MONEY AND CREDIT IN THE SHORT RUN*

by

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February 1981

N° 8104

* This is a preliminary draft of the second chapter of a forthcoming monograph on Neoclassical Monetary Theory. Partial support from the University of Bonn and from the National Science Foundation under grant # SOC 78-06157 to the Center for Analytic Research in Economics and the Social Sciences at the University of Pennsylvania is gratefully acknowledged.

MONEY AND CREDIT IN THE SHORT RUN

The previous lecture was devoted to the study of an economy where consumers could only save by holding nonnegative money balances. There was no money creation or destruction, since the Government was assumed not to engage in any fiscal policy (no taxes or subsidies, no governmental intervention on the goods markets). The stock of money was accordingly constant over time. This case is often called *Outside Money* in the literature.

We introduce now in the model the possibility for the consumers to borrow against future income by selling *short term* bonds to a governmental agency, called the *Bank*. Since there is money creation whenever the Bank grants loans by buying bonds from the consumers, the money stock can then vary over time according to the needs of the economy.

A useful distinction between "inside" and "outside" money is often developed in credit money economies of this type [(1)]. The part of the money stock which is "backed" by the Bank's claîms on the private sector, that is, which has been issued by the Bank when purchasing consumers' bonds, is called *Inside Money*. The other part of the money stock, which is not "backed" by private debts, is called *Outside Money*.

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(1) The distinction has been introduced by Gurley and Shaw (1960).

Our primary concern in the present chapter will be the study of the economic forces which may bring into equilibrium the markets of such an economy in the short run. The analysis will take into account the fact that the Bank may try to influence economic activity through a deliberate policy on the bond market. Two specific policies will be actually considered : the case where the Bank pegs the interest rate which is charged on loans at some given level, and the case where the Bank seeks controlling the money supply, that is, the amount of money which it creates when buying bonds from the private sector.

Neoclassical theorists were faced with serious difficulties when attempting to deal with the problem. There are, according to this view point, two main equilibrating mechanisms in such an economy.

The first one, which is called the real balance, or *wealth effect*, is generated by a change of the current money prices of goods, the interest rate being given, and is defined in such a context as the consequence upon a trader's demand for goods which results from the change of the "real value" of the trader's initial money stock or debt. It is this mechanism which is supposed to bring the economy into equilibrium when the Bank pegs the interest rate at a given level.

The wealth effect works in *opposite* directions for a creditor, who holds initially a positive money balance, and for a debtor, who holds a negative one. Its consequences upon aggregate demand are therefore ambiguous. Neoclassical theorists claimed nevertheless that the wealth effect is an operational regulating mechanism as long as outside money is positive, and that the Bank can then peg the interest rate at any level, i.e., that it has full control over the interest rate. The empirical relevance of this claim has been seriously disputed, however, on the ground that a great part of the money stock has usually a direct or indirect counterpart in private debts, so that one would need quite large variations of money prices to make the wealth effect operational.

The other equilibrating mechanism considered by Neoclassical theorists is generated by a variation of the interest rate, the current money prices of goods being fixed. This variation is assumed to induce a *substitution* between current and future goods, by modifying every trader's optimum consumption savings ratio. It is this mechanism, together with the wealth effect, which is supposed, according to the Neoclassical viewpoint, to enable the Bank to exercise a control over the money supply by varying the interest rate. The empirical relevance of this conclusion has been seriously disputed too, as many economists believe that a variation of the interest rate has little effect in the short run upon aggregate demand in actual economies.

These issues have generated important controversies among theorists, which do not appear to be settled as yet. The purpose of the present chapter is to have a fresh look at the problem, by incorporating explicitly in the analysis the intertemporal choices which the traders face. The main finding of our investigation will be that, contrary to what Neoclassical theorists used to believe, the wealth effect is likely to be too weak to equilibrate the market, even in the case where outside money is positive and large. A credit money economy will be found, as a matter of fact, particularly vulnerable in inflationary situations.

At the root of the phenomenon is the fact that even a small group of traders can "destabilize" the whole economy if they forecast a large rate of inflation. For they may be led then to borrow a lot from the Bank, and by spending the money borrowed on current goods, to generate an excess demand on the corresponding markets at all current prices and interest rates. The conclusion which will emerge from our analysis will be therefore, that *the market "invisible hand" is likely to go astray in a credit money economy with flexible prices, especially in inflationary situations, and that a short run equilibrium where money has positive value may not exist, no matter which policy the Bank chooses to implement.*

The second issue we shall be concerned with is the possibility for the Bank to influence in the short run the money supply, or more generally economic activity in such a context. Our conclusion will be that, *even in the favourable case where there is a short run equilibrium for each interest rate, that is, where the Bank can fully control this variable, the Bank may have a little influence on the money supply or on equilibrium prices goods,* because the intertemporal substitution effect generated by a variation of the interest rate is too weak. Such a circumstance will occur whenever a rise of the interest rate is offset, partially or completely, by a correlative increase of expected prices. It is thus not unlikely to be observed in actual economies, as an increase of the nominal rate of interest may be interpreted by the traders as a sign that inflationary tensions are building up in the economy, and conversely. *The ability of monetary policy to influence, by means of a variation of the interest rate, the amount of money created by monetary authorities when granting loans to the private sector, appears accordingly somewhat dubious.*

Our study will show that the essential short run regulating mechanism of a credit money economy is not the wealth effect, as Neoclassical theorists mistakenly believe, but the *intertemporal substitution effect* which is generated by the relative variations of current and expected prices of goods, or by a variation of the current interest rate. One of the purposes of the present analysis will be to discover the kind of conditions on expectations which are needed to make these substitution effects operational, that is, which ensure the existence of a short run equilibrium subject to the policy chosen by the Bank. This will essentially require that the trader's expectations are *inelastic* with respect to current prices and/or the interest rate. This sort of condition can hardly be expected to prevail in reality. In particular, *all traders' expected prices will have to be insensitive to a great extent, to large increases of current prices of goods, which is quite unlikely to be observed in inflationary environments.*

The chapter is organized as follows. Section 1 is devoted to the description of the institutional set-up of the model. We give in Section 2 a brief account of the Neoclassical views about the short run determination of prices and of the interest rate in our framework. The consumers' short run demand and supply functions are derived in Section 3 from an explicit intertemporal decision making problem. Section 4 is devoted to the case where the Bank pegs the interest rate at some arbitrary level. Finally, the case where the Bank attempts to control the money supply is considered in Section 5.

1. STRUCTURE OF THE MODEL.

The real part of the model is the same as in the preceding chapter. There are accordingly ℓ non storable consumption goods available in each period, whose (positive) equilibrium money prices $p = (p_1,\ldots,p_\ell)$ are to be determined by the market at every date.

The monetary part of the model differs somewhat since a governmental agency is introduced, which performs (some of) the usual functions of a *Bank*. Money takes the form of *Bank deposits*, which may or may not bear interest. Consumers have thus the opportunity to save at each date by keeping a positive money balance at their Bank account. The other roles of the Bank are to grand short term loans to consumers and to receive repayments in money of their past debts. Consumers can thus borrow against future income by selling to the Bank *short term bonds*, a unit of bond being a promise made by the issuer to pay back one unit of money one period later. At each date, the money price s of these bonds determines the nominal *borrowing interest rate* r by the relation $s = \frac{1}{1+r}$. The Bank is supposed to follow a "competitive" policy, i.e., to maintain the equality in every period between the interest rate paid on deposits with the borrowing rate. Since the money price of bonds must be finite, the interest rate must exceed -1 in every period. In this highly stylized model, where money takes the *only* form of Bank deposits on which a negative interest rate can be paid, this is in fact the only constraint which restricts the rate of interest [1].

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(1) If there were an additional sort of money, namely *paper money* on which the interest rate is by definition 0, as it is the case in actual economies, an additional constraint would appear, since the interest rate should then be nonnegative. Under the assumption of full convertibility of the two sorts of money, paper money disappears immediately within the Bank if the interest rate paid on deposits is positive. If this interest rate is zero, paper money and deposits are perfect substitutes. The model studied in this chapter applies therefore directly to this case, with the only additional restriction that the interest rate cannot be negative.

It is assumed, for the sake of simplicity, that there are no other Governmental interventions than those which it makes through its Banking Department, as they were described above. In particular, there are no Government purchases or sales of goods, and no transfer payments such as taxes or subsidies.

The transactions patterns in such an economy can be described as follows. In any period, consumers exchange among themselves their exogenously given endowments of goods. Payments of the purchases are made by transfering the corresponding amounts of money from the buyers' accounts to the sellers' accounts within the Bank. On the other hand, those consumers who wish to borrow sell short term bonds to the Bank, and receive money in exchange. The monetary counterparts of these transactions, together with the reimbursements made by the consumers who are initially debtors, determine the consumers' money holdings at the end of the period.

Our main concern in this Chapter will be the study, in this framework, of the short run determination of equilibrium prices and of the interest rate at some date, say date 1, or the "current period". The Bank may try to influence the equilibrium position by varying the interest rate or its money supply. Our study will be carried out, therefore, subject to the short run policy which the Bank wishes to implement on the credit market. Two specific policies will be actually considered : one where the Bank seeks to peg the interest rate, and another one where the Bank tries to control the amount of money which it creates when granting loans to consumers.

In order to proceed to such a study, one must specify the short run characteristics of every consumer a living at the date under considerations. The consumers' short run "real" characteristics are, as in Chapter I, Section 2, the length n_a of their remaining lefetime, their preferences u_a among consumption streams, and their endowments of goods e_{at} , for $t = 1 , \ldots , n_a$ (1).

The following two assumptions are made throughout the chapter. They are the same as assumptions (*a*) and (*b*) of I.2.

(*a*) *The utility function* u_a *is continuous, increasing and strictly quasi-concave, for every* a .

(*b*) *All components of the endowment vectors,* e_{at} , *are positive, for every* a *and* t .

The other characteristics of a consumer are his money stock $\bar{m}_a \geqq 0$ at the beginning of the period, which is supposed to include any interest payment, and the amount of the money he owes initially to the Bank, $\bar{b}_a \geqq 0$, which is equal to the number of bonds which he supplied at the preceding date. One can assume without loss of generality that either $\bar{m}_a = 0$ or $\bar{b}_a = 0$, and summarize this information by looking at the consumer's "initial net credit position" $\bar{\mu}_a = \bar{m}_a - \bar{b}_a$, which is positive in the case of a creditor, and negative otherwise. The *initial aggregate money stock* M is then, by definition, equal to $\sum_a \bar{m}_a$, while the *initial value of outside money* is described by the economy's

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(1) It is assumed, needless to say, that there are consumers whose horizon extends beyond the current period, i.e. for whom $n_a \geqq 2$.

"aggregate net credit position", that is $\sum_a \bar{\mu}_a$. No specific assumptions will be made on the traders' initial money holdings or debts. They can be, in particular, all equal to zero.

2. NEOCLASSICAL VIEWS ON INSIDE AND OUTSIDE MONEY.

It is useful to begin the analysis with a brief summary of the Neoclassical views about the short run determination of equilibrium prices and of the interest rate in the present context.

According to this viewpoint, the decisions taken at date 1 by a typical consumer a can be considered as functions of the current money prices of goods, which are described by a vector of positive prices p_1, of the current interest rate r_1, and of his initial net credit position $\bar{\mu}_a$. A typical trader's vector of excess demands for goods is then written $z_a(p_1, r_1, \bar{\mu}_a)$, while his demand for money and his supply of bonds are denoted $m_a^d(p_1, r_1, \bar{\mu}_a)$ and $b_a^s(p_1, r_1, \bar{\mu}_a)$, respectively. On the other hand, the amount of money which the consumer gives back to the Bank in reimbursement of his initial debt, if any, is denoted $R_a(p_1, r_1, \bar{\mu}_a)$. It is equal to $\bar{b}_a$ whenever the consumer succeeds in reimbursing the totality of his debt. It may be less otherwise, i.e. when the trader is bankrupt.

Neoclassical theorists usually assume that these functions display a few simple properties.

First, every consumer faces a budget constraint, which states that the value of his excess demand for goods plus his final money holding should be financed either out of his initial money holding $\bar{m}_a$ (net of any reimbursement to the Bank) or from the proceeds of the sale of bonds. Formally,

$$p_1 z_a(.) + m_a^d(.) = \bar{m}_a - R_a(.) + b_a^s(.)/(1+r_1)$$

for every p_1 and r_1, where the symbol $(.)$ stands for $(p_1\ ,\ r_1\ ,\ \bar{\mu}_a)$. As we shall see, these identities imply that the economic system should satisfy *Walras Law.*

Second, the excess demand functions z_a should be homogenous of degree 0 in $(p_1\ ,\ \bar{\mu}_a)$, while the functions m_a^d , b_a^s and R_a should be homogenous of degree 1 in the same variables. Neoclassical theorists justify traditionally these assumptions by arguing, somewhat loosely, that "only real money balances matter", or by assuming, explicitly or implicitly, that price expectations are unit elastic with respect to current prices.

The equilibrium conditions for goods, money and bonds are now easy to formulate. Equilibrium of the goods markets requires that aggregate excess demands for goods should be zero :

$$(C) \qquad \textstyle\sum_a z_a(p_1, r_1, \bar{\mu}_a) = 0 \ .$$

The Bank's net money supply is equal to the amount of money created through the bonds purchases, say ΔM, minus the money paid back to the Bank in reimbursement of past debts, i.e. $\sum_a R_a(p_1, r_1, \bar{\mu}_a)$. The equilibrium condition for money states, then, that the aggregate demand for money must be equal to the initial aggregate money stock M, plus the Bank's net money supply. This yields

$$(D) \qquad \sum_a m_a^d(p_1, r_1, \bar{\mu}_a) = M + \Delta M - \sum_a R_a(p_1, r_1, \bar{\mu}_a) \ .$$

With these notations, the Bank's demand for bonds is given by $(1+r_1)\Delta M$. Equilibrium of the bond market then requires that this expression should be equal to the consumers' aggregate bond supply :

$$(E) \qquad (1+r_1)\Delta M = \sum_a b_a^s(p_1, r_1, \bar{\mu}_a) \ .$$

These equations make clear how the money stock adapts itself to the needs of the economy through the variations of the amount of credit distributed. They have also interesting implications concerning the evolution of outside money over time. Remark first that the expression $\sum_a \left[m_a^d(.) - b_a^s(.)/(1+r_1) \right]$ represents the value of outside money at the end of the period, before any interest payment. Equations $(\mathcal{D})$ and (E) imply that this expression must be equal, in equilibrium, to $M - \sum_a R_a(.)$; or equivalently, to the sum of the initial value of outside money, $\sum_a \bar{\mu}_a$, and of the extent of bankruptcy, i.e., $\sum_a \left[\bar{b}_a - R_a(.) \right]$. Outside money increases within the period by the extent of bankruptcy, which can be interpreted as the Bank's "deficit" in the period under consideration. If one considers on the other hand the value of outside money at the beginning of the next date, which includes interest payments and is thus equal to $\sum_a \left[(1+r_1)\, m_a^d(.) - b_a^s(.) \right]$, one sees that outside money must grow from date 1 to date 2, at a rate equal to the short term interest rate r_1.

It is useful to see, as an incidental remark, how equation $(\mathcal{D})$ is modified when we let the Government to be active on the goods markets and/or to levy taxes and pay subsidies to consumers, for instance through a separate Treasury Department. The value of this new

Department's deficit (the money value of its net purchases of goods, minus net tax receipts) has then to be covered by an issue of money $\Delta\mu$ which must appear on the right hand side of $(\mathcal{D})$ as part of the Government's net supply of money. According to $(\mathcal{D})$ and $(\mathcal{E})$, the increase of outside money from the beginning to the end of the market process at date 1 is then equal to the sum of the extent of bankruptcy and of $\Delta\mu$, that is, to the "deficit" of the Government as a whole. If one considers the additional effects of the interest rates on the growth of outside money, one can say generally, therefore, that outside money at any point of time is equal to the cumulative sum of the past global deficits of the Government, each deficit being compounded by the interest rates which prevailed between the period of its appearance and the date under consideration [(1)].

Since consumers face a budget constraint, the system $(\mathcal{C})$, $(\mathcal{D})$, $(\mathcal{E})$ should satisfy *Walras Law*, which states that the sum of the value of excess demand over all markets is identically equal to zero :

$$p_1 \sum_a z_a(.) + \left[\sum_a m_a^d(.) - M - \Delta M + \sum_a R_a(.)\right] + \left[\Delta M - \sum_a (b_a^s(.)/(1+r_1))\right] = 0.$$

for every p_1 and r_1, where the symbol $(.)$ stands for $(p_1, r_1, \bar{\mu}_a)$. This identity implies as usual that one of the equations may be "eliminated" when studying the equilibrium of the system. There are, therefore, at most $(\ell+1)$ independent equations to determine the $(\ell+2)$ unknown parameters of the system, namely p_1, r_1 and ΔM.

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(1) A negative value of initial outside money at date 1 is thus conceivable. This would mean that the Government ran an overall surplus prior to that date, e.g., by levying taxes from consumers. In such a case, the assumption that the Government does not intervene on the goods markets and that it is inactive on the fiscal side, applies, strictly speaking, only to the current period and the following ones.

The Bank can thus hope *a priori* to be able to influence the equilibrium position of the economy at date 1 by choosing a *short run monetary policy* on the credit market, that is, by specifying how its money creation ΔM varies with current economic observables, e.g. the current prices system p_1 and the current rate of interest r_1 . There are evidently infinitely many ways to do this. Two particular and important policies will be considered in this lecture. In the first case, the Bank will peg the interest rate at an arbitrary level r_1, and let its money supply to adapt itself so as to equilibrate the credit market. In the other case, the Bank will fix its money creation ΔM at a given level, and let the interest rate r_1 free to vary.

Pegging the Interest Rate

Let us first consider the case where the Bank fixes the interest rate at an arbitrary level r_1 .

Since the Bank pegs the interest rate, its credit supply is assumed to be infinitely elastic : all the bonds supplied by the consumers are automatically bought by the Bank. The money creation ΔM is then endogenous, and is in fact defined by the bond market equation (E) as a function of the current prices of goods p_1. Replacing ΔM by this expression in $(\mathcal{D})$ yields the following system of equations :

$$(C) \qquad \sum_a z_a(p_1,r_1,\bar{\mu}_a) = 0$$

$$(\mathcal{D}_1) \qquad \sum_a m_a^d(p_1,r_1,\bar{\mu}_a) = M + \sum_a \left[(b_a^s(p_1,r_1,\bar{\mu}_a)/(1+r_1)) - R_a(p_1,r_1,\bar{\mu}_a) \right]$$

where the rate of interest r_1 is fixed, and where the only variables left are the current prices of goods p_1. .

Equation (C) and $(\mathcal{D}_1)$ explain according to Neoclassical theorists, the short run determination of equilibrium prices when the Bank pegs the rate of interest. Their postulates imply that these equations should display the following properties.

As we have seen, the system satisfies *Walras Law,* which reads

$$p_1 \sum_a z_a(.) + \sum_a m_a^d(.) = M + \sum_a\left[(b_a^s(.)/(1+r_1)) - R_a(.)\right]$$

for every p_1 , where the symbol (.) stands for $(p_1, r_1, \bar{\mu}_a)$. By a now familiar argument, this identity implies that the *Classical Dichotomy* between the real and the money sectors is invalid in the short period, since any solution of (C) satisfies $(\mathcal{D}_1)$ as well.

The homogeneity properties of short run demand and supplies assumed by Neoclassical theorists, imply that multiplying all initial net credit positions $\bar{\mu}_a$ by a positive number λ multiplies by the same factor the level of equilibrium money prices, but leaves unaltered equilibrium relative prices and real quantities. This is the old *Quantity Theory*, reformulated in the present context so as to apply to the short period.

More importantly, these homogeneity postulates imply that the main short run regulating mechanism, when the Bank pegs the rate of interest, is the "wealth effect". To see this point it is most convenient to ignore the complications arising from modifications of relative prices, or alternatively to consider the simple "macroeconomic" case where there is only one good (ℓ=1). Equations $(\mathcal{C})$ and $(\mathcal{D}_1)$ are then equivalent, and one can focus the attention on the good market alone.

In that case, each trader's excess demand for the good can be written $z_a(1\ ,\ r_1\ ,\ \frac{\bar{\mu}_a}{p_1})$. The interest rate being fixed, movements of the current price p_1 affect the trader's desired consumption only through their influence on his initial real monetary wealth $\frac{\bar{\mu}_a}{p_1}$. An increase of p_1 is thus likely to yield an increase of the trader's desired consumption in the case of a creditor, since $\bar{\mu}_a$ is then positive. It has the opposite consequence in the case of a debtor, i.e. when $\bar{\mu}_a$ is negative. Variations of p_1 have no influence when $\bar{\mu}_a = 0$.

The consequence upon aggregate consumption of current price variations through the wealth effect are thus ambiguous, since it depends on the relative magnitudes of the creditors' and the debtors' marginal propensities to consume out of real wealth (the derivatives of the functions z_a with respect to $\frac{\bar{\mu}_a}{p_1}$) and on the sizes and the distribution among traders of their initial net monetary wealths $\bar{\mu}_a$.

Most theorists, however, following the tradition set up by Pigou, Patinkin and others, have ignored deliberately such "distributional effects", and have focused the attention on the response of aggregate demand to changes in the community's aggregate "real net wealth" as the main equilibrating mechanism of a credit money economy. In our context, this viewpoint amounts to saying that aggregate excess demand in (C) behaves qualitatively as if it depended solely on the real value of outside money $\frac{\sum_a \bar{\mu}_a}{p_1}$. Many theorists (Hicks, Patinkin, Johnson) were led accordingly to the apparently natural conclusion that the wealth effect is an operational equilibrating mechanism in the case where outside money is positive. Indeed, according to this line of thought, the community's real net wealth $\frac{\sum_a \bar{\mu}_a}{p_1}$ is large when p_1 is low, which should lead to an aggregate excess demand on the good market. Conversely, aggregate real net wealth is low for large prices, in which case an excess supply for good should appear. By continuity, an equilibrium should exist in between, and moreover, such an equilibrium position should be stable in any *tâtonnement* process where prices respond positively to excess demand. The proposition, which dates back at least to Wicksell, that the levels of money prices are indeterminate in a "pure" inside money economy, i.e. when outside money $\sum_a \bar{\mu}_a$ is zero, which seems to be an immediate consequence of this sort of analysis, appears in many writings too [(1)].

The theoretical and empirical relevance of the wealth effect in a credit money economy has been, and still is on the front stage of the dispute between Keynesian and Neoclassical economists. Part of Keynes's theoretical work can be (and has been) regarded as an attack against the widespread belief that there are built-in stabilizers in a competitive monetary economy with flexible prices, which would automatically eliminate excess supplies or demands without any Government intervention. The traditional Neoclassical answer, which was initiated by Pigou and developed by Patinkin and others, has been that the wealth effect as described above provides indeed such an automatic stabilizer.

This claim appears to be acknowledged as a valid theoretical proposition by many theorists today. Its empirical relevance, however, has been seriously disputed on the basis that outside money is usually

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(1) A few theorists (e.g. Pesek and Saving) claimed that the whole money stock (here $M = \sum_a \bar{m}_a$), or at least a positive fraction of it, should be counted as part of the economy's private net wealth, on the basis that money renders services of social value by facilitating exchanges. For these economists, the wealth effect is an operational regulating mechanism (as long as the money stock is positive), independently of the value of outside money. This position, however, does not seem to have been generally accepted by the profession (see, e.g. Patinkin (1972), Chap. 9, or Crouch (1972), p. 135 or 378), although one can find some traces of it even in recent writings or textbooks.

a small part of the money stock, so that one would need enormous price variations in order to make the wealth effect operational. Many theorists believe nowadays that the wealth effect is usually so weak in practice that it can be neglected.

A few dissenting voices question the theoretical validity of the Neoclassical position as well, by remarking that the elimination of distribution effects which is involved in the "cancelling out" of private money holdings and debts, may be quite misleading. As early as in the thirties, Irving Fisher insisted on the increasing burden which a price decline imposed upon debtors, and concluded that deflation was likely to weaken aggregate demand. Quite recently, James Tobin developed the same kind of arguments, on the grounds that the largest part of private monetary assets has a direct or indirect counterpart in private debts, and that the marginal propensities to consume out of wealth are probably greater for debtors than for creditors (1). In such circumstances, a price increase is likely to yield an increase of aggregate demand, and conversely. The wealth effect appears in that case to be an automatic "destabilizer", in contradiction to what Neoclassical theorists claim.

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(1) See J. Tobin (1980), Chap. I, especially pp 9-11, and the references to I. Fisher therein.

Controlling the Money Supply

What precedes concerned the case where the Bank pegged the interest rate at some arbitrary level r_1. We consider now what Neoclassical theory has to say in the case where the Bank seeks to control the money supply, by fixing *a priori* the amount of money which it creates by granting loans to consumers at an arbitrary level $\Delta M > 0$. The interest rate r_1 is then free to vary, together with the money prices of goods p_1 , in order to equilibrate the markets at date 1.

The equations which determine the equilibrium values of p_1 and r_1 are thus given by the market clearing conditions $(\mathcal{C})$, $(\mathcal{D})$, $(\mathcal{E})$ which were described at the beginning of this section :

$$(\mathcal{C}) \qquad \sum_a z_a(p_1,r_1,\bar{\mu}_a) = 0 \quad ,$$

$$(\mathcal{D}) \qquad \sum_a m_a^d(p_1,r_1,\bar{\mu}_a) = M + \Delta M - \sum_a R_a(p_1,r_1,\bar{\mu}_a) \quad ,$$

$$(\mathcal{E}) \qquad (1+r_1)\ \Delta M = \sum_a b_a^s(p_1,r_1,\bar{\mu}_a) \quad ,$$

where the Bank's money creation $\Delta M > 0$ is fixed exogenously.

As we have seen, this system of equations satisfies *Walras Law* : the sum accross all markets of the values of excess demands is zero for all p_1 and r_1. As a consequence, if one succeeds in achieving equilibrium of all markets but one, the remaining market is in equilibrium as well. On the other hand, the homogeneity properties of short run demands and supplies which Neoclassical theorists assume imply that an equiproportionate change of all initial net credit positions $\bar{\mu}_a$ and of the money supply ΔM multiplies by the same factor all equilibrium money prices, but leaves unchanged equilibrium "real" quantities such as relative prices, and the interest rate. This is again the old *Quantity Theory*, reformulated so as to apply to the short run in the present context [1].

An important issue is to know whether the foregoing system of equations has a solution for every ΔM , for it conditions the Bank's ability to control the money supply. In our framework, where the Banking sector is wholly in the hands of the Government, Neoclassical monetary theory gives an affirmative answer to this question, since many economists of this school claim that monetary authorities can always control the quantity of their own liabilities [2].

In order to uncover the regulating mechanisms which such a claim presupposes, it is convenient to proceed in two steps, by considering successively the real and the money sectors of the model.

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(1) By contrast, a change of the money supply ΔM alone, the $\bar{\mu}_a$'s being fixed, will have "real" effects, according to this line of thought.

(2) See, e.g., Friedman (1969), Chap. 5. There have been some controversies concerning a Central Bank's ability to control the money supply by manipulating the quantity of its own liabilities (sometimes called "base money"), or reserve requirements in the presence of commercial banks. These controversies are not relevant here, since there is no private banking sector in the model.

Let us first "solve" in p_1 the equilibrium conditions (C) for the goods markets, given the interest rate r_1. One can thus associate in such a way to every r_1 the amount of money which the consumers wish to borrow from the Bank at the corresponding solution of (C), i.e. $\sum_a b_a^s(p_1, r_1, \bar{\mu}_a)/(1+r_1)$ [(1)]. Equilibrium of the bonds market will be achieved if one succeeds in finding a value of r_1^* such that this expression is equal to the Bank's money supply ΔM . By Walras Law, the interest rate r_1^* , together with the corresponding solution p_1^* of (C), will then bring the whole system into equilibrium.

The way of looking at the problem makes clear that the existence of a solution to (C), (D) and (E) depends, according to the Neoclassical viewpoint, on the presence and the intensity of two regulating mechanisms. Solving (C) in p_1 for a given r_1 is in fact equivalent to finding the equilibrium prices of goods which would arise if the Bank chose to peg the interest rate at this level. We have seen that, at this stage, the essential regulating mechanism, for Neoclassical theorists, is the wealth effect which is generated by a variation of the current prices of goods. The other mechanism comes into play in the second phase of the procedure, when the interest rate is made to vary. What is needed here is that a change of r_1 causes a significant *substitution* between current and future consumption, and thus generates a sizeable variation of the consumers' desired consumption savings ratio. Large interest rates should generate large savings, so that the amount of money which the consumers

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(1) We ignore in this heuristic discussion the complications arising from the possible existence of multiple solutions of (C) for a given value of the interest rate.

wish to borrow at such interest rates be much lower than the Bank's money supply ΔM. Symmetrically, a low interest rate, by reducing the cost of credit, should incite consumers to borrow a lot. By continuity, one would then be able, by varying the interest rate, to equate the amount of money which the consumers wish to borrow from the Bank with the given money supply ΔM , and thus to bring the whole system into equilibrium. As the argument does not depend upon the value taken by ΔM , the Bank would have in fact, according to this viewpoint, full control over the money supply.

The Neoclassical argument rests essentially in the present context, therefore, upon the presence and the intensity of the wealth effect, which is generated by a variation of the current prices of goods, and of the intertemporal substitution effect which is caused by change of the interest rate. We gave already a brief account of the controversies which concerned the theoretical and empirical relevance of the wealth effect. The impact of a variation of the interest rate upon the demand for credit has generated numerous controversies as well. A significant number of economists doubt that changing the cost of credit has an important influence upon the desired amount of borrowing in the short run. For these economists, the ability of monetary authorities to control the money supply through a variation of the interest rate is rather limited.

*

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This brief (and somewhat oversimplified) review shows that the literature on the subject is a little confusing, as one is faced by a number of conflicting theoretical statements concerning the mechanisms at work in a credit money economy. Although a great deal of work has been done, a precise integration of money and value theory in an economy involving inside and outside money is still badly needed. The goal of the remainder of this chapter is to make progress in that direction.

3. SHORT RUN DEMAND AND SUPPLY FUNCTIONS.

The purpose of the present section is to derive precisely the behavior of consumers in the short run from an explicit analysis of the intertemporal choices they have to make. Let us go back to the simple institutional framework which was described in Section 1, and consider a typical consumer (we drop his index a for convenience), who observes in the current period a price system p_1 and an interest rate equal to r_1 , and who expects the price system p_t and the interest rates r_t to prevail in the future, for $t = 2 ,\dots, n$.

This trader must choose his current consumption $c_1 \geqq 0$, his money balance $m_1 \geqq 0$, his supply of short term bonds $b_1 \geqq 0$, and the amount of money $R_1 \geqq 0$ which he gives back to the Bank in reimbursement of his initial debt $\bar{b}$, if any. The consumer has to plan as well the same quantities c_t , m_t , b_t and R_t for every future period $t = 2 ,\dots, n$. The trader's choices must satisfy the current and expected budget constraints :

$$p_1c_1 + m_1 - \frac{b_1}{1+r_1} = p_1e_1 + \bar{m} - R_1 \quad ,$$

and

$$p_tc_t + m_t - \frac{b_t}{1+r_t} = p_te_t + m_{t-1}(1+r_{t-1}) - R_t$$

for every $t = 2 ,\dots, n$.

The borrowing and the lending rates being equal at every date, the consumer cares in fact only for his net credit position at the Bank in every period, which is equal to $\mu_t = m_t - \frac{b_t}{1+r_t}$ before interest payment. One can therefore impose without any loss of generality that the trader is either a creditor or a debtor but not both at each date, or equivalently, that either m_t or b_t is equal to zero for every t. It will be assumed moreover that a consumer is never allowed to borrow in the last period of his life. This constraint will be expressed here by the condition $b_n = 0$, or equivalently , $\mu_n \geqq 0$.

In order to focus on the essentials, we shall adopt here a set of assumptions concerning a consumer's behaviour towards bankruptcy which is as simple as possible [(1)]. The underlying idea is that there are heavy extra economic penalties associated with bankruptcy, which will be left unspecified, for simplicity. It will be assumed accordingly that in order to avoid these penalties, a consumer never plans to be banrupt in the future [(2)], and that he seeks to reimburse in the current period the maximum possible of his initial debt.

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(1) The issue of bankruptcy is a complex one, which would necessitate a study on its own. To go deeper into this question would have led us too far from our main argument.

(2) This does not preclude, of course, the possibility that the consumer will be actually bankrupt in the future if his expectations are falsified.

The first condition means that for every future date $t = 2, \ldots, n$, the planned reimbursement R_t is equal to the debt b_{t-1}. The current reimbursement R_1 is then easy to find. The maximum amount of money which the consumer can borrow in the current period (i.e. the maximum value of $\frac{b_1}{1+r_1}$), while avoiding the prospect of defaulting in the future, is equal to the sum of the trader's discounted expected incomes, i.e., to $\sum_2^n \beta_t p_t e_t$, where the discount factors β_t are equal to $\frac{1}{(1+r_1)\ldots(1+r_{t-1})}$. The maximum amount of cash which is potentially available to the consumer at date 1 is thus given by $\bar{m} + \sum_1^n \beta_t p_t e_t$, where by convention $\beta_1 = 1$. When this expression is greater than or equal to the consumer's initial debt $\bar{b}$, his reimbursement to the Bank R_1 is equal to $\bar{b}$. Otherwise, the trader is bankrupt, and R_1 is equal to $\bar{m} + \sum_1^n \beta_t p_t e_t$.

These conditions specify complety the constraints under which the consumer is making his choices. By using the net credit positions before interest payment $\mu_t = m_t - \frac{b_t}{1+r_1}$, the consumer's decision problem can thus be formulated as :

(I)

Maximize $u(c_1, \ldots, c_n)$ *with respect to* $(c_1, \ldots, c_n) \geqq 0$ *and* $(\mu_1, \ldots, \mu_n)$, *with* $\mu_n \geqq 0$, *subject to the budget constraints* :

$$p_1 c_1 + \mu_1 = p_1 e_1 + \bar{m} - R_1$$

$$p_t c_t + \mu_t = p_t e_t + \mu_{t-1}(1+r_{t-1})$$

for $t = 2, \ldots, n$, *where the reimbursement* R_1 *is equal to the minimum of* $\bar{b}$ *and of* $\bar{m} + \sum_1^n \beta_t p_t e_t$

The consumer's choices which arise from (I) can in fact be obtained very simply by using the following procedure. Let us add the trader's budget constraints, the constraint of period t being multiplied by the discount factor β_t . Taking into account the condition $\mu_n \geqq 0$ yields

$$\sum_1^n \beta_t\ p_t c_t \leqq \bar{m} + \sum_1^n \beta_t\ p_t e_t - R_1 ,$$

or equivalently,

$$(*) \qquad \sum_1^n \beta_t\ p_t c_t \leqq \mathrm{Max}(\bar{\mu} + \sum_1^n \beta_t\ p_t e_t\ , 0)$$

Maximizing the utility function u under the intertemporal budget constraint (*) determines the trader's optimal current and future assumptions $c_1 , \ldots , c_n$. The corresponding demand for money m_1 and the amount of money borrowed $\frac{b_1}{1+r_1}$ are then deduced from the current budget constraint :

$$\mu_1 = m_1 - \frac{b_1}{1+r_1} = \bar{m} - R_1 - p_1(c_1-e_1) ,$$

together with the condition that either m_1 or b_1 is equal to zero. The optimum values of m_t and $\frac{b_t}{1+r_t}$ are finally obtained recursively from the future budget constraints (1).

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(1) There may be some consumers whose horizon does not extend beyond the current period, i.e. for whom n = 1. The problem faced by the consumers is particularly simple, since then the expected budget constraints vanish. The presence of such consumers is important when one wishes to proceed to a dynamic analysis of the economy, by embedding it, for instance, in an overlapping generation model, as we shall do in the next chapter. The present short run analysis does not depend, purposedly, upon the presence or the absence of consumers of this type at the date under consideration. We shall not mention them explicitly in the sequel.

The procedure makes clear that *(I)* has a solution, which is then unique, if and only if current and expected prices are positive, and current and expected interest rates exceed -1. More significantly, *the consumer's current excess demands for goods* $c_1 - e_1$ *, his demand for money* m_1 *, the amount of money borrowed* $\frac{b_1}{1+r_1}$ *and the reimbursement* R_1 *which arise from (I), depend only upon the initial net credit position* $\bar{\mu}$ *, on current prices* p_1 *and on the discounted expected prices* $\beta_2 p_2, \ldots, \beta_n p_n$.

This statement is the analogue of an assumption which is often made in the macroeconomic literature, namely that a consumer's behaviour depends only on his expected "real" rate of interest. In order to see this point, let us specialize the model to the case where there is only one good, and where a consumer expects a constant rate of inflation π as well as a constant rate of interest r_1 for the future ($p_t = (1+\pi)^{t-1} p_1$, and $r_t = r_1$ for $t = 2, \ldots, n$). Discounted expected prices are then equal to $p_1/(1+\rho)^{t-1}$, where the expected "real" interest rate ρ is given by $1+\rho = (1+r_1)/(1+\pi)$. A consumer's short run behaviour depends indeed in such a case only on $\bar{\mu}$, on p_1 , and on ρ , as announced.

The homogeneity of degre 1 of the intertemporal budget constraint (*) with respect to $\bar{\mu}$, current and expected discounted prices yields immediately the following *Absence of Money Illusion* property :

(1) (Absence of Money Illusion) *The excess demands for goods* $c_1 - e_1$ *resulting from* (I) *are homogenous of degree* 0 *with respect to* $\bar{\mu}$ *, and to current and expected discounted prices,* p_1 , $\beta_2 p_2$,...., $\beta_n p_n$. *The associated demand for money* m_1 *, the amount of money borrowed* $\frac{b_1}{1+r_1}$*, and the reimbursement* R_1 *are homogenous of degree* 1 *in the same variables.*

In order to specify completely how a consumer's current decisions depend upon his environment, one must describe how his expectations, in fact his *discounted expected prices* $\beta_2 p_2$,...., $\beta_n p_n$, depend upon his information about the current state of the economy (the influence of his knowledge of past history is kept implicit, as in Chapter I, Section 3). By assumption, this information consists only of the current price system p_1 , and of the current interest rate r_1 . Expected discounted prices will be thus written as functions of p_1 and r_1 , which will take the specific form $\frac{\psi_t^*(p_1,r_1)}{1+r_1}$ for $t = 2 ,\ldots, n$, in order to isolate the "direct" effect of the current interest rate r_1 upon discounting (1).

When expected discounted prices $\beta_t p_t$ are replaced by their expressions in function of p_1 and r_1 , one gets a current excess demand for goods, a demand for money, a bond supply and a reimbursement of the initial debt which depend on $\bar{\mu}$, p_1 and r_1 only. They will be denoted respectively $z_a(p_1,r_1,\bar{\mu}_a)$, $m_a^d(p_1,r_1,\bar{\mu}_a)$, $b_a^s(p_1,r_1,\bar{\mu}_a)$ and $R_a(p_1,r_1,\bar{\mu}_a)$, reintroducing finally the index a of the consumer.

These functions are linked by the consumers' current budget constraints, which imply

$$p_1 z_a(.) + m_a^d(.) = \bar{m}_a + b_a^s(.)/(1+r_1) - R_a(.)$$

for every p_1 and r_1 , where the symbol (.) stands for $(p_1,r_1,\bar{\mu}_a)$. As we have seen, these identities are the basis for *Walras Law.*

It is interesting to see under which circumstances short run demands and supplies display the homogeneity properties which are assumed in Neoclassical theory, namely the homogeneity of degree 0 of the excess demands for goods $z_a(.)$ with respect to current prices p_1 and initial money wealth $\bar{\mu}_a$, and the homogeneity of degree 1 of the functions $m_a^d(.)$, $b_a^s(.)$ and $R_a(.)$ with respect to the same variables.

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(1) $\psi_t^*(p_1,r_1)$ represents accordingly the price system which the trader expects to prevail in period t , discounted back to date 2, by using the expected interest rates.

In view of the result on *Absence of Money Illusion* stated in *(1)*, these homogeneity properties are valid if, and in general only if, expected discounted prices $\frac{\psi^*_{at}(p_1,r_1)}{1+r_1}$ are unit elastic with respect to current prices, or more precisely, if $\psi^*_{at}(\lambda p_1,r_1) = \lambda\psi^*_{at}(p_1,r_1)$ for every p_1 , λ and t. This would be the case in particular if undiscounted expected prices were proportional to, and expected interest rates independent of current prices p_1 . It is clear, however, that this hypothesis is too specific and thus unacceptable, since expectations depend not only on current prices, but also on the sequence of past prices and interest rates.

It remains to see how changes of the current price system p_1 and of the current interest rate r_1 influence a typical consumer's demands and supplies, in particular his excess demands for goods $z_a(p_1,r_1,\bar{\mu}_a)$.

Consider first, say, an equiproportionate increase of current prices of goods from p_1 to λp_1 , with $\lambda > 1$, the interest rate r_1 (and initial money wealth $\bar{\mu}_a$) being fixed. Under the Neoclassical hypothesis on expectations, that is, if expected discounted prices were unit elastic with respect to current prices, this change would result in a pure *wealth effect*. For in that case $z_a(p_1,r_1,\bar{\mu}_a)$ would be homogenous of degree 0 in p_1 and $\bar{\mu}_a$, and thus

$$z_a(\lambda p_1,r_1,\bar{\mu}_a) = z_a(p_1,r_1,\frac{\bar{\mu}_a}{\lambda}) \ .$$

The increase of current prices of goods would then be equivalent to a proportional decrease of the absolute value of the initial money wealth $\bar{\mu}_a$. The wealth effect is then likely to lead to a decrease of the consumer's excess demands for goods in the case of a creditor, i.e. when $\bar{\mu}_a$ is positive. It works in the opposite direction in the case of a debtor. It has no influence upon current consumption when $\bar{\mu}_a = 0$.

Expected discounted prices are not in general unit elastic with respect to current prices. Typically, therefore, an increase of current prices generates, in addition to the wealth effect, an *intertemporal substitution effect* which is due to the modification of the ratios of expected discounted prices to current ones (1). If, for instance, expected discounted prices rise proportionately more than current prices, future goods become more expensive relatively to current ones, and the substitution effect is likely to yield an increase of current consumption. In such circumstances, the substitution effect counteracts the wealth effect in the case of creditor, i.e. when $\bar{\mu}_a$ is positive, and reinforces it in the case of a debtor. The important point to note at this stage is that contrary to the wealth effect, the consequences upon current consumption of the intertemporal substitution effect do not depend *a priori* upon the sign of the consumer's initial net credit position $\bar{\mu}_a$.

We consider next, say, an increase of the interest rate r_1 , current prices of goods p_1 (and the initial money wealth $\bar{\mu}_a$) being fixed If the functions $\psi^*_{at}(p_1,r_1)$ were actually independent of the interest rate r_1, the increase of r_1 would generate a *direct intertemporal substitution effect* by reducing mechanically expected discounted prices $\dfrac{\psi^*_{at}(p_1,r_1)}{1+r_1}$. The direct substitution effect would then probably

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(1) We ignore in this heuristic discussion the possible modifications of relative expected discounted prices.

yield a decrease of current consumption, independently of the trader's initial net credit position $\bar{\mu}_a$.

In addition to this direct effect, however, there is typically an *indirect intertemporal substitution effect*, which is due to the modification of the forecasts $\psi^*_{at}(p_1,r_1)$ generated by the increase of the interest rate r_1 . The indirect substitution effect is thus likely to counteract the direct effect when the $\psi^*_{at}(p_1,r_1)$ rise, and to reinforce it otherwise. In particular, the direct and indirect effects cancel each other whenever expected discounted prices $\frac{\psi^*_{at}(p_1,r_1)}{1+r_1}$ are actually independent of r_1 . In that case, variations of the current interest rate have no influence upon current consumption. (1)

We proceed now to the description of an example which permits a graphical illustration of the arguments of this section, and which will be used repeatedly at later stages of this lecture. Consider the simple "macroeconomic" case where there is only one good (ℓ=1) and where a typical consumer is planning only one period ahead (n=2). If the current price and interest rate are p_1 and r_1 , and if the consumer expects the price p_2 to prevail in the future, his optimum current and future consumptions are obtained,as we have seen, by maximizing the trader's utility function under the intertemporal budget constraint :

$$p_1 c_1 + \frac{p_2}{1+r_1} c_2 \leqq \mathrm{Max}(\bar{\mu} + p_1 e_1 + \frac{p_2}{1+r_1} e_2 \ , \ 0)$$

The consumer's demand for money and bond supply are deduced in turn from the current budget constraint.

---

(1) Intuitively, what happens there is that a variation of the nominal rate of interest r_1 does not alter the consumer's "expected real interest rate", hence his short run behaviour.

This maximization problem is easily represented in the plane (c_1, c_2). The intertemporal budget constraint is described by a line which is perpendicular to the vector of discounted prices $(p_1 , \frac{p_2}{1+r_1})$. It passes through the points α and β of coordinates $(e_1 , e_2 + \frac{(1+r_1)\bar{\mu}}{p_2})$ and $(e_1 + \frac{\bar{\mu}}{p_1} , e_2)$ in the case of a net creditor, i.e. when $\bar{\mu} \geqq 0$ (Fig. 1.a), and in the case of a net debtor $(\bar{\mu} < 0)$ who is not bankrupt, i.e. such that $\bar{\mu} + p_1 e_1 + \frac{p_2}{1+r_1} e_2 \geqq 0$ (Fig. 1.b). The budget line passes through the origin otherwise.

Fig. 1.a

Fig. 1.b

The respective roles of the wealth and of the intertemporal substitution effects appear quite clearly on the diagrams. Consider first an increase of the current price p_1 , the interest rate r_1 and $\bar{\mu}$ being unchanged. In the increase of p_1 generates a proportional increase of the expected price p_2 , there is an horizontal displacement of the point β towards the left in the case of a creditor, and towards the right in the case of a debtor, but the slope of the intertemporal budget line is unchanged. The consequence on the optimum consumption pattern is then similar to a pure income, or wealth effect. But if the increase of the current price p_1 causes a change of the expected price p_2 which is not proportional to the variation of p_1 , there is in addition a rotation of the budget line, since the ratio $\frac{p_2}{(1+r_1)p_1}$ is modified. This rotation generates a substitution effect between present and future consumption which may weaken, or reinforce the wealth effect.

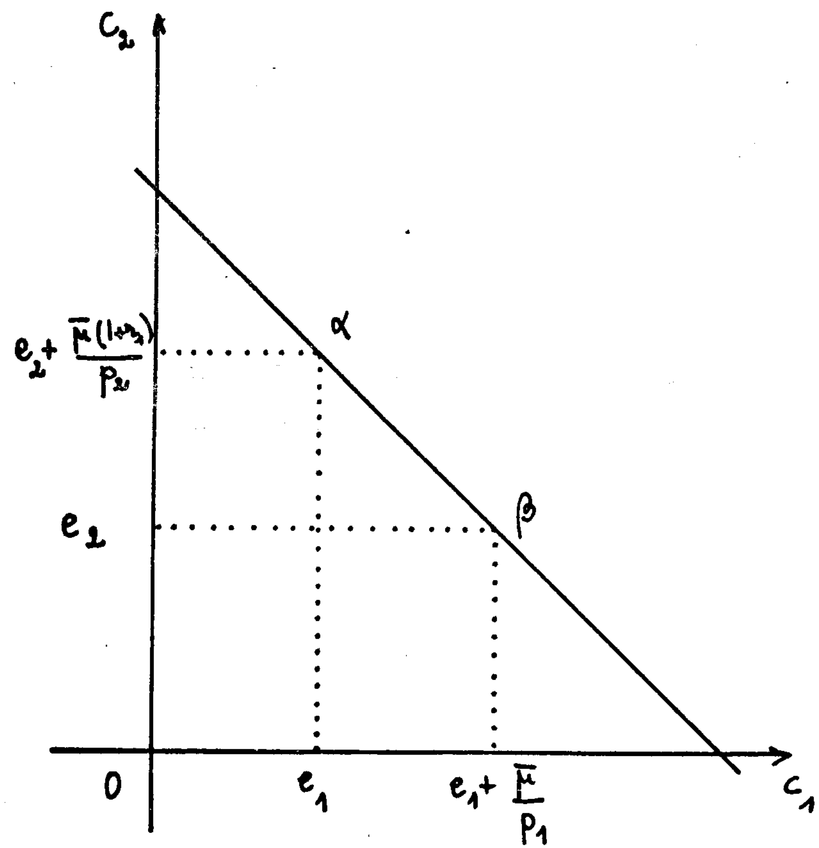
c_2
$e_2 + \frac{\bar{\mu}(1+r_2)}{p_2}$
e_2
α
β
0
e_1
$e_1 + \frac{\bar{\mu}}{p_1}$
c_1

Fig. 1.a

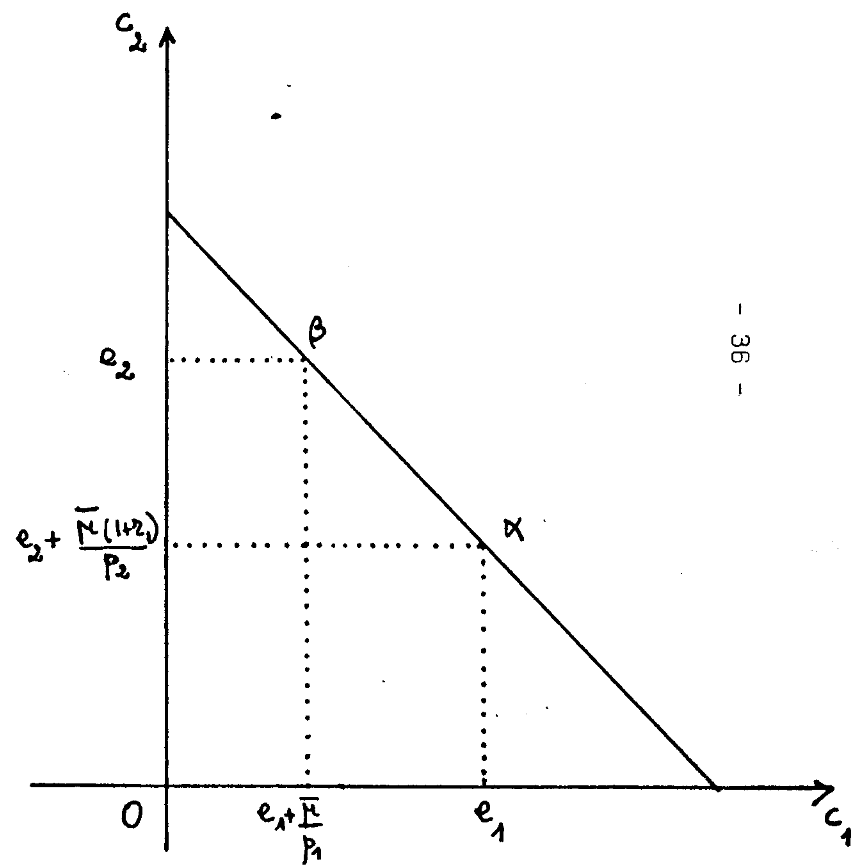
c_2
e_2
$e_2 + \frac{\bar{\mu}(1+r_2)}{p_2}$
β
α
0
$e_1 + \frac{\bar{\mu}}{p_1}$
e_1
c_1

Fig. 1.b

Let us consider next the consequence of a variation of the interest rate r_1, the current price p_1 and $\bar{\mu}$ being fixed. If this variation leaves unchanged the expected price p_2, there is a rotation of the intertemporal budget line around the point β, which generates what we have called the "direct" intertemporal substitution effect. If the variation of r_1 changes the expected price p_2, there is an additional rotation of the budget line around β which generates the "indirect" substitution effect, and which may weaken or reinforce the direct effect. In the particular case where the expected discounted price $\frac{p_2}{1+r_1}$ is independent of r_1, the slope is the budget line actually unchanged, and variations of the interest rate r_1 have no influence upon the trader's optimum consumption pattern.

Remark. It is possible to construct for every consumer, as in Chapter I, Section 5, an expected utility index v, depending upon his current consumption c_1, on his current net credit position $\mu_1 = m_1 - \frac{b_1}{1+r_1}$, and on p_1 and r_1, which, when it is maximized under the current budget constraint, yields the consumer's short run demand and supply functions. Such an expected utility index is defined as the maximum level of utility which the consumer can expect to achieve over his lifetime, when c_1 and μ_1 are given. The study of the properties of this expected utility index is left as an exercise to the reader.

4. PEGGING THE INTEREST RATE.

The aim of this section is to study the short run regulating mechanisms which may bring the economy into equilibrium at date 1, when the Bank pegs the interest rate at some given value r_1 .

It is quite easy to write down the equations which the corresponding equilibrium prices of goods p_1 must satisfy by using the short run demand and supply functions which were derived in the previous section. Equilibrium of the goods markets requires

$$(C) \qquad \sum_a z_a(p_1 \, , \, r_1 \, , \, \bar{\mu}_a) = 0 \quad .$$

On the other hand, the interest rate being pegged at r_1 , the Bank's credit supply is assumed to be infinitely elastic. The bond market is therefore in equilibrium at all current prices p_1 . The equilibrium condition for the money market then states that the aggregate demand for money must be equal to the initial money stock M , plus the Bank's net money supply :

$$(\mathcal{D}_1) \qquad \sum_a m_a^d(p_1,r_1,\bar{\mu}_a) = M + \sum_a \left[\frac{b_s^s(p_1,r_1,\bar{\mu}_a)}{1+r_1} - R_a(p_1,r_1,\bar{\mu}_a) \right] \; .$$

In view of the traders' budget constraints, the system satisfies *Walras Law*, that is :

$$p_1 \sum_a z_a(.) + \sum_a m_a^d(.) = M + \sum_a \left[\frac{b_a^s(.)}{1+r_1} - R_a(.) \right]$$

for every p_1 , where the symbol (.) stands for $(p_1, r_1, \bar{\mu}_a)$. Walras Law has the familiar implication that if all markets but one are brought into equilibrium, the remaining one is in equilibrium too. In particular, solving the equations for the real sector alone (equations (C)) determine not only equilibrium relative prices, but the equilibrium levels of money prices as well. The *Classical Dichotomy* is invalid in the short run.

The foregoing system of equations has thus the same formal structure as the Neoclassical system (C) and $(\mathcal{D}_1)$ which was written down in Section 2, when summarizing the views of this school. It differs from the Neoclassical system in one important respect, however, since we have not assumed any homogeneity properties of the traders' short run demand and supply functions (1). A change of current prices p_1 will thus influence here aggregate demand for goods, essentially through two effects : the wealth effect, which is generated by an equiproportionate variation of current and expected prices, and the intertemporal substitution effect between present and future consumption, which is due to the relative variations of current and expected prices.

Neoclassical theorists, we recall, considered exclusively the wealth effect, and regarded it as an operational regulating mechanism as long as the value of outside money $\sum_a \bar{\mu}_a$ is positive. As we said, this claim appears to be accepted as a valid theoretical proposition by many theorists today, and the controversies have focused mostly on its empirical relevance.

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(1) As a consequence, the short run version of the *Quantity Theory* (see Section 2) is no longer valid under our assumptions.

As we are going to see, the Neoclassical viewpoint is in fact wrong at the theoretical level. It will be shown indeed, by means of a series of examples which are valid even in the case price expectations are unit elastic with respect to current prices, and where outside money is positive and large, that there may be a persistent excess demand on the goods markets when expectations are biased upwards, and a persistent excess supply when expectations are biased downwards. In these cases, which are likely to obtain whenever a significant inflation or deflation has been observed in the past, the goods markets equations (C) have no solution in current prices p_1 , given the interest rate r_1 , and no short run equilibrium can exist. A credit economy will be found, as a matter of fact, particularly vulnerable in inflationary situations, because even a small group of consumers can "destabilize" the whole economy if they forecast a large rate of inflation. For they may be led then to borrow a lot from the Bank, and by spending the money borrowed on current goods, to generate an excess demand on the corresponding markets at all current prices.

In order to present these examples, it is most convenient to consider the simple "macroeconomic" case where there is only one good (ℓ=1) , and where every consumer makes plans for the current period and the next one only (n=2). A typical consumer's utility function $u(c_1,c_2)$ will be written as $w(c_1) + \frac{1}{1+\rho} w(c_2)$, where $\rho \geqq 0$ is his rate of time preference, and where w is differentiable, increasing and strictly concave. It will be assumed furthermore, in order to simplify the exposition, that $w'(o) = +\infty$.

Examples 1 : Persistent Excess Demand

We give first an example where there is an aggregate excess demand on the good market at all current prices p_1 .

Let ω_1 be the *total* amount of good which is available in the current period, i.e., the sum of all consumers' current endowments. Consider a consumer who is not a debtor, that is, such that $\bar{\mu} \geqq 0$, and let e_1 and e_2 be his endowments of good in the current period and the next one. The purpose of the example is to show that this single consumer's demand for consumptionc_1 will exceed ω_1 for every value of p_1 , if this consumer forecasts a large enough rate of inflation. In such a case, there will be an aggregate excess demand on the good market at all current prices, and no equilibrium can exist.

Let us first proceed to a preliminary analysis of this concumer's choices in the plane (c_1,c_2) (Fig. 2.a). It is clear that the consumer's marginal rate of substitution u_2'/u_1' decreases from $+\infty$ to a positive value when one moves certically from the point of coordinates $(\omega_1,0)$ to the point (ω_1,e_2). Thus there is a unique point γ on the segment joining these two points, such that the tangent at the indifference curve at γ goes through the endowment point (e_1,e_2).

<u>Fig. 2.a</u> <u>Fig. 2.b</u>

Let λ be the value of the marginal rate of substitution u_2'/u_1' at the point γ , and assume that the ratio of the consumer's expected discounted price $\dfrac{p_2}{1+r_1}$ to the current one p_1 , exceeds λ .

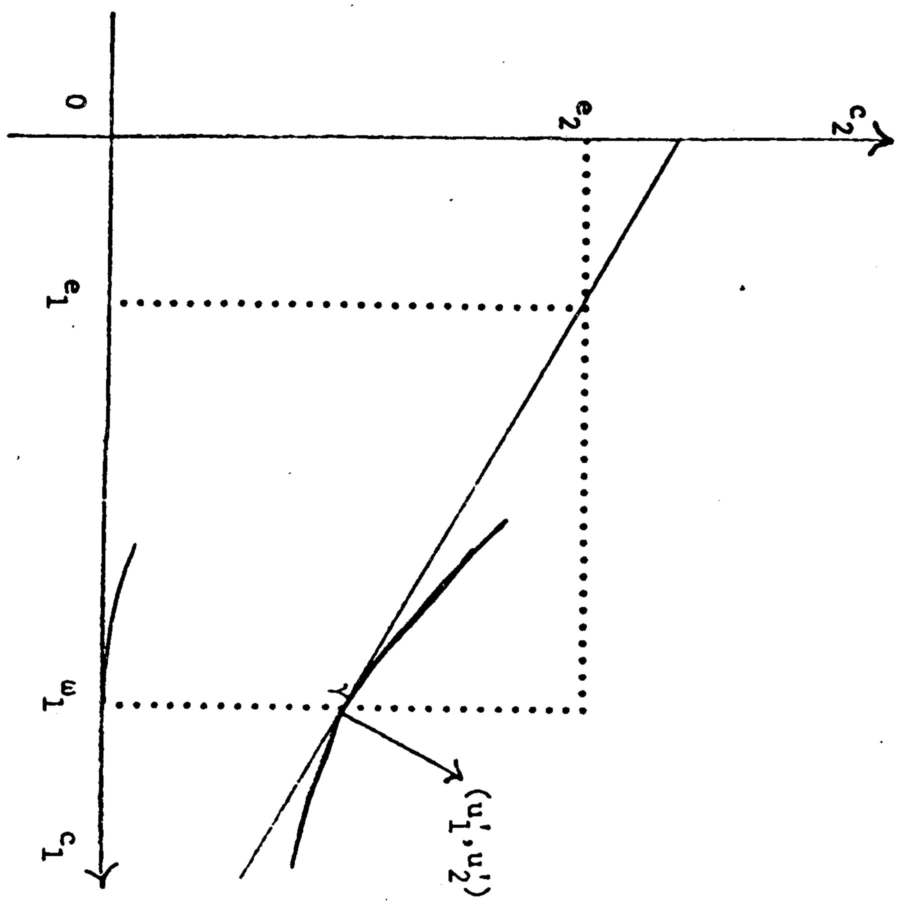

Fig. 2.a

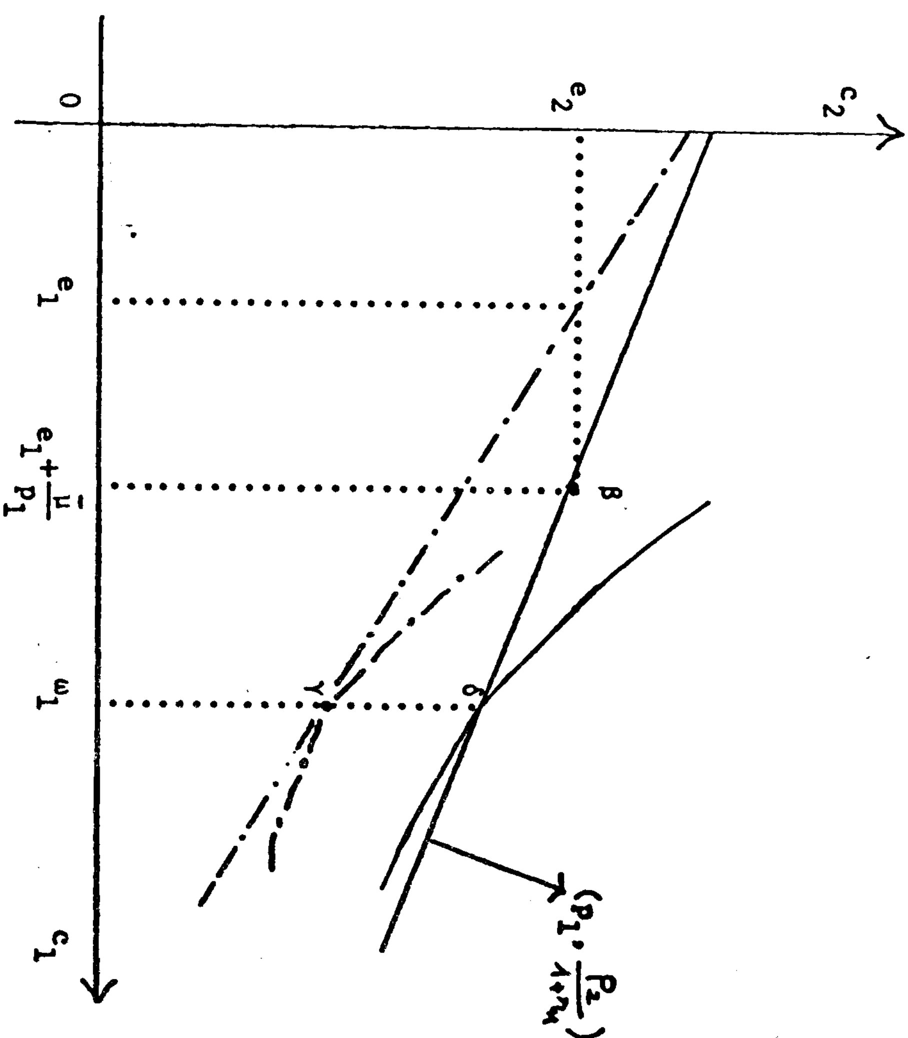

Fig. 2.b

The intertemporal budget line of equation

$$p_1c_1 + \frac{p_2}{1+r_1}c_2 = \bar{\mu} + p_1e_1 + \frac{p_2}{1+r_1}e_2$$

intersects then the vertical line going through $(\omega_1,0)$ at a point δ which is above γ, and where the marginal rate of substitution is thus less than λ (Fig. 2.b). Since $\frac{p_2}{(1+r_1)p_1}$ exceeds λ, the trader's current optimum consumption c_1 must be greater than ω_1.

If this *single* consumer's expectations are biased upwards so that $\frac{p_2}{(1+r_1)p_1} > \lambda$ for all p_1, there will be an *aggregate* excess demand on the good market at all current prices, and no equilibrium can exist.

It should be noted that the example is valid independently of the value of outside money $\sum_a \bar{\mu}_a$. The "disequilibrium" phenomenon which it describes can occur in particular when the expected discounted price $\frac{p_2}{1+r_1}$ is proportional to p_1. The wealth effect, which is then the sole regulating mechanism of the economy, is too weak to equilibrate the market, contrary to what Neoclassical theorists claim.

The above example describes an extreme case where a single creditor's demand for consumption exceeds the total amount of good currently available ω_1. It can be adapted, of course, to less extreme situations. Consider for instance a group of q creditors. Replacing ω_1 by $\frac{\omega_1}{q}$ in the foregoing argument shows that a single creditor's demand for consumption will exceed $\frac{\omega_1}{q}$ if the ratio of his forecast $\frac{p_2}{1+r_1}$ to the current price p_1 is greater than some appropriate value. If the price expectations of all creditors of this group are biased upwards in this way, there will be an aggregate excess demand on the good market at all prices p_1.

The example can be adapted also to the case of a debtor, i.e. when $\bar{\mu} < 0$. Starting from Fig. 2.a, let us assume that the debtor's price expectations are such that the intertemporal budget line of equation

$$p_1 c_1 + \frac{p_2}{1+r_1} c_2 = \bar{\mu} + p_1 e_1 + \frac{p_2}{1+r_1} e_2$$

intersects the vertical line going through the point $(\omega_1, 0)$ at a point δ which is above γ . If (ω_1 , γ_2) are the coordinates of γ , this means

$$p_1 \omega_1 + \frac{p_2}{1+r_1} \gamma_2 \leqq \bar{\mu} + p_1 e_1 + \frac{p_2}{1+r_1} e_2 \quad .$$

Since $\lambda = \dfrac{\omega_1 - e_1}{e_2 - \gamma_2}$, this inequality can be restated :

$$\frac{p_2}{1+r_1} \geqq \lambda p_1 - \frac{\bar{\mu}}{e_2 - \gamma_2}$$

In such a case, the ratio $\dfrac{p_2}{(1+r_1)p_1}$ exceeds λ , which is itself greater than or equal to the marginal rate of substitution at δ . The debtor's optimum current consumption is then larger than ω_1 . If this single debtor's expectations are biased upwards in this way for all p_1 , an aggregate excess demand appears on the good market at all current prices, and no equilibrium exists.

Example 2 : Persistent Excess Supply

We wish to show that if *all* traders' prices expectations are biased downwards for all p_1 , there may be an excess supply on the good market at all current prices.

Let us again consider a typical consumer, with $\bar{\mu}$ positive or negative, and assume that the marginal rate of substitution u_2'/u_1' is bounded below by some value $\nu > 0$ when one moves up the vertical line going through the point $(e_1, 0)$ (Fig. 3). We claim that if $\frac{p_2}{(1+r_1)p_1}$ is less than ν, the trader's optimum current consumption c_1 must be less than e_1.

This is obviously the case if the consumer is bankrupt, since then $c_1 = 0$. In all other cases, the equation of the intertemporal budget line is

$$p_1 c_1 + \frac{p_2}{1+r_1} c_2 = \bar{\mu} + p_1 e_1 + \frac{p_2}{1+r_1} e_2 .$$

If $\bar{\mu} + \frac{p_2}{1+r_1} e_2$ is negative, current consumption c_1 must be less than e_1. If this expression is nonnegative, the marginal rate of substitution at the point α of coordinates $(e_1 , e_2 + \frac{\bar{\mu}(1+r_1)}{p_2})$ exceeds the ratio $\frac{p_2}{(1+r_1)p_1}$, and again, the trader's optimum current consumption c_1 is less than his endowment e_1.

Fig. 3

If *all* consumers have expectations which are biased downwards in this way for all p_1, there will be an aggregate excess supply on the good market at all current prices, and no equilibrium corresponding to the given interest rate r_1 can exist.

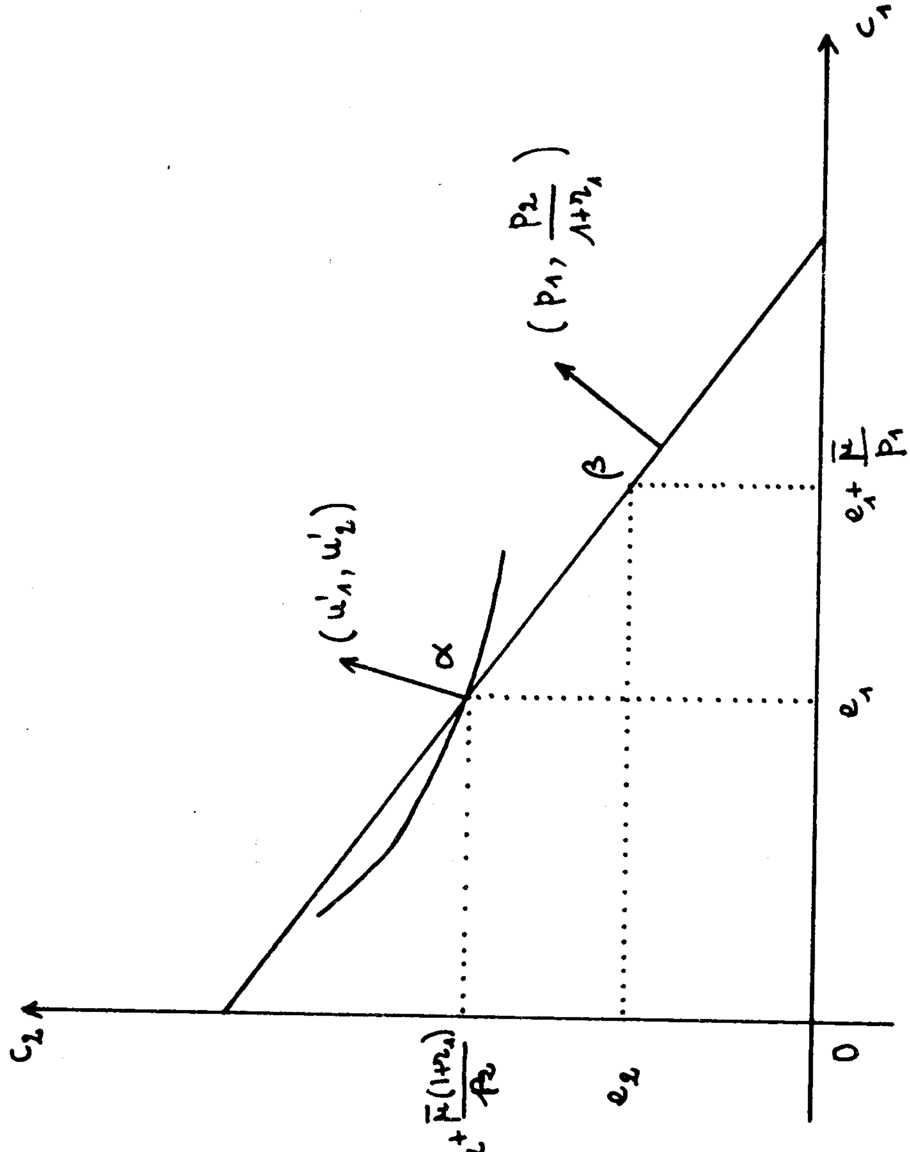
c_2
$e_2 + \frac{\bar{F}(1+r_1)}{p_2}$
e_2
0
e_1
$e_1 + \frac{\bar{F}}{p_1}$
c_1
α
β
(u'_1, u'_2)
$(p_1, \frac{p_2}{1+r_1})$

Fig. 3

The "disequilibrium" phenomenon described in this example is here again independent of the value of outside money $\sum_a \bar{\mu}_a$, and it can occur in particular when the ratio $\frac{p_2}{(1+r_1)p_1}$ is independent of p_1 , i.e. when price expectations are unit elastic with respect to current prices. It contradicts therefore again the Neoclassical position, which claims that the wealth effect is an operational regulating mechanism when outside money is positive.

Remark. The example shows that a persistent excess supply appears on the good market if all consumers are debtors, and if the expression $\bar{\mu} + \frac{p_2}{1+r_1} e_2$ is negative for all p_1 and every consumer. This case should be taken of course only as a *curiosum*, since it is unreasonable to assume all consumers to be debtors.

The first class of examples teaches us that, the interest rate r_1 being fixed, even a *small* group of consumers can "destabilize" the whole economy by generating an excess demand for goods at all current prices p_1 , if they forecast a large rate of inflation. Expectations of this sort are likely to obtain whenever consumers have observed a significant inflation in the past. *A credit money economy appears therefore to be greatly vulnerable in inflationary situations, since then a short run equilibrium is likely not to exist.*

The second example shows that *a short run equilibrium corresponding to a given interest rate r_1 may not exist when deflation has been observed in the past, since then, the consumers' price expectations are likely to be biased downwards.* A credit money economy seems however to be somewhat les vulnerable in a deflationary situation than in the case of inflation, since *all* consumer's price expectations have to be biased downwards in order that a persistent excess supply appears on the good market.

These examples have been developed in the case of a fixed interest rate. The circumstances which they describe may obtain of course for *every* interest rate. For instance, the argument of the first example shows that if a single creditor's price expectations are such that $\frac{p_2}{(1+r_1)p_1}$ exceeds λ for all p_1 and r_1 , an aggregate excess demand appears on the good market at all current prices and interest rates. In a such a case , there can be no value of p_1 and r_1 which achieves the equilibrium of the real sector. No short run equilibrium exists, no matter which interest rate is chosen by the Bank. The argument of the second example can be adapted in the same way.

To sum up, these examples teach us that, *contrary to what most Neoclassical theorists used to believe, the market's "invisible hand" is likely to go astray in a credit money economy with flexible prices, especially in inflationary situations, and that a short run equilibrium may not exist, no matter which value of the interest rate is chosen by the Bank.*

Inelastic Expectations

We have already noted that the above examples of the non-existence of a short run equilibrium corresponding to a given interest rate were valid, in particular, when expected prices are unit elastic with respect to current prices, and that they were independent of the value of outside money $\sum_a \bar{\mu}_a$. This means that the wealth effect, which is the sole regulating mechanism in such circumstances, cannot in general be counted on to bring the economy into equilibrium. The argument shows on the contrary that, in order to be sure of the existence of a short run equilibrium for a given interest rate, we have to rely upon the flexibility of expected prices relatively to current prices, that is, upon the *intertemporal substitution effect* as the main regulating mechanism of the market.

Our purpose is to study now the kind of conditions on expectations which are needed to make this substitution effect operational. As one can anticipate beforehand in view of the examples given above, these conditions be quite strong.

If we wish an aggregate excess supply to appear on the goods markets when current prices p_1 are large, we need a condition on expectations which prevents the phenomena described in the first example to occur. Such a condition is that *for all consumers, expected prices are bounded above when current prices increase indefinitely,* for then, the intertemporal substitution effect is bound to favor more and more future consumption relatively to current consumption, leading eventually to the appearance of an excess supply in the current markets for goods.

On the other hand, in order to be sure that an excess demand appears on the goods markets when current prices are low, expected prices must behave in such a way that the phenomena described in the second example are excluded. Such a condition on expectations is that *there is at least one consumer, who is not a debtor, for whom expected prices are bounded away from zero when current prices go to zero*. For this particular consumer, the intertemporal substitution effect then favors more and more current consumption relatively to future consumption. His desired current consumption must actually tend then to infinity, leading eventually to the appearance of an aggregate excess demand on the goods markets [1]. Intuitively, when these conditions on the consumers' expectations are satisfied, one must be able, by moving from low to large current prices p_1 , the interest rate r_1 being fixed, to find by continuity a value of p_1 which brings the good markets, and therefore the whole system, into equilibrium.

We present now a formal result on the existence of a short run equilibrium associated with a given interest rate r_1 , which makes use of the conditions on expectations which we just described heuristically. Let us say that, whenever $n_a \geqq 2$, consumer a's price expectations are *continuous in current prices* if, given r_1 , the functions $\psi^*_{at}(p_1,r_1)$ are continuous with respect to p_1 , for every t . We shall say that a consumer's price expectations are *bounded above* if there exists a vector $\eta(r_1)$, with all its components positive, which may

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(1) We need such a condition for a consumer *who is not a debtor* in order to avoid the phenomenon described in the *Remark* at the end of the second example. It is quite easy to illustrate these statements by means of a graphical analysis in the plane (c_1,c_2), in the simple case where there is only one good and where every consumer makes plans for the current period and the next one only. This left as an exercice to the reader.

depend upon the interest rate, such that $\psi^*_{at}(p_1,r_1) \leqq \eta(r_1)$ for every t and p_1. They are *bounded away from zero* if there is a vector $\varepsilon(r_1)$ with all its components positive, which again may depend upon r_1, such that $\varepsilon(r_1) \leqq \psi^*_{at}(p_1,r_1)$ for every p_1 and t. These boundedness conditions garantee the presence of a strong stabilizing intertemporal substitution effect when current prices vary. Needless to say, they are incompatible with the Neoclassical postulate of unit elastic price expectations. These boundedness conditions are the key for the following existence result [1].

(1) *Let the interest rate* r_1 *be fixed. Assume (a) and (b) of Section* 1 *, and that every consumer's price expectations are continuous with respect to current prices. Assume moreover that every consumer's price expectations are bounded above, and that there is at least one consumer a with* $n_a \geqq 2$ *and* $\bar{\mu}_a \geqq 0$ *, whose price expectations are bounded away from zero.*

Then, given r_1 *, the system of equations (C) and* (D_1) *has a solution.*

It should be noted that the result does not rest at all on any assumption about the value or the sign of outside money, $\sum_a \bar{\mu}_a$. In particular, the result presented is valid even in the case where all consumers are initially neither creditors nor debtors, i.e., when $\bar{\mu}_a = 0$ for every a. In such a case, the only adjustment mechanism in the economy is the intertemporal substitution effect between current and future consumption.

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(1) A formal proof of the result is given in Appendix C.

The fact that the existence of a short run equilibrium is guaranteed, under the assumptions of (1), without any reference to the sign and the size of outside money, is intuitively understandable, since the intertemporal substitution effect generated by the relative variations of current and expected prices works a priori in the same direction for all consumers, whether they are initially creditors or debtors.

The conditions ensuring the presence of a stabilizing substitution effect, for a given interest rate, are evidently quite strong, since they require essentially *all consumer's* price expectations to be insensitive to large increases of current prices, and *some consumer's* expectations not to be affected by large decreases of p_1 . Such inelasticity conditions cannot be expected to be met in practice, since price forecasts are likely to depend much, in general, upon the prices that are currently observed. This qualitative conclusion confirms our earlier finding that a short run equilibrium was not likely to exist in actual credit money economies.

Remark. Here again, a short run equilibrium is meant to be one where money has a positive value in exchange. Under our assumptions (*a*) and (*b*) of Section 1, there are always nonmonetary equilibria, where the price of money is zero. These are in fact identical to the nonmonetary equilibria associated with the outside money economy which was studied in the first chapter, since the real parts of the models are identical.

5. CONTROLLING THE MONEY SUPPLY.

We studied in the preceding section the case where the Bank tried to peg the interest rate at date 1. The purpose of the present section is to look at the case where the Bank lets the interest rate to vary, and seeks controlling the money supply.

Assume accordingly that the Bank fixes the amount of money which it creates at date 1 by granting loans to consumers, at some given value, say $\Delta M > 0$. The equations which the equilibrium values of p_1 and r_1 must satisfy are then easy to describe with the help of the consumers' short run demand and supply functions.

The equilibrium conditions for the goods markets read as before :

$$(C) \qquad \sum_a z_a(p_1, r_1, \bar{\mu}_a) = 0$$

where, by contrast with the previous section, the interest rate is now free to vary. Next, the equation for money states that the aggregate demand for money should be equal in equilibrium to the initial money stock M , augmented by the Bank's net money supply, which is described in this case by $\Delta M - \sum_a R_a(p_1, r_1, \bar{\mu}_a)$. This yields

$$(D) \qquad \sum_a m_a^d(p_1, r_1, \bar{\mu}_a) = M + \Delta M - \sum_a R_a(p_1, r_1, \bar{\mu}_a) \ .$$

The Bank's demand for bonds is given by $(1+r_1)\Delta M$. Equilibrium of the bond market then requires :

$$(E) \qquad (1+r_1)\Delta M = \sum_a b_a^s(p_1, r_1, \bar{\mu}_a) \ .$$

Since each consumer satisfies a budget constraint, this system of equations satisfies *Walras Law*, which states that the sum across all markets of the values of aggregate excess demands must be identically equal to zero :

$$p_1 \sum_a z_a(.) + \left[\sum_a m_a^d(.) - M - \Delta M + \sum_a R_a(.)\right] + \left[\Delta M - \sum_a (b_a^s(.)/(1+r_1))\right] = 0$$

for every p_1 and r_1 , where the symbol (.) stands for $(p_1, r_1, \bar{\mu}_a)$. The system has thus the same formal structure as the Neoclassical system which was described in Section 2. There is here again an important difference, since the Neoclassical homogeneity postulates are not assumed here, as we wish to stress the importance of the intertemporal substitution effects associated with the relative variations of current and expected prices in the regulating process of the economy (1).

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(1) As before, the Neoclassical propositions about the validity of the *Quantity Theory* in the short period, are accordingly no longer correct.

In order to study the question of the existence of a solution (p_1, r_1) to the foregoing system of equations, it is convenient to use the following heuristic procedure. Solving first in p_1, for a given r_1, the equations (C) for the goods markets, amounts to looking at the equilibrium (if any), which would arise if the Bank chose to peg the interest rate at this level. Whenever such a solution p_1 exists, it involves a creation of money by the Bank, i.e. $\sum_a \frac{b_a^s(p_1, r_1, \bar{\mu}_a)}{1+r_1}$, which is denoted $\Delta M(p_1, r_1)$. Solving the bond market equation (E), and thus, by *Walras Law,* the whole system, is then equivalent to finding a value of the interest rate r_1, and a corresponding solution p_1 of (C), such that the associated money issue $\Delta M(p_1, r_1)$ is equal to the given money supply ΔM.

The Neoclassical argument for asserting the existence of such a solution (p_1, r_1) was briefly described in Section 2. It relies essentiallly, we recall, upon the presence and the intensity of the wealth effect resulting from a change of current prices of goods, and of the intertemporal substitution effect generated by a variation of the interest rate. The first effect ensures, according to Neoclassical theorists, that the goods markets can be brough into equilibrium for each r_1 by movements of current prices, at least when outside money is positive. The second effect ensures enough variation of $\Delta M(p_1, r_1)$, this money issue being allegedly very small for high interest rates, and, conversely, quite large for a low value of r_1. Thus, according to the Neoclassical viewpoint, there should exist, by continuity, a rate of interest r_1 such that $\Delta M(p_1, r_1) = \Delta M$. As the conclusion does not depend upon the value of ΔM , the Bank would then have full control of the money supply.

The Neoclassical argument was found already to be fautly on one count, since we saw in the preceding section that the wealth effect was typically too weak to bring the good markets into equilibrium, given the interest rate. As a matter of fact, examples were provided there which showed that there may be no pair (p_1,r_1) which satisfies equations (C). In such a case, the system (C), (D) , (E) has no solution, no matter which money supply ΔM is chosen by the Bank.

We are going to see that there is another important source of "disequilibrium" in the present model, which can occur in the favorable case where the goods markets equations (C) have a solution in current prices for each interest rate. The Bank has then full control over the interest rate, and the relation between r_1 and the money issue $\Delta M(p_1,r_1)$, which is implied by (C), can be meaningfully defined. It will be found, however, that $\Delta M(p_1,r_1)$ may vary very little with the interest rate if the intertemporal substitution effect resulting from a change of this variable is weak. In such circumstances, the Bank has little control, if any, over the money supply. The likelihood that the Bank chooses mistakenly a value of ΔM which is incompatible with the equilibrium of the goods markets, i.e. for which a solution to the system (C), (D) , (E) does not exist, is then great.

The Unvariability of the Money Supply and of Equilibrium Prices.

We wish to describe now a case where the Bank is unable to exercise *any* influence on the money supply, although it may have full control over the interest rate.

Let us consider the case where the goods markets equations (*C*) can be solved in current prices p_1 for every interest rate, for instance because the assumptions of (*1*) of Section 4 are satisfied for each r_1 . Suppose in addition that every consumer's expected discounted prices $\psi^{*}_{at}(p_1,r_1)/(1+r_1)$ are independent of r_1 . Every trader believes then that the only consequence of a change of the interest rate from 0 to r_1 is to multiply equilibrium prices of goods by $1+r_1$ in all subsequent periods, so that "expected real interest rates" are left invariant. It was shown in Section 3 that a consumer's excess demands for goods, his money demand, the amount of money he borrows, as well as his reimbursement to the Bank, depended solely on his initial net money wealth $\bar{\mu}_a$, and on current and expected discounted prices. Under our assumptions, therefore, the functions $z_a(p_1,r_1\bar{\mu}_a)$, $m^d_a(p_1,r_1,\bar{\mu}_a)$, $b^s_a(p_1,r_1\bar{\mu}_a)/(1+r_1)$ and $R_a(p_1,r_1,\bar{\mu}_a)$ are actually independent of the interest rate.

This fact implies immediately that the equilibrium prices p_1 which satisfy (*C*) for a given r_1 , and the corresponding money issue $\Delta M(p_1,r_1)$, are independent of the interest rate too. Any attempt from the Bank to set the money supply ΔM at a different level will be defeated by the market, in the sense that the system (*C*) , (*D*) , (*E*) will then have no solution. On the other hand, if ΔM was set by coincidence at a level which is compatible with the equilibrium of the goods markets, a solution to the whole system of equations would then exist indeed, but the equilibrium interest rate would be indeterminate.

(1) *Assume that the expected discounted prices* $\psi^*_{at}(p_1,r_1)/(1+r_1)$ *are independent of the interest rate, for every* t. *Then, the functions* $z_a(p_1,r_1,\bar{\mu}_a)$, $m^d_a(p_1,r_1,\bar{\mu}_a)$, $b^s_a(p_1,r_1,\bar{\mu}_a)/(1+r_1)$ *and* $R_a(p_1,r_1,\bar{\mu}_a)$ *do not vary with* r_1 . *As a consequence, if* p_1 *brings the goods markets into equilibrium for a particular interest rate, then* (p_1,r_1) *is a solution of* (C) *for all* r_1 , *and the associated money creation* $\Delta M(p_1,r_1)=\sum_a b^s_a(p_1,r_1,\bar{\mu}_a)/(1+r_1)$ *is independent of the rate of interest.*

In this example, the Bank has full control over the interest rate, but is unable to have any influence on the money supply, or on equilibrium prices. The origin of the phenomenon is evidently the fact that the intertemporal substitution effect generated by a change of the interest rate, vanishes in this case. We saw in Section 3 that a variation of r_1 induces a *direct* substitution effect by altering the discount factor $\frac{1}{1+r_1}$, as well as an *indirect* one, by influencing the forecasts $\psi^*_{at}(p_1,r_1)$. In the example, the two effects cancel each other, because an increase of the interest rate generates a rise of expected prices which offsets the decrease of the discount factor.(1)

The example shows that *if the intertemporal substitution effect generated by a variation of the interest rate is weak, the money creation* $\Delta M(p_1,r_1)$ *which is implied by the equilibrium of the goods markets, may vary little with the interest rate.* Such a circumstance will obtain whenever an increase of the interest rate induces a rise of expected prices which compensates to a large extent the

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(1) Here again, what happens is that the consumer's "expected real interest rates" are not influenced by a variation of the nominal rate r_1 .

direct effect upon discounting. It is thus not unlikely to be observed in actual economies, as an increase of the nominal rate set up by monetary authorities may be interpreted as a sign that inflationary tensions are building up in the economy, and conversely. In such a case, *the range of money supplies* ΔM *which the Bank can successfully impose, i.e. for which the system* (C), (D), (E) *has a solution, will be quite small. The Bank has then little influence on the money supply, although it may have full control over the interest rate.*

An Existence Result

The foregoing discussion shows that the set of money supplies ΔM for which the system of equations (C), (D), (E) has a solution, may be empty, when it is impossible to find a couple (p_1, r_1) which brings the goods markets into equilibrium, or quite small, when the intertemporal substitution effect generated by a variation of the interest rate is weak. We wish to study now the conditions on expectations which ensure that the Bank has full control over the money supply, i.e. which guarantee the existence of a solution of the system (C), (D), (E) for every $\Delta M > 0$. As one can anticipate beforehand, in view of our previous analysis, these conditions will be quite strong, and cannot be expected to prevail in actual credit money economies.

To discuss this problem, it is convenient once again to look at a value of current prices p_1 which solve the equations (C) for a given r_1, and to consider the Bank's corresponding money issue

$\Delta M(p_1,r_1) = \sum_a b_a^s(p_1,r_1,\bar{\mu}_a)/(1+r_1)$. Such a procedure will be valid, provided that we assume, for instance, that the conditions of *(1)* of Section 4 are satisfied for every r_1 .

Our strategy will be to find conditions which guarantee that $\Delta M(p_1,r_1)$ tends to 0 when the interest rate increases without bound, and that $\Delta M(p_1,r_1)$ tends to $+\infty$ as the interest rate approaches -1 , the price system p_1 moving at the same time so as to maintain the equilibrium on the goods markets. Intuitively, there should exist then, by continuity, a value of the interest rate r_1 , and an associated price system p_1 , which achieves the equality of $\Delta M(p_1,r_1)$ with the given money supply ΔM , and thus which satisfies the whole system of equations *(C)*, *(D)* , *(E)*.

In order to look at this issue more closely, it is useful to consider again the simple case where there is only one good, and where every consumer makes plans for the current period and the next one only, which we have employed repeatedly.

In such a framework, we recall, a typical consumer's optimum current and future consumptions are obtained by the maximization of his utility function under the intertemporal budget constraint

$$p_1c_1 + \frac{p_2}{1+r_1} c_2 = \mathrm{Max}(\bar{\mu} + p_1e_1 + \frac{p_2}{1+r_1} e_2 , 0) .$$

His corresponding money demand and bond supply are in turn given by consideration of the current budget constraint, or equivalently, by looking at the expected budget constraint :

$$p_2c_2 = p_2e_2 + (1+r_1)\, m_1 - b_1 ,$$

together with the condition that either m_1 or b_1 is equal to 0 .

If we want to be sure that $\Delta M(p_1,r_1)$ tends to 0 as r_1 goes to infinity, we need a condition on expectations which guarantess that the direct effect upon discounting of an increase of the interest rate, is not hampered by a correlative rise of expected prices, so as to avoid the phenomenon of the unvariability of the money creation which we described above. To do this, it suffices, as we are going to see, to strengthen the conditions of *(1)* of Section 4, by assuming that *all consumers' expected prices* $\psi^*_{a2}(p_1,r_1)$ *are uniformly bounded above by some positive number* η .

Indeed, a typical consumer's expected budget constraint implies that his bond supply b_1 must not exceed his expected income p_2e_2 . Under the new assumption, therefore, the aggregate bond supply must be less than or equal to $\eta \sum_a e_{a2}$. The money issue $\Delta M(p_1,r_1)$ cannot exceed $(\eta\sum_a e_{a2})/(1+r_1)$, which tends evidently to 0 when the interest rate increases without bound. Actually, a stronger result is true, since then $\Delta M(p_1,r_1)$ *tends to* 0 *at least as fast as* $1/(1+r_1)$, *when* r_1 *goes to* $+\infty$.

It remains to look at the conditions which ensure that $\Delta M(p_1,r_1)$ increases without bound when the interest rate tends to -1. A moment of reflexion shows that this must involve *two different sorts of conditions.*

Indeed, we saw in Section 4 that if p_1 and r_1 verify equations *(C)*, they must satisfy equation *(D_1)* too, by virtue of *Walras law* :

$$(\mathcal{D}_1) \quad \sum_a m_a^d(p_1,r_1,\bar{\mu}_a) = M + \sum_a \left[\frac{b_a^s(p_1,r_1,\bar{\mu}_a)}{1+r_1} - R_a(p_1,r_1,\bar{\mu}_a)\right]$$

This implies that whenever $\Delta M(p_1,r_1)$ tends to $+\infty$, the corresponding aggregate demand for money

$$M^d(p_1,r_1) = \sum_a m_a^d(p_1,r_1,\bar{\mu}_a)$$

tends to infinity as well, and conversely. The fact that $\Delta M(p_1,r_1)$ increases without bound means that there is at least one consumer who wishes to borrow larger and larger amounts of money from the Bank. But this means at the same time that there must be another consumer who is willing to hold more and more money. Guaranteeing the presence in the economy of these opposite behaviors must clearly involve two different sorts of conditions. This finding should, in fact, be no surprise. For if all consumers were identical, they would be either all borrowers, or all savers, and the only value which $\Delta M(p_1,r_1)$ could take would be zero.

If we wish to be sure that there is a consumer who is willing to borrow large amounts of money as r_1 tends to -1, we need that, for this consumer, the direct effect of the decrease of the interest rate upon discounting be not hampered by a correlative decrease of expected prices. This will be achieved by strengthening the conditions of *(1)* of Section 4, that is, by assuming that *there is a consumer a , who is not a debtor, whose expected prices* $\psi^*_{a2}(p_1,r_1)$ *are uniformly bounded below by some positive number* ε *.*

It is not difficult to see that this assumption, together with the condition that expected prices are bounded above, implies that *when the interest rate* r_1 *approaches* -1, *the corresponding price* p_1 *which brings the good market into equilibrium must tend to infinity like* $1/(1+r_1)$. Or equivalently, that the product $(1+r_1)\ p_1$ must be bounded above and below by two positive numbers.

In order to see this point, let us represent this particular consumer's intertemporal budget constraint in the plane (c_1,c_2) (Fig. 4).

Fig. 4

Since the consumer's expected price p_2 is bounded below by ε , the point α in this diagram must tend to the endowment point (e_1,e_2) as the interest rate decreases to -1. Now, if the product $(1+r_1)\ p_1$ tended to 0, the intertemporal budget line would become more and more horizontal. By substitution between current and future goods, the consumer's optimum current consumption would increase without bound, thereby generating eventually an aggregate excess demand on the good market. This would violate the assumption that the couple (p_1,r_1) always satisfies equation (C). Thus, the product $(1+r_1)\ p_1$ must remain bounded away from zero.

It remains to check that $(1+r_1)\ p_1$ cannot tend to infinity either. Indeed, if it were the case, the intertemporal budget line would become more and more vertical, since the consumer's expected price p_2 is bounded above. By substitution between current and

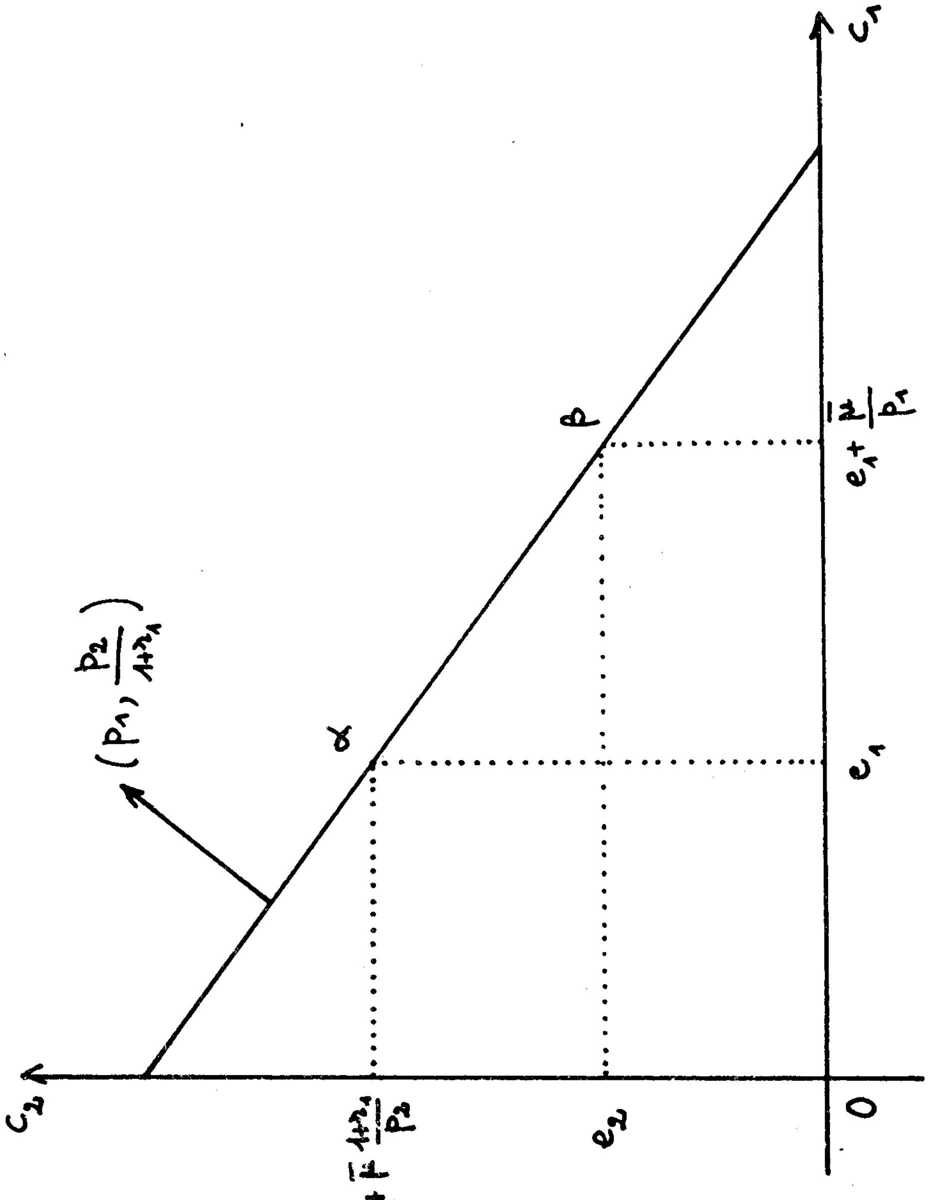

Fig. 4

and future goods, the consumer's expected future consumption c_2 would tend to infinity. This would imply, according to the consumer's expected budget constraint,

$$p_2 c_2 = p_2 e_2 + (1+r_1)\ m_1 - b_1\ ,$$

that the product of his money demand m_1 and of $(1+r_1)$ should increase without bound, since this consumer's expected price is bounded below by ε . In view of $(\mathcal{D}_1)$, therefore, the aggregate bond supply $\sum_a b_a^s(p_1,r_1,\bar{\mu}_a)$ should increase without limit too. But this is impossible, for, as we have seen, this expression cannot exceed $\eta \sum_a e_{a2}$ whenever the consumers' expected prices are bounded above by η . Thus the product $(1+r_1)\ p_1$ must be bounded away from infinity too.

We need, as we said, an additional condition to make sure that there is another consumer, say b , who is demanding more and more money as the interest rate approaches -1. For such a consumer, the ratio $p_2/((1+r_1)\ p_1)$ should tend to zero so that substitution of future against current consumption induces him to save eventually. Since $(1+r_1)\ p_1$ is bounded away from zero and from infinity, this means that this consumer's expected price must tend to 0. As a matter of fact, the precise condition which we shall need is that *there is a consumer* b *whose expected price* $\psi_{b2}^*(p_1,r_1)$ *tends to* 0 , *but not faster than* $(1+r_1)$, *when the interest rate* r_1 *decreases to* -1.

In order to see this point, let us look at the graphical representation of this consumer's decision making problem, as in

Fig. 4 [1]. Since p_1 increases without bound when r_1 decreases to -1 , the point β in the diagram approaches the endowment point (e_1,e_2). On the other hand, the intertemporal budget line becomes more and more vertical, since p_2 tends to zero and $(1+r_1)\ p_1$ is bounded below by a positive number. Hence, the consumer's optimum future consumption c_2 increases without limit. It follows then from the consumer's expected budget constraint,

$$\frac{p_2}{1+r_1}\ (c_2-e_2) = m_1 - \frac{b_1}{1+r_1} \quad ,$$

that his demand for money m_1 increases without bound too, since the consumer's expected discounted price $p_2/(1+r_1)$ is, under our assumption, bounded away from zero. Accordingly, the aggregate demand for money $M^d(p_1,r_1)$, or equivalently $\Delta M(p_1,r_1)$, tends to infinity when the interest rate r_1 approaches -1 , and when p_1 moves correlatively so as to maintain the equilibrium of the good market, as we claimed.

This heuristic discussion has permitted us to discover the kind of conditions which we needed to ensure the existence of a solution to the system $(\mathcal{C})$, $(\mathcal{D})$, $(\mathcal{E})$ for every positive money supply ΔM. In order to state a formal result along these lines, it is necessary to give first a few precise definitions.

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(1) If the consumer is initially a debtor, we are sure that he is not bankrupt eventually, since p_1 tends to infinity. Fig. 4 then applies, with $\bar{\mu} < 0$.

Let us say that a consumer's price expectations are *continuous* if the functions $\psi^*_{at}(p_1, r_1)$ are continuous with respect to p_1 and r_1, for every t. We shall say that a consumer's *expected prices are uniformly bounded above* if there exists a vector η, with all its components positive, such that $\psi^*_{at}(p_1, r_1) \leqq \eta$ for all p_1 and r_1, and every t. A consumer's expected prices will be said *uniformly bounded away from zero* if there is a vector ε, with all its components positive, such that $\psi^*_{at}(p_1, r_1) \geqq \varepsilon$ for all p_1 and r_1, and every t. Finally, let us say that a consumer's *expected prices tend to* 0, *but not faster than* $(1+r_1)$, *when the interest rate decreases to* -1, if for each sequence (p_1^k, r_1^k) such that r_1^k converges to -1, the sequences $\psi^*_{at}(p_1^k, r_1^k)$ tend to 0 for each t, and if there is a vector λ, with all its components positive, which may depend upon the sequence, such that $\psi^*_{at}(p_1^k, r_1^k)/(1+r_1^k) \geqq \lambda$ for every k and all t.

Our existence result can now be stated, with these definitions [(1)] :

(2) *Assume* (a) *and* (b) *of Section* 1, *and that every consumer's price expectations are continuous. Assume moreover that every consumer's expected prices are uniformly bounded above, and that there is at least one consumer* a *with* $n_a \geqq 2$ *and* $\bar{\mu}_a \geqq 0$, *whose expected prices are uniformly bounded away from zero. Assume finally that there is at least another consumer* b, *with* $n_b \geqq 2$, *whose expected prices tend to* 0, *but not faster than* $(1+r_1)$, *when the interest rate tends to* -1.

Then, the system (C), (D), (E) *has a solution for every* $\Delta M > 0$, i.e. *the Bank has full control over the money supply.*

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(1) A rigorous proof of the result is given in Appendix C.

It goes without saying that these conditions are extremely specific, and that they cannot be expected to prevail in reality. Our analysis shows in particular, that the effectiveness of large increases of the rate of interest as an antiinflationary measure, by reducing the money supply, is subordinated to the condition that *all* traders' expected prices are uniformly bounded above ..... a condition which is quite unlikely to obtain in an inflationary environment.

On the other hand, the effectiveness of large decreases of the interest rate as a reflationary measure is subject to conditions which are less stringent, but are nevertheless quite specific. But there exists a much more severe limitation to the scope of such a policy in actual economies. For in the real world, the mere presence of paper money imposes, as we have seen, the additional restriction that the interest rate must be nonnegative (1).

The qualitative conclusion which emerges from this analysis is therefore, that, *even in the favorable case where there is a short run equilibrium for each rate of interest, the ability of monetary authorities to have, by varying the cost of credit, a significant influence on the money supply or on equilibrium prices, is rather problematic.*

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(1) See Footnote 1 at the beginning of Section 1.

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