N° 8014

EFFECTIVE POLICY TOOLS AND QUANTITY CONTROLS

By

Roger GUESNERIE
CEPREMAP

and

Kevin ROBERTS
St. Catherine's College, Oxford

July 1980

In a perfectly competitive economy with no externalities and an optimal distribution of income, *an optimum optimorum* has been reached. No matter how sophisticated, available policy tools will never be utilized by a benevolent government or Central Planner. Here, a more realistic situation is envisaged; in particular, the purpose of this paper is to analyze the usefulness of policy tools in a suboptimal world.

Over the last decade a large body of literature has built up dealing with the usefulness of commodity taxes. These are tools which act through prices, augmenting the decentralized competitive system by driving a wedge between agents. Given the welfare connotations of competition, it is not surprising that attention should have focussed on tools which act to augment rather than replace prices. In contrast to such work, this paper gives emphasis to the role that can be played by quantity controls : rationing, redistribution-in-kind, etc. In particular, it will be contended that although quantity controls will not in general be utilized when the *optimum optimorum* is achievable, in second-best situations they are likely to prove an invaluable aid in promoting a socially desirable state of affairs.

The investigation conducted here starts from a given initial situation, which is away from the first best, and examines the desirability of implementation of quotas : why, and more importantly in which directions, should quantity constraints operate from such a situation ? The approach taken is abstract in the sense that the characteristics of the starting point as well as of the policy tools available are described in a general way encompassing many different situations. Such an abstract point of view is justified by the fact that in a broad class of problems, the welfare effects of quotas can be analyzed in terms of social opportunity costs without explicit reference to the specific set of policy tools which are available. Our results are in nature local and primarily concern the desirability of small quota policies; however they do have consequences which are emphazised in different realistic problems when the implementation of large quotas is considered.

This paper is based upon, and is an extension of two independently written papers: "La Gratuité, Outil de Politique Economique" [1978] by Roger GUESNERIE, and "The Treatment of the Poor under Tax/Transfer Schemes" [1978] by Kevin ROBERTS.

I. A SIMPLE EXAMPLE

We will first look at an example which introduces and illustrates some of the major ideas developed in the rest of the paper.

Consider a small society which can transact as much as it desires with the outside world. The vector of prices associated with the n existing commodities is $p = (p_1,\dots,p_n)$ and it is normalized so that $p_1 = 1$. Since the society can exchange as much as it wants at these prices, the corresponding vector can be unambiguously considered as defining the relative "social opportunity costs" of commodities.

Suppose an individual consumer faces a price system $q = (q_1,\dots,q_n)$ (normalized by $q_1 = 1$) which is different for some reason (think of taxes) from the vector p. This consumer is a price taker and equates his marginal rates of substitution and relative prices.

Consider now the following question : the Government is able to force the consumer to consume one more unit of some commodity (say commodity n), or to consume one unit less of this commodity. In other words, the Government is able to enforce a positive quota (one unit more of n) or a negative quota, i.e. a ration (one unit less of n). Is one of these (small) quotas desirable from the social welfare point of view ?

In the case under consideration, the answer is simple if one makes the additional assumption that the utility function of the consumer has the following form :

$$u(x) = x_1 + \sum_{\ell=2}^{n} u_\ell(x_\ell).$$

With such a utility function, if the consumer is forced to consume one unit more of commodity n, he will not modify his decisions in the other markets $\ell = 2,\dots,n-1$; he will finance his extra consumption of commodity n by giving up $-q_n$ units of the numeraire. It should be noted that the consumer is not injured (to a first order approximation) by the small quota imposed upon him. This is an intuitive fact which can be seen as the consequence of the envelope theorem, according to which if one starts from a situation which is optimal with respect to a given environment then an adaptation to a small change in the environment implies only second order gains (or losses).

./.

Now let us look at the operation from the point of view of society. The cost to society is p_n, the social cost of one unit of commodity n, minus q_n, the cost of q_n units of the numeraire. Let us call $t_n = q_n - p_n$, the "tax" on commodity n. If $t_n > 0$, the cost of the operation $- t_n$ is negative to the society when the consumer under consideration is indifferent. Clearly, the implementation of the corresponding (personal) positive quota is desirable in a strong sense to society. The same argument shows that $t_n < 0$ implies the desirability of a negative quota.

From this analysis two facts can be emphasized :

- When for some reason the prices faced by consumers differ from the "social" costs of the commodities, the implementation of quotas is desirable.
- When there are discrepancies between social and individual costs, consumers make choices which, although the best given the signals they face, are not the best for society. Quotas can correct these undesirable choices. Hence, when commodity n of the example is taxed too heavily consumers purchase an insufficient amount of the commodity and a positive quota is desirable. Conversely, if n is subsidized then its consumption is too high (from the point of view of society) and a reduction is obtained through a negative quota.

The main purpose of this paper is to understand the degree of truth and generality of the tentative conclusions suggested by this analysis and to evaluate their implications for more realistic quantity control policies. Clearly, the above analysis has two main weaknesses :

First, the prices p of the example define social values of commodities unambiguously. In some circumstances one is unable to identify any reasonable concept of social values, and in others, there will be values that it will be reasonable to label "social", although the corresponding vector will have weaker properties than the vector of world prices in the example.

Second, the special utility function which has been considered leads us to ignore the spillover effects associated with the imposition of quotas. The study of spillover effects is in fact a central issue in view of the implementation of quantity controls and will be a central concern of this paper.

One should also mention that the conjectures suggested by the example apply to small quotas acting on a single individual when, in practice, policy quotas act anonymously and may be large. It will also be one

./.

of the purposes of this paper to understand, from the welfare analysis of small individualized quotas, the role of anonymous quotas.

We are now in position to give a description of the structure of the paper.

Section II consist in preliminaries. The assumptions of a reference model are discussed in section II.1. The existence of social opportunity costs possessing strong properties, is assumed. The partial equilibrium analysis of the spillover effects of quantity constraints is presented in section II.2.

In section III, a general proposition on the desirability of small personalized quotas is stated, proved and discussed. The theorem generalizes the conclusions inferred from our introductory example. A first comparison of the respective role of prices and quotas is presented.

Section IV is concerned with the role of anonymous quotas and sufficient conditions are given for the desirability of authoritarian redistribution-in-kind or rationing in a complex economy with taxes. Additional insights are provided into the economic role of quotas.

Section V considers two possible directions of generalization of the preceding results : extension to a wider class of situations where a weaker concept of social values can be identified and extension to the case where consumers are already constrained in the initial situation. In both cases, it is shown that the main results still hold, although necessarily in a weaker form.

An application of the previous ideas to the specific problem in public finance of the optimal taxation of consumers at the end-points of non-linear tax schedules is considered in section VI and, finally, section VII forms the conclusion.

II. PRELIMINARIES

II.1. Model and assumptions

In this section a very general model is considered. We shall be interested in determining conditions under which sets of policy tools are capable of bringing about a social improvement.

./.

We will examine an economy with n commodities, indexed by $\ell = 1,\ldots,n$, m consumers, indexed by $i = 1,\ldots,m$, q firms, indexed by $j = 1,\ldots,q$. As in the general framework adopted by Guesnerie [1979] individuals decisions are made on the basis of some vector of signals denoted s. A *feasible state* of the model is then defined by a sequence of consumption bundles (x_i) and production plans (y_j) associated with the vector s such that :

$$
\begin{array}{ll}
x_i = \chi_i(s) & i = 1,\ldots, m \\
y_j \in y_j(s) & j = 1,\ldots, q \\
\psi_k(s) \leq 0 & k = 1,\ldots, \nu \\
\sum_i x_i \leq \sum_j y_j + \omega &
\end{array}
$$

where $\chi_i(.)$, $y_j(.)$ are respectively the demand function of consumer i, the supply correspondence of firm j and $\psi_k (k = 1,\ldots, \nu)$ defines some constraint on s. We will write the vector of signals $s = (u, v)$ where in v we single out the variables associated with the implementation of quantity constraints.

In this section we focus attention on quantity constraints associated with one commodity, commodity n, and affecting only one consumer h. v, the amount of "forced consumption" of commodity n for consumer h, will be some real number.

To be precise, the demand functions have the following form :

(2.1.) $$x_{h\ell} = \chi_{h\ell}(u, v) \qquad \ell = 1,\ldots, n-1$$

$$x_{hn} = \chi_{hn}(u) + v$$

(2.2.) $$x_i = \chi_i(u) \qquad i \neq h$$

Two remarks. First, the quota on agent h acts as an additive disturbance on his unconstrained demand. Second, the demand of $i \neq h$ does not depend upon v, the amount of "forced consumption" imposed upon h.

We consider this general model and an initial situation, denoted 0, in which there is no quota on consumer h in the sense that $v = 0$. This initial situation is denoted (x_i^o), (y_j^o), (u^o), $(v^o = 0)$ and the following assumptions are made

./.

(A1) $\forall$ i, there exists a $q_i^0 \in \mathbb{R}^n$ such that a consumption change Δx_i from x_i^0 increases i's utility if

(2.3) $q_i^0 \, \Delta x_i > f_i(\Delta x_i)$

where f_i is a given function such that

$$\frac{f_i(\Delta x_i)}{\|\Delta x_i\|} \to 0 \qquad \text{as} \; \|\Delta x_i\| \to 0.$$

The essence of this assumption is that q_i^0 is a *"virtual" price* which supports the allocation x_i^0 (see Neary and Roberts (1980)). If the economy under consideration embodies price-taking behavior then q_i^0 will be the prices taken. As it is being assumed that any *small* change that costs more is desirable, there is an implicit assumption that indifference curves are suitably smooth, i.e. differentiable.

If the Government acts to change the prevailing allocation, there exists the problem that some changes will not be feasible on production grounds. Something must therefore be said about feasible changes in aggregate demand. Corresponding to (A1) we have

(A2) There exists a vector $\rho^0 \in \mathbb{R}^n_+$ with the following property. By using the available policy tools and through an adequate Δu, the Government can induce any change ΔZ in net aggregate excess demand which satisfies

(2.4) $\rho^0 . \Delta Z \leq g(\Delta Z)$

(where g is a function such that $\frac{g(\Delta Z)}{\|\Delta Z\|} \to 0$ as $\|\Delta Z\| \to 0$), without affecting the consumption bundles of consumers.

Clearly (A2) holds when, as in the above introductory example, society can transact at world prices ρ^0. The assumption also holds when the production departments of the Government can produce any commodity and when, in the initial situation, they are faced with the same vector of shadow prices i.e. ρ^0. The reader familiar with the optimal taxation literature will also notice that in a standard Diamond-Mirlees (1971) model without public firms, (A2) is satisfied (through changes in commodity taxes) with ρ^0, as the production price vector. In fact, the results obtained in this section only require a weaker assumption (a2) which replaces the last sentence of (A2) "without affecting the consumption bundles of consumers" by "without affecting the utility levels to a first-order approximation". [1]

./.

The vector ρ^o, the existence of which is assumed in (A2) (and which is determined up to a positive scalar), indicates *(relative) social opportunity costs for commodities*. Later on, in section V, with assumption (A'2) imposed, we will refer to a much less demanding concept of social opportunity cost, but obviously with a weaker assumption we will obtain less powerful results.

The third assumption concerns the possibility of making arbitrarily small demand changes; in fact it will be simpler, without relinquishing generality, to assume differentiability.

(A3) If $\chi_i(s)$ is i's consumption bundle when policy parameters are set at s, then $\chi_i(s)$ is differentiable in s.

It should be noted that differentiability (hence continuous partial derivatives) is assumed both with respect to u and v.(2)

Of particular interest is a tool which is universally agreed to be desirable.

(A4) There exists a policy parameter s_k such that small increases (or decreases) in s_k either strictly increase the utility of all individuals or strictly decrease the utility of all individuals, i.e. either

$$(2.5) \qquad q_i^o \left(\frac{\partial \chi_i}{\partial s_k}\right) > 0 \quad \forall\, i \text{ or } q_i^o \left(\frac{\partial \chi_i}{\partial s_k}\right) < 0 \quad \forall\, i$$

Examples of such a policy parameter are a poll-subsidy, a universally liked public good or the price of a good supplied by all individuals. As with these examples, such a policy change does not need to be feasible(3).

Some implications of assumptions (A1) - (A4) will be derived in the following two lemmas which will be used in the sequel.

Lemma 1.

If (A1) - (A4) are satisfied and if there exists a policy tool, with parameter s_o, that acts only on h, then a Pareto improvement is possible if

$$(2.6) \qquad q_h^o \cdot \left(\frac{\partial \chi_h}{\partial s_o}\right) \geq 0 \quad \text{and} \quad \rho^o \cdot \frac{\partial \chi_h}{\partial s_o} < 0$$

./.

Proof :

In the proof $\sigma(\Delta s)$ designates any function tending to zero as $\|\Delta s\| \to 0$.

Consider increasing s_o by Δs_o and increasing s_k by $\Delta s_k (\Delta s_k > 0)$ (where this is in the direction of universal approval) with $\Delta s_k = \beta \Delta s_o$ where β is a fixed number. By A3,

$$q_h^o \cdot \Delta x_h = q_h^o \cdot \left[\frac{\partial \chi_h}{\partial s_o} \Delta s_o + \frac{\partial \chi_h}{\partial s_k} \Delta s_k\right] + \|\Delta s\| \, \sigma(\Delta s)$$

Given the assumption (A4)

$$q_h^o \cdot \Delta x_h \geq q_h^o \cdot \left(\frac{\partial \chi_h}{\partial s_k}\right) \Delta s_k + \|\Delta s\| \, \sigma(\Delta s)$$

so that

$$\frac{q_h^o \cdot \Delta x_h}{\|\Delta x_h\|} \geq \left(q_h^o \cdot \frac{\partial \chi_h}{\partial s_k}\right) \frac{\Delta s_k}{\|\Delta x_h\|} + \frac{\|\Delta s\|}{\|\Delta x_h\|} \, \sigma(\Delta s)$$

For $\|\Delta s\|$ small enough, $\dfrac{\Delta s_k}{\|\Delta x_h\|}$ has a positive lower bound (consider

$\left\|\dfrac{\partial \chi_h}{\partial s_o} \dfrac{1}{\beta} + \dfrac{\partial \chi_h}{\partial s_k}\right\|^{-1}$) as well as $\left\|\dfrac{\Delta s}{\Delta x_h}\right\|$, so that :

$$\frac{q_h^o \cdot \Delta x_h}{\|\Delta x_h\|} \geq a' \qquad\qquad a' > 0, \text{ (because of A4)}$$

and $q_h^o \, \Delta x_h \geq f_h(\Delta x_h)$ for $\|\Delta x_h\|$ small enough and hence for Δs small enough. (If $\Delta s_k > 0$ is small enough, and $\Delta s_k = \beta \Delta s_o$, h is made better off as is everybody else (by (A4)).

Consider now

$$\rho^o \cdot \Delta Z = \rho^o \cdot \left[\frac{\partial \chi_h}{\partial s_o} \Delta s_o + \sum_i \frac{\partial \chi_i}{\partial s_k} \Delta s_k\right] + \|\Delta s\| \, \sigma(\Delta s)$$

For obtaining feasibility, we can choose β small enough so that

$$0 < \left|\rho^o \cdot \sum_i \frac{\partial \chi_i}{\partial s_k} \Delta s_k\right| < -\frac{1}{2} \rho^o \cdot \frac{\partial \chi_h}{\partial s_o} \Delta s_o$$

./.

then we obtain :

$$\rho^{o}\ \Delta Z \leq \frac{1}{2}\left|\rho^{o}\ \frac{\partial x_h}{\partial s_o}\ \Delta s_o\right| + \Delta s_o\ \sigma(\Delta s_o)$$

An argument similar to the preceding one shows that $\frac{\rho^{o} \cdot \Delta Z}{\|\Delta Z\|}$ is hence greater than $g(\Delta Z)$ for Δs_o small enough. With (A2) the conclusion follows.||

Lemma 1 describes the simplest case of how a policy paremeter change can bring about a social improvement : if the direct effect of the change injures nobody and releases resources which can be distributed to consumers then the change must be desirable[4].

Now let us consider the situation where there exist two policy tools which act upon consumer h. Obviously each tool can be checked independently against the criterion given in (2.6). But even where both tools fail the test, it will usually be possible to bring about an improvement.

Lemma 2.

If (A1) - (A4) are satisfied and if there exist two independent policy tools, with parameters s_o and s_1, that act only on h then a Pareto improvement is possible if

$$q_h^o \cdot \left(\frac{\partial x_h}{\partial s_o}\right) \neq 0$$

and

$$\rho^{o}\left(\frac{\partial x_h}{\partial s_1}\right) \cdot q_h^o\left(\frac{\partial x_h}{\partial s_o}\right) \neq \rho^{o}\left(\frac{\partial x_h}{\partial s_o}\right) \cdot q_h^o\left(\frac{\partial x_h}{\partial s_1}\right).$$

Proof :

We convert the situation back to a single parameter system by making both s_o and s_1 functions of some s_2.

Let $$\alpha = \left[\rho^{o}\left(\frac{\partial x_h}{\partial s_1}\right) \cdot q_h^o\left(\frac{\partial x_h}{\partial s_o}\right) - \rho^{o}\left(\frac{\partial x_h}{\partial s_o}\right) q_h^o\left(\frac{\partial x_h}{\partial s_1}\right)\right]$$

and let $$\frac{\partial s_o}{\partial s_2} = \alpha\ q_h^o\left(\frac{\partial x_h}{\partial s_1}\right)$$

$$\frac{\partial s_1}{\partial s_2} = -\ \alpha\ q_h^o\left(\frac{\partial x_h}{\partial s_o}\right) .$$

./.

It is easy to check that

$$q_h^o\left(\frac{\partial x_h}{\partial s_2}\right) = 0 \quad \text{and} \quad \rho^o\left(\frac{\partial x_h}{\partial s_2}\right) < 0.$$

Considering (2.6) then gives the desired result. || [5]

II.2. Quantity constrained demand functions.

To proceed further, we have to be more specific concerning the effect of quantity constraints on demand i.e. we have to explore the properties of the quantity constrained demand functions which were introduced in (2.1) above. For this purpose, we will consider a single consumer and examine the choice he makes for rationed goods when his consumption of some good (good n) is strictly controlled. The argument presented here can be related both to Guesnerie [1978] who considers one rationed good or to Neary and Roberts [1980] who provide a more complete discussion and an extension to a case where a collection of goods is rationed (such generality is not required here). [6]

Assume that preferences are representable by a twice-differentiable, strictly quasi-concave utility function $\mathcal{U}$. With an income of M and prices $\tilde{q}$ for the first n-1 goods and q_n for the n-th good, a fully unrationed demand system is derived by solving :

$$(2.7) \qquad \text{Max } \mathcal{U}(\tilde{x}, x_n) \text{ s.t } \tilde{q}.\tilde{x} + q_n.x_n \leq M \; ^{(7)}$$

Let $\tilde{D}(q, M)$ and $D_n(q, M)$ be this unrationed (or notional) demand system which will be taken to be a differentiable function of prices and income (indifference surfaces are assumed to possess strict Gaussian curvature).

Next, assume that the consumer is forced to purchase y of good n. He now faces the problem :

$$(2.8) \qquad \text{Max } \mathcal{U}(\tilde{x}, x_n) \text{ s.t } \tilde{q}.\tilde{x} + q_n x_n \leq M \text{ and } x_n = y$$

Let $\bar{D}(\tilde{q}, q_n, M, y)$ be the n-1 dimensional vector which solves (2.8). $\bar{D}$ is the demand system for unrationed goods when rationing operates. What is the connection between $\tilde{D}(\tilde{q}, q_n, M)$ and $\bar{D}(\tilde{q}, q_n, M, y)$? Note first that if $\mathcal{U}$ is quasi concave, the bundle $(\bar{x} = \bar{D}(\tilde{q},...,y), y)$ would be chosen in an unrationed situation under some price/income configuration. In

./.

fact, as marginal utilities of unrationed goods will be proportional to prices, it is clear that the prices which support $(\bar{x}, y)$ are of the form $(\tilde{q}, \phi_n)$ where ϕ_n is the "virtual" price or "shadow" price measuring the marginal utility of the rationed good[8]. Then for some ϕ_n we have :

(2.9) $\quad \bar{D}(\tilde{q}, q_n, M, y) = \tilde{D}(\tilde{q}, \phi_n, M + (\phi_n - q_n)\, y)$

(2.10) $\quad y = D_n(\tilde{q}, \phi_n, M + (\phi_n - q_n)\, y)$

(with prices $(\tilde{q}, \phi_n)$ it requires an income of $M + (\phi_n - q_n)\, y$ to attain the point $(\bar{x}, y)$).

The remarkable feature of (2.9) and (2.10) is that (2.10) can be used to solve for ϕ_n which may be inserted into (2.9) to give the rationed demand system. This gives a straightforward way of studying the connection between rationed and unrationed demand systems.

By total differentiation of (2.9) and (2.10) - or equivalently by considering (2.9) as an identity when ϕ_n is the implicit function $\phi_n(\tilde{q}, q_n, M, y)$ defined by (2.10)[9] - the reader can check that

(2.11)
$$\left(\frac{\partial \bar{D}}{\partial y}\right)_{(.)} = \frac{\left(\frac{\partial \tilde{D}^c}{\partial q_n}\right)_{(*)}}{\left(\frac{\partial D^c_n}{\partial q_n}\right)_{(*)}} + \left(\frac{\partial \bar{D}}{\partial M}\right)_{(*)} (\phi_n(.) - q_n)$$

and

(2.12)
$$\left(\frac{\partial \bar{D}}{\partial M}\right)_{(.)} = \left(\frac{\partial \tilde{D}}{\partial M}\right)_{(*)} - \frac{\left(\frac{\partial D_n}{\partial M}\right)_{(*)}}{\left(\frac{\partial D^c_n}{\partial q_n}\right)_{(*)}} \left(\frac{\partial \tilde{D}^c}{\partial q_n}\right)_{(*)}$$

where superscript c denotes that demands are Hicksian compensated demands and derivatives are taken in $(.) = (\tilde{q}, q_n, M, y)$ and $(*) = [\tilde{q}, \phi_n(.), M + (\phi_n(.) - q_n)\, y]$.

Premultiplying (2.11) and (2.12) by marginal utilities gives utility changes

(2.13) $\quad \left(\frac{d\mathcal{U}}{dy}\right)_{(.)} = \mu(\phi_n(.) - q_n)$

./.

$$(2.14)\qquad \left(\frac{d\mathcal{U}}{dM}\right)_{(.)} = \mu$$

where μ is the marginal utility of income of a consumer who chooses the bundle $(\bar{x}, y)$ when unrationed. Notice that if a ration is introduced into an initially unrationed situation then $\phi_n = q_n$ and, from (2.13) *to the first order there will be no loss in utility to the consumer.* This is explained by the fact that in unrationed situations, a consumer's indifference curve will be tangential to his budget line; if he is forced to move along the budget line then, initially, this is equivalent to moving along the indifference curve.

Combining (2.11) - (2.14), one can obtain the "compensated" rationed demand function which gives the change in the demand induced by a change in the ration level, accompanied by a change of income (proportional to $\phi_n - q_n$) which would compensate for the change in ration. We obtain (with obvious notation)

$$(2.15)\qquad \left(\frac{\partial\bar{D}^c}{\partial y}\right)_{(.)} \overset{\text{def}}{=} \frac{\partial\bar{D}}{\partial y} - \frac{\partial\bar{D}}{\partial M}(\phi_n - q_n) = \frac{\left(\frac{\partial\tilde{D}^c}{\partial q_n}\right)_{(*)}}{\left(\frac{\partial D^c_n}{\partial q_n}\right)_{(*)}}$$

Thus the effect on demand of a compensated change in the ration level is directly equivalent to a compensated change in the price of the rationed good sufficient to make the demand for that good equal to the new ration level ; in compensated terms, the effect of quantity controls acting on a consumer are directly equivalent to price changes of the goods being controlled. This analysis gives us an intuitive understanding as well as full information of the spillover effects of rationing. We are thus in position to generalize the informal analysis sketched in section I.

III. THE DESIRABILITY OF SMALL PERSONALIZED QUOTAS.

III.1. A general proposition.

In this section, we will make two additional assumptions with respect to the general model of section II. First, the behavioural aspects of the model will be made more precise and we will assume that there is some consumer price vector faced by all consumers for their transactions which is

./.

denoted $q = (\tilde{q}, q_n)$. Second we will assume that, in the initial situation, all consumers are unconstrained price takers with respect to the prevailing price system.

With regard to the notation adopted in section II, where u is the vector of general signals and v the quota, we give to these assumptions the following precise meaning.

(3.1.) (B) $\chi_{i\ell}(u) = D_{i\ell}(q(u), M_i(u)) \qquad \forall\, \ell, \quad \forall\, i \neq h$

$$\chi_{hn}(u, v) = D_{hn}(\tilde{q}(u), q_n(u), M_h(u)) + v$$

$$\chi_{h\ell}(u, v) = \overline{D}_{h\ell}(\tilde{q}(u), q_n(u), M_h(u), \chi_{hn}(u, v)), \qquad \ell \neq n$$

(I) In the initial situation, consumer h is unconstrained

$$x^o_{hn} = D_{hn}(\tilde{q}^o, q^o_n, M^o) \qquad (q^o = q(u^o))$$

$$x^o_{h\ell} = D_{h\ell}(\tilde{q}^o, q^o_n, M^o) \;(= \overline{D}_{h\ell}(\tilde{q}^o, q^o_n, M^o, D_{hn}(q^o, M^o))$$

In fact, the form assumed en (3.1) for $\chi_{i\ell}(u)$ $(i \neq h)$ is used only for the sake of consistent interpretation of the model, but does not play any role in the proof (as the reader can verify). A relaxation of both assumptions (B) and (I) is provided in section IV.

Given our notation and definitions, it is clear that

$v > 0$ defines *a positive quota* : it determines a *forced consumption* of commodity n, the extra consumption being paid for at market prices q_n

$v < 0$ defines *a negative quota* : it is obtained through a *rationing* of commodity n, the consumer market price being unchanged.

In addition to (B) and (I), the assumptions (A1) - (A4) of the preceding section will now be assumed to hold in the initial situation.

Consider

$$t^o_\ell = q^o_\ell - \rho^o_\ell \qquad \ell = 1,\ldots, n-1$$

$$t^o_n = q^o_n - \rho^o_n$$

$t^o = (\ldots\, t^o_\ell \,\ldots)$ measures the discrepancies between social values of commodities and consumer prices. It will be termed the vector of *fictitious taxes* (since it may differ from the vector of real taxes).

We are now in a position to state the main result of this section.

./.

Theorem 1.

Suppose that (B) holds and that in the initial situation (A1) - (A4) are satisfied, consumer h faces no quantity control (I) and

$$(3.3) \qquad \sum_{1}^{n} t^{o}_{\ell} \left(\frac{\partial D^{c}_{hn}}{\partial q_{\ell}}\right)_{(o)} < 0 \qquad\qquad (\text{resp.} > 0)$$

Then a Pareto improvement is achievable through the implementation of a (small) positive (resp. negative) quota.

Proof.

We should first notice that there is no contradiction between the differentiability assumption A3 and the formulation of the quota policy in (B).

We will apply lemma 1 of subsection II.1, with $s_o = v$.

Clearly $q^o . \left(\frac{\partial x_h}{\partial v}\right)_{(o)}$ is proportional to $\left(\frac{\partial \mathcal{U}_h}{\partial v}\right)_{(o)}$ which, according to (B) and (2.13) and since $\phi^o_n = q^o_n$, is equal to zero.

Now (with obvious notation) $\rho^o . \left(\frac{\partial x_h}{\partial v}\right)_{(o)} = \rho^o_n + \tilde{\rho}^o . \left(\frac{\partial \bar{D}_h}{\partial v}\right)_{(o)}$

Applying (2.11) $\frac{\partial \bar{D}}{\partial v} = \frac{\frac{\partial \tilde{D}^c}{\partial q_n}}{\frac{\partial D^c_n}{\partial q_n}}$

Hence $\rho^o . \left(\frac{\partial x_h}{\partial v}\right)_{(o)} < 0$ is equivalent to

$$(3.4) \qquad \sum_{\ell=1}^{n} \rho^o_{\ell} \left(\frac{\partial D^c_{h\ell}}{\partial q_n}\right)_{(o)} > 0$$

Introducing taxes (3.4) is equivalent to

$$(3.5) \qquad \sum_{\ell=1}^{n} t^o_{\ell} \cdot \left(\frac{\partial D^c_{h\ell}}{\partial q_n}\right)_{(o)} < 0.$$

(3.3) follows by invoking symmetry of the Slutsky matrix. ||

./.

The interesting feature of this result is that the criterion (3.3) will almost always be non-zero, given that there is no *systematic* reason for ρ^0 to be related to the price faced by h. Thus, subject to this important caveat, it will almost always be desirable to introduce quantity controls into an economic system from which they are excluded. The caveat is required because if ρ^0 is directly proportional to the prices faced by h then (3.3) will be zero : if h faces social opportunity costs as prices then, as one would expect, theorem 1 gives no recommendation for the introduction of quantity controls.

Thus, in general, quantity controls are desirable outside a first best optimum. This should not be viewed as a surprising conclusion, but rather as a natural one. However theorem 1 tells us much more than that. It gives us a precise criterion for deciding whether the quantity control should be a positive or a negative quota. We will now investigate when the criterion signals one policy rather than another.

A first interpretation of the criterion can be given by looking first at (3.5). The formula gives the change in (fictitious) tax revenue when there is an income compensated increase in the price of good n. As a compensated price increase is equivalent to a compensated quota decrease, (3.5) says that h should be forced to consume more of a good if, thereby, "tax revenue" increases. This fictitious revenue then allows all individuals to be made better-off.

The formula (3.3) has another interpretation : it gives the change in the compensated demand for good n that would be induced by an intensification of the tax system, all taxes being increased proportionally to their initial values. Alternatively, the formula in (3.3) is a local measure of the degree to which the demand for a good is discouraged by the introduction of taxes. For this reason, Mirrlees (1976) adopted the idea of an index of discouragement which is the formula in (3.3) divided by demand. The criterion given in the theorem then states that *a consumer should be forced to consume more (less) of a good if the demand for that good has been discouraged (encouraged) by the fictitious tax system.* In essence, as suggested by the introductory example, quantity controls should be applied to help minimize distortions in the economic system.

If the demand for a good has been discouraged then it is tempting to say that the good has been taxed too heavily. However, great care must

./.

be taken when discussing the notion of high or low taxes. For, as support prices for consumers and producers may both be independently scaled, it is also possible to say that

$$(3.6) \qquad \begin{aligned} t^o_\ell &= \lambda q^o_\ell - (1-\lambda)\,\rho^o_\ell \qquad \ell = 1,\ldots,n-1 \\ t^o_n &= \lambda q^o_n - (1-\lambda)\,\rho^o_n \end{aligned}$$

defines an admissible fictitious tax system $(0 < \lambda < 1)$.

When λ is close to unity, taxes are higher on goods with high consumer support prices; when λ is close to zero, taxes are higher on goods with low producer support prices. Despite these problems, it is possible to say, in a meaningful way, that the tax on one good is greater than another.

<u>Definition</u> : ℓ <u>is taxed at least as highly as</u> ℓ' if

$$(3.7) \qquad \frac{q_\ell}{\rho_\ell} \geq \frac{q_{\ell'}}{\rho_{\ell'}} \qquad (10)$$

Thus a good is taxed higher than another if the percentage increase in price induced by the tax introduction is higher. The advantage of the definition is that it is unaffected by renormalizations of producer or consumer prices. Further, the relation induced is an ordering being complete, reflexive and transitive. Given this definition, it is now possible to present a result with the flavour that highly taxed goods should be "forced", through quantity controls, into consumers.

<u>Theorem 2</u>. If the assumptions of Theorem I are satisfied and if for agent h :

1) n is less (*resp.* more) fictitiously taxed than its complements
2) n is more (*resp.* less) fictitiously taxed than its substitutes

   Hence a Pareto improvement is achievable by imposing a positive (*resp.* negative) quota upon individual h.

<u>Proof</u>. Given theorem 1 it must be shown that (using (3.3, and dropping index h)

$$\sum_{\ell=1}^{n} t^o_\ell \frac{\partial D^c_n}{\partial q_\ell} < 0 \qquad (\textit{resp.} > 0).$$

./.

Prices may be normalized so that $q_n^o = p_n^o$. We then have from the definition of more heavily taxed and the assumptions of the theorem :

1) if $t_\ell^o > 0$ (*resp.* < 0), then $\frac{\partial D_n^c}{\partial q_\ell} < 0$;

2) if $t_\ell^o < 0$ (*resp.* > 0), then $\frac{\partial D_n^c}{\partial q_\ell} > 0$.

Thus each term in the summation is negative (*resp.* positive) and the result follows. ||

III.2. <u>Quotas versus prices</u>.

The above results on the desirability of quotas are not surprising in view of our partial equilibrium analysis of spillover effects in section II.2. This analysis emphasized that (small) quotas on some commodity n were equivalent to a compensated price change of this commodity. In fact, formula (3.3) is exactly similar to formulae obtained in the study of the desirability of compensated price changes (in a one consumer economy); in particular, a price analogue of theorem 2 has been given by Dixit [1975][(11)].

We now come to the question of comparing quotas and prices. For a fair evaluation of their respective merits we must compare the implementation of personalized quotas considered here with "personalized prices". Our conclusion is that personalized quotas are equivalent to personalized *compensated* price changes; this suggests that (personalized) quotas should be useful in different circumstances.

1) although (personalized) price changes are allowed, they cannot be accompanied by compensating changes of incomes

2) although compensating changes of incomes are allowed, the (personalized) price changes are constrained

3) both (personalized) price changes and compensating income changes are forbidden.

The two first cases are particularly interesting and we will illustrate them in the context of a standard Diamond-Mirrlees model with one consumer[(12)]. In such an economy price changes and quotas are both anonymous and personalized, and although the example is rather academic[(13)] it is particularly useful for our illustrative purpose. ./.

Let us recall briefly the constituents of the model .

1) *Producers*. With prices p, producers are price-takers. This gives rise to a competitive supply function y(p). All profits are, for convenience, assumed to be taxed away by the Government.

2) *Consumers*. These are as in the general model of section II except that now it is further assumed that (identical) consumers are price-takers at prices q. Consumers also receive a poll-subsidy M from the Government.

3) *Government*. The Government chooses a level of public production y_g from some production set Y_g, the poll-subsidy M, p and q (and so, indirectly, taxes t = q - p) subject to the constraint of feasibility, i.e. aggregate demand being met by aggregate supply.

To this model, we add the possibility that the amount consumed of good n is subject to the control of the Government.

Let us start by a preliminary remark : As soon as an assumption of type (A4) is accepted, and if there are no constraints on taxes, it is well known that aggregate production efficiency is desirable. Now with convex production sets, the shadow prices of public firms should be equal to the production prices faced by private firms in a situation which is a second best optimum. The corresponding price vector p is then, straightforwardly, the vector of social opportunity costs defined in A2. Furthermore if the initial situation is not an optimum, but if commodity taxes can be varied freely, it is easy to check that any move of aggregate demand satisfying $p.dz \leq 0$ can be matched by an appropriate change in taxes and p remains a vector of social opportunity costs in the sense of (A2)(14).

Turning to consumers, q supports all consumers' consumption bundles in the initial situation so that it is also the support price of (A1). Thus, when taxes can be freely varied for all consumers in the economy, fictitious taxes of Theorem I are defined by t = q - p and correspond exactly to the actual taxes levied by the Government. We may now illustrate the above assertions on the complementarity of taxation and quotas.

(1) *If price changes cannot be accompanied by compensating income changes, quotas are desirable as "complements" of taxes.*

Suppose that the poll subsidy M is constrained by a certain number $\overline{M}$, that the "initial situation" that we are considering is a maximum

./.

of some social welfare function (*) and that at this optimum the constraint $M = \overline{M}$ is binding. Hence it is known that the optimal solution is such that

$$\sum_{\ell=1}^{n} t_\ell^* \frac{\partial D_k^c}{\partial q_\ell} = \mu x_k^* \qquad k = 1,\dots, n$$

Multiplying the k^{th} equality by t_k^*, summing and taking into account the negative semi-definiteness of the matrix of compensated demand, it is easy to see that μ has the opposite sign of $\sum_k t_k^* x_k^*$, the total receipts of indirect taxation.

Hence, in the case where the total indirect receipts are positive, it would be *desirable to impose a positive quota on any consumption good* (such that $x_n^* > 0$). This conclusion is in line with our previous discussion. When total fiscal receipts are positive, all consumption goods are too heavily (although optimally) taxed, and their consumption should be encouraged through quotas. Quotas help to correct the distortions induced by taxes and play a "complementary" role.

(2) *If price changes are constrained, although income transfers are not, quotas are desirable as "substitutes" of taxes.*

Let us consider the optimality condition when the constraint on the poll tax is removed, but the tax or subsidy on some commodity n is constrained. If this constraint is binding at the optimum of some social welfare function we have for the considered commodity n

$$\sum_{\ell=1}^{n} t_\ell^* \frac{\partial D_n^c}{\partial q_\ell} = \eta_n$$

(t_ℓ^* being no longer real but fictitious taxes)

where η_n is positive when the constraint has the form $t_n \leq \overline{t}_n$

η_n is negative when the constraint has the form $t_n \geq \overline{t}_n$

Applying our previous results, a negative quota is desirable in the first case and a positive quota in the second. Quotas appears here as *substitutes* for price changes : a ration is a substitute for an increase in a tax (supposed to be impossible) and forced consumption is a substitute for a subsidy (which cannot be made as large as is desirable).

IV. THE DESIRABILITY OF ANONYMOUS QUOTAS.

The analysis of the preceding section applies to small additive personalized quotas. Quotas used in the real world have different features : first, they do not take the form of additive disturbances, second, they are not necessarily small, third, they act anonymously on the agents. However, we shall see that the preceding sections give us adequate tools and conclusions for thinking about anonymous quotas; in particular, they provide sufficient conditions for the desirability of such quotas (section IV.A). In section IV.B we will be in a position to complete our discussion on prices versus quotas by comparing anonymous quotas and optimal taxes.

Throughout this section, we will assume that we are in a world analogous to the one introduced in subsection III.2., with the difference that constraints on taxes and subsidies (which will be assumed to take the simple form $\underline{t}_\ell \leq t_\ell < \overline{t}_\ell$) may exist. In the absence of quantity controls, a feasible state of the model is associated with (x_i); (y) (y_g), p, q, M such that :

(4.1) $x_i = D_i(q, M)$

(4.2) $y = y(p)$

(4.3) $f(y_g) \leq 0$

(4.4) $\underline{t}_\ell \leq t_\ell \leq \overline{t}_\ell \qquad \ell = 1, \ldots, n$

(4.5) $\sum\limits_i x_i \leq y + y_g$

The argument to be developed does not depend crucially upon the particular assumptions that we are considering, and can be transposed to different contexts.

IV. A. Sufficient conditions for the desirability of anonymous quotas.

We will introduce two different kinds of policy tools; authoritarian redistribution-in-kind and rationing :

1. Authoritarian redistribution-in-kind :

The Government selects some commodity, commodity n, to be redistributed. Each consumer receives tickets. Each ticket gives a right to one

./.

unit of the free commodity. The amount of tickets is $w > 0$, it is the same for each consumer and it is a control variable. With tickets, consumers can obtain commodities freely either from a private firm or from a public firm. Private firms are refunded by the Government at price p_n, the production price for commodity n. In addition to the quantity w obtained freely, consumers can obtain an extra amount of commodity n from the market where it is sold at consumption price $q_n(q_n = p_n + t_n)$.

The authoritarian aspect of the process lies in the fact that no transaction in tickets is allowed; tickets are personal and cannot be sold. In others words, a "white" market is forbidden. Clearly, there are enforcement problems for such a policy and this is one reason why it has been rarely used. However, it should be noted that if the white market in tickets is allowed, (we have then a "liberal" redistribution-in-kind) then there is no role left for redistribution-in-kind when uniform lump sum transfers can be implemented. This equivalence of liberal redistribution in kind and money income redistribution is fairly intuitive and is confirmed in the more precise analysis of Guesnerie [1978] to which the reader is referred for more details.

2. Rationing :

The Government chooses to ration commodity n. The control variable is w, the maximum amount (the same for everybody) which can be obtained in the market for commodity n. So x_{in}, consumption of commodity n, satisfies $x_{in} \leq w$. Such a rationing scheme is anonymous and non manipulable in the sense of Benassy [1976]. However its actual implementation creates incentives for black markets and as for redistribution-in-kind, we assume that these pressures are supressed. It should be noted that in this rationing process (as well as for the authoritarian redistribution-in-kind just described) only consumers are constrained and express an effective demand; firms are always unconstrained and express notional demand.

In order to introduce the two different anonymous quota policies which have just been described in the formal model of (4.1) - (4.5) it is enough to modify the demand function of (4.1) as follows.

For redistribution-in-kind (4.6) $x_i = \Xi_i(q, M, w)$

./.

with $\Xi_i(q, M, w) = D_i(q, M + q_n w)$ if $D_{in}(q, M + q_n w) \geq w$

$\tilde{\Xi}_i(q, M, w) = \overline{D}_i(q, M + q_n w, w)$ if $D_{in}(q, M + q_n w) \leq w$

For rationing (4.7) $x_i = \Xi'_i(q, M, w)$

with $\Xi'_i(q, M, w) = D_i(q, M)$ if $D_{in}(q, M) \leq w$

$\tilde{\Xi}'_i(q, M, w) = \overline{D}_i(q, M, w)$ if $D_{in}(q, M) > w$

Now consider some initial situation (x_i^o) (y^o) (y_g^o) p^o, q^o, M^o which is a feasible state of the initial model (4.1) - (4.5). Pick some commodity, commodity n; would it be desirable to redistribute it in kind or to ration it ? The relevance of our previous analysis for answering this question results from the two following remarks.

- Authoritarian redistribution-in-kind acts in such a way that some people consume an amount of commodity n greater than the amount they would like to consume (they would prefer to sell their tickets at markets prices q_n). It is analogous to the "forced consumption" of section III. On the other hand, rationing implies that some people consume less than what they would like and hence acts as a negative quota in the sense of section III.
- In order to prove that quantity controls are desirable, it is enough to prove that around the initial free market situation, it is socially beneficial to implement "small" quantity controls. It follows that quantity controls should be used at the optimum, even if we ignore how far from the free market is the optimum.

With these remarks in mind the following is unsurprising.

Theorem 3.

Let us call $\rho^o = (\nabla f)_{\left(y_g^o\right)}$, the gradient of the production function of the public sector and $t_\ell^o = \rho_\ell^o - q_\ell^o$.

Let h(n) and h'(n), be two consumers such that

$$x^o_{h(n),n} = \operatorname*{Min}_i x^o_{i,n} \quad , \quad x^o_{h'(n),n} = \operatorname*{Max}_i x^o_{in}$$

(h(n) and h'(n) are assumed to be unique). Then,

./.

a) if

$$(4.8) \qquad \sum t_\ell^o \left(\frac{\partial D^c_{h(n),n}}{\partial q_\ell}\right)_{(o)} < 0,$$

b) if

$$(4.9) \qquad \sum t_\ell^o \; \frac{\partial D^c_{h'(n),n}}{\partial q_\ell}_{(o)} > 0,$$

then a Pareto improvement (with respect to the initial situation) can be obtained through the use of a) authoritarian redistribution-in-kind,
b) rationing.

Proof.

- First consider the desirability for h(n) and h'(n) of small individualized quotas in the sense of section III.1. We note that the assumptions of subsection II.1 hold. It is straightforward for (A1) (A3) (as soon as the competitive demand function is assumed to be differentiable). (A2) holds with $\rho^o = (\nabla f)_{(y_g^o)}$ and (A4) with $s_k = M$.

It follows from theorem 1 that, a small personalized positive (resp. negative) quota is desirable for h(n) (resp. h'(n)). Let us call them v and v' respectively.

- Second, we have to prove that these small desirable quotas (which act additively on the unconstrained demand function) can be implemented through the anonymous quotas policies which are considered.

- For redistribution-in-kind we proceed as follows : first we reduce the uniform lump sum transfer M^o to $M'^o = M^o - q_n^o\, x^o_{h(n),n}$ and distribute an amount of tickets $w^o = x^o_{h(n),n}$; clearly the actual allocation is unchanged. Consider v, the desirable positive quota on t(n), which can be made as small as desired. It results from the proof of the theorem, that the Pareto superior final situation is close to the initial one. Hence if v is small enough, the consumption $x_{h(n),n}$ remains the smallest one. But by an adequate small change in the distribution of tickets from w^o, we can realize the final situation as a feasible state of the model (4.2) - (4.6).

- For rationing, the argument parallels the preceding one. We take v' the the desirable negative quota on h'(n), small enough so that in the Pareto superior situation, h'(n) still has the highest consumption of n. With an adequate amount of tickets (close to $x^o_{h'(n),n}$), we can realize this situation as a feasible state of the model (4.2) - (4.5)[15]. ||

./.

Corollaries of theorem 3 could be obtained straightforwardly by applying the statements of theorem 2 to the analysis. The conclusions have the same flavour as the ones of section III. Highly taxed commodities are good candidates for (authoritarian) redistribution-in-kind, when highly subsidized commodities should be rationed. However we should note that the analysis which proceeds through the exploration of the neighbourhood of an initial free market situation does not say anything on the optimal amount of rationing or redistribution-in-kind.

IV.B. Optimal taxation versus anonymous quotas

The preceding analysis did not make any assumption about the initial situation. In particular, it could have been optimal with respect to some social welfare function. If this social welfare function is egalitarian, common sense suggests that optimal taxation will lead to a subsidization of necessities and to a taxation of luxuries. Loosely speaking, the preceding analysis would have the implication that necessities should be rationed and luxuries redistributed-in-kind.

We will now discuss this conclusion more precisely. Consider a social welfare optimum of the model (4.1) - (4.5). At the optimum, the first order conditions associated with the maximisation of the unconstrained tax on commodity ℓ and the uniform lump sum transfer M involve

$$(4.10) \qquad \sum_i \lambda_i \, q.\frac{\partial D_i}{\partial q_\ell} = \gamma \sum_i p.\frac{\partial D_i}{\partial q_\ell}$$

$$(4.11) \qquad \sum_i \lambda_i \, q.\frac{\partial D_i}{\partial M} = \gamma \sum_i p.\frac{\partial D_i}{\partial M}$$

where all derivatives and vectors have to be taken at the optimum and where γ is a positive number. Manipulations of (4.10) and (4.11) give rise to the following tax rule (cf Diamond [1975]).

$$(4.12) \qquad \frac{\gamma}{m} \sum_i t \, . \frac{\partial D_i^c}{\partial q_\ell} = \text{cov}\,(\beta_i, x_{i\ell})$$

where β_i, the net social marginal utility of transfer of individual i, is given by

$$(4.13) \qquad \beta_i = \lambda_i - \gamma \left(1 - t.\frac{\partial D_i}{\partial M}\right)$$

./.

(t is a vector of shadow taxes which identify with real taxes when constraints 4.4. do not bind).

Comparing (4.12) and (4.8) - (4.9) it is seen that optimal taxes depend on aggregate "indices of discouragement" whereas the desirability of quantity controls depends upon individual "indices of discouragement". In general, little can be said concerning the sign of the left hand side of (4.8) and (4.9) even if we know the sign of the left hand side of (4.12).

However, if there is no systematic deviation between compensated demand effects across consumers, (4.12) can be utilized. Assume that all consumers have identical compensated demand derivatives (at the optimum). Inserting (4.12) in (4.8) (4.9) gives the result that authoritarian redistribution-in-kind for commodity ℓ is certainly desirable when

(4.14) $\text{cov}\ (\beta_i, x_{i\ell}) < 0$

and rationing is desirable when the inequality is reversed. As it is to be expected that, with an egalitarian social welfare function, λ_i will be lower for individuals with higher income, there is some presumption from (4.13) that, "normally", β^i will also fall with income. Thus (4.14) says that individuals should be forced to buy more of goods that are consumed in greater amount by the rich. With optimal taxation, it would actually be the case that necessities should be rationed and luxuries redistributed in kind. However our argument stresses that this conclusion, although not "unreasonable", is not fully correct : first, individual and aggregate indices of discouragement may be at variance and second, it is not necessarily true that optimal taxes lead to high taxes for luxuries and subsidies for necessities[16].

V. EXTENSIONS

V.A. The case where opportunity costs derive from a social welfare function.

The assumption (A2) on social opportunity costs is undoubtedly strong and although it is valid in a class of models and situations of economic interest it may fail in many other interesting situations. Here we will introduce a different assumption (A'2) which rests upon a less demanding concept of social opportunity costs.

./.

Suppose that there is some exogeneous social welfare function W which is used to evaluate the state of the economy (A2) then becomes

(A'2) : In the initial situation, there is a vector $\rho^o >> 0$ such that given any change ΔZ in the scarcity constraints, the Government can choose a change Δu in policy tools such that $\Delta W \geq \rho^o.\Delta Z + \sigma(\Delta Z)$

where $\dfrac{\sigma(\Delta Z)}{\|\Delta Z\|}$ tends to zero with ΔZ.

With this assumption, a lemma similar to lemmas 1-2 subsection II.1 can be proved.

Lemma 3. If (A1), (A'2), (A3), (A4) are satisfied and if there exists a policy tool, with parameter s_o that acts only on h, then a *welfare improvement* with respect to W is possible if

$$q^o_h.\left(\frac{\partial x_h}{\partial s_o}\right) \geq 0 \qquad \text{and} \qquad \rho^o.\left(\frac{\partial x_h}{\partial s_o}\right) < 0$$

The proof is a variant of the proof of lemma 1 above, which is left to the reader.

It is then straightforward to check that the conclusions of theorem I can be adapted to this new set of assumptions.

Theorem 4. Suppose that (B) holds and consider an initial situation where (A1), (A'2), (A3), (A4) are satisfied and when consumer h faces no quantity control initially. Define the vector of fictitious taxes $t^o = q^o - \rho^o$ as above.

If $$\sum_1^n t^o_\ell \left(\frac{\partial D^c_{hn}}{\partial q_\ell}\right)_{(o)} < 0 \qquad \text{(resp. } > 0)$$

(or if the commodity n has the properties assumed in theorem 2), then a *welfare improvement*(17) is achievable through the implementation of a (small) positive quota (resp. negative) of commodity n.

(A'2) is likely to hold if the initial situation is a social optimum with respect to W : in such a situation, if ρ^o is the vector of Lagrange multipliers associated with the scarcity constraints then it is a basic result of programming that, in general, ρ^o is the derivative of the objective function with respect to a vector of perturbations of the right

./.

hand side of the scarcity constraints (see for example Varian [1978]. When this is exactly true, we will say that we are in a regular optimum.

Definition. The initial situation is a regular welfare optimum, if

a) it is the maximum of the social welfare function W (under constraints excluding the use of quotas policies)

b) there exists a vector ρ^o of (pseudo) Lagrange multipliers with the following property : $\frac{\partial W}{\partial \omega} = \rho^o$ where $\frac{\partial W}{\partial \omega}$ is the derivative of the objective function as a function of the total initial endowments ω.

At this stage, it should be mentioned that the welfare optima found in second best problems are "likely" to be regular if enough smoothness assumptions are introduced in the model(18). Clearly, when the initial situation is a regular optimum, (A'2) is satisfied with the vector of pseudo Lagrange multipliers. It follows easily that :

Theorem 5.

Theorem 4 is true if (A'2) is replaced by : the initial situation is a regular welfare optimum (associated with the vector of pseudo Lagrange multipliers ρ^o).

In particular our analysis of anonymous quotas can straightforwardly be adapted.

Theorem 6.

Theorem 3 remains true if the initial situation is a regular welfare optimum, if ρ^o is the vector of pseudo Lagrange multipliers which are associated with it, and if Pareto improvement is replaced by welfare improvement.

The above theorems are relevant for the understanding of the usefulness of quantity controls when all other policy tools have been used. Hence the extension of the previous results is of considerable economic relevance. However this extension has been obtained at the cost of a weakening of conclusions : Quotas policies no longer result in Pareto improvement but in welfare improvement. All the comments and discussions of the previous sections should be reconsidered with these remarks in mind.

./.

V.B. Relaxation of the assumptions on the initial situation.

The argument of section III, as well as the subsequent argument of subsection IV.1, required that the agents affected by rationing were unconstrained price takers in the initial situation. This assumption guaranteed that the quota policy (forced purchase and sale) had only a second order welfare effect on the consumer. We will relax this assumption here. Formally the assumptions of section III will be modified as follows

(B') a) $x_{i\ell}(u) \quad = D_{i\ell}(\phi_i(u),\ M_i(u)) \quad \forall\ i \neq t, \quad \forall\ \ell$

b) $x_{hn}(u,\ v) = \tilde{D}_{hn}(\tilde{q}(u),\ \phi_{hn}(u),\ M'_h(u)) + v$

c) $x_{h\ell}(u,\ v) = \overline{D}_{h\ell}(\tilde{q}(u),\ q_n(u),\ M_h(u),\ x_{hn}(u,\ v))$

We do not make any specific assumption on consumers $i \neq t$ who behave as if they were competitive with respect to some shadow price system ϕ_i and shadow income M_i, which both depend on u and not on v.

Consumer h is implicitely faced with the linear price system $(q(u) = \tilde{q}(u),\ q_n(u))$ but is already constrained on the market for commodity n for which its shadow price ϕ_{hn} differs from the market price q_n (and hence $M_h \neq M'_h$). Let us notice that formally it would not make any difference to consider that consumer h, as others, is faced with a shadow price system $\tilde{\phi}_h(u)$ (instead of $\tilde{q}(u)$).(19)

The quota policy acts as an additive perturbation of the demand for commodity n.

In such a framework personalized quotas have a significant welfare effect on consumer h and cannot be expected to be Pareto improving even with the strong assumptions of section III. However, a result of this type could be expected if this welfare effect were offset by adequate transfers. The existence of such transfers is postulated in *assumption* (B") where (a) of assumption (B') still hold and (b) and (c) are replaced by

(b) $x_{hn}(u,\ v',\ v) = D_{hn}(\tilde{q}(u),\ \phi_{hn}(u,\ v'),\ M'_h(u,\ v')) + v$

(c) $x_{h\ell}(u,\ v',\ v) = \overline{D}_{h\ell}[\tilde{q}(u),\ q_n(u),\ M_h(u) + v',\ x_{hn}(u,\ v,\ v')]$

v' is a real addition to income $M_h(u)$, and affect indirectly ϕ_{hn} and M' but not $\tilde{q}$. (ϕ_{hn} and M'_h are strictly speaking different functions

./.

than those introduced in (B'), but for simplicity we do not use a different notation).

Consider an initial situation with $v^o = v^{o'} = 0$. An *income compensated quota* is thus defined as a move v, v' from the initial situation. When v > 0 (resp. < 0), the quota is said positive (resp. negative).

Assume that (A1) - (A4) holds in the initial situation and consider putting

$$t^o_{h\ell} = q^o_\ell (u^o) - p^o_\ell \qquad \ell \neq n$$

$$t^o_{hn} = \phi^o_{hn}(u^o) - p^o_n$$

Using the results on the effect of rationing of section II.2, one sees that with lemma 2 one can transpose the argument of section III. It is left to the reader to obtain :

Theorem 7. Suppose that (B") holds and that in the initial situation A1-A4 holds. If

$$(4.15) \qquad \sum_{\ell=1}^{n} t^o_{h\ell} \left(\frac{\partial D^c_{hn}}{\partial q_\ell}\right)_{(o)} < 0 \qquad \text{(resp. } > 0)$$

(where $(o) = (\tilde{q}(u^o),\ \phi_{hn}(u^o),\ M'_h(u^o))$)

then, there exists a Pareto improving positive (resp. negative) income compensated quota.

This theorem is a generalization of theorem 1. It is left to the reader to give another version of this theorem where (A2) would be replaced by (A'2) so generalizing theorems 4, 5.

As a simple application of the above statement let us consider the case where agents face a set of real prices $q = (\tilde{q}, q_n)$ and where in the initial situation there are distortions only in the market for commodity n, $t^o_{h\ell} = 0$ ($\ell \neq n$). Clearly, the sign of the index of discouragement of equation (4.15) is the opposite of $t^o_n = \phi^o_{hn} - p^o_n$. Hence

Corollary. If (A1) - (A4) are satisfied and consumer h faces no distortions in markets other than for good n and

$$p^o_n < q^o_n \leq \phi^o_{hn} \qquad \text{(resp. } p^o_n > q^o_n \geq \phi^o_{hn})$$

./.

then a Pareto improvement is achievable by forcing h to consume more (less) of good n through the use of an income compensated quota.

VI. NONLINEAR TAXATION.

Both the conventional (linear) price system and anonymous quantity controls are special examples of nonlinear price systems where marginal prices faced depend upon quantity purchased. With this interpretation, rationing, for instance, is equivalent to where a nonlinear price is constant up to a point and then infinite.

With optimal nonlinear prices there can be no gain from introducing anonymous quantity controls. The analysis of earlier sections has demonstrated that simple nonlinear price systems often dominate linear systems. Here we show that the foregoing analysis can be used to throw light upon the general structure of fully nonlinear systems.

The analysis of the introduction of an anonymous quota into a nonlinear price (or tax) system is similar to the analysis of section IV which dealt with linear systems, i.e. an anonymous quota is visualized as a personalized quota on the individual purchasing the least amount of a good. This idea will be examined in the context of the two-commodity nonlinear income tax model of Mirlees (1971). It will be shown that the result which states that the lowest income individual should face a zero marginal tax rate on income follows from our analysis. This is the purpose of VI.A to look at the problem when there are a finite number of consumers; a formal proof for a continuum of agents is given in VI.B.

VI.A. A finite number of consumers.

A simplified version of the general model will be examined. There are two goods, a consumption good c and labour e. There is a constant returns to scale production technology and, by a suitable choice of units, it may be assumed that one unit of labour produces one unit of consumption. Under competitive conditions the pre-tax wage is unity so that e is also pre-tax income. If a (possibly linear) income tax T(.) is imposed then an individual with preferences representable by a utility function u(c, e) will be faced with the problem :

$$\max_{c,e} u(c, e) \quad \text{s.t.} \quad e \geq T(e) + c \tag{6.1}$$

Figure 6.1 shows the choice made by individual h who chooses to obtain the lowest income.

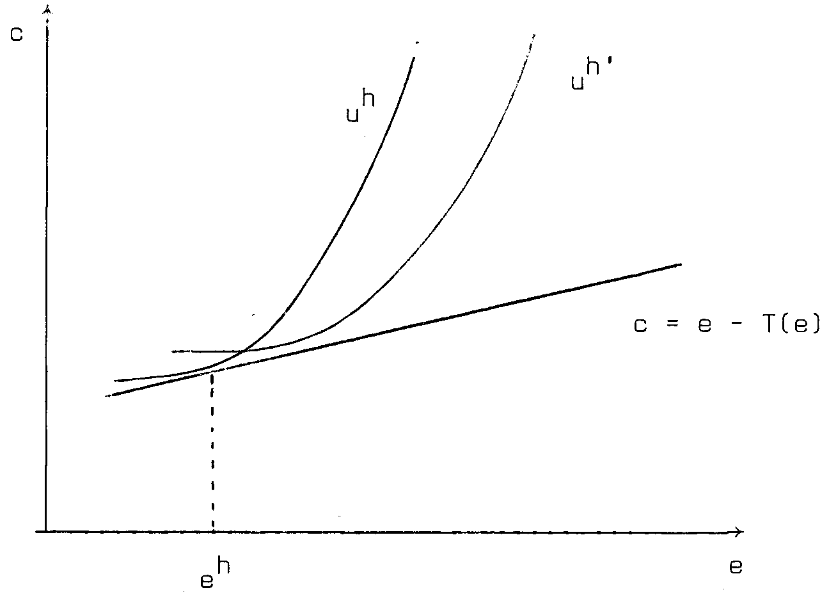

Figure 6.1.

In this model the (pre-tax) price vector (1, 1) is the vector of social opportunity costs in the sense of (A2) and, if T is differentiable; $(1, 1-T'(e^h))$ is the support price, in the sense of (A1), for h. Consider the imposition of a quota which forces h to move up the tax schedule. For a small change, this corresponds to an income compensated quota of subsection V.B. Applying the corollary to theorem 7, h should be forced to work harder if $1 - T'(e^h) < 1$ or, alternatively, if $T'(e^h) > 0$. Similarly, h should be forced to work less if $T'(e^h) < 0$.

If, a figure 6.1, h' is the consumer with the second lowest income then it is clear that the tax schedule can be modified to force h along the tax schedule. Thus, as any change in the *optimum* tax schedule cannot be desirable, the above analysis suggests that h must face a zero marginal income tax rate.

It is not difficult to show that identical results go through when h and h' both choose to obtain the some income level. However, the argument can break down when the tax schedule is not differentiable so that a situation like the one portayed in figure 6.2 arises (which is often the case under standard assumptions on preferences and productivities (see next footnote)).

./.

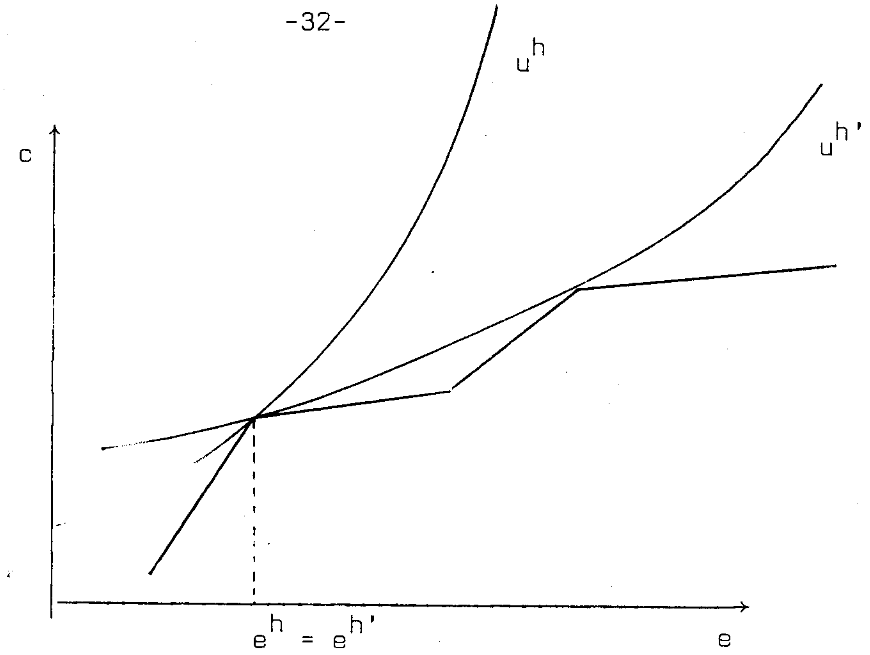

Figure 6.2

Consider forcing h to work harder. An income compensated quota to h will be strictly preferred by h' who, in situations based upon anonymity, cannot be prevented from enjoying benefits. The technique used to study anonymous quotas clearly needs to be amended to cope with this situation[20].

VI.B. A continuum of consumers

We now consider the amendments required when there is continuum of consumers. The essence of the different circumstances so created is that as a small quota impinges upon only an infinitesimal group the gain to all made possible by the quota is insufficient to compensate those who are "forced" away from their chosen position. The results we derive are of general applicability to the case where quotas are small but not necessarily infinitesimal.

Let h be a real valued index for the continuum of consumers and let f(h) be the density function of h. The lowest h value is 0 ($f(o) > 0$) and it is assumed that, at each point in (c, e) space, consumers with a higher h have indifference curves with less slope, i.e. high h consumers always choose to have a high income. Assuring differentiability, we prove :

Theorem 8. If $\frac{dT(e^o)}{de} = T'(e^o) \neq 0$ then there exists a (quantity constraining) tax schedule which increases the value of a utilitarian social welfare function $W = \int u^h f(h)\, dh$.

./.

Proof. The argument for $T' < 0$ is simple and, in fact, can be proven by amending the analysis of Sadka (1976). For $T' > 0$, consider moving to a new tax schedule $\tilde{T}_s$, $s \geq 0$, defined by

$$\begin{aligned} \tilde{T}_s(e) &= \infty & e &< e^o + s \\ \tilde{T}_s(e) &= T(e) & e &\geq e^o + s \end{aligned} \tag{6.2}$$

By prohibitive taxation, everybody is forced to move up the tax schedule and earn income $e^o + s$. If $h(s)$ is that consumer who originally earned income $e^o + s$ then consumers from $h = 0$ to $h = h(s)$ are the ones affected by the change in tax system. In this model, feasibility is ensured if tax revenue R does not fall. Under $\tilde{T}_s$, welfare and revenue are given by

$$\tilde{W}_s = \int_0^{h(s)} u^h(e^o + s - T(e^o + s),\ e^o + s)\ f(h)dh + \int_{h(s)}^{\infty} u^h(e^h - T(e^h),\ e^h)\ f(h)dh \tag{6.3}$$

$$\tilde{R}_s = \int_0^{h(s)} T(e^o + s)\ f(h)dh + \int_{h(s)}^{\infty} T(e^h)\ f(h)dh \tag{6.4}$$

where e^h is the income earned by h when the tax schedule is T (under $\tilde{T}_s$, everybody chooses to earn at least $e^o + s$).

With an atomless f, it is straightforward to check that (where W and R are "initial welfare and revenue, respectively)

$$\tilde{W}_o = W \quad \text{and} \quad \tilde{R}_o = R \tag{6.5}$$

$$\left.\frac{d\tilde{W}}{ds}\right|_{s=o} = 0 \quad \text{and} \quad \left.\frac{d\tilde{R}}{ds}\right|_{s=o} = 0 \tag{6.6}$$

$$\left.\frac{d^2\tilde{W}}{ds^2}\right|_{s=o} = 0 \quad \text{and} \quad \left.\frac{d^2\tilde{R}}{ds}\right|_{s=o} = T'(e^o)\ f(0) \left.\frac{dh}{ds}\right|_{s=o} \tag{6.7}$$

The zero second-order welfare effect in (6.7) follows because an infinitesimal group has utility affected only to a second order. (6.5) - (6.7) may be used to give a Taylor expression of (6.3) and (6.4).

$$\tilde{W}_s = W + \sigma(s^2) \tag{6.8}$$

./.

$$\tilde{R}_s = R + \frac{s^2}{2} T'(e^o) f(0) \left.\frac{dh}{ds}\right|_{s=o} + \sigma(s^2) \qquad (6.9)$$

where $\frac{\sigma(s^2)}{s^2} \to 0$ as $s^2 \to 0$.

As (6.9) is strictly positive and it is of an order of magnitude greater than (6.8), there is no difficulty in showing that by choosing a tax schedule $\overline{T}_s = \tilde{T}_s - ks^2$, it is posible, by suitable choice of k, to obtain $\overline{W}_s > W$ and $\overline{R}_s \geq R$ for small s. This proves the result. ||

It should be mentioned that we have not shown that the original tax schedule is Pareto dominated by another schedule and, as mentioned above, that is what differentiates theorem 8 from the other results of this paper. Of course, if the revenue gain pointed to (6.9) could be distributed to the consumers directly affected by the quota then, as this group is of size $\int_0^{h(s)} f(h)\, dh = s\, f(0) \left.\frac{dh}{ds}\right|_{s=o}$, a Pareto improvement would clearly be possible. The difficulty is that this selective transfer is not achievable through an anonymous procedure.

In conclusion, it has been shown that if the optimum income tax schedule is such that indifference curves are smoothly tangential to the budget set then the optimum marginal tax rate at the lower end of the schedule is zero. Although this result has been derived previously by application of the transversality conditions derived from the appropriate variational analysis used to solve for the optimum tax (cf Seade (1977)) piecemeal approach of this paper fully exposes the reasons for why this remarkable result should hold. With straightforward amendment, a similar construction can also be applied to show that the consumer who chooses to earn the largest income should face a zero marginal income tax rate.

VII. CONCLUSION.

The general purpose of this paper has been to analyse and determine rules for desirable changes in an undistinguished suboptimal world. With decentralization through a price system it is usually possible to use prices as the value placed upon goods by the agents in the economy. Although the rules of this paper have been presented in terms of social opportunity

./.

costs and support prices, the exact assumptions concerning the existence of such entities have been carefully laid out. For, there is no general guarantee that implicit prices with the desired properties can be defined.

The more specific purpose has been to move away from the study of tools that act through the price system and, instead, to investigate the possible role of quantity controls. As every undergraduate is taught, quantity controls like rationing and redistribution-in-kind are Pareto inefficient; however, such a result is not relevant when the initial situation is, itself, Pareto inefficient. The results of this paper strongly support the use of quantity controls in an economy which is away from its first-best and it was shown that even in a world of optimal commodity taxation, quantity controls could generally be important. For unlike anonymous taxes, anonymous quantity controls act on particular subset of consumers and, furthermore, small quotas or rations act in an income compensated way upon agents. With many groups in society having veto power, this latter feature implies that quantity controls may be one of the few tools available with any real potence.

o

o o

(1) This assumption could be precisely stated :

(a2) There exists a vector $\rho^{o} \in R^{\ell}$ which has the following property :

given any exogenous change in aggregate net demand ΔZ with the property that :

$\rho^{o}.\Delta Z \leq g(\Delta Z)$

for some function g where $\frac{g(\Delta Z)}{\|\Delta Z\|} \to 0$ as $\|\Delta Z\| \to 0$

there is a change Δu in the policy tools available to the Government (quantity control excluded) such that the corresponding changes in consumption Δx_i and the change in production Δy satisfy

$$\sum_i \Delta x_i + \Delta y = \Delta Z$$

$$q_i^{o}.\Delta x_i \geq f'_i(x_i) \quad \text{where } f'_i \text{ is such that } \frac{f'_i(\Delta x_i)}{\|\Delta x_i\|} \to 0$$

It is left to the reader to see the modifications which should be brought into the proof of theorem 1 in order to take into account this assumption instead of (A.2). (a2) is weaker than (A2) but it still requires that enough policy tools be available to the Government.

(2) In fact this assumption does not make any problem with the formulation of the quota policy adopted here where v is "forced consumption" and not a fixed quota. In this latter case, there is a discontinuity in the derivatives of demand with respect to prices at the point where the quantity constraint binds. This is a source of difficulty in the study of anonymous quotas which are considered in section III.

(3) (A4) is similar to the assumption used by Diamond and Mirrlees (1971) to show that in a world of optimal taxation, aggregate productive efficiency is desirable.

(4) With regard to the likelihood of the existence of such a tool it may be noted that if ρ^{o} is directly proportional to q_h^{o} then the analysis can never apply whereas, when this is not the case, there always exists a change in h's consumption which, if it could be induced by a policy parameter change, would lead to a social improvement. Notice, however, that it will not in general be possible to satisfy (2.6) by adopting policies which act to confiscate or hand-out goods. Finally, it may be noted that, trivially, the analysis continues to apply, in a revised form, when the inequalities in (2.6) are reversed.

(5) This result can be understood in terms of number of instruments versus number of targets. In the model as presently constructed, there are two targets : individual h's utility and resources which can be used to make everybody better-off. (2.6) shows that it is almost always the case that two instruments will be sufficient to bring about an improvement. Notice again that, for an improvement to be possible, it is necessary that ρ^{o} and q_h^{o} are not directly proportional.
Before proceeding further, it may be noted that there are straightforward many-person generalizations of the results that have been obtained. In particular, if $m + 1$ policy tools act upon m individuals then, extending lemma 2 , a Pareto improvement will generally be achievable. As the policy tool for which there is universal agreement (assumption (A.4)) is only needed to make individuals who are not directly affected by policy changes better-off, such an assumption is not in general required when m + 1 policy tools act upon all m individuals in the economy.

(6) Although the analysis of constrained demand around an initial situation which is unconstrained has already been explored in the past (see for example[1950];the analysis of a more general situation only seems to have been undertaken recently. Related results in various directions and with different methods (and apparently independently) have been recently obtained by Drèze[1977],Guesnerie[1978] Laroque[1978],Neary-Roberts [1980]. Slutsky[1980].The most systematic approach is that of Neary-Roberts [1980].

(7) In the following when $x \in \mathbb{R}^n$, $x = (\tilde{x}, x_n)$ where $\tilde{x}$ is a truncated vector of $\mathbb{R}^{n-1}$ and x_n is the n^{th} coordinate of x.

(8) See Neary-Roberts [1980] for a rigorous argument.

(9) See Guesnerie [1978] for more details.

(10) Suppose that we normalize prices such that commodity ℓ be the numeraire. ℓ is taxed as highly as ℓ' in the sense of the above definition if and only if the tax on ℓ' is smaller or equal than zero when ℓ is the numeraire.

(11) Although there are some differences in the underlying framework.

(12) Or equivalently with identical consumers.

(13) Since there would be no informationnal obstacles to the attainment of a first best optimum.

(14) For a further discussion of the desirability of aggregate productive efficiency, see Diamond and Mirrlees (1971).

(15) The reader should realize that there is a difficulty in this proof because the function Ξ_i has partial derivatives (with respect to q for examples) which are discontinuous in w, when the general analysis supposes continously differentiable reaction functions. The solution given here rests on the proof that there is (locally) a one to one correspondence between the so called positive and negative quotas of section III and the tickets allocations. The proof given in Guesnerie [1978] for the part a of the theorem is slightly more complicated but faces directly the difficulty created by the discontinuity of partial derivatives.

(16) It is known that more precise conclusions can be obtained in a two class economy. For example Mirrlees has shown [1976] that if the total amount of taxes paid by the rich exceeds the total amount of taxes paid by the poor, then it is actually true that the left hand side of (4.13) is negative (resp. positive) for commodities for which the rich (resp. the poor) has the highest consumption. (This condition on tax receipts is likely to be obtained when there is a strong social priority for the poor). The homogeneity of the signs of indices of discouragements is obtained when individuals have separable utility functions for the most taxed good. The interested reader will reestablish these assertions and give a precise statement on the desirability of redistribution in kind for this "most taxed" commodity.

(17) With respect to the social welfare function W introduced in assumption (A'2).

(18) The existence of "generalized" Lagrange multipliers is quite general in second best models (see Guesnerie [1979]).However for the equality $\frac{\partial W}{\partial \omega} = \rho^o$ to be true (which is "almost always" the case in convex programming) we need
i) that there is no discontinuity in the optimal solution for small perturbations of the right hand sides of scarcity constraints
ii) that the reactions functions introduced in the model be sufficiently smooth.

./.

(19) However this may not be a very significant extension; when the choice set of the consumer is not a linear manifold, $\tilde{\phi}_h$ should be taken both a function of u and v.

(20) The fact that it may be optimal for these to be no smooth tangency between the tax schedule and the lowest income consumer's indifference curve should be recognised. As Guesnerie and Seade [1980] have shown, this is almost always the case when finite populations are considered. Even with a continuum of individuals, Mirrlees [1971] has suggested that it will often be desirable for some individuals not to work (a corner solution) and, in Roberts [1979] , there are example of nonlinear pricing schedules where consumers at the lower end of the price schedule are bunched together. Seade [1977] , who considers the zero marginal tax argument in detail, assumes smooth tangencies throughout.

BIBLIOGRAPHIE

BENASSY J.P. (1975)
"Neo keynesian disequilibrium theory in a monetary economy" Review of Economic Studies.

DIAMOND P. (1975)
"A many person Ramsey tax rule" Journal of Public Economics 4, 335-342.

DIAMOND P. - MIRRLEES J. (1971)
"Optimal Taxation an Public Production I and II" American Economic Review 6, 18-27.

DIXIT A. (1971)
"Welfare effects of tax and price changes"
Journal of Public Economics 4, 103-123.

DREZE J. (1977)
"Demand Theory under quantity rationing" Core D.P.

GUESNERIE R. (1978)
"La gratuité outil de politique économique ?"
CEPREMAP discussion paper 7807(March 1978).

GUESNERIE R. (1979)
"General statements on second best Pareto optimality"
Journal of Mathematical Economics 6, 169-194.

GUESNERIE R. - SEADE J. (1980)
"Nonlinear pricing in a finite economy" mimeo.

LAROQUE G. (1978) "The fixed equilibria : some results in local comparative statics" Econometrica 46, 1127-1155?

MIRRLEES J. (1971) "An exploration in the theory of optimum income taxation" Review of Economic Studies 38, 175-208.

MIRRLEES J. (1976)
"Optimal tax theory, a synthesis"
Journal of Public Economics.6, 327-359.

MIRRLEES J. (1975)
"Optimal taxation in a two class economy"
Journal of Public Economics 4, 27-34.

NEARY J. - ROBERTS K.(1980)
"The theory of household behaviour under rationing"
European Economic Review ( ), 25-42.

ROBERTS K. (1978)
"The treatment of the poor under tax - Transfer schemes"
M.I.T.

ROBERTS K. (1979)
"The welfare considerations of nonlinear pricing"
Economic Journal.

SADKA E. (1976) "On income distribution, incentives effects and optimal income taxation" Review of Economic Studies 43, 261-267.

SEADE J. (1977) "On the shape of optimal tax schedules" Journal of Public Economics 7, 203-236.

TOBIN J. - HOUTHAKER H. (1950) "The effects of Rationing on Demand Elasticities" Review of Economic Studies 18, 140-153.

VARIAN H. (1978) Microeconomic Analysis, Norton.

°

° °