CLASSICAL STATIONARY STATES WITH MONEY AND CREDIT *

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CLASSICAL STATIONARY STATES

WITH MONEY AND CREDIT

The aim of this lecture is to study the existence and the properties of steady states in the credit money economy which was analysed in the preceding chapter, when population is stationary. Such steady states are defined as sequences of short run equilibria (in the sense of the previous lecture), where the nominal interest rate, the relative prices of goods and the rate of inflation are constant over time. Although we have shown that the existence of a short run equilibrium was doubtful in economies of this type, steady states are of independent interest, because they may obtain as long run equilibria of other dynamic (e.g. disequilibrium) processes.

It will be established that the *Quantity Theory* and the *Classical Dichotomy* are valid propositions when applied to steady states. Real magnitudes (the relative prices of goods, the real rate of interest, the traders' consumptions) are determined by the equilibrium conditions of the goods markets. Nominal values (the money prices of goods, the rate of inflation, the nominal interest rate) are determined in turn by looking at the money sector, including the Bank's monetary policy. In particular, the money prices of goods are proportional at any time to the level of monetary aggregates such as outside money or the Bank's money supply.

These findings imply that the Bank cannot peg in the long run a real variable such as the real interest rate by its interventions on the credit market. Any attempt to do so is self-defeating, in the sense that the economy will never be able then to approach a monetary steady state. In such circumstances, the only stationary equilibrium which can obtain must be a nonmonetary one. This would mean the eventual breakdown of the monetary institution.

If the Bank wishes to preserve the monetary character of the economy, it must therefore control nominal variables only, e.g. by pegging the nominal interest rate at a constant level, or the rate growth of its money supply. These interventions on the credit market are however *neutral* in the long run, in the sense they will have no influence on the set of steady state real quantities.

It will be shown that there are two types of monetary steady states which typically coexist in a credit money economy, namely,

. *Golden Rule steady states*, where the real interest rate is equal to the rate of population growth, which is here 0, and where the consumers' aggregate net credit position is permanently positive, or permanently negative, and

. *Balanced steady states*, where outside money is zero at any time, and where the real interest rate differs typically from 0.

The reasons underlying this duality are easily understood by using the following heuristic argument. We have shown in the preceding lecture that outside money increases within each period by the extent of bankruptcies, and that it grows mechanically from the end of a given period to the beginning of the next at a rate equal to the nominal interest rate (see the discussion of equations (C), (D), (E) in II.2). It is natural to assume that consumers do not make forecasting errors, and thus, that bankrupcies do not occur along a steady state. Therefore, outside money grows at a rate equal to the nominal interest rate along a steady state. On the other hand, the Quantity Theory implies that outside money and the prices of goods grow at the same rate along a steady state. The apparent contradiction can be resolved only if the nominal rate of interest is equal to the rate of inflation, or if outside money is permanently equal to 0.

Two important classes of economies will be distinguished. When consumers have larger real incomes in the late periods of their lifes, every Golden Rule steady state involves a negative outside money. In "well behaved" economies, the real interest rate should then be positive for all Balanced steady states. This is sometimes called the *Classical* case, since it is the kind of situations which were described by Classical theorists like I. Fisher. On the other hand, if the consumers have larger real incomes in their youth, and if they do not discount too much the future, every Golden Rule steady state implies a positive outside money, while the real interest rate should be negative for all Balanced steady states. This is sometimes called the *Samuelson* case, as such situations were considered by this author in his seminal paper on the "social contrivance" of money [1].

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(1) The terminology is borrowed from D. Gale's excellent article on the subject (1973). The reader may wish to begin with a preliminary reading of this paper.

1. CONSUMERS' CHARACTERISTICS.

The institutional set up of the economy which we consider is the same as that of the preceding chapter. Since we wish to study steady states, however, the demographic structure of the model has to be made now precise.

We shall use the convenient framework of an overlapping generation model without bequest and with a constant population, as in the the first Chapter (see I.6). We recall briefly the characteristics of the overlapping generation model, for the sake of completeness.

There are various "types" of consumers. A consumer of type i is described by :

(*i*) the number of periods of his lifetime, $n_i \geqq 2$,

(*ii*) his real income, or endowments of goods during his lifetime. His endowment of goods when he is of age τ , i.e., in the τ-th period of his life, is described by a vector $e_{i\tau}$, with ℓ components.

(*iii*) His intertemporal preferences, which are represented by a utility function $u_i(c_{i1}, \dots, c_{in_i})$, where $c_{i\tau} \geqq 0$ is the trader's vector of consumption goods when he is of age τ .

At each date, a newborn consumer of each type comes into the market. Thus there are n_i consumers of type i in activity in any period, each of them being in a different period of his life. The characteristics of a consumer are supposed to be independent of time, that is, they are independent of the date of his "birth".

We shall make the following assumptions throughout this lecture. They are the same as assumptions *(a)* and *(b)* of I.6.

(a) *The utility function* u_i *is continuous, increasing and strictly quasiconcave for every* i .

(b) *The endowment vectors* $e_{i\tau}$ *have all their components positive, for all* i *and* τ .

2. LONG RUN DEMANDS AND SUPPLIES.

Steady states are characterized by the fact that the relative prices of goods and the nominal rate of interest are constant over time, and that all money prices of goods are growing at a constant rate. The purpose of the present section is to look at the consumers' behaviour in such an environment.

To this effect, consider the economy at some date, which we shall call the "current period", where the price system for goods is p, and the nominal interest rate r. Assume moreover that the interest rate was constant and equal to r in the past, and that prices of goods have been growing at a constant rate π. In other words, past prices of goods were equal to $(1+\pi)^{-1} p$, $(1+\pi)^{-2} p$, and so on.

We shall assume that in such a situation, a consumer, no matter which type he belongs to and which age he has, forecasts that the same interest rate r will prevail and that prices will continue to grow at the rate π in the future. This assumption is natural, and implies that consumers have *rational* expectations along a steady state (1).

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(1) Note that the assumption concerns the dependence of a consumer's forecasts with respect to the *sequence* of current and past prices and interest rates. It is thus compatible with the conditions which were used in the preceding Chapter to guarantee the existence of a short run equilibrium, since these conditions concerned the dependence of a consumer's expectations with respect to the *current* prices and interest rate only.

Consider a consumer of a given type, say i , who is just "born" at the date under consideration. This consumer must choose, for every period τ of his life, his consumption $c_\tau \geqq 0$, his money balance $m_\tau \geqq 0$ and his supply of bonds $b_\tau \geqq 0$, or more concisely his net credit position at the Bank before interest payments, $\mu_\tau = m_\tau - b_\tau/(1+r)$. The net credit position which he plans to have at the end of his life must be nonnegative, i.e., $\mu_\tau \geqq 0$ for $\tau = n_i$. On the other hand, he begins his life without any credit or any debt at the Bank, since there are no bequests in the model.

The consumer seeks to maximize his lifetime utility under his current and expected budget constraints. His behavior is thus described by the following program, which is simply the transposition of *(I)* of II.3 to the case at hand.

(I) *Maximize* u_i *with respect to* $c_\tau \geqq 0$ *and* μ_τ *, for* $\tau = 1, \ldots, n_i$ *, subject to* $\mu_{n_i} \geqq 0$ *and to the budget constraints :*

$$(1+\pi)^{\tau-1}\, p\, c_\tau + \mu_\tau = (1+\pi)^{\tau-1}\, p\, e_{i\tau} + (1+r)\, \mu_{\tau-1}$$

for $\tau = 1, \ldots, n_i$ *, (with the convention* $\mu_o = 0$*)* .

It is convenient to reformulate this problem by dividing each budget constraint by $(1+\pi)^{\tau-1}$. If one defines the *real rate of interest* ρ by $1+\rho = (1+r)/(1+\pi)$, and if one sets $\mu'_\tau = \mu_\tau/(1+\pi)^{\tau-1}$, this yields :

$$pc_{\tau} + \mu'_{\tau} = pe_{i\tau} + (1+\rho)\ \mu'_{\tau-1}$$

for every τ . This way of looking at the consumer's decision problem makes clear that the optimum values of $c_{\tau} - e_{i\tau}$ and of $\mu'_{\tau} = \mu_{\tau}/(1+\pi)^{\tau-1}$ which arise from (I) depend only on the price system p , and on the real interest rate ρ . These optimum values will be denoted $z_{i\tau}(p,\rho)$ and $\mu_{i\tau}(p,\rho)$, respectively (1). With these notations, the consumer's excess demands for goods, and his desired net credit position in the τ-th period of his life, are given by $z_{i\tau}(p,\rho)$ and $(1+\pi)^{\tau-1}\ \mu_{i\tau}(p,\rho)$. The homogeneity properties of the budget constraints imply immediately that the functions $z_{i\tau}(p,\rho)$ and $\mu_{i\tau}(p,\rho)$ are homogenous of degree 0 and 1, respectively, with respect to p.

An equivalent way to get the functions $z_{i\tau}(p,\rho)$ and $\mu_{i\tau}(p,\rho)$, which we shall use later on, is to remark that the optimum consumption program (hence the functions $z_{i\tau}(p,\rho)$) can be computed by maximizing u_i under the intertemporal budget constraint

$$\sum_{\tau} \frac{pc_{\tau}}{(1+\rho)^{\tau-1}} = \sum_{\tau} \frac{pe_{i\tau}}{(1+\rho)^{\tau-1}}$$

which is obtained by eliminating the financial variables μ_{τ} (or μ'_{τ}) from the above sequence of current and expected budget constraints, and by taking into account the fact that it is nevel optimal for a consumer to keep a positive money balance at the end of his life. The expressions

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(1) These expressions represent in fact the consumer's excess demands for goods and his desired net credit position in each period of his life, if the price system had been constant in the past and equal to p , and if the interest ρ was permanently paid on money balances and charged on loans.

$\mu_{i\tau}(p,\rho)$ are then determined recursively from the consumer's budget constraints associated with each τ , i.e. from

$$pz_{i\tau}(p,\rho) + \mu_{i\tau}(p,\rho) = (1+\rho)\ \mu_{i,\tau-1}(p,\rho) \ .$$

According to our assumptions on expectations, consumers make no forecasting errors when the prices of goods are growing at a constant rate π , and when the interest rate r is constant over time. In such an environment, consumers do carry out, during their lifetime, the plans which they made at the date of their "birth". What a consumer of type i and of age τ actually does in the current period is therefore what he intended to do during that period, when he formulated his plans as a newborn consumer, $(\tau-1)$ periods before. At that time the prevailing price system was $p/(1+\pi)^{\tau-1}$. What such a consumer does in the current period is thus given by the optimum values of $c_\tau - e_{i\tau}$ and μ_τ of Problem (I), when p is replaced by $p/(1+\pi)^{\tau-1}$. In view of the homogeneity properties of the solution of (I) with respect to the prices of goods, these optimum values are $z_{i\tau}(p,\rho)$ and $\mu_{i\tau}(p,\rho)$ respectively.

Consider now all consumers of type i who participate in the market on the current period. The demographic structure which as postulated implies that there are n_i of them, each of whom is in a different period of his life. The excess of their aggregate consumption over their aggregate endowment of goods in the current period (their "long run" excess demand for goods) is simply :

$$z_i(p,\rho) = \sum_\tau z_{i\tau}(p,\rho) \quad ,$$

where the summation sign runs from $\tau = 1$ to $\tau = n_i$. Similarly, their desired net credit position is :

$$\mu_i(p,\rho) = \sum_\tau \mu_{i\tau}(p,\rho) \quad .$$

The net credit position which a consumer of type i and of age τ wishes to have at the end of the current period determines his corresponding money demand $m_{i\tau}$ and his bond supply $b_{i\tau}$ by the relation :

$$\mu_{i\tau}(p,\rho) = m_{i\tau} - b_{i\tau}/(1+r)$$

together with the condition that either $m_{i\tau}$ or $b_{i\tau}$ is equal to 0. Summing over all consumers of type i who live in the current period shows that their aggregate demand for money is a function of p and ρ which will be noted $m_i(p,\rho)$. The amount of money which they wish to borrow in the aggregate (the sum of the $b_{i\tau}/(1+r)$) is a function of p and ρ too,

$$\beta_i(p,\rho) = m_i(p,\rho) - \mu_i(p,\rho) \quad ,$$

and thus, their corresponding aggregate bond supply is $(1+r)\ \beta_i(p,\rho)$.

To sum up, these expressions define the aggregate behaviour of all consumers of a given type who participate in the market in a given period when prices are growing at a constant rate and when the nominal interest rate is constant over time, in function of the prices of goods p which prevail at the date under consideration and of the real rate of interest ρ . The homogeneity properties of the consumers' budget constraints imply, as we have already noted, that *the "long run" excess demands for goods* $z_i(p,\rho)$ *and the desired net credit positions* $\mu_i(p,\rho)$ *are homogenous of degree* 0 *and* 1 , *respectively, with respect to the prices of goods.*

In addition, the consumers' budget constraints imply that :

$$p\, z_{i\tau}(p,\rho) + \mu_{i\tau}(p,\rho) = (1+\rho)\, \mu_{i,\tau-1}(p,\rho)$$

for every τ . Adding these relations, while taking into account the fact that $\mu_{i\tau}(p,\rho) = 0$ for $\tau = n_i$, yields :

$$p\, z_i(p,\rho) + \mu_i(p,\rho) = (1+\rho)\, \mu_i(p,\rho)$$

for every p and ρ [1]. Summing over all types i shows that the consumers' long run demands and supplies satisfy what can be viewed as a general version of *Say's Law,* that is

$$p \sum_i z_i(p,\rho) - \rho \sum_i \mu_i(p,\rho) = 0$$

for every price system p *and every real interest rate* ρ .

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(1) This identity can be obtained directly if it is viewed as an aggregate budget restraint for the consumers of type i. The expression $pz_i(p,\rho) + \mu_i(p,\rho)$ should be equal, for every p and ρ , to these consumers' aggregate net monetary wealth at the beginning of the period, including interest payments. This aggregate monetary wealth is given by $(1+r)\ \mu_i(p/(1+\pi)\ ,\ \rho)$, since $p/(1+\pi)$ is the price system which prevailed at the preceding date. Taking into account the homogeneity of degree 1 of μ_i with respect the prices of goods yields the result.

3. THE EQUATIONS OF STEADY STATES.

The purpose of this section is to specify the system of equations which characterize steady states and to study its properties [1]. It is will be established in particular that the *Classical Dichotomy* and the *Quantity Theory* are valid propositions when applied to steady states. Relative prices of goods and the real rate of interest are determined by the equilibrium of the goods markets. Nominal magnitudes such as the level of money prices, the nominal rate of interest, the rate of inflation are in turn determined by the consideration of the monetary sector. In particular, the level of money prices will be shown to be proportional to monetary aggregates such as outside money, the Bank's money supply, etc ... This findings will imply that monetary authorities cannot influence, in the long run, the "real" evolution of the economy by controlling the nominal rate of interest or the rate of growth of its money supply. Pure monetary policy is *neutral* in the long run.

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(1) It must be emphasized that the steady states which are analyzed in the sequel are *monetary* steady states, i.e. money has positive value in exchange at every date. Under the assumptions (*a*) and (*b*) of Section 1, there exists in addition *nonmonetary* steady states, where the price of money is zero in every period. Nonmonetary steady states are characterized by the fact that consumers make no intertemporal value transfers. In the particular case where there is only one good, a nonmonetary steady state is the autarchic one, where every consumer consumes his own endowment of good in each period of his life.

It will be shown in addition that there are two different sorts of (monetary !) steady states which coexist in a credit money economy :

. *Golden Rule Steady States*, where the real rate of interest rate is equal to the rate of population growth, which is here equal to zero, and where outside money typically differs from zero at any point of time, and

. *Balanced Steady States*, where the aggregate net credit position of the private sector is permanently 0, and where the real interest rate typically differs from the population growth rate.

A steady state is defined, we recall, as a sequence of short run equilibria where the nominal rate of interest is constant over time, say r , and where prices of goods grow at a constant rate (the "rate of inflation"), say π . If p_o denotes the vector of goods prices which prevail at date 0 , then the price system at date t is equal to

$$p_t = (1+\pi)^t \, p_o$$

for every t.

As we have seen in the preceding section, in such an environment, the aggregate behaviour of the consumers of type i can be described in any period t by their excess demands for goods $z_i(p_t,\rho)$, where ρ is the real interest rate determined by r and π , and by their desired net credit position at that time, excluding the payment of the interest r , i.e. $\mu_i(p_t,\rho)$, or more precisely, their demand for money $m_i(p_t,\rho)$ and their bond supply $(1+r)\ \beta_i(p_t,\rho)$.

The conditions which describe the equilibrium of the various markets along a steady state are then easily expressed by using these "long run" demands and supplies. Indeed, equilibrium of the goods markets requires that aggregate excess demands for goods is zero in every period, that is

$$(A_t) \qquad \sum_i z_i(p_t,\rho) = 0$$

for every t , where the summation sign runs over all types i of consumers.

The condition for money states that the aggregate "long run" demand for money at date t , i.e. $\sum_i m_i(p_t,\rho)$, is equal to the outstanding money stock at that time M_t . It is in fact convenient to express the money stock M_t as a function of the value of outside money at the beginning of the period, and of the amount of money created by the Bank when granting loans to consumers at date t , i.e. $\Delta M_t \geqq 0$. A moment of reflexion shows that M_t is equal to the initial money stock at the date under consideration, including the payment of the interest rate r , i.e. to $(1+r)\ M_{t-1}$, to which is added the Bank's money issue ΔM_t , net of the consumers' reimbursements of their past debts. As there are no forecasting errors along a steady state according to our assumptions on the consumers' expectations, the aggregate reimbursement is indeed equal to the aggregate bond supply at the preceding date B_{t-1} . One obtains therefore

$$M_t = (1+r)\ M_{t-1} + \Delta M_t - B_{t-1} \quad ,$$

or equivalently,

$$M_t = (1+r)\ \mu_{t-1} + \Delta M_t$$

where μ_{t-1} is the value of outside money at the beginning of period t , before interest payments. With these notations, the equilibrium condition for money reads

$$(\mathcal{B}_{t1}) \qquad \sum_i m_i(p_t,\rho) = (1+r)\ \mu_{t-1} + \Delta M_t$$

for every t.

The Bank's demand for bonds is given by $(1+r)\ \Delta M_t$ in every period, while the consumers' aggregate bond supply is $(1+r) \sum_i \beta_i(p_t,\rho)$. Equilibrium of the bond market at each date thus necessitates

$$(\mathcal{B}_{t2}) \qquad \Delta M_t = \sum_i \beta_i(p_t,\rho)$$

for every t.

It is clear from the homogeneity of degree 1 of the functions $m_i(p_t,\rho)$ and $\beta_i(p_t,\rho)$ with respect to the prices of goods, that the monetary aggregates ΔM_t and μ_t must grow at the rate π over time. This is obvious for ΔM_t , in view of the bond market conditions $(\mathcal{B}_{t2})$. On the other hand, the value of μ_t is given along a steady state by

$$\sum_i \mu_i(p_t,\rho) = \sum_i \left[m_i(p_t,\rho) - \beta_i(p_t,\rho)\right]$$

which grows evidently at the rate π too [1]. The dynamic evolution of the two monetary aggregates ΔM_t and μ_t is thus determined once one specifies their values ΔM_o and μ_o at date 0.

A steady state is therefore characterized by a set of parameters $(p_o, \pi, r, \mu_o, \Delta M_o)$ such that the associated prices $p_t = (1+\pi)^t p_o$, as well as the monetary aggregates $\mu_t = (1+\pi)^t \mu_o$ and $\Delta M_t = (1+\pi)^t \Delta M_o$, satisfy the equilibrium conditions (A_t), (B_{t1}) and (B_{t2}) for every t. By virtue of the homogeneity properties of the consumers' long run demands and supplies, these equilibrium conditions are fulfilled in every period if and only if they are satisfied at a single date, say date 0. One can state accordingly

A steady state is characterized by the set of parameters $(p_o, \pi, r, \mu_o, \Delta M_o)$ *which satisfy*

$$(A) \qquad \sum_i z_i(p_o,\rho) = 0$$

$$(B_1) \qquad \sum_i m_i(p_o,\rho) = (1+\rho)\,\mu_o + \Delta M_o$$

$$(B_2) \qquad \Delta M_o = \sum_i \beta_i(p_o,\rho)$$

where the real interest rate ρ *is given by* $1+\rho = (1+r)/(1+\pi)$ [2].

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(1) It is intuitively clear how these statements must be modified when the population grows at the rate γ. For then the aggregate demand for money and the aggregate bond supply should grow at a rate λ given by $1+\lambda = (1+\gamma)(1+\pi)$. The monetary aggregates ΔM_t and μ_t grow then at the rate λ too.

(2) Equation (B_1) is obtained from (B_{01}) by using the fact that $\mu_{-1} = \mu_o/(1+\pi)$.

The foregoing equilibrium conditions characterize a steady state without making any reference to the monetary policy which the Bank may wish to implement. They must be therefore supplemented by a specification of the parameters which the Bank seeks to control, in the long run, by its interventions on the credit market. Examination of the system (A), (B_1), (B_2) shows that the monetary authority can *a priori* hope to control at most two variables among those which define a steady state, since this system involves $(\ell+2)$ equations while there are $(\ell+4)$ unknowns. For instance, the Bank may choose to peg the level and the rate of growth of its money supply : this would fix exougenously the parameters ΔM_o and π in the above system. Or alternatively, it may choose to peg the nominal interest rate permanently at the level r . The Bank's money issue ΔM_t would then be endogenous at every date. The Bank may even ambition to peg a "long run" real variable such as the real interest rate ρ , by linking for instance the nominal rate of interest which it wishes to impose at each date with the past growth rates of the money supply.

The system of equations (A), (B_1), (B_2), together with a specification of the values of the parameters which the Bank seeks to control in the long run by its intervention of the bond market, define completely steady states which are associated with the Bank's policy. We proceed now to the study of the qualitative properties of such a system.

As we have seen, the long run excess demands $z_i(p_o,\rho)$ appearing in this system are homogeneous of degree 0, while the functions $m_i(p_o,\rho)$ and $\beta_i(p_o,\rho)$ are homogenous of degree 1 with respect to the prices p_o of goods. Moreover, these long run demands and supplies satisfy *Say's Law*, that is

$$p_o \sum_i z_i(p_o,\rho) - \rho \sum_i \mu_i(p_o,\rho) = 0$$

for every p_o and ρ .

These properties imply that the *Classical Dichotomy* and the *Quantity Theory* are valid propositions in the present model. The ℓ equations (A) for the goods markets are homogenous of degree 0 with respect to p_o , and thus define *a priori* the set of equilibrium relative prices of goods and of equilibrium real interest rates. Equilibrium of the real sector determines real equilibrium magnitudes. Once a particular solution of (A) has been selected, the corresponding equilibrium nominal values can be determined in turn by looking at the monetary part of the model. Indeed, the homogeneity properties of (B_1) and (B_2) show that the equilibrium level of money prices of goods is proportional to the level of the monetary aggregates μ_o and ΔM_o . Finally, the determination of the nominal rate of interest r and of the rate of inflation π (or equivalently the rate of growth of the monetary aggregates), given the real interest rate ρ obtained from (A), is achieved by taking into account the Bank's monetary policy. For instance, if the Bank chose to peg the nominal rate, this determines r , and thus the inflation rate π by $1+\rho = (1+r)/(1+\pi)$. If the Bank chose to peg the rate of growth of its money supply, this determines π , and thus the nominal interest rate r by the same relation.

These considerations have important implications concerning what monetary policy can achieve, and most significantly, what it cannot do. If one takes as granted the existence of a solution to the equations (A) (we shall see that this is true under quite general conditions), long run real equilibrium magnitudes (the relative prices of goods, the real rate of interest, the traders' consumptions) are determined by the equilibrium of the real sector, independently of the Bank's monetary policy. The Bank can in principle peg the level of money prices of goods and the rate of inflation π (or alternatively the nominal interest rate r) at predetermined values either by pegging the level and the growth of its money supply ΔM_t , or by fixing permanently the nominal rate of interest r at an appropriate level [(1)]. Such a control is purely nominal however, in the sense that it does not affect real equilibrium quantities in the long run. In particular, given a particular solution of (A), and thus a real interest rate ρ , a permanent increase of the nominal rate r induces only in the long run a correlative increase of the rate of inflation π so as to leave the real rate $(1+r)/(1+\pi)$ unchanged. Similarly, a permanent increase of the rate of growth of the money supply ΔM_t leads only in the long run to an equal increase of the rate of inflation π , and to a correlative rise of the nominal rate r so as to maintain the

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(1) The Bank has full control over the rate of inflation only if the nominal rate r can assume any value between -1 and $+\infty$. If r cannot be negative (we saw in II.1 that this is the case if paper money is assumed to be present in the model), there is a lower bound on the rate of inflation, given a particular solution of (A), since it must then satisfy $1+\pi \geqq 1/(1+\rho)$.

ratio $(1+r)/(1+\pi)$ constant. Variations of the nominal rate r or of the growth rate of the money supply are *neutral* in the long run (1).

An immediate corollary of this analysis is that the Bank cannot use its control over nominal magnitudes to peg a real quantity in the long run. Of course, the Bank may always attempt to do so through a deliberate policy on the credit market. For instance, it may decide to peg the real interest rate at some predetermined value $\bar{\rho}$, by setting a nominal interest rate r_t in each period which is related to the rate growth of the money supply observed in the immediate past, e.g. by using the relation

$$1+r_t = (1+\bar{\rho})\ (\Delta M_{t-1}/\Delta M_{t-2})$$

If the chosen value $\bar{\rho}$ is not compatible with the equilibrium of the real sector (A), such a policy will be defeated by the market, in the sense that the economy will never reach a monetary steady state. If it ever approached a steady state, it should then be a nonmonetary one, where money has no value in exchange. This would mean a complete breakdown of the monetary institutions under consideration.

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(1) This statement concerns only the consequence of a change of the growth rate of the money supply by means of monetary policy. If this change was brought about by means of fiscal policy, i.e. by levying taxes from and paying subsidies to consumers, it would have typically real effects in the long run. The reader will easily check this fact by working out by himself the simple example where there is only one good, and one type of consumers who live two periods, with a Cobb-Douglas utility function, $u(c_1,c_2) = c_1c_2$.

The analysis which precedes shows that all the information about the real characteristics of steady states is embodied in the equilibrium conditions (A) for the goods markets. The purpose of the next to sections is to investigate more closely the properties of the solutions of these equations.

It turns out that the set of these solutions displays a remarkable and quite simple structure. This is almost evident if one considers *Say's Law*, which claims that

$$p_o \sum_i z_i(p_o, \rho) = \rho \sum_i \mu_i(p_o, \rho)$$

for every p_o and ρ . Any solution (p_o, ρ) of (A) must therefore satisfy either $\rho = 0$, or $\mu_o = \sum_i \mu_i(p_o, \rho) = 0$. Another way which is even simpler, to obtain this result, is to consider the money and the bond equations (B_{t1}) and (B_{t2}) above. By adding them and rearranging, one gets

$$\mu_t = \sum_i [m_i(p_t, \rho) - \beta_i(p_t, \rho)] = (1+r)\ \mu_{t-1}$$

for every t. These relations imply that outside money μ_t must grow along a steady state at the rate r as well as at the rate π (by virtue of the homogeneity of degree 1 of the functions m_i and β_i with respect to prices). This apparent contradiction can only be resolved if either the nominal interest rate r is equal to the rate of inflation, in which case the real interest rate ρ is equal to zero, or if outside money μ_t is permanently zero along the steady state (1).

There are therefore at most two types of steady states :

. *Golden Rule Steady States*, where the real rate of interest is equal to zero, and where the consumers' aggregate net credit position is typically permanently positive or permanently negative, and

. *Balanced Steady States*, where the consumers' aggregate net credit position is zero at any point of time, and where the real rate of interest differs typically from zero.

These two types of steady states will in fact coexist under quite general conditions, as we are going to see in the next two sections.

Remark. It should be emphasized that the results of this section borrow very little from the particular features of the overlapping generation model. The general argument used only the homogeneity of the consumers' long run demands and supplies, and *Say's Law*, which is a simple accounting identity. The results should therefore be valid in any reasonably well specified model of an exchange credit money economy.

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(1) It is intuitively clear how these statements must be modified when population grows at the rate γ . For then outside money should grow at the same time at the rate r and at the rate λ which is defined by $1+\lambda = (1+\pi)(1+\gamma)$. Thus either the real interest rate is equal to the rate of population growth, or outside money is zero.

4. GOLDEN RULE STEADY STATES.

We study first Golden Rule steady states, which involve by definition a real rate of interest ρ equal to 0. If we set $z_i^*(p) = z_i(p,0)$, the equilibrium of the real sector for such steady states is described by the equations

$$(A^*) \qquad \sum_i z_i^*(p_o) = 0$$

The existence of a solution to this system is really a straighforward matter. Indeed, the functions z_i^* are homogenous of degree 0 with respect to the prices of goods, and *Say's Law* reduces here to $p_o \sum_i z_i^*(p_o) = 0$ for every p_o . (A^*) looks like a traditional Walrasian system, and should have accordingly a solution under the usual conditions of continuity and convexity of the consumers' preferences [1].

(1) *Assume (a) and (b) of Section 1. The system (A^*) has then a solution. Every solution p_o involve positive prices, and is defined up to a positive real number.*

Our main concern in this section will be the study of the conditions which determine the sign of the consumers' aggregate net credit position along Golden Rule steady states. If we denote $\mu_i^*(p) = \mu_i^*(p,0)$, this amounts to looking at the sign of the expression $\sum_i \mu_i^*(p_o)$, where p_o is a solution of (A^*).

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(1) A proof of the result is given in Appendix D.

It is clear that the sign of outside money along Golden Rule steady states will depend on the intertemporal profiles of the consumers' real incomes during their lifetime, and on how they discount the future. Intuitively, one should expect that if the consumers have on the average larger real incomes in their "youth" than in their old age, and if they do not discount too much the future, they should be in the aggregate net creditors, that is, Golden Rule outside money should be positive. Conversely, if the consumers are on the average richer in the late periods of their lifes, they should be net debtors in the aggregate, i.e. Golden Rule outside money should be negative.

The remainder of this section is devoted to a precise formulation of these heuristic statements. To this effect, it is useful to begin the analysis by considering the simple case where there is only one good, and where consumers live only two periods. The value of p_o which arises from (A^*) is then indeterminate, and can be fixed arbitrarily.

According to our previous study of the consumers' behaviour along a steady state (Section 2), the functions $z^*_{i1}(p) = z_{i1}(p,0)$ and $z^*_{i2}(p) = z_{i2}(p,0)$ are the optimum values of $c_1 - e_{i1}$ and of $c_2 - e_{i2}$ which result from the maximization of the utility function u_i under the consumer's intertemporal budget constraint, which takes here the form

$$p(c_1+c_2) = p(e_{i1}+e_{i2})$$

Therefore, if the consumer does not discount the future i.e. if his marginal rate of substitution between present and future consumption is equal to one whenever $c_1 = c_2$, he is going to consume $\frac{e_{i1} + e_{i2}}{2}$ in every period (see Fig. 1.a and 1.b).

Fig. 1.a

Fig. 1.b

In such a case, $\mu_i^*(p)$ is equal to the consumer's saving when he is young, that is, to :

$$\mu_i^*(p) = p(e_{i1} - c_1) = p\,\frac{e_{i1} - e_{i2}}{2} \quad .$$

If the consumer's income is larger when he is young than when he is old $(e_{i1} > e_{i2})$, then saving is positive. If he is on the contrary richer in his old age $(e_{i2} > e_{i1})$, then $\mu^*(p)$ is negative. On the other hand, discounting of the future generates a clockwise "rotation" of the indifference curves around the points of the diagram such that $c_1 = c_2$. This should lead to an increase of c_1 , and thus to a decrease of the consumer's desired net credit position $\mu_i^*(p)$.

Therefore, if the consumers are in the aggregate richer in their youth, i.e. if $\sum_i e_{i1} > \sum_i e_{i2}$, and if they do not discount "too much" the future, outside money is positive along the Golden Rule steady state. If the consumers are richer when they are old in the aggregate, i.e. if $\sum_i e_{i1} < \sum_i e_{i2}$, outside money is negative along the Golden Rule steady state.

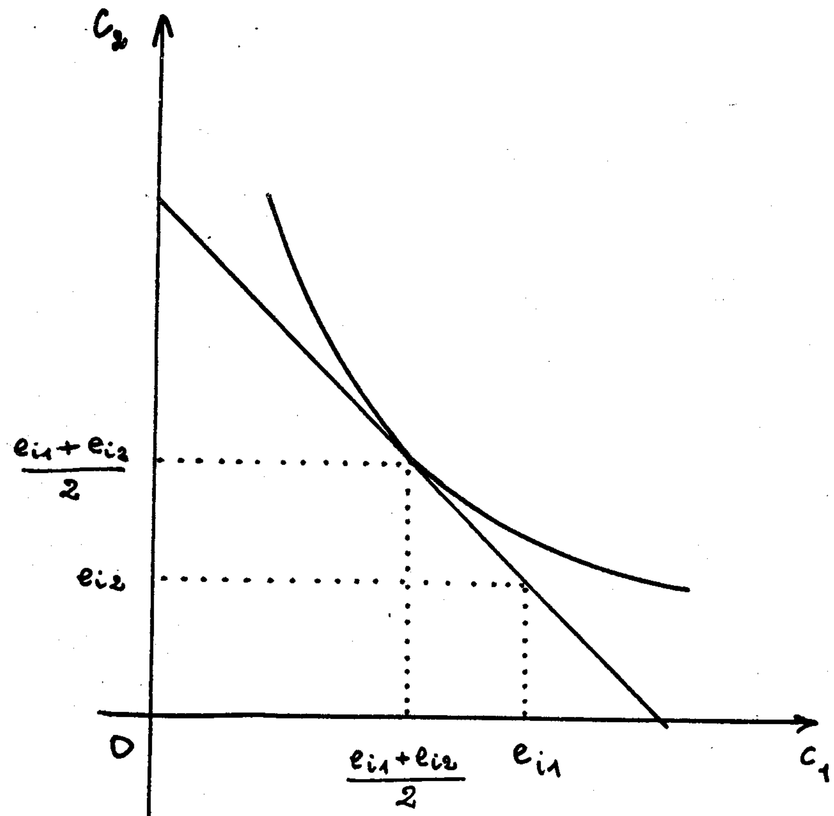
c_2
$\frac{e_{i1}+e_{i2}}{2}$
e_{i2}
0
$\frac{e_{i1}+e_{i2}}{2}$
e_{i1}
c_1

Fig 1. a.

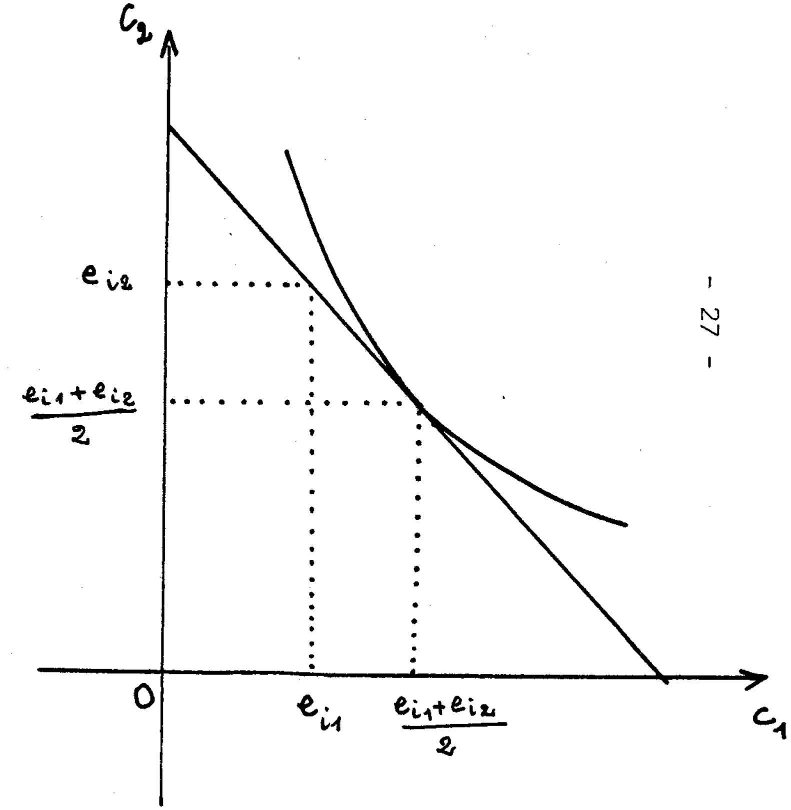
c_2
e_{i2}
$\frac{e_{i1}+e_{i2}}{2}$
0
e_{i1}
$\frac{e_{i1}+e_{i2}}{2}$
c_1

Fig. 1. b.

This analysis can be generalized to the case where there are several goods and where consumers live for an arbitrary number of periods. In order to make this precise, we must first speak in a meaningful way of the consumers' rates of time preference. We shall assume therefore that every consumer's utility function is separable, with a constant discount rate. Specifically, assumption (*a*) of Section 1 is replaced by :

(*a'*) *The utility function* u_i *is of the form* $\sum_\tau \delta_i^{\tau-1} w_i(c_t)$ *, where* w_i *is increasing, continuous and strictly concave, and where* $0 < \delta_i \leqq 1$ *, for every* i .

We have next to find an appropriate "measure" of the difference between the consumers' real incomes in their youth and their real incomes in their old age. Consider the consumers of type i , and let $\varepsilon_{i\tau}$ be the difference between their endowments of goods in the τ first periods and in the last τ periods of their life :

$$\varepsilon_{i\tau} = (e_{i1} + \ldots + e_{i\tau}) - (e_{n_i-\tau+1} + \ldots + e_{n_i}) .$$

Define next the vector ε_i by :

$$\varepsilon_i = \sum_1^q \varepsilon_{i\tau}$$

when n_i is odd and equal to $2q+1$, and by :

$$\varepsilon_i = \sum_1^{q-1} \varepsilon_{i\tau} + \frac{1}{2}\varepsilon_{iq}$$

when n_i is even and equal to $2q$.

This "measure" is at first sight adequate. If the consumers are richer in their youth, the vectors $\varepsilon_{i\tau}$, and thus the vector ε_i , will tend to have its components positive. Conversely, if they are richer when they are old, the components of the vectors $\varepsilon_{i\tau}$, and thus those of ε_i , will tend to be negative. This "measure" is further justified by the following fact [(1)].

(2) *Assume (a'). If there is no rate of time preference* ($\delta_i = 1$), *then* $\mu_i^*(p) = p\,\varepsilon_i$ *for every price system* p . *If there is a positive rate of time preference* ($\delta_i < 1$) , *then* $\mu_i^*(p) \leqq p\,\varepsilon_i$ *for every* p .[(2)]

The argument has been developed up to now at the level of a single type of consumers. Finding the sign of outside money along Golden Rule steady states is then easy by considering all types together.

Let us define the vector ε as $\sum_i \varepsilon_i$, and consider first the case where the consumers have on the average larger real incomes in the early periods of their lifes, so that the vector ε has all its components nonnegative, with some of them positive. In view of (2), if there is no

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(1) A proof of this fact is given in Appendix D.

(2) Actually, there is strict inequality under assumption (*b*) of Section 1, if w is differentiable, and if the partial derivative of w with respect to the h-th good, $\frac{\partial w}{\partial c_h}$, is infinite whenever $c_h = 0$. See Appendix D.

preference for present consumption ($\delta_i = 1$ for every type), the consumers wish to be net creditors in the aggregate, since $\sum_i \mu_i^*(p)$ is equal to $p\varepsilon$ and is thus positive for every p. Outside money is then positive for every solution of (A^*). It is intuitively clear that the same conclusion should hold, by continuity, when the consumers do not discount too much the future, i.e., when the parameters δ_i are close to 1. This argument justifies the following proposition [1].

(3) *Assume (a') of the present section, and (b) of Section 1. If the vector $\varepsilon = \sum_i \varepsilon_i$ has all of its components nonnegative, with some of them positive, outside money is positive along every Golden Rule steady state, when the parameters δ_i are close enough to 1.*

The case where the consumers are on the average richer when they are old is even simpler. Let us assume that the vector ε has all of its components nonpositive, with some of them negative. In view of (2), the desired aggregate net credit position $\sum_i \mu_i^*(p)$ is then negative for every p, independently of the consumers' rates of time preference. Outside money is thus negative for every solution of (A^*).

(4) *Assume (a') of the present section, and (b) of Section 1. If the vector $\varepsilon = \sum_i \varepsilon_i$ has its components nonpositive, with some of them negative, outside money is negative along every Golden Rule steady state.*

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(1) A proof of the proposition (essentially a continuity argument) is given in Appendix D.

5. BALANCED STEADY STATES.

We proceed now to the study of Balanced steady states, for which outside money is by definition equal to zero in every period. It will be shown that Balanced steady states do exist under quite general conditions, namely *(a)* and *(b)* of Section 1. Moreover, it will be argued that such steady states are likely to involve a negative real rate of interest if consumers have larger real incomes in their youth and if they do not discount too much the future, and that the real interest rate is likely to be positive whenever consumers are richer in the late periods of their lifes.

Real equilibrium quantities along a Balanced steady state are given by the following set of equations, which express that the goods markets clear and that outside money is equal to zero :

$$(A) \qquad \sum_i z_i(p_o,\rho) = 0$$

$$(B) \qquad \sum_i \mu_i(p_o,\rho) = 0$$

The functions $z_i(p_o,\rho)$ and $\mu_i(p_o,\rho)$ appearing in this system are homogenous of degree 0 and 1, respectively with respect to p , and they are linked by *Say's Law*, that is

$$p_o \sum_i z_i(p_o,\rho) - \rho \sum_i \mu_i(p_o,\rho) = 0$$

for every p and ρ .

The equations expressing the equilibrium of the goods markets are identical to the system (A^*) of the previous section when the real interest rate is set equal to 0. Therefore, every $(\bar{p}_o, 0)$ such that $\bar{p}_o$ is a solution of (A^*) satisfies (A). It should be noted however, that Say's Law does not generally imply $\sum_i \mu_i(\bar{p}_o, 0) = 0$, since $\rho = 0$. In fact this circumstance will occur only by an unlikely coincidence. For instance, we know from the analysis of the preceding section that, if the consumers have a zero rate of time preference, $\sum_i \mu_i(\bar{p}_o, 0)$ is equal to $\bar{p}_o \sum_i \varepsilon_i$ (see (2) of Section 4). When there is only one good this expression is equal to zero if and only if $\sum_i \varepsilon_i = 0$, a condition which is satisfied only in very special cases.

This means that if we want to find a solution to the whole system, we must typically look for solutions (p_o, ρ) of (A) such that ρ differs from zero. By Say's Law, such solutions will satisfy the equation (B) as well.

In order to understand intuitively how changes in the real rate of interest permit us to find a solution to the complete system (A), (B), it is convenient to look at the simple case where there is only one good. The value of p_o is then indeterminate and can be fixed arbitrarily. We are thus left with two equations to determine the real rate of interest ρ , which is the sole unknown of the system. The previous argument shows that $\rho = 0$ is always a solution of (A), but that typically, it does not satisfy (B). In that case, finding an equilibrium value of the real interest rate can be achieved by looking for solutions of (A) such that $\rho \neq 0$, or by looking directly at equation (B), since by Say's Law, any solution of (B) satisfies (A) too.

A typical heuristic argument for asserting the existence of such a solution is to say that an increase of the real interest rate should favour savings, and should thus lead to an increase of the consumers' desired aggregate net credit position $\sum_i \mu_i(p_o,\rho)$. One can then reasonably expect that this expression is positive for large real rates of interest, and that it becomes negative when ρ approaches -1. By continuity, therefore there should be in between a value of the real interest rate which satisfies $(\mathcal{B})$.

The foregoing argument can be easily visualized if one considers the simple case where consumers live only two periods. According to our analysis of the consumers' behaviour along a steady state (Section 2), the expressions $z_{i1}(p_o,\rho)$ and $z_{i2}(p_o,\rho)$ are the optimum values of $c_1 - e_{i1}$ and of $c_2 - e_{i2}$ which result from the maximization of the utility function u_i under the intertemporal budget constraint

$$p_o \, c_1 + \frac{p_o}{1+\rho} c_2 = p_o \, e_{i1} + \frac{p_o}{1+\rho} e_{i2} \quad .$$

The desired net credit position $\mu_i(p_o,\rho)$ is then equal to the value of the consumer's saving when he is young,

$$\mu_i(p_o,\rho) = - p_o \, z_{i1}(p_o,\rho) \quad .$$

As the real rate of interest ρ varies, the line representing the intertemporal budget constraint (see Fig. 2), rotates around the endowment point $(e_{i1} \, , \, e_{i2})$.

<u>Fig. 2</u>

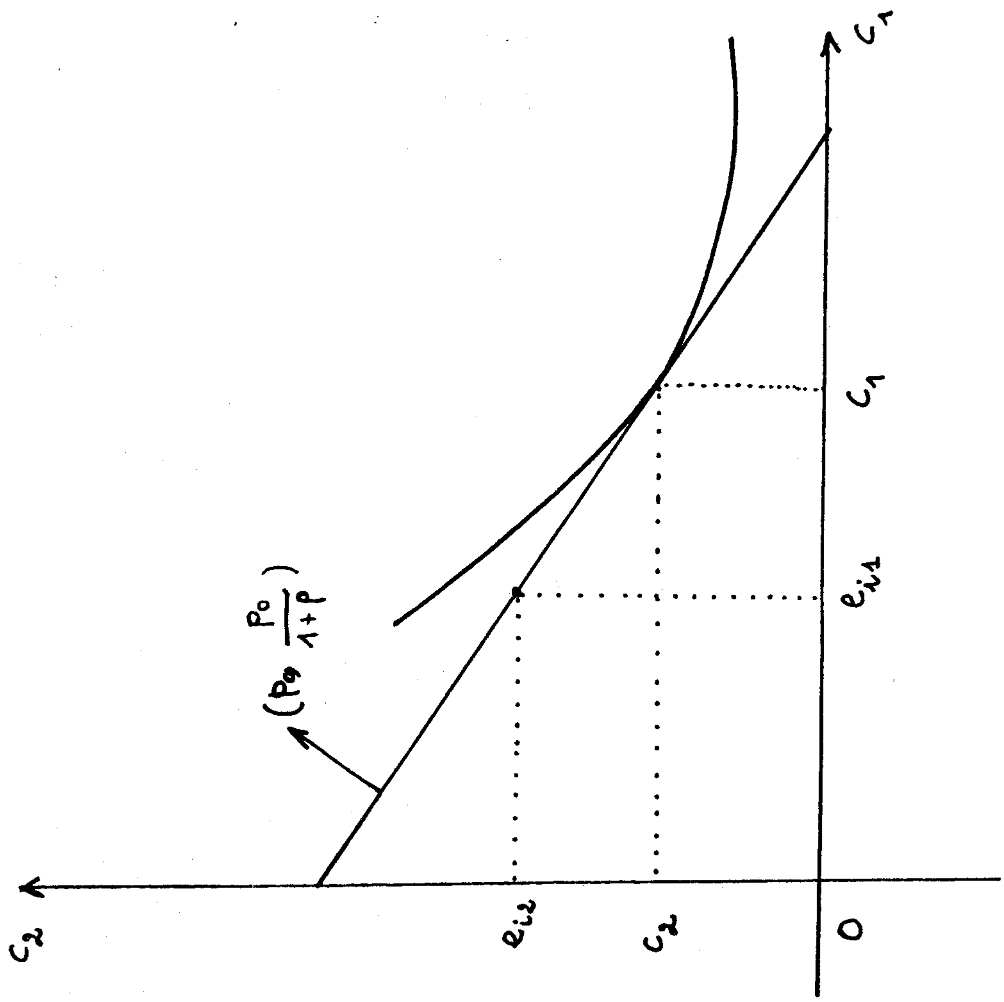

Fig. 2

When the real interest rate tends to -1, this line becomes almost horizontal. Optimum consumption c_1 must go to infinity, and thus net saving $\mu_i(p_o, \rho) = p_o(e_{i1} - c_1)$ tends eventually to $-\infty$. If the real rate of interest tends to $+\infty$, the intertemporal budget line becomes more and more vertical. Optimum consumption c_1 must be ultimately less then e_{i1} , and therefore $\mu_i(p_o, \rho)$ becomes positive. Aggregating over all types of consumers yields the desired properties of $\sum_i \mu_i(p_o, \rho)$.

The foregoing heuristic argument shows that one can reasonably expect the system (A), (B) to have a solution under rather general conditions. It turns out indeed that it has one under assumptions (a) and (b) of Section 1 (1).

(1) *Assume (a) and (b) of Section* 1 . *Then* (A) *has a solution. Every solution* (p_o, ρ) *involves a positive vector of prices, which is defined up to a positive real number, and a real interest rate* $-1 < \rho < +\infty$.

We have seen that the expression $\sum_i \mu_i(p_o, \rho)$ is likely to be increasing with the real rate of interest, for each value of p_o . If this property is taken as granted, it becomes possible to get more information about the probable sign of the real interest rate associated with a Balanced steady state, by relying upon the analysis of the preceding section.

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(1) A proof of the result is given in Appendix D.

Let us assume that the consumers have separable utility functions, as in (a') of Section 4. Consider first the case where the consumers have larger real incomes in their youth, so that the components of the vector $\varepsilon = \sum_i \varepsilon_i$ are all nonnegative, with some of them positive. We know from (2) of Section 4 that, if the consumers do not discount the future, their desired aggregate net credit position $\sum_i \mu_i(p_o,\rho)$ is equal to $p_o\varepsilon$ and is thus positive for every p_o whenever $\rho = 0$. Thus, if the expression $\sum_i \mu_i(p_o,\rho)$ is increasing with ρ , any solution of (A), (B) should imply a negative real rate of interest. It is intuitively clear that by continuity, the same result should hold when the consumers discount the future, provided that their rates of time preference are close to 0. *If the consumers are richer in their youth, and if they do not discount too much the future, any Balanced steady state is likely to involve a negative real rate of interest.*

Let us consider the other case where the consumers are richer in the late periods of their lifes, so that the components of the vector ε are all nonpositive, with some of them negative. We know from (2) of Section 4 that $\sum_i \mu_i(p_o,\rho)$ is then negative for every p_o , independently of the consumers' rates of time preference, whenever $\rho = 0$. If the expression $\sum_i \mu_i(p_o,\rho)$ is increasing with ρ , any solution of (A), (B) implies therefore a positive real interest rate. *If the consumers are richer when they are old, any Balanced steady state is likely to involve a positive real rate of interest.*

Remark. It should be emphasized that Balanced steady states are monetary equilibria. Money has positive value at every date, and is actually used by the consumers to make intertemporal income transfers. The fact that outside money is zero simply means that at any point of time, the money balances of the consumers are exactly matched by their debts. Balanced steady states are thus essentially different from the long run equilibria which would arise in this economy if the price of money is zero, since such nonmonetary equilibria are characterized by the fact that non intertemporal income transfers take place.

6. CONCLUDING REMARKS.

Our analysis showed that there are two different sorts of monetary steady states which coexist in a credit money economy. There are first Golden Rule steady states, where, when the population is stationary, the rate of inflation equals the nominal interest rate, and where the consumers' aggregate net credit position is permanently positive, or permanently negative. And Balanced steady states, where outside money is zero at any time, and where the real interest rate is positive, or negative.

Two particular cases were distinguished. Whenever the consumers have larger real incomes in the late periods of their lifes, every Golden Rule steady state involves a negative outside money. The real interest rate should then be positive for each Balanced steady state, at least in "well behaved" economies. This is sometimes called the *Classical* case in the literature, since this is the kind of situations which were depicted by Classical writers, like I. Fisher, when arguing that real interest rates should be positive. On the other hand, if the consumers have larger real incomes in their youth, and if they do not discount too much the future, every Golden Rule steady state implies a positive outside money, while the real rate of interest should be negative, in "well behaved" economies, for all Balanced steady states. This is sometimes called the *Samuelson* case, as such situations were considered by this author in his seminal paper on the "social contrivance" of money. One gets then the following qualitative table :

Characteristics of the economy	Golden Rule steady states	Balanced steady states
Classical	$\rho = 0$ $\mu < 0$	$\rho > 0$ $\mu = 0$
Samuelson	$\rho = 0$ $\mu > 0$	$\rho < 0$ $\mu = 0$

In addition to monetary steady states, there is of course a nonmonetary stationary equilibrium where money has no value in exchange, and where there are accordingly no intertemporal income transfers at the individual level, contrary to what happens along a monetary steady state.

The mere multiplicity of possible long run equilibria raises important questions concerning the nonsteady behaviour of the model. More precisely, consider an arbitrary date, where the (nonsteady) past history of the economy is given. Assume that the Government intervenes from now on only on the credit market through its Banking Department, by manipulating the nominal interest rate or the money supply, and that the evolution of the economy is described, following the Neoclassical tradition, by a sequence of short run equilibria, subject to the (stationary) monetary policy chosen by the Bank in each period. Will the economy tend to a steady state, and, if yes, which one ?

In view of the validity of the *Classical Dichotomy* in the present model, if the Bank implements a policy which aims at pegging a real variable (e.g. the real interest rate) at a predetermined level which is incompatible with the long run equilibrium of the real sector (that is, with the equations (A) of Section 3), the corresponding sequence of short run equilibria can only tend to a nonmonetary equilibrium if it converges at all towards a steady state. This would mean the eventual breakdown of the monetary institutions. One can hope therefore to preserve the monetary character of the economy only by restricting the attention to the case where the Bank seeks to control nominal variables, e.g. by pegging the nominal interest rate at a constant level, or the rate of growth of the amount of credit distributed at every date.

The dynamics of the model is a complex matter, which will depend, presumably crucially, on how the consumers' forecasts of future prices and interest rates are functions at any moment of their information on the current and past states of the economy. Very little is in fact known on the subject, and this is a topic where a significant effort of research is needed in the future. A moment of reflexion shows nevertheless that a few dynamic configurations are impossible, or unlikely.

Let us start with the simple remark that along a sequence of short run equilibria, outside money increases within each period by the Bank's deficit (the extent of bankruptcy), and that it grows in addition mechanically from the end of a period to the beginning of the next at a rate equal to the nominal interest rate [1]. Therefore if outside money is initially positive, it is bound to remain positive afterwards, in which case the economy will never converge to a Golden Rule steady state involving a negative outside money (as it occurs in the Classical case). Symmetrically, if outside money is initially negative, it is likely to remain negative onwards, if one neglects the possibility that bankruptcies make its sign reversed. In such circumstances, the trajectory of the economy is likely to stay away from all Golden Rule steady states which involve a positive outside money (as they do in the Samuelson case).

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(1) See the discussion of the equations $(\mathcal{C})$, $(\mathcal{D})$, $(\mathcal{E})$ in II.2. It should be noted that this statement relies upon a simple accounting argument which is independent of the particular notion of equilibrium which is employed in the short run. This statement, and the considerations which follow, are accordingly valid even if the evolution of the economy is governed by a dynamic "disequilibrium" process where trade occurs at any moment at nonmarket-clearing prices.

Balanced steady states, where outside money is by definition equal to 0, can be obtained as the limit of a nonsteady sequence of short run equilibria only if nominal outside money grows eventually at rate which is less that the limiting rate of inflation, so as to ensure that "real" outside money tends actually to 0. Now, nominal outside money grows eventually at a rate equal to the nominal rate of interest (expectations become almost rational as one gets closer to the steady state, and there are thus no bankruptcies ultimately). Accordingly, a Balanced steady state which is the limit of a particular nonsteady trajectory involves necessarily a negative real interest rate, if one neglects the coincidental case where nominal outside money is permanently equal to 0 along the trajectory under consideration (1). By contrast, a Balanced steady state where the real interest rate is positive will be typically unstable.

These heuristic arguments may give in some cases a few qualitative insights about the dynamic of the model. In the "Classical" case, a trajectory for which outside money is initially positive will typically stay away from monetary steady states. If the trajectory approaches in the long run a steady equilibrium, it must then be a nonmonetary one, in which case the monetary institutions are eventually destroyed.

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(1) Note that convergence towards a Balanced steady state where the real interest rate is negative is conceivable in particular when the Bank sets a nominal interest rate equal to 0 in every period, in which case nominal outside money is constant overtime, if we neglect the effect of bankruptcies. *Sustained inflation is conceivable in that class of models with a constant level of nominal outside money, without implying any disruption of the monetary system.*

In most cases, however, there is still a multiplicity of possible long run equilibria. The study of the nonsteady behaviour of the economy requires then to model precisely the consumers' process of expectations formation. As we said, this is an area where our ignorance is great, and where a great deal of research is needed.

Remark. There have been a few studies of the monetary dynamics of the overlapping generation model when expectations are *rational*, i.e. when consumers forecast correctly future prices and interest rates even out of steady states (Cass and Yaari (1966), Gale (1973). See also Kareken and Wallace, Eds. (1980) and Hahn (1980)). Gale's work suggests that, in "well behaved" environments, assuming a unique steady state in each class :

. in the Samuelson case, the Balanced steady state is stable,

. in the Classical case, the Golden Rule path is stable if outside money is initially negative, while the only Rational expectations equilibrium is the nonmonetary one if outside money is negative.

Whether or not these qualitative conclusions hold when consumers do not have rational expectations but learn over time the dynamics of prices and interest rates, is an unresolved question.

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