N° 7917

ONYMOUS CONSISTENT VOTING SYSTEMS *

by Roy GARDNER

C E P R E M A P

and

Iowa State University

* The author wishes to thank Douglas Blair, Don Brown, and Allan Feldman for helpful suggestions, and the latter for permission to cite some unpublished results. Preliminary versions of this paper were presented at the 1979 meetings at the Public Choice Society, and at the Center for Mathematical Studies in Economics and Management Science, Northwestern University. Research support by CNRS and the Iowa State University Foundation is gratefully acknowledged.

Abstract

This paper studies the strategic consistency of various onymous voting systems. The first result is the following extension of the Gibbard-Satterthwaite theorem: all stable, non-imposed/neutral social choice (set) functions are collegial polities. The converse of this result is not true, even if stability is replaced by exact and strong consistency in the sense of Peleg. The relationship between consistency and the distribution of voting power is examined. Finally, it is shown that the sincere outcome of a collegial polity is in the core.

I. Introduction

In practically all economic systems, certain economic decisions are made by voting. Modern interest in this question has enjoyed a great revival since the famous result of Arrow (1963): if a preference aggregation procedure has a universal domain and satisfies independence of irrelevant alternatives, Pareto optimality, and transitivity, then it is dictatorial. More recently, Brown (1975) has provided a major extension of Arrow's result: if a preference aggregation procedure satisfies the Arrow conditions, with transitivity replaced by acyclicity, then the procedure is a collegial polity. The difference between dictatorship and collegial polity consists in the fact that a dictator is decisive by himself, while in a collegial polity there is a set of voters (the collegium) which is part of every decisive coalition, but is not decisive by itself. Example of collegial polities are given by the Roman republic and the United Nations Security Council (pre-1965), with the collegium consisting of the tribunes in the former, the Great Powers in the latter.

A related literature now exists on the question of the extent to which the outcomes of voting depends upon strategic considerations. For example, let there be three individuals $\{1, 2, 3\}$ and four outcomes being voted upon $\{x, y, z, w\}$. The voting method being used is Borda's rule: four points for first place, three points for second place, two points for third place, one point for fourth place, the alternative with the most points winning. Suppose the individual preferences are xzwy, yxzw, yxzw respectively.[1] Then if all vote honestly the winner is x, with ten points, while y is second with nine points. However, individuals 2 and 3 can guarantee that y wins, by voting yzwx, yxwz instead. Thus, it is in the best interests of individuals 2 and 3 to manipulate the election by misrepresenting their preferences.[2]

There exists a remarkable relationship between the existence of dictators and immunity of voting systems to manipulation. It has been independently shown by Gibbard (1973) and Satterthwaite (1975) that if a voting system is not imposed and not manipulable by individuals acting alone, it is dictatorial.[3] In particular, Gibbard arrives at this result as consequence of Arrow's theorem; by first showing that a non-imposed nonmanipulable voting system must generate an Arrow social welfare function. An important feature of the Gibbard-Satterthwaite Theorem is that the voting system make unique choices--no ties are allowed. Kelly (1977) has shown that if this feature is relaxed, there do exist non-dictatorial voting systems which are non-manipulable also. The next section of this paper specifies the class of voting systems so generated. If a voting system is neutral, not imposed, allows ties, and cannot be manipulated by any coalition, it is a collegial polity. Just as Gibbard's result uses Arrow's theorem, this result relies on Brown's theorem.

Arrow's theorem has always been disturbing because of its distributional implications--the dictator has all the voting power. Anonymous voting systems, on the other hand, imply an even distribution of voting power.[4] Peleg (1978a) has shown that for anonymous voting systems which do not allow ties, the sincere outcome does not correspond to a strong equilibrium in the associated voting game, unless an (m - 1)/m majority is required to defeat an alternative. In the third section of this paper, a similar condition is found for collegial polities. Moreover, there exists a direct connection between the distribution of power and the possibility of manipulation. It is also shown in this section that if one adopts the notion of consistency as belonging to the core, then the sincere outcome of a collegial polity is again consistent. The implications of these results for further work are addressed in the conclusion.

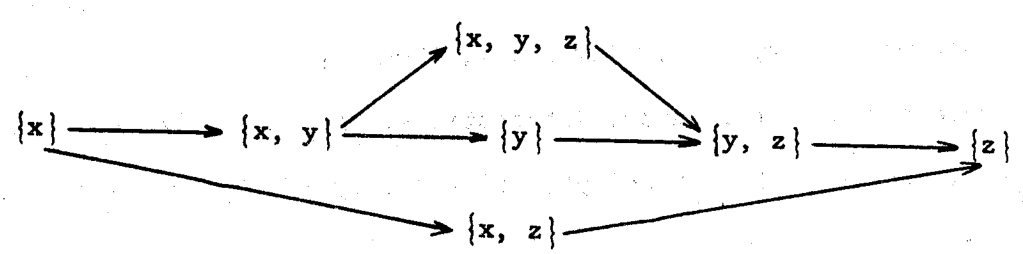
II. Stability and Collegial Polity

Let X be the set of social states, with typical members x, y, z. Let $N = \{1, 2, \ldots, n\}$ be the set of voters. $\mathcal{R}$ is the space of complete, reflexive, and transitive orderings of X; $\mathcal{P} \subset \mathcal{R}$ is the subspace of linear orderings. A social choice function is a mapping F, from $\mathcal{R}^n$ to X such that $F(a) \neq \phi$, $F(a) \subset X$ for any situation $a = (R_1, R_2, \ldots, R_n)$.

The strategic voting model views a social choice function as the outcome function of a cooperative game in normal form. Thus, each player i has as his strategy space $\mathcal{R}$, his true preferences being a point R_i^* in that space. In order for a voter to assess the results of different strategies, it is necessary to compare different choice sets. The set relation chosen to do this is drawn from Feldman (1979):

<u>Definition</u>. Let A and B be non-empty subsets of X, $A \not\subset B$, and R_i a preference ordering of X. Then $A \underline{R}_i B$ if for all x in $A - B$, y in $A \cap B$, and z in $B - A$, xP_iyP_iz.

This ordering represents an extension of Kelly's ordering (1977). For example a voter preferring x to y to z has the $\underline{R}$ relation,



the arrows denoting set preference.

Let $\underline{a}$ be a given situation, with $F(a) = B$. F is stable at $\underline{a}$ if there exists no coalition S, $\phi \neq S \subseteq N$, and situation $b \neq a$, such that

(i) $a_i = b_i$ for all i in N-S

(ii) $F(b) \underline{R}_i F(a)$ for all i in S.

If F is stable at every situation, F is stable.

F is exactly and strongly consistent (Peleg, 1978a) if there exists a mapping H: $R^n \to R^n$ such that

(i) H(a) is stable

(ii) $FoH(a) = F(a)$

for every situation a. Clearly if F is stable, F is exactly and strongly consistent: simply take H to be the identity mapping. An exactly and strongly consistent social choice function need not be stable; but, given the transformation H, stability is achieved.

A social choice function F is not imposed if for every non-empty subset of alternatives $B \subseteq X$, there exists a situation a such that $F(a) = B$.

Let $T : X \to X$ be a 1-1 transformation of the set of social states. Then a social choice function F is neutral if, at any situation a , $F(a) = T(F(T_a))$.

In addition to social choice functions, it is convenient to introduce a social preference relation P , again a binary relation on the set of social states. Some useful conditions on P are as follows :

Acyclicity : x_1Px_2 , x_2Px_3 , $x_{n-1}Px_n$ then $\sim (x_nPx_1)$.

Pareto optimality: xP_iy for all i, then xPy

Independence of Irrelevant Alternatives. Let a, a' be two situations. If xP_iy if and only if $xP_i'y$ and yP_ix if and only if $yP_i'x$ for all i, then xPy if and only if xP'y.

One usually thinks of P us avising from a group preference function f , i.e. P = f(a) for any situation a. Then given P , one can define the social choice function F such that

(1) $F(a) = \{x \text{ in } X : \text{there is no } y \text{ in } X \text{ such that } yPx\}$

which definition is meaningful as long as P is acyclic and X is finite.

A coalition S is decisive if, whenever all the members of S prefer x to y, then xPy. Pareto optimality thus means that the grand coalition is decisive. A prefilter η is a collection of subsets of N such that : (a) $N \in \eta$, (b) $\phi \notin \eta$, (c) $S_1 \in \eta$, $S_2 \in \eta$ then $S_1 \cap S_2$ is non-empty, (d) $S_1 \in \eta$, $S_1 \subset S_2$, then $S_2 \in \eta$.

A basic result connecting these various ideas is the following theorem due to Brown (1975).

Theorem (Brown). The sets decisive with respect to a group preference function satisfying the conditions of acyclicity, Pareto optimality, and independence of irrelevant alternatives form a prefilter.

The non-empty intersection of the sets in a prefilter one can identify as the collegium of collegial polity. In this intersection is itself a decisive set, one has an oligarchy. In particular, if the oligarchy consists of a single individual, one has dictatorship.

One can now assert the following result :

<u>Theorem 1</u>. Let $n \geq 2$, $m \geq 3$. Then all stable, non-imposed social choice functions are collegial polities.[5]

<u>Proof</u>. The idea of the proof is to show that a stable, non-imposed social choice function generates a social preference relation satisfying the hypothesis of Brown's theorem. The proof itself is modeled after that of Gibbard (1973).

The proof begins by restricting all situations to $\mathcal{P}^n$. This restriction is relaxed at the end. Let Q in $\mathcal{P}^n$ be fixed. Then $a^*(x, y)$ denotes that x and y have been moved to the top of each ordering in situation a, preserving the ordering between them if it is strict, while the rest of each ordering agrees with Q. Ties between x and y are also broken by Q.

Define the social preference relations relative to situation a, P(a) and I(a) by

(2) $xP(a)y$ if and only if $x \neq y$ & $\{x\} = F(a^*(x, y))$.

(3) $xI(a)y$ if and only if $x \neq y$ & $\{x, y\} = F(a^*(x, y))$.

These definitions are meaningful as long as F is neutral and non-imposed.[6]

Some properties of P(a) are now generated by a series of lemmas.

Lemma 1. Let $a = (R_1, R_2, \ldots, R_n)$, $b = (R_1', R_2', \ldots, R_n')$ be two situations such that for all i, xP_iy if and only if $xP_i'y$ and yP_ix if and only if $yP_i'x$. Then $xP(a)y$ if and only if $xP(b)y$.

Proof. By construction, $a^*(x, y) = b^*(x, y)$; hence, $F(a^*(x, y)) = F(b^*(x, y))$. Therefore, by (3) $xP(a)y$ if and only if $xP(b)y$.

Lemma 2. Given situation a and alternatives x, y, suppose a' is another situation such that

(i) yP_ix, then $R_i = R_i'$ for all i

(ii) xP_iy or yP_ix for all i

(iii) $\sim xP(a)y$.

Then $\{x\} \neq F(a')$.

Proof. Suppose on the contrary that $\{x\} = F(a')$. It is first shown that $\{x\} = F(a'^*(x, y))$. By construction, at situation $a'^*(x, y)$, xP_iz, yP_iz for all i and all z in $X - \{x, y\}$. It follows then that $\{z\} \not\subset B = F(a'^*(x, y))$. For if $\{z\} = B$, there exists by non-imposition a situation b at which $F(b) = \{x\}$, and so F is unstable at $a'^*(x, y)$. If $\{x, z\} = B$, then there exists a situation b at which $F(b) = \{x\}$. Since $\{x\}\ \underline{R}\ \{x, z\}$ for all individuals at $a'^*(x, y)$, F is again unstable there. A similar argument holds for $B = \{y, z\}$. Finally, if $B = \{x, y, z\}$, there exists a situation b at which $F(b) = \{x, y\}$ and $\{x, y\}\ \underline{R}\ \{x, y, z\}$ for all individuals at $a'^*(x, y)$, making F unstable. Therefore, B must be either $\{x\}$ or $\{x, y\}$ or $\{y\}$. Since $a'(x, y)$ agrees with a' on the pair (x, y), Lemma 1 requires that $\{x\} = B$.

Now consider the situation $a^*(x, y) = a''$. By lemma 1 and the argument just made, $F(a'') = \{y\}$ or $= \{x, y\}$. By construction, a'' and $a'^*(x, y)$ have $R_i' = R_i$ for all i preferring y to x. If $F(a'') = \{y\}$, then F is unstable at a'', since the agents preferring x to y have altered preferences to reach a'(x, y), and for those agents x $\underline{R}$ y. Likewise, if F(a") = $\{x, y\}$, F is unstable at a", since $\{x\}$ $\underline{R}$ $\{x, y\}$ for agents preferring x to y.

This contradiction proves that $\{x\} \neq F(a')$.

Corollary 1. If $xP_i y$ for all i in situation a, then xP(a)y.

Proof. There exists a situation a' at which F(a') = $\{x\}$. For any situation a satisfying the hypothesis, conditions (i) and (ii) of lemma 2 are satisfied, but the conclusion is contradicted. Therefore, $\sim\sim$ xP(a)y, that is, xP(a)y.

Corollary 2. $\sim xI_i y$ for all i and $\sim$ xP(a)y, then $\{x\} \neq F(a)$.

Proof. Conditions (ii) and (iii) of lemma 2 are satisfied. Let a' = a. Then condition (i) is satisfied. Hence. $F(a') = F(a) \neq \{x\}$.

Lemma 3. P(a) is acyclic.

It is sufficient to show that for $a' = a^*(x_1, x_2, \ldots, x_n)$, $x_1P(a')x_2$, $x_2P(a')x_3$, ..., $x_{n-1}P(a')x_n$ imply $\sim x_nP(a')x_1$.

We show first that F(a') must be either $\{x\}$ or $\{x,z\}$.

Let S = F(a') be any other subset of $\{x_1, x_2, \ldots, x_n\}$. If $x_1 \not\in S$, let $x_i \in S$ be the social state with the least index i. Consider the situation $a'' = a'^*(x_{i-1}, x_i)$. By lemma 1, $F(a'') = \{x_i\}$. Then by corollary 2 to lemma 2, $x_i\, P(a'')x_{i-1}$. But a" agrees with a' on (x_{i-1}, x_i), contradicting the hypothesis. Similarly, if $x_n \not\in S$, let $x_i \in S$ be the social state with the greatest index i, and consider the situation $a'' = a'^*(x_{i-1}, x_i)$ as before. Finally, if both $x_1 \in S$ and $x_n \in S$, pick $x_i \in S$, for any $1 < i < n$ and again consider the situation $a'' = a'^*(x_{i-1}, x_i)$.

Having thus exhausted all other cases, F(a') is either $\{x_1\}$ or $\{x_1, x_n\}$. Let a" = $a'^{*}(x_1, x_n)$. By lemma 1, F(a") = F(a'). If F(a") = $\{x_1\}$, then $x_1 P(a') x_n$. If F(a") = $\{x_1, x_n\}$, then $x_1 I(a') x_n$. In either case, $\sim x_n P(a') x_1$. Hence, P(a) is acyclic.

By lemmas 1-3, the group preference function induced by F satisfies the hypothesis of Brown's theorem, and thus the decisive sets with respect to F form a prefilter.

Finally, we relax the restriction to situations in $\mathcal{P}^n$. Here the proof follows Gärdenfors (1977).

Let S be decisive for F restricted to $\mathcal{P}^n$. We show that S is decisive on all of $\mathcal{R}^n$; i.e., that for any situation a = $(R_1, R_2, \ldots, R_n)$, F(a) is the set of R_S- maximal elements. Given a, let B denote the set of R_S- maximal elements. Let b = $(P'_1, P'_2, \ldots, P'_n)$ be a situation in $\mathcal{P}^n$ such that for all alternatives $x \in B$ and $y \in X-B$, one has xP_iy for $i \in S$ and yP_i for $i \in N-S$; and furthermore that F(b) = B. If F(a) = B, we are done, so suppose that F(a) = $B_1 \neq B$. Now consider $B_2 = F(P'_S, R_{N-S})$. If $B_2 \neq B$, then F is unstable at situation b, since by construction $B_2 \underline{R} B$ for all agents in N-S. Therefore $B_2 = B$. But then F is unstable at situation a, since by construction $B \underline{R} B_1$ for all agents in S. Therefore, $B_1 = B$ and S is decisive for all of $\mathcal{R}^n$, which completes the proof.

If one requires in addition that the social choice function in this theorem be decisive, one can then show full transitivity of the social preference relation, not just acyclicity. This case then implies that the collegial polity is dictatorship. This is the sense in which theorem 1 generalizes the Gibbard-Satterthwaite Theorem.

III. Strategic Consistency of Collegial Polities

This section reverses the argument of the last section and asks to what extent, given a collegial polity, one can expect strategic consistency of outcomes, in which the sincere outcome corresponds to an equilibrium in the associated voting game. Cooperative games in normal form will be considered first, with the equilibrium concept being strong equilibrium.

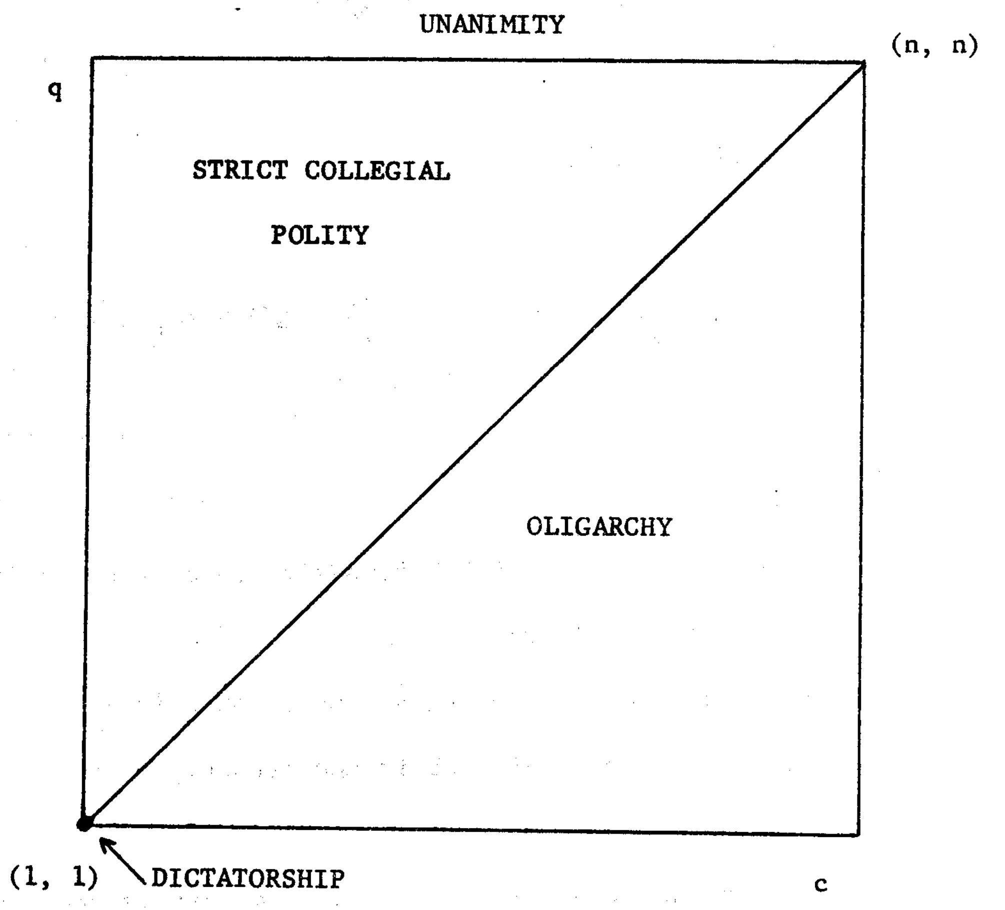
The following parameterization of collegial polities will prove useful.[7] Let $C \subset N$ be the collegium, with $\mu(C)$, the number of members of C, equal to c. Let q be the quota required for a coalition to be decisive. Then the decisive coalitions of the collegial polity are precisely the collection $\eta = \{S: \ S \subset N, \ C \subset S, \ \mu(S) \geq q\}$. Given n voters, the various possible collegial polities are described by the pair (c, q), where $1 \leq c, q \leq n$. For example, in the Roman republic, there were three hundred senators and ten tribunes, with a simple majority of senators and all the tribunes being needed for a decisive coalition; hence, $n = 310$, $q = 160$, and $c = 10$. For the United Nations Security Council (pre-1965), the corresponding parameter values are $n = 11$, $q = 7$, and $c = 5$. In terms of the parameterization, $c < q < n$ means strict collegial polity; $c \geq q$ represents oligarchy (in particular, unanimity when $q = n$); and $c = q = 1$ means dictatorship. These various possibilities are depicted in figure 1.

One can now state the following

<u>Theorem 2</u>. Let there be n voters choosing among m alternatives, within an (c, q) collegial polity. Then the sincere outcome of the collegial polity is not necessarily stably if

(4) $\quad c \geq q(m - 1) - n(m - 2).$

Figure 1. Varieties of Collegial Polity



Proof. Label the alternatives $\{x_1, x_2, \ldots, x_m\}$. Let $(C, S_1, S_2, \ldots, S_{m-1})$ be a partition of n with the following preferences:

for all i in C, $x_m\ x_1\ x_2 \ldots x_{m-1}$

for all $i \in S_1$, $x_1\ x_2 \ldots x_m$

for all $i \in S_{m-1}$ $x_{m-1}\ x_m\ x_1 \ldots x_{m-2}$.

Denote this situation a.

Suppose $\mu(C \cup S_2 \cup \ldots S_{m-1}) \geq q$; then $x_m P x_1$. Similarly, for any alternative x_j, $j \neq m$,

(5) $\mu(C \cup S_k) \geq q$ implies $x_m P x_j$.
$k \neq j$

Suppose (5) holds for all x_j, $j \neq m$. Let $\mu(S_i) = \mu(S_j) = \mu$ for any i, j. Then (5) implies

$$c + (m - 2)\mu \geq q.$$

At the same time, $c + (m - 1)\mu = n$, by the construction of the partition. Substituting for μ, one has condition (4).

Now the sincere outcome of the collegial polity at a is x_m. We shall show that the collegial polity is not exactly and strongly consistent at a, hence unstable.

Assume per absurdum that the collegial polity is exactly and strongly consistent at a. Then there exists a mapping H: $R^n \to R^n$ such that H(a) is a strong equilibrium and $F \circ H(a) = \{x_m\}$. For the social choice to be $\{x_m\}$ alone, all the members of C must rank x_m first. Now, wherever H maps the strategies of $S_1 \cup S_2 \ldots \cup S_m$, it must be the case that $\{x_m\}$ is chosen. Let $S_1 \cup S_2 \ldots \cup S_m$ all announce the preferences of S_{m-1}, $x_{m-1}\ x_m\ x_1 \ldots x_{m-2}$. Then the social choice is $\{x_{m-1}, x_m\}$ and $\{x_{m-1}, x_m\}\ \underline{R}_i\ \{x_m\}$ for all i in $S_1 \cup S_2 \ldots \cup S_m$, contradicting the hypothesis of stability.

Applying formula (4) to the Roman republic, one has that for all $m \geq 3$, the voting system is open to strategic manipulation on the part of coalitions. It is an open question whether this low threshold value of m helps to explain the turbulent politics of what republic's last century of existence. The corresponding threshold value for the pre-1965 United Nations Security Council is $m = 5$. Denoting this threshold value of m by m*, one has that $m^* = (2n - c - q)/(n - q)$. For fixed c and n, then, increases in the quota q increase m*; the higher the quota, the higher the threshold for instability.

An important feature of the proof of Theorem 2 is that ties are allowed. Peleg (1978b) has shown that if a collegial polity makes unique choices, then there does correspond to it an exactly and strongly consistent social choice function. Even without unique choices, one can show that dictatorship, oligarchy, and strict collegial polities not satisfying (4) are exactly and strongly consistent.[8]

There is an interesting relationship between formula (4) and the distribution of power in a collegial polity, where voting power is measured by the Shapley-Shubik (1954) index. This index computes the probability that a voter is pivotal in a randomly drawn coalition. A voter is pivotal if the coalition is decisive if it includes him, but not if it excludes him. Intuitively, a member of the collegium should have more voting power than an outside voter. Denoting the Shapley-Shubik index for voter i by $\phi(i)$, this intuition is borne out by the following results for the (c, q) collegial polities represented in figure 1:

Dictatorship $\quad \phi(i) = \begin{cases} 1 \text{ if i is dictator} \\ 0 \text{ otherwise} \end{cases}$

Oligarchy $\quad \phi(i) = \begin{cases} 1/c \text{ if i is an oligarchy} \\ 0 \text{ otherwise} \end{cases}$

Strict Collegial Polity[9]

$$\phi(i) = \begin{cases} \frac{1}{c} - \frac{(q-1)!(n-c)!}{n!(q-c-1)!c} & \text{if } i \text{ is in the collegium} \\ \frac{(q-1)!(n-c-1)!}{n!(q-c-1)!} & \text{otherwise} \end{cases}$$

Unanimity $\phi(i) = 1/n$ for all i.

In these expressions, total voting power is normalized to one. Applying these measures to the Roman republic, one has a voting power of .09675 for a tribune, and .00011 for a senator. Thus, a tribune is about 88 times as powerful as an individual senator. The corresponding measures for the pre-1965 United Nations Security Council are .19740 for a country with veto power, and .00216 for a country without such power. Here, a Great Power is about 91 times as powerful as a non-Great Power member of the Security Council.

For strict collegial polity, with c and n fixed, it is clear that a voter outside the collegium becomes more powerful as the quota rises, approaching a maximum of 1/n in the case of unanimity. Also, the distribution of power becomes unambiguously more even as the quota rises. Figure 2 shows the Lorenz curves of the distribution of power for the c = 1, n = 5 collegial polity as the quota q varies from 1 to 5, 1 being dictatorship and 5 being unanimity.

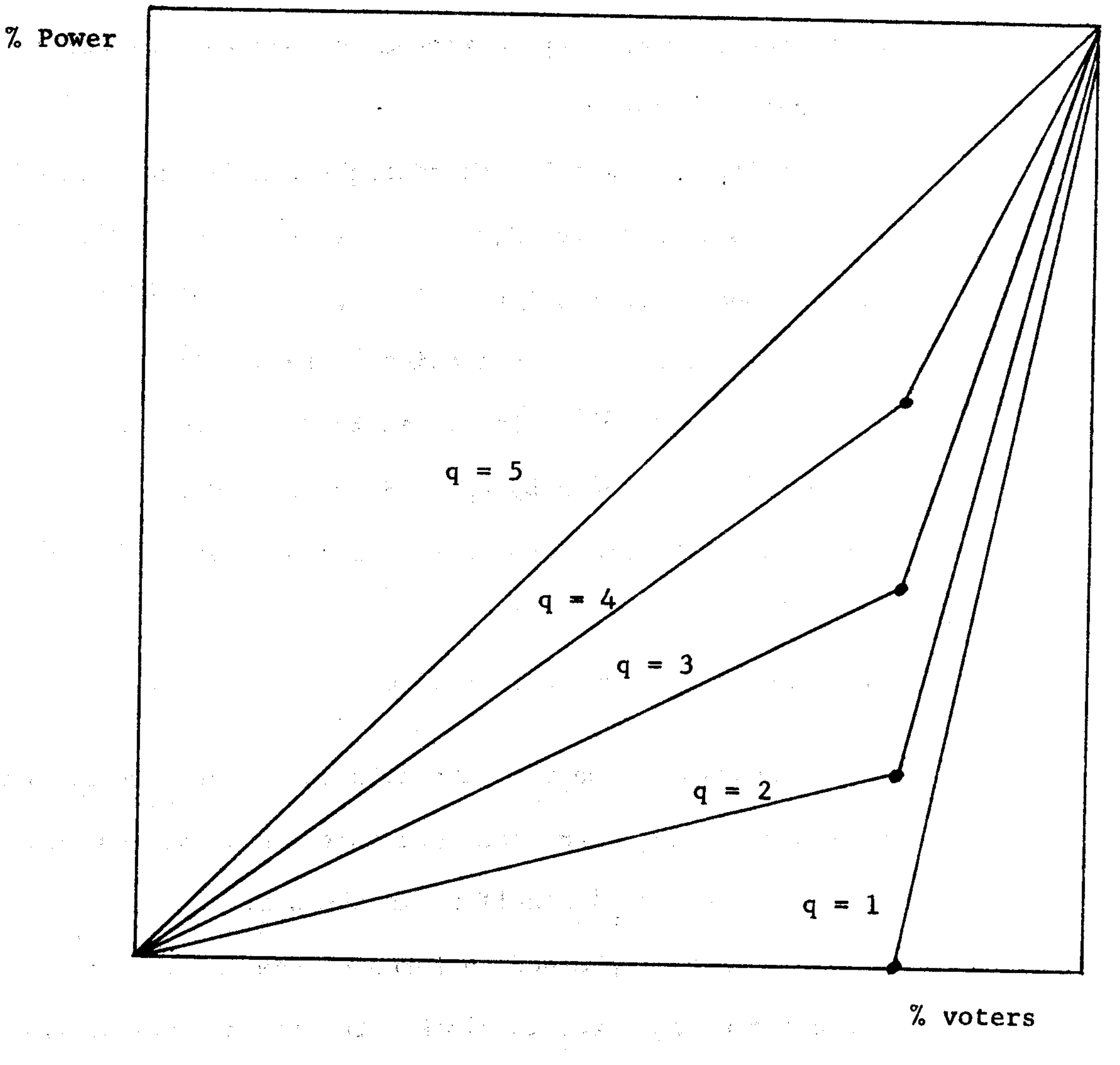
Returning to formula (4), it is now apparent that for fixed (c, n, m), the larger the quota q the more likely it is that

$$q(m - 1) - n(m - 2) > c.$$

Thus, the more even the distribution of power, the more likely it is that the instability noted in Theorem 2 is avoided.

It is common in economic applications to consider cooperative games, not only in normal form, but also in characteristic function form, especially from

Figure 2. Lorenz Curves for Voting Power, n = 5, c = 1.



the standpoint of the core. For example, a competitive equilibrium cannot be improved upon by any coalition, and thus is in the core of the associated characteristic function form game. A like phenomenon occurs for collegial polities.

Theorem 3. The sincere outcome of a (c, q) collegial polity is in the core.

Proof. We distinguish two cases, according to whether or not the collegial polity is exactly and strongly consistent.

Suppose the collegial polity is exactly and strongly consistent. Then, as in Peleg (1978b), given situation $\underline{a}$, let $F(a) = B$. If B is not in the core, then there exists a decisive coalition $S \in \eta$ and $B' \subset X$, such that $B' \underline{R}_i B$ for all i in S. Let $b = H(a)$ be a strong equilibrium corresponding to situation $\underline{a}$, such that $F(b) = B$. For all i in S, let $B' \underline{R}_i \{z\}$ for all z in $X - B'$. Denote this vector of preferences by b'_S. Since S is winning, $F(b_{N-S}, b'_S) = B'$, contradicting the fact that b is a strong equilibrium point.

A similar argument works in the second case also.

As an illustration of Theorem 3, consider the situation described in the proof of Theorem 2. Here one can show that the core consists of all subsets of X containing x_m. In particular, $\{x_m\}$ itself is in the core.

This result on the core can be explained in terms of the difference between the power to enforce and the power to block. Coalitions outside the collegium, by manipulating, can allow more alternatives into the sincere choice set, but cannot delete alternatives already there. The sincere choice set already includes all the outcomes that can be enforced. Therefore, the core is, if anything, more extensive than the sincere choice set.

IV. Conclusion

This paper has considered the extent to which onymous voting systems--those in which the distribution of voting power is uneven--are immune to strategic manipulation of preferences. All social choice functions which are immune to such manipulation are collegial polities. Moreover, if the quota is sufficiently small (the distribution of power sufficiently uneven), a strict collegial polity is open to manipulation of preferences by coalitions. However, if the voting system generated by a collegial polity is viewed as a cooperative game in characteristic function form, that game has a non-empty core, including the sincere outcome.

There are several directions in which these results could stimulate further research. So far, no structure has been imposed upon the set of alternatives, save that it be finite. It would be interesting to see what is true when there are several dimensions of alternatives, as in income distribution models, or a vector space of alternatives, as is usually assumed in electoral competition models. It is known for example (Aumann and Kurz (1976)) that for anonymous voting systems, majority rule leads to a definite tax system. It would be nice to know what sort of taxation to expect in various onymous voting systems. The author intends to pursue this topic in the sequel.

FOOTNOTES

1. The individual preference relation, individual i prefers x to y, will be written xP_iy. When no confusion will result, the P_i will be supressed.
2. This is not an isolated example, but rather a pervasive phenomenon in voting systems like Borda's rule. See Gardner (1977) for details.
3. For the case of manipulation by coalitions of voters, this result has been established by Batteau and Blin (1976).
4. Voting power is being measured as in Shapley-Shubik (1954).
5. This result does not contradict that of Gibbard (1977), where a much less demanding notion of manipulability than R is used. A similar result can be found in Ferejohn-Grether (1979), who, however, deal with a substantially different framework.
6. I am grateful to Allan Feldman for this observation.
7. This is not the most general parameterization possible.
8. For the unanimity rule, thus by analogy for any oligarchy, this result is alluded to by Kelly (1977) and proved by Feldman (1979).
9. This formula is due to Douglas Blair.

REFERENCES

Arrow, Kenneth J. Social Choice and Individual Values. New Haven: Yale University Press, 1963.

Aumann, Robert J. and Mordecai Kurz. "Power and Taxes." Econometrica 45 (1977), 1137-1161.

Batteau, Pierre and Jean Marie Blin. "On Group Manipulability of Voting Procedures." Discussion Paper #242, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1976.

Brown, Donald J. "Aggregation of Preferences." Quarterly Journal of Economics, 89 (1975), 456-469.

Dutta, Bhaskar and Prasanta K. Pattanaik. "On Nicely Consistent Voting Systems." Econometrica 46 (1978), 163-170.

Feldman, Allan. "Manipulation and the Pareto Rule." Working Paper No. 77-24, Department of Economics, Brown University, 1979.

Ferejohn, John A. and David M. Grether. "Stable Voting Procedures and the Theory of Social Choice." Social Science Working Paper 210, California Institute of Technology, 1979.

Gärdenfors, Peter. "A Concise Proof of Theorem on Manipulation of Social Choice Functions." Public Choice 32 (1977), 137-142.

__________. "On Definitions of Manipulation of Social Choice Functions," in Aggregation and Revelation of Preferences (ed. J. J. Laffont). Amsterdam: North-Holland, 1978.

Gardner, Roy. "The Borda Game." Public Choice 30 (1977), 43-50.

Gibbard, Allan. "Manipulation of Schemes that Mix Voting with Chance." Econometrica 45 (1977), 665-680.

__________. "Manipulation of Voting Schemes: A General Result." Econometrica 41 (1973), 587-601.

Guilbaud, G. Th. "Les théories de l'intérêt général et le problème logique de l'agrégation." Économie appliquée 5 (1952), 501-584.

Hansson, Bengt. "The Existence of Group Preference Functions." Public Choice 28 (1976), 89-98.

Kelly, Jerry S. "Strategy-Proofness and Social Choice Functions without Singlevaluedness." Econometrica 45 (1977), 439-446.

Peleg, Bezalel. "Consistent Voting Systems." Econometrica 46 (1978a), 153-162.

__________. "Representations of Simple Games by Social Choice Functions." International Journal of Game Theory 7 (1978b), 81-94.

Satterthwaite, Mark. "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions." Journal of Economic Theory 10 (1975), 187-217.

Shapley, Lloyd S. and Martin Shubik. "A Method for Evaluating the Distribution of Power in a Committee System." American Political Science Review 48 (1954), 787-792.