r} / \frac{\partial M^d}{\partial r}\right|$ is small, which will occur if the influence of the interest rate on private consumption and on investment demand is weak compared to its impact on the demand for money. A general tax cut increases the demand for consumption and the demand for money. The equilibrium interest rate goes up, while the effect on output is ambiguous (it will rise if γ is small). A shift of taxes from workers to rentiers will initially increase consumption, and depress the demand for money. The equilibrium output will go up, while the consequence on the interest rate are ambiguous (it will go down if γ is small). Finally, an increase of the current money supply causes output to rise, and the interest rate goes down. The announcement in the current period that the Government will implement an expansionary policy of that kind in the near future will generate an increase in the current aggregate demand for the good and a rise of the current money demand, and will accordingly generate similar consequences : the current interest rate increases, while output goes up if γ is small.

All these expansionary policies cause the Keynesian region to skrink (at least when γ is small). In Fig. 1, the Walrasian equilibrium W moves on L_1 rightwards, while the curves L_2 and L_3 shift towards the north-east.

Finally, income policies that attempt to alter the price p or the wage w have ambiguous consequences. An increase of w , by shifting income from rentiers to workers, generates initially a rise of private demand for consumption, and a decrease of the demand for money. But this move has no clear effects an investment. If substitution of capital to labour is predominant, investment initially goes up. The overall effect will be to make the equilibrium output level to rise (the interest rate will go down if γ is small). But if a rise of w causes investment to go down because its profitability has decreased, one may witness the opposite. Finally, a proportional increase of p and w has initially, under our assumptions, a depressing impact on consumption, investment and increases the demand for money. Output goes down. The interest rate decreases when γ is small.

4. CLASSICAL UNEMPLOYMENT.

We study now the case where the equilibrium regime associated with a given configuration (p , w) , involves an excess supply of labour and an excess demand of good.

In that case, the firm's production department is not constrained in any market. The actual production program satisfies

$$Y = Y(\frac{w}{p}) < \bar{Y} \qquad \text{and} \qquad E = E(\frac{w}{p}) < \bar{E} \ .$$

On the other hand, the investment department faces a ration $\bar{I}$ that may be binding. Realized investment I will be the minimum of $I(\bar{Y} , p , w , r)$ and of $\bar{I}$.

If we look now at the household's sector, the worker faces a ration $\bar{L}$ which is binding, i.e. $\bar{L} < L^*$. His actual labour supply is then $L = \bar{L}$. The households' effective demands will be than $C_a(p , r , R_a)$, where

$$\left.\begin{aligned} pR_1 &= w\bar{L} + \theta_1 D + M_{o1} + (1 + \frac{1}{r}) B_{o1} - T_1 \\ pR_2 &= \theta_2 D + M_{o2} + (1 + \frac{1}{r}) B_{o2} - T_2 \end{aligned}\right\} \qquad (4.1)$$

and where

$$D = pY - wE - B_{of} - T_f = pY(\frac{w}{p}) - wE(\frac{w}{p}) - B_{of} - T_f$$

The households' realized demands C_a will be the minimum of $C_a(p , r , R_a)$ and of the rations $\bar{C}_a$ that they face on the good market — which may be binding since there is an excess demand on that market.

Their associated demands for money will then be $M_a^d(p , r , R_a , \bar{C}_a)$.

Finally, the Government faces on the good market a ration $\bar{G}$ which may be binding. Realized public consumption G will be the minimum of the target demand of G^d and of $\bar{G}$.

We turn next to the determination of the equilibrium values of the rations $\bar{Y}$, $(\bar{C}_a)$, $\bar{I}$, $\bar{G}$, $\bar{L}$ and of the interest rate. In order to achieve this goal, by virtue of Walras Law, we have only to consider the equilibrium conditions for the good, labour and for money.

The equilibrium relation for the labour market, E = L , yields immediately the value of the ration $\bar{L} = E(\frac{w}{p})$. On the other hand, the equilibrium ration $\bar{E}$ is naturally equal to L^* . Stating that there is an excess supply of labour is described by $\bar{L} < L^*$, or equivalently $\bar{E} > E(\frac{w}{p})$.

$$\bar{L} = E(\frac{w}{p}) < L^* \qquad \text{and} \qquad \bar{E} = L^* \tag{4.2}$$

Remark that (4.2) implies $\frac{w}{p} > F'(L^*)$, or equivalently $Y(\frac{w}{p}) < Y^*$. We shall assume henceforth that (4.2) holds.

We turn next to the good market. We wish to state the conditions on the rations $\bar{Y}$, $\bar{C}_a$, $\bar{I}$, $\bar{G}$ and on the interest rate r which express that this market balance (*ex-post*), and that it is in excess demand.

First, the fact that the ration $\bar{Y}$ is not binding is described by

$$\bar{Y} > Y(\frac{w}{p}) \tag{4.3}$$

In that case, the firm's effective supply, as well as its actual output Y , is equal to $Y(\frac{w}{p})$. Remark next that for any $\bar{Y}$ satisfying (4.3), and any interest rate r , the agents' effective demands of good are well defined. The Government's demand is G^d , while the firm's investment effective demand is $I(\bar{Y}, p, w, r)$. For the households, they are described by $C_a(p, r, R_a)$ where the R_a's are given by (4.1) , with $\bar{L} = E(\frac{w}{p})$. The households' aggregate effective demand is then equal to $C(Y(\frac{w}{p}), p, w, r)$. By definition, the rationing scheme which operates on the demand side of the good market associates to the firm's effective supply $Y(\frac{w}{p})$ and to the other agents' effective demands, well defined rations $\bar{C}_a$, $\bar{I}$ and $\bar{G}$. Two points are important to note at this stage. First, the rations $\bar{C}_a$, $\bar{I}$ and $\bar{G}$ are completely determined — given the rationing scheme — by $\bar{Y}$, r and, of course, by p and w . Second, when there is excess demand, i.e., when

$$Y(\frac{w}{p}) < C(Y(\frac{w}{p}), p, w, r) + I(\bar{Y}, p, w, r) + G^d \qquad (4.4)$$

the rationing scheme ensures, by definition, that *ex-post* transactions balance : constrained demands (that is, for each demander, the minimum of his effective demand and of this ration) do sum to $Y(\frac{w}{p})$.

The equilibrium conditions for the good market are now easy to formulate. We shall require, naturally, that in equilibrium, the ration $\bar{Y}$ is equal to aggregate effective demand :

$$\bar{Y} = C(Y(\frac{w}{p}), p, w, r) + I(\bar{Y}, p, w, r) + G^d \qquad (4.5)$$

It is then easily seen that equilibrium of the good market is achieved when $\bar{Y}$ and r the equality (4.5) and the inequality (4.3) (assuming, as we did, that (4.2) holds). For in that case, (4.4) is trivially satisfied, and from our discussion, *ex-post* transactions on the good market balance.

To simplify notations, let us define

$$\bar{S}(\bar{Y} , p , w , r) = \bar{Y} - C(Y(\frac{w}{p}) , p , w , r) - I(\bar{Y} , p , w , r) \tag{4.6}$$

Then (4.5) is equivalent to

$$\bar{S}(\bar{Y} , p , w , r) = G^d \tag{4.7}$$

It remains to state the equilibrium condition for money. To this effect, recall that for any $\bar{Y} > Y(\frac{w}{p})$, the rations $\bar{C}_a$, $\bar{I}$, $\bar{G}$, that are perceived by the demanders of good are completely determined by the rationing scheme on that market. In particular, the rations $\bar{C}_a$ can be viewed as functions of $\bar{Y}$, p , w and r , which we shall write $\bar{C}_a = \varphi_a(\bar{Y},p,w,r)$. The households'demands for money are then described by $M_a^d(p , r , R_a , \bar{C}_a)$ where the R_a's are given by (4.1) (with $\bar{L} = E(\frac{w}{p})$) , and the rations $\bar{C}_a$ depend on $\bar{Y}$, p , w , r in the way we just described. Then the households' aggregate demand for money is a function of $\bar{Y}$, p , w , r , which we shall note $\bar{M}^d(\bar{Y} , p , w , r)$. With this notation, the equilibrium condition for money reads

$$\bar{M}^d(\bar{Y} , p , w , r) = M \tag{4.8}$$

To sum up, *consider a configuration* (p , w) *such that* $\frac{w}{p} > F'(L^*)$. *Then, looking for a classical unemployment equilibrium associated with* (p , w) *is equivalent to finding* $\bar{Y}$ *and* r *that satisfies the following system*

$$\begin{cases} \bar{S}(\bar{Y} , p , w , r) = G^d \\ \bar{M}^d(\bar{Y} , p , w , r) = M \\ \bar{Y} > Y(\frac{w}{p}) \end{cases}$$

Before studying the conditions under which such a system has indeed a solution, we may note that it is analogous the standard Keynesian IS-LM model. But there are important differences. In the foregoing system, the unknown ration $\bar{Y}$, which represents the level of aggregate effective demand, no longer coincides with the output level as it did in the Keynesian unemployment case (of course, in a Classical Unemployment regime, output is equal to the firm's notional supply $Y(\frac{w}{p})$) . The second important difference is that the functions $\bar{S}$ and $\bar{M}^d$ in the Classical system differ essentially from those which appeared in the Keynesian IS-LM equations.

We next spend some time spelling out a few properties of these functions.

The "Classical" function $\bar{S}(\bar{Y} , p , w , r)$ is given by (4.6). It is thus increasing with r . It increases too with $\bar{Y}$. As a matter of fact,

$$\frac{\partial \bar{S}}{\partial \bar{Y}} \geq 1 - \alpha > 0$$

where α is the upper bound of the marginal propensity to invest. This shows that $\bar{S}$ goes to $+ \infty$ when $\bar{Y}$ increases indefinitely. Moreover, it is clear that the Classical function $\bar{S}$ is equal to its Keynesian counterpart when $\bar{Y} = Y(\frac{w}{p})$,

$$\bar{S}(Y(\frac{w}{p}) , p , w , r) = S(Y(\frac{w}{p}) , p , w , r) .$$

Let us look now at the "Classical" money demand function $\bar{M}^d(\bar{Y} , p , w , r)$. By definition,

$$\bar{M}^d(\bar{Y} , p , w , r) = \sum_a M_a^d(p , r , R_a , \bar{C}_a)$$

where the R_a's are given by (4.1), with $\bar{L} = E(\frac{w}{p})$, and $\bar{C}_a = \varphi_a(\bar{Y},p,w,r)$ depends on $(\bar{Y}$, r) through the rationing scheme on the demand side of the good market.

Clearly, each household' current real wealth R_a is independent of $\bar{Y}$. The influence of $\bar{Y}$ on $\bar{M}^d$ goes exclusively through its impact on rationing of the households' demands of good (forced savings). In fact, we must have

$$\bar{M}^d(\bar{Y} , p , w , r) \geqq M^d(Y(\frac{w}{p}) , p , w , r)$$

with inequality if none of the rations $\bar{C}_a$ is binding. The following result provides more information about the function $\bar{M}^d$. To this effect, we shall need an assumption on the rationing scheme on the good market.

(f) *Consider the rationing scheme which operates on the demand side of the good market. Each household's ration* $\bar{C}_a$ *is assumed to be a nondecreasing function of the firm's effective supply, and a nonincreasing function of each agent's (including himself) effective demand of good.*

Part of this assumption is innocuous. If the firm's supply goes up, an household's ration $\bar{C}_a$ should not go down. If the effective demand of another agent goes up, the ration $\bar{C}_a$ should not go up. The last part, which states that an household cannot raise his own ration by increasing his own demand,is more restrictive. It simply says that the rationing scheme is *non-manipulable*.

(1) *The Classical demand for money* $\bar{M}^d(\bar{Y}\ ,\ p\ ,\ w\ ,\ r)$ *is a non-decreasing function of* $\bar{Y}$ *. It is a decreasing function of* r *, which goes to* $+\infty$ *as* r *tends to* 0 *, and to* 0 *when* r *tends to* $+\infty$ *.*

Proof. We first check that $\bar{C}_a = \varphi_a(\bar{Y}\ ,\ p\ ,\ w\ ,\ r)$ is non-increasing with $\bar{Y}$, and non-decreasing with r . Indeed, if $\bar{Y}$ goes up, investment demand $I(\bar{Y}\ ,\ p\ ,\ w\ ,\ r)$ goes up. In view of assumption (*f*), $\bar{C}_a$ cannot go up. If r goes up, investment demand and each household's effective demand goes down. Again, (*f*) implies that $\bar{C}_a$ cannot go down.

It is then easy to prove (1), if we consider

$$\bar{M}^d(\bar{Y}\ ,\ p\ ,\ w\ ,r) = \sum_a M_a^d(p\ ,\ r\ ,\ R_a\ ,\ \bar{C}_a)\ .$$

When $\bar{Y}$ rises, each R_a is invariant, while the $\bar{C}_a$'s do not increase. In view of assumption (*e*) (Section 2), $\bar{M}^d$ cannot go down. On the other hand, $\bar{M}^d$ decreases with r on three counts. First, each M_a^d in the above expression directly decreases with r . Second, the R_a's go down since B_{oa}/r diminishes. Finally, either each $\bar{C}_a$ does not move or rises. The result follows again from (*e*). It is finally quite easy to check that, from (*e*), $\bar{M}^d$ goes to $+\infty$ when r tends to 0 (each R_a diverges then to $+\infty$ as B_{oa}/pr), and to 0 when r increases without bound. Q.E.D.

We are now well equiped to study the existence of a Classical unemployment equilibrium. In fact we wish to show :

(2) *There exists a Classical unemployment equilibrium corresponding to the configuration* (p, w) *if and only if* $\frac{w}{p} > F'(L^*)$ *and* $S^*(Y(\frac{w}{p}), p, w) < G^d$ *. The equilibrium is then unique.*

Proof. Our discussion showed that, given (p, w), a Classical unemployment equilibrium exists if and only if (i) $\frac{w}{p} > F'(L\ )$ and (ii) there exists a pair $(\bar{Y}, r)$, with $\bar{Y} > Y(\frac{w}{p})$ that is solution of

$$\bar{S}(\bar{Y}, p, w, r) = G^d$$
$$\bar{M}^d(\bar{Y}, p, w, r) = M$$

We know from *(1)* that $\bar{M}^d$ decreases from $+\infty$ to zero when r varies from 0 to $+\infty$. For each $\bar{Y}$, there is a unique value of the interest rate which satisfies $\bar{M}^d(\bar{Y}, p, w, r) = M$. Reporting this value of r in $\bar{S}$, yields a function which depends on $\bar{Y}$, p, w (and implicitly on M), which we may note $\bar{S}^*(\bar{Y}, p, w)$. If $\frac{w}{p} > F'(L^*)$, looking for a Classical unemployment equilibrium then is equivalent to finding $\bar{Y} > Y(\frac{w}{p})$ such that

$$\bar{S}^*(\bar{Y}, p, w) = G^d$$

The function $\bar{S}^*$ is easily seen to be increasing with $\bar{Y}$. In fact, straightforward differenciation of the Classical system with respect to $\bar{Y}$ and r yields

$$\frac{\partial \bar{S}^*}{\partial \bar{Y}} = \frac{\partial \bar{S}}{\partial \bar{Y}} - \frac{\partial \bar{S}}{\partial r}\left(\frac{\partial \bar{M}^d}{\partial \bar{Y}} \Big/ \frac{\partial \bar{M}^d}{\partial r}\right) \geqq \frac{\partial \bar{S}}{\partial \bar{Y}} \geqq 1 - \alpha > 0$$

Thus $\bar{S}^*$ increases without bound as $\bar{Y}$ diverges towards $+\infty$. We can therefore conclude that a Classical unemployment equilibrium exists if and only if (i) $\frac{w}{p} > F'(L^*)$ and (ii) $\bar{S}^*(Y(\frac{w}{p}), p, w) < G^d$.

We are not yet done, since wish in fact to show that $\bar{S}^*$ can equivalently be replaced in (ii) by the Keynesian excess supply $S^*(Y(\frac{w}{p}) , p , w)$.

Consider now the unique r_1 such that

$$\bar{M}^d(Y(\frac{w}{p}) , p , w , r_1) = M$$

Then (ii) is equivalent to

$$\bar{S}(Y(\frac{w}{p}) , p , w , r_1) < G^d$$

Since $\bar{S} = S$ whenever $\bar{Y} = Y(\frac{w}{p})$, the foregoing inequality reads

$$S(Y(\frac{w}{p}) , p , w , r_1) < G^d \tag{4.9}$$

Consider next the unique r_o such that

$$M^d(Y(\frac{w}{p}) , p , w , r_o) = M$$

and remark that $S^*(Y(\frac{w}{p}) , p , w) < G^d$ means simply

$$S(Y(\frac{w}{p}) , p , w , r_o) < G^d \tag{4.10}$$

We wish to show that (4.9) and (4.10) are equivalent. Now, clearly, $r_1 \geqq r_o$. Thus, since S is increasing with r , (4.9) implies (4.10). Suppose that conversely, (4.10) holds. If

$$\lim_{r \to +\infty} S(Y(\frac{w}{p}) , p , w , r) \leqq G^d ,$$

(4.9) follows trivially. Otherwise, there exists a unique r_2 such that

$$S(Y(\frac{w}{p}) , p , w , r_2) = G^d$$

One may remark that when $\bar{Y} = Y(\frac{w}{p})$ and $r = r_2$, the rations $\bar{C}_a = \varphi_a(\bar{Y},p,w,r)$ cannot be binding, since then effective supply $Y(\frac{w}{p})$ is equal to aggregate effective demand. Therefore

$$\tilde{M}^d(Y(\frac{w}{p}) , p , w , r_2) = M^d(Y(\frac{w}{p}) , p , w , r_2)$$

But, since (4.10) holds, $r_2 > r_o$. This implies

$$\tilde{M}^d(Y(\frac{w}{p}) , p , w , r_2) < M^d(Y(\frac{w}{p}) , p , w , r_o) = M$$

and thus $r_2 > r_1$ too . Hence (4.9) holds.

Unicity is obvious. Q.E.D.

If we go back to Fig. 1, this result shows quite simply, that a Classical unemployment equilibrium obtains if and only the configuration (p , w) belongs to the region which is above the line L_1 and on the left of the curve L_2 .

The disequilibrium which obtains on the labour market at a (p , w) in the Classical region, is measured by $L^* - E(\frac{w}{p})$. The locus of all (p , w) in the Classical region that are associated to a constant disequilibrium of size $\delta = L^* - E(\frac{w}{p}) > 0$, is described by a line where the real wage $\frac{w}{p}$ is constant and equal to $F'(L^* - \delta)$. The portion of the line L_1 which joins the origin to the Walrasian equilibrium corresponds to full employment (Fig. 3.a).

Similarly, disequilibrium on the good market may be measured by the difference between the equilibrium level $\bar{Y}$ of aggregate effective demand and the firm's effective supply $Y(\frac{w}{p})$. The locus of all (p , w) in the Classical region that are associated with a constant "inflationary gap" $\delta > 0$ on the good market is then described by a curve of equation

$$\bar{S}^*(Y(\frac{w}{p}) + \delta , p , w) = G^d$$

Such a curve is a "translation" of L_2 towards the left (Fig. 3.b).

Policy Shifts

Consider a pair (p , w) in the Classical region. Output and employment are then given by $Y(\frac{w}{p})$ and $E(\frac{w}{p})$. The policy implications are thus clear. Contrary to what happened in the Keynesian case, any expansionary policy through a rise of public spending, a tax cut, a shift of taxes from workers to rentiers, an increase of the money supply M , will have no effect on output and employment. Their only consequences will be on the inflationary gap on the good market, on the interest rate, and on investment.

To take only two examples, consider the effects of an increase of G^d , or of M. A rise of G^d has two initial consequences. First, it increases effective aggregate demand. Second, it may tighten the rationing experienced by the other demanders, and particular by households. As a result, the demand for money $\bar{M}^d(\bar{Y} , p , w , r)$ may increase initially. The equilibrium interest rate goes up, while the effect on aggregate effective demand $\bar{Y}$ is ambiguous. It will rise if the demand for money $\bar{M}^d$ is much more sensitive to the interest rate than consumption and investment demand. In that case, since supply is unchanged, the inflationary gap on the good market goes up. The demand for investment may rise or fall. And more importantly, realized investment may actually go down since it may be more severely rationed as a result of the rise of the Government's demand.

An expansionary monetary policy (an increase of M) is easily seen to make aggregate effective demand to go up and the interest rate to go down. Here again, the supply of good does not change, so the inflationary gap on the good market increases. Effective demand for investment and for consumption both go up, but actual investment may fall, as it may be rationed more tightly.

Without going into details, one may say generally that an expansionary policy that succeeded in increasing output in a Keynesian regime will simply increase the inflationary gap on the good market without altering actual output and employment in the Classical case. The consequences on the interest rate will be similar in the two regimes.

We end up by looking at income policies. A decrease of w or a rise of definitely increases output and employment. Finally, a proportional increase of p and w has no effect on output. But the consequences of these moves on aggregate demand and on the interest rate are ambiguous.

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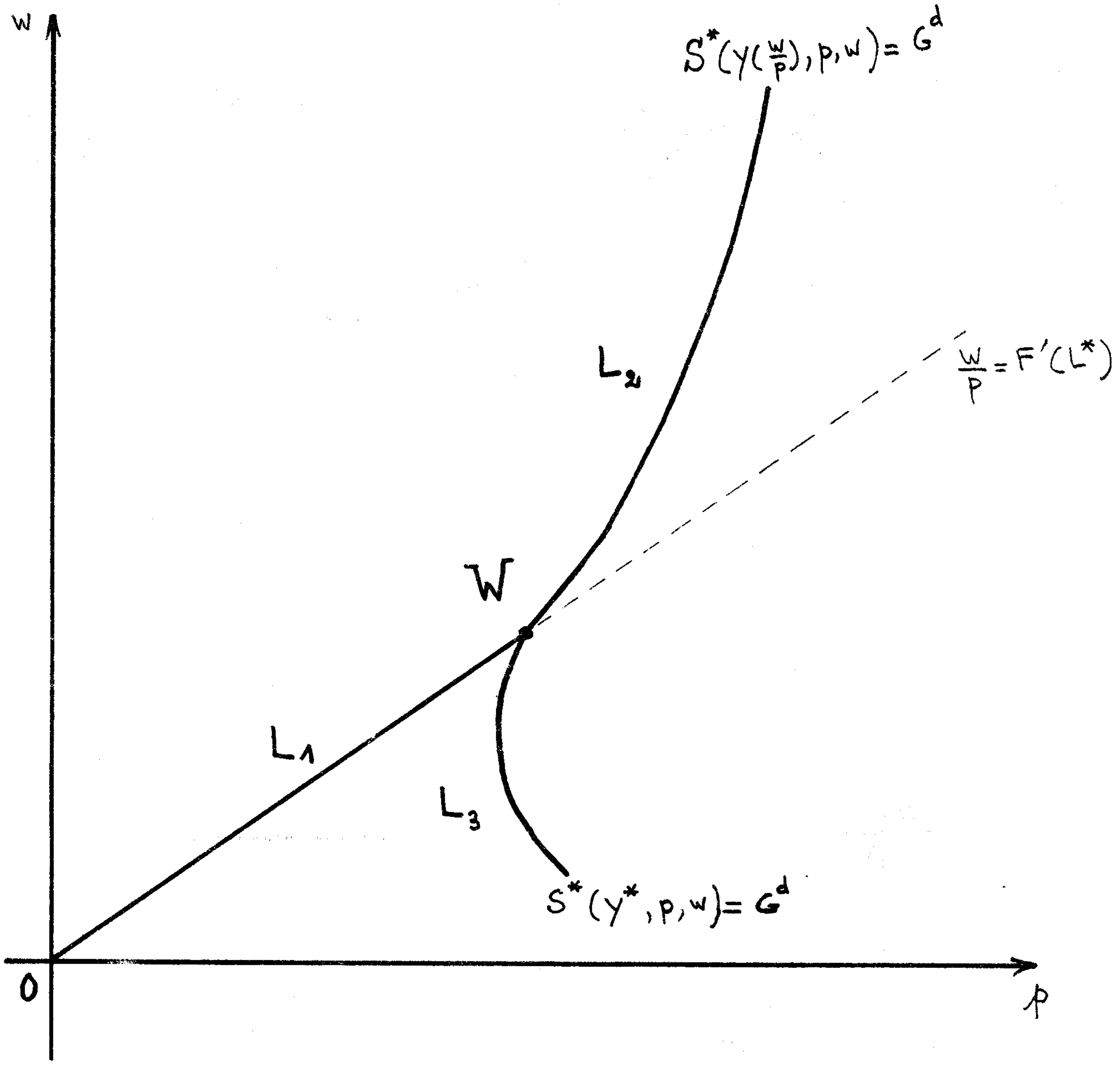

Fig. 1

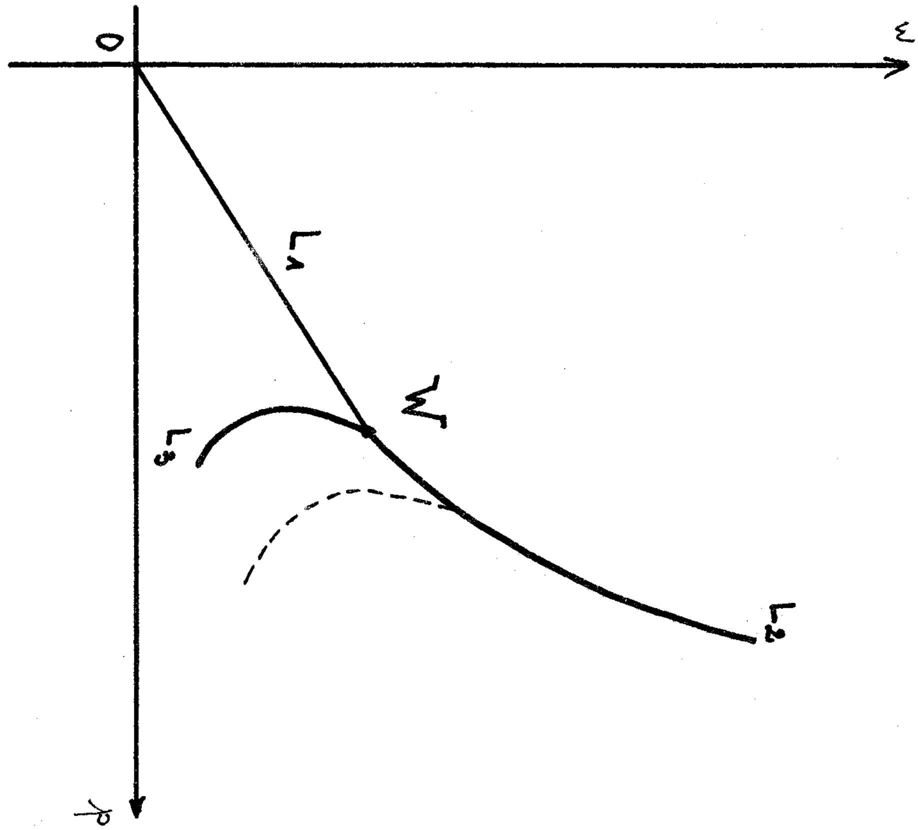

Fig. 2.a

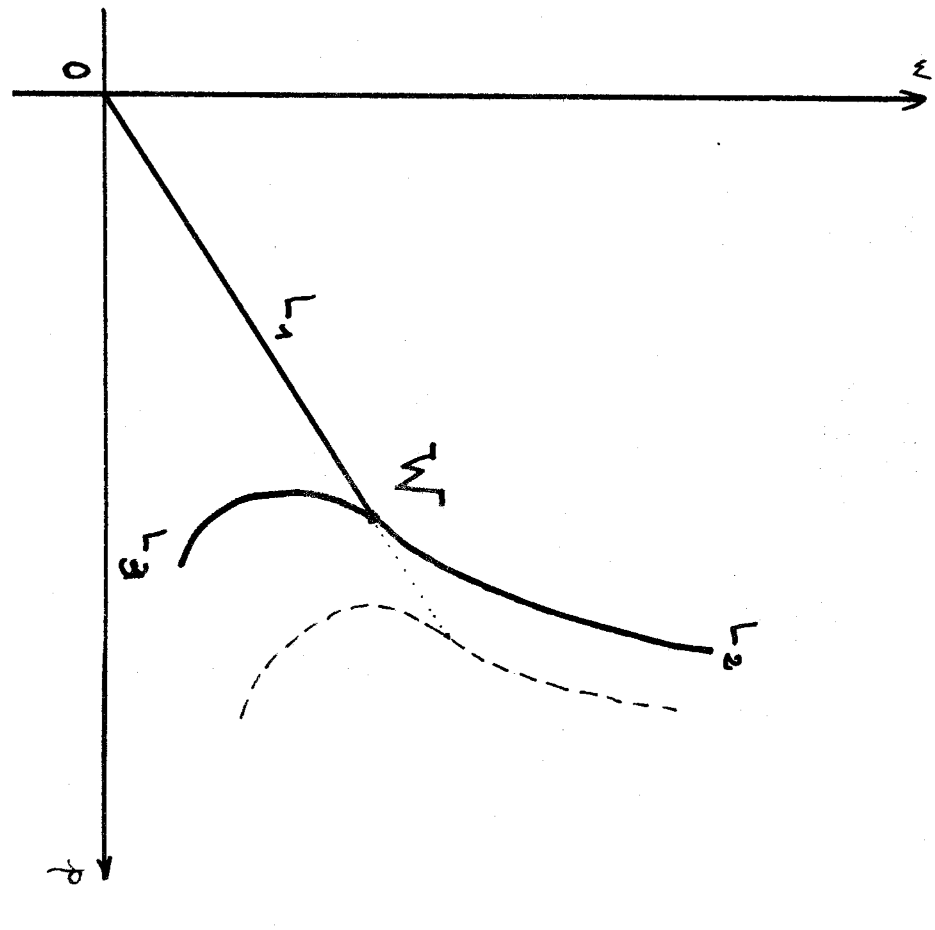

Fig. 2.b

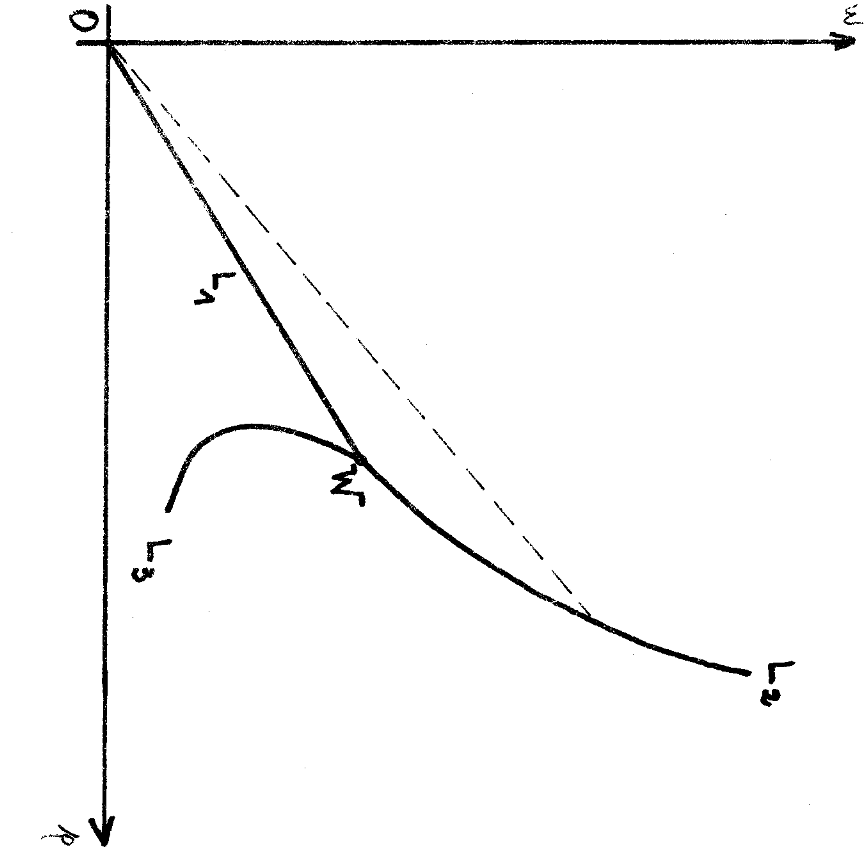

Fig. 3.a

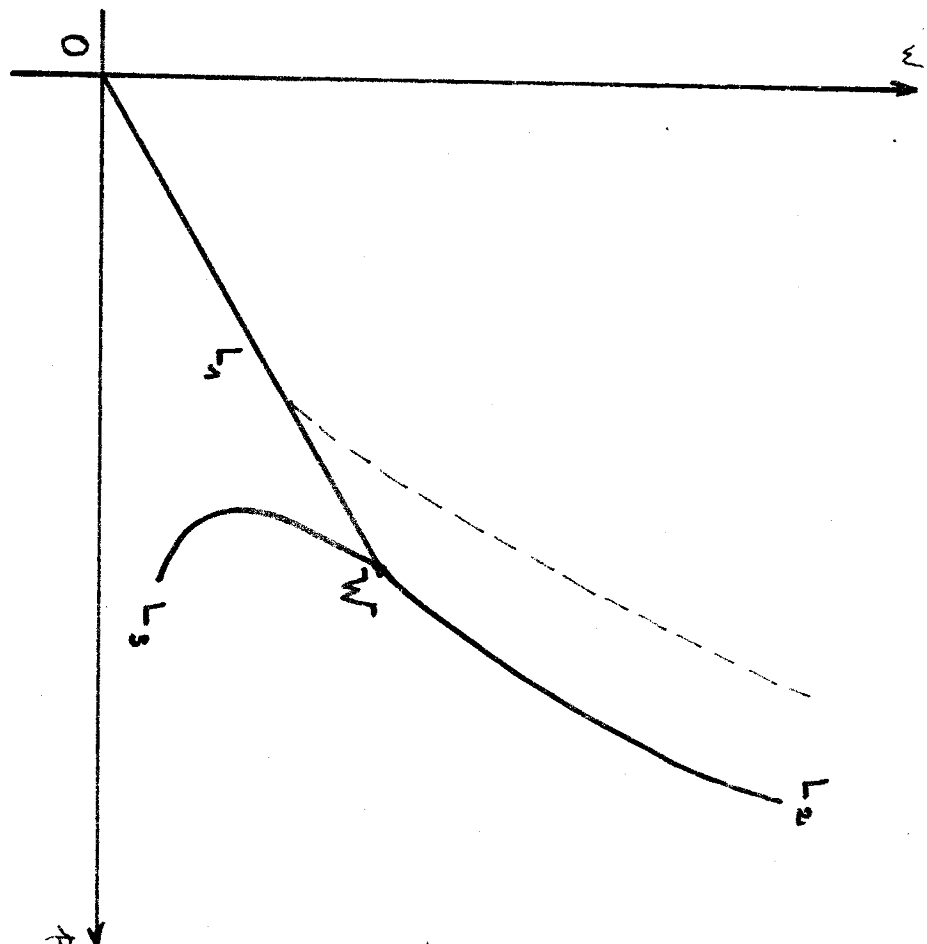

Fig. 3.b