OPEN MARKET POLICIES

AND

LIQUIDITY *

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OPEN MARKET POLICIES AND LIQUIDITY

In the real world, there are many channels through which a banking system may influence economic activity by implementing a specific monetary policy. One way, which was considered in the previous two chapters, is to intervene on the credit market by trying to manipulate the cost of borrowing, or the amount of money which is created when granting loans to the private sector. Another one, which we shall study presently, is that the Bank attempts to influence the economy's "liquidity" by exchanging "illiquid" assets such as long term bonds for "liquid" assets such as short term bonds or money.

The model which we shall use to take into account this kind of phenomena is quite simple, and bears some resemblance with popular keynesian macroeconomic models. The real part of the model is the same as in the previous lectures. As in Chapter I, consumers have to decide in each period how much to consume and to save (no borrowing is allowed). But consumers can now save by holding two sorts of assets, instead of one : paper money and perpetuities, which are both issued by a governmental agency, the Bank. In such a context, the Bank can in principle engage in open market operations by trading perpetuities for money, and vice versa. In particular, the Bank may wish to peg the interest rate, i.e. the reciprocal of the money price of perpetuities, or the money supply.

We shall be interested here with the determination of short run equilibrium prices and interest rate at a given date , called the "current" period, subject to the Bank's policy.

Conventional Neoclassical Theory claims that a short run equilibrium typically exists, and that, therefore, the Bank has full control over the interest rate or the money supply in a money economy with flexible prices like this one. There is however considerable disagrement about the underlying regulating mechanisms. "Monetarists" underscore the role of wealth effects, and of the intertemporal substitution which is engineered by a variation of the interest rate. Others emphasize the substitution between money and bonds which is embodied in the Keynesian Liquidity Preference Theory. It will be shown, and this should by now be no surprise, that although both viewpoints contain some elements of truth, they miss an important part of the story, i.e. intertemporal substitution effects.

In order to make intertemporal substitution effects operative, we shall have to assume that some consumers' prices and interest rates forecasts are to a large extent insensitive to the current values of these variables. The restrictive character of such a condition makes the existence of a short run equilibrium unlikely, contrarily to what Neoclassical theorists used to believe.

Another issue we shall be concerned with is the presence of a "Liquidity Trap". It will be shown that such a phenomenon does exist in the present model whenever the Bank has full control over the money stock, in the sense that, the lower the interest rate, the less sensitive it becomes to a given variation of the Bank's money supply.

This lecture is organized as follows. Neoclassical theory is briefly reviewed in Section 1. The basic assumptions of the model are recalled in Section 2, and the behaviour of the consumers is made precise in Section 3. The question of the existence of a short run equilibrium is analysed next, first when the Bank attempts to peg the interest rate (Section 4), and second, when it tries to control the money supply (Section 5). Lastly, the "Liquidity Trap" phenomenon is studied in Section 6.

1. NEOCLASSICAL THEORY REVISITED.

We begin with a brief review of what conventional theory has to say in the present context.

The real part of the model is the same as before. There are thus ℓ non durable goods available in each period, whose (positive) equilibrium money prices p have to be determined at every date. Consumers decide at every moment how much to consume and to save : for simplicity, borrowing is not allowed. On the other hand, consumers can save by holding two kinds of monetary assets. There is first paper money, on which no interest is paid by assumption (1). And second, long term bonds which take here the form of *perpetuities*, i.e. promises to pay to the holder one unit of money in each period. At any date, the (positive) money price of perpetuities s determines the rate of interest, which is defined as the value of the short term interest rate r which equalizes the discounted value of interest payments with the current price, i.e.

$$s = \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots = \frac{1}{r} \quad .$$

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(1) The assumption that money does not bear interest is made only for convenience. The analysis which follows can be easily transposed to the case where money is replaced by any safe short term interest bearing asset. This is left as an exercise to the reader.

Both assets are issued by a Governmental agency, which is called the Bank. This agency's interventions are by assumption restricted to open market operations, i.e. to exchanges of bonds for money. An issue of perpetuities by the Bank will reduce the amount of money held by consumers ; conversely, the Bank's purchases of bonds will increase the money stock. The Bank can then aim at influencing systematically the economy's liquidity by implementing a specific policy, such as pegging the interest rate or the money supply. We shall be concerned with the existence of a short run equilibrium at a given date, say date 1, or the "current period", subject to the Bank's policy.

According to Neoclassical theorizing, a consumer a's actions at date 1 can be viewed as functions of the current money prices of goods p_1 , of the current interest rate on bonds r_1 , and of his initial money wealth, $\bar{m}_a + (\bar{b}_a/r_1)$, where $\bar{m}_a$ is the consumer's initial money holdings (including interest payments on bonds), and $\bar{b}_a$ stands for his initial stock of perpetuities. The consumer's excess demands for goods, his money and bond demands are then denoted $z_a(p_1,r_1,\bar{m}_a + (\bar{b}_a/r_1))$, $m_a^d(p_1,r_1,\bar{m}_a + (\bar{b}_a/r_1))$, $b_a^d(p_1,r_1,\bar{m}_a + (\bar{b}_a/r_1))$ respectively.

Summing these individual demand functions over all consumers yields an aggregate excess demand for goods, and an aggregate demand for money and for bonds :

$$Z(p_1,r_1) = \sum_a z_a(p_1,r_1,\bar{m}_a + (\bar{b}_a/r_1)) \ ,$$

$$M^d(p_1,r_1) = \sum_a m_a^d(p_1,r_1,\bar{m}_a + (\bar{b}_a/ r_1)) \ ,$$

$$B^d(p_1,r_1) = \sum_a b_a^d(p_1,r_1,\bar{m}_a + (\bar{b}_a/r_1)) \ ,$$

where the influence of initial holdings of money and of perpetuities is kept implicit, for notational convenience.

Writing down the equilibrium conditions for all markets at date 1 is easy by using these expressions. Equilibrium of the goods markets requires as usual

$$(C) \qquad Z(p_1, r_1) = 0 \quad .$$

The equation for money states that the aggregate demand for money must be equal in equilibrium to the initial money stock $M = \sum_a \bar{m}_a$, to which is added the Bank's money creation through the purchase of bonds. If we take momentarily this money creation ΔM as a parameter of the system, the money condition reads :

$$(D) \qquad M^d(p_1, r_1) = M + \Delta M \quad .$$

The fact that the Bank is issuing the quantity ΔM of money, means that it is purchasing $r_1\ \Delta M$ perpetuities on the bond market. Equilibrium of that market requires therefore

$$(E) \qquad B^d(p_1, r_1) + r_1\ \Delta M = B \quad ,$$

where $B = \sum_a \bar{b}_a$ is the initial aggregate stock of bonds.

These equations imply, trivially, that the Bank's money supply ΔM is bounded below by $-\Delta M$, and above by (B/r_1). The first inequality simply means that the final money stock has to be nonnegative. The second that the Bank's money creation cannot exceed the value of outstanding bonds. Moreover, if one eliminates ΔM from equations $(\mathcal{D})$ and $(\mathcal{E})$, one gets

$$(\mathcal{D}_1) \qquad M^d(p_1,r_1) = M + \frac{1}{r_1}\left[B - B^d(p_1 \ , \ r_1)\right]$$

This equality implies that the Bank's open market interventions do influence in the present model the economy's "liquidity", i.e. the composition of the consumers' portfolios, but cannot alter, within the period, private aggregate money wealth (1).

Examination of the above equations $(\mathcal{C})$, $(\mathcal{D})$, $(\mathcal{E})$ shows that the Bank has, as usual, one degree of freedom when choosing its money supply, and that it can try to influence the current short run equilibrium position by linking ΔM with current economic observables (p_1,r_1). We shall restrict ourselves, for simplicity, to two specific policies, one in which the Bank pegs the interest rate, and the other in which it pegs the money supply ΔM .

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(1) Aggregate money wealth will increase however from one period to the next, owing to interest payments made by the Bank on outstanding bonds.

Before looking at what conventional Neoclassical theory has to say in such cases, we review the particular properties of individual demand functions which are usually postulated in that kind of theorizing. The first property is *Walras Law*, which states that the sum accross markets of the values of excess demands is identically zero :

$$p_1 \; Z(p_1,r_1) + \left[M^d(p_1,r_1) - M - \Delta M\right] + \frac{1}{r_1}\left[B^d(p_1,r_1) + r_1 \; \Delta M - B\right] = 0$$

for every p_1 and r_1 . This identity is a consequence of the consumers' budget constraints.

The second class of assumptions are, as usual, homogeneity postulates stating that, for each consumer a , the excess demand functions z_a , the demands for money and for bonds m_a^d and b_a^d , are homogenous of degree 0 and 1, respectively, with respect to the current prices of goods p_1 and the money wealth $\bar{m}_a + (\bar{b}_a/r_1)$. Such postulates are traditionally justified by the argument that "only real magnitudes matter", or by the assumption that expected prices of goods are unit elastic with respect to current prices p_1 .

Pegging the Interest Rate

We first consider the case where the Bank pegs the interest rate r_1 , and thus has an infinitely elastic demand (supply) of perpetuities. The Bank's money supply ΔM is then endogenously determined by the equilibrium condition (E) of the bond market. Reporting this value of ΔM into the money market equation $(\mathcal{D})$ yields, as we have seen, the equilibrium condition $(\mathcal{D}_1)$.

The short run equilibrium of the system is thus described by the equations $(\mathcal{C})$ and $(\mathcal{D}_1)$, where r_1 is fixed, and the vector p_1 is variable. As a matter of fact, Walras Law implies that one of these equations is redundant, so that we can focus the attention on the goods markets conditions $(\mathcal{C})$.

The Neoclassical homogeneity postulates imply the short run *Quantity Theory :* a doubling of all initial money stocks $\bar{m}_a$ and bond holdings $\bar{b}_a$, the interest rate r_1 being fixed, doubles the equilibrium prices p_1 , but has no influence upon equilibrium "real" magnitudes.

On the other hand, the same postulates imply that, given r_1 , the main regulating mechanism in the short run, is the wealth effect. Indeed, if the model is specialized to the case where there is one good, each individual' excess demand for good can be written as $z_a(1,r_1,(\bar{m}_a/p_1)+(\bar{b}_a/p_1r_1))$. A variation of p_1 thus influences the excess demand for the good exclusively by changing every individual's "real wealth".

If the good is normal, every consumer's excess demand, and thus the aggregate excess demand $Z(p_1,r_1)$ is a decreasing function of p_1 . The Neoclassical argument for asserting the existence of an equilibrium, given the interest rate, is then, as usual, that a large (resp. low) value of p_1 should lead to an aggregate excess supply (resp. demand) on the good market. By continuity, there must be an equilibrium in between. Moreover, the equilibrium is unique, and stable in any tatonnement process which responds positively to excess demand.

As we have seen before, the theoretical validity of this sort of argument appears to have been accepted by many theorists today, but its empirical relevance has been seriously disputed. We shall see, and this should be no surprise to the reader, that the argument is in fact theoretically incorrect, because it neglects an important regulating mechanism, namely the intertemporal substitution effects which are generated by a variation of current prices.

Pegging the Money Supply

We now turn to the case where the Bank pegs its money supply ΔM at a value such that $\Delta M + M > 0$, this condition meaning that the final money stock must be positive. The associated equilibrium values of p_1 and r_1 are then defined by the system of equations

(*C*) $$Z(p_1,r_1) = 0 \quad ,$$

(*D*) $$M^d(p_1,r_1) = M + \Delta M \quad ,$$

(*E*) $$B^d(p_1,r_1) + r_1 \, \Delta M = B \quad ,$$

where ΔM is given exogenously.

As an incidental remark, it is easily checked that the Neo-classical homogeneity postulates imply the following version of the short run *Quantity Theory* : a doubling of all initial money stocks $\bar{m}_a$ and bond holdings $\bar{b}_a$, and of the Bank's money supply ΔM, leads to a doubling of the equilibrium prices p_1 , but leaves unchanged the equilibrium interest rate r_1 and all equilibrium "real" magnitudes. By contrast, a change of ΔM alone will have "real" effects, according to this viewpoint.

Most theorists would be willing to admit readily that the above system of equations has a solution for arbitrary values of ΔM , i.e. that the Bank has full control over the money supply, at least in this simple context where there is no private banking sector. There is considerable disagreement, however, about the mechanisms which make possible to achieve an equilibrium. We briefly reviewed the debate concerning the wealth effect associated with a variation of current prices. The variety of opinions on the possible consequences of a variation of the interest rate is even more confusing.

The origin of this variety can be understood if one considers the impact on demand functions of an increase of r_1 . There is first a wealth effect, since each consumer's money wealth $\bar{m}_a + (\bar{b}_a/r_1)$ goes down, which should decrease both consumption and savings. On the other hand, an increase of r_1 alters the terms at which present goods can be exchanged for future goods. This intertemporal substitution effect should decrease current consumption, and increase savings : the demands for money and for bonds should both go up. Lastly, the increase of the interest rate should make perpetuities more attractive by comparison with money. Substitution between bonds and money should yield accordingly an increase of the demand for bonds, and a decrease of the demand for money.

According this heuristic argument, the aggregate excess demand for goods $Z(p_1,r_1)$ should be inversely related to the interest rate. The impact of a variation of r_1 on the demands for money and for bonds is more ambiguous, as it depends on the importance given to the various effects we just discussed.

At one end of the spectrum, there is the "monetarist" view that money demand displays little sensitivity to the interest rate. An extreme version of a model of this type would be to say that $M^d(p_1,r_1)$ is actually independent of r_1 . In the simple "macroeconomic" case where there is one good, the equilibrium price p_1 could be viewed as determined by the money equation $(\mathcal{D})$. The equilibrium level of r_1 would be found in turn by looking, say, at the good equation $(\mathcal{C})$. From such a viewpoint, variations of the interest rate make possible to achieve an equilibrium essentially through wealth and intertemporal substitution effects.

At the other end of the spectrum, there are a number of economists who discount the wealth and intertemporal substitution effects of a variation of the interest rate, and who believe that substitution between money and bonds, which is at the heart of the Keynesian "liquidity preference" theory, is an important part of the story. An extreme version of this viewpoint would be to say that $Z(p_1,r_1)$ is actually independent of r_1 . When there is one good, the equilibrium price p_1 can then be found by solving the good market equation $(\mathcal{C})$. The existence of such a solution is asserted by appealing to the wealth effect associated with a variation of p_1 . Given such a value of p_1 , and an arbitrary rate of interest r_1 , the equilibrium of the other markets would require a money creation by the Bank, say $\Delta M(p_1,r_1)$, which is given by the two following and, by Walras Law, equivalent expressions :

$$\Delta M(p_1,r_1) = M^d(p_1,r_1) - M$$

$$= \frac{1}{r_1}\left[B - B^d(p_1,r_1) \right] \quad .$$

The problem is then to find a value of r_1 such that $\Delta M(p_1,r_1)$ is equal to the given money supply ΔM . The argument rests essentially here upon an alleged substitution between money and bonds. Since an increase of r_1 should make bonds more attractive realtively to money, $\Delta M(p_1,r_1)$ should be a decreasing function of the rate of interest rate. Moreover, if r_1 is close to zero, "almost everybody prefers cash to holding a debt which yields so low a rate of interest"(1). The product $r_1 \; \Delta M(p_1,r_1)$ should then be approximately equal to B , implying that $\Delta M(p_1,r_1)$ largely exceeds ΔM . On the other hand, if r_1 is large, almost nobody should hold money. $\Delta M(p_1,r_1)$ would then be close to $-M$, which is less than the money supply ΔM . It is clear that, by continuity, an equilibrium value of r_1 should then exist in between, and that furthermore, such an equilibrium value is stable in any Walrasian tatonnement process. As the argument applies to an arbitrary value of ΔM , the Bank would have in fact full control over the money supply (2).

Most economists fall in between these two categories, and believe that an equilibrium can be achieved as a result of the various mechanisms which we described. Most theories give a more or less predominant role to the substitution between money and bonds which is

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(1) See Keynes (1936, Chap. 15).

(2) However, in this extreme "Keynesian" version, a variation of ΔM has no effect upon the equilibrium of the real sector.

engineered by a variation of the rate of interest. It is therefore worthwhile to have a quick look at the usual microeconomic explanations of the phenomenon.

The first explanation was provided by Keynes himself when introducing the notion of liquidity preference, and is based on the assumption that consumers have certain and inelastic expectations about future interest rates [(1)]. The explanation is in fact still popular in modern macroeconomic textbooks [(2)].

The argument is quite simple. Consider a consumer who has to decide in which form to hold his savings. If the current interest rate is r_1 , and if the consumer expects with certainty the interest rate r_2 to prevail in the future, it is clear that savings will be invested wholly in bonds if $(1/r_2) + 1 > (1/r_1)$, and wholly in money if the inequality is reversed. Now suppose that there is a continuum of infinitesimal consumers who hold different expectations, and that these expectations do not depend upon the current rate, for instance because consumers believe that current variations of r_1 are only transitory. The aggregate demand for money will then be a smooth, decreasing function of the current interest rate, since as r_1 rises, there will be more transactors who switch from money into bonds. Moreover, if r_1 is close to zero, almost everybody prefers cash, while nobody wishes to hold money when r_1 is large.

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(1) See Keynes (1936, Chaps 13,15).

(2) See, e.g. Crouch (1972, Chap. 4) or Ott, Ott and Yoo (1975).

This explanation is useful in the sense that it provides a justification of the theory of liquidity preference at the macroeconomic level. It has some drawbacks at the microeconomic level, since it predicts that every individual holds either bonds or money, but never both assets. The difficulty has been eliminated by Tobin's seminal contribution who showed that an expected utility maximizing risk averse transactor would diversify his portfolio if he is uncertain about the future interest rate[1]. Assuming that all consumers' probability distributions over future bond prices are inelastic with respect to the current rate of interest yields then the desired properties of the aggregate demand for money.

Both explanations give a rationale to the Keynesian theory of liquidity preference via inelastic expectations. As appealing as it is, the argument has been criticized, however, on the ground that it requires exceedingly strong assumptions, since *all* consumers' interest rate expectations must apparently be inelastic.

More importantly, from our viewpoint, all the reasonnings which we reviewed suffer from the defect of being of a partial equilibrium nature.The widespread belief that monetary authorities can control the money supply is to be taken accordingly at best as an act of faith, for it is not the result of a coherent general equilibrium analysis. Our purpose in the sequel is to built such a theory, thereby uncovering the sort of assumptions which are needed in order to ensure the existence of a short run equilibrium in the simple economy under consideration.

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(1) See J. Tobin (1958), and also K. Arrow (1970, Chap. 3).

2. CONSUMERS' CHARACTERISTICS.

The real part of the model is, as we said, the same as in the previous chapters. The consumers' "real" characteristics at date 1 are thus as before the length n_a of their remaining lifetime, their preferences u_a among consumption streams, and their endowments of goods e_{at} $t = 1, \dots, n_a$ (1).

We make the usual assumptions :

(a) *The utility function* u_a *is continuous, increasing and strictly quasi-concave, for each* a .

(b) *All components of the endowments vectors* e_{at} *are positive, for each* a *and* t .

Each consumer owns at the beginning of date 1 a stock of money $\bar{m}_a$, which is assumed to include interest payments on bonds, and a stock of perpetuities $\bar{b}_a$. The initial aggregate stocks of money $M = \sum_a \bar{m}_a$, and of bonds $B = \sum_a \bar{b}_a$, will be assumed to be positive in the sequel.

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(1) The usual caveat applies here. It is assumed that there are consumers whose lifetime extends beyond the current period ($n_a \geqq 2$). There may be consumers for whom $n_a = 1$, but the present short run analysis will not rely on them explicitly.

3. SHORT RUN DEMANDS.

We begin the analysis by looking at an individual's behaviour, and consider to this effect a typical consumer (we drop the index a momentarily), who is faced at date 1 by the price system p_1 and the interest rate r_1 (or equivalently the price of bonds $s_1 = 1/r_1$). We wish to describe how the consumer chooses his current consumption $c_1 \geqq 0$, and his demands for money $m_1 \geqq 0$, and for perpetuities $b_1 \geqq 0$.

The consumer must plan as well his demands $(c_t , m_t , b_t) \geqq 0$ over his horizon, for $t = 2, \ldots, n$. His choices will thus depend crucially on how his expectations are formulated. We noticed already when reviewing the literature, that a consumer will decide to hold either bonds or money but not both when he has certain expectations about future interest rates, and that, on the contrary, he may diversify his portfolio if he has uncertain expectations. Although the latter assumption is surely more realistic, we shall work with deterministic expectations in the sequel. This is merely a convenience which keeps the mathematical exposition as simple as possible. Assuming probabilistic expectations would yield indeed the same qualitative conclusions (1).

Let p_t be the consumer's expected prices of goods, and let r_t stand for his forecasts of future interest rates (or equivalently, $s_t = 1/r_t$ the expected prices of perpetuities). The consumer will seek

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(1) For a mathematical analysis of a similar model with probabilistic expectations, the reader may consult Grandmont and Laroque (1976), or Grandmont (1977, Section 3.3).

to maximize his utility under the current and expected budget constraints. This decision problem can be formulated as :

(I)

Maximize u *with respect to* $(c_t , m_t , b_t) \geqq 0$ *for* $t = 1 ,\ldots, n$, *subject to the budget constraints* :

$$p_1 c_1 + m_1 + s_1 b_1 = p_1 e_1 + \bar{m} + s_1 \bar{b} \quad ,$$

$$p_t c_t + m_t + s_t b_t = p_t e_t + m_{t-1} + (s_t+1) b_{t-1} \quad ,$$

for $t = 2 ,\ldots, n$.

This problem has a solution when current and expected prices, and current and expected interest rates are positive. Moreover, the utility function being strictly quasi-concave, the optimum consumption program, and thus current consumption c_1 , is uniquely determined. Consider now the *expected yield* g of perpetuities at date 1 which is implied by the consumer's expectations, and which is defined by

$$1 + g = (s_2 + 1)/s_1 \quad .$$

It is clear that if $g > 0$, the consumer prefers to hold his savings, if any, in the form of bonds. The corresponding optimum current portfolio is unique, and involves a money demand m_1 equal to zero. If on the other hand $g < 0$, the optimum current portfolio is again unique, but involves this time a zero demand for bonds. Lastly, when the expected yield g is zero, the optimum value of current savings $m_1 + s_1 b_1$ is uniquely defined, but the consumer is indifferent between holding money or bonds.

The *Absence of Money Illusion* property takes here the following simple form, as the reader will easily check. If (c_t , m_t , b_t) is a solution of (I) above, then $(c_t , \lambda m_t , \lambda b_t)$ will be a solution of (I) too whenever the initial stocks of assets $\bar{m}$ and $\bar{b}$, and current and expected prices $p_1 ,\ldots, p_n$ are multiplied by the positive parameter λ .

In order to complete the description of the consumer's behaviour, the dependence of expectations upon the trader's information has to be specified. As in the previous models, we shall keep implicit the influence of his information on past history, and single out the impact of the current prices and of the current interest rate. Expected prices and expected interest rates will thus be denoted $\psi_t(p_1 , r_1)$ and $\rho_t(p_1 , r_1)$ respectively, for $t = 2 ,\ldots, n$.

When expectations are replaced by these expressions, the solutions of (I) depend only on initial money wealth $\bar{m} + (\bar{b}/r_1)$, and on p_1 and r_1 . If one reintroduces at this stage the consumer's index a , this yields an excess demand function for goods $z_a(p_1 , r_1 , \bar{m}_a + (\bar{b}_a/r_1))$. The corresponding demands for money and bonds will be denoted $m^d_a(p_1 , r_1 , \bar{m}_a + (\bar{b}_a/r_1))$ and $b^d_a(p_1 , r_1 ,\bar{m}_a + (\bar{b}_a/r_1))$, respectively. We remarked previously that the solution of (I) exhibited some indetermination when the expected yield on bonds was zero. In that case, the preceding expressions will represent an arbitrary member of the solution set.

In view of the Absence of Money Illusion property stated above, it is clear that the Neoclassical homogeneity postulates, i.e. the homogeneity of degree 0 (respectively 1) of the excess demand for goods (respectively the demands for money and bonds) with respect to the initial money wealth $\bar{m}_a + (\bar{b}_a/r_1)$ and current prices p_1 , hold if, and in general only if, expected prices p_t are proportional to current prices and if expected interest rates are independent of them. These assumptions will not be retained here, however, as they will be shown to be typically inconsistent with the existence of a short run equilibrium.

A proportional increase of current prices from p_1 to λp_1 will thus have more complex consequences on a consumer's behaviour than is usually posited in a Neoclassical World. There is first a "wealth effect" which is due to the change in the "purchasing power" of initial money wealth $\bar{m}_a + (\bar{b}_a/r_1)$. This wealth effect would occur alone if expected prices moved proportionately to current prices and if expected interest rates were unchanged. As these assumptions on expectations are typically not satisfied, the change of current prices will yield an additional "intertemporal substitution effet" which is due to the alteration of the terms at which future goods can be exchanged for current ones. Lastly, there may be a substitution between money and bonds if the sign of the consumer's expected yield on perpetuities is reversed. Similarly, an increase of the current interest rate will yield a wealth effect (through the change of the initial money wealth $\bar{m}_a + (\bar{b}_a/r_1)$) as well as an intertemporal substitution effect, and a possible substitution between bonds and money.

Summing all individual demands over all consumers gives an aggregate excess demand for goods, and an aggregate demand for money and bonds :

$$Z(p_1 , r_1) = \sum_a z_a(p_1 , r_1 , \bar{m}_a + (\bar{b}_a/r_1)) ,$$

$$M^d(p_1 , r_1) = \sum_a m_a^d(p_1 , r_1 , \bar{m}_a + (\bar{b}_a/r_1)) ,$$

$$B^d(p_1 , r_1) = \sum_a b_a^d(p_1 , r_1 , \bar{m}_a + (\bar{b}_a/r_1)) ,$$

where the influence of the initial stocks of assets is kept implicit.

If one takes momentarily the Bank's money creation through the purchase of perpetuities as a parameter ΔM , the market clearing conditions at date 1 read then :

$(\mathcal{C})$ $$Z(p_1 , r_1) = 0 ,$$

$(\mathcal{D})$ $$M^d(p_1 , r_1) = M + \Delta M ,$$

$(\mathcal{E})$ $$B^d(p_1 , r_1) + r_1 \Delta M = B .$$ (1)

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(1) The conditions $(\mathcal{D})$ and $(\mathcal{E})$, and in fact any condition involving $M^d(p_1 , r_1)$ and/or $B^d(p_1 , r_1)$ have to be interpreted with some care, since, with a finite number of consumers, the aggregate demands for money and bonds may exhibit some degree of indeterminacy. They mean that there exist particular choices from every individual's set of solutions of $(\mathcal{I})$ for the given configuration (p_1 , r_1), such that the market clearing conditions are satisfied.

It is possible to remove this aggregate indeterminacy by assuming, as it is often done in macroeconomic textbooks, a continuum of infinitesimal consumers who hold diverse expectations about future interest rates. This approach would have yielded the same results. We preferred not to use it, in order to avoid unnecessary technicalities.

This system is formally identical to the Neoclassical system which was discussed in Section 1 when reviewing the literature. In view of the consumers' current budget constraints, it satisfies *Walras Law* :

$$p_1 Z(p_1 , r_1) + \left[M^d(p_1 , r_1) - M - \Delta M\right] + \frac{1}{r_1}\left[B^d(p_1 , r_1) + r_1 \Delta M - B\right] = 0$$

for every p_1 and r_1 . It differs from the Neoclassical system in one important respect, however, for we did not assume any homogeneity properties. We shall see that in order to ensure the existence of a short run equilibrium, one has to make assumptions on expectations which violate the Neoclassical homogeneity postulates. The various versions of the short run *Quantity Theory* which were discussed in Section 1 are then no longer valid.

4. PEGGING THE INTEREST RATE.

We are interested first in the case where the Bank pegs the interest rate r_1 . The Bank's supply or demand of perpetuities is then assumed to be infinitely elastic : the money creation ΔM becomes engogenously determined by the equilibrium of the bond market. The equilibrium conditions associated with this policy are thus obtained by eliminating ΔM between $(\mathcal{D})$ and $(\mathcal{E})$, which yields :

$$(\mathcal{C}) \qquad Z(p_1,r_1) = 0 \quad ,$$

$$(\mathcal{D}_1) \qquad M^d(p_1,r_1) = M + \frac{1}{r_1}\left[B - B^d(p_1,r_1)\right] \quad ,$$

where the interest rate is fixed. By Walras Law, one of these conditions is actually redundant, and one can focus the attention on the real sector $(\mathcal{C})$ when looking for an equilibrium.

Neoclassical theorists claim that a short run equilibrium typically exists in such case, by appealing to the wealth effect which results from a variation of the current prices p_1 . We shall see that such a statement is theoretically incorrect, and that one must reinforce the wealth effect with a strong stabilizing intertemporal substitution effect to guarantee the existence of an equilibrium.

In order to see this point, it is convenient here again to look at the simple case where there is only one good, and where each transactor makes plans for the current and the next periods only. A typical consumer's current and expected budget constraints then read

$$p_1c_1 + m_1 + s_1b_1 = p_1e_1 + \bar{m} + s_1 \bar{b}$$

$$p_2c_2 + m_2 + s_2b_2 = p_2e_2 + m_1 + (s_2+1)\ b_1 \quad .$$

If the consumer's expected yield on perpetuities g , which is defined by $1+g = (s_2+1)/s_1$, is positive, he will decide to hold his savings wholly in bonds. One can then set without any loss of generality $m_1 = 0$ in the foregoing constraints. The optimum consumption program is obtained in such a case by maximizing the trader's utility function subject the following inequalities, which are obtained from the budget constraints by eliminating the asset variables :

$$p_1c_1 + \frac{p_2c_2}{1+g} \leqq p_1e_1 + \frac{p_2e_2}{1+g} + \bar{m} + s_1 \bar{b}$$

$$p_1c_1 \leqq p_1e_1 + \bar{m} + s_1 \bar{b} \quad .$$

The corresponding region of feasible current and future consumptions is pictured in Fig. 1.a below.

When the expected yield g is negative, the same procedure applies with $b_1 = 0$. The associated feasible region for (c_1,c_2) is then given by :

$$p_1 c_1 + p_2 c_2 \leqq p_1 e_1 + p_2 e_2 + \bar{m} + s_1 \bar{b}$$

$$p_1 c_1 \leqq p_1 e_1 + \bar{m} + s_1 \bar{b}$$

and is represented in Fig. 1.b [1].

Fig. 1.a

Fig. 1.b

Suppose now that the consumer's wealth $\bar{m} + s_1 \bar{b}$ is positive. It is clear from Fig. 1.a that, when the expected yield g is positive, the optimum current consumption will exceed e_1 if and only if the ratio $p_2/(1+g)\, p_1$ is greater than the marginal rate of substitution u_2'/u_1' at the point α . Since p_2/p_1 exceeds $p_2/(1+g)\, p_1$ in such a case, this condition reads :

$$\frac{p_2}{p_1} > \frac{p_2}{(1+g)p_1} > \frac{u_2'}{u_1'}\bigg|_{\alpha} .$$

A similar reasoning on Fig. 1.b shows that when g is negative, $c_1 > e_1$ if and only if

$$\frac{p_2}{(1+g)p_1} > \frac{p_2}{p_1} > \frac{u_2'}{u_1'}\bigg|_{\alpha} .$$

---

(1) Both diagrams apply of course when g = 0 .

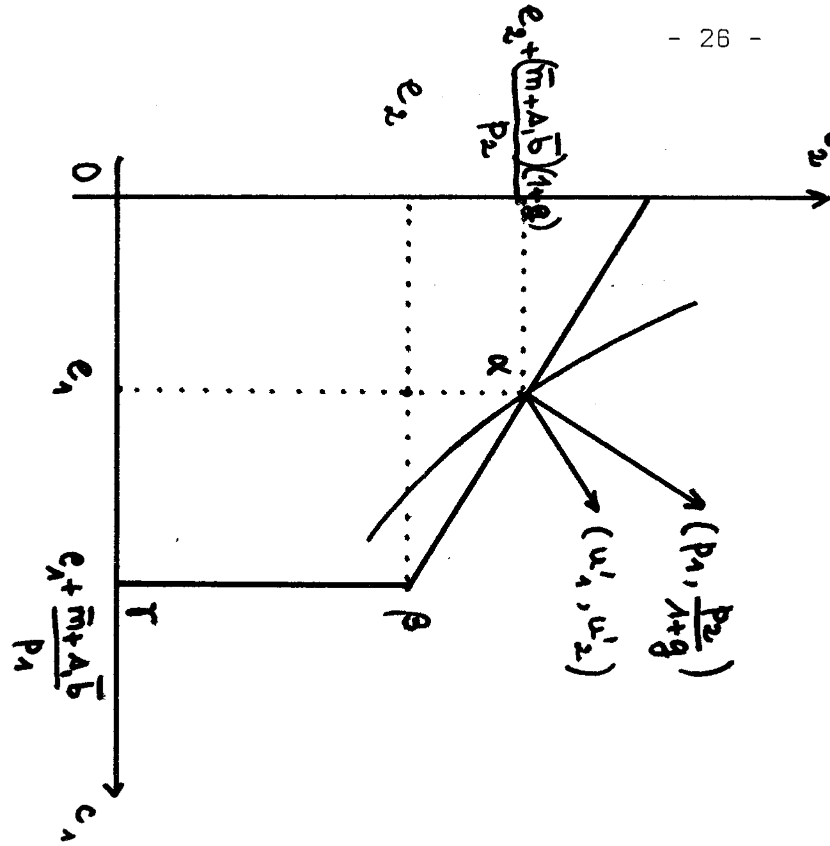
c_2
$e_2 + \frac{\bar{m}+A_1\bar{b}}{p_2}(1+g)$
e_2
0
e_1
$e_1 + \frac{\bar{m}+A_1\bar{b}}{p_1}$
c_1
α
β
γ
$(p_1, \frac{p_2}{1+g})$
(u'_1, u'_2)

Fig. 1.a

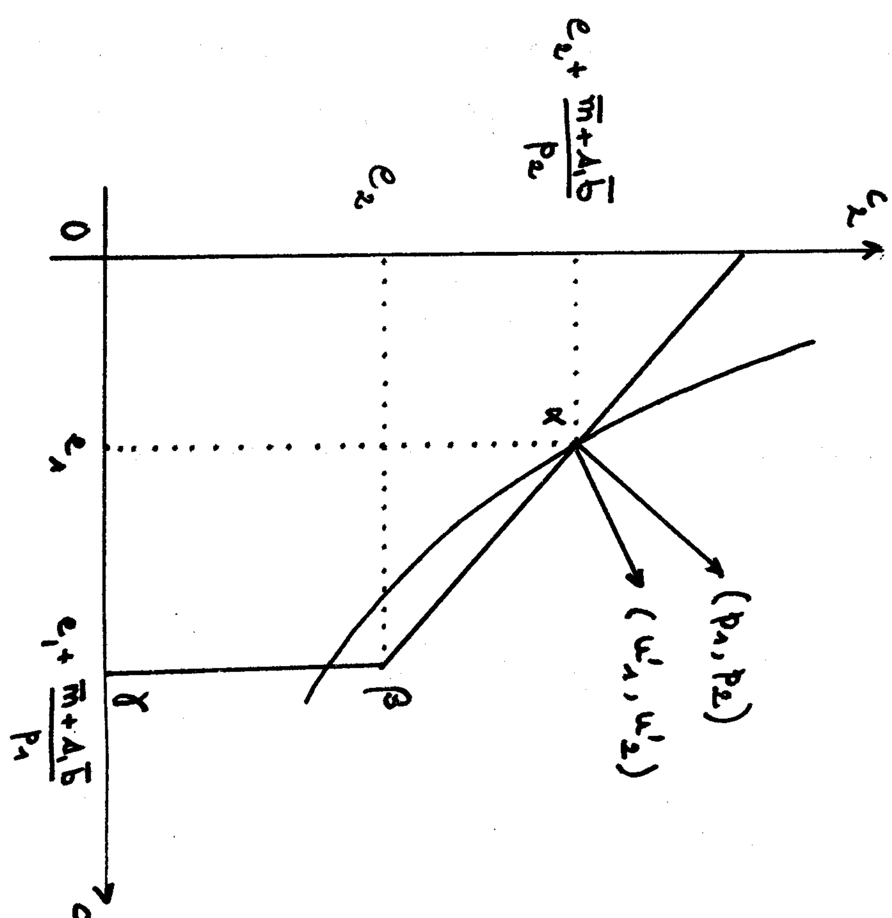
c_2
$e_2 + \frac{\bar{m}+A_1\bar{b}}{p_2}$
e_2
0
e_1
$e_1 + \frac{\bar{m}+A_1\bar{b}}{p_1}$
c_1
α
β
γ
(p_1, p_2)
(u'_1, u'_2)

Fig. 1.b

It is easy to construct from these premises an example where there is an *aggregate excess demand* on the good market at all current prices. Assume that the typical consumer's utility function is of the form $w(c_1) + \delta w(c_2)$, where w is strictly concave and differentiable, and $0 < \delta \leqq 1$. This implies that the marginal rate of substitution u_2'/u_1' is a decreasing function of future consumption. Whether g is positive or not, the individual desired consumption c_1 exceeds then e_1 if both ratios p_2/p_1 and $p_2/(1+g)\ p_1$ are greater than or equal to the marginal rate of substitution at the endowment point, i.e. $\delta\ w'(e_2)/w'(e_1)$. If *all* consumers' expectations concerning future prices and interest rates, as functions of p_1 and r_1 , are biased upwards in this way, there will be an aggregate excess demand on the good market at all current prices p_1 . One can make then the size of aggregate excess demand arbitrarily small, through wealth effects, by increasing the current price p_1 , since the point β on the diagrams converge to the endowment point (e_1,e_2) as p_1 tends to infinity. But there does not exist a short run equilibrium where money has positive value.

It is equally easy to construct an example of a persistent aggregate *excess supply* on the good market. Assume that for all consumers, the marginal rate of substitution is bounded below by $\nu > 0$ when one moves up the vertical line going through the endowment point (e_1,e_2). It is straightforward to check that, if each consumer's ratio p_2/p_1 , as a function of p_1 and r_1 , is bounded above by ν , there is an aggregate excess supply of the good at all values of p_1 [(1)]. Again, there can be no short run equilibrium.

---

(1) The argument works equally well by using the ratio $p_2/(1+g)\ p_1$.

The two examples apply in particular in the "Neoclassical" case, where expected prices are unit elastic with respect to current prices, and expected interest rates are independent of them. They are in addition, independent of the sizes of the initial stocks of assets M and B. They invalidate, therefore, the Neoclassical position, which claims that one should be able, given the interest rate, to equilibrate the market through the wealth effect which is engineered by a variation of current prices.

The origin of the inexistence phenomenon which we just described is to be found of course in the rigidities of the individual ratios p_2/p_1 and $p_2/(1+g)\ p_1$, i.e. in the absence of a strong, stabilizing substitution between current and future consumption. In order to make such a substitution effect operational, what is needed, here again, is some insensitivity of expectations with respect to current prices.

In order to check this point, let us go back to the simple case represented in the foregoing diagrams, and look at what happens when p_1 tends to infinity. Each consumer's excess demand c_1-e_1 is bounded above by his real wealth $(\bar{m} + s_1\ \bar{b})/p_1$, which goes to 0 . It suffices therefore that c_1-e_1 becomes negative for at least one consumer to get eventually an aggregate excess supply on the good market. Now suppose that there is an insensitive consumer whose expected price p_2 and expected interest rate r_2 do not vary with the current price p_1 . This consumer's expected yield g is then actually independent of p_1 , which leads to consider Fig. 1.a if it is positive, and Fig. 1.b otherwise. In either case, the intertemporal budget line $\alpha\beta$ rotates around the point α , and tends to be almost vertical, which yields the desired result. Intertemporal substitution, together with the wealth effect, ensures the appearance of an aggregate excess supply on the good market as p_1 tends to ∞ .

Consider next the case where p_1 tends to 0. The insensitive consumer's intertemporal budget line rotates again around α and becomes almost horizontal. This trader's desired current consumption is then bound to go to infinity, thereby generating an aggregate excess demand on the good market. Thus, by continuity, one should be able to equilibrate the good market, and therefore the whole system.

The insensitive consumer ensures the presence of a stabilizing intertemporal substitution which, together with the wealth effect, guarantees the existence of an equilibrium for a given interest rate [1]. We state now a general result along this line.

Consider a consumer a whose horizon extends beyond the current period ($n_a \geqq 2$). We shall say that his expectations are *continuous in current prices* if the functions $\psi_{at}(p_1,r_1)$ and $\rho_{at}(p_1,r_1)$ are continuous, given r_1, with respect to p_1 , for every t . Expectations will be said *bounded with respect to current prices* if, given r_1, there are two vectors $\varepsilon(r_1)$ and $\eta(r_1)$, with all their components positive, such that $\varepsilon(r_1) \leqq \psi_{at}(p_1,r_1) \leqq \eta(r_1)$, and if there are two positive numbers $\varepsilon'(r_1)$ and $\eta'(r_1)$ such that $\varepsilon'(r_1) \leqq \rho_{at}(p_1,r_1) \leqq \eta'(r_1)$, for all p_1 and t .

----

(1) The argument shows too that substitution between money and bonds need not play any role in the equilibrating process if the interest rate is given.

Such boundedness conditions lead of course to a violation of the Neo-classical homogeneity postulates, and are the key for the following existence result [1].

(1) *Let the interest rate* r_1 *be fixed. Assume (a) and (b) of Section 2, and that every consumer's expectations are continuous in current prices. Assume moreover that there is a consumer with* $n_a \geqq 2$ *and* $\bar{m}_a + (\bar{b}_a/r_1) > 0$ *, whose expectations are bounded with respect to current prices.*

Then, given r_1 *, the system (C), (*D_1*) has a solution, i.e. the Bank can peg the interest rate at the level* r_1 .

To sum up, this analysis has shown that, in contradiction with conventional theory, the wealth effect resulting from a variation of current prices, the interest rate being fixed, may be too weak to equilibrate the market, and that intertemporal substitution effects must be taken into account. However, in order to make such substitutions operational, we had to make the strong assumption that some consumer's expectations are to a large extent insensitive to a variation of current prices. Here again, one is entitled to conclude from the restrictive character of that kind of assumption that the existence of a short run equilibrium is problematic.

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(1) A proof of this result is given in Appendix E.

5. CONTROLLING THE MONEY SUPPLY.

We turn now to the case where the Bank attempts to peg its money creation at a given level $\Delta M > -M$. The short run equilibrium current prices and interest rate (p_1, r_1) are then the solutions of the following set of conditions :

(C) $$Z(p_1, r_1) = 0 \quad ,$$

(D) $$M^d(p_1, r_1) = M + \Delta M \quad ,$$

(E) $$B^d(p_1, r_1) + r_1 \, \Delta M = B \quad ,$$

where M is given.

This system, of course, needs not admit a solution. The examples which were developed in the preceding section showed that an equilibrium might not exist on the goods markets, for a given interest rate, if the consumers' expectations were biased upwards, or downwards. These inexistence examples can be extended, trivially, so as to be valid for all interest rates. In that case, there can be no pair (p_1, r_1) which satisfies equations (C).

Conventional theory asserts nevertheless that the foregoing system has a solution for every ΔM, i.e. that the monetary authority has complete control of the money supply. The purpose of the present section is to uncover the kind of assumptions which validate such a proposition.

It is convenient, to this effect, to use once again the following heuristic procedure. Given an arbitrary r_1 , let p_1 be a corresponding solution of (C). As this configuration (p_1,r_1), one needs a creation of money $\Delta M(p_1,r_1)$ by the Bank in order to bring the two other markets into equilibrium, which is given by the following and, according to Walras Law, equivalent expressions :

$$\Delta M(p_1,r_1) = M^d(p_1,r_1) - M$$

$$= \frac{1}{r_1}\left[B - B^d(p_1,r_1)\right] \quad .$$

Solving the whole system amounts then to finding a value of r_1 , and a corresponding solution p_1 of (C), such that $\Delta M(p_1,r_1)$ equals the given money supply ΔM .

The validity of the procedure presupposes that (C) can be solved in p_1 for each interest rate. We shall assume that it is indeed the case, for instance because the conditions of (1) of Section 4 are fulfilled for each r_1 .

Our strategy will be to find conditions which ensure that $\Delta M(p_1,r_1)$ approaches -M when the interest rate is large, and conversely, that $\Delta M(p_1,r_1)$ becomes very large when r_1 tends to 0 , the prices p_1 moving correlatively in each case so as to maintain the equilibrium of the real sector [(1)]. Intuitively, by continuity, one should then be able to equilibrate the entire system.

It is easily seen that $\Delta M(p_1,r_1)$ is equal to -M , or equivalently that $M^d(p_1,r_1)$ is equal to 0 in the present model, when the interest rate is large. Indeed, if $r_1 \geqq 1$, or alternatively if the price s_1 of perpetuities is less than or equal to unity, every consumer's expected yield on bonds is positive. Nobody wishes accordingly to hold money.

It remains to see when $\Delta M(p_1,r_1)$ has the desired property as r_1 gets close to zero. The "Keynesian" viewpoint of this matter has been recalled in Section 1. It supposes that *all* consumers' interest rate expectations are inelastic with respect to a variation of the current rate. With a finite number of transactors as here, every individual's expected yield on perpetuities becomes then negative as r_1 goes to 0 . Everybody switches eventually from money into bonds, in which case $\Delta M(p_1,r_1)$ is equal to B/r_1 , and thus tends to infinity.

Some substitution between money and bonds is clearly needed in order to get the desired result as the interest rate goes down.

---

(1) In the remainder of this section, it will be always assumed that changes of the interest rate are accompanied by such compensating moves of prices.

Otherwise, $\Delta M(p_1,r_1)$ would be equal to $-M$ for all r_1 . The foregoing Liquidity Preference argument is however too demanding, and is accordingly based upon exceedingly strong assumptions on expectations, for it requires *full* substitution between money and perpetuities eventually. But this is obviously asking too much. The money creation

$$\Delta M(p_1,r_1) = \frac{1}{r_1}\left[B - B^d(p_1,r_1) \right]$$

may well tend to infinity as r_1 decreases, without the aggregate demand for bonds going to 0 .

One may expect therefore to be able to weaken significantly the assumptions underlying the Keynesian Liquidity Preference theory. It will shown that it is indeed the case, provided that wealth and intertemporal substitution effects are properly integrated in the analysis. In fact, it will be enough to strengthen the conditions of (*1*) of Section 4, by postulating the presence of *a single consumer who holds initially some bonds, and whose price and interest rate expectations are insensitive to variations of* p_1 *and of* r_1 .

In order to see this point more precisely, it is convenient to specialize the model to the case where there is one good, and where all consumers make plans for the current and the next dates only. Assume that there is a particular consumer (we drop his index a to ease the exposition) with $\bar{b} > 0$, whose price and interest rate expectations , p_2 and r_2 , are actually independent of p_1 and r_1 .

There is evidently substitution between bonds and money at the level of this particular consumer. If r_1 is large enough, this consumer's demand for money, like everybody else's, is equal to zero. If r_1 gets close to 0 , this consumer's demand for bonds vanishes (but others may behave differently).

At such low interest rates r_1 , given p_1 , this consumer's optimum actions result from the maximization of his utility function under the budget constraints :

$$p_1c_1 + m_1 = p_1e_1 + \bar{m} + \frac{\bar{b}}{r_1} \quad ,$$

$$p_2c_2 = p_2e_2 + m_1 \quad ,$$

where the demand for bonds b_1 , as well as final money and bond holdings m_2 and b_2 , have been set equal to zero, without any loss of generality.

We shall show now that under our assumption, this particular consumer's money demand m_1 tends to infinity as r_1 goes to 0 , while p_1 moves to maintain the equilibrium of the good market. This will imply that under these circumstances, aggregate money demand $M^d(p_1,r_1)$, and thus $\Delta M(p_1,r_1)$, tends to infinity as well.

We distinguish two cases.

. *The product* p_1r_1 *tends to* 0 . The current budget constraint can then be written

$$p_1r_1c_1 + r_1m_1 = p_1r_1e_1 + r_1\bar{m} + \bar{b} \quad .$$

The individual's current consumption c_1 is surely bounded, since the good market has to be in equilibrium. Thus the product $r_1 m_1$ approaches $\bar{b}$, which is positive : the consumer's demand for money m_1 goes to $+\infty$. The essential mechanism here is the wealth effect induced by a variation of r_1 .

. *The product* $p_1 r_1$ *is bounded away from* 0 . Then, the region of the feasible consumption programs (c_1, c_2) is obtained by eliminating the variable m_1 from the budget constraints. It is thus defined by

$$p_1 c_1 + p_2 c_2 = p_1 e_1 + p_2 e_2 + \bar{m} + \frac{\bar{b}}{r_1} ,$$

$$p_1 c_1 \leqq p_1 e_1 + \bar{m} + \frac{\bar{b}}{r_1} ,$$

and is represented below in Fig. 2.

Fig. 2

Since $p_1 r_1$ is bounded away from 0 , the point β on the diagram stays at a finite distance of the endowment point (e_1, e_2). On the other hand, the point α moves vertically towards infinity. The intertemporal budget line $\alpha\beta$ becomes more and more vertical, implying that the individual's planned consumption c_2 tends to $+\infty$. Since :

$$m_1 = p_2 (c_2 - e_2) ,$$

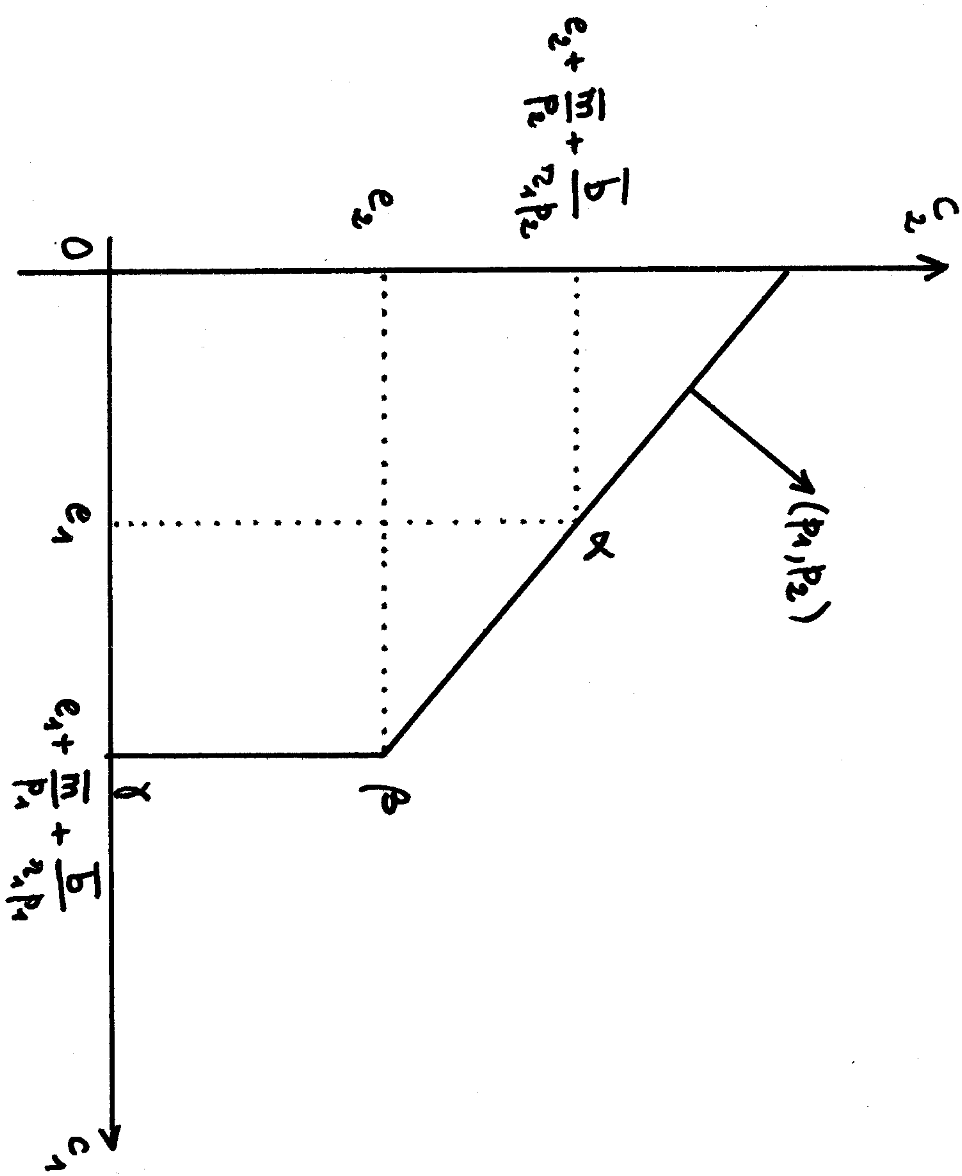
0
c_2
e_2
$e_2 + \frac{\bar m}{p_2} + \frac{\bar b}{r_1 p_2}$
c_1
e_1
$e_1 + \frac{\bar m}{p_1} + \frac{\bar b}{r_1 p_1}$
α
β
γ
(p_1, p_2)

Fig. 2

this consumer's demand for money goes to infinity as well. The essential mechanism here is the intertemporal substitution effect.

The foregoing heuristic analysis shows that the monetary authority has complete control over the money supply in the present model, provided that there is a consumer, with $\bar{b}_a > 0$, whose interest rate and price expectations are insensitive to a variation of p_1 and r_1 . We give now a general result along this line.

We shall say that a consumer's expectations are *continuous* if the functions $\psi_{at}(p_1,r_1)$ and $\rho_{at}(p_1,r_1)$ are continuous with respect to p_1 and r_1 , for all t . They will be said to be *bounded* if there are two vectors ε and η , with all their components positive, such that $\varepsilon \leqq \psi_{at}(p_1,r_1) \leqq \eta$, and two positive numbers ε' and η' such that $\varepsilon' \leqq \rho_{at}(p_1,r_1) \leqq \eta'$, for all p_1 and r_1 , and all t . This inelasticity condition, which violates the Neoclassical homogeneity postulates, is essential for the following existence result (1).

(1) *Assume (a) and (b) of Section 2, and that all consumers' expectations are continuous. Assume moreover that there is a consumer, with* $n_a \geqq 2$ *and* $\bar{b}_a > 0$ *, whose expectations are bounded.*

Then, the system (C), (D), (E) has a solution for every $\Delta M > -M$ *, i.e. the Bank has complete control over the money supply.*

This result shows that the inelasticity assumptions underlying the Keynesian Liquidity Preference theory can be much weakened. One needs only a limited substitution between money and bonds if proper account is taken of wealth and intertemporal substitution effects. Full control of the money supply by means of an open market policy still requires, apparently, very strong assumptions on expectations. The qualitative conclusion of our analysis is thus, once again, that the possibility of such a control seems quite problematic.

6. THE LIQUIDITY TRAP.

It is sometimes claimed by Keynesian theorists that open market policies are ineffective when the interest rate is close to zero. We show in this section that such a "Liquidity Trap" phenomenon does exist in the present model whenever the Bank has full control over the money supply, in the sense that the lower the interest rate, the less sensitive it becomes to a given variation of the Bank's money supply.

The Liquidity Trap phenomenon is often viewed as a property of the aggregate demand for money, which is supposed to tend to infinity as the interest rate goes to zero. This assertion is usually justified by appealing to Liquidity Proference theory. At low interest rates, it is argued, almost everybody switches from bonds into cash, and thus aggregate money demand increases without bound.

That sort of argument is at best partial. It first contemplates only the consumers' choices between money and bonds, and thus neglects consumption savings decisions. A related point is that it does not say anything about how the prices of goods are supposed to behave when the interest rate goes down. Finally, it is based upon the assumptions underlying the Liquidity Preference theory, which, as we have seen, are uncessarily strong.

There is in the literature another interpretation of the Liquidity Trap, which views it as a property of the relation between short run equilibrium interest rates and equilibrium money stocks [1]. We are going to see that, with this interpretation, there is indeed a Liquidity Trap in the present model whenever the Bank has full control of the money supply.

The property can be stated differently if the Bank pegs the interest rate, or if it controls the money supply. In the first case, given r_1 , one looks at the price system p_1 which satisfies the goods markets clearing conditions (C). The corresponding equilibrium money stock is then $M^d(p_1,r_1)$, while the associated Bank's equilibrium money creation $\Delta M(p_1,r_1)$ is given by :

$$\begin{aligned}\Delta M(p_1,r_1) &= M^d(p_1,r_1) - M \quad ,\\ &= \frac{1}{r_1}\left[B - B^d(p_1,r_1)\right] \quad .\end{aligned}$$

We shall say that there is a Liquidity Trap if $M^d(p_1,r_1)$ or equivalently $\Delta M(p_1,r_1)$, tends to infinity as the interest rate decreases to 0 , the prices p_1 adjusting to clear the goods markets. But we have seen in the preceding section that such a property was true precisely under the conditions which gave the Bank complete control of the money supply, namely under the assumptions of *(1)* of Section 5. We can therefore state without further argument [2]

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(1) See, e.g. Patinkin (1965, Ch. XIV : 3).

(2) A proof of this statement is given in Appendix E.

(1) *Consider an infinite sequence of prices and interest rates* (p_1^k, r_1^k) *which satisfy the goods markets equations* (C) *for all* k , *such that* r_1^k *tends to* 0 . *Let* $M^d(p_1^k, r_1^k)$ *be a corresponding sequence of equilibrium money stocks.*

Then, under the assumptions of (1) of Section 5, $M^d(p_1^k, r_1^k)$ *tends to* $+\infty$.

When the Bank pegs its money supply, ΔM is given, and one looks at a corresponding solution (p_1, r_1) of the system (*C*), (*D*), (*E*). One can say then that there is a Liquidity Trap if equilibrium interest rates r_1 tend to 0 when the money creation ΔM increases indefinitely.

That statement presupposes obviously that there is indeed a short run equilibrium as the money supply tends to infinity. In that case, the argument is almost trivial. For the money creation ΔM is bounded above by the money value of the initial stock of bonds B/r_1 . Thus interest rates must go to 0 (at least as fast as $B/\Delta M$) when ΔM tends to infinity. Formally,

(2) *Consider an infinite sequence of money supplies* ΔM^k *which tends to* $+\infty$, *and let* (p_1^k, r_1^k) *be a corresponding sequence of equilibrium prices and interest rates. Then* r_1^k *tends to* 0 .

The above two statements show that if one is willing to admit that the Bank can fully control the money stock, then a Liquidity Trap phenomenon exists in principle, in the sense that the relation between the Bank's equilibrium creation of money ΔM and the equilibrium interest rate r_1 becomes almost "flat" when ΔM is large or when r_1 is low (see Fig. 3). In such circumstances, the impact of open market policies on interest rates, or on the economy's "liquidity", becomes relatively weak. In this model, the Liquidity Trap is however a limit phenomenon, which occurs asymptotically when the interest rate gets close to zero, or the money supply is large. The model has little to say about its actual strength, and thus about its practical relevance. One can remark nevertheless that the product of r_1 with ΔM, or with $\Delta M(p_1,r_1)$, cannot exceed, in equilibrium, the initial stock of bonds B. The curve representing the relation between equilibrium interest rates and equilibrium money creations is thus bound to lie below the hatched region in Fig. 3.

Fig. 3

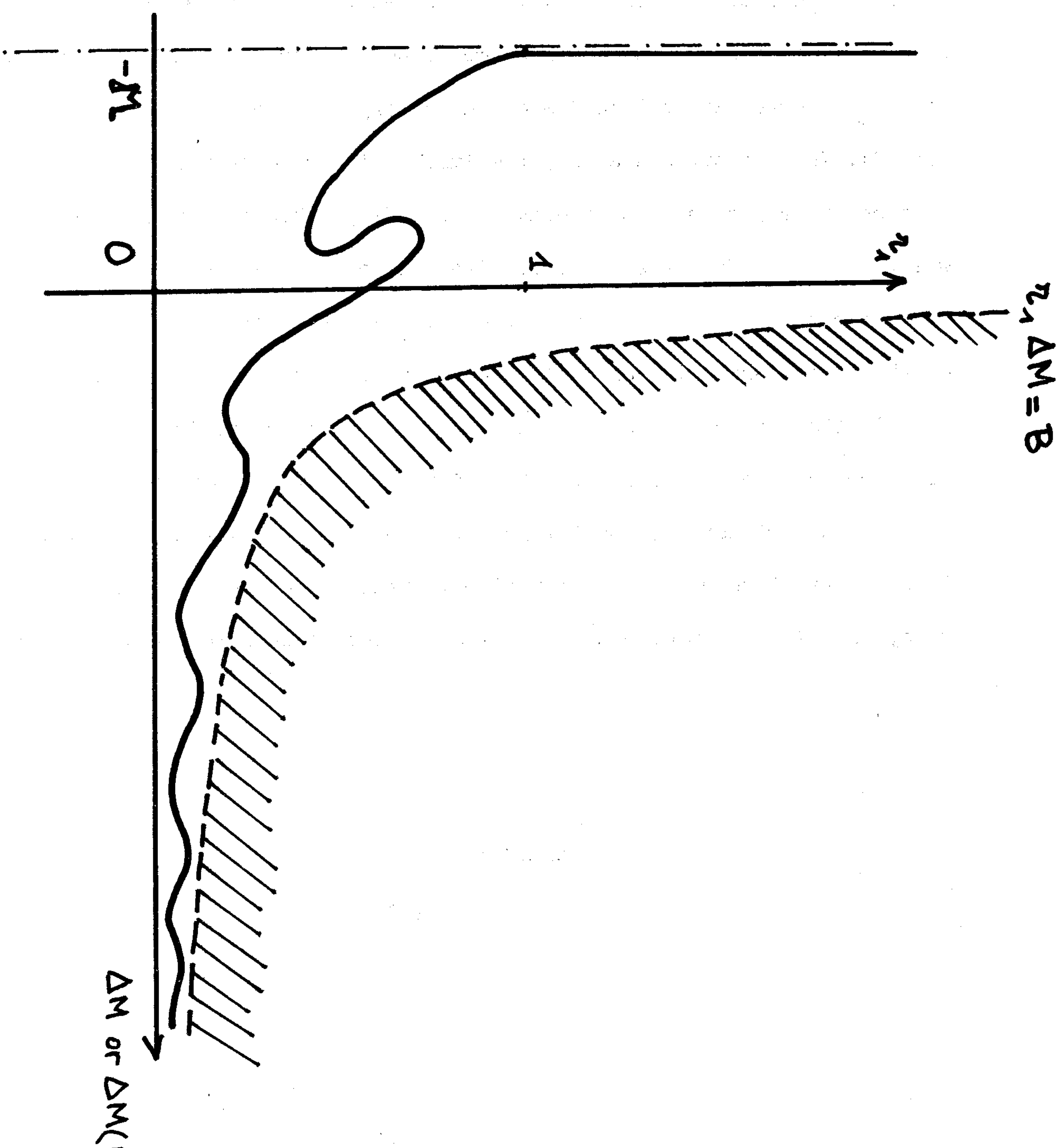

Fig. 3

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